Solving Quantitative Reasoning Problems with Language Models

Anonymous Author(s)
Affiliation
Address
email

Abstract

Language models have achieved remarkable performance on a wide range of tasks that require natural language understanding. Nevertheless, state-of-the-art models have generally struggled with tasks that require quantitative reasoning, such as solving mathematics, science, and engineering problems at the college level. To help close this gap, we introduce Minerva, a large language model pretrained on general natural language data and further trained on technical content. The model achieves strong performance in a variety of evaluations, including state-of-the-art performance on the MATH dataset. We also evaluate our model on over two hundred undergraduate-level problems in physics, biology, chemistry, economics, and other sciences that require quantitative reasoning, and find that the model can correctly answer nearly a quarter of them.

1 Introduction

Artificial neural networks have seen remarkable success in a variety of domains, such as computer vision, speech recognition, audio and image generation, translation, playing games, and robotics. In particular, large language models have achieved excellent performance across a variety of natural language tasks including common-sense reasoning, question answering, and summarization [1, 2, 3, 4, 5]. However, these models have struggled with tasks that require quantitative reasoning, such as solving mathematics, science, and engineering problems [6, 7].

Quantitative reasoning problems are an interesting domain of application for language models because they test the capability of models on several fronts. They require the solver to correctly parse a natural language input, potentially recall world knowledge that pertains to the problem, and apply an algorithm or series of computations to the information provided in order to arrive at a correct solution. They also require that the solver is able to correctly parse and generate precise sequences of mathematical tokens, as well as apply a computational procedure to tokens via symbolic and numerical manipulation. Finally, such problems are a proving ground for research toward robust quantitative reasoning solvers that are useful in supporting the work of humans in scientific and technical fields.

Previous research has shown that large language models achieve impressive performance on math and programming questions after training on domain specific datasets [8, 9, 10]. In this work, we apply this approach to quantitative reasoning problems, in which the model is required to provide a complete and self-contained solution, without relying on external tools. Such tasks include mathematics word problems, competition mathematics evaluations, and many problems in science and engineering.

Question: A line parallel to $y = 4x + 6$ passes through $(5, 10)$. What is the y-coordinate of the point where this line crosses the y-axis?

Target: Since the line is parallel to $y = 4x + 6$, we know that its slope is 4. Thus the equation of the line is $y = 4x + b$ where b is the y-intercept. Plugging $(5, 10)$ into this gives $10 = 4 \cdot 5 + b \Rightarrow b = \boxed{-10}$ which is what we wanted.

Model output: The line is parallel to $y = 4x+6$, which means that it has the same slope as $y = 4x + 6$. Since the line passes through $(5, 10)$, we can use the point-slope form of a line to find the equation of the line:

$$\begin{aligned} y - 10 &= 4(x - 5) \\ y - 10 &= 4x - 20 \\ y &= 4x - 10 \end{aligned}$$

Now that we have the equation of the line, we can find the y-coordinate of the point where the line crosses the y-axis by substituting $x = 0$ into the equation:

$$y = 4 \cdot 0 - 10 = \boxed{-10}.$$

Question: Assume that variance of first n natural numbers is 10 and variance of first m even natural numbers is 16. Compute $m + n$.

Model output: Variance of first n natural numbers is $\frac{n^2-1}{12}$. Variance of first m even natural numbers is $\frac{2^2(m^2-1)}{12}$. So, $\frac{n^2-1}{12} = 10$ and $\frac{2^2(m^2-1)}{12} = 16$. Solving for n and m, we get $n = 11$ and $m = 7$. So, $m + n = 18$.

Question: Prove that for every a, b, $b \neq a$, holds $\frac{a^2+b^2}{2} > \left(\frac{a+b}{2}\right)^2$.

Model output: We have

$$\begin{aligned} \frac{a^2 + b^2}{2} &> \left(\frac{a + b}{2}\right)^2 \\ \frac{a^2 + b^2}{2} &> \frac{a^2 + 2ab + b^2}{4} \\ 2a^2 + 2b^2 &> a^2 + 2ab + b^2 \\ a^2 - 2ab + b^2 &> 0 \\ (a - b)^2 &> 0, \end{aligned}$$

which is true for all a and b.

Figure 1: Example questions with solutions from Minerva 62B. These solutions were selected by majority vote over 256 samples, or maj1@256. All questions were prompted with the same prompt (a 4-shot prompt from MATH; see appendix). **Left:** Question from the MATH dataset (Algebra 4). The model derives the solution by writing down a line equation, simplifying it, substituting $x = 0$, and solving for y. The model outputs valid LaTeX along with natural language explanations. This detailed approach is different from the one used in the ground truth target. **Top right:** Example question from the Joint Entrance Examination Main Math 2020 exam taken each year by almost 2M Indian high-school students intended to study engineering and similar fields. **Bottom right:** Example question from the National Math Exam in Poland (May 2022). The exam is taken by approximately 270K high-school students every year.

