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# NeuroLKH: Combining Deep Learning Model with Lin-Kernighan-Helsgaun Heuristic for Solving the Traveling Salesman Problem (Appendix)

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## A Experiments for TSP

To verify the quality of the edge candidate set learned by NeuroLKH, we report two metrics for the edge candidate set attained by different methods, i.e., the average ranking of the optimal edges and the percentage of optimal edges missed in the set, respectively. Regarding the sensitivity analysis of the Minimum Spanning Tree with the subgradient optimization in LKH algorithm, 0.68% and 0.67% of the optimal edges are missed in the candidate set for TSP100 and TSP500, respectively, where the average rankings of optimal edges are 1.670 and 1.681. The ideal average ranking would be 1.5 since the two optimal edges for each node would be the first and the second in the ranks. NeuroLKH reduces the average ranking to 1.557 and 1.597 where only 0.05% and 0.09% of the optimal edges are missed in the set, which justifies the effectiveness of NeuroLKH in learning desirable edge candidates.

For TSP, we choose the number of directed edges pointed from one node in the sparse edge set  $E^*$  as  $\gamma = 20$  to include most of the edges in the optimal tours into the sparse graph, which results in only 0.01% of the optimal edges missed in the sparse graph for the training dataset. In our experiments with  $\gamma = 10, 20, 30$  (trained with 20% of the training samples to save time), 0.643%, 0.209% and 0.208% of the optimal edges are missed in the candidate set with the average ranking of the optimal edges 1.653, 1.646 and 1.640 for TSP500, respectively. With  $\gamma > 20$  (i.e. numbers of edges), it only improves the average ranking marginally with similar percentages of optimal edges but obviously increases the computational time. Pertaining to other routing problems, we find similar results therefore we use  $\gamma = 20$  for consistency. We find that the network can hardly give a high edge score to an edge with considerably large Euclidean distance and include it into the candidate set. Therefore larger  $\gamma$  is not needed which does not impact the performance much as long as it is not too small (e.g. less than 20).

The model outputs the node penalties within the range of  $[-C, C]$  with  $C = 10$ . In the original LKH algorithm, a subgradient optimization process is used to optimize the node penalties iteratively until convergence for each instance. In this process for the training instances where the coordinates are always between 0 and 1, we find that the penalties are usually between -10 and 10 (for different sizes). While testing for instances with different coordinate ranges, we scale the instances to make the coordinates between 0 and 1. The aspect ratio is fixed so that the objective value is just scaled by a constant. Therefore, we use  $C = 10$  in our experiments.

In Table S.1, we compare NeuroLKH with other recently proposed Deep Learning based methods on TSP100. Notably, most of them can hardly handle problems with more than 100 nodes. One exception is the method in [8], which is tested on large problems but the performance deteriorates rapidly with the increase of problem size and is still inferior to LKH. We adopt the results from their original works where the datasets tested on might be different but are sampled from the same distribution. Therefore the optimality gap is a more important measure than the objective value. The running time is reported for solving 1000 instances in total with the assumption that it is linearly related to the number of instances. Apparently, NeuroLKH significantly outperforms other methods with a short running time. And more importantly, as shown in Table 1 and Table 2, NeuroLKH generalizes well to large TSP with up to 5000 nodes.

Table S.1: Comparative results on TSP100. Here we report three results of NeuroLKH with different time limits from Table 1.

Method	Time(s)	Gap(‰)	Method	Time(s)	Gap(‰)	Method	Time(s)	Gap(‰)
GCN greedy [18]	36	838.000	AM Greedy [21]	0.6	453.000	AM sampling [21]	360	226.000
Wu [33]	720	142.000	GCN bs [18]	240	139.000	CVAE-Opt-RS [15]	50500	135.000
da Costa [5]	246	87.000	CVAE-Opt-DE [15]	55100	34.000	POMO [22]	6	14.000
Fu [8]	90	4.000	DPDP 10k [20]	456	0.900	DPDP 100k [20]	990	0.400
<b>NeuroLKH</b>	33	0.111	<b>NeuroLKH</b>	127	0.030	<b>NeuroLKH</b>	938	0.000

## B Experiments for TSPLIB

NeuroLKH is trained using only the instances with nodes generated from the uniform distribution. With the same training dataset size, we trained another model NeuroLKH\_M using a mixture of instances with uniformly distributed nodes, clustered nodes with 3-8 clusters, half uniform and half clustered nodes following [30]. Following the convention for TSPLIB in [12, 36], the number of trials is set to be the number of nodes and the algorithms are run 10 times for each instance. During each run, the algorithm will stop when the optimal solution is found and the number of trials actually conducted is reported. Here we show the results of LKH, VSR-LKH, NeuroLKH and NeuroLKH\_M for each instance in Table S.2, Table S.3 and Table S.4. The optimal tour distance is shown under the instance name. We report the success times where the optimal solution is found, the best performance (tour distance) during the runs, the average performance, the average running time (seconds) and the average number of trials actually conducted. The results of LKH are the same as reported in [36] (except the running time where we run all the algorithms on our machine for a fair comparison) while the results of VSR-LKH are slightly different due to behaviour uncontrolled by the random seed in the code.

## C Experiments for Other Routing Problems

Here we briefly introduce the Capacitated Vehicle Routing Problem (CVRP), the Pickup and Delivery Problem (PDP) and CVRP with Time Windows (CVRPTW). For PDP, the customers contain pairs of pickup and delivery nodes. The vehicle starts from the depot, visits each customer node once and returns to the depot with the constraint that the pickup node must be visited before the corresponding delivery node. For CVRP, multiple routes can be planned. In each route, the vehicle starts from the depot, visits some customers and returns to the depot. The total demand of the customers in each route cannot exceed the vehicle capacity and each customer must be visited once. CVRPTW generalizes CVRP with an additional constraint that each customer must be visited within the corresponding time window. The time will be spent on traveling between the nodes and serving the customers. The goal of all three problems is to minimize the tour distance.

