

EVOLUTION-AWARE POSITIVE-UNLABELED LEARNING FOR PROTEIN DESIGN

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ABSTRACT

We consider prediction of protein function, focusing on protein functionalities that enhance survival for one or more organisms. Sequencing these organisms provides plentiful positive training examples editdue to survivorship bias. In contrast, synthesizing and characterizing a protein with a mutation unseen in nature requires time-consuming wet lab experiments, making negative training examples scarce. Thus, datasets are often imbalanced, hindering classifier accuracy outside the training data. Positive-unlabeled (PU) learning attempts to address this issue by considering unlabeled protein sequences to be part of the data and modeling them as positive with a probability called the class prior. This class prior is often constant. Our insight is that an understanding of evolution suggests a novel sequence-dependent class prior when learning from sequencing data. We propose Evo-PU, a PU learning framework that integrates our novel class prior to create a likelihood for training classifiers. We evaluate Evo-PU on multiple real-world tasks on influenza hemagglutinin protein. Using influenza genomic surveillance data and held-out laboratory assays of mutants unseen in nature, Evo-PU outperforms state-of-the-art PU learning, one-class classification (OCC), and **deep generative model-based methods (DGM) on these real-world problems**, demonstrating the benefit of combining evolutionary modeling with data-driven learning for protein design. **We further assess Evo-PU on standard ProteinGym benchmarks, focusing on protein overall fitness prediction.** Evo-PU outperforms existing PU-learning and OCC baselines, while remains competitive to DGM-based approaches.

1 INTRODUCTION

A fundamental challenge in protein design is accurately classifying amino acid sequences according to whether they possess a particular biochemical functionality. Sequencing living organisms provides many examples of functional sequences, particularly when the functionality confers an evolutionary advantage – **a phenomenon described as survivorship bias** (Bermúdez-Guzmán et al., 2020; Thomas et al., 2022). As examples, mussel foot proteins are promising adhesives (Kord Forooshani & Lee, 2017), proteins from polar fish are promising for cryogenic storage of tissues (He et al., 2018), and viral fusion proteins are promising drug delivery vehicles (Brown et al., 2024). Evolutionary pressure guides natural selection to discover functional proteins that would be otherwise unlikely to arise and widespread sequencing gives researchers access to these sequences. However, the resulting datasets are imbalanced: non-functional protein mutants are absent because they inhibit survival and some functional sequences may also be missing. When the functionality of interest is essential for survival, all sequenced proteins carry a positive label. This imbalance complicates training accurate classifiers.

Positive-unlabeled (PU) learning (Liu et al., 2003; Bekker & Davis, 2020) provides a natural framework for training classifiers when only a subset of positive examples is labeled, there are no labeled negative examples, and the rest of the data is unlabeled. The unlabeled set may contain both positive and negative instances, but their labels are unknown. In protein design, sequenced functional proteins can be treated as labeled positives, while all other proteins of interest form an unlabeled set. Within this framework, Protein-PU (Song et al., 2021) is a specialized PU method developed for protein design using deep mutational scanning (DMS) data, where mutants are generated in the laboratory, typically one amino acid away from a parental sequence. Protein-PU models a functional protein detection assay as a labeling process in which all functional sequences have an equal chance of being labeled positive (the “detection probability”). While effective for DMS, this assumption does not hold

054 for naturally-occurring sequences: evolutionary selection and mutational accessibility make some
 055 sequences much less likely to be detected, especially those distant from common natural variants.
 056 Furthermore, standard PU methods not designed for protein design (Bekker & Davis, 2020) often
 057 identify reliable negatives based on their similarity to known positives and use them, along with the
 058 positives, to train a classifier. Such approaches are ill-suited here, because even a few mutations can
 059 make a protein non-functional.

060 Alternative approaches to prediction with positive-only protein data include one-class classification
 061 (OCC) (Tax & Duin, 2001; Perera et al., 2021), which trains only on positive sequences to identify
 062 outliers, and deep generative models (DGMs) trained on multiple sequence alignments (MSA) to
 063 capture evolutionary conservation and estimate functional likelihood (Meier et al., 2021; Frazer et al.,
 064 2021; Thadani et al., 2023). **While DGMs effectively predict the overall fitness score of whole protein**
 065 **sequences, they are challenged to capture the exact functional nuances of short, locally acting peptide**
 066 **motifs within these sequences, which are more relevant for domain scientists to understand and**
 067 **design protein functions for applications such as disease surveillance and therapeutic developments.**
 068 A detailed review of existing methods aligned with these approaches is provided in Appendix A.

069 To overcome the limitations of PU-learning, OCC, and deep generative model-based approaches
 070 for protein design, we model protein evolution within a PU learning approach and develop a novel
 071 *sequence-dependent* detection probability model. Our essential insight is that functional nucleotide
 072 sequences close to sequences prevalent in nature are more likely to be detected than functional
 073 sequences far away. This is because a nearby sequence has a higher chance of being formed by
 074 mutation from an existing sequence. Using this detection probability model, we propose a novel
 075 framework for PU learning from sequencing of naturally occurring nucleotide sequences. We
 076 demonstrate Evo-PU’s effectiveness on three real-world tasks: screening viral epitopes for immune
 077 evasion, identifying peptides with binding specificity to a human receptor and classifying peptides
 078 with viral fusion activity. Using influenza genomic surveillance data and held-out laboratory assays
 079 of mutants not observed in nature, Evo-PU achieves superior performance compared to state-of-
 080 the-art PU learning, OCC, and DGM-based methods. **We further evaluate Evo-PU on standard**
 081 **ProteinGym benchmarks (Notin et al., 2023), which focus on predicting overall fitness rather than**
 082 **specific biochemical properties, and find that Evo-PU improves upon PU-learning and OCC baselines,**
 083 **while DGMs that designed for fitness prediction perform best in this regime.**

084 2 METHOD

085 In this section, we present the Evo-PU framework, where protein sequences are modeled via a
 086 data-generating process inspired by natural selection. This process defines the likelihood of observing
 087 sequences based on functionality and the emergence of their nucleotide variants. Exact computation
 088 is infeasible due to the large sequence space, so we use approximations that focus on biologically
 089 plausible sequences. The resulting likelihood is then used to train a probabilistic classifier. **We also**
 090 **compare Evo-PU to classic binary and Protein-PU likelihoods, showing how it generalizes them**
 091 **via sequence-dependent observation probabilities, and highlight key distinctions from DGM-based**
 092 **methods.**

093 2.1 NATURAL SELECTION AS A DATA-GENERATING PROCESS

094 Proteins are sequences of amino acids that perform functional tasks in living organisms. Let \mathcal{A} denote
 095 the set of 20 natural amino acids, and consider proteins of length L , so that the set of all amino acid
 096 sequences is $\mathcal{X} = \mathcal{A}^L$. Each amino acid is encoded by a codon of three nucleotides drawn from
 097 \mathcal{N} (either $\{A, C, G, T\}$ for DNA or $\{A, C, G, U\}$ for RNA). Some codons are stop signals and do
 098 not encode amino acids; we define the set of valid nucleotide sequences of length $3L$ as $\mathcal{Y} \subset \mathcal{N}^{3L}$,
 099 with each $y \in \mathcal{Y}$ translating deterministically to $x \in \mathcal{X}$ via $T : \mathcal{Y} \rightarrow \mathcal{X}$. For any $x \in \mathcal{X}$, we denote
 100 $\mathcal{Y}(x) = T^{-1}(x) \subset \mathcal{Y}$ as the set of all nucleotide sequences encoding x .

101 Each amino acid sequence x is associated with a binary variable $A(x)$ indicating whether the
 102 corresponding protein exhibits a property of interest (e.g., immune evasion, bind to host cell membrane
 103 receptor or fuse with host cell membrane). Specifically, $A(x) = 1$ if the protein exhibits the property
 104 and 0 otherwise. In Evo-PU, we model $\mathbb{P}(A(x) = 1)$ via a parametrized function $p_a(x; \theta)$, where

108 $\theta \in \Theta$ is a model parameter to be estimated by maximum likelihood estimation (MLE) along with
 109 other nuisance parameters.

110 To model sequence observation, we consider substitution-only evolution at the nucleotide level, so
 111 sequences remain length $3L$ and their corresponding proteins length L . Let $E(y)$ be a binary variable
 112 indicating whether nucleotide sequence y has emerged. By “emergence”, we mean that mutation
 113 has created the nucleotide sequence in an organism. We model the emergence probability of y using
 114 a parametrized function $p_e(y; \alpha)$ detailed in Section 2.4, so that $\mathbb{P}(E(y) = 1) = p_e(y; \alpha)$. In our
 115 framework, we treat α as a nuisance parameter to be estimated via MLE.

116 Since we consider the properties of such proteins to be crucial for an organism’s survival and
 117 ultimately subject to evolutionary surveillance, it is *necessary* for a nucleotide sequence y to be
 118 observed that (1) y has emerged ($E(y) = 1$) and (2) the corresponding protein $T(y)$ possesses the
 119 property $(A(T(y)) = 1)$. Observation is not guaranteed even if these conditions hold, due to sampling
 120 limitations or ecological prevalence. We capture this using a nucleotide-level observability variable
 121 $O_y(y)$, which is 1 if the sequence y is detected and 0 otherwise. Formally, we model

$$123 \quad O_y(y) \mid A(T(y)), E(y) \sim \text{Bernoulli}(q(y)),$$

124 with

$$125 \quad q(y) = \begin{cases} p_o(y), & E(y) = 1 \text{ and } A(T(y)) = 1, \\ 126 \quad 0, & \text{otherwise.} \end{cases}$$

127 For simplicity, we often take $p_o(y) = p_o$ constant across all sequences, although it could be sequence-
 128 specific in general. We treat p_o as a nuisance parameter to be estimated via MLE, together with θ
 129 and α . This definition captures the fact that an amino acid sequence can only be observed in the
 130 dataset if at least one of its corresponding nucleotide sequences is detected. Define $O_{\mathcal{X}}(x)$ to be the
 131 observability binary random variable at amino acid level. As a direct consequence of this model, any
 132 observed amino acid sequence must be functional: $O_{\mathcal{X}}(x) = 1 \implies A(x) = 1$.

133 Finally, under model parameters θ , p_o , and α , we assume that $\{A(x) : x \in \mathcal{X}\}$ are mutually
 134 independent, $\{E(y) : y \in \mathcal{Y}\} \cup \{O_y(y) : y \in \mathcal{Y}\}$ are conditionally independent given $\{A(x) : x \in$
 135 $\mathcal{X}\}$. As a result, the amino acid-level observabilities, $\{O_{\mathcal{X}}(x)\}$ are independent across x , which
 136 allows the likelihood of observing amino acid sequences to factorize into a product form, as we will
 137 show in the next section.

139 2.2 BIOLOGICALLY-REALISTIC LIKELIHOOD FUNCTION

140 Using the model from Section 2.1, we can express the probability of observing an amino acid
 141 sequence x . While it is possible to define observability at the nucleotide level, we focus on amino
 142 acid-level observability due to its smaller, more tractable search space. Nonetheless, the framework
 143 can be generalized to nucleotide sequences.