1.1 Our Contribution

We present Minerva, a language model that achieves strong performance on many quantitative reasoning tasks. Our model is able to process scientific and mathematical questions formulated in natural language, and generate step-by-step solutions using correct LaTeX notation. Figures 1 and 2 show example responses from Minerva to mathematics and physics questions.

Minerva is based on a pretrained general language model (which we call BaseModel[1]) that is further trained on a high-quality dataset containing scientific and mathematical data. We start with 8B and 62B parameter models and continue training them on our technical content dataset. We achieve state-of-the-art performance on the MATH dataset and strong performance on other math tasks formulated in natural language, including GSM8k, while using less compute than previous models. Notably, our models can achieve robust performance in a few-shot setting without being explicitly trained on these evaluation datasets.

The main novelty of this paper is a large training dataset that juxtaposes natural language with the correct use of formal language, such as equations and diagrams. The data is collected from the arXiv preprint server and from web pages that we carefully process to minimise the loss of mathematical content. In addition to establishing a new SOTA result on the MATH dataset, this work sets a

[1]We omit the name of the model for the purpose of anonymization.

Question: A particle moves so that it is at $\left(3\sin\frac{t}{4}, 3\cos\frac{t}{4}\right)$ at time t. Find the speed of the particle, measured in unit of distance per unit of time.

Target: The parametric curve $(x, y) = \left(3\sin\frac{t}{4}, 3\cos\frac{t}{4}\right)$ describes a circle with radius 3. Furthermore, it makes a full revolution at time $t = 8\pi$. *(figure)* Therefore, the speed of the particle is $\frac{6\pi}{8\pi} = \boxed{\frac{3}{4}}$.

Model output: The speed is the magnitude of the velocity vector. Therefore, we must first find the velocity vector, which is the derivative of the position vector. We have

$$\mathbf{v} = \frac{d}{dt}\left(3\sin\frac{t}{4}, 3\cos\frac{t}{4}\right) = 3\left(\cos\frac{t}{4}, -\sin\frac{t}{4}\right)\frac{1}{4} = \left(\frac{3}{4}\cos\frac{t}{4}, -\frac{3}{4}\sin\frac{t}{4}\right).$$

Therefore, the magnitude of the velocity vector is

$$|\mathbf{v}| = \sqrt{\left(\frac{3}{4}\cos\frac{t}{4}\right)^2 + \left(-\frac{3}{4}\sin\frac{t}{4}\right)^2} = \sqrt{\frac{9}{16}\cos^2\frac{t}{4} + \frac{9}{16}\sin^2\frac{t}{4}}$$
$$= \frac{3}{4}\sqrt{\cos^2\frac{t}{4} + \sin^2\frac{t}{4}} = \frac{3}{4}.$$

Therefore, the speed of the particle is $\boxed{\frac{3}{4}}$.

Figure 2: Example question from the MATH dataset (Precalculus 4), along with the ground truth target and the Minerva solution. The model correctly takes the derivative of a position vector to compute the velocity. It then computes the norm of the resulting vector, and uses a trigonometric identity to obtain a final numerical answer. Minerva takes a completely different approach from the ground truth solution. The model output was not modified, except for a few newlines that were removed to save space.

new lower bound for the performance achievable on quantitative reasoning benchmarks simply by increasing data quality.

Existing benchmarks are limited with respect to quantitative reasoning; for this reason, we build a dataset of over 200 undergraduate-level questions in science and mathematics from MIT's OpenCourseWare (OCW); this provides a measure of our model's quantitative reasoning abilities in a chain-of-thought context beyond a pure mathematical setting.

1.2 Related Works

Solving quantitative reasoning problems expressed in natural language has been an active area of study [11, 12]. Prompting language models using chain-of-thought solutions can lead them to output step-by-step reasoning to unseen problems [13]. The GSM8k dataset [7] showed that training verifiers to rerank model outputs can lead to improved performance. The original version of GSM8k included special syntax for algebraic calculations, which were processed by a calculator. In this work we focus on self-contained models without access to external tools.

The standard method for evaluating language models on generative tasks is to greedily sample one solution per problem. Recent works [8, 14, 15, 16] have shown that it is advantageous to sample multiple solutions per problem, and then filter those down to a final answer. We find that majority voting [16] significantly improves performance over greedy decoding.

Code generation. Applying code generating models to mathematical problems has been an active area of exploration. PaLM [5] showed that a large language model with code in its training dataset can achieve good performance on a code version of GSM8k. Furthermore, by manually constructing prompts [10], the Codex model [8] can generate code solutions to MATH [6] problems. These solutions often rely on external libraries to perform mathematical operations such as solving equations or taking limits. This is a complementary approach to ours, in which we directly probe the model's capability to arrive at an answer by relying only on its own reasoning capability.

Formal mathematics. Mathematics developed as a discipline based in natural language, but its axiomatic fundamentals make it possible to simulate mathematical thinking. This can be achieved using specialized programming languages that facilitate the simulation of logical and mathematical thinking using a computer, such as Coq [17], Isabelle [18], HOL4 [19], Lean [20], Metamath [21] and Mizar [22]. Work on automation of proof assistants and automated theorem provers such as E [23], leanCoP [24], and Vampire [25] has substantially benefited from integration with machine learning methods [26, 27, 28, 29, 30].

