HYPERNCA: GROWING DEVELOPMENTAL NETWORKS WITH NEURAL CELLULAR AUTOMATA

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ABSTRACT

In contrast to deep reinforcement learning agents, biological neural networks are grown through a self-organized developmental process. Here we propose a new hypernetwork approach to grow artificial neural networks based on neural cellular automata (NCA). Inspired by self-organising systems and information-theoretic approaches to developmental biology, we show that our *HyperNCA* method can grow neural networks capable of solving standard reinforcement learning tasks. Finally, we explore how the same approach can be used to build developmental *metamorphosis networks* capable of transforming their weights to solve variations of the initial RL task.

1 INTRODUCTION

Reinforcement learning agents are predominantly trained either with policy gradient methods (Arulkumaran et al., 2017) or evolved with evolutionary strategies (Salimans et al., 2017) or genetic algorithms (Such et al., 2017; Risi & Stanley, 2019). In both of these approaches, the neural network weights are directly optimised, meaning that the search space is the weight space. An alternative approach to finding optimal policies are the so-called *indirect encodings* Stanley et al. (2019); Stanley & Miikkulainen (2003). Indirect encoding methods introduce an intermediate step into the agent training process which decouples the optimisation space from the policy weight space: instead of directly searching for optimal policy weights, we optimise a model whose output is the policy weights.

A well known example of indirect encoding found in nature is the genotype-phenotype relation between DNA sequences and the proteins they encode. For instance, we know that the information needed to grow a fully-functioning human brain must be contained in the human genome. However, DNA does not encode the final position of every neuron or the presence of every synapse. We know this simply because a back-of-the-envelope calculation shows that the amount of information in the human DNA sequence – approximately 1 gigabyte – would not be sufficient to explicitly encode the 10^{15} synapses present in the human brain. In other words, your brain adjacency matrix is nowhere to be found in your DNA (Zador, 2019).

Unlike RL agents, animals grow their neural circuitry through a developmental process in which neurons determine their synaptic connections solely based on local interactions. In biological system, the growth of the nervous system, as any other tissue, is governed by gene regulatory networks: a single set of rules encoded in the cells DNA which, depending on the cells state, will produce different developmental outcomes.

The fundamental insight acting as the foundation of this paper is the connection between the notion of *computational irreducibility* and biological developmental processes. Some dynamical systems —notably chaotic ones, but not only— may have the interesting co-occurrence of two properties: being deterministic and unpredictable simultaneously. In order to determine the state after n steps of such a system, one must evolve the system for n steps according to its dynamical equation; in other words, there does not exist an analytical expression describing the state of the system for any step n. Such systems are said to be computationally irreducible (Zwirn & Delahaye, 2011). It is conjectured that the developmental process of biological systems is computationally irreducible: the only way to resolve the final configuration of the neural circuitry of the brain is to let unfold the genetic information present in the DNA, a process named *algorithmic growth* (Hiesinger, 2021). The advantage of algorithmic growth consists in not requiring to explicitly codify each of the details about the final configuration of the system, instead, a small amount of information encoding the growth process is capable of giving rise to much bigger system purely through the self-organised local interactions of it dynamics.

Cellular automata (CA) are a family of computational models able to exhibit computational irreducibility (Wolfram, 2002) and which display complex dynamical patterns emerging exclusively from the local interaction of simple rules. Recently, neural cellular automata (NCA) which replace the update rule of the CA by a neural network have gained popularity due to their potential to being trained using modern deep learning tools.

Inspired by the ideas of algorithmic growth and computational irreducibility, we aim to grow neural networks using neural cellular automata to solve reinforcement learning tasks. We introduce *Hyper-NCAs*, NCAs acting as hypernetworks, and show that they are capable of producing policy networks with a significantly bigger number of parameters able to solve modern reinforcement learning tasks such as a discrete action environment (e.g. Lunar Lander) and a quadrupedal robot locomotion task. Unlike existing hypernetwork approaches (Ha et al., 2016; Stanley et al., 2009; Carvelli et al., 2020), our proposed indirect-encoding relies on the developmental process guiding the emergence of the final policy network. Finally, we propose the notion of *metamorphosis networks*: a NCA-based approach to morph neural networks such that the network information is preserved and re-used. We demonstrate how metamorphosis networks can be used to morph the weights of a policy network in order to adapt to differently damaged morphologies.