144 Since possessing functional of interest is the prerequisite to be surveilled in tasks discussed in this
 145 work, we have

$$146 \quad \mathbb{P}(O_{\mathcal{X}}(x) = 1) = \mathbb{P}(O_{\mathcal{X}}(x) = 1 \mid A(x) = 1) \mathbb{P}(A(x) = 1).$$

147 Conditional on $A(x) = 1$, the nucleotide observabilities $\{O_y(y) : y \in \mathcal{Y}(x)\}$ are independent.
 148 Therefore, this gives

$$149 \quad \begin{aligned} \mathbb{P}(O_{\mathcal{X}}(x) = 1 \mid A(x) = 1) &= \mathbb{P}(\exists y \in \mathcal{Y}(x) : O_y(y) = 1 \mid A(x) = 1) \\ 150 \quad &= 1 - \mathbb{P}(\forall y \in \mathcal{Y}(x) : O_y(y) = 0 \mid A(x) = 1) \\ 151 \quad &= 1 - \prod_{y \in \mathcal{Y}(x)} (1 - \mathbb{P}(O_y(y) = 1 \mid E(y) = 1, A(x) = 1) \mathbb{P}(E(y) = 1)) \\ 152 \quad &= 1 - \prod_{y \in \mathcal{Y}(x)} (1 - p_o p_e(y; \alpha)). \end{aligned}$$

153 Thus, the marginal probability of observing amino acid sequence x is

$$154 \quad \mathbb{P}(O_{\mathcal{X}}(x) = 1) = p_a(x; \theta) \left[1 - \prod_{y \in \mathcal{Y}(x)} (1 - p_o p_e(y; \alpha)) \right].$$

162 Since the amino acid-level observabilities $\{O_{\mathcal{X}}(x)\}$ are independent across x , the likelihood of a
 163 dataset can be written as a product over sequences. Given observed amino acids $\mathcal{D}_n \subseteq \mathcal{X}$ and its
 164 complement $\mathcal{D}'_n = \mathcal{X} \setminus \mathcal{D}_n$, the log-likelihood therefore becomes
 165

$$\begin{aligned} 166 \quad \ell(\theta, \alpha, p_o; \mathcal{D}_n) &= \sum_{x \in \mathcal{D}_n} \log \mathbb{P}(O_{\mathcal{X}}(x) = 1) + \sum_{x' \in \mathcal{D}'_n} \log \mathbb{P}(O_{\mathcal{X}}(x') = 0) \\ 167 \\ 168 \quad &= \sum_{x \in \mathcal{D}_n} \left[\log p_a(x; \theta) + \log \left(1 - \prod_{y \in \mathcal{Y}(x)} (1 - p_o p_e(y; \alpha)) \right) \right] \\ 169 \\ 170 \quad &+ \sum_{x' \in \mathcal{D}'_n} \log \left[1 - p_a(x'; \theta) \left(1 - \prod_{y' \in \mathcal{Y}(x')} (1 - p_o p_e(y'; \alpha)) \right) \right]. \end{aligned} \quad (1)$$

173 This log-likelihood can then be maximized to estimate θ , α , and p_o . Evo-PU maximizes this objective
 174 to train a probabilistic classifier.
 175

176 2.3 COMPARISON OF EVO-PU WITH PU-LEARNING AND DGM-BASED METHODS

178 In this section, we compare our Evo-PU likelihood in Eq. 1 to the two existing PU-learning likelihood
 179 formulations and discuss the central distinctions between Evo-PU and DGM-based methods.
 180

181 Comparison to exiting PU-learning likelihoods

182 Here we present the two related likelihood formulations within the PU-learning framework:

- 183 • **Classical binary classifier** likelihood, which assumes unobserved sequences lack the
 184 chemical property:

$$186 \quad \sum_{x \in \mathcal{D}_n} \log p_a(x; \theta) + \sum_{x' \in \mathcal{D}'_n} \log(1 - p_a(x'; \theta)); \quad (2)$$

- 189 • **Protein-PU** likelihood proposed by (Song et al., 2021), which incorporates a fixed labeling
 190 efficiency parameter $q \in (0, 1)$:

$$191 \quad \sum_{x \in \mathcal{D}_n} \log q p_a(x; \theta) + \sum_{x' \in \mathcal{D}'_n} \log(1 - q p_a(x'; \theta)). \quad (3)$$

194 All three likelihoods share a similar structure: a sum over observed sequences in \mathcal{D}_n , which are
 195 treated as positives in the classical framework or labeled in the Protein-PU framework, and a second
 196 sum over sequences not in \mathcal{D}_n .

197 The classical likelihood can be viewed as a special case of both Evo-PU and Protein-PU. Specifically,
 198 setting $p_o p_e(y; \alpha) = 1$ for all $y \in \mathcal{Y}(x)$ in Eq. 1 implies that every functional amino acid sequence is
 199 always observed whenever it exists, reducing Evo-PU to the classical likelihood in Eq. 2. Likewise,
 200 setting $q = 1$ in the Protein-PU likelihood in Eq. 3 means that every functional sequence is always
 201 labeled, which also recovers the classical form.

202 Comparing Evo-PU and Protein-PU directly, Protein-PU models labeling efficiency or class prior with
 203 a constant parameter q , representing the probability that a functional sequence is labeled. In contrast,
 204 Evo-PU models class prior as sequence-dependent through the term: $1 - \prod_{y \in \mathcal{Y}(x)} (1 - p_o p_e(y; \alpha))$,
 205 which reflects how likely a sequence is to emerge through mutational processes. This sequence-
 206 dependent formulation captures variability in the observation process that cannot be explained
 207 by a fixed efficiency parameter. As we will show in our numerical studies, this leads to better
 208 alignment with the natural data-generating process and improved predictive performance compared
 209 to approaches with constant labeling efficiency.

210 211 Distinctions between Evo-PU and DGM-based methods

212 We highlight here the key distinctions between Evo-PU and DGM-based approaches. First, DGM-
 213 based methods primarily capture patterns associated with overall evolutionary fitness, whereas Evo-
 214 PU is designed to identify sequence features that control a specific biochemical property essential
 215 for organismal survival. Second, Evo-PU bases its predictions on modeling why sequences are
 observed or missing, not solely on the distribution of sequences that appear in databases. While

DGMs infer constraints from observed sequences, they do not model the evolutionary forces and surveillance processes that determine the presence or absence of variants. By incorporating a sequence-dependent class prior that reflects mutational accessibility, Evo-PU directly leverages the evolutionary mechanism underlying sequence observability, offering a principled way to exploit this information for property-specific protein-function prediction.

2.4 NUCLEOTIDE EMERGENCE PROBABILITY MODEL

We now describe the nucleotide emergence probability $p_e(y; \alpha)$, a key component of the likelihood in Eq. 1. Let \mathcal{D}_N denote the set of observed nucleotide sequences at the current step, and \mathcal{D}_N^p the set of sequences that emerged naturally in the previous step (see Section 3.3 for details on estimating \mathcal{D}_N^p).

Between steps, evolution occurs at the nucleotide level. For each sequence $y^p \in \mathcal{D}_N^p$, let $c(y^p)$ denote its (unobserved) total count in the previous step, and let $\mathbb{P}(y^p \rightarrow y')$ be the probability that a host carrying y^p transmits a mutated sequence y' to the next host. Because mutations must overcome within-host bottlenecks to establish dominance (Petrova & Russell, 2018), we introduce a parameter $\alpha \in (0, 1)$ representing the probability that a mutated sequence successfully establishes dominance within the host.

For an unobserved sequence $y' \notin \mathcal{D}_N$, we model the emergence probability as

$$p_e(y'; \alpha) = 1 - \prod_{y^p \in \mathcal{D}_N^p} (1 - \mathbb{P}(y^p \rightarrow y') \alpha)^{c(y^p)}. \quad (4)$$

Here, $(1 - \mathbb{P}(y^p \rightarrow y') \alpha)^{c(y^p)}$ represents the probability that y' fails to emerge from all replications of y^p . Assuming independent mutation events across replications, the product gives the probability that y' fails to emerge from any sequence in the previous step, and its complement gives the probability that y' emerges at least once.

In practice, both $\mathbb{P}(y^p \rightarrow y')$ and α are typically small as the mutation rate is low, while the counts $c(y^p)$ are large. Using the classical approximation $(1 + x)^a \approx e^{ax}$ for $|x| \ll 1$ and $|ax| \gg 1$, we obtain

$$p_e(y'; \alpha) \approx 1 - \exp \left(- \sum_{y^p \in \mathcal{D}_N^p} \mathbb{P}(y^p \rightarrow y') \alpha c(y^p) \right). \quad (5)$$

For sequences already observed in the current step, $y \in \mathcal{D}_N$, we set $p_e(y; \alpha) = 1$, since their emergence is already established.

2.5 ESTIMATING THE LIKELIHOOD FUNCTION VIA PROTEIN EVOLUTION

Evaluating the likelihood in Eq. 1 requires summing over all unobserved amino acid sequences x' of length L in \mathcal{D}'_n , the full set of sequences not present in \mathcal{D}_n . This set is exponentially large and makes exact computation of the likelihood intractable. Moreover, for each amino acid sequence (both observed and unobserved) x , evaluating the sequence-dependent class prior also requires a product over $\mathcal{Y}(x)$, a set of all nucleotide sequences that can translate to x , which can also be large for long amino acid sequences. Since in the reality mutations are rare, most of nucleotide sequences in $\mathcal{Y}(x)$ that have not been observed in the nucleotide dataset \mathcal{D}_N have negligible emergence probability $p_e(y; \alpha) \approx 0$, contributing minor to the likelihood.

To reduce computation, we approximate the likelihood by considering a smaller subset of amino acid sequences $\hat{\mathcal{D}}'_n \subset \mathcal{D}'_n$, generated by the observed nucleotide sequence dataset \mathcal{D}_N , and containing only sequences likely to emerge naturally, yet unobserved. Specifically, we construct $\hat{\mathcal{D}}'_n$ by first generating a set of unobserved nucleotide sequences $\hat{\mathcal{D}}_N$ that contains nucleotide sequences with one point mutation away from any observed nucleotide sequence in observed set \mathcal{D}_N . In $\hat{\mathcal{D}}_N$, we include only those unobserved nucleotide sequences whose emergence probability $p_e(y'; \alpha) > \epsilon$ for some fixed $\epsilon, \alpha > 0$. Then, we construct $\hat{\mathcal{D}}'_n = \{T(y') \in \mathcal{X} : y' \in \hat{\mathcal{D}}_N \text{ and } T(y') \notin \mathcal{D}_n\}$.

Moreover, to further reduce the computation in the term for class prior $\prod_{y \in \mathcal{Y}(x)} (1 - p_{\text{op}}(y; \alpha))$, we restrict $\mathcal{Y}(x)$ for any $x \in \mathcal{D}_n \cup \hat{\mathcal{D}}'_n$ to $\hat{\mathcal{Y}}(x) = \mathcal{Y}(x) \cap (\mathcal{D}_N \cup \hat{\mathcal{D}}'_N)$.

