On the Distributed Multi-robot Herding

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***Abstract*— Multi-robot herding deals with the problem of steering a group of non-cooperative entities towards desired locations by means of a team of robots. Herding demands high levels of coordination due to the complex nonlinear dynamics of such evasive entities, and the wild environments preclude the use of central coordination units. These aspects lead to the need of distributed estimation and graph-based control protocols. In this work, we propose CIC (Coordinated Implicit Control), a distributed control protocol that is composed by two main blocks. The first is an Implicit Control module that is able to stabilize any continuous input-nonaffine system, tackling the complex nature of the herd dynamics. The second is an Extended Certifiable Optimal Distributed Kalman Filter that, by defining the multi-robot team as a distributed communication graph, allows each robot to rapidly recover the information necessary to compute the control input, while accounting for potential uncertainties. Different experiments validate the proposal.**

I. Introduction

The problem of multi-robot herding [1] consists of using a team of cooperative robots (also called herders) to drive a group of non-cooperative entities (also called evaders) to desired locations or trajectories, by considering the potentially adversarial behaviors of the evaders with respect to the herders' movements. The importance of multi-robot herding comes from the fact that many multi-robot applications, such as entrapment [2], hunting [3] or escorting [4], can be reformulated as a herding problem. Herding has also motivated several interdisciplinary research [5]–[7], arriving at the same conclusion: herding involves complex distributed coordination and local nonlinear behaviors. To address both issues, we propose CIC (Coordinated Implicit Control), a distributed algorithm composed by (i) a local Implicit Control (IC) [8] module that stabilizes the evaders' dynamics to converge to the desired herd references and (ii) an Extended Certifiable Optimal Distributed Kalman Filter (CO-DKF) [9] that achieves the desired distributed coordination among robots, conveniently defined over a communication graph.

Among the works that address the multi-robot herding problem, a typical assumption is that all the evaders go to the same desired location [1], [10]–[12]. In contrast, we propose a solution that allows to specify individual desired locations for each evader. This feature is achieved in some works by steering the evaders one by one [13], [14], while our solution is able to steer all the evaders simultaneously. Another classical assumption is that of linear [15]–[17] and/or homogeneous dynamics [1], [18], which might not reflect the highly complex non-cooperative nature of the

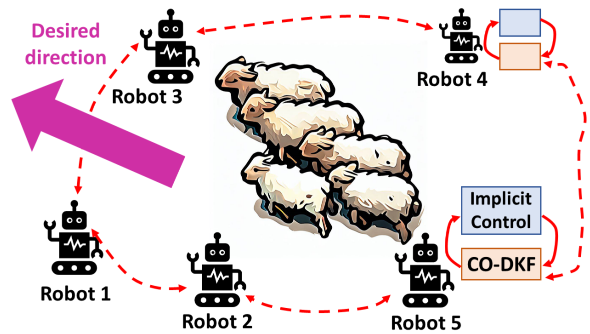

Fig. 1: In herding, the robotic herders drive the evaders towards desired trajectories. IC leverages the repulsive forces to stabilize the herd dynamics and drive the evaders towards their desired locations. The herders use CO-DKF to communicate and fuse relevant estimated global information.

evaders. Thanks to IC, our proposal is capable to deal with both input-nonaffine evaders' dynamics and potential heterogeneous behaviors of the evaders. Besides, the CO-DKF module accounts for the uncertainty in the sensors and the dynamics, increasing the robustness of the solution. Finally, beyond the literature of centralized solutions [8], [19], the works that propose distributed cooperative solutions [1], [11], [12] do not consider the estimation problem, that can both resolve sensor issues and the lack of global information for a more effective control. The proposed CO-DKF estimator solves this, achieving the desired distributed properties for a scalable protocol.

In what follows, we briefly introduce the problem formulation (section II) and continue with the description of the proposed CIC algorithm (section III). The algorithm is composed by the IC module (III-A) and the CO-DKF module (III-B), which are appropriately presented. After that, we show illustrative experiments (section IV) to conclude that CIC solves the multi-robot herding problem successfully for different evaders' behaviors and team sizes.