The goal of this work is not to reach the ever-elusive *state-of-the-art* at any given RL tasks, but rather, to introduce a new kind of indirect-encoding that puts forward the value of self-organisation and algorithmic growth to generate neural policies for reinforcement learning agents.

2 Related work

Indirect encodings. Indirect encodings are inspired by the biological process of mapping a genotype to a phenotype and have been primarily studied in the context of neuroevolution (Floreano et al., 2008) (i.e. evolving artificial neural networks) but more recently were also optimized through gradient-descent based approaches (Ha et al., 2016). Even before the success of deep RL, these methods have been successful in solving challenging car navigation tasks from pixels alone (Koutník et al., 2013).

A highly influential indirect encoding is HyperNEAT (Stanley et al., 2009). HyperNEAT employs an indirect encoding called compositional pattern producing networks (CPPNs) that *abstracts away the process of growth* and instead describes the connectivity of a neural network through a function of its geometry. An approach building on this idea, called HyperNetworks (Ha et al., 2016), has more recently shown that networks generating the weights of another network can also be trained end-to-end through gradient descent. Hypernetworks have been shown to be able to generate competitive CNNs (Zhmoginov et al., 2022) and RNNs in a variety of tasks while using a smaller set of trainable parameters. However, while both HyperNEAT and Hypernetworks are indirect encodings that have been applied to reinforcement learning problems (Stanley et al., 2009; Carvelli et al., 2020), they do not rely on the process of development over time, which is hypothesised to be an important ingredient of biological systems (Hiesinger, 2021).

Neural cellular automata (NCA). Cellular automata are a class of computational models whose outcomes emerge from the local interactions of simple rules. Introduced by Neumann et al. (1966) as a part of his quest to build a self-replicating machine or *universal constructor*, a CA consist of a lattice of computing cells which iteratively update their states based exclusively on their own state and the states of their local neighbours. On a classical CA, each cell’s state is represented by an integer and adjacent cells are considered neighbours. Critically, the update rule of a cellular automaton is identical for all the cells.

Wulff & Hertz (1992) showed that a recurrent single-layer neural networks could be trained with the delta-rule algorithm to approximate the dynamical patterns generated by chaotic one-dimensional CA. While Wulff and Hertz work was the first one to combine neural networks with cellular automata, they limited themselves to top-down study of classical CA’s dynamics with a neural network rather than replacing the CA rule by a neural neural network and studying their bottom-up emerging properties. Tavares et al. (2006) proposed a *Neuro-Cellular Automata* model which replaces the symbolic if-then rules of classical CA by a neural network and represents the states by real numbers rather than integers. However, similarly to Wulff & Hertz (1992), they limited themselves to showing that a small feedfoward NCA with only 8 neurons trained through backprop could reproduce one-dimensional CA patterns such as the Turing complete *Rule 110*. Gilpin (2018) formulated NCA as a convolutional network architecture and showed that they could model complex dynamical CA patterns such as Conway’s Game of Life. Critically, by formulating a NCA as a convolutional neural network, Gilpin enabled NCA to be easily trainable through backpropation exploiting the modern deep learning toolkit.

Mordvintsev et al. (2020) demonstrated how NCA can work as robust models of morphogenesis, i.e. the process by which single stem cells grow into fully-formed organisms. They represent the lattice space as a 16-dimensional grid with the first 3 channels representing the visible colours, one channel representing the living state of the cell, and the remaining hidden channels representing undefined quantities that the NCA can exploit to encode information. Their neural rule encompasses a convolutional architecture with Sobel filters as kernels which compute spatial gradients of the cells states, before feeding the resulting activations to feed-forward layers whose output determine the new state of each cell. Our work builds upon this architecture, and similarly to previous work by Sudhakaran et al. (2021), allowing the convolutional layers to be trained along with the rest of the network parameters.