Language models applied to formal and synthetic mathematical problems. Previous work trained language models to predict mathematical expressions [31, 28, 29, 32, 33]. In turn, such a predictive model can be used to guide a proof search, as done by [29]. Large language models excel in modelling natural language, though in the case of formal languages, models that facilitate retaining information about the graph structure of a given mathematical formula, such as GNNs, are still very competitive.

Modelling mathematics as a discipline of natural language. New benchmark datasets [6, 34] cover more advanced mathematical topics. In this domain language models are facing limited competition from other classes of models.

2 Training and Evaluation

2.1 Mathematical Training Dataset

Our models were trained on a dataset that includes 38.5B tokens from webpages filtered for mathematical content and from papers submitted to the arXiv preprint server. In addition, the dataset includes a general natural language dataset, which is the same on which BaseModel was pretrained. Our mathematical webpage dataset was constructed by collecting pages that contain mathematical expressions in MathJax format. The pages underwent a cleaning process that removes most HTML tags but preserves mathematical notation, including LaTeX symbols and formatting, such that mathematical formulae like $e^{\pi i} + 1 = 0$ or $E = mc^2$ are presented in full to the model. This procedure makes it possible for the model to perform well on tasks that require calculation and symbolic manipulation. Table 1 provides a breakdown of the training dataset. See appendix for more details.

Table 1: Proportion of data, and number of tokens, from each source in the technical training dataset. The general natural language dataset is a subset of the dataset used to pretrain the model.

Data source	Proportion of data	Tokens	Present during pretraining
Math Web Pages	45%	17.5B	No
arXiv	45%	21.0B	No
General Natural Language Data	10%	>100B	Yes

2.2 Models and Training Procedure

Our approach is to finetune pretrained decoder-only Transformer language models on our mathematical dataset using an autoregressive objective. Table 2 contains the main model and training hyperparameters. Additional details can be found in the appendix.

Table 2: Model architecture and training hyperparameters. Model training was resumed from the pretrained BaseModel, and the number of steps quoted refers only to continued training on the technical dataset.

Model	Layers	Heads	d_{model}	Parameters	Steps	Tokens
8B	32	16	4096	8.63B	624k	164B
62B	64	32	8192	62.50B	416k	109B

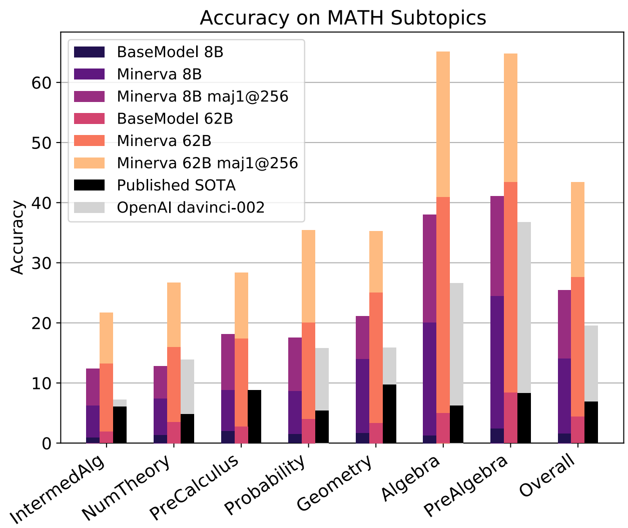

Figure 3: Performance on MATH dataset. `maj1@256` denotes evaluations where 256 samples were selected for each problem and only the most common answer was selected [16].

2.3 Evaluation Datasets

We focus on few shot evaluation. See appendix for a discussion of finetuning. For evaluation, we truncate the inputs from the left to 1024 tokens and we generate up to 512 tokens. When sampling once per problem, we sample greedily. When sampling multiple times per problem we use nucleus sampling [35] with temperature $T = 0.6$, $p = 0.95$. For generative tasks, the model produces a chain-of-thought and demarcates a final answer. We evaluate a solution as correct if the final answer matches with the ground truth solution, independent of the quality of the chain-of-thought preceding it. See appendix for further evaluation details. To evaluate correctness, we parse the final answers and compare them using the `SymPy` library [36]. This is done in order to correctly identify answers that are mathematically equivalent such as $1/\sqrt{3}$ and $\sqrt{3}/3$. See appendix for further details.

The existing datasets on which we focus are:

- MATH: a dataset of 12K middle school and high school mathematics problems [6]. Problem statements are written in LaTeX. We prompt the model with a fixed 4-shot prompt (listed in the appendix). This prompt includes four random examples from the training dataset whose ground truth targets are not too long.
- GSM8k: middle school math word problems [7]. Models are evaluated using the chain-of-thought prompt from Wei et al. [13]. Previous models evaluated on GSM8k made use of an external calculator. In this work, our model does not have access to any external tools.
- MMLU-STEM: subset of the MMLU dataset [37] focused on science, technology, engineering, and mathematics (STEM). We use the 5-shot prompt from the development set for each task. It is multiple choice and does not include a chain-of-thought.

