Bayesian Policy Optimization for Model Uncertainty

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Abstract

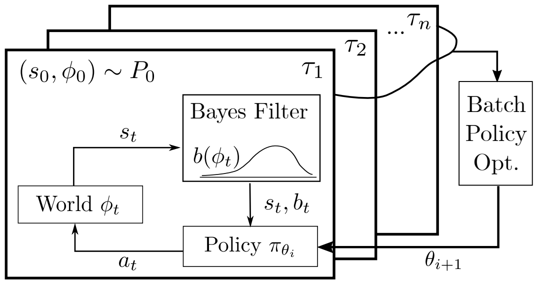
Addressing uncertainty is critical for autonomous systems to robustly adapt to the real world. We formulate the problem of model uncertainty as a continuous Bayes-Adaptive Markov Decision Process (BAMDP), where an agent maintains a posterior distribution over the latent model parameters given a history of observations and maximizes its expected long-term reward with respect to this belief distribution. Our algorithm, Bayesian Policy Optimization, builds on recent policy optimization algorithms to learn a universal policy that navigates the exploration-exploitation trade-off to maximize the Bayesian value function. To address challenges from discretizing the continuous latent parameter space, we propose a policy network architecture that independently encodes the belief distribution from the observable state. Our method significantly outperforms algorithms that address model uncertainty without explicitly reasoning about belief distributions, and is competitive with state-of-the-art Partially Observable Markov Decision Process solvers.

1 Introduction

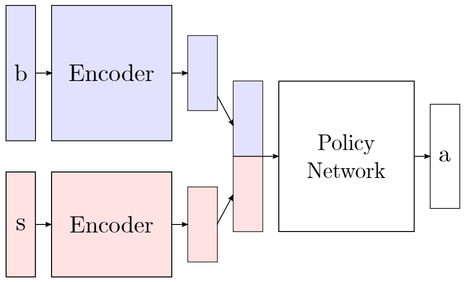
At its core, real-world robotics is about operating under uncertainty. An autonomous car must drive alongside human drivers it has never met, under road conditions which are different from the day before. An assistive home robot must simultaneously infer a user's intended goal as it is helping them. A robot arm must recognize and manipulate various objects. All these examples share a few common themes: (1) an underlying dynamical system with unknown *latent parameters* (road conditions, human goals, object identities), (2) an agent that can probe the system via *exploration*, while ultimately (3) maximizing the expected long-term reward via *exploitation*.

The Bayes-Adaptive Markov Decision Process (BAMDP) framework (Ghavamzadeh et al., 2015) elegantly captures the exploration-exploitation dilemma that the agent faces. Here, the agent maintains a *belief*, which is a posterior distribution over the latent parameters ϕ given a history of observations. A BAMDP can be cast as a Partially Observable Markov Decision Process (POMDP) (Duff & Barto, 2002) whose state is (s, ϕ), where s corresponds to the observable world state. By planning in the belief space of this POMDP, the agent balances explorative and exploitative actions. In this paper, we focus on BAMDP problems in which the latent parameter space is either a discrete finite set or a bounded continuous set that can be approximated via discretization. For this class of BAMDPs, the belief is a categorical distribution, allowing us to represent the belief with a vector of weights.

The core problem for BAMDPs with continuous state-action spaces is the exploration of the reachable belief space. In particular, discretizing the latent space can result in an arbitrarily large belief vector, which causes the belief space to grow exponentially. Approximating the value function over the reachable belief space is challenging: although point-based value approximations (Kurniawati et al., 2008; Pineau et al., 2003) have been largely successful for approximating value functions of discrete POMDP problems, these approaches do not easily extend to continuous state-action spaces. Monte-Carlo Tree Search approaches (Silver & Veness, 2010; Guez et al., 2012) are also prohibitively expensive in continuous state-action spaces: the width of the search tree after a single iteration is too large, preventing an adequate search depth from being reached.



(a) Training procedure



(b) Network structure

Figure 1: An overview of Bayesian Policy Optimization. The policy is simulated on multiple latent models. At each timestep of the simulation, a black-box Bayes filter updates the posterior belief and inputs the state-belief to the policy (Figure 1a). Belief and state are independently encoded before being pushed into the policy network (Figure 1b).