Because of their emergent nature, CA and NCA are capable of generating complex dynamical patterns with few model parameters as shown by recent work (Kaiser & Sutskever, 2015; Mordvintsev et al., 2020; Ruiz et al., 2020; Sudhakaran et al., 2021).

3 Hyper Neural Cellular Automata

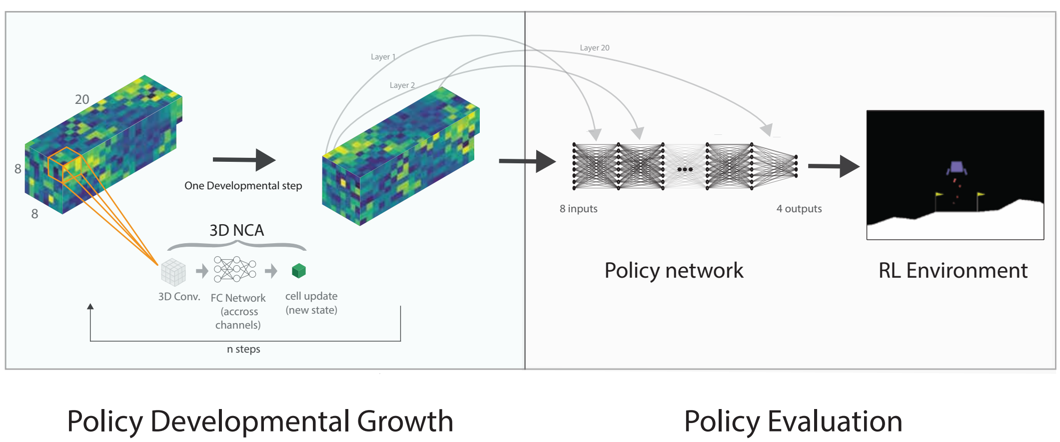

Figure 1: **Hyper Neural Cellular Automata (HyperNCA).** During the **Developmental Growth phase** *(left)*, an initial random seed is updated for a finite number of steps using a 3D Neural Cellular Automata (NCA). The seed can have a single or several channels, here a single channel is represented. **Policy evaluation** *(right)*: Once development is concluded, the first channel of the grown pattern is mapped to the weight matrix of a policy network which is evaluated on a reinforcement learning task —in this example safely landing the spacecraft on the ground. The size of the seed is chosen such that it matches the size of the RL agent observation space while non-matching values of the last layer are simply zeroed-out. The number of hidden layers of the policy can be arbitrarily chosen using a bigger seed, in the shown example the seed/policy has 20 layers.

Our *Hypernetwork Neural Cellular Automata* (HyperNCA) consists of a neural cellular automata (NCA) which operates on a high-dimensional substrate whose values are interpreted as the weights of another network. Based on previous work (Mordvintsev et al., 2020; Sudhakaran et al., 2021), we design a NCA using a convolutional architecture which operates on a 3-dimensional substrate where each of the three dimensions has N channels, resulting in a substrate of dimension $3N$. The NCA consist of 3D convolutions followed by a dense layer. The convolutions have kernel size 3, hence preserving the locality of the cellular automata transition rule $\mathcal{T}$.

In order to generate a network we proceed as follows: **1.** We randomly initialise a substrate sampling from random uniform distribution. **2.** The NCA is applied to the substrate for a finite number of steps. **3.** The values of one of the channels of the substrate are interpreted as the weights of a policy network $\mathcal{P}$. **4.** The resulting policy $\mathcal{P}$ is evaluated in a reinforcement learning task $\mathcal{T}$. **5.** The resulting fitness of $\mathcal{P}$ in task $\mathcal{T}$ guides the evolution of the NCA weights. The substrate has shape L x C x W x W, where L is the number of layers of the policy network, C is the number of channels of convolutional layers of the NCA, and W is the size of the observation space of the task. This generic approach enables us to grow networks of any size —in depth and width— by simply adjusting the shape of the substrate. The algorithmic description of the approach is provided in Algorithm 1.