Similarly, we plot the performance of the LKH and NeuroLKH algorithms for solving CVRP, PDP and CVRPTW in Figure S.1, which shows similar trends as those in Figure 2. The time limits are set to the longest ones used in Table 3, i.e., the running time of LKH algorithm with 10000 trials.

For the results reported in Table 3, almost all the improvements of NeuroLKH over LKH on different sizes and with different time limits are statistically significant with confidence levels larger than 99%. The only exceptions are the performance for the smallest size of each problem and the longest time limits (the running time of LKH with 10000 trials), where the confidence levels are 98.7%, 98.9% and 77.9% for CVRP100, PDP40 and CVRPTW40, respectively. The confidence level for CVRPTW40 with the time limit of LKH with 10000 trials is relatively low because CVRPTW with 40 nodes solved by LKH is already fairly close to the optimality with such a long time limit. Therefore the improvement room left for NeuroLKH is small.

## D Experiments on CVRPLIB and Solomon Benchmark

CVRPLIB [30] contains various sized CVRP instances with a combination of 3 depot positioning, 3 customer positioning and 7 demand distributions. We train one network using CVRP instances ranging from 101 to 300 nodes. The instances are generated from this mixture of distributions

proposed in [30] and we generate  $120000/|V|$  instances for each size  $|V|$  in the training dataset, resulting in approximately 120000 instances in total.

Solomon benchmark [28] contains CVRPTW instances with 100 customers and various distributions of time windows. An additional constraint for this benchmark is to minimize the number of routes. Therefore the goal is to minimize the tour distance using the minimum number of routes. We choose R2-type as the testbed in our experiment. We generate a training dataset of instances with 100 customers. The node coordinates are generated independently from the uniform distribution ranging from 0 to 80. The demands are generated from a Gaussian distribution with mean 15 and standard deviation 10 and the capacity is fixed as 1000. The serving time  $s$  for each customer is fixed as 10. The center of time window for node  $i$  is generated from the uniform distribution with the interval  $[dist, 1000 - s - dist]$ , where  $dist$  is the distance between node  $i$  and the depot. And the width of time window is generated from a Gaussian distribution with the mean and standard deviation set to 115 and 35, 240 and 0, 350 and 160, 150 and 380, 470 and 70, respectively. For each of the first two sets of parameters, four different types are generated with 0%, 25%, 50% and 100% of the customers receiving the time windows. And for the last three sets of parameters, all customers are receiving the time windows, resulting in 11 types of instances in total. We generate 5000 instances for each type in the training dataset. Please refer to the code for more details.

As the running time is all relatively short, we run both LKH and NeuroLKH for 100 times on each instance. The results of LKH and NeuroLKH are shown in Table S.5, Table S.6 and Table S.7, while the time limits are set to the running time of LKH with 100, 1000 and 10000 trials. The optimal tour distance is shown under the instance name. We report the average running time (seconds), the best performance (tour distance) during the runs, the average performance, the success times when the optimal solution is found.

