Finite-Time Command Filtered Adaptive Control for Robot Manipulators in Random Vibration Environment

Xinyu Song, Lin Zhao and Guozeng Cui

***Abstract*—This paper introduces a finite-time tracking control algorithm for robot manipulator systems in a random vibration environment, which addresses the challenges of parameter uncertainty and input saturation. The algorithm combines command filtered adaptive backstepping with neural networks to approximate unknown nonlinear dynamics and avoid the singularity problem of traditional finite-time backstepping methods. An error compensation mechanism based on the fractional power function is also introduced to improve trajectory tracking accuracy, and the algorithm is shown to ensure practical finite-time stability in mean square. Numerical simulations demonstrate that the effectiveness of proposed method.**

***Index Terms*—Adaptive neural control, command filtered backstepping, stochastic robot manipulators, finite-time convergence**

I. INTRODUCTION

Robotic manipulators are increasingly used in industrial production, agriculture, and other fields [1], [2]. Achieving high-precision tracking of a given manipulator trajectory is critical, as manipulators are not limited to fixed-point operation [3]–[7]. Backstepping methods combined with adaptive neural/fuzzy techniques [4], [5] are commonly used to control manipulator systems (MSs), which are modeled as nonlinear systems with strong uncertainty. However, the complexity explosion problem (CEP) of backstepping methods has prompted the development of improved methods, such as command filtered backstepping (CFB) [8]. In this method, a command filter is added to roughly represent the differentiation of the virtual signal. It is then applied to asymptotic control for flexible robotic manipulators [7], with an error-compensation mechanism to lessen approximation error. However, these algorithms mainly consider the deterministic case for MSs.

Stochastic disturbances during manipulator motion in transportation equipment can invalidate or reduce the performance of controllers designed according to deterministic models. [9], [10] described the stochastic Lagrangian dynamics equations for multi-joint MSs in random vibration environments and designed a controller based on backstepping. While [11] designed an adaptive CFB controller for stochastic multi-joint manipulator systems (SMJMSs), it requires a completely known inertia matrix, making the uncertainty model and control algorithm conservative. Besides, the control algorithms in [9]–[11] can only achieve asymptotic convergence.

Compared to asymptotic control algorithms, finite-time control provides stronger robustness and higher control accuracy [12], and also requires proper fractional power based controller design. However, existing finite-time control algorithms for robot arms, such as [13], [14], do not consider random vibrations, and it is a challenge to design algorithms that enable finite-time control of stochastic manipulators. Although the finite-time backstepping control (FTBC) can achieve mean-square finite-time stability of stochastic nonlinear systems [15], [16], the singularity problem caused by the fractional power function in the virtual control signal differentiation renders the controller unusable. In order to enable high-precision tracking of SMJMSs with uncertain parameters and input saturation, we present a new finite-time command filtered adaptive neural backstepping (FTCFANB) controller in this study.

1) Compared with the asymptotic control methods based on traditional backstepping and CFB in [9]–[11], this paper proposes a novel finite-time CFB control method for SMJMSs, a new finite-time command filter backstepping control method is proposed in this paper. A new virtual control signal and error compensation mechanism based on the fractional power function are designed to improve convergence speed and steady-state performance while ensuring finite-time convergence.

2) Compared with the asymptotic control in [11], the uncertainty of the inertia matrix is taken into account as well. The inertia matrix is finely divided, and the system's uncertain nonlinear dynamics are approximated using neural networks (NNs). In addition, compared with [9]–[11], this paper additionally takes input saturation into account and addresses its effects by adding an auxiliary system, making the MSs under consideration in this paper more applicable in real-world settings.

This work was supported in part by the National Natural Science Foundation of China under Grant 61603204, and in part by the Natural Science Foundation of Shandong Province under Grant ZR2021MF046. (*Corresponding author: Lin Zhao; Guozeng Cui.*)

X. Y. Song and L. Zhao are with the College of Automation, Qingdao University, Qingdao, 266071, China (e-mail: xysong7664@163.com; zhaolin1585@163.com).

G. Z. Cui is the School of Electronic and Information Engineering, Suzhou University of Science and Technology, Suzhou 215009, China (email: guozengcui@gmail.com).

II. PRELIMINARIES AND SYSTEM DESCRIPTIONS

A. Preliminaries and some lemmas

The continuous-time stochastic system can be described as follows:

$$d\chi(t) = F(\chi(t))dt + \Phi(\chi(t))dw(t), \chi(t_0) = \chi_0, \tag{1}$$

This article has been accepted for publication in IEEE Transactions on Circuits and Systems--II: Express Briefs. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2023.3282208

where $w(t) \in \mathbb{R}^m$ is an independent standard Wiener process, $\chi(t) \in \mathbb{R}^n$ is state variable of the stochastic system, χ_0 is the initial condition.

Definition 1: [15] If there exists a positive constant ϵ and a stochastic settling time $T(\epsilon, \chi_0) < \infty$ such that $\forall t > t_0 + T$, the system (1) satisfies $E[|\chi(t)|^2] < \epsilon$, then the equilibrium $\chi(t) = 0$ is practically finite-time stable in mean square (PFTSMS).

Definition 2: [15] We define a differential operator L associated with the stochastic system (1) as follows: For a function $V(\chi) \in C^2$, $LV(\chi)$ is given by $\frac{\partial V}{\partial \chi} F(\chi(t)) + \frac{1}{2}\mathrm{Tr}\{\Phi(\chi(t))\frac{\partial^2 V}{\partial \chi^2}\Phi(\chi(t))\}$, where Tr is the trace of matrix.