Our key insight is that we can bypass learning the value function and directly learn a policy that maps beliefs to actions by leveraging the latest advancements in batch policy optimization algorithms (Schulman et al., 2015; 2017). Inspired by previous approaches where the learning algorithms are trained with an ensemble of models (Rajeswaran et al., 2017; Yu et al., 2017), we examine model uncertainty through the lens of the BAMDP framework. Although our approach provides only locally-optimal policies, we believe that it offers a practical and scalable solution for continuous BAMDPs.

Our method, which we refer to as Bayesian Policy Optimization (BPO), is a batch policy optimization method which utilizes a black-box Bayesian filter and augmented state-belief representation. During the offline training period, BPO simulates the policy on multiple latent models sampled from the source distribution (Figure 1a). At each timestep of the simulation, it computes the posterior belief with a Bayes filter and inputs the state-belief pair (s, b) to the policy. Our algorithm only needs to update the posterior along the simulated trajectory in each sampled model, rather than branching at each possible action and observation as in MCTS-based approaches.

As the latent space is discretized more finely and the size of the belief vector grows, naïvely concatenating the observable state and belief to produce the policy input results in degraded performance. To address this challenge, we introduce two networks to independently encode belief and observable state (Figure 1b). These equal-sized lower-dimensional embeddings are concatenated and pushed into the policy network, and the encoder networks and policy network are jointly trained.

Our key contribution is the following. We introduce a Bayesian policy optimization algorithm to learn policies that directly reason about model uncertainty while maximizing the expected long-term reward (Section 4). To address the challenge of large belief representations, we introduce two encoder networks to balance the size of belief and state embeddings in the policy network. In addition, we show that although our method is designed for BAMDPs, it can be applied to continuous POMDPs when a compact belief representation is available (Section 5). Through experiments on classical POMDP problems and BAMDP variants of OpenAI Gym benchmarks, we show that BPO significantly outperforms algorithms that address model uncertainty without explicitly reasoning about beliefs and is competitive with state-of-the-art POMDP algorithms (Section 6).

2 Preliminaries: Bayesian Reinforcement Learning

The Bayes-Adaptive Markov Decision Process framework (Duff & Barto, 2002; Ross et al., 2008; Kolter & Ng, 2009) was originally proposed to address uncertainty in the transition function of an MDP. The uncertainty is captured by a latent variable $\phi \in \Phi$ which parametrizes the transition function, e.g. $\phi_{sas'} = T(s, a, s')$, or a more general parameterization, e.g. physical properties of the system driving the MDP. The latent variable is either fixed or has a known transition function. We extend the previous formulation of ϕ to address uncertainty in the reward function as well.

Formally, a BAMDP is defined by a tuple $\langle S, \Phi, A, T, R, P_0, \gamma \rangle$, where S is the observable state space of the underlying MDP, Φ is the latent space, and A is the action space. T and R are the parameterized transition and reward functions, respectively. The transition function is defined as the following: $T(s, \phi, a, s', \phi') = P(s', \phi'|s, \phi, a) = P(s'|s, \phi, a)P(\phi'|s, \phi, a, s')$. Lastly, the initial distribution over (s, ϕ) is given by $P_0 : S \times \Phi \to \mathbb{R}^+$, and γ is the discount.

Bayesian Reinforcement Learning (BRL) considers the long-term expected reward with respect to the uncertainty over ϕ, rather than the true (unknown) value of ϕ. The uncertainty is represented as a *belief distribution* $b \in B$ over latent variables ϕ. BRL maximizes the following Bayesian value function, which is the expected value *given the uncertainty*:

$$\begin{aligned} V_\pi(s, b) &= R(s, b, a) + \gamma \sum_{s' \in S, b' \in B} P(s', b'|s, b, a) V_\pi(s', b') \\ &= R(s, b, a) + \gamma \sum_{s' \in S, b' \in B} P(s'|s, b, a) P(b'|s, b, a, s') V_\pi(s', b') \end{aligned} \tag{1}$$

where the action is $a = \pi(s, b)$.[1]

The Bayesian reward and transition functions are defined in expectation with respect to ϕ: $R(s, b, a) = \sum_{\phi \in \Phi} b(\phi) R(s, \phi, a)$, $P(s'|s, b, a) = \sum_{\phi \in \Phi} b(\phi) P(s'|s, \phi, a)$. The belief distribution can be maintained recursively: starting from the initial belief b_0, a Bayes filter performs the posterior update from b to b':

$$b'(\phi'|s, b, a, s') = \eta \sum_{\phi \in \Phi} b(\phi) T(s, \phi, a, s', \phi')$$

where η is the normalizing constant. The belief $b_t(\phi)$ is the posterior distribution over ϕ given $(s_0, a_1, s_1, ..., s_t)$, the history of states and actions.