270 By replacing \mathcal{D}'_n with the subset $\hat{\mathcal{D}}'_n$ and $\hat{\mathcal{Y}}(x)$ with $\hat{\mathcal{Y}}(x)$ in Eq. 1, and using the emergence
 271 probabilities $p_e(y; \alpha) = 1, \forall y \in \mathcal{D}_N$ and $p_e(y'; \alpha)$ as defined in Eq. 5 for all $y' \in \hat{\mathcal{D}}'_N$, we
 272 approximate the log-likelihood function by:
 273

$$\begin{aligned} 274 \ell_n(\theta, p_o, \alpha; \mathcal{D}_n) &\approx \sum_{x \in \mathcal{D}_n} \left[\log p_a(x; \theta) + \log \left(1 - \prod_{y \in \hat{\mathcal{Y}}(x)} (1 - p_o p_e(y; \alpha)) \right) \right] \\ 275 &+ \sum_{x' \in \mathcal{D}'_n} \log \left[1 - p_a(x'; \theta) \left(1 - \prod_{y' \in \hat{\mathcal{Y}}(x')} (1 - p_o p_e(y'; \alpha)) \right) \right] := \hat{\ell}_n(\theta, p_o, \alpha; \mathcal{D}_n). \\ 276 & \\ 277 & \\ 278 & \\ 279 & \end{aligned} \tag{6}$$

280 We then train the probabilistic classifier $p_a(x; \theta)$ by jointly estimating the classifier parameters θ
 281 and two nuisance parameters: nucleotide observability efficiency p_o , and the probability that an
 282 emerged sequence becomes dominant α by minimizing the loss function defined as the negative of
 283 this approximated log-likelihood:
 284

$$285 (\theta^*, p_o^*, \alpha^*) \in \arg \min_{(\theta, p_o, \alpha) \in \Theta \times (0, 1) \times (0, 1)} -\hat{\ell}_n(\theta, p_o, \alpha; \mathcal{D}_n). \tag{7}$$

288 3 NUMERICAL EXPERIMENTS

290 Here, we present numerical experiments on both real-world influenza datasets and standard Prote-
 291 inGym (Notin et al., 2023) benchmarks to evaluate the performance of Evo-PU. These two sets of
 292 tasks serve distinct purposes. The influenza tasks focus on identifying functional mutations in specific
 293 peptide motifs for downstream applications—precisely what Evo-PU is designed for—whereas the
 294 ProteinGym benchmarks assess model’s ability to predict overall protein fitness, a broader objective.
 295

296 3.1 PREDICTING FUNCTIONAL MOTIFS OF THE INFLUENZA HEMAGGLUTININ PROTEIN

297 **Problem background:** Influenza causes over 500,000 deaths worldwide each year (Stöhr, 2002;
 298 Thompson et al., 2009; Nair et al., 2011). Three critical drivers of influenza virus’s ability to infect
 299 hosts are its ability to evade the human immune system, to bind with host cells, and to fuse with
 300 host cell membrane, all mediated by the functions of peptide domains located at different regions
 301 within its viral protein hemagglutinin (Epand, 2003). The Ca1 epitope (a combined of 11 amino acid
 302 residues) is one of five antigenic sites located in H1 subtype hemagglutinin (positions 169–173 and
 303 206–208 and 238–240 per H3 numbering) (Wu & Wilson, 2017; Sriwilaijaroen & Suzuki, 2012), and
 304 it often mutates to escape the recognition of human immune response. Upon successful evasion of
 305 immune system, influenza binds with host cell via the binding domain in hemmaglutinin (positions
 306 134–138, 186–195 and 221–228 per H3 numbering) (Wiley et al., 1981; Yang et al., 2007), here
 307 in this study we combine the 23 amino acid residues and refer them as binding peptide. Upon
 308 binding, influenza can fuse with host cell membrane via hemagglutinin fusion domain, a consecutive
 309 23-amino-acid-sequence named fusion peptide (Wiley & Skehel, 1987; Luo, 2012). Our goal for
 310 these experiments is to test the ability of Evo-PU in predicting mutants with vastly different functions,
 311 i.e. immune evasion, human receptor binding and membrane fusion.
 312

313 **Dataset:** We obtained the prevalence data on host-infecting hemagglutinin protein nucleotide se-
 314 quences collected between year 2001 and year 2024 (NCBI, 2024; Shu & McCauley, 2017). We
 315 extracted 7,383 unique nucleotide sequences that encode 504 unique amino acid sequences for fusion
 316 peptide mutants. In the binding peptide case, only human-infecting hemagglutinin protein nucleotide
 317 sequences were used, since different hemmaglutinin subtypes can bind with non-human hosts via
 318 their affinities with other types of influenza receptors (Matrosovich et al., 2009). We identified
 319 3,862 unique nucleotide sequences encoding 1,458 distinct binding peptide protein mutants. For the
 320 "evasion peptide" (Ca1 epitope) case, only human-infecting H1 hemagglutinin nucleotide sequences
 321 collected between year 2001 and year 2024 were used. We identified 497 unique nucleotide se-
 322 quences encoding 181 distinct protein sequences located at the Ca1 antigenic site. In our framework,
 323 we designate the nucleotide datasets as the observed nucleotide dataset \mathcal{D}_N used to compute the
 324 emergence probability presented the model proposed in Section 2.4 as a part of the approximated
 325 log-likelihood function in Eq. 6. We designate the amino acid datasets translated from the observed
 326 nucleotide sequences as the observed amino acid sequence set \mathcal{D}_n .
 327

The held-out test dataset for fusion peptides was from studies examining the fusion properties of previously unseen influenza fusion peptide mutants via site-directed mutagenesis (Han et al., 1999; Qiao et al., 1999; Tamm et al., 2002; Lai et al., 2006; Su et al., 2008; Cross et al., 2009). It contains 76 unique amino acid sequences, of which 46 exhibit the fusion property (positive samples) and 30 show impaired fusion (negative samples). Similarly, the test dataset for binding peptides comprises 33 lab-generated mutagenesis results (Yang et al., 2007; Martín et al., 1998; Maines et al., 2011; Chen et al., 2012) and 13 newly observed functional binding peptides from 2025. Among the 46 test sequences, 25 show binding affinity to human influenza receptors, while the remaining 21 sequences show no binding. For the evasion task, the test set contains 51 peptide sequences collected in 2025 and labeled as evasive (functional). To form the non-evasive class, we randomly sampled 51 observed nucleotide sequences and introduced nine mutations to produce unobserved variants, which were then translated to amino acids. With this mutation distance, these sequences are sufficiently dissimilar from the functional set and are unlikely to be evasive, so we treat them as negatives in the test set.

3.2 PREDICTING PROTEIN OVERALL FITNESS: PROTEINGYM BENCHMARKS

ProteinGym (Notin et al., 2023) is a large-scale benchmark for protein-fitness prediction, featuring standardized DMS assays and curated clinical datasets with annotated mutation effects. We evaluate Evo-PU on two ProteinGym datasets: (1) PSAE_PICP2 (PSAE) and (2) A0A247D711_LISMN (A0). For each task, Evo-PU is trained on the associated MSA (1,785 sequences for PSAE; 57 for A0) and tested on the corresponding DMS substitution datasets (1,581 sequences for PSAE; 1,653 for A0). Because our evolutionary model operates at the nucleotide level and requires prevalence information—data not available in ProteinGym—we randomly sample a nucleotide sequence encoding each amino-acid MSA sequence and assume equal prevalence across all sequences when running Evo-PU. Although Evo-PU is not designed for general protein-fitness prediction, the goal of this experiment is precisely to assess how well it performs in this broader setting.

3.3 Evo-PU: MODEL CHOICES

To demonstrate the flexibility of our framework with different probabilistic classifiers, we evaluate Evo-PU using two model classes: (1) standard logistic regression (LR), and (2) a neural network classifier inspired by the Wide and Deep network architecture (WD) Cheng et al. (2016). We provide the description of the neural network in Appendix B.

To train the Evo-PU classifier, we first construct the set of unobserved nucleotide sequences $\hat{\mathcal{D}}_{\mathcal{N}}$ as outlined in Section 2.4. We approximate the set of previously emerged nucleotide sequences $\mathcal{D}_{\mathcal{N}}^p$ by the observed dataset $\mathcal{D}_{\mathcal{N}}$ and estimate the total count $c(y^p)$ of each $y^p \in \mathcal{D}_{\mathcal{N}}^p$ as $Tf(y^p)$, where $f(y^p)$ is the empirical frequency of y^p in $\mathcal{D}_{\mathcal{N}}$ and T is the estimated number of infected hosts in the preceding period. For the influenza datasets, $\mathcal{D}_{\mathcal{N}}$ spans 24 years (2001–2024), we assume the previous step covers the same duration. Using the global estimate of 1 billion influenza cases per year (Nair et al., 2011), we set $T = 24B$. **For the ProteinGym experiments, the underlying datasets do not include temporal coverage or prevalence information. To maintain consistency in our implementation of Evo-PU, we therefore adopt the same estimate of $T = 24B$ for these tasks.**

From each $y^p \in \mathcal{D}_{\mathcal{N}}^p$, we generate all possible single-nucleotide mutants y' , considering both transition and transversion mechanisms (Luo et al., 2016) (See Appendix C for nucleotide mutation patterns). Following prior studies (Wakeley, 1996; Stoltzfus & Norris, 2016; Pauly et al., 2017; Acevedo et al., 2014), we assume mutation probabilities of $\mathbb{P}(y^p \rightarrow y') \approx 2.6 \times 10^{-5}$ for transitions and 1.4×10^{-7} for transversions.

We then construct the candidate set $\hat{\mathcal{D}}'_{\mathcal{N}}$ of likely emergent but unobserved nucleotide sequences by retaining only those y' satisfying $p_e(y'; \alpha) \geq 1 - \exp(-10)$ under $\alpha = 1$. For the influenza tasks, this procedure yields 30,433; 17,366; and 1,927 unique unobserved nucleotide sequences for the fusion, binding, and evasion peptides, respectively. **For the ProteinGym datasets, it yields 302,933 sequences for PSAE and 40,101 for A0.** Some of these nucleotide sequences translate into amino acid sequences already present in the data. After removing such duplicates, the remaining sequences give rise to 1,916; 5,203; and 549 unique amino acid sequences for the three influenza tasks, **and 167,308 and 26,153 for the PSAE and A0 problems**, respectively. These resulting counts determine the cardinality of $\hat{\mathcal{D}}'_{\mathcal{N}}$ used in the likelihood approximation in Eq. 6. The generated nucleotide set

378 $\hat{\mathcal{D}}'_N$ is used to approximate, for each amino acid sequence x , the set of nucleotide sequences that can
 379 translate to it, denoted $\hat{\mathcal{Y}}(x)$, which is required to compute the product term in the Evo-PU likelihood.
 380

381 Directly optimizing the loss in Eq. 7 over discrete amino acid sequences is intractable. To make the
 382 optimization feasible, we encode each amino acid using three physicochemical properties that are
 383 known to correlate with influenza peptide function (Moon & Fleming, 2011; Foulquier, 2001)—and
 384 construct continuous sequence representations by concatenating these encodings. **For consistency, we**
 385 **apply the same representation to the ProteinGym problems, although these properties are not tailored**
 386 **to general DMS functional characterization and may therefore limit performance in that setting.**

387 3.4 COMPARISON METHODS AND METRIC

389 We evaluate Evo-PU against several baselines, including the closest PU learning framework for
 390 protein design: Protein-PU (Song et al., 2021), a standard PU learning method (2-Step) (Bekker &
 391 Davis, 2020). To ensure a fair comparison, we generate unlabeled sets containing 1,865 (for fusion
 392 task), 5,535 (for binding task), 461 (for evasion task), **167,308 (for PSAE task)** and **26,153 (for A0**
 393 **task)** amino acid sequences—matching the size of $\hat{\mathcal{D}}'_n$ used in Evo-PU. We explore two strategies to
 394 generate this unlabeled dataset: (1) random sampling (RAND) and (2) evolutionary knowledge-based
 395 generation (E-GEN), where the latter one uses the evolutionary knowledge and generates exactly
 396 the same set $\hat{\mathcal{D}}'_n$ as used in our Evo-PU framework. Comparing Evo-PU with PU methods using
 397 RAND unlabeled data highlights the benefits of incorporating protein evolution knowledge, while
 398 comparisons with those using E-GEN unlabeled data emphasize the advantages of our novel loss
 399 function, which integrates the natural selection process.