Lemma 1: [6] Given a continuous function $f(z) \in \mathbb{R}^n$ on a compact set Ω, there exists a radial basis function (RBF) NN $H^{\mathrm{T}}B(z)$ such that the maximum approximation error is bounded by $\varepsilon > 0$, i.e., $\sup_{z\in\Omega}\left|f(z) - H^{\mathrm{T}}B(z)\right| \leq \varepsilon$. The RBF basis function vector is defined as $B(z) = [b_1(z), b_2(z), ..., b_K(z)]^{\mathrm{T}}$, where K is the number of NNs nodes, and $H \in \mathbb{R}^{K\times n}$ is the weight matrix. Each basis function is defined as $B_i(z) = \exp[\frac{-(z-\gamma_i)^{\mathrm{T}}(z-\gamma_i)}{w_i^2}], i = 1, 2, ..., K$, where $\gamma_i = [\gamma_{i1}, ..., \gamma_{in}]^{\mathrm{T}}$ is the center vector and w_i is the width.

Lemma 2: [8] There exists $(\sum_{i=1}^{K}|z_i|)^\kappa \leq \sum_{i=1}^{K}|z_i|^\kappa \leq K^{1-\kappa}(\sum_{i=1}^{K}|z_i|)^\kappa, i = 1, \cdots, K$ for $z_i \in \mathbb{R}$ and $0 < \kappa \leq 1$.

Lemma 3: [15] For $p, q \in \mathbb{R}$, $a > 0, b > 0$ and $\alpha(p,q) > 0$, $|p|^a|q|^b \leq \frac{a\alpha(p,q)|s|^{a+b}}{a+b} + \frac{b\alpha(p,q)^{-\frac{a}{b}}|q|^{a+b}}{a+b}$.

Lemma 4: [15] If there are three positive constants $\Delta, \Gamma, \kappa \in (0,1)$ and two $\kappa_\infty-$functions β_1 and β_2 ,which make a C^2 function $V(\chi(t))$ such that

$$\begin{cases} \beta_1(\|\chi(t)\|) \leq V(\chi(t)) \leq \beta_2(\|\chi(t)\|), 0 \leq s \leq t, \\ W(\chi(t)) - W(\chi(t)) \leq -\Delta\int_s^t W^\kappa(\chi(v))dv + \Gamma(t-s). \end{cases}.$$

Then, for $\forall t \geq T$, there holds $\|\chi(t)\| < \epsilon$, where $T = \frac{1}{(1-\kappa)\sigma\Delta}\left[V^{1-\kappa}(\chi(0)) - \left(\frac{\Gamma}{(1-\sigma)\Delta}\right)^{(1-\kappa)/\kappa}\right]$, $\epsilon = \beta_1^{-1}\left[([\Gamma/((1-\sigma)\Delta)])^{1/\kappa}\right], 0 < \sigma < 1$.

Lemma 5: [17] For nonsingular matrix $A \in \mathbb{R}^{n\times n}$, if it exists $A = \bar{A} + \Delta A$, there holds

$$A^{-1} = \bar{A}^{-1} + \Delta\tilde{A}, \tag{2}$$

where $\Delta\tilde{A} = -\bar{A}^{-1}\Delta A(I_n + \bar{A}\Delta A)^{-1}\bar{A}^{-1}$.

Lemma 6: [9] The inequality $x^T y \leq \frac{\epsilon^p}{p}|x|^p + \frac{1}{q\epsilon^q}|y|^q$ holds for any vectors $x, y \in \mathbb{R}^n$ and any positive scalar ϵ and $p > 1$, where $q = \frac{p}{p-1}$.

B. System descriptions

Following the same line as in [9], [10], we consider the dynamics model of SMJMSs as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + h(q) = \mathrm{sat}(u) + \Lambda(q)\xi, \tag{3}$$

where $q \in \mathbb{R}^n$ represents the generalized coordinates of the system, $M(q) \in \mathbb{R}^{n\times n}$ is the symmetric positive definite inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis/centrifugal matrix, $h(q) \in \mathbb{R}^n$ is the potential force, $\Lambda(q) \in \mathbb{R}^n$ and $\Lambda(q)\xi$ is the random excitation force caused by the white noise $\xi \in \mathbb{R}^m$ with bounded variance.

$u \in \mathbb{R}^n$ is the control force acting on the system and $\mathrm{sat}(u) = [\mathrm{sat}(u_1), \mathrm{sat}(u_2), \cdots, \mathrm{sat}(u_n)]^T$ is the input saturation function vector satisfies $\mathrm{sat}(u_i) = \begin{cases} u_i, |u_i| \leq u_i^* \\ \mathrm{sign}(u_i) * u_i^*, |u_i| > u_i^* \end{cases}$, where $u_i^* > 0$ are known constant. Define $\eta(u_i) = u_i^* * \tanh(\frac{u_i}{u_i^*})$, then u can be expressed as $u = \mathrm{sat}(u) = \eta(u) + \tilde{\eta}(u)$, where $\eta(u), \tilde{\eta}(u) \in \mathbb{R}^n$, and $|\tilde{\eta}(u_i)| = |\mathrm{sat}(u_i) - \eta(u_i)| \leq \max\{u_{i,\max}(1-\tanh(1)), u_{i,\min}(1-\tanh(1))\} \triangleq \bar{\eta}_i$, and denote $\bar{\eta} = [\bar{\eta}_1, \bar{\eta}_2, \cdots, \bar{\eta}_n]$.

Assumption 1: Suppose ξ is a white noise vector with bounded variance.

Assumption 2: M is positive and symmetrical, and M can be divided into the nominal part and the unknown part.