II. Problem Statement

A team of n robots is in charge of herding m evaders, moving in a 2D space. Herders are indexed by i and evaders by j. The goal of the paper is to control the position of the evaders, $\mathbf{x}_j$, gathered in the vector $\mathbf{x} = \begin{bmatrix}\mathbf{x}_1^T & \dots & \mathbf{x}_m^T\end{bmatrix}^T$, by positioning the herders, $\mathbf{u}_i, \mathbf{u} = \begin{bmatrix}\mathbf{u}_1^T & \dots & \mathbf{u}_n^T\end{bmatrix}^T$, over time. The dynamics of each evader is $\dot{\mathbf{x}}_j = f_j(\mathbf{x}, \mathbf{u})$, while the joint system dynamics is

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \tag{1}$$

where $f(\mathbf{x}, \mathbf{u})$ simply comes from stacking all $f_j(\mathbf{x}, \mathbf{u})$. This formulation allows to consider heterogeneous herds,

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with different number of evaders and motion models. The herders are defined over a communication graph $\mathcal{G} = \{V, E\}$. Herders i and i' communicate with each other if $(i, i') \in E$. The set $\mathcal{J}_i = \{i' | (i, i') \in E\} \cup \{i\}$ denotes the neighbors of robot i.

Our goal is to herd the evaders towards the desired positions $\mathbf{x}^*$ or trajectories $\dot{\mathbf{x}}^*$ simultaneously, using only the local information available from the neighboring robots. Finally, to keep the generality of the solution, in this work we assume that the maximum velocity of both herders and evaders is $v_{\max} > \dot{\mathbf{x}}^* > 0$.

III. Coordinated Implicit Control

In this section we described CIC (Coordinated Implicit Control), the proposed algorithm to solve the distributed multi-robot herding problem. In section III-A we present the theoretical background of IC and frame the formulation in the context of herding. After that, in section III-B we present and describe the extended CO-DKF, the distributed estimator that addresses the centralized limitations of the IC.

A. Implicit Control

The idea behind IC [8], [19] is that, in systems that are nonlinear with respect to the input, the classical analytical expressions lead to implicit equations, so they do not have a closed-form. Instead, Implicit control considers an analytical equation in terms of the time derivative of the input. Let

$$h(\mathbf{x}, \mathbf{u}) = f(\mathbf{x}, \mathbf{u}) - f^*(\mathbf{x}) - \dot{\mathbf{x}}^*, \tag{2}$$

with $f^*(\mathbf{x})$ is the desired closed-loop behavior of the herd. The roots of $h(\cdot)$ correspond to the solutions of the control input of the problem. Since the analytical solution of this equation might not exist, the solution proposed by IC is to consider h as a dynamic system and design $\dot{\mathbf{u}}$ such that h converges to 0. If $\mathbf{u}$ is such that $h = 0$, then $\dot{\mathbf{x}}$ follows $f^*(\cdot) - \dot{\mathbf{x}}^*$; and since $f^*(\cdot)$ is stable, $\mathbf{x}$ converges to the desired tracking reference. If the desired evolution of function $h(\cdot)$ is defined as

$$dh(\mathbf{x}, \mathbf{u})/dt = h^*(\mathbf{x}, \mathbf{u}), \tag{3}$$

with $h^*(\cdot)$ such that $h(\cdot)$ is stable, then the input dynamics

$$\dot{\mathbf{u}} = \mathbf{J}_{\mathbf{u}}^+ \left(h^*(\mathbf{x}, \mathbf{u}) - \mathbf{J}_{\mathbf{x}} f(\mathbf{x}, \mathbf{u}) \right) \tag{4}$$

imposes Eq. (3), and $h(\cdot)$ converges to $\mathbf{0}$. In Eq. (4), $\mathbf{J}_{\mathbf{x}}$ is the Jacobian of $h(\cdot)$ with respect to $\mathbf{x}$, $\mathbf{J}_{\mathbf{u}}$ is the Jacobian of $h(\cdot)$ with respect to $\mathbf{u}$ and $^+$ is the pseudoinverse of a matrix. The use of the pseudoinverse is because $m \neq n$ in general. Eqs. (1) and (4) yield to the explicit system

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \quad \dot{\mathbf{u}} = \mathbf{J}_{\mathbf{u}}^+ \left(h^*(\mathbf{x}, \mathbf{u}) - \mathbf{J}_{\mathbf{x}} f(\mathbf{x}, \mathbf{u}) \right). \tag{5}$$

The control problem is then reduced to analyzing the stability of the system in (5). In particular, in this paper it is considered that

$$f^*(\mathbf{x}) = -\mathbf{F}_1 \mathbf{x} \qquad \text{and} \qquad h^*(\mathbf{x}, \mathbf{u}) = -\mathbf{F}_2 h(\mathbf{x}, \mathbf{u}). \tag{6}$$

$\mathbf{F}_1, \mathbf{F}_2$ are two matrices chosen such that $\begin{pmatrix} -\mathbf{F}_1 & 0.5\mathbf{I} \\ 0.5\mathbf{I} & -\mathbf{F}_2 \end{pmatrix}$ is negative definite [8].