The use of (s, b) casts the partially-observable BAMDP as a fully-observable MDP in belief space, which allows any policy gradient method to be applied. However, we highlight that a reactive Bayesian policy in belief space is equivalent to a policy with memory in observable space (Kaelbling et al., 1998). In our work, the complexity of memory is delegated to a Bayes filter that computes a sufficient statistic of the history.

2.1 Partially Observable Markov Decision Processes and Mixed-Observability MDPs

A Partially Observable Markov Decision Process (POMDP) problem is the tuple $\langle S, A, O, T, Z, R, P_0, \gamma \rangle$ where S, A, and O are the state, action, and observation space, respectively. T is the state transition function, $T(s, a, s') = P(s'|s, a)$, and Z is the observation function, $Z(s, a, o) = P(o|s, a)$, of observing o after taking action a at state s. P_0 is the initial state distribution.

Mixed-observability MDPs (MOMDP) (Ong et al., 2010) are similar to BAMDPs: their states are (s, ϕ) where s is observable and ϕ is latent. Although any BAMDP problem can be cast as a POMDP or a MOMDP problem (Duff & Barto, 2002), the source of uncertainty in a BAMDP is usually from the transition function rather than the unobservability of the state as with MOMDPs and POMDPs.

3 Related Work

There is a long history of research in belief-space reinforcement learning and robust reinforcement learning. We highlight the works that are most relevant to our approach, and refer the reader to Ghavamzadeh et al. (2015) and Shani et al. (2013); Aberdeen (2003) for more comprehensive reviews of the Bayes-Adaptive and Partially Observable MDP literatures.

[1] The state space S can be either discrete or continuous and the belief space B is always continuous, but we use $\sum$ notation for notational simplicity.

Belief-Space Reinforcement Learning Planning in belief space, where part of the state representation is a belief distribution, is intractable (Papadimitriou & Tsitsiklis, 1987). This is a consequence of the curse of dimensionality: the dimensionality of belief space over a finite set of variables is equal to the size of that set, so the size of belief space grows exponentially. Many approximate solvers focus on one or more of the following: 1) value function approximation, 2) compact, approximate belief representation, or 3) direct mapping of belief to an action. QMDP (Littman et al., 1995) assumes full observability after one step to approximate Q-value. Point-based solvers like SARSOP (Kurniawati et al., 2008) and PBVI (Pineau et al., 2003) exploit the piecewise-linear-convex structure of POMDP value functions (under mild assumptions) to approximate the value of a belief state. Sampling-based approaches such as BAMCP (Guez et al., 2012) and POMCP (Silver & Veness, 2010) combine Monte-Carlo sampling and simple rollout policies to approximate Q-values at the root node in a search tree. Except for QMDP, these approaches target discrete POMDPs and cannot be easily extended to continuous spaces. Sunberg & Kochenderfer (2018) extend POMCP to continuous spaces by utilizing double progressive widening. Model-based trajectory optimization methods (Platt et al., 2010; van den Berg et al., 2012) have also been successful for navigation on systems like unmanned aerial vehicles and other mobile robots.

Neural-network variants of POMDP algorithms are well suited for compressing high-dimensional belief states into compact representations. For example, QMDP-Net (Karkus et al., 2017) jointly trains a Bayes-filter network and a policy network to approximate Q-value. Our method is closely related to Exp-GPOMDP (Aberdeen & Baxter, 2002), a model-free policy gradient method for POMDPs, but we take advantage of model knowledge from the BAMDP and revisit the underlying policy optimization method with recent advancements.

Robust (Adversarial) Reinforcement Learning One can bypass the burden of maintaining belief and still find a robust policy by maximizing the return for worst-case scenarios. Commonly referred to as Robust Reinforcement Learning (Morimoto & Doya, 2001), this line of work utilizes a min-max objective and is conceptually equivalent to H-infinity control (Başar & Bernhard, 2008) from classical robust control. Recently, this objective has often translated to training the agent against various external disturbances and adversarial scenarios, with which several algorithms (Pinto et al., 2017; Bansal et al., 2018; Pattanaik et al., 2018) have successfully produced robust agents with complex behaviors. Instead of training against an adversary, the agent can train with an ensemble of models to be robust against model uncertainty. For example, Ensemble Policy Optimization (EPOpt) (Rajeswaran et al., 2017) trains the agent on multiple MDPs and focuses on improving worst-case performance by concentrating rollouts on MDPs where the current policy performs poorly. Ensemble-CIO (Mordatch et al., 2015) optimizes trajectories across a finite set of MDPs.