Algorithm 1: HyperNCA: Growing neural networks with Neural Cellular Automata & CMAES

Input: Reinforcement learning task $\mathcal{T}$, NCA model, number of developmental steps δ, training hyper-parameters Ω, fitness function $\mathcal{F}$.
Output: Optimal agent policy weights $\mathcal{W}$.

Sample random substrate seed $\mathcal{S}$ from an uniform distribution $\mathcal{U}(-1, 1)$;
while *Fitness* $\mathcal{F}$ < *target fitness* **do**
    while *generation* < *generations limit* **do**
        Generate a population of NCA using CMA-ES sampler;
        for *NCA **in** NCA population* **do** `// Parallelized across CPU cores`
            Let NCA update the initial seed for δ steps;
            The grown values of the first channel are interpreted as the weights of the agent policy $\mathcal{W}$;
            The grown policy is evaluated on the RL task $\mathcal{T}$;
        end
        if *mean population fitness* $\mathcal{F}$ < *early-stopping threshold* $\in \Omega$ **then**
            break;
        end
        Use population fitness values to update CMA-ES sampler's distribution mean and covariance matrix;
    end
    Save solution and mean solution with highest fitness;
end

4 Experiments and results

4.1 Optimisation details

We use CMA-ES (Covariance Matrix Adaptation Evolution Strategy) (Hansen & Ostermeier, 1996), a black-box population-based optimisation algorithm to train the HyperNCA. Evolutionary strategies (ES) algorithms have been shown to be capable of finding performing solutions for reinforcement learning tasks, both through directly optimising the policy weights Salimans2017Mar and with encodings of the policy through local learning rules (Najarro & Risi, 2020). Black-box methods such as CMA-ES have the advantage of not requiring to compute gradients and being easily parallelizable. Experimentes are run on a single machine with a *AMD Ryzen Threadripper 3990X* CPU with 64 cores. We choose a population size of 64, such that we can run one generation in parallel, —each core running an evaluation of the environment—, and adopt an initial variance of 0.1. Finally, we employ an early stopping method to discard and reset unpromising training runs.

All the code necessary to train the HyperNCA as well as the trained models will soon be made openly available.

4.2 Hypernetwork baseline

As a baseline, we evaluate a Linear Hypernetwork, similar to the one presented in Ha et al. (2016). The smallest hypernetwork is composed of a set of linear embeddings and a single linear weight generating matrix. The number of trainable parameters can be calculated by $(N * E_{dim} + E_{dim} * M$, where N equals the number of embeddings and $M = num\_target\_params/N$. Typically, each embedding in the hypernetwork corresponds to a layer in the target network. However, because the target networks are small (288 and 1792 parameters), we opt to include many embeddings and concatenate the weight generator outputs to create the final parameter vector, to ensure a smaller hypernetwork.

4.3 Results

First, we demonstrate that our method allows to generate policy networks capable of solving tasks with both continuous and discrete action spaces: a simulated 3D quadruped whose goal is to learn to walk as far as possible along one direction and Atari’s *Lunar Lander*, a discrete task whose goal is to smoothly land on a procedurally generated terrain.

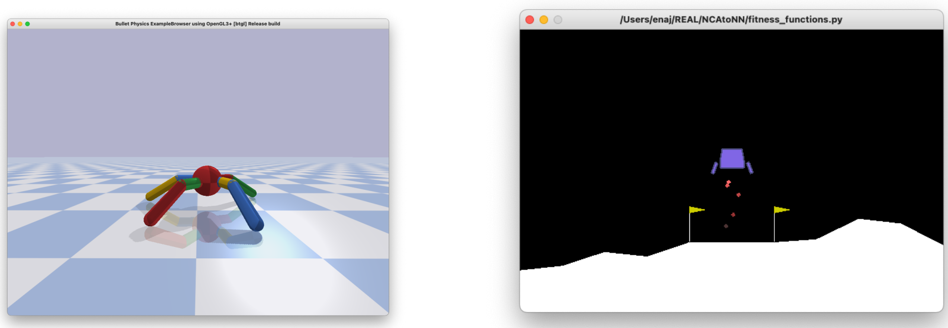

Figure 2: *Left*: Quadruped environment *AntBulletEnv-v0*. *Right*: Discrete task *LunarLander-v2*.