2.4 Undergraduate-Level STEM Problems

To evaluate the scientific reasoning capabilities of Minerva, we harvested a set of STEM problems at the undergraduate level, most of which involve multi-step reasoning, which we refer to in this paper as OCWCourses. From publicly-available course materials offered by MIT (OpenCourseWare), we collected problems with automatically-verifiable solutions (either numeric or symbolically verifiable via `SymPy`) from courses including 'solid-state chemistry', 'information and entropy', 'differential equations', and 'special relativity.' These were processed by contractors to be self-contained and to have a clearly-delineated final answer (i.e., problems asking for a proof or open-ended short answer were not included). In total we curated 272 problems, 191 of which have numeric solutions and 81 with symbolic solutions. In the appendix, we detail the contributions from each course, and the process of converting these course materials into a format suitable for processing by language models. We also provide the text of all problems. We plan to release these as part of an open-source dataset which will be detailed in an upcoming manuscript.

2.5 Inference-Time Techniques

We find that we can considerably outperform greedy decoding by sampling $k > 1$ solutions (with a non-zero temperature) and selecting one using majority voting [16]. This consists of grouping predictions with respect to their final answer and selecting the most common answer. Following the notation of previous work [14], we denote this as `maj1@k`. A variation of this algorithm, denoted `majn@k`, involves selecting the n most common answers.

Intuitively, the reason this reranking algorithm improves performance is that while there are many ways to answer a question incorrectly, there are typically very few ways to answer correctly. This is in contrast to `pass@k`, where a task is considered solved if any single sample solves it. See Section 4.2 for more details about `pass@k` . In the appendix, we report how performance depends on k for different metrics.

The results in Table 3 are based on sampling with `maj1@k`; we report on `maj5@k` in the appendix. Even when `pass@k` continues to improve as k is increased, majority voting saturates faster: 97% of the large k accuracy is achieved at $k = 64$ for MATH and $k = 16$ for GSM8k. This is because majority voting selects the most common answer in the modeled distribution, and the error of this estimate decreases with increasing k. This is in contrast to `pass@k` where the performance improvement comes from the tail of the distribution, which can keep improving as k is increased.

An alternative is to rerank samples with respect to their negative log-likelihood. We found that majority voting performs significantly better than log-likelihood reranking.

3 Results

Table 3 summarizes the results for our 8B and 62B models on the evaluation datasets described in Section 2.3. The inference techniques we tried were described in Section 2.5, and in Table 3 we present only the best-performing technique. We use $k = 64$ for OCWCourses, $k = 256$ for MATH, and $k = 100$ for GSM8k. Figure 3 presents a breakdown of the MATH dataset results by subtopic. We present model output samples in Figures 1 and 2. In addition, we evaluated Minerva 62B on the National Math Exam in Poland and found that it achieves a score of 57%, which happens to be the national average in 2021 [38, p. 23].

We include results on the latest publicly available language model from OpenAI, davinci-002, evaluated using the OpenAI API with temperature set to the official recommendation ($T = 0.2$). Note that Minerva 62B outperforms davinci-002 and significantly outperforms the state-of-the-art on MATH, one of the hardest of quantitative reasoning datasets to date. GSM8k is an easier dataset, and while our mathematical training helps significantly, it does not enable the model to achieve state-of-the-art performance, which was achieved by a model with 9x the number of parameters. MMLU-STEM is a multiple choice dataset, on which Minerva achieves similar performance to Chinchilla [39]. Chinchilla is a considerably stronger base model as it uses roughly twice as much compute during pretraining as BaseModel, but our mathematical training helps bridge the gap.

While our main focus is on few shot evaluation, we also tried to finetune Minerva on MATH. While we did not observe any improvement, we found that finetuning BaseModel on MATH did give a significant improvement, which suggests that the marginal utility of standard finetuning decreases as the quality and diversity of the unsupervised finetuning dataset improves. Further details can be found in the appendix.

4 Performance Analysis

4.1 Model Mistakes

To better understand the types of mistakes our models make, we compare the performance of Minerva 8B and Minerva 62B on 216 problems with high confidence majority decisions of both

Table 3: **Model performance on several quantitative reasoning datasets.** For majority voting we use $k = 256$ samples for MATH, $k = 64$ for OCWCourses, and $k = 100$ for GSM8k. We evaluated datasets that did not have published results on recent models on OpenAI davinci-002. MMLU-STEM is a multiple choice task for which majority voting is not necessary. Superscripts denote results that are quoted from previous work: ^{a} GPT-2 [6], ^{b} PaLM 540B `maj1@40` [16], and ^{c} Chinchilla [39].

