# **Recurrent Memory Transformer**

**Anonymous Author(s)** Affiliation Address email

# Abstract

Transformer-based models show their effectiveness across multiple domains and 1 tasks. The self-attention allows to combine information from all sequence ele-2 ments into context-aware representations. However, global and local information 3 4 has to be stored mostly in the same element-wise representations. Moreover, the 5 length of an input sequence is limited by quadratic computational complexity of 6 self-attention. In this work, we propose and study a memory-augmented segmentlevel recurrent Transformer (Recurrent Memory Transformer). Memory allows 7 to store and process local and global information as well as to pass information 8 between segments of the long sequence with the help of recurrence. We implement 9 a memory mechanism with no changes to Transformer model by adding special 10 11 memory tokens to the input or output sequence. Then Transformer is trained to control both memory operations and sequence representations processing. Results 12 of experiments show that our model performs on par with the Transformer-XL 13 on language modeling for smaller memory sizes and outperforms it for tasks that 14 require longer sequence processing. This makes Recurrent Memory Transformer a 15 16 promising architecture for applications that require learning of long-term depen-17 dencies and general purpose in memory processing, such as algorithmic tasks and 18 reasoning.

#### Introduction 1 19

Transformers (Vaswani et al., 2017) have been widely 20 adopted across multiple domains and tasks (Radford 21 et al., 2018; Dong et al., 2018; Devlin et al., 2019; 22 Dosovitskiy et al., 2021; Ramesh et al., 2021; Jaegle 23 et al., 2021). The key component of Transformer 24 layer is a self-attention. Self-attention allows to up-25 date each sequence element representation with in-26 27 formation from all other elements in the sequence. 28 As a result, rich contextual representation for every element is generated at the end of encoding. This 29 way, global sequence-level and local information are 30 stored in a single representation. However, this mix-31 ing of two types of information in a single represen-32 tation has limitations. Distributed storage of global 33 features across all sequence elements results in global 34 features "blurring" and makes it harder to access them. 35



Figure 1: Recurrent Memory Transformer. Memory is added as tokens to the input sequence and memory output is passed to the next segment. During training gradients flow from the current segment through memory to the previous segment.

- Another well-known deficiency of Transformers is poor scaling of self-attention with input sequence 36
- 37 length that hurts its applications to long inputs (Child et al., 2019; Guo et al., 2019; Dai et al., 2019;
- Beltagy et al., 2020; Ainslie et al., 2020; Zaheer et al., 2020; Wang et al., 2020; Choromanski et al., 38
- 2020). 39

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Our work introduces a memory-augmented segment-level recurrent Transformer named Recurrent 40 Memory Transformer (RMT). RMT uses a memory mechanism based on special memory tokens (Burt-41 sev et al., 2020) added to the input sequence. Memory tokens provide additional reserved capacity to 42 the model that could be used to process information which is not directly representing any element in 43 the input sequence. To process long sequences, we split them into segments and pass memory states 44 from a previous to a current segment. This memory passing makes the model recurrent and removes 45 the input sequence length limitations. RMT model can theoretically work with infinite lengths but, in 46 practice, it is limited by memory capacity and the efficiency of memory access/update operations. 47 Our implementation of both memory and recurrence in RMT requires no changes to the Transformer 48 model because modifications are made only to the input and output sequences of the model. 49 We tested RMT on the tasks that require a global information about the whole input sequence to be 50

solved. We use copy, reverse, and associative retrieval tasks in the setting where the input sequence
 is split into segments. RMT and Transformer-XL perfectly solve these tasks, but exceeding some
 value of sequence length, RMT starts to outperform Transformer-XL. Also, we experimentally show
 that the proposed Recurrent Memory Transformer requires less memory size to perform closely to
 Transformer-XL on language modeling tasks. RMT code and experiments are available<sup>1</sup>.

# 56 Contributions

- In this study we augment Transformer with token based memory storage and segment-level recurrence.
- We experimentally evaluate proposed architecture as well as vanilla Transformer and Transformer-XL on memory-intensive tasks such as copy, reverse, associative retrieval and language modeling. We show that RMT outperforms Transformer-XL for sequence processing tasks and on par with Transformer-XL on language modeling but requires less memory.
- We analysed how the Transformer model learns to use memory. Specific interpretable memory read-write patterns of attention are shown.