Assumption 3: For the nominal part $\bar{M}$, there are bounds such that $\bar{M}_{\min}\|x\|^2 \leq x^{\mathrm{T}}\bar{M}x \leq \bar{M}_{\max}\|x\|^2$, where $x \in \mathbb{R}^n$, The positive constants $\bar{M}_{\max}$ and $\bar{M}_{\min}$ are known. For an unknown component ΔM, there exists an unidentified $\Delta M^* > 0$ such that $\|\Delta M\| \leq \Delta M^*$.

According to Assumption 2 and Lemma 5, it can be known that $M^{-1}(q) = \bar{M}^{-1} + \Delta\tilde{M}$. After $x_1 = q, x_2 = \dot{q}$ are defined, the equations in (3) may be rewritten as follows:

$$\begin{aligned} dx_1 &= x_2 dt \\ dx_2 &= [\bar{M}^{-1}\mathrm{sat}(u) + \varphi]dt + \tilde{\varphi}\varpi dw \\ y &= x_1, \end{aligned} \tag{4}$$

where $\varphi = -M^{-1}(q)[C(q,\dot{q})x_1 + h(q)] + \Delta\tilde{M}\mathrm{sat}(u)$, $\tilde{\varphi} = M^{-1}(q)\Lambda(q)$. w is an m-dimensional independent standard Wiener process. The power spectral density of the white noise ξ is $\frac{1}{2\pi}\varpi$, where $\varpi \in \mathbb{R}^{m\times m}$ is a positive matrix.

Remark 1: This paper considers the dynamic model of SMJMSs proposed in reference [9], [10] is considered, and it is assumed that the random vibration is caused by white noise ξ. However the variance of ξ is not assumed to be bounded in [9], [10], which is usually required in the actual environment. This paper considers the design of finite-time tracking controller under Assumption 1.

III. Controller design

The tracking errors z_i is defined as follows:

$$z_1 = x_1 - q_d, z_2 = x_2 - \bar{\alpha}, \tag{5}$$

in which $\bar{\alpha} = [\psi_{1,1}, \psi_{2,1}, \cdots, \psi_{n,1}]^{\mathrm{T}} \in \mathbb{R}^n$ is the output of finite-time command filter (FTCF) with the virtual signal $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_n]^{\mathrm{T}}$ as the input. $q_d \in \mathbb{R}$ is the desired trajectory and its first derivative $\dot{q}_d$ are known, smooth and bounded functions. The FTCF is designed as

$$\begin{aligned} \dot{\psi}_{i,1} =& \zeta_i \\ \zeta_i =& -\varrho_1|\psi_{i,1} - \alpha_i|^{\frac{1}{2}}\mathrm{sign}(\psi_{i,1} - \alpha_i) + \psi_{2,i} \\ \dot{\psi}_{i,2} =& -\varrho_2\mathrm{sign}(\psi_{i,2} - \zeta_i). \end{aligned} \tag{6}$$

Remark 2: It should be noted that [18] provides a method for choosing the parameters ϱ_1 and ϱ_2, according to which ϱ_1 and ϱ_2 should both be of adequate size and ϱ_2 is selected first.

Lemma 7: [18] If the parameters ϱ_1, ϱ_2 are chosen appropriately, $\psi_{i,1} = \alpha_{i,0}$ and $\zeta_i = \dot{\alpha}_{i,0}$ can be reached in a finite time without input noise, where $\alpha_i = \alpha_{i,0}$. The inequalities

$|\psi_{i,1}-\alpha_{i,0}| \leq \Theta_i \Xi_i = \pi_1$ and $|\chi_i - \dot{\alpha}_{i,0}| \leq \Upsilon_i \Xi_i^{\frac{1}{2}} = \pi_2$ hold in finite time if the input noise satisfies $|\alpha_i - \alpha_{i,0}| \leq \Xi_i$, where $\Theta_i > 0$ and $\Upsilon_i > 0$ are constants.

The virtual signal α and controller u are designed as

$$\alpha = -c_1 {v_1}^{\kappa} - \frac{3}{4}(1+k_1)v_1 + \dot{q}_d - \rho, \tag{7}$$

$$u = \bar{M}(-c_2 {v_2}^{\kappa} - (\frac{3}{4}k_2 + \frac{5}{4})v_2 - \frac{\hat{\theta} v_2 B^T B}{4\tau} - \rho), \tag{8}$$

where $0 < \kappa < 1$, $k_i > 0, c_i > 0, (i = 1, 2)$ are designed constants. The Matrix B is the basis function of RBF NNs, $\theta = \max\{\|H\|^2\}$ is a constant, where $\|\cdot\|$ is the 2-norm of the vector, $H \in \mathbb{R}^{K\times n}$ is the weight matrix. Define $\hat{\theta}$ as an estimate of θ, and the updating process for $\hat{\theta}$ is as follows:

$$\dot{\hat{\theta}} = -\iota\hat{\theta} + \frac{\lambda}{4\tau}(v_2^T v_2)^2 B^{\mathrm{T}} B, \tag{9}$$

where $\iota > 0$ is a constant.

Additionally, we incorporate an error compensation mechanism, denoted by ς_i, as defined by

$$\begin{aligned} \dot{\varsigma}_1 &= -k_1\varsigma_1 - c_1\varsigma_1^{\kappa} + (\bar{\alpha} - \alpha) + \varsigma_2 \\ \dot{\varsigma}_2 &= -k_2\varsigma_2 - c_2\varsigma_2^{\kappa} \end{aligned} \tag{10}$$

with $\varsigma_i(0) = 0 (i = 1, 2)$.$\rho$ is an auxiliary system which has the function of anti-saturation. It is defined as follows

$$\dot{\rho} = -\rho + \bar{M}^{-1}\eta(u) - \bar{M}^{-1}u. \tag{11}$$

Remark 3: Based on the FTCF in (6), the $\bar{\alpha}$ and its first-order derivative $\dot{\bar{\alpha}}$ are obtained using the virtual controller α as input. Actually, the FTCF is a first-order Levant differentiator presented in [18], which can not only achieve the fast filtering for α, but also ensure the stability in finite time. And the FTCF was first applied to the finite-time control of stochastic manipulator systems, addressing the singularity issue encountered by traditional FTBC in [15], [16].