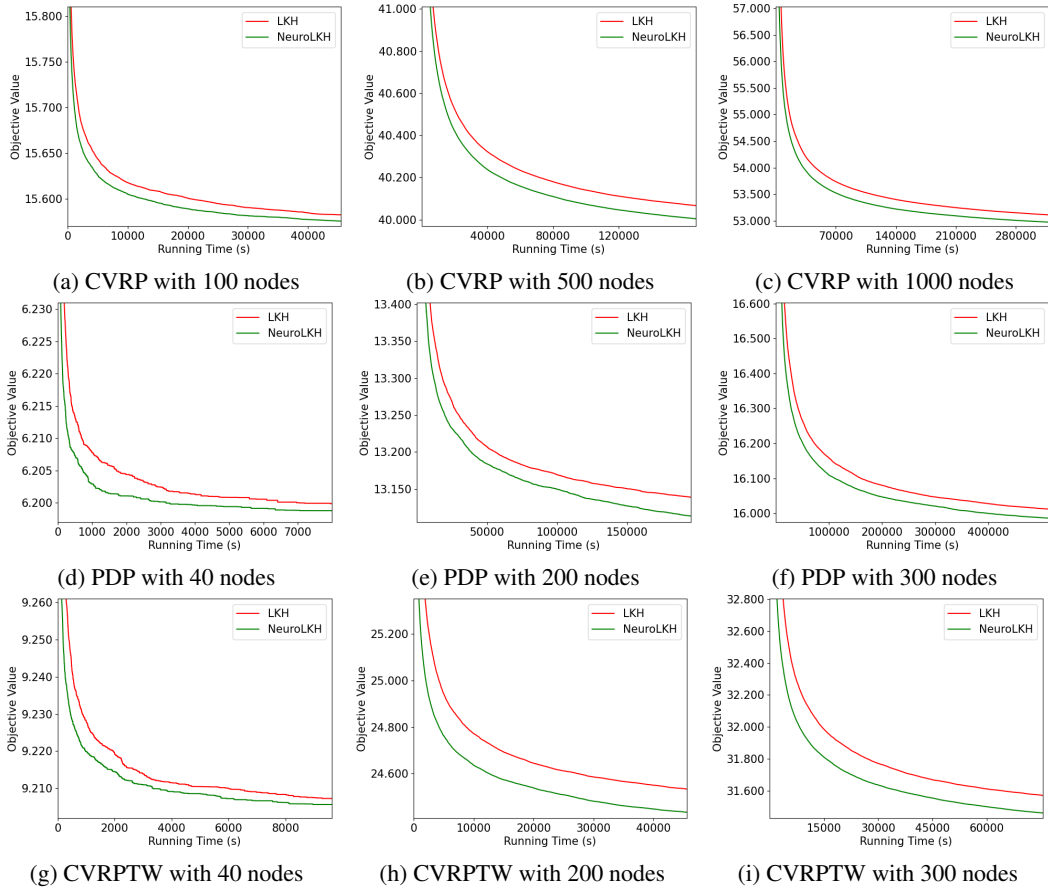


Figure S.1: Performances of LKH and NeuroLKH for solving CVRP, PDP and CVRPTW with different sizes against different running time

Table S.2: TSPLIB results for each hard instance

Method	Name	Success	Best	Average	Time	Trials	Name	Success	Best	Average	Time	Trials
LKH	kroB150	2/10	26130	26131.6	0.32	128.4	rat195	9/10	2323	2323.5	0.22	55
VSR-LKH		4/10	26130	26131.2	0.21	106.3		9/10	2323	2323.5	0.36	69.5
NeuroLKH_R		10/10	26130	26130	0.07	9.8		10/10	2323	2323	0.11	8.4
NeuroLKH_M		10/10	26130	26130	0.12	22.1		10/10	2323	2323	0.06	3.9
LKH	pr299	9/10	48191	48194.3	0.4	51.7	d493	6/10	35002	35002.8	4.71	219.6
VSR-LKH		10/10	48191	48191	0.43	13.6		10/10	35002	35002	0.5	8.8
NeuroLKH_R		10/10	48191	48191	0.25	10.1		6/10	35002	35032.2	6.73	320.5
NeuroLKH_M		10/10	48191	48191	0.22	13.2		10/10	35002	35002	0.67	27.5
LKH	rat575	2/10	6773	6773.8	3.23	526.9	pr1002	8/10	259045	259045.6	4.53	549
VSR-LKH		6/10	6773	6773.4	3.2	310.6		10/10	259045	259045	0.72	16
NeuroLKH_R		9/10	6773	6773.1	1.91	179		10/10	259045	259045	8.46	330.6
NeuroLKH_M		7/10	6773	6773.3	3.87	345.3		10/10	259045	259045	1.05	34
LKH	u1060	5/10	224094	224107.5	101.76	663.3	vm1084	3/10	239297	239372.6	46.16	824.1
VSR-LKH		10/10	224094	224094	3.52	19.1		7/10	239297	239312.6	49.41	474.8
NeuroLKH_R		10/10	224094	224094	35.07	206.9		1/10	239297	239379.5	23.4	1028.9
NeuroLKH_M		10/10	224094	224094	10.05	75.4		7/10	239297	239315.1	21.29	439.7
LKH	pcb1173	4/10	56892	56895	5.37	844	rl1304	3/10	252948	253156.4	18.28	1170
VSR-LKH		8/10	56892	56893	7.07	436.9		10/10	252948	252948	1.44	17.9
NeuroLKH_R		9/10	56892	56892.5	5.32	410.4		9/10	252948	252953.1	9.26	370.8
NeuroLKH_M		8/10	56892	56893	6.48	378.2		8/10	252948	252958.2	11.36	600.6
LKH	rl1323	6/10	270199	270219.6	12.57	718.8	nrw1379	6/10	56638	56640	9.84	759.3
VSR-LKH		10/10	270199	270199	9.08	189.7		9/10	56638	56638.5	12.84	253.7
NeuroLKH_R		7/10	270199	270247.9	16.59	742.2		9/10	56638	56638.5	15.28	372.4
NeuroLKH_M		8/10	270199	270204.4	11.13	538.5		10/10	56638	56638	7.85	260.8
LKH	fl1400	1/10	20127	20160.3	2703.75	1372.9	fl1577	0/10	22254	22260.6	965.98	1577
VSR-LKH		1/10	20127	20160.3	3323.31	1380.6		0/10	22254	22255.8	3095.13	1577
NeuroLKH_R		0/10	20165	20235.5	356.77	1400		1/10	22249	22256.6	652.75	1445.8
NeuroLKH_M		0/10	20164	20169.4	754.03	1400		0/10	22254	22302.8	522.49	1577
LKH	vm1748	9/10	336556	336557.3	17.62	1007.9	u1817	1/10	57201	57251.1	63.28	1817
VSR-LKH		10/10	336556	336556	5.42	37.8		7/10	57201	57212	159.43	967
NeuroLKH_R		5/10	336556	336628	38.16	1282.9		2/10	57201	57227.3	238.86	1803.4
NeuroLKH_M		10/10	336556	336556	13.65	460.2		2/10	57201	57225.2	126.01	1691.5
LKH	rl1889	0/10	316549	316549.8	59.31	1889	d2103	0/10	80454	80462	111.69	2103
VSR-LKH		4/10	316536	316569	143.58	1393.9		0/10	80454	80454.2	619.38	2103
NeuroLKH_R		0/10	316638	316648.7	141.23	1889		4/10	80450	80452.4	339.12	1560.3
NeuroLKH_M		3/10	316536	316619.4	81.93	1485.6		3/10	80450	80454.6	213	1614.7
LKH	u2152	3/10	64253	64287.7	88.79	1614	pcb3038	4/10	137694	137701.2	79.22	2078.6
VSR-LKH		7/10	64253	64270.1	178.54	1334.7		7/10	137694	137695.5	214.24	1422.2
NeuroLKH_R		9/10	64253	64258.7	56.63	520.9		8/10	137694	137695	151.91	1104
NeuroLKH_M		8/10	64253	64255.2	66.85	878.1		8/10	137694	137695	99.23	1084.6
LKH	fl3795	0/10	28813	28813.7	34045.95	3795	fnl4461	9/10	182566	182566.5	31.89	923.1
VSR-LKH		0/10	28831	28831	75405	3795		10/10	182566	182566	19.94	89.1
NeuroLKH_R		0/10	28999	29010.6	80797.24	3795		10/10	182566	182566	27.91	171.5
NeuroLKH_M		0/10	29488	29495.3	1329.72	3795		10/10	182566	182566	19.26	151.5
LKH	rl5915	0/10	565544	565581.2	221.29	5915	rl5934	0/10	556136	556309.8	371.79	5934
VSR-LKH		1/10	565530	565580.8	896.59	5354.9		4/10	556045	556099.6	923.66	4804.7
NeuroLKH_R		0/10	565585	565969.9	658.32	5915		8/10	556045	556059.5	376.57	3470.2
NeuroLKH_M		1/10	565530	565579.5	365.82	5352.9		10/10	556045	556045	143.34	1529.8