# 66 2 Related work

In our study we add a memory to general purpose attention based neural architecture. Memory is 67 a recurrent topic in neural networks research. It had started from the early works (McCulloch and 68 Pitts, 1943; Stephen, 1956) and significantly progressed in 90's with introduction of Backpropagation 69 Through Time learning algorithm (Werbos, 1990) and Long-Short Term Memory (LSTM) (Hochreiter 70 and Schmidhuber, 1997) neural architecture. Today memory-augmented neural networks (MANNs) 71 usually rely on some kind of recurrent external-memory which is separate from the model's pa-72 rameters. Neural Turing Machines (NTMs) (Graves et al., 2014) and Memory Networks (Weston 73 et al., 2014) are equipped with a storage for vector representations that can be accessed with an 74 attention mechanism. Memory Networks (Weston et al., 2014; Sukhbaatar et al., 2015) were designed 75 to enable reasoning by sequential attention over to the content of a memory. NTMs followed by 76 Differentiable Neural Computer (DNC) (Graves et al., 2016) and Sparse DNC (Rae et al., 2016) 77 are implemented as recurrent neural networks able to write to memory storage over time. All these 78 models are differentiable and can be trained via backpropagation through time (BPTT). Parallel line 79 of research extends recurrent neural networks such as LSTM with data structures like stacks, lists, 80 or queues (Joulin and Mikolov, 2015; Grefenstette et al., 2015). MANN architectures with a more 81 advanced addressing mechanisms such as address-content separation and multi-step addressing were 82 proposed in (Gulcehre et al., 2016, 2017; Meng and Rumshisky, 2018). The Global Context Layer 83 model (Meng and Rumshisky, 2018) uses the idea of address-content separation to solve the difficulty 84 of training content-based addressing in the canonical NTM. 85

86 Recent rise of Transformer models also resulted in introduction of a number of new memory archi-

87 tectures. Transformer-XL (Dai et al., 2019) introduces a segment-level recurrence at the level of

<sup>88</sup> hidden representations. These representations of a sequence are computed and stored in the cache

to be reused as an extended context for the next segment. Compressive Transformer (Rae et al.,

<sup>&</sup>lt;sup>1</sup>anonymous link. Code, raw experiments results and hyperparameters are provided in supplementary materials.

2019) adds the second layer of memory to Transformer-XL. This memory compresses and stores 90 information from the cache.  $\infty$ -former (Martins et al., 2021) utilizes continuous-space attention and 91 represents input sequence as a continuous signal to make long-term memory unbounded. *Memory* 92 Layers (Lample et al., 2019) model has a product key memory layer instead of a feed-forward layer 93 within Transformer block to increase model capacity. 94 In many variations of Transformer different sorts of global representations are added. Among them 95 are Star-Transformer (Guo et al., 2019), Longformer (Beltagy et al., 2020), GMAT (Gupta and Berant, 96 2020), Extended Transformer Construction (ETC) (Ainslie et al., 2020) and Big Bird (Zaheer et al., 97 2020). All these architectures re-design self-attention mechanism to reduce it computational com-98

<sup>99</sup> plexity with and ensure input coverage with the help of global representations. *Memory Transformer* 

(Burtsev et al., 2020) keeps Transformer model intact and adds memory by extending input sequence
 with special memory tokens. Perceiver IO (Jaegle et al., 2021) maps an entire arbitrary input to the

fixed number of latent representations. Transformer layers do further processing over latent memory
 representations only.

Segment-level recurrence in Transformers is actively explored in a number of studies. Transformer-104 XL, Compressive Transformer keep previous states and re-use them in subsequent segments. Ernie-105 Doc (Ding et al., 2021) improves processing by using same-layer recurrence instead of attending to 106 previous layer outputs of a precedent segment. Memformer (Wu et al., 2020) introduces a dedicated 107 memory module to keep previous hidden states in summarized representations. Memformer uses two 108 special layers added to the Transformer model. Memory cross-attention layer reads from memory 109 and memory slot attention layer updates it. MART (Lei et al., 2020) has a similar approach as 110 Memformer but uses memory update rules analogous to LSTM (Hochreiter and Schmidhuber, 1997) 111 and GRU (Cho et al., 2014). FeedBack Transformer (Fan et al., 2020) goes further with full, and not 112 segment-level, recurrence. FeedBack Memory merges past hidden representations from all layers 113 into a single vector and makes it accessible to the computations at any layer. The disadvantage of full 114 recurrence is that it is less parallelizable. FeedBack Memory requires every sequence element to be 115 processed sequentially. In segment-level recurrent models, all elements of a segment are processed by 116 Transformer layers in parallel. Only segments are processed sequentially. Staircase Transformer (Ju 117 et al., 2021) combines segment-level recurrence and depth recurrence. Staircase models use the 118 output for previous segments and pass them as input for the next segment. Our Recurrent Memory 119 Transformer is based on special memory tokens similar to Memory Transformer, segment-level 120 recurrence as in Transformer-XL, and depth-recurrent mechanism for memory processing similar to 121 Staircase. 122

# **3 Recurrent Memory Transformer**



Figure 2: **Comparison of Recurrent Memory Transformer and Transformer-XL architectures.** Recurrent Memory Transformer augments Transformer with global memory tokens and passes them to allow a segment-level recurrence. Special read/write memory tokens are added to the input sequence. Multiple memory tokens can be used in each read/write block. Updated representations of write memory are passed to the next segment. During training, RMT uses BPTT to propagate gradient to previous segments through memory tokens representation. Recurrence with memory has no limitations on effective context length, whereas Transformer-XL can use only finite context with cached states. All RMT memory/recurrence operations are made on the input and output level of the Transformer model.