Implicit Control facilitates the design of controllers for general input-nonaffine systems. This opens the possibility of controlling heterogeneous herds regardless of the complexity of the evaders' dynamics. In our case, $\mathbf{x}$ is the position of the evaders, and $f^*(\mathbf{x})$ the function describing their desired dynamics. Meanwhile, Eq. (5) determines the derivative of the herders' positions $\mathbf{u}$. Note that the computation of the herders' position dynamics depends on the positions of all evaders and herders. In the next section we propose a distributed estimator to provide each robotic herder with the state and input information needed to compute the IC output.

B. Certifiable Optimal Distributed Kalman Filter

Up to now, the IC requires that herders know the state and input of the system perfectly. To overcome this limitation, we develop an Extended Certifiable Optimal Distributed Kalman Filter (CO-DKF) [9], tailored to work with the IC. The objective is, for all the herders, to have an accurate estimation of evaders' and herders' position, aggregated in the vector $\xi = \begin{bmatrix} \mathbf{x}_1^T & \dots & \mathbf{x}_m^T & \mathbf{u}_1^T & \dots & \mathbf{u}_n^T \end{bmatrix}^T$. Notice that, in contrast with typical estimation problems, the control inputs must also be estimated. In typical control solutions this is not possible because the input has no dynamics, it is just an algebraic expression. IC addresses this issue by providing the input dynamics.

Particularizing to the CO-DKF, as in every Kalman filter, each robot has a predicted and updated version of ξ, denoted by $\bar{\xi}_i$ and $\hat{\xi}_i$ respectively. Associated to them, robots keep track of $\bar{\mathbf{P}}_i$ and $\hat{\mathbf{P}}_i$, the predicted and updated error covariance matrices associated to $\bar{\xi}_i$ and $\hat{\xi}_i$. At a given instant, the robots first compute, locally, the prediction step of an extended Kalman filter

$$\begin{aligned} \bar{\xi}_i &= (\mathbf{I} + \Delta_T \mathbf{J}_\xi) \hat{\xi}_i, \\ \bar{\mathbf{P}}_i &= \hat{\mathbf{P}}_i + \Delta_T (\mathbf{J}_\xi \hat{\mathbf{P}}_i + \hat{\mathbf{P}}_i \mathbf{J}_\xi^T + \mathbf{Q}) \end{aligned} \tag{7}$$

where $\mathbf{J}_\xi$ denotes the Jacobian of the expanded system in (5) evaluated at $\hat{\xi}_i$, Δ_T is the discretization sample period and $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ is a zero mean Gaussian noise with covariance $\mathbf{Q}$, which can be tuned according to the confidence on the evaders' model.

After prediction, robots exchange the information from the prediction and the sensors:

$$\mathbf{c}_i = \mathbf{H}_i^T \mathbf{R}_i^{-1} \mathbf{z}_i, \mathbf{C}_i = \mathbf{H}_i^T \mathbf{R}_i^{-1} \mathbf{H}_i, \bar{\mathbf{s}}_i = \bar{\mathbf{P}}_i^{-1} \bar{\xi}_i, \bar{\mathbf{S}}_i = \bar{\mathbf{P}}_i^{-1},$$

with

$$\mathbf{z}_i = \mathbf{H}_i(\xi) \xi + \mathbf{v}_i, \tag{8}$$

where $\mathbf{z}_i$ is the measurement taken by herder i, $\mathbf{v}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_i)$ is the measurement noise with covariance $\mathbf{R}_i$, and $\mathbf{H}_i$ depends on ξ. It is important to note that the herders' measurements will depend on its position, i.e., a herder will only be able to measure the entities that are close to it.