Remark 4: An auxiliary system (11), which can ensure that the real control input u can be designed, is included to the controller design in order to combat the impact of input saturation on control performance.

IV. Finite-Time Stability Analysis

Theorem 1: Consider the SMJMSs (3) subject to Assumptions 1-3, by selecting FTCF as in (6) and constructing virtual signals α and controller u as in (7)-(8), an adaptive updating law is designed as in (9), along with an error compensation mechanism in (10) and an auxiliary system in (11), then all signals in the closed-loop system are mean-square bounded in finite time, and the tracking error z_1 is PFTSMS.

Proof: Establish the compensated tracking error signal v_i to complete the finite-time stability proof

$$v_1 = z_1 - \varsigma_1, v_2 = z_2 - \varsigma_2 - \rho. \tag{12}$$

According to Itô formula we get

$$\begin{aligned} dv_1 &= (x_2 - \dot{q}_d - \dot{\varsigma}_1)dt \\ dv_2 &= (\bar{M}^{-1}\mathrm{sat}(u) + \varphi - L\bar{\alpha} - \dot{\varsigma}_2 - \dot{\rho})dt + \bar{\varphi}\varpi dw. \end{aligned} \tag{13}$$

where $\bar{\varphi} = \tilde{\varphi} - \frac{\partial\bar{\alpha}}{\partial x_1}$.

Step 1**:** Thinking about the stochastic system (4), the stochastic Lyapunov function is created as $V_1 = \frac{1}{4}({v_1}^{\mathrm{T}} v_1)^2$. Based on Definition 2, there exists

$$LV_1 = {v_1}^{\mathrm{T}} v_1 {v_1}^{\mathrm{T}}(\alpha + z_2 + (\bar{\alpha} - \alpha) - \dot{q}_d - \dot{\varsigma}_1). \tag{14}$$

Substituting virtual signal (7), error compensation mechanism (10) into (14) and according to the Lemma 6, we have

$$\begin{aligned} LV_1 \leq &- c_1 {v_1}^{\mathrm{T}} v_1 {v_1}^{\mathrm{T}} {v_1}^{\kappa} + c_1 {v_1}^{\mathrm{T}} v_1 {v_1}^{\mathrm{T}} {\varsigma_1}^{\kappa} \\ &+ \frac{1}{4}({v_2}^{\mathrm{T}} v_2)^2 + \frac{k_1}{4}({\varsigma_1}^{\mathrm{T}}\varsigma_1)^2. \end{aligned} \tag{15}$$

According to the Lemma 2 and Lemma 3, we have

$${v_1}^{\mathrm{T}} v_1 {v_1}^{\mathrm{T}} {v_1}^{\kappa} \leq ({v_1}^{\mathrm{T}} v_1)(\sum_{s=1}^{n} v_{1,s})^{1+\kappa} = ({v_1}^{\mathrm{T}} v_1)^{\frac{3+\kappa}{2}}. \tag{16}$$

$${v_1}^{\mathrm{T}} v_1 {v_1}^{\mathrm{T}} {\varsigma_1}^{\kappa} \leq \frac{3}{\kappa+3}({v_1}^{\mathrm{T}} v_1)^{\frac{3+\kappa}{2}} + \frac{\kappa}{\kappa+3}({\varsigma_1}^{\mathrm{T}}\varsigma_1)^{\frac{3+\kappa}{2}}. \tag{17}$$

Substituting (16)-(17) into (15), one has

$$\begin{aligned} LV_1 \leq &- \frac{c_1\kappa}{\kappa+3}({v_1}^{\mathrm{T}} v_1)^{\frac{3+\kappa}{2}} + \frac{c_1\kappa}{\kappa+3}({\varsigma_1}^{\mathrm{T}}\varsigma_1)^{\frac{3+\kappa}{2}} \\ &+ \frac{1}{4}({v_2}^{\mathrm{T}} v_2)^2 + \frac{k_1}{4}({\varsigma_1}^{\mathrm{T}}\varsigma_1)^2. \end{aligned} \tag{18}$$

Step 2**:** Choose $V_2 = V_1 + \frac{1}{4}({v_2}^{\mathrm{T}} v_2)^2 + \frac{\tilde{\theta}^2}{2\lambda}$ as the Lyapunov function, where $\lambda > 0$ is a constant. Substitute (10)-(11) and (13), we have

$$\begin{aligned} LV_2 = &LV_1 + {v_2}^{\mathrm{T}} v_2 {v_2}^{\mathrm{T}}(\bar{M}^{-1}u + \varphi - L\bar{\alpha} + \rho + k_2\varsigma_2 + c_2{\varsigma_2}^{\kappa} \\ &+ \bar{M}^{-1}\tilde{\eta}(u)) - \frac{\tilde{\theta}\dot{\hat{\theta}}}{\lambda} + \frac{1}{2}\{\bar{\varphi}^{\mathrm{T}}(2v_2{v_2}^{\mathrm{T}} + {v_2}^{\mathrm{T}} v_2 I)\bar{\varphi}\}. \end{aligned} \tag{19}$$