#### 3.1 Background: Transformer-XL 124

Transformer-XL (Dai et al., 2019) extends Transformer model with state re-use cache mechanism 125 for segment-level recurrence and relative position encoding. Input sequence is split on segments 126 processed sequentially. Hidden states computed for the previous segment  $M^n$  are cached for each 127 transformer layer n. The input of the layer n consists of the last m states from the cached memory 128 and output of previous Transformer layer for the current segment  $\tau$ :

129

$$\tilde{H}_{\tau}^{n-1} = [SG(M_{-m:}^{n-1}) \circ H_{\tau}^{n-1}], \tag{1}$$

here, SG stands for stop-gradient, o denotes concatenation. Cached states allow to increase effective 130 context size of Transformer model and save on compute operations. 131

Then,  $\tilde{H}_{\tau}^{n-1}$  goes to Transformer layer to produce layer n outputs for segment  $\tau$ : 132

$$H_{\tau}^{n} = \text{TransformerLayer}(Q_{\tau}^{n}, K_{\tau}^{n}, V_{\tau}^{n}),$$
$$Q_{\tau}^{n} = W_{q}^{n} H_{\tau}^{n-1}; K_{\tau}^{n} = W_{k}^{n} \tilde{H}_{\tau}^{n-1}; V_{\tau}^{n} = W_{v}^{n} \tilde{H}_{\tau}^{n-1}.$$

In Transformer-XL, self-attention layers are modified to use relative position encodings to improve 133 generalization to longer attention lengths. The overall architecture is shown in the Figure 2. 134

#### 3.2 Memory and recurrence 135

Memory augmented Transformers such as GMAT, ETC, Memory Transformer (Gupta and Berant, 136 2020; Ainslie et al., 2020; Burtsev et al., 2020) proposed to use special global tokens as storage 137 for representations. Usually, memory tokens are added to the beginning of the input sequence. 138 However, in decoder-only architectures the causal attention mask makes impossible for memory 139 tokens at the start of the sequence to collect information from the subsequent tokens. On the other 140 hand, if memory tokens are placed at the end of the sequence then preceding tokens unable to 141 access their representations. To solve this problem we add a recurrence to the sequence processing. 142 Representations of memory tokens placed at the end of the segment are used as an input memory 143 representations at the start as well as at the end of the next segment. 144

145 In the Recurrent Memory Transformer input is augmented with special [mem] tokens, processed in a standard way along with the sequence of tokens. Each memory token is a real-valued vector. m146 memory tokens are added at the beginning of the segment tokens representations  $H^0_{\tau}$  and the same m 147 tokens are added at the end: 148

$$\begin{split} \dot{H}_{\tau}^{0} &= [H_{\tau}^{mem} \circ H_{\tau}^{0} \circ H_{\tau}^{mem}], \\ \bar{H}_{\tau}^{N} &= \text{Transformer}(\tilde{H}_{\tau}^{0}), \\ [H_{\tau}^{read} \circ H_{\tau}^{N} \circ H_{\tau}^{write}] &:= \bar{H}_{\tau}^{N}, \end{split}$$

here N is a number of Transformer layers. 149

The starting group of memory tokens functions as a read memory that allows sequence tokens to 150 attend to memory states produced at the previous segment. The ending group works as a write 151 memory that can attend to all current segment tokens and update representation stored in the memory. 152 As result,  $H_{\tau}^{write}$  contains updated memory tokens for the segment  $\tau$ . 153

Segments of the input sequence are processed sequentially. To enable recurrent connection between 154

segments, we pass outputs of the memory tokens from the current segment to the input of the next 155 segment: 156

$$\begin{split} H^{mem}_{\tau+1} &:= H^{write}_{\tau},\\ \tilde{H}^0_{\tau+1} &= [H^{mem}_{\tau+1} \circ H^0_{\tau+1} \circ H^{mem}_{\tau+1}]. \end{split}$$

Both memory and recurrence in the RMT are based only on global memory tokens. It allows to 157 keep the backbone Transformer unchanged and make RMT memory augmentation compatible with 158 any model from the Transformer family. Memory tokens operate only on the input and output of 159

the model. In this study we implement RMT on top of the original Transformer-XL code. Botharchitectures are shown in Figure 2.

Recurrence in the RMT is different compared to the Transformer-XL because the former stores only 162 m memory vectors per segment. On the other hand, the Transformer-XL stores  $m \times N$  vectors per 163 segment. Also, in the RMT model memory representations from the previous segment are processed 164 by Transformer layers together with the current segment tokens. This makes memory part of RMT 165 effectively deeper in a number of applied Transformer layers  $\tau \times N$ . Additionally, we allow all 166 memory tokens in the read/write block to access all other tokens in the same block. The causal 167 attention mask is applied only to tokens of the input sequence. The RMT attention mask is shown in 168 Figure 6 (d). 169

We train the RMT with Backpropagation Through Time (BPTT). During backward pass, unlike in Transformer-XL, memory gradients are not stopped between segments. The number of previous segments to backpropagate is a hyperparameter of a training procedure. We vary BPTT unroll in our experiments from 0 to 4 previous segments. Increasing this parameter is computationally expensive and requires a lot of GPU RAM. However, such techniques as gradient checkpointing could be used to alleviate this problem.