Continuous 3D locomotion RL task This task consists of a robot simulated using the Bullet physics engine (Coumans & Bai, 2016); the quadruped has 13 rigid links, including four legs and a torso, along with 8 actuated joints (Duan et al., 2016). It is modeled after the ant robot in the MuJoCo simulator and constitutes a common benchmark in RL (Finn et al., 2017). The robot has an input size of 28, comprising the positional and velocity information of the agent and an action space of 8 dimensions, controlling the motion of each of the 8 joints. The fitness of the quadruped agent depends on the distance travelled during a period of $1,000$ time-steps along a fixed axis. We made the environment deterministic by instantiating the robot on the same initial position at each episode.

We follow the method described in Algorithm 1 to grow a feed forward policy network consisting of three layers with $[28, 28, 8]$ neurons respectively, no bias and hyperbolic tangent as activation function. This policy network therefore has $1,792$ weight parameters. By contrast, the HyperNCA used to grow the policy has 314 trainable parameters, roughly 5.7 times less parameters than the resulting policy network. The grown network yields a reward of $1,192$ meters walked.Obtaining SoTA performance on these environments is out of the scope of this paper, hence neither the evolutionary CMA-ES hyperparameters nor the HyperNCA architecture have been optimised.

In comparison, the small baseline hypernetwork has 704 parameters and achieves a lower score of 898 ± 43 fitness over 100 trials. We also evaluated a large hypernetwork with $5,280$ parameters using ES (Salimans et al., 2017). Interestingly, here the network achieves a fitness $> 1,400$, which demonstrates that the hypernetwork can in fact perform well with a large enough number of parameters but is challenged to compress the weights to the same extend than the NCA. Future work will have to investigate these results closer to draw definite conclusions.

Figure 3 shows an example of the developmental steps for a policy network created by an evolved HyperNCA. The weight pattern quickly self-organizes into a particular global structure after which it appears to be mostly fine-tuned for the remaining developmental steps.

Growing deeper policy networks In order to demonstrate the flexibility and potential of our approach, we evolve a deeper policy consisting of 30 hidden layers and $22,960$ parameters which is generated by a HyperNCA with 548 trainable parameters, that is, a phenotype ~ 40 times bigger than its genotype. The resulting policy reaches a fitness reward of $1,075$. While such a big policy is not necessary to solve the quadruped task, this result demonstrates that the developmental compres-

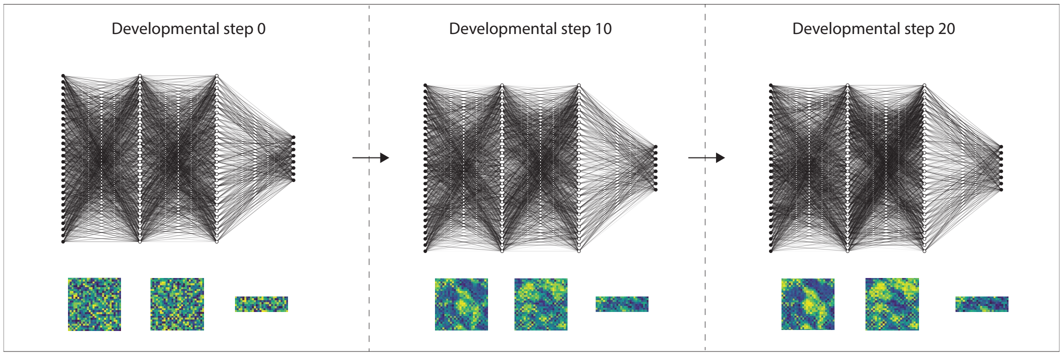

Figure 3: Quadruped policy networks at three developmental steps: initial random step, after 10 and after 20 developmental steps. Below each layer, the connections are visualised as weight matrices.

sion exhibited by the HyperNCA can encode high-dimensional phenotype. Crucially, this enables the optimisation algorithm to operate on a much smaller parameter space, allowing us to rely upon optimisation techniques —like covariance-based approaches— that would struggle with memory issues in higher dimensions.