400 Moreover, we also compare Evo-PU against the two OCC methods: (1) a standard OC-
 401 SVM (Schölkopf et al., 2001) and (2) iForest (Liu et al., 2008). These methods use only the
 402 observed set \mathcal{D}_n , without incorporating any generated unlabeled data. For consistency, we use the
 403 same classifiers (LR or WD) with the same CHEM sequence representation across all PU learning
 404 and OCC baselines, matching those used in Evo-PU.

405 We further benchmark Evo-PU against three DGM-based baselines: (1) the evolutionary model
 406 of variant effect (EVE) (Frazer et al., 2021), (2) zero-shot predictions from the protein language
 407 model ESM-1v (Meier et al., 2021), and (3) a similarity-based classifier using ESM2 embeddings
 408 with k-nearest neighbors (kNN x ESM2) (Esmaili et al., 2025). For kNN x ESM2, we incorporate
 409 generated unlabeled data as negative sequences.

410 All models are evaluated on the same test datasets and compared using the area under the receiver
 411 operating characteristic curve (AUC) and average precision (AP), where for both metrics, the higher
 412 values indicate better classification performance. **A full description of all baseline methods, and**
 413 **optimization details are provided in Appendix D and Appendix E, respectively.**

415 3.5 RESULTS AND DISCUSSION

417 In this section, we report AUC results for two influenza tasks (fusion and binding) and one ProteinGym
 418 benchmark (PSAE). For methods involving randomness, we report the mean AUC with error bars.
 419 Full results, including AP metrics, are provided in Appendix F. Figure 1 summarizes performance
 420 across all methods using LR (top row) and WD classifiers (bottom row). Evo-PU achieves the highest
 421 performance in nearly all influenza settings and remains competitive with DGM-based baselines on
 422 the ProteinGym benchmarks.

423 The influenza datasets and ProteinGym benchmarks reflect fundamentally different prediction goals.
 424 The influenza tasks focus on a single, well-defined biochemical property (e.g., membrane fusion or
 425 receptor binding) of defined local protein motif that is essential for viral survival, while ProteinGym
 426 evaluates global protein fitness shaped by multiple biochemical factors simultaneously. Evo-PU is
 427 designed to model the data-generating process for one specific property and therefore aligns naturally
 428 with the influenza tasks, where observed sequences reflect natural selection acting on that property.
 429 Conversely, DGM-based methods-trained to approximate the overall sequence-fitness landscape-are
 430 naturally stronger on ProteinGym, a benchmark centered on global fitness prediction. We note that
 431 the AUC values of DGM-based methods for the binding task fall below 0.5. We hypothesize that this
 results from the nature of the test data, which contains lab mutagenesis measurements derived from

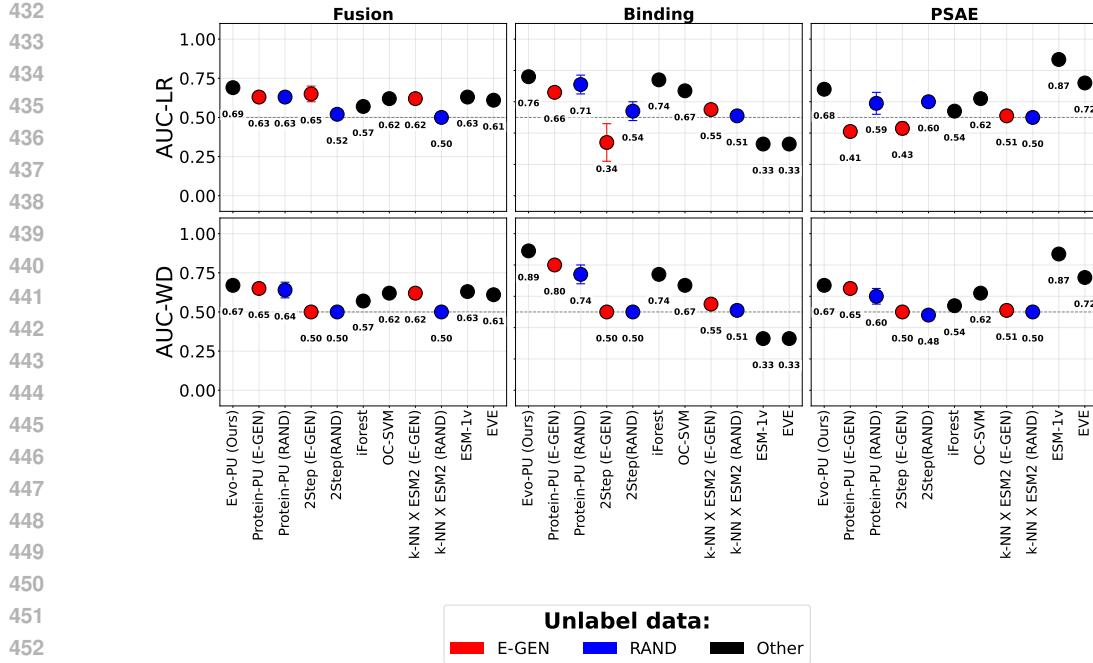


Figure 1: Best average AUC values with error bars across all methods for fusion peptide (left), receptor binding peptide (middle), and PSAE protein (right) classification. Top row: LR classifier with Evo-PU, Protein-PU, and Two-Step. Bottom row: WD classifier with the same methods.

multiple distinct wild-type backgrounds. Such data do not match the assumptions of DGM-based approaches, which typically model variation around a single natural sequence.

For the influenza datasets, Evo-PU consistently surpasses the closest PU-learning baseline (Protein-PU with E-GEN unlabeled data). This improvement comes from two key advantages: 1) Evo-PU uses sequence-dependent class priors that better reflect each sequence’s probability of emergence; and 2) it trains a classifier using our novel loss function, which incorporates the effects of natural selection.

Although Evo-PU underperforms DGM-based methods on ProteinGym datasets, this gap may reflect limitations in our protein representation and simplified nucleotide-prevalence assumptions, suggesting opportunities for improvement. Overall, these results show that Evo-PU provides strong predictive performance for tasks involving specific biochemical properties and offers a principled framework for modeling evolutionary accessibility in PU-learning.

4 CONCLUSION

We introduced Evo-PU, an evolution-informed positive–unlabeled framework for predicting protein functions critical to organism survival. Evo-PU embeds protein evolution and natural selection at the nucleotide level to assign sequence-dependent class priors and define a biologically grounded likelihood of observing amino acid sequences. As exact likelihood computation is intractable, we use an efficient approximation that focuses on biologically plausible nucleotide-derived variants. We evaluate Evo-PU on three influenza tasks and two ProteinGym benchmarks, where it outperforms state-of-the-art methods on influenza and remains competitive on broader fitness prediction.

While promising, Evo-PU leaves room for improvement. The current model does not consider insertions or deletions, and experimental validation of top predictions would strengthen its practical utility. Extending Evo-PU to high-throughput experimental datasets lacking prevalence information and refining its emergence model offer exciting directions for future work. Additionally, training Evo-PU on human-infecting variants may aid in assessing the avian-to-human transmission risk of emerging avian influenza strains. Together, these avenues highlight Evo-PU’s adaptability and potential impact in protein design and biomedical research.

486 REFERENCES
487

488 Ashley Acevedo, Leonid Brodsky, and Raul Andino. Mutational and fitness landscapes of an rna
489 virus revealed through population sequencing. *Nature*, 505(7485):686–690, 2014.

490 Jessa Bekker and Jesse Davis. Learning from positive and unlabeled data: A survey. *Machine
491 Learning*, 109(4):719–760, 2020.

492 Luis Bermúdez-Guzmán, Gabriel Jimenez-Huezo, Andrés Arguedas, and Alejandro Leal. Mutational
493 survivorship bias: The case of pnkp. *PLoS One*, 15(12):e0237682, 2020.

494 Douglas W Brown, Ping Wee, Prakash Bhandari, Amirali Bukhari, Liliya Grin, Hector Vega, Maryam
495 Hejazi, Deborah Sosnowski, Jailal Ablack, Eileen K Clancy, et al. Safe and effective in vivo
496 delivery of dna and rna using proteolipid vehicles. *Cell*, 187(19):5357–5375, 2024.

497 Li-Mei Chen, Ola Blixt, James Stevens, Aleksandr S Lipatov, Charles T Davis, Brian E Collins,
498 Nancy J Cox, James C Paulson, and Ruben O Donis. In vitro evolution of h5n1 avian influenza
499 virus toward human-type receptor specificity. *Virology*, 422(1):105–113, 2012.

500 Heng-Tze Cheng, Levent Koc, Jeremiah Harmsen, Tal Shaked, Tushar Chandra, Hrishi Aradhye, Glen
501 Anderson, Greg Corrado, Wei Chai, Mustafa Ispir, et al. Wide & deep learning for recommender
502 systems. In *Proceedings of the 1st workshop on deep learning for recommender systems*, pp. 7–10,
503 2016.

504 Karen J Cross, William A Langley, Rupert J Russell, John J Skehel, and David A Steinhauer.
505 Composition and functions of the influenza fusion peptide. *Protein and peptide letters*, 16(7):
506 766–778, 2009.

507 Richard M. Epand. Fusion peptides and the mechanism of viral fusion. *Biochimica et Biophysica
508 Acta (BBA) - Biomembranes*, 1614(1):116–121, 2003.

509 Farzaneh Esmaili, Yongfang Qin, Duolin Wang, and Dong Xu. Kinase-substrate prediction using an
510 autoregressive model. *Computational and Structural Biotechnology Journal*, 27:1103–1111, 2025.

511 Elodie Foulquier, 2001. URL [https://www.imgt.org/IMGTeducation/
512 Aide-memoire/_UK/aminoacids/charge/#:~:text=3%20amino%20acids%20\(arginine%2C%20lysine,atoms%20in%20their%20side%20chain](https://www.imgt.org/IMGTeducation/Aide-memoire/_UK/aminoacids/charge/#:~:text=3%20amino%20acids%20(arginine%2C%20lysine,atoms%20in%20their%20side%20chain).

513 Jonathan Frazer, Pascal Notin, Mafalda Dias, Aidan Gomez, Joseph K Min, Kelly Brock, Yarin Gal,
514 and Debora S Marks. Disease variant prediction with deep generative models of evolutionary data.
515 *Nature*, 599(7883):91–95, 2021.

516 Xing Han, David A Steinhauer, Stephen A Wharton, and Lukas K Tamm. Interaction of mutant
517 influenza virus hemagglutinin fusion peptides with lipid bilayers: probing the role of hydrophobic
518 residue size in the central region of the fusion peptide. *Biochemistry*, 38(45):15052–15059, 1999.

519 Zhiyuan He, Kai Liu, and Jianjun Wang. Bioinspired materials for controlling ice nucleation, growth,
520 and recrystallization. *Accounts of chemical research*, 51(5):1082–1091, 2018.

521 Pegah Kord Forooshani and Bruce P Lee. Recent approaches in designing bioadhesive materials
522 inspired by mussel adhesive protein. *Journal of Polymer Science Part A: Polymer Chemistry*, 55
523 (1):9–33, 2017.

524 Alex L Lai, Heather Park, Judith M White, and Lukas K Tamm. Fusion peptide of influenza
525 hemagglutinin requires a fixed angle boomerang structure for activity. *Journal of Biological
526 Chemistry*, 281(9):5760–5770, 2006.

527 Bing Liu, Yang Dai, Xiaoli Li, Wee Sun Lee, and Philip S Yu. Building text classifiers using positive
528 and unlabeled examples. In *Third IEEE international conference on data mining*, pp. 179–186.
529 IEEE, 2003.