Table S.3: TSPLIB results for each easy instance

Method	Name	Success	Best	Average	Time	Trials	Name	Success	Best	Average	Time	Trials		
LKH	eil51	10/10	426	426	0	1	berlin52	10/10	7542	7542	0.01	0		
VSR-LKH		10/10	426	426	0	1		10/10	7542	7542	0.02	0		
NeuroLKH_R		10/10	426	426	0	1		10/10	7542	7542	0.02	0		
NeuroLKH_M		10/10	426	426	0	1		7542	10/10	7542	7542	0.02	0	
LKH	st70	10/10	675	675	0.01	1	eil76	10/10	538	538	0	1		
VSR-LKH		10/10	675	675	0.01	1		10/10	538	538	0	1		
NeuroLKH_R		10/10	675	675	0.01	1		10/10	538	538	0	1		
NeuroLKH_M		675	10/10	675	675	0.01		1	538	10/10	538	538	0	1
LKH	pr76	10/10	108159	108159	0.02	1	rat99	10/10	1211	1211	0	1		
VSR-LKH		10/10	108159	108159	0.02	1		10/10	1211	1211	0	1		
NeuroLKH_R		108159	10/10	108159	108159	0.02		1	1211	10/10	1211	1211	0.01	1
NeuroLKH_M		10/10	108159	108159	108159	0.02		1	10/10	1211	1211	0	1	
LKH	kroA100	10/10	21282	21282	0.02	1	kroB100	10/10	22141	22141	0.03	1.2		
VSR-LKH		10/10	21282	21282	0.01	1		10/10	22141	22141	0.04	2.5		
NeuroLKH_R		21282	10/10	21282	21282	0.01		1	22141	10/10	22141	22141	0.03	1
NeuroLKH_M		10/10	21282	21282	21282	0.01		1	10/10	22141	22141	0.03	1	
LKH	kroC100	10/10	20749	20749	0.01	1	kroD100	10/10	21294	21294	0.02	1.8		
VSR-LKH		10/10	20749	20749	0.02	1		10/10	21294	21294	0.02	1		
NeuroLKH_R		20749	10/10	20749	20749	0.02		1	21294	10/10	21294	21294	0.02	1
NeuroLKH_M		10/10	20749	20749	20749	0.02		1	10/10	21294	21294	0.02	1	
LKH	kroE100	10/10	22068	22068	0.03	3.2	rd100	10/10	7910	7910	0	1		
VSR-LKH		10/10	22068	22068	0.06	8.5		10/10	7910	7910	0	1		
NeuroLKH_R		22068	10/10	22068	22068	0.03		1	7910	10/10	7910	7910	0.01	1
NeuroLKH_M		10/10	22068	22068	22068	0.04		4.8	10/10	7910	7910	7910	0.01	1
LKH	eil101	10/10	629	629	0	1	lin105	10/10	14379	14379	0	1		
VSR-LKH		10/10	629	629	0	1		10/10	14379	14379	0	1		
NeuroLKH_R		629	10/10	629	629	0		1	14379	10/10	14379	14379	0	1
NeuroLKH_M		10/10	629	629	629	0		1	10/10	14379	14379	0	1	
LKH	pr107	10/10	44303	44303	0.13	1	pr124	10/10	59030	59030	0.04	1		
VSR-LKH		10/10	44303	44303	0.13	1		10/10	59030	59030	0.04	1		
NeuroLKH_R		44303	10/10	44303	44303	0.14		1.1	59030	10/10	59030	59030	0.07	1
NeuroLKH_M		10/10	44303	44303	44303	0.13		1	10/10	59030	59030	0.06	1	
LKH	bier127	10/10	118282	118282	0.01	1	ch130	10/10	6110	6110	0.03	1		
VSR-LKH		10/10	118282	118282	0.02	1		10/10	6110	6110	0.07	7.3		
NeuroLKH_R		118282	4/10	118282	118300.6	0.13		102.5	6110	10/10	6110	6110	0.02	1.1
NeuroLKH_M		10/10	118282	118282	118282	0.01		1	10/10	6110	6110	6110	0.03	2.1
LKH	pr136	10/10	96772	96772	0.08	1	pr144	10/10	58537	58537	0.37	1		
VSR-LKH		10/10	96772	96772	0.08	1		10/10	58537	58537	0.43	1		
NeuroLKH_R		96772	10/10	96772	96772	0.15		4.5	1/10	58537	58584.7	2.6	131.8	
NeuroLKH_M		10/10	96772	96772	96772	0.11		1	2/10	58537	58614	2.31	122.3	
LKH	ch150	10/10	6528	6528	0.04	1.7	kroA150	10/10	26524	26524	0.05	3.8		
VSR-LKH		10/10	6528	6528	0.02	1		10/10	26524	26524	0.04	1		
NeuroLKH_R		6528	10/10	6528	6528	0.02		1.1	26524	10/10	26524	26524	0.04	2.6
NeuroLKH_M		10/10	6528	6528	6528	0.02		1.1	10/10	26524	26524	0.02	1	
LKH	pr152	10/10	73682	73682	0.48	29.4	u159	10/10	42080	42080	0.01	1		
VSR-LKH		8/10	73682	73709.2	0.69	47		10/10	42080	42080	0.01	1		
NeuroLKH_R		73682	8/10	73682	73709.2	1.44		59.6	42080	10/10	42080	42080	0.01	1
NeuroLKH_M		9/10	73682	73695.6	0.87	38.7		10/10	42080	42080	42080	0.01	1	