Discrete RL task *Lunar lander* is an single-player Atari game where the player's goal is to smoothly land the lander on a procedurally generated terrain. There are four discrete actions available: do nothing, fire engine towards the left, fire engine vertically and fire engine towards the right. There are 8 states: the 2D position coordinates of the lander, its linear velocities, its angle, its angular velocity, and two booleans representing whether each leg is in contact with the ground or not (Brockman et al. (2016)). Reward for moving from the top of the screen to the landing pad and coming to rest is about 100-140 points. If the lander moves away from the landing pad, it loses reward. If the lander crashes, it receives an additional -100 points. If it comes to rest, it receives an additional $+100$ points. Each leg with ground contact is $+10$ points. Firing the main engine is -0.3 points each frame. Firing the side engines -0.03 points each frame. The task is considered solved if the player attains $200+$ reward over 100 evaluations.

To solve this task, we grow a five-layers feedforward policy with $[8, 8, 8, 8, 3]$ nodes per layer respectively, no bias and hyperbolic tangent as activation function. The resulting policy has 288 parameters and is generated by a HyperNCA with 226 trainable parameters, that is HyperNCA genotype with roughly 24% less parameters than the resulting policy phenotype. The policy is grown for 20 developmental steps and yields a reward of 252 ± 63 evaluated over one hundred runs, therefore solving the task according to the pre-establish criterion of 200 points. Similarly to the previous task, the hyperparameters have not been optimised. For comparison, in this domain the baseline hypernetwork has 252 trainable parameters and achieves 264 ± 28 fitness over 100 trials.

4.4 METAMORPHOSIS NEURAL NETWORKS

Metamorphosis is a developmental phenomena by which an organism experiences substantial changes in its morphology during post-embryonic life —a biological process first appeared in pterygote insects 400 million years ago (Belles, 2020). We draw inspiration from this biological phenomena to build *metamorphosis networks*: an extension of our HyperNCA approach to morph policy networks in order make the agent capable of solving different tasks. We demonstrate our approach with the quadruped RL task described in the previous section.

The approach is akin to the one we use to grow policy networks, however, rather than evaluating on a single quadruped morphology, we create a set of three morphologies variations: the standard Ant quadruped *M1* and two damaged morphologies *M2* and *M3* with partially mutilated legs. Consequently, we use a NCA to develop functional weights out of the initial seed for 10 developmental steps and use the resulting policy to control an agent solving the first morphology *M1*, subsequently, we let the same NCA further developed the grown weights for 20 developmental steps and evaluate the result network on the second morphology *M2*, same procedure for the third morphology: the weights develop for another 20 steps and use the resulting network to solve morphology *M3*.

The resulting policy networks yield $1,329$, $1,343$ and $1,266$ rewards (distance walked) on each morphology respectively. The evolved NCA has $1,480$ trainable parameters, while each of the policy it develops has $1,792$ parameters. Notice that it is a single NCA model that give rise to the three final policy networks.

In order to demonstrate that the policy weights are indeed morphing (i.e. the model is not using the same weights to solve all three morphologies), we show in Fig. 4 the relative changes in the weights between each policy as the continuous weight trajectories represented in 3-dimensional PCA space.