While adversarial and ensemble model approaches have been shown to be robust even to unmodeled effects, when the worst-case scenario is too extreme, these approaches may result in an overly conservative behavior. In addition, since these methods do not infer or utilize uncertainty, they tend to perform poorly when explicit information-gathering actions are required. Our approach is fundamentally different from these approaches because it internally maintains a belief distribution. As a result, it is capable of producing policies that outperform robust policies in many scenarios.

Adaptive Policy Methods Some approaches can adapt to changing model estimates without operating in belief space. Adaptive-EPOpt (Rajeswaran et al., 2017) retrains the agent with an updated source distribution after some interactions with real world. PSRL (Osband et al., 2013) samples from a source distribution, executes an optimal policy for the sample for a fixed horizon, and then re-samples from an updated source distribution. These approaches can work well for scenarios in which the latent MDP is fixed throughout multiple episodes. Universal Policy with Online System Identification (UP-OSI) (Yu et al., 2017) learns to predict the maximum likelihood estimate ϕ_{MLE} and trains a universal policy that maps (s, ϕ_{MLE}) to an action. However, without a notion of belief, both PSRL and UP-OSI can over-confidently execute policies that are optimal for the single estimate, which may result in poor performance in expectation over different MDPs.

Algorithm 1 Bayesian Policy Optimization

Require: Bayes filter $\psi(\cdot)$, initial belief $b_0(\phi)$, P_0, policy π_{θ_0}, horizon H, $n_{\text{itr}}, n_{\text{sample}}$

```
1: for $i = 1, 2, \cdots, n_{\text{itr}}$ do
2:     for $n = 1, 2, \cdots, n_{\text{sample}}$ do
3:         Sample latent MDP $M$: $(s_0, \phi_0) \sim P_0$
4:         $\tau_n \leftarrow \texttt{Simulate}(\pi_{\theta_{i-1}}, b_0, \psi, M, H)$
5:     Update policy: $\theta_i \leftarrow \texttt{BatchPolicyOptimization}(\theta_{i-1}, \{\tau_1, \cdots, \tau_{n_{\text{sample}}}\})$
6: return $\pi_{\theta_{best}}$

7: procedure SIMULATE$(\pi, b_0, \psi, M, H)$
8:     for $t = 1, \cdots, H$ do
9:         $a_t \leftarrow \pi(s_{t-1}, b_{t-1})$
10:        Execute $a_t$ on $M$, observing $r_t, s_t$
11:        $b_t \leftarrow \psi(s_{t-1}, b_{t-1}, a_t, s_t)$
12:    return $(s_0, b_0, a_1, r_1, s_1, b_1, \cdots, a_H, r_H, s_H, b_H)$
```

4 Bayesian Policy Optimization

We propose Bayesian Policy Optimization, a simple policy gradient algorithm for BAMDPs (Algorithm 1). The agent learns a stochastic Bayesian policy that maps a state-belief pair to a probability distribution over actions $\pi : S \times B \rightarrow P(A)$. During each training iteration, BPO collects trajectories by simulating the current policy on several MDPs sampled from the prior distribution. During the simulation, the Bayes filter updates the posterior belief distribution at each timestep, and the updated state-belief pair is provided to the Bayesian policy. By simulating on MDPs with different latent variables, BPO observes the evolution of the state-belief throughout multiple trajectories. Since the state-belief representation makes the partially observable BAMDP a fully observable Belief-MDP, any batch policy optimization algorithm (e.g. Schulman et al. (2015; 2017)) can be used to maximize the Bayesian Bellman equation (Equation 1).

We represent the policy with a deep neural network. To address the high-dimensionality of belief space, we introduce two separate networks to independently encode state and belief (Figure 1b). The belief encoder and state encoders are composed of multiple layers of nonlinear (e.g. ReLU) and linear operations, and output a compact representation of belief and state, respectively. We construct the two encoders such that their outputs are the same size, then concatenate them as an input to the policy network. The encoder networks and policy network are jointly trained by the batch policy optimization.

As with most policy gradient algorithms, BPO provides only a locally optimal solution. Nonetheless, BPO is capable of producing robust policies that scale to problems with high-dimensional observable states and beliefs, as we empirically verify in Section 6.