# **176 4 Experiments**

We designed our experiments to evaluate the ability of Recurrent Memory Transformers to preserve
long-term dependencies across multiple input segments. The first set of experiments includes copy,
reverse, associative retrieval and quadratic equations tasks. The second addresses language modeling
task for word-level on WikiText-103 (Merity et al., 2017) and for character-level on enwik8 (Mahoney,
2006). We compare Recurrent Memory Transformer with Transformer and Transformer-XL models.

# 182 4.1 Algorithmic Tasks

Firstly, we evaluate RMT on algorithmic tasks that require information about the whole input sequence to be solved successfully. In a recurrent setting, the model has to keep information about all previous

185 segments to make predictions.

In the copy task, an input sequence should be repli-186 cated twice after a special start-to-generate token. In 187 the reverse task, an input sequence should be gen-188 erated in a reverse order. Input for the associative 189 retrieval task consists of N key-value pairs. Then one 190 key is randomly selected, and the task is to produce 191 an appropriate value for the selected key. Another 192 task is to solve quadratic equations. One example 193 consists of an equation, its solution with discriminant, 194 and an answer. The task is to generate a solution and 195 answer, while only answer quality is evaluated. 196



Figure 3: Reverse task in one and four segments setting for decoder-only models. Dotted lines show segment borders.

For all tasks, input and output sequences are split
 into segments and processed by models sequentially. Datasets for algorithmic tasks were randomly
 pre-generated, and the same data was used in all experiments.

Transformer-XL and RMT are decoder-only Transformer models. We do not compute loss over
 the input sequence before the start-to-generate token. The loss is computed over target sequence
 segments only. This procedure is illustrated in Figure 3.

#### 203 4.2 Language Modeling

We use two standard benchmarks for language modeling: WikiText-103 and enwik8. WikiText-103 (Merity et al., 2017) is used for word-level language modeling and contains 103M words from English Wikipedia articles. Enwik8 (Mahoney, 2006) is used for character-level and consists of 10<sup>8</sup> first bytes of XML text dump of the English Wikipedia. We compare Recurrent Memory Transformer with decoder-only Transformer and Transformer-XL as baselines. Model size and training parameters are selected to match Transformer-XL paper. For Wikitext-103 an input context length was set to 150 tokens, and for enwik8 it was set to 512 characters.

Another set of experiments inspected how RMT handles long-term dependencies and recurrence. We increased the number of segments and recurrent steps by making segments smaller (50 tokens for WikiText-103, 128 characters for enwik8). The increased number of recurrent steps makes language modeling tasks harder for RMT because information has to be stored in the same amount of memory for more time steps.

# 216 4.3 Implementation details

Our RMT implementation is based on Transformer-XL repository<sup>2</sup>. We also use WikiText-103, enwik8 data and processing from this repository. Language modeling experiments follow the same model and training hyperparameters as Transformer-XL. WikiText-103 experiments use 16-layer Transformers, enwik8 – 12 layer Transformers. We refer to Transformer-XL with memory size equal to zero as a Baseline.

With this experimental setup we were able to reproduce results for the Transformer-XL model close to the original paper (see Appendix A.4 for enwik8 and Table 2 for WikiText-103).

# 224 5 Results

Baseline, Tr-XL, RMT perform perfectly in the single segment setting on copy and reverse tasks (Figure 4). In this case models do not need recurrence because the whole sequence is available. When the number of segments is larger than one, non-recurrent baseline struggles to solve tasks, but both memory models demonstrate ability to retain required information from the previous segments in memory.



Figure 4: **RMT outperforms Transformer-XL on Copy and Reverse tasks as number of segments increases.** Panels show test set per-character accuracy on copy, reverse, and associative retrieval tasks (from left to right). Memory/cache size equals to the length of a segment for the both models. RMT does not pass gradients between segments in this experiment. MT results are the same as for the Baseline. Source/target sequence lengths for copy, reverse and associative retrieval tasks: 24/48, 24/24, 10/1.

As the number of segments increases, Recurrent Memory Transformer starts to outperform Transformer-XL with memory sizes less than the number of all previous tokens. With the number of segments up to 6 mean accuracy of Transformer-XL drops by up to 0.2 points, and with 9 segments

score plunges to an accuracy score of 0.2, close to the baseline without memory.

Associative retrieval results are similar with the number of segments up to 4. RMT manages to solve the task with Transformer-XL closely behind. However, in the setting with 5 segments, RMT performance slightly decreases and Transformer-XL average accuracy rises higher.

On the quadratic equations task (Table 1) we have checked that it is possible to solve the task with the Transformer baseline and no segmentation used. The baseline in this case defines upper bound

<sup>&</sup>lt;sup>2</sup>https://github.com/kimiyoung/transformer-xl

<sup>239</sup> for this task. Enabling recurrence with multiple segments RMT solves the task perfectly, while

240 Transformer-XL finds the task challenging.

241 Results of experiments on word-level language

modeling on WikiText-103 are shown in Table 2.

In the first section with a segment length of 150,Tr-XL and RMT outperform the baseline and

Memory Transformer (MT) by a large margin.

It shows the significance of increased effective

context length by Tr-XL cache or RMT memory

<sup>248</sup> for language modeling.