530 Fei Tony Liu, Kai Ming Ting, and Zhi-Hua Zhou. Isolation forest. In *2008 eighth ieee international
531 conference on data mining*, pp. 413–422. IEEE, 2008.

540 Guang-Hua Luo, Xiao-Huan Li, Zhao-Jun Han, Zhi-Chun Zhang, Qiong Yang, Hui-Fang Guo, and
 541 Ji-Chao Fang. Transition and transversion mutations are biased towards gc in transposons of chilo
 542 suppressalis (lepidoptera: Pyralidae). *Genes*, 7(10):72, 2016.

543

544 Ming Luo. *Influenza Virus Entry*, pp. 201–221. Springer US, Boston, MA, 2012.

545 Taronna R Maines, Li-Mei Chen, Neal Van Hoeven, Terrence M Tumpey, Ola Blixt, Jessica A Belser,
 546 Kortney M Gustin, Melissa B Pearce, Claudia Pappas, James Stevens, et al. Effect of receptor
 547 binding domain mutations on receptor binding and transmissibility of avian influenza h5n1 viruses.
 548 *Virology*, 413(1):139–147, 2011.

549

550 Javier Martín, Stephen A Wharton, Yi Pu Lin, Darin K Takemoto, John J Skehel, Don C Wiley, and
 551 David A Steinhauer. Studies of the binding properties of influenza hemagglutinin receptor-site
 552 mutants. *Virology*, 241(1):101–111, 1998.

553 M Matrosovich, J Stech, and H Dieter Klenk. Influenza receptors, polymerase and host range. *Revue
 554 scientifique et technique*, 28(1):203, 2009.

555 Joshua Meier, Roshan Rao, Robert Verkuil, Jason Liu, Tom Sercu, and Alex Rives. Language models
 556 enable zero-shot prediction of the effects of mutations on protein function. *Advances in neural
 557 information processing systems*, 34:29287–29303, 2021.

558

559 C Preston Moon and Karen G Fleming. Side-chain hydrophobicity scale derived from transmembrane
 560 protein folding into lipid bilayers. *Proceedings of the National Academy of Sciences*, 108(25):
 561 10174–10177, 2011.

562 Harish Nair, W Abdullah Brooks, Mark Katz, Anna Roca, James A Berkley, Shabir A Madhi,
 563 James Mark Simmerman, Aubree Gordon, Masatoki Sato, Stephen Howie, et al. Global burden
 564 of respiratory infections due to seasonal influenza in young children: a systematic review and
 565 meta-analysis. *The Lancet*, 378(9807):1917–1930, 2011.

566

567 NCBI. Influenza virus data hub, 2024. https://www.ncbi.nlm.nih.gov/labs/virus/vssi/#/virus?SeqType_s=Nucleotide&VirusLineage_ss=taxid:197911&VirusLineage_ss=taxid:197912&VirusLineage_ss=taxid:197913&VirusLineage_ss=taxid:1511083 [Accessed: (2024-10-11)].

568

569

570

571 Pascal Notin, Aaron Kollasch, Daniel Ritter, Lood Van Niekerk, Steffanie Paul, Han Spinner, Nathan
 572 Rollins, Ada Shaw, Rose Orenbuch, Ruben Weitzman, et al. Proteingym: Large-scale benchmarks
 573 for protein fitness prediction and design. *Advances in Neural Information Processing Systems*, 36:
 574 64331–64379, 2023.

575

576 Matthew D Pauly, Megan C Procario, and Adam S Lauring. A novel twelve class fluctuation test
 577 reveals higher than expected mutation rates for influenza a viruses. *Elife*, 6:e26437, 2017.

578

579 Pramuditha Perera, Poojan Oza, and Vishal M Patel. One-class classification: A survey. *arXiv
 preprint arXiv:2101.03064*, 2021.

580

581 Velislava N Petrova and Colin A Russell. The evolution of seasonal influenza viruses. *Nature Reviews
 Microbiology*, 16(1):47–60, 2018.

582

583 Hui Qiao, R Todd Armstrong, Grigory B Melikyan, Fredric S Cohen, and Judith M White. A specific
 584 point mutant at position 1 of the influenza hemagglutinin fusion peptide displays a hemifusion
 585 phenotype. *Molecular biology of the cell*, 10(8):2759–2769, 1999.

586

587 Bernhard Schölkopf, John C Platt, John Shawe-Taylor, Alex J Smola, and Robert C Williamson.
 588 Estimating the support of a high-dimensional distribution. *Neural computation*, 13(7):1443–1471,
 589 2001.

590

591 Yuelong Shu and John McCauley. Gisaid: Global initiative on sharing all influenza data—from vision
 592 to reality. *Eurosurveillance*, 22(13):30494, 2017.

593

594 Hyebin Song, Bennett J Bremer, Emily C Hinds, Garvesh Raskutti, and Philip A Romero. Inferring
 595 protein sequence-function relationships with large-scale positive-unlabeled learning. *Cell systems*,
 596 12(1):92–101, 2021.

594 Nongluk Sriwilaijaroen and Yasuo Suzuki. Molecular basis of the structure and function of h1
 595 hemagglutinin of influenza virus. *Proceedings of the Japan Academy, Series B*, 88(6):226–249,
 596 2012.

597

598 Klaus Stöhr. Influenza—who cares. *The Lancet infectious diseases*, 2(9):517, 2002.

599

600 Arlin Stoltzfus and Ryan W Norris. On the causes of evolutionary transition: transversion bias.
 601 *Molecular biology and evolution*, 33(3):595–602, 2016.

602

603 Y Su, Xingguo Zhu, Y Wang, M Wu, and P Tien. Evaluation of glu11 and gly8 of the h5n1 influenza
 604 hemagglutinin fusion peptide in membrane fusion using pseudotype virus and reverse genetics.
 605 *Archives of virology*, 153:247–257, 2008.

606

607 Lukas K Tamm, Xing Han, Yinling Li, and Alex L Lai. Structure and function of membrane fusion
 608 peptides. *Peptide Science: Original Research on Biomolecules*, 66(4):249–260, 2002.

609

610 David MJ Tax and Robert PW Duin. Uniform object generation for optimizing one-class classifiers.
 611 *Journal of machine learning research*, 2(Dec):155–173, 2001.

612

613 Nicole N Thadani, Sarah Gurev, Pascal Notin, Noor Youssef, Nathan J Rollins, Daniel Ritter, Chris
 614 Sander, Yarin Gal, and Debora S Marks. Learning from prepandemic data to forecast viral escape.
 615 *Nature*, 622(7984):818–825, 2023.

616

617 Adam Thomas, Benjamin D Evans, Mark van der Giezen, and Nicholas J Harmer. Survivor bias
 618 drives overestimation of stability in reconstructed ancestral proteins. *bioRxiv*, pp. 2022–11, 2022.

619

620 William W Thompson, Eric Weintraub, Praveen Dhankhar, Po-Yung Cheng, Lynnette Brammer,
 621 Martin I Meltzer, Joseph S Bresee, and David K Shay. Estimates of us influenza-associated deaths
 622 made using four different methods. *Influenza and other respiratory viruses*, 3(1):37–49, 2009.

623

624 John Wakeley. The excess of transitions among nucleotide substitutions: new methods of estimating
 625 transition bias underscore its significance. *Trends in ecology & evolution*, 11(4):158–162, 1996.

626

627 DC Wiley, IA Wilson, and JJ Skehel. Structural identification of the antibody-binding sites of hong
 628 kong influenza haemagglutinin and their involvement in antigenic variation. *Nature*, 289(5796):
 629 373–378, 1981.

630

631 Don C Wiley and John J Skehel. The structure and function of the hemagglutinin membrane
 632 glycoprotein of influenza virus. *Annual review of biochemistry*, 56(1):365–394, 1987.

633

634 Nicholas C Wu and Ian A Wilson. A perspective on the structural and functional constraints for
 635 immune evasion: insights from influenza virus. *Journal of molecular biology*, 429(17):2694–2709,
 636 2017.

637

638 Zhi-Yong Yang, Chih-Jen Wei, Wing-Pui Kong, Lan Wu, Ling Xu, David F Smith, and Gary J
 639 Nabel. Immunization by avian h5 influenza hemagglutinin mutants with altered receptor binding
 640 specificity. *Science*, 317(5839):825–828, 2007.

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Appendix

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A LITERATURE REVIEW

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657 In this section, we review existing methods relevant to our Evo-PU framework. We first discuss
658 general approaches, including PU learning, one-class classification (OCC), and deep generative
659 model-based methods, and then highlight specific studies that directly address protein applications,
660 which are most relevant to our work.661
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665 **Positive-unlabeled learning:** Typically, PU learning methods involve two primary steps: (1) identifying
666 some unlabeled data as reliable negatives and (2) training a final classifier model using the
667 positive data and reliable negatives. Examples of such methods include Spy-EM (Liu et al. (2002))
668 and Roc-SVM (Li et al. (2010)). Alternatively, some PU learning methods treat unlabeled data as
669 negative but assign greater importance to positive data by penalizing incorrect predictions of positive
670 instances. Examples include biased-SVM (Liu et al. (2003)) and weighted logistic regression (Lee
671 and Liu (2003)). A particularly relevant PU learning method for our study is the PU learning for
672 protein design (Protein-PU) framework (Song et al. (2021)), which fits a logistic regression model
673 using positive and unlabeled data through a custom loss function that incorporates prior knowledge
674 about the distribution of labeled data.675
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680 **One-class classification:** OCC methods can be divided into One-class Support Vector Machine
681 (OSVM)-based and non-OSVM-based approaches (Khan and Madden (2014)). Pioneer OSVM-
682 based methods build a smallest hyper-sphere that encloses positive samples (SVDD) (Tax and Duin
683 (1999a;b; 2001)) or a hyper-plane that separates positive data from the origin (OC-SVM) (Schölkopf
684 et al. (2001)). Recent advances in OSVM-based methods have used neural network for feature
685 extraction and apply traditional OSVM approaches over the extracted features (Erfani et al. (2016);
686 Ghafoori and Leckie (2020)). Examples of non-OSVM-based methods includes the ones using neural
687 network models (Manevitz and Yousef (2001); Skabar (2003); Chalapathy (2018)), decision trees
688 (Liu et al. (2008); Désir et al. (2012); Xu et al. (2023)), nearest neighbors (Munroe and Madden
689 (2005)) and Bayesian classifiers (Wang and Stolfo (2003)). OCC framework has been tailored to
690 protein-related applications. For example, Mei and Zhu (2015) considered a problem of predicting
691 protein-protein interaction and proposed to use OSVM-based method to sample negative data first
692 and then use the two-class SVM as a final classifier. Yousef and Charkari (2015) proposed to use
693 SVDD together with physicochemical property-based representations of proteins to classify genes
694 with diseases of interest.695
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701 **Protein classification using deep generative models:** Recent methods for protein classification
702 leverage deep generative models trained on multiple sequence alignments to capture amino acid
703 distributions and evolutionary conservation. [For example, zero-shot prediction via the protein-
704 language-model ESM-1v \(Meier et al. \(2021\)\) that computes the fitness likelihood of a queried
705 sequence with respect to a wild type sequence.](#) The Evolutionary Model of Variant Effect (EVE)
706 (Frazer et al. (2021)) predicts pathogenicity by training a variational autoencoder (VAE) on MSA-
707 derived sequences of a human protein of interest. The VAE estimates the relative likelihood of each
708 single amino acid variant compared to the wild type, producing evolutionary indices. These indices
709 are then used to fit a two-component Gaussian mixture model that outputs pathogenicity probabilities.
710 Another example is EVEscape (Thadani et al. (2023)), which extends the EVE framework by
711 combining information from evolutionary scores from EVE with protein structural and chemical
712 information and using logistic functions to predict the likelihood of immune escape in viral variants.713
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B A WIDE AND DEEP NEURAL NETWORK ARCHITECTURE