5 Generalization to POMDP

Although Bayesian Policy Optimization is designed for BAMDP problems, it can naturally be applied to POMDPs. In a general POMDP where the state is unobservable, we only need $b(s)$, so we remove the state encoder network.

Knowing the transition and observation functions, we can construct a Bayes filter which computes the belief b over the hidden state:

$$b_t(s_t) = \psi(b_{t-1}, a_t, o_t) = \eta \sum_{s_{t-1} \in S} b_{t-1}(s_{t-1}) T(s_{t-1}, a_t, s_t) Z(s_{t-1}, a_t, o_t)$$

where η is the normalization constant and Z is the observation function as described in Section 2. Then, BPO optimizes the following Bellman equation:

$$V_\pi(b) = \sum_{s \in S} b(s) R(s, \pi(b)) + \gamma \sum_{b' \in B} P(b'|b, \pi(b)) V_\pi(b')$$

For general POMDPs with large state spaces, however, discretizing the state space to form the belief state is impractical. We believe that this generalization is best suited for beliefs with conjugate distributions, e.g. Gaussians.

6 Experimental Results

We evaluate Bayesian Policy Optimization on discrete and continuous POMDP benchmarks to highlight the usage of information gathering actions. We also evaluate BPO on BAMDP problems constructed by varying physical model parameters on OpenAI benchmark problems (Brockman et al., 2016).

We compare BPO mainly against EPOpt and UP-MLE, robust and adaptive policy gradient algorithms, respectively. For UP-MLE, we have replaced the online system identification (OSI) network in UP-OSI, which learns to predict the maximum likelihood estimate (MLE) of ϕ via supervised learning, with the MLE estimate from the same Bayes filter we use for BPO. This allows us to directly compare the performance when a full belief distribution is used (BPO) rather than a point estimate (UP-MLE). For the OpenAI BAMDP problems, we also compare with a policy trained with TRPO on an environment with the mean values of the initial belief distribution over latent parameters.

All the policy gradient algorithms (BPO, EPOpt, UP-MLE) use TRPO as the underlying batch policy optimization subroutine. We used the implementation of TRPO provided by Duan et al. (2016). For all algorithms, we compare the results from the seed with the highest mean reward across multiple random seeds (Table 1). Although these are the most relevant algorithms which utilize batch policy optimization to address model uncertainty, we emphasize that none of them formulate the problems as BAMDPs.

Note that for all BAMDP problems with continuous latent spaces (`Chain` and `MuJoCo`), the latent parameters are sampled in the continuous space, regardless of discretization.

Tiger (Discrete POMDP) In the `Tiger` problem, originally proposed by Kaelbling et al. (1998), a tiger is hiding behind one of two doors. The agent chooses between three actions: listen, or open one of the two doors. When the agent listens, it receives a noisy observation of the tiger's position. If the agent opens the door and reveals the tiger, it receives a penalty of -100. Opening the door without the tiger results in a reward of 10. Listening incurs a penalty of -1. In this problem, the optimal agent listens until its belief about which door the tiger is behind is substantially higher than the other. Chen et al. (2016) show that `Tiger` can be converted into a BAMDP problem with two latent states, one for each position of the tiger.

The difference in performance (Table 1) demonstrates the benefit of planning in state-belief space when information gathering is required to reduce model uncertainty. Since the EPOpt policy does not maintain a belief distribution, it only sees the most recent observation. Without the full history of observations, EPOpt only learns that opening doors is risky; because it expects the worst-case scenario, it always chooses to listen. UP-MLE leverages all past observations to estimate the tiger's position. However, without the full belief distribution, the policy cannot account for the confidence of the estimate. Once there is a higher probability of the tiger being on one side, the UP-MLE policy prematurely chooses to open the safer door. Despite this bad decision, providing the MLE estimate to the policy results in higher performance than EPOpt. BPO significantly outperforms both of these algorithms, learning to listen until it is extremely confident in the tiger's location. In fact, BPO nearly achieves the approximately-optimal return found by SARSOP (19.0 ± 0.6), a state-of-the-art offline POMDP solver that approximates the optimal value function rather than performing policy optimization (Kurniawati et al., 2008).