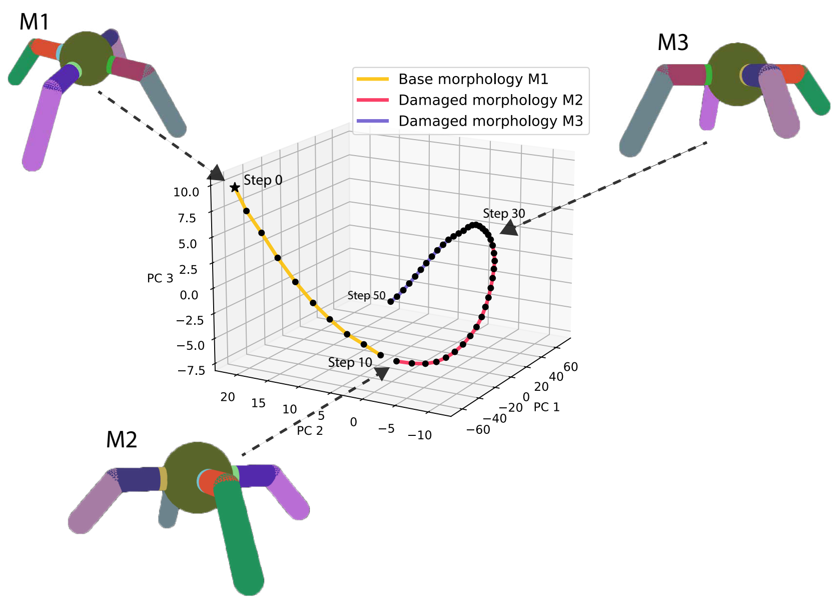

Figure 4: Low-dimensional representation of the policy weights at each developmental step. The steps pointed by each of the morphologies *M1*, *M2* and *M3* represent the weights used by the agents during evaluation. M1 is the standard Mujoco Ant morphology, M2 has one front leg and back leg mutilated and M3 has the two front legs mutilated. The dimensionality reduction is computed with PCA. The initial seed is represented with a star symbol $\star$.

Finally, we evaluate each of the policies on the inappropriate morphologies and show in Table 1 that the NCA-guided metamorphosis process has indeed a positive impact on the policies performance on each morphology reward.

	M1	M2	M3
Policy M1	**1,329**	827	747
Policy M2	21	**1,343**	916
Policy M3	13	1,140	**1,266**

Table 1: Rewards obtained from cross evaluation of the three policies grown by the same Hyper-NCA at three developmental steps on each of the three morphologies M1, M2 and M3. The reward value represent the distance walked by the quadruped from its initial position. The values in the diagonal (shown in bold) correspond to the rewards the policies were grown to perform well on, i.e. policy M1 was evaluated on morphology M1 at developmental step 10, policy M2 was evaluated on morphology M2 at developmental step 30 and policy M3 was evaluated on M3 at step 50. The diagonal values are the rewards obtained by each quadruped morphology at their corresponding developmental steps. The values outside of the diagonal are the rewards obtained when evaluating the policies at the developmental steps which do not match the morphologies at that step; this produce a decrease in the obtained reward demonstrating that the policy weights are significantly morphing.

5 Discussion and Future work

Our HyperNCA approach is able to solve the tested RL tasks, reaching a comparable performance to other indirect encoding methods. Since it has to undergo a developmental process, our model yet seems currently harder to optimise, although more experiments are necessary to confirm this hypothesis. To optimize these highly dynamical and self-organizing systems, more open-ended search methods such as quality diversity (QD) (Pugh et al., 2016) or intrinsically-motivated learning approaches (Baranes & Oudeyer, 2013) could be of particular interest. These methods might be more suited to navigate the likely highly deceptive fitness landscapes of self-organizing systems.

The goal of this paper is not necessarily to reach SotA in one RL benchmark, but rather to propose a novel developmental framework to grow a neural network, through local interactions. Contrasting to the previous literature on hypernetworks and indirect encodings, our encoding follows a developmental process, inspired from biological growth and self-organisation. While currently not outperforming direct encoding approaches, research in neuroscience suggests that this type of "genomic bottleneck" (Zador, 2019) has an important regularizing constraint, allowing animals to generalize better to new situations. Promising results suggest that artificial agents can also benefit from such a compression for better generalization (Pedersen & Risi, 2021) and the HyperNCA approach presented in this paper could be a promising method to study this question in more detail.

Future important work includes extending the approach to more complex tasks and environments, which might more clearly benefit from the HyperNCA's ability to grow deep policy networks. Additionally, we are particularly excited to further investigate the metamorphic promises of such developmental encoding. For example, the changes in an organism's form in biological metamorphosis are often striking (e.g. a caterpillar turning into a butterfly). Could a HyperNCA facilitate similar radical changes in the morphologies and nervous systems of artificial robots?

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