MT is slightly better than the Transformer base-249 line. This is due to the fact that MT adds special 250 memory tokens only to the beginning of an in-251 put sequence. Autoregressive MT has no way 252 to write to memory because of the causal at-253 tention mask and, therefore, is unable to pass 254 information between segments. Thus, in the au-255 toregressive setting MT could be seen as equiv-256 alent to the prefix/promt-tuning (Li and Liang, 257 2021; Lester et al., 2021). 258

RMT improves over MT memory mechanism 259 with read/write blocks. The best RMT models 260 with memory size 10 and 25 show similar perfor-261 mance as Transformer-XL with a memory size 262 equal to 75. RMT learns to use smaller memory 263 more effectively than Transformer-XL. Addi-264 tionally, the smaller memory size of RMT leads 265 to reducing required GPU memory for running 266 the model. 267

Decreasing the size of segments to 50, we force 268 models to work with longer recurrent dependen-269 cies as the number of recurrent steps increases. 270 RMT with memory consisting of a single vec-271 tor shows similar results to Transformer-XL 272 with memory size 10. It is worth noting that 273 274 Transformer-XL memory consists of hidden representations from all layers (in this case, it is 275  $10 \times 16$  vectors) when RMT memory is only 276

Table 1: Test set solve rate for quadratic equations. Input sequence length is 180 tokens and consists of quadratic equation, solution, and answer. The input sequence is split into a number of segments with an answer as the last segment. Accuracy is 1.0 if the full answer is predicted correctly.

MODEL	MEMORY	SEGMENTS	$Acc_{\pm std}$
BASELINE	0	1	$0.99 \pm 0.01$
TR-XL	30	6	$0.93 \pm \text{NA}$
RMT	30	6	$0.99 \pm 0.002$

Table 2: Test set perplexity on WikiText-103. Average perplexity for the best performed variations of RMT models reported (see full results in Appendix A.5). Underlined values show Tr-XL and RMT models with close results. RMT models with smaller memory sizes achieve similar scores to Tr-XL models with larger memory.

Model	MEMORY	SEGMENT LEN	$\text{PPL}_{\pm \text{STD}}$
BASELINE	0	150	$29.95 \pm 0.15$
MT	10	150	$29.63 \pm 0.06$
TR-XL (PAPER)	150	150	24.0
TR-XL (OURS)	150	150	$24.12 \pm 0.05$
TR-XL	75	150	$24.68 \pm 0.01$
TR-XL	25	150	$\overline{25.57} \pm 0.02$
RMT BPTT-3	10	150	$25.04 \pm 0.07$
RMT BPTT-2	25	150	$24.85 \pm 0.31$
BASELINE	0	50	$39.05\pm0.01$
TR-XL	100	50	$25.66 \pm 0.01$
TR-XL	50	50	$26.54 \pm 0.01$
TR-XL	25	50	$\overline{27.57} \pm 0.09$
TR-XL	10	50	$28.98 \pm 0.11$
RMT BPTT-1	1	50	$\bar{28.71} \pm 0.03$
RMT BPTT-3	10	50	$\underline{26.37} \pm 0.01$

277 memory\_size vectors. Transformer-XL with memory size 50 and RMT with memory size 5 shows
278 similar perplexity values.

On enwik8 RMT models with memory size 5 and Transformer-XL with memory size 40 show similar results. Confirming that RMT learns to use smaller amounts of memory representation more effectively. All experiments results on enwik8 dataset are in Appendix A.4 and Table 3.

We analyze how number of segments, sequence length, length of training context, and memory size 282 affect models' performance on different tasks in Figure 5. As we split sequence into more segments 283 it becomes more crucial to be able to pass information between segments. For the copy task, we 284 split 360 tokens sequence into multiple segments. In Figure 5a we observe that Tr-XL performance 285 286 starts to degrade and eventually falls to the baseline model performance as the number of segments increases. In contrast, RMT continues to solve the task perfectly. In the more extreme setting, when 287 we keep memory size fixed, but increase the total length of sequence Tr-XL fails shortly, while RMT 288 starts to degrade on 720 tokens sequence length (Figure 5b). 289

Recurrent Memory Transformer learns to make predictions depending on #BPTT\_unrolls previous segments +1 current segment. Transformer-XL does not use BPTT and relies only on memory\_size cached states and current segment making in total: memory\_size + segment\_length tokens. In

7



(a) Increasing number of (b) Increasing sequence (c) Dependency of test set (d) Increased memory size segments for a fixed se- length up to 1080 tokens perplexity on visible con- and deeper recurrence requence length of 360 to- with 120 tokens segment text length at training time. sult in better performance. kens. and memory size 60.

Figure 5: (a) RMT is able to solve copy task perfectly with multiple recurrent steps, while Tr-XL fails. (b) RMT learns to use memory of the same fixed size more effectively than TR-XL as sequence length increases. (c&d) WikiText-103 with 50 tokens segment length. (c) Marker size corresponds to memory size. Visible context at training time can be increased by enlarging Tr-XL cache or using more BPTT unrolls for RMT. Increasing visible context leads to lower perplexity for both models. (d) Test set perplexity for different memory sizes. When memory size is zero Tr-XL and RMT are just baseline Transformer models without recurrence.

Figure 5c, we compare RMT and Tr-XL according to the described value of visible context at training time.