The exchanged quantities are aggregated using the information provided by the covariances. In the case of the measurements, this is direct because they are independent: $\mathbf{y}_i = \sum_{j \in \mathcal{J}_i} \mathbf{c}_j$ and $\mathbf{Y}_i = \sum_{j \in \mathcal{J}_i} \mathbf{C}_j$. On the other hand, the aggregation of predictions is harder due to the correlations between them. To solve it, CO-DKF uses the outer Löwner-John method [20], which leads to the optimization problem

$$\bar{\mathbf{S}}_i^*, \boldsymbol{\lambda}_i^* = \underset{\bar{\mathbf{S}}, \boldsymbol{\lambda}}{\arg\max} \;\; \mathrm{Tr}(\bar{\mathbf{S}}) \tag{9a}$$

$$s.t. \;\; \mathbf{0} \prec \bar{\mathbf{S}} \preceq \sum_{j=1}^{|\mathcal{J}_i|} \lambda_j \bar{\mathbf{S}}_j, \;\; \sum_{j=1}^{|\mathcal{J}_i|} \lambda_j \leq 1, \;\; \lambda_j \geq 0, \tag{9b}$$

where $\mathrm{Tr}(\cdot)$ is the trace operator, and $\prec$ and $\preceq$ denote definiteness and semidefiniteness. It is proven [9] that, by means of the above formulation, the solution of (9) can be certified as optimal in the Mean Square Error (MSE) sense, locally and at each instant. It is also proven that this implies that the estimation is optimal in the MSE sense under unknown correlations. The output of (9) is used to aggregate the predictions: $\bar{\mathbf{P}}_i^* = (\bar{\mathbf{S}}_i^*)^{-1}, \quad \bar{\xi}_i^* = \bar{\mathbf{P}}_i^* \sum_{j\in\mathcal{J}_i} \lambda_j^* \bar{\mathbf{s}}_j$. With the data aggregated, each robot calculates $\hat{\mathbf{P}}_i = (\bar{\mathbf{S}}_i^* + \mathbf{Y}_i)^{-1}$ and then the estimate is updated

$$\hat{\xi}_i = \bar{\xi}_i^* + \hat{\mathbf{P}}_i(\mathbf{y}_i - \mathbf{Y}_i \bar{\xi}_i^*). \tag{10}$$

The updated estimate is the one employed in the IC module to compute the control input, locally and at each robot.

In summary, the CIC algorithm first estimates the position of evaders and herders in a distributed way by means of the Extended CO-DKF. Then, the current estimate is employed to compute the control input by means of the IC module. The integration between both modules is tight, in the sense that the same IC provides the dynamic equations to the extended version of the CO-DKF.

IV. Results

To validate the CIC algorithm, in this section we conduct simulated experiments, validating the IC and the Extended CO-DKF modules separately. After that, we show experiments with the complete CIC algorithm

A. Simulations of the IC module

The first case of study is the herding of 3 evaders by 3 herders. We set $\mathbf{F}_1 = 0.25\mathbf{I}_{2m}$, leading to a settling time of 12s with an exponential transient according to $2m$ first order independent systems. To ensure convergence, $\mathbf{F}_2 = 50\mathbf{I}_{2m}$. We assume that $v_{\max} = 0.4$m/s. The control is computed every 10ms. To verify the generality of the proposal, we employ two standard and highly nonlinear evaders' models from the literature, called Inverse and Exponential model [8]. Their parameters are tuned as in [8]. The desired herding configuration evolves with $\dot{x}_j^* = v_j^*$ and $\dot{y}_j^* = 0.5w_j^* \cos(w_j^* t + 2\pi/j)$, with $\mathbf{w}^*$=[0.05, 0.1, 0.02]rad/s, $\mathbf{v}^*$=[0.05, 0.05, 0.05]m/s.

The IC module is capable of steering each evader simultaneously through their associated sinusoidal references. This yields to herders' trajectories surrounding and modulating the interaction forces with the evaders. With the evaders in their desired trajectories, the system reaches a steady-state behavior where the periodic movement of the evaders is shared by the herders.

TABLE I: List of symbols in the figures.