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721 We customized a neural network structure inspired by (Cheng et al. (2016)) integrates linear memo-
722 rization with nonlinear generalization for protein function classification. The model takes an input
723 feature vector and the input is processed through two parallel branches: a wide component, consisting
724 of a single fully connected layer that projects the input into a 64-dimensional space, and a deep com-
725 ponent, implemented as a two-layer perceptron with 32 and 16 hidden units, each followed by batch
726 normalization, ReLU activation, and dropout ($p=0.3$). The outputs of the wide and deep branches
727 are concatenated into an 80-dimensional joint feature representation, which is then mapped to a

702 single sigmoid output neuron for binary classification. Weights are initialized with Kaiming-normal
 703 initialization.
 704

706 C RNA NUCLEOTIDE MUTATIONS

708 Consider the set of four RNA nucleotides: adenine (A), guanine (G), cytosine (C), and uracil (U).
 709 Possible RNA nucleotide mutations via transition and transversion (Luo et al. (2016)) are summarized
 710 in Table 1. Specifically, one nucleotide can mutate to another specific nucleotide via transition and
 711 two other nucleotides via transversion.
 712

714 Table 1: Possible scenarios of RNA nucleotide mutations

| RNA Mutations | |
|---------------|--------------------|
| Transition | Transversion |
| (A)→(G) | (A)→(C) (A)→(U) |
| (G)→(A) | (G)→(C) (G)→(U) |
| (C)→(U) | (C)→(G) (C)→(A) |
| (U)→(C) | (U)→(A) (U)→(G) |

729 D DETAILS OF BASELINE METHODS

731 D.1 PU-LEARNING METHODS

733 **2Step:** In 2Step, 20% of the positive samples are randomly selected and inserted into the unlabeled
 734 set as “spies.” These spies and the unlabeled data are temporarily treated as negatives, while the
 735 remaining 80% of the positives are used as labeled positives. A primary classifier is trained on this
 736 combined dataset (spies + unlabeled as negatives, remaining positives as positives). After training,
 737 the primary model assigns a probability of being positive to each sequence. The lowest probability
 738 among all spy sequences is used as a threshold: any unlabeled sequence with a lower score than
 739 this threshold is labeled a reliable negative. The final classifier is then trained using these reliable
 740 negatives and all original positives, and is used for final prediction.

741 **Protein-PU:** Protein-PU (Song et al. (2021)) trains a single classifier on the full positive and unlabeled
 742 sets using a custom loss function (Eq. 3) with a constant class prior q as discussed in Section 2.2. Here,
 743 we estimate q using the ratio of positive to unlabeled samples, following the original formulation.
 744 This yields $q = 0.56, 0.57$, and 0.58 for the fusion, binding, and evasion tasks, respectively, and
 745 $q = 0.50$ for the both PSAE and A0 ProteinGym problems.

747 D.2 OCC METHODS

749 For these OCC baselines, we do not incorporate any of the generated sequences. The models are
 750 trained using only positive observed sequences for influenza tasks and only provided MSA sequences
 751 for ProteinGym benchmarks.

752 **OC-SVM:** Standard OC-SVM (Schölkopf et al. (2001)) learns a hyperplane separating the training
 753 data from the origin.
 754

755 **iForest:** iForest (Liu et al. (2008)) scores anomalies based on the number of splits needed to isolate
 them.

756 D.3 DEEP GENERATIVE MODEL-BASED METHODS
757758 **EVE:** In EVE (Frazer et al. (2021)), we follow the procedures described in the original paper. For
759 the influenza tasks, we first choose the most frequently observed sequence as the wild type, retrieve
760 similar sequences from the UniRef90 database, construct an MSA, and create the training set by
761 concatenating the relevant MSA segments. For the ProteinGym problems, we directly use the curated
762 MSA datasets provided by the benchmark. We then train a variational autoencoder (VAE) on the
763 one-hot encoded MSA sequences and use it to compute an evolutionary index for each test sequence
764 relative to the wild type. These indices are subsequently modeled with a two-component Gaussian
765 mixture model (GMM) to predict the functional class of each sequence.
766766 **Zero-shot:** For the influenza tasks, we use the most frequently observed sequences as wild-type
767 references and compute the fitness likelihood difference between each test sequence and the wild type
768 using the ESM-1v model, following Eq. 1 in Meier et al. (2021). For the ProteinGym benchmarks,
769 we use the provided wild-type sequences and follow the same likelihood computation.
770771 **Similarity-based method (k-NN x ESM2):** In this baseline, we first embed each protein sequence,
772 including positive sequences, unlabeled generated sequences and test sequences into a latent space
773 via a protein language model ESM2 (Lin et al. (2023)). Then, we train k-nearest neighbor for
774 predictions. We vary the value of k from 2 to 10 and report the one that yields highest AUC. The
775 similar implementation has also been considered for example in Esmaili et al. (2025).
776776 E OPTIMIZATION DETAILS
777778 We implement Evo-PU and all PU-learning baselines in PyTorch (Paszke et al. (2019)) using the
779 Adam optimizer. For Evo-PU, the bounds of α are set to $(0.00075, 1)$ for fusion and two ProteinGym
780 benchmarks, $(0.00025, 1)$ for binding, and $(0.0001, 1)$ for evasion; the bounds of p_o are fixed to
781 $(0.01, 0.99)$ for all tasks. We apply L_2 regularization with a penalty of 50, train for 2000 epochs, and
782 use a learning rate of 0.01. For Protein-PU, we tune the learning rate in the range $[10^{-6}, 10^{-2}]$ with
783 0.1 step size due to its sensitivity, and report the best results.
784785 For Protein-PU with RAND data, we generate 10 unlabeled datasets and report mean AUC and AP
786 and errors across them. For 2Step with E-GEN data, where spy assignment introduces randomness,
787 we report mean AUC, AP, and standard errors across 10 runs. For 2Step with RAND data, we use
788 the same 10 unlabeled datasets as in Protein-PU; for each dataset, we run 10 independent trials with
789 different spy assignments and report mean AUC and AP with error bars across all runs.
790791 For OC-SVM, iForest and k-NN (with ESM2 representation), we use the Scikit-learn implementations
792 (Pedregosa et al. (2011)). The EVE model is run using the official implementation: <https://github.com/OATML-MarksLab/EVE>
793793 F FULL NUMERICAL RESULTS
794795 In this section, we present the full numerical results. The results for influenza tasks (fusion, binding
796 and evasion) are presented in Figures 2 - 4 while the results for ProteinGym (PSAE and A0) are
797 presented in Figure 6 and 5. For each figure, top row presents the results where LR classifier is used
798 and bottom row presents the results where WD classifier is used. The left column reports AUC values
799 while the right column reports AP values.
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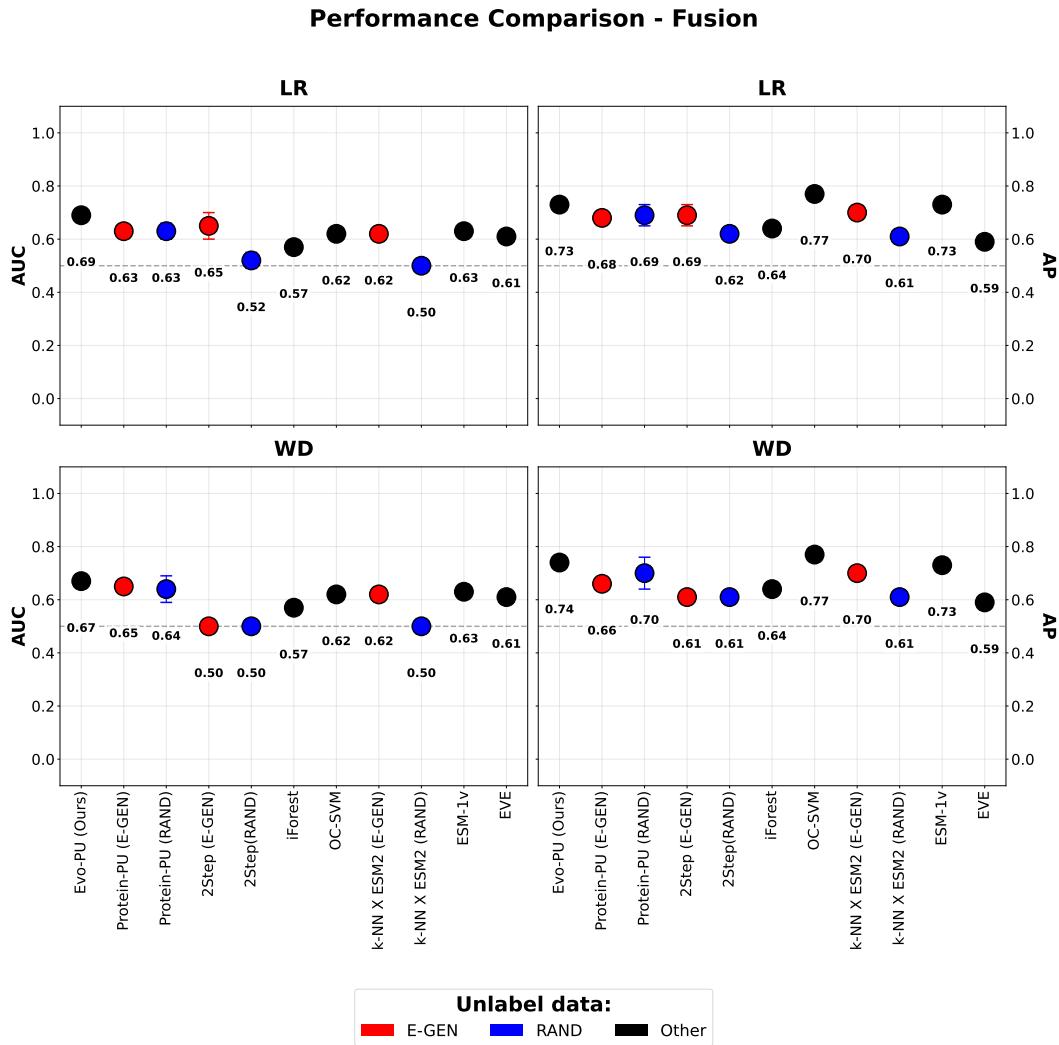


Figure 2: Performance comparison on the fusion tasks. The left column reports AUC values, and the right column reports AP values. The top row shows results using the LR classifier, while the bottom row shows results using the WD classifier.