	MATH	OCWCourses	GSM8k	MMLU-STEM
BaseModel 8B	1.5%	1.5%	4.1%	22%
Minerva 8B	14.1%	7.7%	16.2%	36%
Minerva 8B, maj1@k	25.4%	12.5%	28.4%	-
BaseModel 62B	4.4%	5.9%	33.0%	39%
Minerva 62B	27.6%	12.9%	52.4%	54%
Minerva 62B, maj1@k	**43.4%**	**23.5%**	68.5%	-
OpenAI davinci-002	19.1%	14.8%	-	-
Published SOTA	6.9%^{a}	-	**74.4%**^{b}	**55%**^{c}

models. Specifically, we selected examples where the top answer received at least 15% of votes, and that either Minerva 8B was correct and Minerva 62B was incorrect (15 samples), or vice versa (201 samples). The categories and examples for each category are described in the appendix.

As shown in Table 4, the prevailing errors of the 8B model were related to incorrect reasoning or calculations. Many of the calculation errors were relatively benign mistakes in arithmetic. Solutions that were too short were relatively rare (in these cases, the model immediately produces an incorrect answer without any intermediate reasoning steps). Finally, in a few cases, the model hallucinates an equation or mathematical fact that is not real.

In the samples where the 62B model was incorrect, the dominating failure modes were again incorrect reasoning and incorrect calculations.

In summary, we find that the 62B Minerva model retains most of the skills of the 8B model and improves upon both reasoning and calculation robustness.

Table 4: Failure modes of the 8B Minerva model, out of 201 samples which the 62B model solved correctly and the 8B model did not.

Type of mistakes	Occurrences
Incorrect reasoning	82
Incorrect calculation	70
Misunderstands question	22
Uses incorrect fact	16
Solution too short	4
Hallucinated math objects	4
Other mistakes	3

4.2 False Positives

In our approach to solving quantitative reasoning problems, we are able to automatically verify whether the final answer to a problem is correct, but we do not have an automatic way to verify the model's chain of reasoning. This leaves open the possibility of false positives: samples which have the correct final answer, but for which the reasoning is incomplete or incorrect.

We selected 100 random questions from MATH (20 per difficulty level), along with answers sampled at zero temperature from the 62B model. We then manually inspected the answers to determine the false positive rate, which is the ratio between number of false positive examples and number of examples for which the final answer is correct; see Table 5. We found that the overall false positive rate is low, though it does increase with difficulty level.

Our focus on pass@1 and majority voting as the primary evaluation metrics is due in part to the fact that they are less susceptible to false positives than pass@k [14]. While the pass@256 accuracy is 84.5% for the 62B model, false positives account for part of it. We inspected the samples that failed in majority voting but passed on pass@k due to a single correct answer, and estimate the false positive rate for pass@256 to be 30% among samples selected in this way. After removing false positives, we estimate that the pass@256 accuracy to be bigger than 68%; see appendix for details.

Table 5: Estimated false positive rates of the 62B model on the MATH dataset, by difficulty level. The average is the estimated false positive rate on the MATH dataset, given by the average of per-level false positive rates weighted by positive rates.

	Difficulty level					
	1	2	3	4	5	Average
False positive rate	$< 5\%$	10%	$< 5\%$	15%	30%	8%

5 Memorization

A central question in interpreting Minerva's solutions is whether performance reflects genuine analytic capability or instead rote memorization. This is especially relevant as there has been much prior work indicating that language models often memorize some fraction of their training data [40, 41, 42]. When examining model solutions, we find that memorization of intermediate facts, such as numerical values of square roots or trigonometric identities, are crucial elements of model solutions. Truly strong performance would combine recall of intermediate facts with genuine solution synthesis. We would like to investigate a strong form of memorization, where model performance is a result of memorizing the explicit problems and solutions in our evaluation set, but also a weaker form, where the model has memorized alternate answers to the same questions.

In order to evaluate the degree to which our models solve problems by recalling information memorized from training data, we conduct three analyses on the MATH dataset. First we directly search for problems and solutions in our training corpus. Next, we generate modified versions of problems and evaluate our models' robustness to these changes. Finally, we measure the degree of overlap between the ground truth solutions and solutions generated by our model and measure the effect of this similarity on model performance. Overall, we find little evidence that the model's performance can be attributed to memorization.

5.1 Training and Evaluation Dataset Overlap

We selected the problems for which our 62B parameter model produced a correct answer, and filtered them to the 100 problems with the highest majority vote score, expecting that problems with a high majority vote score are more likely to have been memorized. For each of these question-answer pairs, we compute the BLEU score across chunks of 500 characters in our Math Web Pages dataset (a histogram of the BLEU scores is shown in the appendix). We then manually inspect the 250 documents with the highest BLEU scores. While many of the top matches were from homework help sites with math questions and solutions, none of the questions matched the questions in the subset of MATH under consideration. We have included these 250 segments in the appendix. We note that some problems from MATH can be found on the web. Nevertheless, this analysis concludes that these problems did not make it through our data collection process.