Symbol		Meaning	
		Desired positions	
Symbol	Meaning	Symbol	Meaning
	Initial position herders		Initial position evaders
	Final position herders		Final position evaders
	Trajectories herders		Trajectories evaders

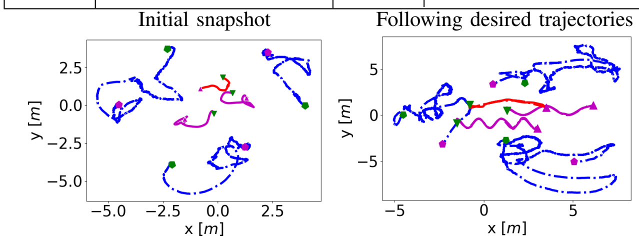


Fig. 2: Three robotic herders herding three heterogeneous evaders. The magenta trajectories correspond to Inverse evaders while the red trajectory is an Exponential evader. The other symbols follow the convention in Table I.

To validate the flexibility of the IC module, we consider a showcase with 50 evaders and 5 herders. The control goal is to steer the centroid of the herd. To maintain cohesion of the herd and avoid collisions, we add very weak repulsive and coalition forces among evaders

$$\dot{\mathbf{x}}_j = f_j^{inv/exp}(\mathbf{x}, \mathbf{u}) + \vartheta \sum_{j'=1}^{m} \mathbf{d}_{jj'} \left(\frac{1}{||\mathbf{d}_{jj'}||^3} - ||\mathbf{d}_{jj'}||^2 \right) \tag{11}$$

with $\mathbf{d}_{jj'} = \mathbf{x}_j - \mathbf{x}_{j'}$ and $\vartheta = 2 \times 10^{-4}$.

Fig. 3 shows how the five herders can steer the whole herd towards desired regions. After an initial shared goal, the herders can split the herd in two sub-herds and steer each of them, simultaneously.

B. Simulations of the CO-DKF module

Now we validate the performance of the CO-DKF module against other DKFs, to assess the improvement in estimation error. The validation is conducted by comparing with: (i) *AtA-CO-DKF*, our proposal but assuming all-to-all communication, (ii) *OCDFK*, Algorithm 3 from [21], (iii) *TCDFK*, Algorithm 2 from [22], (iv) *HDfKF*, complete algorithm in [23], (v) *HADfKF*, simplified algorithm in [23], (vi) *CKF*, centralized equivalent KF. The target system is:

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}, \; \mathbf{A} = \begin{pmatrix} 1 & 0 & \sin w_p T & \cos w_p T - 1 \\ 0 & 1 & 1 - \cos w_p T & \sin w_p T \\ 0 & 0 & \cos w_p T & \sin w_p T \\ 0 & 0 & \sin w_p T & \cos w_p T \end{pmatrix}, \tag{12}$$

$w_p = 0.5$rad/s, $T = 0.1$s and $\mathbf{Q} = 2 \times 10^{-6}\mathbf{I}_4$.

To do the comparison, we analyze two scenarios. Experiment 1 initializes a random sensor network, with appropriate parameters to obtain a sparse connected topology. Then, a random uniform distribution decides the quantities sensors measure among two options: measuring x or y. $\mathbf{H}_i$ is picked from a uniform distribution in the range $[1, 3]$. A Bernoulli

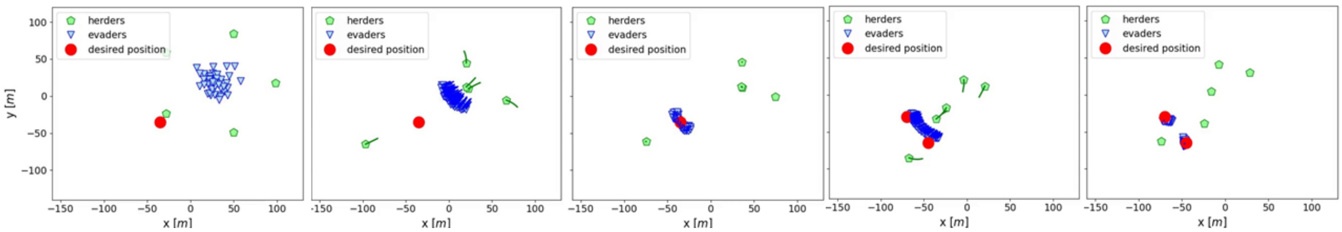
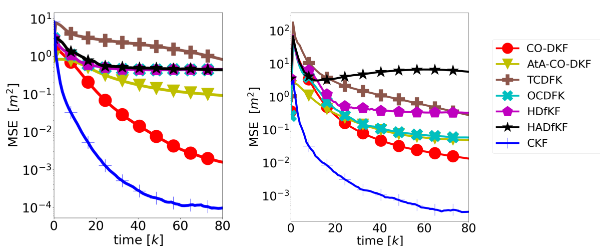