RMT with a single memory vector could be trained to achieve lower perplexity as Transformer-XL with memory size 10. It means that RMT can learn to store information from previous observations more compactly. Another observation is that RMT with memory sizes 10 and 25 performs worse but closely to Transformer-XL even when Transformer-XL has access to more non-compressed states (50, 100, 200) from previous segments. Furthermore, we observed instabilities and out-of-memory issues during RMT training for a larger number of BPTT unrolls and memory sizes.

Recurrent Memory Transformer does not benefit from increasing memory size from 5 to 50, but results of Transformer-XL better scale with memory size (Figure 5d). RMT models with memory size 5 have close results to Transformer-XL with cache 50, confirming that RMT learns to store more compact representations. Dynamic of RMT perplexity suggests that there is some optimal memory size for RMT to solve the task, and further increase does not add much. Training RMT with one BPTT unroll drastically improves its results showing the importance of BPTT training (Figure 5d).



Figure 6: Selected attention map patterns of memory models. (a) - write to memory, (b) - read from memory (RMT, segment length=24, memory size=24), (c) - rewrite from read memory to write memory (RMT, segment length=8, memory size=8), (d) - read from previous hidden states (Transformer-XL, segment length=24, memory size=24)

To get an understanding of memory operations, learned by models during training algorithmic tasks, it is useful to look at attention maps. Figure 6 shows some heatmaps from attention layers of models trained on copy and reverse. The darkness of each pixel shows how much the element from the corresponding row "attends" to its column.

In each RMT attention map sequence tokens are preceded by read memory, located at the top left corner, and followed by write memory at the bottom right. Lines at the central part of (a), top image shows classic attention of token sequence to itself, but the bottom line represents the operation of writing of sequence tokens to memory in straight order. When completing reverse, the (a), bottom image model learns to write the sequence in the reversed order, which is in line with common sense.

When it comes to reproducing the target sequence, model accesses memory, Figure 6 (b) and writes to the output sequence. Another operation (a) is requiring from read memory to write memory. It is

to the output sequence. Another operation (c) is rewriting from read memory to write memory. It is commonly used by RMT in settings with larger number of segments to keep information about recent segments longer.

Transformer-XL mechanism of accessing memory (d) does not allow straightforward writing to memory without changing sequence token representations. Sequential reading from cache is represented by straight lines on Transformer-XL attention maps.

Using token representations as storage harms model performance in tasks with larger number of segments. On reverse with 4 segments Transformer-XL with limited memory size 6, Figure 8 (b) attempts to mix representations of tokens and read multiple symbols from one cached state in next segments. This results in the average accuracy of 0.8 on the given task. Despite having the same memory size, RMT manages to compress the whole segment in memory tokens Figure 8 (a) and achieve mean accuracy 1.0.

Visualizations from Figure 6 and Figure 8 provide evidence to support our hypotheses that Tr-XL has to mix representations from previous and current segments in the same hidden states to pass information between segments. Also, visualizations show how memory tokens in RMT help mitigate such kind of mixing.

RMT ability of sequence compression to memory is illustrated in Figure 7. For copy with 6
 segments RMT compresses and then reads the sequence of 12 tokens with just 6 memory tokens.
 For Transformer-XL decreasing memory size harms the accuracy score significantly with number of
 segments larger than 2.

# 337 6 Conclusions

In this paper we introduced Recurrent Memory Transformer a simple recurrent memory augmentation of Transformer model. RMT is implemented by extension of an input sequence with special global memory tokens and segment-level recurrence.

In our experiments we compared RMT with Transformer baseline and Transformer-XL which is a well-known modification of Transformer for long sequences. RMT almost perfectly solves Copy, Reverse as well as quadratic equations tasks for sequences consisting of multiple segments outperforming Transformer-XL. It also demonstrates quality for associative retrieval task on par with Transformer-XL. As expected, baseline Transformer fails to solve these tasks for multi-segment settings.

RMT trained as language model significantly ahead of Transformer baseline and shows quality 347 metrics similar to Transformer-XL but for up to 10 times smaller memory size. Experimental results 348 demonstrate that for fixed memory size backpropagating gradients for more segments improves 349 performance of RMT. Analysis of attention maps suggests that better RMT performance can be 350 related to more effective storage of input representations in dedicated memory tokens compared to 351 352 mixing representations storage in Transformer-XL. Overall, results of the study show that dedicated memory storage and recurrence provided by Recurrent Memory Transformer make it a promising 353 architecture for applications that require learning of long-term dependencies and general purpose 354 in-memory processing, such as algorithmic tasks and reasoning. Furthermore, we believe that RMT 355 could open the way for adding memory and recurrence to other models in the Transformer family. 356