Table S.4: TSPLIB results for each easy instance (continued)

Method	Name	Success	Best	Average	Time	Trials	Name	Success	Best	Average	Time	Trials
LKH	d198	10/10	15780	15780	0.57	1	kroA200	10/10	29368	29368	0.06	1.7
VSR-LKH		10/10	15780	15780	0.43	1		10/10	29368	29368	0.06	1.5
NeuroLKH_R		0/10	15789	15825	2.54	198	29368	10/10	29368	29368	0.05	1
NeuroLKH_M	15780	10/10	15780	15780	0.87	1		10/10	29368	29368	0.04	1
LKH	kroB200	10/10	29437	29437	0.02	1	ts225	10/10	126643	126643	0.04	1
VSR-LKH		10/10	29437	29437	0.03	1		10/10	126643	126643	0.02	1
NeuroLKH_R		10/10	29437	29437	0.02	1	126643	10/10	126643	126643	0.06	1
NeuroLKH_M	29437	10/10	29437	29437	0.02	1		10/10	126643	126643	0.06	1
LKH	tsp225	10/10	3916	3916	0.06	1	pr226	10/10	80369	80369	0.08	1
VSR-LKH		10/10	3916	3916	0.07	1		10/10	80369	80369	0.1	13.3
NeuroLKH_R		10/10	3916	3916	0.06	1	80369	6/10	80369	80381.7	1.34	146.2
NeuroLKH_M	3916	10/10	3916	3916	0.06	1		10/10	80369	80369	0.22	5.9
LKH	gil262	10/10	2378	2378	0.14	10.6	pr264	10/10	49135	49135	0.24	14.4
VSR-LKH		10/10	2378	2378	0.05	1.7		10/10	49135	49135	0.19	1
NeuroLKH_R		10/10	2378	2378	0.13	8	49135	10/10	49135	49135	0.13	6.2
NeuroLKH_M	2378	10/10	2378	2378	0.05	2.2		10/10	49135	49135	0.09	2.4
LKH	a280	10/10	2579	2579	0.03	1	lin318	10/10	42029	42029	0.23	27.9
VSR-LKH		10/10	2579	2579	0.02	1		10/10	42029	42029	0.09	1.8
NeuroLKH_R		10/10	2579	2579	0.02	1	42029	10/10	42029	42029	0.18	3.6
NeuroLKH_M	2579	10/10	2579	2579	0.03	1		10/10	42029	42029	0.15	5.9
LKH	rd400	10/10	15281	15281	0.23	33	fl417	10/10	11861	11861	2.69	7.3
VSR-LKH		10/10	15281	15281	0.23	11.6		10/10	11861	11861	1.91	3.7
NeuroLKH_R		10/10	15281	15281	0.11	3.9	11861	5/10	11861	11867.6	16.64	337.2
NeuroLKH_M	15281	10/10	15281	15281	0.12	4.7		9/10	11861	11861.1	16.7	51.7
LKH	pr439	10/10	107217	107217	0.59	39.5	pcb442	10/10	50778	50778	0.16	8.2
VSR-LKH		10/10	107217	107217	0.44	22.1		10/10	50778	50778	0.07	3
NeuroLKH_R		3/10	107217	107267.4	1.64	320.1	50778	10/10	50778	50778	0.11	3.8
NeuroLKH_M	107217	9/10	107217	107224.2	0.71	90.3		10/10	50778	50778	0.18	6.9
LKH	u574	10/10	36905	36905	0.8	149.9	p654	10/10	34643	34643	7.04	22.9
VSR-LKH		10/10	36905	36905	0.39	29.2		10/10	34643	34643	4.28	9
NeuroLKH_R		10/10	36905	36905	0.2	3.8	34643	1/10	34643	34765.8	40.27	619
NeuroLKH_M	36905	10/10	36905	36905	0.11	1.9		10/10	34643	34643	2.63	7
LKH	d657	10/10	48912	48912	0.48	33.5	u724	10/10	41910	41910	1.53	125.4
VSR-LKH		10/10	48912	48912	0.44	21		10/10	41910	41910	0.85	23.3
NeuroLKH_R		5/10	48912	48912.5	6.65	511.5	41910	10/10	41910	41910	0.94	46.6
NeuroLKH_M	48912	10/10	48912	48912	0.39	10		10/10	41910	41910	0.64	16.8
LKH	rat783	10/10	8806	8806	0.08	4.2	d1291	10/10	50801	50801	6.27	192.1
VSR-LKH		10/10	8806	8806	0.11	3.9		10/10	50801	50801	2.51	39.5
NeuroLKH_R		10/10	8806	8806	0.14	4.2	50801	9/10	50801	50803.4	5.64	274.4
NeuroLKH_M	8806	10/10	8806	8806	0.21	12.2		7/10	50801	50808.2	9.46	437.4
LKH	u1432	10/10	152970	152970	0.43	5.3	d1655	10/10	62128	62128	5.44	176
VSR-LKH		10/10	152970	152970	0.55	5		10/10	62128	62128	0.94	9.8
NeuroLKH_R		10/10	152970	152970	0.56	7.1	62128	8/10	62128	62128.2	22.22	870.4
NeuroLKH_M	152970	10/10	152970	152970	0.43	3.8		10/10	62128	62128	7.86	214.1
LKH	u2319	10/10	234256	234256	0.46	3.1	pr2392	10/10	378032	378032	0.4	5.8
VSR-LKH		10/10	234256	234256	0.89	3.9		10/10	378032	378032	0.78	8.7
NeuroLKH_R		10/10	234256	234256	0.67	3.5	378032	10/10	378032	378032	1.22	25.5
NeuroLKH_M	234256	10/10	234256	234256	0.37	2.6		10/10	378032	378032	1.31	25.9