	BPO	BPO-	EPOpt	UP-MLE	TRPO
Tiger	**17.9** ± 0.6	-	-19.9 ± 0.0	-9.8 ± 2.0	-
Chain-3	**259.4** ± 12.2	**269.5** ± 12.7	**267.9** ± 13.1	242.0 ± 11.2	-
Chain-10	**363.6** ± 15.1	352.5 ± 15.7	267.9 ± 13.1	**378.2** ± 15.7	-
Chain-1000	**342.4** ± 15.2	223.0 ± 9.4	267.9 ± 13.1	**342.4** ± 14.9	-
LightDark	**-166.7** ± 2.4	-	-1891.2 ± 45.0	-745.9 ± 22.3	-
Cheetah	**115.6** ± 3.5	109.13 ± 3.1	107.0 ± 2.7	108.9 ± 3.3	64.3 ± 6.1
Swimmer	36.0 ± 0.4	**36.9** ± 0.6	27.9 ± 0.4	**37.6** ± 0.4	29.4 ± 0.6
Ant	**117.0** ± 2.7	**115.9** ± 3.9	112.5 ± 3.1	111.7 ± 2.5	**116.5** ± 7.5

Table 1: Comparison of the 95% confidence intervals of average return for BPO and other benchmark algorithms across all environments. The algorithms with highest average return on each environment are in bold, with multiple algorithms selected if the intervals overlap. BPO achieves the highest return on seven of the eight environments. The combined result of BPO and BPO- achieves the highest return on all environments.

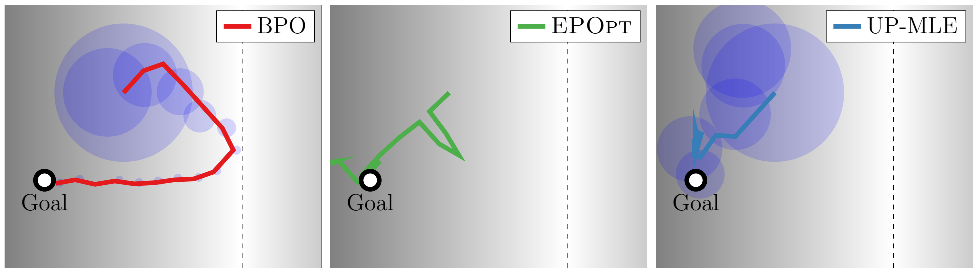


Figure 2: Visualization of different algorithms on the `LightDark` environment. The dashed line indicates the light source, $x = 5$. Blue circles are one standard deviations for per-step estimates. The BPO policy moves toward the light to obtain a better state estimate before moving toward the goal.

Chain (Discrete BAMDP) To evaluate the usefulness of the independent encoder networks, we consider a variant of the `Chain` problem (Strens, 2000). The original problem is a discrete MDP with five states $\{s_i\}_{i=1}^5$ and two actions $\{A, B\}$. Taking action A in state s_i transitions to s_{i+1} with no reward; taking action A in state s_5 transitions to s_5 with a reward of 10. Action B transitions from any state to s_1 with a reward of 2. However, these actions are noisy: in the canonical version of `Chain`, the opposite action is taken with slip probability 0.2. In our variant, the slip probability is uniformly sampled from $[0, 1.0]$ at the beginning of each episode.[2] In our algorithm, we discretize the parameter space with 3, 10, and 1000 uniformly-spaced samples. The state is represented with a one-hot encoding.

BPO- is the version of our algorithm that directly feeds the original state and belief representations to the policy network. At coarser discretizations (3, 10), there is little difference between BPO and BPO-. However, with a large discretization (1000), the performance of BPO- degrades significantly, while BPO maintains comparable performance. Interestingly, the performance of all algorithms degrade if the discretization is too fine. This level of discretization seems to make the problem unnecessarily complex given its simple structure. In this problem, either action provides equal information about the latent parameter. Since active information-gathering actions do not exist, we expect BPO and UP-MLE to achieve similar performance.

Light-Dark (Continuous POMDP) We consider a variant of the `LightDark` problem proposed by Platt et al. (2010), where an agent tries to reach a known goal location while being uncertain about its own position. At each timestep, the agent receives a noisy observation of its location. In our problem, the line $x = 5$ is a light source; the farther the agent is from the light, the noisier its observations. The agent must decide either to reduce uncertainty by

[2] A similar variant has been introduced in Wang et al. (2012).

moving closer to the light, or exploit by moving from its estimated position to the goal. We refer the reader to the appendix for details about the rewards and observation noise model.