According to Assumption 3 and $\|\tilde{\eta}(u)\| \leq \bar{\eta}$, then applying Young's inequality in Lemma 6, one has

$$\begin{aligned} &{v_2}^{\mathrm{T}} v_2 {v_2}^{\mathrm{T}} \bar{M}^{-1}\tilde{\eta}(u) \leq \|v_2\|^3 \|\bar{M}^{-1}\|_F \|\bar{\eta}\| \\ &\leq \frac{3}{4}({v_2}^{\mathrm{T}} v_2)^2 + \|\bar{M}^{-1}\|_F^4(\bar{\eta}^T\bar{\eta})^2 \leq \frac{3}{4}({v_2}^{\mathrm{T}} v_2)^2 + \frac{(\bar{\eta}^T\bar{\eta})^2}{\bar{M}_{\min}^4}, \end{aligned} \tag{20}$$

$$k_2 {v_2}^{\mathrm{T}} v_2 {v_2}^{\mathrm{T}} \varsigma_2 \leq \frac{3k_2}{4}({v_2}^{\mathrm{T}} v_2)^2 + \frac{k_2}{4}({\varsigma_2}^{\mathrm{T}}\varsigma_2)^2. \tag{21}$$

And according to the property of norm, that is

$$\begin{aligned} &\frac{1}{2}\{\bar{\varphi}^{\mathrm{T}}(2v_2{v_2}^{\mathrm{T}} + {v_2}^{\mathrm{T}} v_2 I)\bar{\varphi}\} \leq \frac{m}{2}\|\bar{\varphi}^{\mathrm{T}}(2v_2{v_2}^{\mathrm{T}} + {v_2}^{\mathrm{T}} v_2 I)\bar{\varphi}\|_\infty \\ &\leq \frac{m\sqrt{m}}{2}\|\bar{\varphi}^{\mathrm{T}}(2v_2{v_2}^{\mathrm{T}} + {v_2}^{\mathrm{T}} v_2 I)\bar{\varphi}\| \leq \frac{3m\sqrt{m}}{2}{v_2}^{\mathrm{T}} v_2\|\bar{\varphi}^{\mathrm{T}}\bar{\varphi}\|. \end{aligned} \tag{22}$$

Substituting (20)-(22) into (19), one has

$$\begin{aligned} LV_2 \leq &LV_1 + {v_2}^{\mathrm{T}} v_2 {v_2}^{\mathrm{T}}(\bar{M}^{-1}u + \varphi - L\bar{\alpha} + \rho \\ &+ \frac{3}{4}(1+k_2)v_2 + c_2{\varsigma_2}^{\kappa}) - \frac{\tilde{\theta}\dot{\hat{\theta}}}{\lambda} + \frac{k_2}{4}({\varsigma_2}^{\mathrm{T}}\varsigma_2)^2 \\ &+ \frac{3r\sqrt{r}}{2}{v_2}^{\mathrm{T}} v_2\|\bar{\varphi}^{\mathrm{T}}\bar{\varphi}\| + \frac{(\bar{\eta}^T\bar{\eta})^2}{\bar{M}_{\min}^4}. \end{aligned} \tag{23}$$

Then, letting $\hat{\varphi} = {v_2}^{\mathrm{T}}(\varphi - L\bar{\alpha}) + \frac{3m\sqrt{m}}{2}\|\bar{\varphi}^{\mathrm{T}}\bar{\varphi}\|$, we adopt a RBF NNs $H^{\mathrm{T}}B(X)$ to approximate it. For $\forall\varepsilon > 0$, one has

$$\hat{\varphi} = H^{\mathrm{T}}B(X) + \delta(X), |\delta(X)| \leq \varepsilon. \tag{24}$$

This article has been accepted for publication in IEEE Transactions on Circuits and Systems--II: Express Briefs. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2023.3282208

By using Young's inequality in Lemma 6, we get

$$(v_2{}^{\mathrm{T}}v_2)\hat{\varphi} = (v_2{}^{\mathrm{T}}v_2)H^{\mathrm{T}}B(X) + (v_2{}^{\mathrm{T}}v_2)\delta(X) \le \frac{(v_2{}^{\mathrm{T}}v_2)^2\|H\|^2B^{\mathrm{T}}B}{4\tau} + \tau + \frac{1}{4}(v_2{}^{\mathrm{T}}v_2)^2 + \varepsilon^2, \quad (25)$$

where $\tau > 0$ is a constant. Substituting (8) - (9) and (25) into (23) yields

$$LV_2 \le -\sum_{i=1}^{2}\frac{c_i\kappa}{\kappa+3}(v_i{}^{\mathrm{T}}v_i)^{\frac{3+\kappa}{2}} + \sum_{i=1}^{2}\frac{c_i\kappa}{\kappa+3}(\varsigma_i{}^{\mathrm{T}}\varsigma_i)^{\frac{3+\kappa}{2}} + \sum_{i=1}^{2}\frac{k_i}{4}(\varsigma_i{}^{\mathrm{T}}\varsigma_i)^2 + \frac{\iota}{\lambda}\tilde{\theta}\dot{\hat{\theta}} + \frac{(\bar{\eta}^T\bar{\eta})^2}{\bar{M}_{\min}^4} + \tau + \varepsilon^2. \quad (26)$$