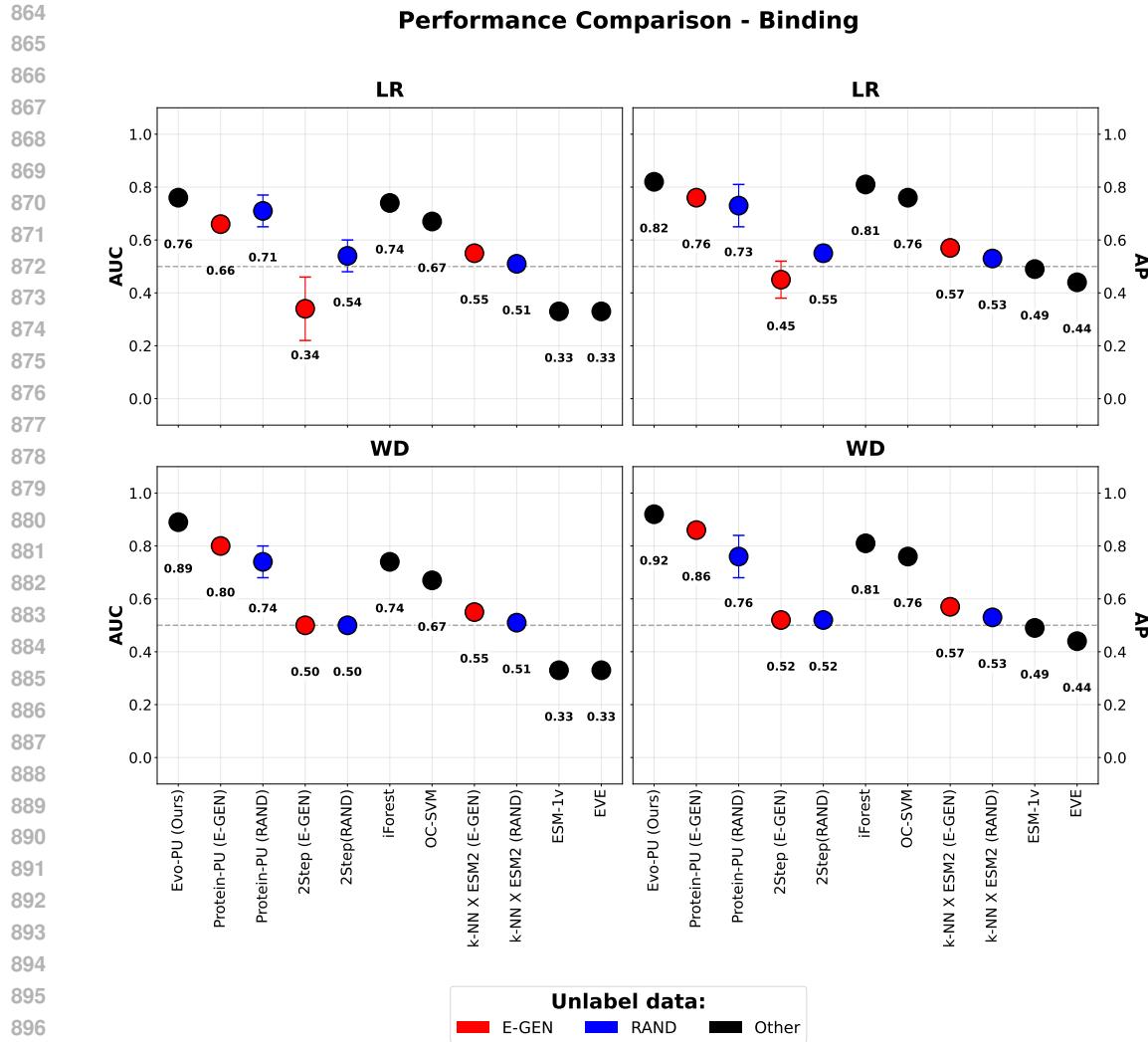


Figure 3: Performance comparison on the binding tasks. The left column reports AUC values, and the right column reports AP values. The top row shows results using the LR classifier, while the bottom row shows results using the WD classifier.

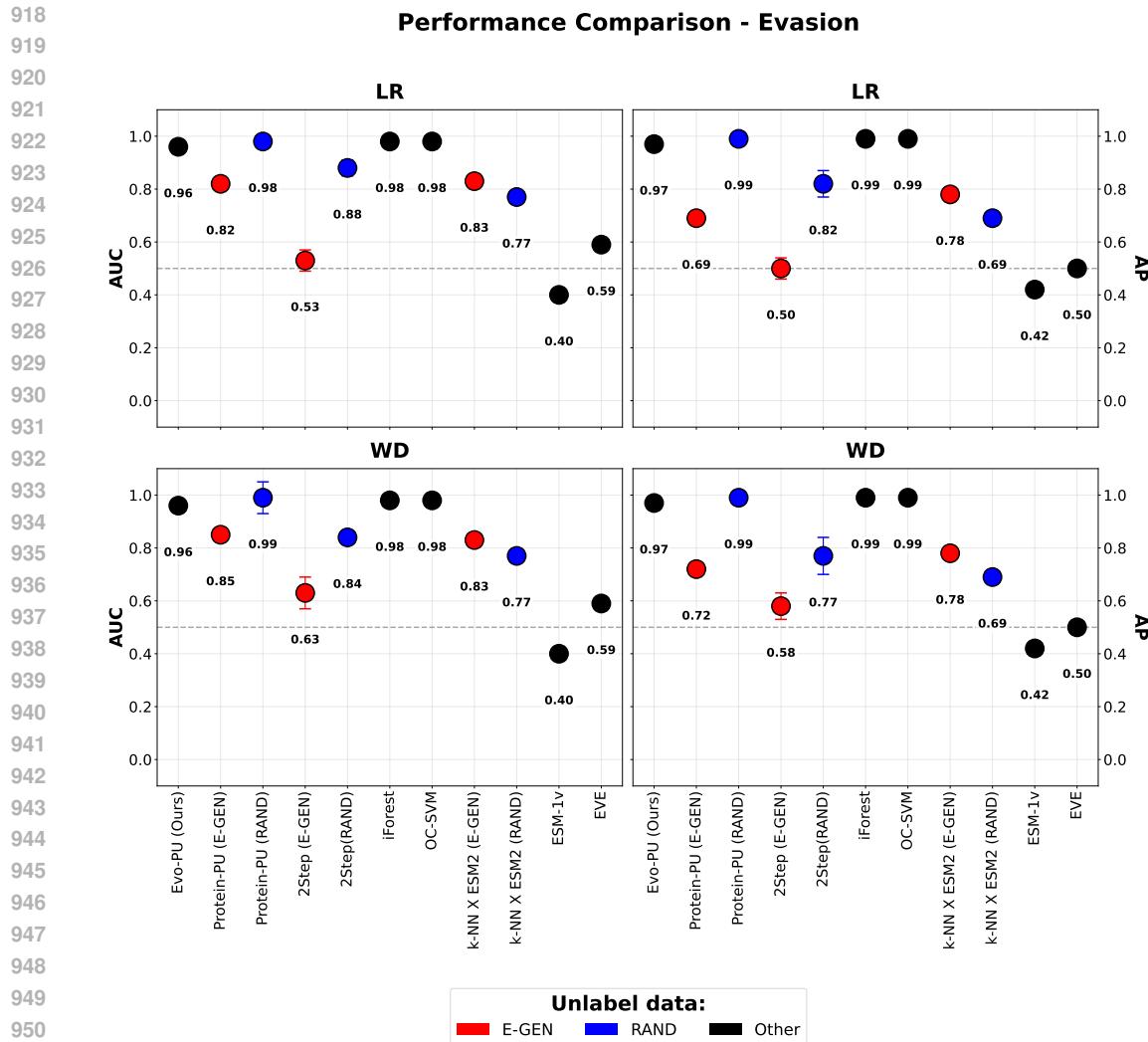


Figure 4: Performance comparison on the evasion tasks. The left column reports AUC values, and the right column reports AP values. The top row shows results using the LR classifier, while the bottom row shows results using the WD classifier.

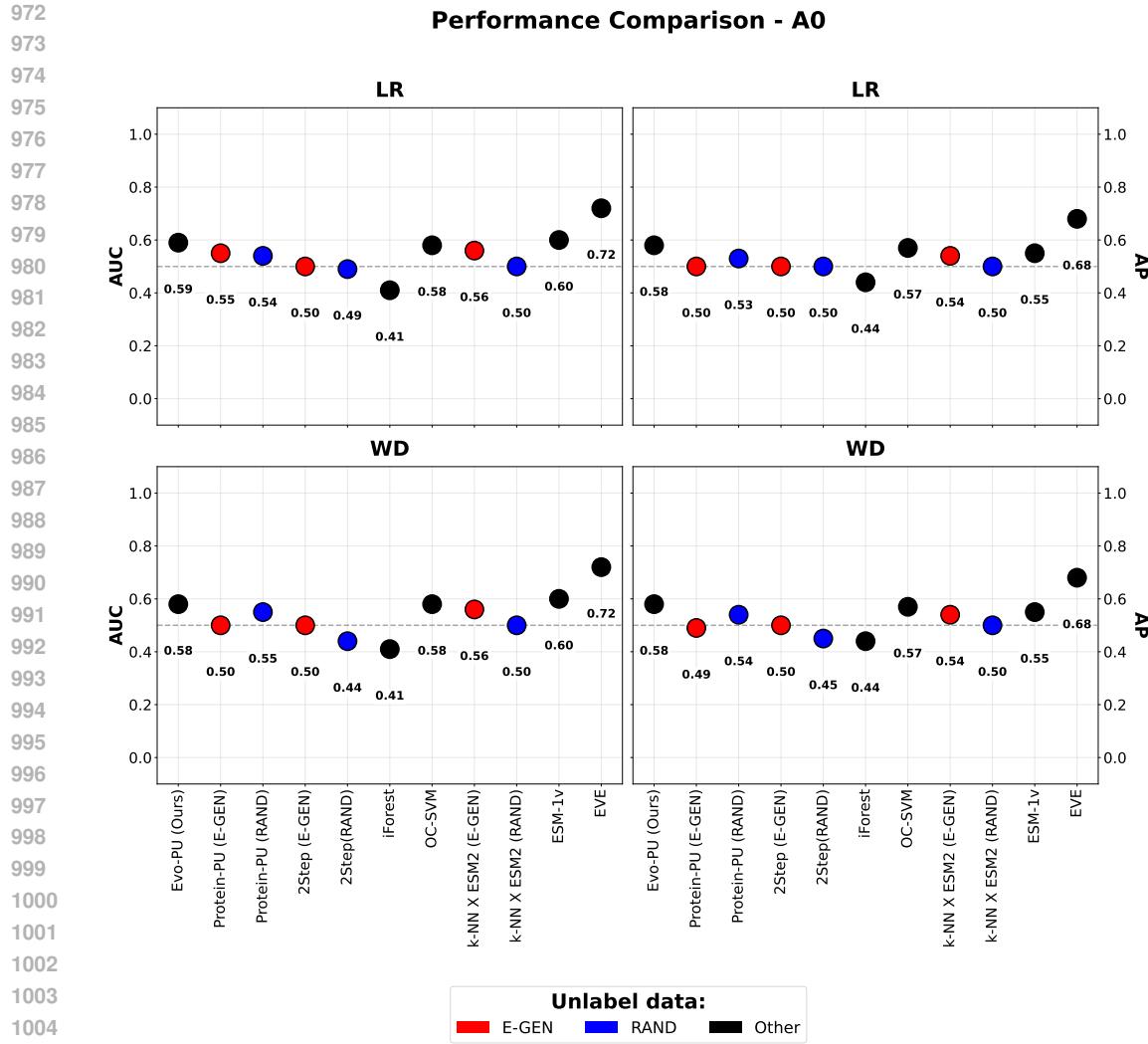


Figure 5: Performance comparison on the ProtienGym-A0. The left column reports AUC values, and the right column reports AP values. The top row shows results using the LR classifier, while the bottom row shows results using the WD classifier.