This example demonstrates how BPO can be applied to general continuous POMDPs (Section 5). The latent state is the continuous pose of the agent. For this example, we parameterize the belief as a Gaussian distribution and perform the posterior update with an Extended Kalman Filter as in Platt et al. (2010).

Figure 2 compares sample trajectories from different algorithms on the `LightDark` environment. Based on its initial belief, the BPO policy moves toward $x = 5$ to acquire less noisy observations. As it becomes more confident in its position estimate, it changes direction toward the light then moves straight to the goal. Both EPOpt and UP-MLE move straight to the goal.

MuJoCo (Continuous BAMDP) Finally, we evaluate the algorithms on a set of simulated benchmarks from OpenAI Gym (Brockman et al., 2016) using the MuJoCo physics simulator (Todorov et al., 2012): `HalfCheetah`, `Swimmer`, and `Ant`. Each environment has several latent physical parameters that can be changed to form a BAMDP. We refer the reader to the appendix for details regarding the model variation and belief parameterization.

The MuJoCo benchmarks demonstrate the robustness of BPO in the face of model uncertainty. For each environment, BPO learns a universal policy that adapts to the changing belief over the latent parameters. Table 2 provides a more in-depth comparison of the long-term expected reward achieved by each algorithm. Across the three MuJoCo benchmarks, BPO is least frequently the worst and most frequently the best of the algorithms (excluding TRPO). In particular, for the `HalfCheetah` environment, BPO has a higher average return than both EPOpt and UP-MLE for most MDPs. Although BPO is slightly worse than UP-MLE on `Swimmer`, we believe that this is largely due to random seeds, especially since BPO- matches UP-MLE's performance (Table 1).

We observe that qualitatively, all three algorithms were capable of producing agents with reasonable gaits. We believe that this is due to several reasons. First, the environments do not require active information-gathering actions to achieve a high reward. Furthermore, for deterministic systems with little noise, the belief collapses quickly; as a result, the MLE is as meaningful as the belief distribution. As demonstrated by Rajeswaran et al. (2017), a universally robust policy for these problems is capable of performing the task. Therefore, even algorithms that do not maintain a history of observations can perform well.

7 Discussion

Bayesian Policy Optimization is a practical and scalable approach for continuous BAMDP problems. We demonstrate that BPO learns policies that achieve comparable performance to state-of-the-art discrete POMDP solvers, while also outperforming state-of-the-art robust policy gradient algorithms that address model uncertainty without formulating it as a BAMDP problem. Our network architecture scales well with respect to the degree of latent parameter space discretization due to the independent encoding of state and belief. We highlight that BPO is agnostic to the choice of batch policy optimization subroutine. Although we have used TRPO in this work, we can take advantage of more recent policy optimization algorithms such as PPO (Schulman et al., 2017), while leveraging improvements in variance-reduction techniques (Weaver & Tao, 2001).

While BPO outperforms algorithms that do not explicitly reason about belief distributions, adaptive algorithms such as UP-MLE seem sufficient when explicit information-gathering actions are not needed. If all actions are informative (as with the MuJoCo and Chain environments) and the posterior belief distribution easily collapses into a unimodal distribution, UP-MLE provides a lightweight alternative. However, we emphasize that our Bayesian approach is necessary for environments where the uncertainty must actively be reduced.

Although BPO scales to a fine-grained discretizations of latent space, our experiments suggest that each problem has an optimal discretization level, beyond which further discretization may degrade performance. As a result, we may want to perform variable-resolution discretization