Table S.5: CVRPLIB results

Name	Method	LKH with 100 trials as time limit				LKH with 1000 trials as time limit				LKH with 10000 trials as time limit			
		Time	Best	Avg	Suc	Time	Best	Avg	Suc	Time	Best	Avg	Suc
X-n101-k25 27591	LKH	1.2	27744	28214.5	0	13	27591	27794.3	6	131	27591	<b>27667.0</b>	33
	NeuroLKH		27665	<b>28146.6</b>	0		27591	<b>27790.3</b>	5		27591	27669.5	30
X-n106-k14 26362	LKH	0.9	26495	26730.1	0	10	26426	26557.6	0	105	26381	26438.3	0
	NeuroLKH		26447	<b>26712.8</b>	0		26396	<b>26528.5</b>	0		26381	<b>26428.5</b>	0
X-n110-k13 14971	LKH	0.4	14971	15216.2	2	3	14971	<b>15073.3</b>	31	29	14971	<b>15020.4</b>	53
	NeuroLKH		14971	<b>15207.3</b>	2		14971	15074.2	32		14971	15022.3	58
X-n115-k10 12747	LKH	0.2	12750	12838.3	0	2	12747	<b>12778.3</b>	14	17	12747	<b>12770.3</b>	46
	NeuroLKH		12747	<b>12837.6</b>	1		12747	12783.9	14		12747	12771.8	40
X-n120-k6 13332	LKH	0.3	13332	13547.4	1	2	13332	13394.3	10	21	13332	13358.6	40
	NeuroLKH		13333	<b>13519.9</b>	0		13332	<b>13389.7</b>	5		13332	<b>13352.9</b>	33
X-n125-k30 55539	LKH	3.1	56167	56690.8	0	31	55733	56041.8	0	335	55546	55813.0	0
	NeuroLKH		56011	<b>56624.7</b>	0		55645	<b>55981.7</b>	0		55539	<b>55779.7</b>	1
X-n129-k18 28940	LKH	0.8	29173	29635.5	0	8	28967	29257.5	0	86	28948	29108.8	0
	NeuroLKH		29160	<b>29566.1</b>	0		28948	<b>29224.3</b>	0		28948	<b>29081.3</b>	0
X-n134-k13 10916	LKH	1.1	11024	11215.7	0	10	10931	11048.8	0	94	10916	10994.8	1
	NeuroLKH		11023	<b>11194.9</b>	0		10941	<b>11044.6</b>	0		10916	<b>10987.1</b>	1
X-n139-k10 13590	LKH	0.4	13670	13894.9	0	3	13605	13713.6	0	33	13590	13660.4	5
	NeuroLKH		13672	<b>13871.1</b>	0		13605	<b>13702.6</b>	0		13590	<b>13657.0</b>	6
X-n143-k7 15700	LKH	0.5	15765	<b>16186.5</b>	0	5	15737	15910.4	0	50	15711	15812.4	0
	NeuroLKH		15781	16208.1	0		15726	<b>15885.9</b>	0		15726	<b>15787.3</b>	0
X-n148-k46 43448	LKH	0.9	43833	44382.4	0	9	43485	43819.2	0	89	43448	43635.2	18
	NeuroLKH		43809	<b>44283.0</b>	0		43514	<b>43818.1</b>	0		43448	<b>43634.7</b>	19
X-n153-k22 21220	LKH	1.7	21328	21559.2	0	15	21236	21326.8	0	156	21225	<b>21263.6</b>	0
	NeuroLKH		21298	<b>21493.7</b>	0		21240	<b>21311.1</b>	0		21225	21272.1	0
X-n157-k13 16876	LKH	0.5	16903	17008.7	0	4	16876	16911.0	8	40	16876	16893.4	40
	NeuroLKH		16900	<b>17006.8</b>	0		16876	<b>16904.9</b>	14		16876	<b>16889.0</b>	52
X-n162-k11 14138	LKH	0.3	14179	<b>14362.6</b>	0	3	14156	<b>14225.2</b>	0	26	14138	<b>14196.8</b>	6
	NeuroLKH		14190	14388.8	0		14161	14245.3	0		14138	14213.9	2
X-n167-k10 20557	LKH	0.6	20826	21319.8	0	7	20583	20863.2	0	65	20557	20749.5	1
	NeuroLKH		20687	<b>21270.5</b>	0		20592	<b>20857.8</b>	0		20557	<b>20740.3</b>	1
X-n172-k51 45607	LKH	1.2	46141	46679.2	0	11	45742	46078.0	0	122	45607	45840.5	5
	NeuroLKH		46134	<b>46533.1</b>	0		45707	<b>45994.7</b>	0		45607	<b>45783.9</b>	3
X-n176-k26 47812	LKH	3.6	48035	48819.7	0	33	47930	<b>48273.6</b>	0	353	47840	<b>48090.3</b>	0
	NeuroLKH		48147	<b>48726.3</b>	0		47950	48279.7	0		47812	48098.9	1
X-n181-k23 25569	LKH	0.5	25677	25829.7	0	4	25611	25691.2	0	42	25582	25645.3	0
	NeuroLKH		25691	<b>25822.8</b>	0		25603	<b>25685.9</b>	0		25577	<b>25641.2</b>	0
X-n186-k15 24145	LKH	1.0	24297	<b>24882.6</b>	0	10	24227	24528.3	0	104	24149	<b>24359.6</b>	0
	NeuroLKH		24511	24911.5	0		24178	<b>24523.0</b>	0		24147	24361.7	0
X-n190-k8 16980	LKH	0.9	17187	17418.0	0	8	17065	17275.4	0	84	16993	17155.2	0
	NeuroLKH		17160	<b>17410.0</b>	0		17041	<b>17259.8</b>	0		16985	<b>17145.1</b>	0
X-n195-k51 44225	LKH	1.4	44911	45594.9	0	11	44437	44799.6	0	117	44334	44558.1	0
	NeuroLKH		44685	<b>45244.5</b>	0		44422	<b>44688.0</b>	0		44237	<b>44524.8</b>	0
X-n200-k36 58578	LKH	4.3	59329	59984.3	0	39	58919	59174.2	0	405	58643	<b>58927.5</b>	0
	NeuroLKH		59229	<b>59803.9</b>	0		58844	<b>59104.6</b>	0		58694	58937.4	0