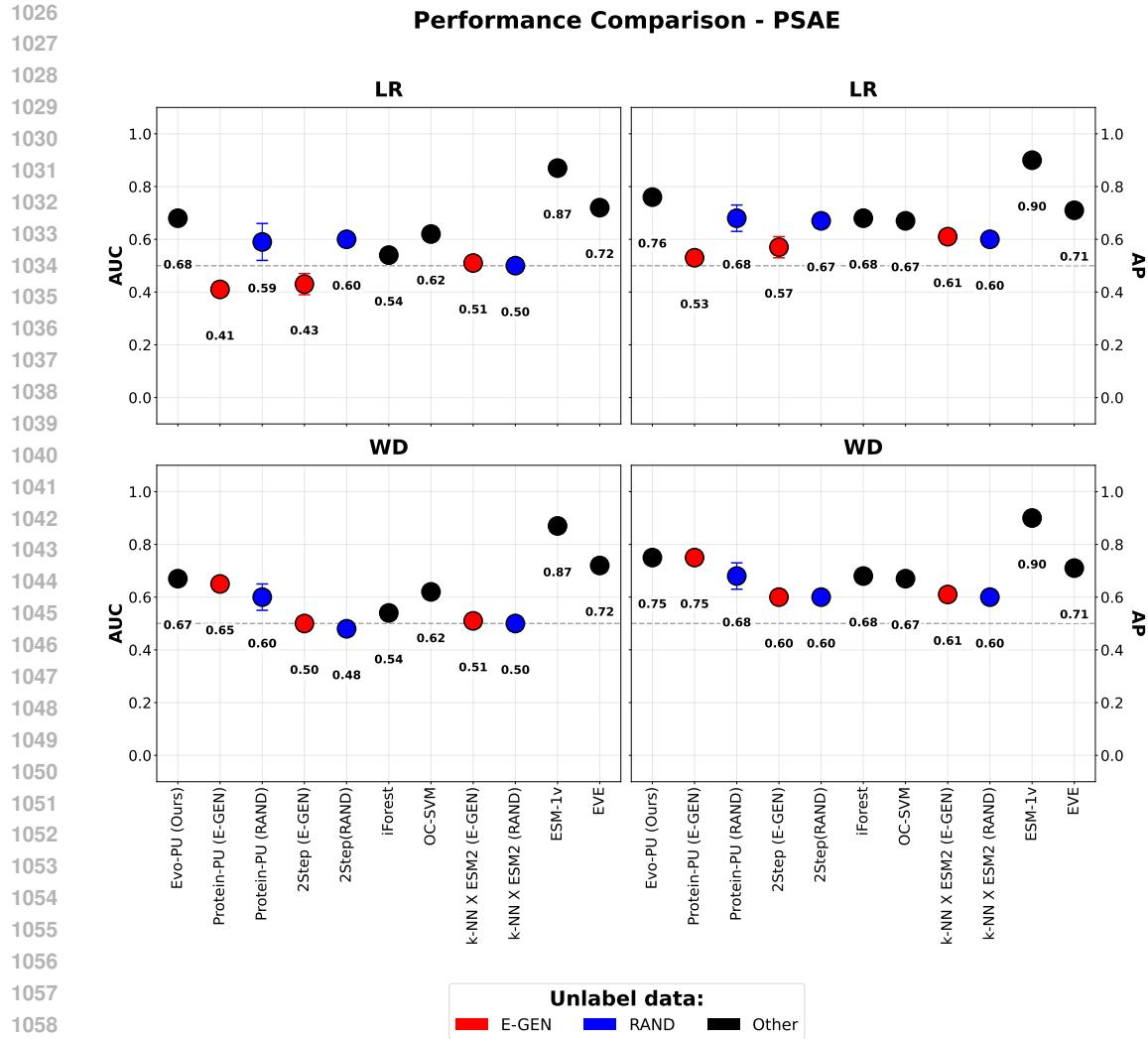


Figure 6: Performance comparison on the ProtienGym-PSAE. The left column reports AUC values, and the right column reports AP values. The top row shows results using the LR classifier, while the bottom row shows results using the WD classifier.

1080 REFERENCES FOR THE APPENDIX
1081

1082 R Chalapathy. Anomaly detection using one-class neural networks. *arXiv preprint arXiv:1802.06360*,
1083 2018.

1084 Heng-Tze Cheng, Levent Koc, Jeremiah Harmsen, Tal Shaked, Tushar Chandra, Hrishi Aradhye, Glen
1085 Anderson, Greg Corrado, Wei Chai, Mustafa Ispir, et al. Wide & deep learning for recommender
1086 systems. In *Proceedings of the 1st workshop on deep learning for recommender systems*, pages
1087 7–10, 2016.

1088 Chesner Désir, Simon Bernard, Caroline Petitjean, and Laurent Heutte. A random forest based
1089 approach for one class classification in medical imaging. In *Machine Learning in Medical Imaging:
1090 Third International Workshop, MLMI 2012, Held in Conjunction with MICCAI 2012, Nice, France,
1091 October 1, 2012, Revised Selected Papers 3*, pages 250–257. Springer, 2012.

1092 Sarah M Erfani, Sutharshan Rajasegarar, Shanika Karunasekera, and Christopher Leckie. High-
1093 dimensional and large-scale anomaly detection using a linear one-class svm with deep learning.
1094 *Pattern Recognition*, 58:121–134, 2016.

1095 Farzaneh Esmaili, Yongfang Qin, Duolin Wang, and Dong Xu. Kinase-substrate prediction using an
1096 autoregressive model. *Computational and Structural Biotechnology Journal*, 27:1103–1111, 2025.

1097 Jonathan Frazer, Pascal Notin, Mafalda Dias, Aidan Gomez, Joseph K Min, Kelly Brock, Yarin Gal,
1098 and Debora S Marks. Disease variant prediction with deep generative models of evolutionary data.
1099 *Nature*, 599(7883):91–95, 2021.

1100 Zahra Ghafoori and Christopher Leckie. Deep multi-sphere support vector data description. In
1101 *Proceedings of the 2020 SIAM International Conference on Data Mining*, pages 109–117. SIAM,
1102 2020.

1103 Shehroz S Khan and Michael G Madden. One-class classification: taxonomy of study and review of
1104 techniques. *The Knowledge Engineering Review*, 29(3):345–374, 2014.

1105 Wee Sun Lee and Bing Liu. Learning with positive and unlabeled examples using weighted logistic
1106 regression. In *ICML*, volume 3, pages 448–455, 2003.

1107 Xiao-Li Li, Bing Liu, and See Kiong Ng. Negative training data can be harmful to text classification.
1108 In *Proceedings of the 2010 conference on empirical methods in natural language processing*, pages
1109 218–228, 2010.

1110 Zeming Lin, Halil Akin, Roshan Rao, Brian Hie, Zhongkai Zhu, Wenting Lu, Nikita Smetanin,
1111 Robert Verkuil, Ori Kabeli, Yaniv Shmueli, et al. Evolutionary-scale prediction of atomic-level
1112 protein structure with a language model. *Science*, 379(6637):1123–1130, 2023.

1113 Bing Liu, Wee Sun Lee, Philip S Yu, and Xiaoli Li. Partially supervised classification of text
1114 documents. In *ICML*, volume 2, pages 387–394. Sydney, NSW, 2002.

1115 Bing Liu, Yang Dai, Xiaoli Li, Wee Sun Lee, and Philip S Yu. Building text classifiers using positive
1116 and unlabeled examples. In *Third IEEE international conference on data mining*, pages 179–186.
1117 IEEE, 2003.

1118 Fei Tony Liu, Kai Ming Ting, and Zhi-Hua Zhou. Isolation forest. In *2008 eighth ieee international
1119 conference on data mining*, pages 413–422. IEEE, 2008.

1120 Guang-Hua Luo, Xiao-Huan Li, Zhao-Jun Han, Zhi-Chun Zhang, Qiong Yang, Hui-Fang Guo, and
1121 Ji-Chao Fang. Transition and transversion mutations are biased towards gc in transposons of chilo
1122 suppressalis (lepidoptera: Pyralidae). *Genes*, 7(10):72, 2016.

1123 Larry M Manevitz and Malik Yousef. One-class svms for document classification. *Journal of machine
1124 Learning research*, 2(Dec):139–154, 2001.

1125 Suyu Mei and Hao Zhu. A novel one-class svm based negative data sampling method for recon-
1126 structing proteome-wide htlv-human protein interaction networks. *Scientific reports*, 5(1):8034,
1127 2015.

1134 Joshua Meier, Roshan Rao, Robert Verkuil, Jason Liu, Tom Sercu, and Alex Rives. Language models
 1135 enable zero-shot prediction of the effects of mutations on protein function. *Advances in neural*
 1136 *information processing systems*, 34:29287–29303, 2021.

1137

1138 Daniel T Munroe and Michael G Madden. Multi-class and single-class classification approaches to
 1139 vehicle model recognition from images. *proc. AICS*, pages 1–11, 2005.

1140

1141 Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor
 1142 Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, et al. Pytorch: An imperative style,
 1143 high-performance deep learning library. *Advances in neural information processing systems*, 32,
 2019.

1144

1145 Fabian Pedregosa, Gaël Varoquaux, Alexandre Gramfort, Vincent Michel, Bertrand Thirion, Olivier
 1146 Grisel, Mathieu Blondel, Peter Prettenhofer, Ron Weiss, Vincent Dubourg, et al. Scikit-learn:
 1147 Machine learning in python. *the Journal of machine Learning research*, 12:2825–2830, 2011.

1148

1149 Bernhard Schölkopf, John C Platt, John Shawe-Taylor, Alex J Smola, and Robert C Williamson.
 1150 Estimating the support of a high-dimensional distribution. *Neural computation*, 13(7):1443–1471,
 2001.

1151

1152 Andrew Skabar. Single-class classifier learning using neural networks: An application to the
 1153 prediction of mineral deposits. In *Proceedings of the 2003 International Conference on Machine*
Learning and Cybernetics (IEEE Cat. No. 03EX693), volume 4, pages 2127–2132. IEEE, 2003.

1154

1155 Hyebin Song, Bennett J Bremer, Emily C Hinds, Garvesh Raskutti, and Philip A Romero. Inferring
 1156 protein sequence-function relationships with large-scale positive-unlabeled learning. *Cell systems*,
 1157 12(1):92–101, 2021.

1158

1159 David MJ Tax and Robert PW Duin. Data domain description using support vectors. In *ESANN*,
 1160 volume 99, pages 251–256, 1999a.

1161

1162 David MJ Tax and Robert PW Duin. Support vector domain description. *Pattern recognition letters*,
 20(11-13):1191–1199, 1999b.

1163

1164 David MJ Tax and Robert PW Duin. Uniform object generation for optimizing one-class classifiers.
 1165 *Journal of machine learning research*, 2(Dec):155–173, 2001.

1166

1167 Nicole N Thadani, Sarah Gurev, Pascal Notin, Noor Youssef, Nathan J Rollins, Daniel Ritter, Chris
 1168 Sander, Yarin Gal, and Debora S Marks. Learning from prepandemic data to forecast viral escape.
Nature, 622(7984):818–825, 2023.

1169

1170 Ke Wang and Salvatore Stolfo. One-class training for masquerade detection. 2003.

1171

1172 Hongzuo Xu, Guansong Pang, Yijie Wang, and Yongjun Wang. Deep isolation forest for anomaly
 1173 detection. *IEEE Transactions on Knowledge and Data Engineering*, 35(12):12591–12604, 2023.

1174

1175 Abdulaziz Yousef and Nasrollah Moghadam Charkari. A novel method based on physicochemical
 1176 properties of amino acids and one class classification algorithm for disease gene identification.
Journal of biomedical informatics, 56:300–306, 2015.

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