5.2 Performance on Modified MATH Problems

To further investigate memorization, we randomly selected twenty problems which the 62B model answered correctly under majority voting. We manually modified each problem either by introducing minor changes to problem wording (framing) or by changing the numbers which appeared in the problem and modifying the solution accordingly. We then compared the accuracy over sampled solutions before and after the modification. Results are shown in Figure 4. In both cases the accuracy before and after modifications are correlated, with no clear bias in favor of the original formulation. This is suggestive of minimal memorization. The modified problems are listed in the appendix.

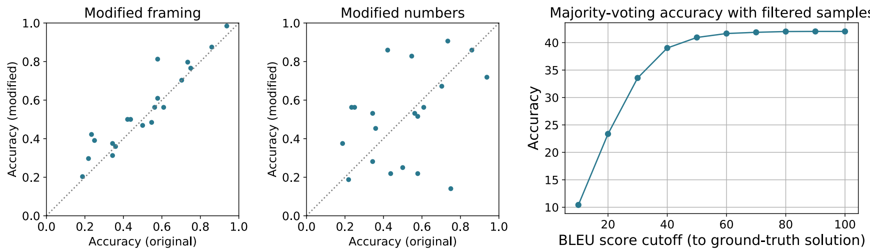

Figure 4: **Results indicating lack of memorization on MATH. Left, Center:** Accuracy of original questions from the MATH dataset and their modified versions. Each point represents a question. The x axis is accuracy on the original question, and the y axis is accuracy on the modified one. **Right:** Majority vote accuracy, computed only on samples with BLEU score to the ground truth solution less than or equal to the x-axis value.

5.3 BLEU Score Between Ground Truth and Generated Solutions

We seek to detect memorization of solutions by computing BLEU score between ground truth answers and model generated answers. We use the 62B model and analyze 256 samples per problem in the MATH dataset. First, we compute overlap statistics for all correct samples. We find that 160 out of 5,000 test questions have a sample with a BLEU score greater than or equal to 80 (see appendix). We note that they tend to be short solutions. To understand the effect of answer similarity on performance, we remove model samples above a certain BLEU score threshold, and recompute the majority vote accuracy. We find that majority vote performance is robust even down to relatively low similarities (see Figure 4), indicating that performance cannot be attributed to model outputs that are very similar to ground truth answers.

6 Conclusions and Discussion

In this work, we take an approach to quantitative reasoning that relies on solving problems using mathematical reasoning expressed in natural language. We show that by training a large language model on a high quality mathematical dataset, we are able to achieve strong performance on tasks that require logical reasoning, numerical calculation, and symbolic manipulation. Our model does not make use of external tools, and at inference time relies exclusively on autoregressive sampling to achieve this performance. Complementary approaches to quantitative reasoning include code-generating models and formal methods. These are all different routes toward a common goal: an agent that can reason about and solve quantitative problems. We believe that such an agent should combine useful elements from all of these approaches.

6.1 Limitations of Our Approach

Our approach to quantitative reasoning has several limitations. First, we have no automatic way of verifying the correctness of the model's answers. This is in contrast to formal approaches, for which automatic verification is intrinsic. Second, our model has no access to external tools such as a calculator or a Python interpreter. It is therefore limited in its ability to perform quantitative reasoning tasks that require complicated numerical calculations. Third, because our model was trained on a large amount of data, we have little direct control over the specific capabilities that the model acquired.

6.2 Societal Impact

Artificial neural networks capable of solving quantitative reasoning problems in a general setting have the potential of substantial societal impact. Minerva, while a step in this direction, is still far from achieving this goal, and its potential societal impact is therefore limited. The model's performance is still well below human performance, and furthermore, we do not have an automatic way of verifying the correctness of its outputs. If these issues could be solved, we expect the impacts of this model to be broadly positive. A direct application could be an accessible and affordable math tutor which could help improve educational inequalities.

References

[1] Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J Liu. Exploring the limits of transfer learning with a unified text-to-text transformer. *arXiv preprint arXiv:1910.10683*, 2019.

[2] Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are few-shot learners. *Advances in neural information processing systems*, 33:1877–1901, 2020.

[3] Jack W Rae, Sebastian Borgeaud, Trevor Cai, Katie Millican, Jordan Hoffmann, Francis Song, John Aslanides, Sarah Henderson, Roman Ring, Susannah Young, et al. Scaling language models: Methods, analysis & insights from training gopher. *arXiv preprint arXiv:2112.11446*, 2021.

[4] Shaden Smith, Mostofa Patwary, Brandon Norick, Patrick LeGresley, Samyam Rajbhandari, Jared Casper, Zhun Liu, Shrimai Prabhumoye, George Zerveas, Vijay Korthikanti, et al. Using deepspeed and megatron to train megatron-turing nlg 530b, a large-scale generative language model. *arXiv preprint arXiv:2201.11990*, 2022.