Then, we choose $\bar{V} = \sum_{i=1}^{2}\frac{(\varsigma_i{}^{\mathrm{T}}\varsigma_i)^2}{4}$ as the Lyapunov function to verify the stability of the error compensation mechanism. Lemma 7 states that the inequality $\|\bar{\alpha} - \alpha\| \le \pi_1$ holds for a finite time. Combining this with Lemma 6, one has

$$L\bar{V} \le -(k_1 - \frac{3}{2})(\varsigma_1{}^{\mathrm{T}}\varsigma_1)^2 - (k_2 - \frac{1}{4})(\varsigma_2{}^{\mathrm{T}}\varsigma_2)^2 - \sum_{i=1}^{2}c_i(\varsigma_i{}^{\mathrm{T}}\varsigma_i)^{\frac{3+\kappa}{2}} + \frac{1}{4}\pi_1{}^4. \quad (27)$$

Moreover, we get $\frac{\iota}{\lambda}\tilde{\theta}\dot{\hat{\theta}} \le -\frac{\iota}{2\lambda}\tilde{\theta}^2 + \frac{\iota}{2\lambda}\theta^2$. Then, we define $V = V_2 + \bar{V}$ and have

$$LV \le -\sum_{i=1}^{2}\frac{c_i\kappa}{\kappa+3}(v_i{}^{\mathrm{T}}v_i)^{\frac{3+\kappa}{2}} - \sum_{i=1}^{2}\frac{3c_i}{\kappa+3}(\varsigma_i{}^{\mathrm{T}}\varsigma_i)^{\frac{3+\kappa}{2}} - \frac{\iota}{2\lambda}\tilde{\theta}^2 - (\frac{3}{4}k_1 - \frac{3}{2})(\varsigma_1{}^{\mathrm{T}}\varsigma_1)^2 - (\frac{3}{4}k_2 - \frac{1}{4})(\varsigma_2{}^{\mathrm{T}}\varsigma_2)^2 + \Gamma', \quad (28)$$

where $\Gamma' = \|\bar{M}^{-1}\|_F^4(\bar{\eta}^T\bar{\eta})^2 + \tau + \varepsilon^2 + \frac{1}{4}\pi_1{}^4 + \frac{\iota}{2\lambda}\theta^2$.

Based on Lemma 3, choosing $p = \frac{\tilde{\theta}^2}{2\lambda}$, $q = 1$, $a = \frac{\kappa+3}{4}$, $b = 1 - a = \frac{1-\kappa}{4}$, $\alpha(p,q) = \frac{4}{\kappa+3}$, it follows that $(\frac{\tilde{\theta}^2}{2\lambda})^{\frac{\kappa+3}{4}} \le \frac{\tilde{\theta}^2}{2\lambda} + \frac{1-\kappa}{4}(\frac{4}{\kappa+3})^{-\frac{\kappa+3}{1-\kappa}}$. Letting $\frac{3}{4}k_1 - \frac{3}{2} = 0$, we obtain $k_1 = 2$, similarly $k_2 = \frac{1}{3}$. Substituting that into (28), it is straightforward to show that

$$LV \le -\sum_{i=1}^{2}\frac{c_i\kappa}{\kappa+3}((v_i{}^{\mathrm{T}}v_i)^2)^{\frac{3+\kappa}{4}} - \sum_{i=1}^{2}\frac{3c_i}{\kappa+3}((\varsigma_i{}^{\mathrm{T}}\varsigma_i)^2)^{\frac{3+\kappa}{4}} - \iota(\frac{\tilde{\theta}^2}{2\lambda})^{\frac{\kappa+3}{4}} + \Gamma, \quad (29)$$

where $\Gamma = \Gamma' + \frac{1-\kappa}{4}(\frac{4}{\kappa+3})^{-\frac{\kappa+3}{1-\kappa}}$

Letting $\Delta = \min\{4^{\frac{\kappa+3}{4}}\frac{c_i\kappa}{\kappa+3}, 4^{\frac{\kappa+3}{4}}\frac{3c_i}{\kappa+3}, \iota\}$, then we have

$$LV \le -\Delta V^{\frac{\kappa+3}{4}} + \Gamma. \quad (30)$$

Denoting $V(x(t)) = V$, the Itô formula allows us to obtain the following result for $0 \le s_1 \le s_2$

$$E[V(x(s_2))] = EV(x(s_1)) + \int_{s_1}^{s_2} E[LV(x(t))]dt. \quad (31)$$

Taking equation (30) into (31) and applying the Jensen's inequality, then substituting it into (31) yields

$$E[V(x(s_2))] - E[V(x(s_1))] \le -\Delta\int_{s_1}^{s_2}[E[V(x(t))]]^{\frac{\kappa+3}{4}}dt + \Gamma(s_2 - s_1). \quad (32)$$

Let $\chi(x(t)) = EV(x(t))$, we obtain the setting time $T = \frac{4}{(1-\kappa)\sigma\Delta}[[EV(x)]^{\frac{1-\kappa}{4}} - (\frac{\Gamma}{(1-\sigma)\Delta})^{\frac{1-\kappa}{\kappa+3}}]$ exists, such that $E[V(x(t))] \le \epsilon$ for $\forall t \ge T$, $\epsilon = 4(\frac{\Gamma}{(1-\sigma)\Delta})^{\frac{1}{\kappa+3}}$. Using the mathematical expectation property, we can conclude that the inequality $\left[E\|v_i\|^2\right]^2 \le E\left(\|v_i\|^4\right) \le E\left(\sum_{i=1}^{2}(v_i{}^Tv_i)^2\right) \le 4\epsilon$ holds for all $t \ge T$. Thus, for $\forall t \ge T$ $E\|v_i\|^2 \le 2\sqrt{\epsilon}, E\|\varsigma_i\|^2 \le 2\sqrt{\epsilon}, E\|\tilde{\theta}\|^2 \le 2\sqrt{\epsilon}$, and $E\|z_1\|^2 \le 2E\|v_1\|^2 + 2E\|\varsigma_1\|^2 \le 4\sqrt{\epsilon}, t \ge T$.