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#### Checklist 477

478	1. For all authors
479 480	(a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
481 482	(b) Did you describe the limitations of your work? [Yes] We mention training instabilities and GPU RAM issues in Section 5.
483 484 485	(c) Did you discuss any potential negative societal impacts of your work? [No] The proposed model and method do not have any specific impacts. All general negative societal impacts applicable to the field could be potentially relative.
486 487	(d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
488	2. If you are including theoretical results
489	(a) Did you state the full set of assumptions of all theoretical results? [N/A]
490	(b) Did you include complete proofs of all theoretical results? [N/A]
491	3. If you ran experiments
492 493 494 495	(a) Did you include the code, data, and instructions needed to reproduce the main exper- imental results (either in the supplemental material or as a URL)? [Yes] We include code, training scripts, and raw experimental data in the supplementary material. The supplemental materials would be published on github with the final version of the paper.
496 497 498	<ul><li>(b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See Section 4.3, Appendix A, and provided supplementary material.</li></ul>
499 500 501	<ul><li>(c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] All the key experiments results are reported with std. Furthermore, we provide raw experimental data in the supplementary materials.</li></ul>
502 503 504 505	<ul><li>(d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes] We used different GPUs depending on the task: 1080Ti, V100, A100. We provide this information in Appendix A for each task.</li></ul>

ramesh21a.html. 461

506	4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets
507 508 509	(a) If your work uses existing assets, did you cite the creators? [Yes] We refer to the original Tr-XL code and Tr-XL paper. We use it for establish baselines and setting our methods. See Section 4.3
510 511	(b) Did you mention the license of the assets? [No] Tr-XL licence is Apache 2.0 and available at its github repo.
512 513	(c) Did you include any new assets either in the supplemental material or as a URL? [Yes] Our code is in the supplemental material.
514 515	(d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [No] We used publicly available Tr-XL code (Apache 2.0) and datasets.
516 517 518	(e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [No] We use either synthetic data or datasets collected from the Wikipedia (Wikitext-103, enwik8).
519	5. If you used crowdsourcing or conducted research with human subjects
520 521	(a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
522 523	(b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
524 525	(c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

# 526 A Training details and additional results

### 527 A.1 Algorithmic tasks

Datasets were randomly generated by uniformly sampling tokens from dictionary into task sequences and generating targets accordingly to the tasks. After generation, datasets are fixed for all experiments.

Copy and reverse use sequences of sizes 24, 40, 120, 240, and 360, making total copy/reverse input length 48/72, 80/120, 240/360, 480/720, 720/1080. The associative retrieval task consists of 4 key-value pairs and one randomly selected key; the answer consists of one value. Train, validation and test sizes of copy 24, reverse 24 and associative retrieval datasets are 100000, 5000 and 10000.

<sup>534</sup> Transformer-XL had the same cache size on training and validation to match RMT.

For training all models on copy and reverse, we used constant learning rate 1e-4 with reduction on plateau with decay factor of 0.5. Copy and reverse were solved by models with 4 layers and 4 heads, associative retrieval models had 6 layers and 4 heads. Models with the same context size and memory

size were trained for the same number of steps and the same training parameters.

Experiments with sequence length 24 were conducted on a single Nvidia GTX 1080 Ti GPU from 1 hour to 2-3 days. Copy and reverse on longer sequence lengths were done on more powerful Tesla

hour to 2-3 days. Copy and reverse on longer sequence lengths were done on  $V_{100}$  using 1.3 days with training time verying from 1 hour to 3.4 days

V100 using 1-3 devices with training time varying from 1 hour to 3-4 days.



Figure 7: Test set per-character accuracy on copy (a), reverse with a sequence length 24 (b) and 120 (c). Memory size is limited to half of the length of a segment for (a) and (b) and to 60 tokens for (c). Target sequence length equals source length for reverse and doubles for the copy.

### 542 A.2 Associative retrieval

<sup>543</sup> We used code for the task dataset generation from (Ba et al., 2016)<sup>3</sup>.

### 544 A.3 Quadratic equations

This dataset consists of equations with integer coefficients with step-by-step solutions using a 545 discriminant. Process of equation generation is started from uniformly sampling real roots  $x_1, x_2$ 546 from -100 to 100. The answer of an equation is represented as  $x_1, x_2$ . Next, we find the equation as 547 multiplication of two parentheses  $(x - x_1)(x - x_2) = 0$ , which is expanded to  $x^2 - (x_1 + x_2)x + x_1 + x_2 + x_2$ 548  $x_1x_2 = 0$ . Next, we multiply all coefficients by a random natural number  $\alpha$  from 1 to 10. The final 549 equation form is  $\alpha x^2 - \alpha (x_1 + x_2)x + \alpha x_1 x_2 = 0$ . A dataset sample is made of these stages in 550 reversed order. We also provide a string with the discriminant calculation to help find the equation 551 roots. 20 percent of equations in the dataset do not have real roots. 552

- 553 Example equation string:
- $-4 x^2 + 392 x 2208 = 0$ ,
- 555 solution string:
- $x^2-98*x+552=0; D=98^2-4*1*552=7396=86^2; x=(98-86)/2=6; x=(98+86)/2=92$
- 557 and answer:
- 558 6,92

Each solution step is tokenized on char level and padded to the length of 30 tokens. The total length of each training sample is 180, the dataset has 100000 training, 10000 validation and 20000 test samples.

For this task we used models with 6 layers, 6 heads and segment sizes 180 and 30. Trained was performed with the same schedule as copy and reverse on a single GTX 1080 ti for 1-2 days. Memory size for RMT and Transformer-XL was chosen equal to the segment length.