[5] Aakanksha Chowdhery, Sharan Narang, Jacob Devlin, Maarten Bosma, Gaurav Mishra, Adam Roberts, Paul Barham, Hyung Won Chung, Charles Sutton, Sebastian Gehrmann, Parker Schuh, Kensen Shi, Sasha Tsvyashchenko, Joshua Maynez, Abhishek Rao, Parker Barnes, Yi Tay, Noam Shazeer, Vinodkumar Prabhakaran, Emily Reif, Nan Du, Ben Hutchinson, Reiner Pope, James Bradbury, Jacob Austin, Michael Isard, Guy Gur-Ari, Pengcheng Yin, Toju Duke, Anselm Levskaya, Sanjay Ghemawat, Sunipa Dev, Henryk Michalewski, Xavier Garcia, Vedant Misra, Kevin Robinson, Liam Fedus, Denny Zhou, Daphne Ippolito, David Luan, Hyeontaek Lim, Barret Zoph, Alexander Spiridonov, Ryan Sepassi, David Dohan, Shivani Agrawal, Mark Omernick, Andrew M. Dai, Thanumalayan Sankaranarayana Pillai, Marie Pellat, Aitor Lewkowycz, Erica Moreira, Rewon Child, Oleksandr Polozov, Katherine Lee, Zongwei Zhou, Xuezhi Wang, Brennan Saeta, Mark Diaz, Orhan Firat, Michele Catasta, Jason Wei, Kathy Meier-Hellstern, Douglas Eck, Jeff Dean, Slav Petrov, and Noah Fiedel. Palm: Scaling language modeling with pathways, 2022. URL https://arxiv.org/abs/2204.02311.

[6] Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset, 2021. URL https://arxiv.org/abs/2103.03874.

[7] Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. Training verifiers to solve math word problems, 2021. URL https://arxiv.org/abs/2110.14168.

[8] Mark Chen, Jerry Tworek, Heewoo Jun, Qiming Yuan, Henrique Ponde de Oliveira Pinto, Jared Kaplan, Harri Edwards, Yuri Burda, Nicholas Joseph, Greg Brockman, Alex Ray, Raul Puri, Gretchen Krueger, Michael Petrov, Heidy Khlaaf, Girish Sastry, Pamela Mishkin, Brooke Chan, Scott Gray, Nick Ryder, Mikhail Pavlov, Alethea Power, Lukasz Kaiser, Mohammad Bavarian, Clemens Winter, Philippe Tillet, Felipe Petroski Such, Dave Cummings, Matthias Plappert, Fotios Chantzis, Elizabeth Barnes, Ariel Herbert-Voss, William Hebgen Guss, Alex Nichol, Alex Paino, Nikolas Tezak, Jie Tang, Igor Babuschkin, Suchir Balaji, Shantanu Jain, William Saunders, Christopher Hesse, Andrew N. Carr, Jan Leike, Josh Achiam, Vedant Misra, Evan Morikawa, Alec Radford, Matthew Knight, Miles Brundage, Mira Murati, Katie Mayer, Peter Welinder, Bob McGrew, Dario Amodei, Sam McCandlish, Ilya Sutskever, and Wojciech Zaremba. Evaluating large language models trained on code, 2021. URL https://arxiv.org/abs/2107.03374.

[9] Jacob Austin, Augustus Odena, Maxwell Nye, Maarten Bosma, Henryk Michalewski, David Dohan, Ellen Jiang, Carrie Cai, Michael Terry, Quoc Le, et al. Program synthesis with large language models. *arXiv preprint arXiv:2108.07732*, 2021.

[10] Iddo Drori, Sarah Zhang, Reece Shuttleworth, Leonard Tang, Albert Lu, Elizabeth Ke, Kevin Liu, Linda Chen, Sunny Tran, Newman Cheng, Roman Wang, Nikhil Singh, Taylor L. Patti,

Jayson Lynch, Avi Shporer, Nakul Verma, Eugene Wu, and Gilbert Strang. A neural network solves, explains, and generates university math problems by program synthesis and few-shot learning at human level, 2021. URL https://arxiv.org/abs/2112.15594.

[11] Rik Koncel-Kedziorski, Hannaneh Hajishirzi, Ashish Sabharwal, Oren Etzioni, and Siena Dumas Ang. Parsing algebraic word problems into equations. *Transactions of the Association for Computational Linguistics*, 3, 2015.

[12] Mohammad Javad Hosseini, Hannaneh Hajishirzi, Oren Etzioni, and Nate Kushman. Learning to solve arithmetic word problems with verb categorization. *EMNLP*, 523533, 2014.

[13] Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Ed Chi, Quoc Le, and Denny Zhou. Chain of thought prompting elicits reasoning in large language models, 2022. URL https://arxiv.org/abs/2201.11903.

[14] Yujia Li, David Choi, Junyoung Chung, Nate Kushman, Julian Schrittwieser, Rémi Leblond, Tom Eccles, James Keeling, Felix Gimeno, Agustin Dal Lago, Thomas Hubert, Peter Choy, Cyprien de Masson d'Autume, Igor Babuschkin, Xinyun Chen, Po-Sen Huang, Johannes Welbl, Sven Gowal, Alexey Cherepanov, James Molloy, Daniel J. Mankowitz, Esme Sutherland Robson, Pushmeet Kohli, Nando de Freitas, Koray Kavukcuoglu, and Oriol Vinyals. Competition-level code generation with alphacode, 2022. URL https://arxiv.org/abs/2203.07814.