Remark 5: When the parameter κ in the control algorithm is in the range $(0, 1)$, the closed-loop system studied in this paper can be achieved finite-time stability, which calls for the inclusion of fractional power functions in both the error-compensation mechanism ς_i and the virtual control signal α. When $\kappa = 1$, the closed-loop system can only converge asymptotically over time, which can be seen as a specific example of finite-time control, but is no longer able to achieve finite-time convergence.

Remark 6: The mean square of tracking error $E\|z_1\|^2$ is less than $4\sqrt{\epsilon}$ when $t > T$, where $\epsilon = 4(\frac{\Gamma}{(1-\tau)\Delta})^{\frac{1}{\kappa+3}}$ and $T = \frac{4}{(1-\kappa)\sigma\Delta}[[EV(x_0)]^{\frac{1-\kappa}{4}} - (\frac{\Gamma}{(1-\sigma)\Delta})^{\frac{1-\kappa}{\kappa+3}}]$. In order to guarantee a lower Γ, we can use tiny λ and ι, while higher parameters c_i and ι can guarantee a larger Δ. Smaller Γ and larger Δ can guarantee a reduced mean squared error $E\|z_1\|^2$ and a faster convergence rate for the closed-loop system.

V. Simulation results

Assuming no air resistance, we considered a system of double-linked manipulators suspended from a randomly vibrating ceiling to verify the proposed method, with the system (3), in which $q = [q_1, q_2]$, $M(q) = [M_{mn}] \in \mathbb{R}^{2\times2}$ and $C(q,\dot{q}) \in \mathbb{R}^{2\times2}$ are given by $M_{11} = (m_1 + m_2)l_1^2, M_{12} = m_2l_1l_2\cos(q_2 - q_1), M_{21} = m_2l_1l_2\cos(q_2 - q_1), M_{22} = m_2l_2^2, C_{11} = 0, C_{12} = -m_2l_1l_2\sin(q_2 - q_1)\ddot{q}_2C_{21} = m_2l_1l_2\cos(q_2 - q_1)\dot{q}_1, C_{22} = 1, h(q) \in \mathbb{R}^2$ and $\Lambda(q) \in \mathbb{R}^{2\times2}$ are given by $h_1 = (m_1 + m_2)gl_1\sin(q_1), h_2 = m_2gl_2\sin(\dot{q_2}), \Lambda_{11} = -m_1l_1\cos(q_1) + 0.5m_2l_1\sin(q_1)\sin(2(q_2 - q_1)), \Lambda_{12} = -m_1l_1\sin(q_1) - 0.5m_2l_1\cos(q_1)\sin(2(q_2 - q_1)), \Lambda_{21} = m_2l_2\sin(q_1)\sin(2(q_2 - q_1)), \Lambda_{22} = -m_2l_2\cos(q_1)\sin(2(q_2 - q_1))$. $[m_1, m_2] = [0.6, 0.8]$ and $[l_1, l_2] = [0.8, 1.2]$ respectively correspond to the mass and length of the pendulums, and the units are kg and m. The value of acceleration of gravity g is 9.8, and the unit is m/s^2. The input saturation for controller $u = [u_1, u_2]$ are considered as $\mathrm{sat}(u_i) = \begin{cases} u_i, |u_i| \le 15 \\ \mathrm{sign}(u_i) * 15, |u_i| > 15 \end{cases}$. Consistent with reference [10], power spectral density ϖ of white noise ξ is considered by $\varpi = \varpi_{2\times2} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$. The control parameters are set as follows: $k_1 = 2, k_2 = 1/3$, $l_1 = 7.5, l_2 = 25$, $\varrho_1 = 750, \varrho_2 = 180$, $\iota = 100$, $b_1 = b_2 = 1$, and $\kappa = 3/5$. The number of neurons for the RBF NNs is 10, the basis function centers are evenly distributed in $[-2, 2] \times \cdots \times [-2, 2]$, and the width is set to 4.

Choosing the reference signal $q_d = [1.5\sin(t), \sin(2t)]^{\mathrm{T}}$ and initial conditions $q(0) = [-0.1, 0.25]^{\mathrm{T}}$, $\dot{q}(0) = [-0.3, 0.45]^{\mathrm{T}}$, the desired signal q_d and the trajectory of q are

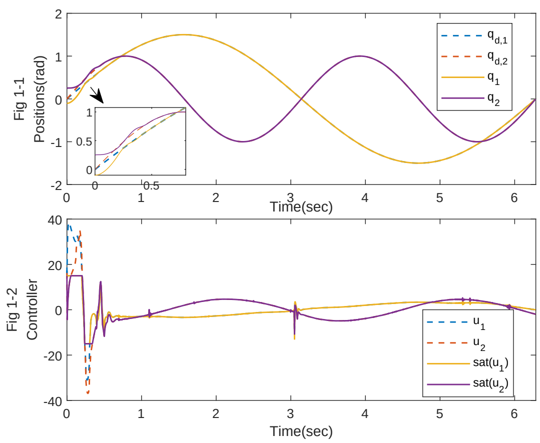

Fig. 1: Manipulator tracking response curves (Fig1-1) and saturation-limited controller response curves (Fig1-2)