# 565 A.4 Enwik8

We verified our experimental setup by reproducing Transformer-XL results on enwik8 dataset (Table 3). We used 12-layer Baseline (Transformer), Transformer-XL, RMT in all enwik8 experiments. All results on enwik8 dataset are in Table 3. We used 2 NVIDIA A100 80Gb GPUs, training time varied from 10 to 30 hours depending on sequence length, memory size, and number of BPTT unrolls.

# 570 A.5 WikiText-103

We used 16-layer models in all experiments on WikiText-103 dataset. Training hyperparameters were used from (Dai et al., 2019) and authors PyTorch scripts<sup>4</sup>. All results on WikiText-103 dataset are in Table 4. In most of the WikiText-103 experiments, we used 2 NVIDIA A100 80Gb GPUs, training time varied from 10 to 30 hours depending on sequence length, memory size, and number of BPTT unrolls.

<sup>&</sup>lt;sup>3</sup>https://github.com/GokuMohandas/fast-weights/blob/539fb10e3c384d5f782af2560bf28631cd0eaa61/ fw/data\_utils.py

<sup>&</sup>lt;sup>4</sup>https://github.com/kimiyoung/transformer-xl

Model	MEMORY	SEGMENT LEN	$BPC_{\pm STD}$
TR-XL (DAI ET AL., 2019)	512	512	1.06
TR-XL (OURS)	512	512	1.071
TR-XL	200	128	1.140
TR-XL	100	128	1.178
TR-XL	75	128	1.196
TR-XL	40	128	$1.230 \pm 0.001$
TR-XL	20	128	1.261
TR-XL	10	128	$1.283 \pm 0.001$
RMT BPTT-1	5	128	$1.241 \pm 0.002$
RMT BPTT-2	5	128	$1.231 \pm 0.002$
RMT BPTT-1	10	128	$1.240 \pm 0.006$
RMT BPTT-2	10	128	$1.228 \pm 0.003$
RMT BPTT-0	20	128	1.301
RMT BPTT-1	20	128	1.229
RMT BPTT-2	20	128	1.222

Table 3: Test set bits-per-character on enwik8. Our experimental setup shows similar scores to the original paper (Dai et al., 2019) with segment length 512.

Model	MEMORY	SEGMENT LEN	$PPL_{\pm STD}$
BASELINE	0	150	$29.95 \pm 0.15$
MT	10	150	$29.63 \pm 0.06$
MT	25	150	$29.67 \pm 0.03$
MT	75	150	$29.69 \pm 0.02$
MT	150	150	$29.82 \pm 0.35$
TR-XL (PAPER)	150	150	24.0
TR-XL (OURS)	150	150	$24.12 \pm 0.05$
TR-XL	75	150	$24.68 \pm 0.01$
TR-XL	25	150	$25.57 \pm 0.02$
RMT BPTT-0	10	150	$26.85 \pm 0.02$
RMT BPTT-1	10	150	$25.92 \pm 1.07$
RMT BPTT-2	10	150	$25.32 \pm 0.61$
RMT BPTT-3	10	150	$25.04 \pm 0.07$
RMT BPTT-0	25	150	29.73
RMT BPTT-1	25	150	24.91
RMT BPTT-2	25	150	$24.85 \pm 0.31$
BASELINE	0	50	$39.05 \pm 0.01$
TR-XL	200	50	25.14
TR-XL	100	50	$25.66 \pm 0.01$
TR-XL	50	50	$26.54 \pm 0.01$
TR-XL	25	50	$27.57 \pm 0.09$
TR-XL	10	50	$28.98 \pm 0.11$
TR-XL	5	50	$30.06 \pm 0.07$
TR-XL	1	50	$32.35 \pm 0.03$
RMT BPTT-0	1	50	$31.33 \pm 1.26$
RMT BPTT-1	1	50	$28.71 \pm 0.03$
RMT BPTT-2	1	50	28.44
RMT BPTT-3	1	50	$28.40 \pm 0.03$
RMT BPTT-0	5	50	$30.32 \pm 0.18$
RMT BPTT-1	5	50	$27.05 \pm 0.20$
RMT BPTT-2	5	50	$26.83 \pm 0.18$
RMT BPTT-3	5	50	$26.75 \pm 0.26$
RMT BPTT-4	5	50	$26.67 \pm 0.03$
RMT BPTT-0	10	50	$30.69 \pm 0.01$
RMT BPTT-1	10	50	$27.95 \pm 1.32$
RMT BPTT-2	10	50	$26.62 \pm 0.34$
RMT BPTT-3	10	50	$26.37 \pm 0.01$
RMT BPTT-4	10	50	$26.25 \pm 0.19$
RMT BPTT-0	25	50	29.75
RMT BPTT-1	25	50	26.32
RMT BPTT-2	25	50	27.31
RMT BPTT-0	50	50	29.75
	50	50	=>e

Table 4: Test set perplexity on WikiText-103. All experiments with RMT and Tr-XL models.

# **B** Operations with Memory



Figure 8: Approaches to compression and decompression of sequence with length 12 and memory with size 6. (a) - RMT, (b) - Transformer-XL.