Table S.6: CVRPLIB results (continued)

Name	Method	LKH with 100 trials as time limit				LKH with 1000 trials as time limit				LKH with 10000 trials as time limit			
		Time	Best	Avg	Suc	Time	Best	Avg	Suc	Time	Best	Avg	Suc
X-n204-k19 19565	LKH	0.6	19795	20159.5	0	5	19718	19880.5	0	49	19662	19777.9	0
	NeuroLKH		19794	<b>20076.7</b>	0		19692	<b>19857.7</b>	0		19583	<b>19776.3</b>	0
X-n209-k16 30656	LKH	0.9	31259	31648.1	0	9	30818	31214.9	0	93	30700	31028.9	0
	NeuroLKH		31163	<b>31555.8</b>	0		30864	<b>31140.2</b>	0		30722	<b>30969.3</b>	0
X-n214-k11 10856	LKH	2.6	11727	12131.2	0	23	11147	<b>11487.5</b>	0	229	10974	<b>11182.9</b>	0
	NeuroLKH		11702	<b>12128.5</b>	0		11235	11498.2	0		10988	11214.2	0
X-n219-k73 117595	LKH	1.4	117821	118242.7	0	10	117595	117790.2	1	101	117595	117684.3	3
	NeuroLKH		117046	<b>117998.3</b>	0		117622	<b>117733.2</b>	0		117595	<b>117654.8</b>	4
X-n223-k34 40437	LKH	1.4	41250	41880.6	0	12	40766	41087.1	0	127	40560	<b>40818.7</b>	0
	NeuroLKH		41066	<b>41662.6</b>	0		40641	<b>41022.8</b>	0		40563	40821.3	0
X-n228-k23 25742	LKH	1.6	26051	<b>26541.4</b>	0	15	25863	26037.7	0	150	25781	25910.4	0
	NeuroLKH		26067	26614.9	0		25835	<b>26030.5</b>	0		25791	<b>25907.7</b>	0
X-n233-k16 19230	LKH	0.5	19615	19885.4	0	4	19379	19599.1	0	39	19305	19477.2	0
	NeuroLKH		19499	<b>19831.7</b>	0		19381	<b>19597.4</b>	0		19324	<b>19473.2</b>	0
X-n237-k14 27042	LKH	0.8	27381	27829.6	0	7	27164	27406.5	0	65	27050	27276.5	0
	NeuroLKH		27324	<b>27789.5</b>	0		27124	<b>27402.8</b>	0		27042	<b>27240.0</b>	1
X-n242-k48 82751	LKH	2.2	84353	85218.4	0	19	83419	83826.9	0	198	83045	83401.3	0
	NeuroLKH		84090	<b>84685.6</b>	0		83299	<b>83743.7</b>	0		83042	<b>83357.2</b>	0
X-n247-k50 37274	LKH	2.8	37681	38206.5	0	26	37353	37701.6	0	280	37289	37457.1	0
	NeuroLKH		37629	<b>38118.3</b>	0		37326	<b>37638.8</b>	0		37292	<b>37454.3</b>	0
X-n251-k28 38684	LKH	1.3	39394	39831.8	0	11	39010	39274.9	0	117	38838	<b>39067.3</b>	0
	NeuroLKH		39277	<b>39720.2</b>	0		38988	<b>39259.8</b>	0		38887	39069.3	0
X-n256-k16 18839	LKH	2.4	19931	20953.7	0	17	19150	19519.9	0	148	18926	19164.9	0
	NeuroLKH		19681	<b>20730.2</b>	0		19046	<b>19433.9</b>	0		18889	<b>19143.3</b>	0
X-n261-k13 26558	LKH	1.2	27395	27891.3	0	13	26966	27367.3	0	150	26686	27104.9	0
	NeuroLKH		27174	<b>27746.3</b>	0		26749	<b>27308.2</b>	0		26661	<b>27074.4</b>	0
X-n266-k58 75478	LKH	4.1	77457	78371.5	0	35	76117	76718.3	0	359	75803	76193.4	0
	NeuroLKH		76864	<b>77879.7</b>	0		76175	<b>76582.7</b>	0		75876	<b>76187.2</b>	0
X-n270-k35 35291	LKH	1.7	35999	36580.2	0	14	35513	35870.2	0	142	35407	35613.1	0
	NeuroLKH		35808	<b>36425.9</b>	0		35509	<b>35817.5</b>	0		35424	<b>35598.2</b>	0
X-n275-k28 21245	LKH	0.7	21455	21784.3	0	5	21304	21524.7	0	50	21245	<b>21422.8</b>	1
	NeuroLKH		21515	<b>21715.7</b>	0		21320	<b>21512.0</b>	0		21281	21424.8	0
X-n280-k17 33503	LKH	2.2	34230	34932.1	0	22	33790	34218.8	0	229	33633	33943.8	0
	NeuroLKH		34071	<b>34844.4</b>	0		33699	<b>34178.3</b>	0		33632	<b>33943.0</b>	0
X-n284-k15 20215	LKH	0.7	20917	<b>21194.0</b>	0	7	20580	20862.8	0	76	20381	20655.2	0
	NeuroLKH		20903	21199.4	0		20609	<b>20849.8</b>	0		20455	<b>20639.5</b>	0
X-n289-k60 95151	LKH	5.9	97877	99666.8	0	53	96381	97129.8	0	557	95687	96226.0	0
	NeuroLKH		97731	<b>99084.0</b>	0		96163	<b>96998.4</b>	0		95754	<b>96154.4</b>	0
X-n294-k50 47161	LKH	2.1	48490	49351.2	0	17	47575	48009.5	0	176	47381	47644.4	0
	NeuroLKH		48093	<b>48990.2</b>	0		47550	<b>47914.8</b>	0		47354	<b>47616.2</b>	0
X-n298-k31 34231	LKH	1.7	35568	36543.4	0	12	34732	35199.4	0	121	34343	34764.9	0
	NeuroLKH		35380	<b>36292.7</b>	0		34656	<b>35113.6</b>	0		34320	<b>34763.9</b>	0