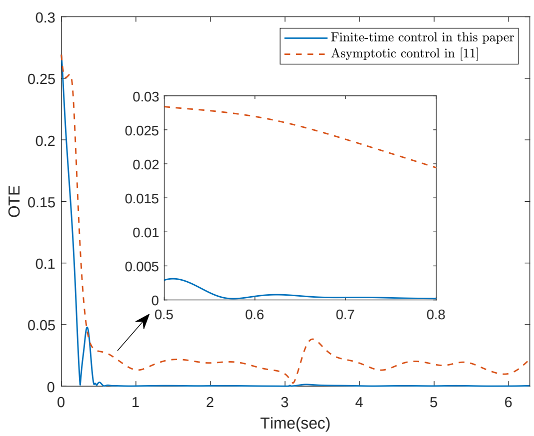

Fig. 2: OTE with the different control algorithms

shown in Fig. 1-1 for the proposed control scheme. As shown in Fig. 1-2 , the controller output does not exceed the limits imposed. We create an overall tracking error OTE $= \|q - q_d\|$ to compare the performance of convergence between the finite-time control technique of this study and the asymptotic control of [11]. Fig. 2 shows the OTE with a different κ while keeping the other parameters the same. The suggested FTCFANB algorithm clearly outperforms asymptotic control in terms of convergence rate and tracking precision.

Remark 7: Based on the above guiding principles for parameter adjustment in Remark 6, we select the simulation parameters according to the trial-and-error method.

VI. Conclusion

This paper presents an adaptive neural control technique based on the CFB approach to address the finite-time tracking control of SMJMSs with parameter uncertainty and input saturation. The suggested approach guarantees that the tracking error is PFTSMS and addresses the CEP and singularities by using traditional backstepping. In the future, the focus will be on the output feedback control of stochastic manipulator systems.

References

[1] Z. Zhang, E. Kayacan, B. Thompson, and G. Chowdhary, "High precision control and deep learning-based corn stand counting algorithms for agricultural robot," Auton. Robot., vol. 44, no. 7, pp. 1289-1302, 2020.

[2] J. Qin, F. Leonard, and G. Abba, "Real-time trajectory compensation in robotic friction stir welding using state estimators," IEEE Trans. Control Syst. Technol., vol. 24, no. 6, pp. 2207-2214, 2016.

[3] X. Zhao, Z. Liu, B. Jiang and C. Gao, "Switched controller design for robotic manipulator via neural network-based sliding mode approach," IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 70, no. 2, pp. 561-565, Feb. 2023.

[4] R. Wai and R. Muthusamy, "Design of Fuzzy-neural-network-inherited backstepping control for robot manipulator including actuator dynamics," IEEE Trans. Fuzzy Syst., vol. 22, no. 4, pp. 709-722, Aug. 2014.

[5] Z. Chen, F. Huang, C. Yang and B. Yao, "Adaptive fuzzy backstepping control for stable nonlinear bilateral teleoperation manipulators with enhanced transparency performance," IEEE Trans. Ind. Electron., vol. 67, no. 1, pp. 746-756, Jan. 2020.

[6] F. Meng, L. Zhao, and J. P. Yu, "Backstepping based adaptive finite-time tracking control of manipulator systems with uncertain parameters and unknown backlash," J. Franklin Inst., vol. 357, no. 16, pp. 11281-11297, 2020.

[7] T. Cheng, B. Niu, G. Zhang, Z. Wang, and P. Duan, "Event-triggered-based adaptive command-filtered asymptotic tracking control for flexible robotic manipulators," Nonlinear Dyn., vol. 105, no. 2, pp. 1543-1557, 2021.

[8] J. A. Farrell, M. Polycarpou, M. Sharma, and D. Wenjie, "Command filtered backstepping," IEEE Trans. Autom. Contr., vol. 54, no. 6, pp. 1391-1395, 2009.

[9] M. Y. Cui, Z. J. Wu, X. J. Xie, and P. Shi, "Modeling and adaptive tracking for a class of stochastic Lagrangian control systems," Automatica, vol. 49, no. 3, pp. 770-779, 2013.

[10] M. Y. Cui, Z. J. Wu, and X. J. Xie, "Output feedback tracking control of stochastic Lagrangian systems and its application," Automatica, vol. 50, no. 5, pp. 1424-1433, 2014.

[11] S. Z. Diao, W. Sun, and S. F. Su, "Neural-based adaptive event-triggered tracking control for flexible-joint robots with random noises," Int. J. Robust Nonlinear Control, vol. 32, no. 5, pp. 2722-2740, 2022.

[12] H. Wang, K. Xu and H. Zhang, "Adaptive finite-time tracking control of nonlinear systems with dynamics uncertainties," IEEE Trans. Autom. Contr., doi: 10.1109/TAC.2022.3226703.

[13] D. Zhai and Y. Xia, "Finite-time control of teleoperation systems with input saturation and varying time delays," IEEE Trans. Syst., Man, Cybern., Syst., vol. 47, no. 7, pp. 1522-1534, July 2017.

[14] G. Li, X. Chen, J. P. Yu and J. Liu, "Adaptive neural network-based finite-time impedance control of constrained robotic manipulators with disturbance observer," IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 69, no. 3, pp. 1412-1416, March 2022.

[15] F. Wang, B. Chen, Y. Sun, Y. Gao and C. Lin, "Finite-time fuzzy control of stochastic nonlinear systems," IEEE Trans. Cybern., vol. 50, no. 6, pp. 2617-2626, June 2020.

[16] H. Wang and Q. Zhu, "Finite-time stabilization of high-order stochastic nonlinear systems in strict-feedback form," Automatica, vol. 54, pp. 284-291, 2015.

[17] K. Lu and Y. Q. Xia, "Finite-time attitude control for rigid spacecraft-based on adaptive super-twisting algorithm," IET Control Theory Appl., vol.8, no.15, pp.1465-1477, 2014.

[18] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," Int J Control, vol. 76, pp. 924C941, 2003.