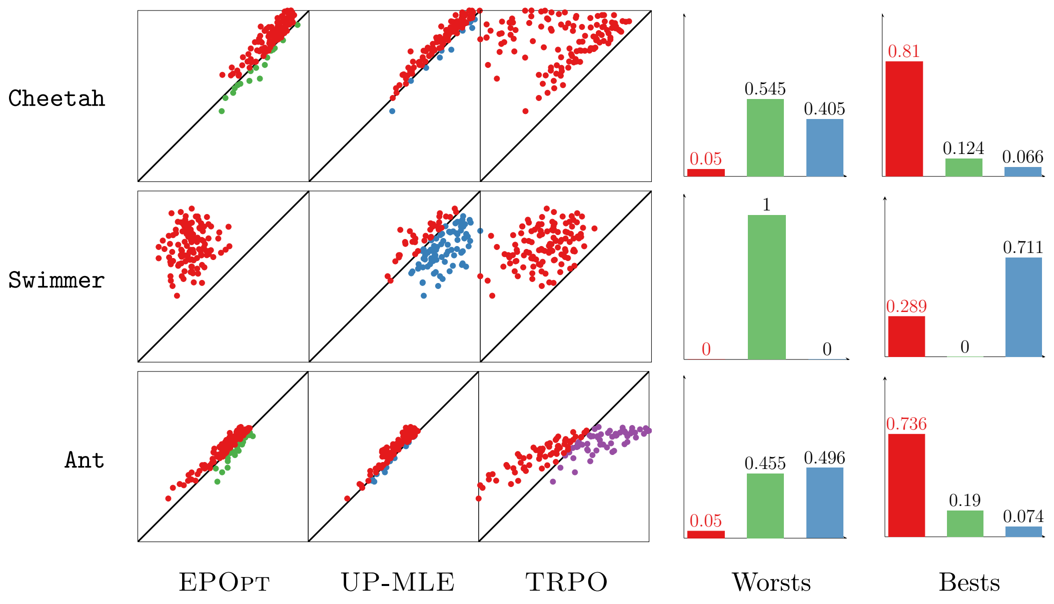


Table 2: Comparison of different algorithms on `MuJoCo` BAMDPs. In the pairwise scatter plots (left), each point represents an MDP, where the x-coordinate is the average long-term reward by a baseline algorithm and the y-coordinate is that by BPO. Points are colored according to which algorithm achieved a higher average return: BPO (red), EPOpt (green), UP-MLE (blue), or TRPO (purple). The diagonal black line is $y = x$; the farther a point is above the line $y = x$, the more BPO outperforms that baseline. The bar chart (right) illustrates the frequency that each of the algorithms produced the worst or best average long-term reward. TRPO is excluded since it only trains on the nominal environment.

rather than performing an extremely fine single-resolution discretization. Adapting iterative densification ideas previously explored in motion planning (Gammell et al., 2015) and optimal control (Munos & Moore, 1999) to the discretization of latent space may yield a more compact belief representation while enabling further improved performance.

Although our decision to discretize latent space is reasonable for BAMDPs where the latent space is typically smaller than the observable state space, this imposes a practical limit to the POMDPs that our approach generalizes to. Rather than deciding on a discretized belief representation and relying on a model-based Bayes filter to perform posterior updates, we can consider directly learning to map a history of observations to a lower-dimensional belief embedding, as proposed by QMDP-Net (Karkus et al., 2017). This would enable the policy to learn a meaningful belief embedding without losing information from our a priori choice of discretization.

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Appendix

Experimental Detail: `LightDark`

After each action, the agent receives a noisy observation of its location, which is sampled from a Gaussian distribution, $o \sim \mathcal{N}([x, y]^\top, w(x))$ where $[x, y]$ is the true location. The noise variance is a function of x and is minimized when $x = 5$: $w(x) = \frac{1}{2}(x - 5)^2 + \text{const.}$ There is no process noise.

The reward function is $r(s, a) = -\frac{1}{2}(\|s - g\|^2 + \|a\|^2)$, where s is the true agent position and g is the goal position. A large penalty of $-5000\|s_T - g\|^2$ is incurred if the agent does not reach the goal by the end of the time horizon, analogous to the strict equality constraint in the original optimization problem (Platt et al., 2010).

The initial belief is $[x, y, \sigma^2] = [2, 2, 2.25]$. During training, we randomly sample latent start positions from a rectangular region $[2, -2] \times [4, 4]$ and observable goal positions from [0,-2] $\times$ $[2, 4]$.

Experimental Detail: `MuJoCo`

For ease of analysis, we vary two parameters for each environment. For `HalfCheetah`, the front and back leg lengths are varied. For `Ant`, the two front leg lengths are varied. `Swimmer` has four body links, so the first two link lengths vary together according to the first parameter and the last two links vary together according to the second parameter. We chose to vary link lengths rather than friction or damping constant because a policy trained on a single nominal environment was capable of performing well across large variations in those parameters. All link lengths vary by up to 20%.

To construct a Bayes filter, the 2D-parameter space is discretized into a 5×5 grid with a uniform initial belief. We assume Gaussian noise on the observation, i.e. $o = f_\phi(s, a) + w$ with $w \sim \mathcal{N}(0, \sigma^2)$, with ϕ being the parameter corresponding to the center of each grid cell. It typically only requires a few steps for the belief to concentrate in a single cell of the grid, even when large σ^2 is assumed.