Table S.7: Solomon benchmark results

		LKH with 100 trials as time limit				LKH with 1000 trials as time limit				LKH with 10000 trials as time limit			
Name	Method	Time	Best	Avg	Suc	Time	Best	Avg	Suc	Time	Best	Avg	Suc
R201 1252372	LKH NeuroLKH	0.6	1252372 1253210	1275464.5 <b>1271983.1</b>	1 0	4	1252372 1252372	1258897.3 <b>1257114.3</b>	8 5	35	1252372 1252372	1254027.4 <b>1253575.6</b>	21 11
R202 1191698	LKH NeuroLKH	4.5	1195297 1193776	1234362.3 <b>1221507.1</b>	0 0	33	1191698 1191698	1207016.3 <b>1204530.3</b>	19 31	283	1191698 1191698	1197334.8 <b>1193964.5</b>	82 87
R203 939504	LKH NeuroLKH	2.1	947357 943363	964214.4 <b>957044.9</b>	0 0	16	941996 941405	948987.2 <b>947506.7</b>	0 0	143	939504 939504	943864.1 <b>943832.2</b>	6 3
R204 825510	LKH NeuroLKH	4.9	836241 838945	879723.0 <b>875614.2</b>	0 0	37	829440 825510	<b>846041.2</b> 846814.1	0 1	320	825510 825510	838430.7 <b>837939.1</b>	2 8
R205 994429	LKH NeuroLKH	1.4	994429 1003685	1046294.2 <b>1038416.6</b>	1 0	11	994429 994429	1024682.4 <b>1022870.4</b>	8 6	95	994429 994429	1014571.8 <b>1009598.9</b>	40 45
R206 906145	LKH NeuroLKH	1.5	913333 913333	942722.6 <b>940668.2</b>	0 0	12	909820 906145	926079.9 <b>925617.6</b>	0 2	104	906145 906145	918597.5 <b>918009.4</b>	19 24
R207 890608	LKH NeuroLKH	6.0	908532 903583	965102.0 <b>956950.3</b>	0 0	51	894793 893384	929064.0 <b>924560.7</b>	0 0	445	890608 890608	915756.4 <b>913160.8</b>	1 5
R208 726817	LKH NeuroLKH	2.0	726817 727258	751164.7 <b>744832.5</b>	2 0	15	726817 726817	736114.7 <b>733925.3</b>	9 6	125	726817 726817	731161.9 <b>730790.3</b>	16 9
R209 909158	LKH NeuroLKH	1.7	918711 914609	946581.2 <b>935769.5</b>	0 0	14	913141 909158	927854.0 <b>922974.0</b>	0 2	113	909158 909158	920110.0 <b>917506.1</b>	7 9
R210 939373	LKH NeuroLKH	1.7	951624 939373	979061.5 <b>967980.8</b>	0 1	13	939373 939373	959573.6 <b>955815.6</b>	1 9	114	939373 939373	953584.1 <b>950722.1</b>	20 39
R211 890930	LKH NeuroLKH	5.1	910853 909830	963151.8 <b>956837.2</b>	0 0	44	893168 892988	926350.7 <b>923050.2</b>	0 0	378	890930 890930	914120.6 <b>912125.8</b>	2 2