Fig. 3: 5 herders herding 50 Inverse evaders. In the first 50 seconds, the herd is steered towards the desired location. After that, the herd is split in two sub-herds and driven towards two desired locations simultaneously.

distribution with $p = 0.5$ decides if the diagonal values of $\mathbf{R}_i$ are in the range $[3, 5] \times 10^{-2}$ or $[3, 5]$, i.e., high-quality or low-quality. This is done 100 times, assessing the results by computing the averaged Mean Square Error MSE over the experiments. Experiment 2 is the same as Experiment 1, but only one sensor is of high quality.

The results of Experiments 1 are shown in Fig. 4a. The best performance among the distributed estimators is obtained by CO-DKF, with a difference of more than an order of magnitude with the other state-of-the-art filters. These differences also hold in Experiment 2 (Fig. 4b).



(a) Experiment 1 (b) Experiment 2

Fig. 4: Averaged MSE for the different estimators.

C. Simulations of the complete CIC algorithm

Now, we combine the IC and the extended version of the CO-DKF module to deal with the herding of 5 evaders by 5 herders. We maintain $\mathbf{F}_1$ and $\mathbf{F}_2$ as in section IV-A. Besides, we set $\mathbf{Q} = 0.02\mathbf{I}_{2m+2n}$ and $\mathbf{R}_i = 0.07\mathbf{I}_{2m+2n}$ $\forall i$ to simulate a scenario with bad sensing capabilities. To be realistic in the use of the communication bandwidth, messages are exchanged every 100ms, while the control is computed every 10ms. Sensing and communication are defined by a circular region centered at each robot, of radius $d_m = d_c = 6.5$m respectively.

The first row of Fig. 5 shows the trajectories followed by herders and evaders for the different test cases. IC is able to herd the evaders successfully, driving each evader simultaneously towards its desired location. The second row of Fig. 5 shows that the Root Mean Square Error (RMSE) between the estimates and real positions of the entities rapidly decreases to the sensor noise level. Before the first 400ms (4 communication rounds) the estimator has converged. Despite the different frequencies between communication and control, the herders are successful in predicting the global state and controlling the herd.

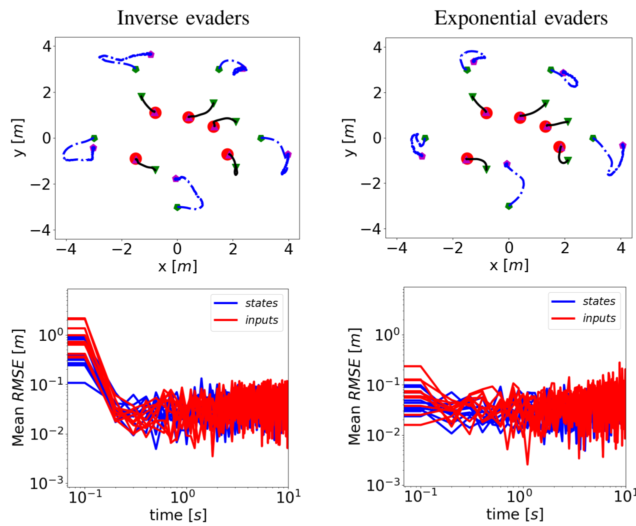


Fig. 5: Simulation results of 5 evaders by 5 herders using CIC. The first row shows the trajectories of herders and evaders (same symbols of Table I). The second row depicts the RMSE in the estimation of evaders' (blue) and herders' (red) positions, averaged over the five herders.

V. Conclusions

This paper has proposed a distributed estimation and control algorithm for the multi-robot herding problem called Coordinated Implicit Control. The algorithm departs from a graph-based modeling of the multi-robot team, and builds upon to main blocks, namely: an Implicit Control module which is able to control general and heterogeneous input-nonaffine non-cooperative evaders' dynamics, and an Extended Certifiable Optimal Distributed Kalman Filter that achieves the desired distributed architecture while recovering the global state and input of the system. Different experiments confirm the generality, flexibility and robustness of the CIC algorithm for wide variety of herding settings, avoiding unnecessary central coordination units and exploiting the distributed communications among robots.

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