[15] Romal Thoppilan, Daniel De Freitas, Jamie Hall, Noam Shazeer, Apoorv Kulshreshtha, Heng-Tze Cheng, Alicia Jin, Taylor Bos, Leslie Baker, Yu Du, YaGuang Li, Hongrae Lee, Huaixiu Steven Zheng, Amin Ghafouri, Marcelo Menegali, Yanping Huang, Maxim Krikun, Dmitry Lepikhin, James Qin, Dehao Chen, Yuanzhong Xu, Zhifeng Chen, Adam Roberts, Maarten Bosma, Vincent Zhao, Yanqi Zhou, Chung-Ching Chang, Igor Krivokon, Will Rusch, Marc Pickett, Pranesh Srinivasan, Laichee Man, Kathleen Meier-Hellstern, Meredith Ringel Morris, Tulsee Doshi, Renelito Delos Santos, Toju Duke, Johnny Soraker, Ben Zevenbergen, Vinodkumar Prabhakaran, Mark Diaz, Ben Hutchinson, Kristen Olson, Alejandra Molina, Erin Hoffman-John, Josh Lee, Lora Aroyo, Ravi Rajakumar, Alena Butryna, Matthew Lamm, Viktoriya Kuzmina, Joe Fenton, Aaron Cohen, Rachel Bernstein, Ray Kurzweil, Blaise Aguera-Arcas, Claire Cui, Marian Croak, Ed Chi, and Quoc Le. Lamda: Language models for dialog applications, 2022. URL https://arxiv.org/abs/2201.08239.

[16] Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models, 2022. URL https://arxiv.org/abs/2203.11171.

[17] The Coq development team. *The Coq reference manual*, 2022. URL http://coq.inria.fr. Version 8.15.

[18] Makarius Wenzel, Lawrence C. Paulson, and Tobias Nipkow. The isabelle framework. In Otmane Ait Mohamed, César Muñoz, and Sofiène Tahar, editors, *Theorem Proving in Higher Order Logics*, pages 33–38, Berlin, Heidelberg, 2008. Springer Berlin Heidelberg. ISBN 978-3-540-71067-7.

[19] John Harrison. Hol light: A tutorial introduction. In Mandayam Srivas and Albert Camilleri, editors, *Formal Methods in Computer-Aided Design*, pages 265–269, Berlin, Heidelberg, 1996. Springer Berlin Heidelberg. ISBN 978-3-540-49567-3.

[20] Leonardo Mendonça de Moura, Soonho Kong, Jeremy Avigad, Floris van Doorn, and Jakob von Raumer. The lean theorem prover (system description). In Amy P. Felty and Aart Middeldorp, editors, *CADE*, volume 9195 of *Lecture Notes in Computer Science*, pages 378–388. Springer, 2015. ISBN 978-3-319-21400-9. URL http://dblp.uni-trier.de/db/conf/cade/cade2015.html#MouraKADR15.

[21] Norman D. Megill and David A. Wheeler. *Metamath: A Computer Language for Pure Mathematics*. Lulu Press, Morrisville, North Carolina, 2019. URL http://us.metamath.org/downloads/metamath.pdf. http://us.metamath.org/downloads/metamath.pdf.

[22] Adam Grabowski, Artur Kornilowicz, and Adam Naumowicz. Mizar in a nutshell. *J. Formalized Reasoning*, 3(2):153–245, 2010. doi: 10.6092/issn.1972-5787/1980. URL https://doi.org/10.6092/issn.1972-5787/1980.

[23] Stephan Schulz. System Description: E 1.8. In Ken McMillan, Aart Middeldorp, and Andrei Voronkov, editors, *Proc. of the 19th LPAR*, volume 8312 of *LNCS*. Springer, 2013.

[24] Jens Otten. leancop 2.0 and ileancop 1.2: High performance lean theorem proving in classical and intuitionistic logic (system descriptions). In Alessandro Armando, Peter Baumgartner, and Gilles Dowek, editors, *Automated Reasoning*, pages 283–291, Berlin, Heidelberg, 2008. Springer Berlin Heidelberg. ISBN 978-3-540-71070-7.

[25] Laura Kovács and Andrei Voronkov. First-order theorem proving and Vampire. In *CAV*, 2013.

[26] Alexander A. Alemi, François Chollet, Niklas Een, Geoffrey Irving, Christian Szegedy, and Josef Urban. Deepmath - Deep Sequence Models for Premise Selection. In *Proceedings of the 30th International Conference on Neural Information Processing Systems*, NIPS'16, pages 2243–2251, USA, 2016. Curran Associates Inc. ISBN 978-1-5108-3881-9. URL http://dl.acm.org/citation.cfm?id=3157096.3157347.

[27] Zarathustra Amadeus Goertzel, Karel Chvalovský, Jan Jakubuv, Miroslav Olsák, and Josef Urban. Fast and slow enigmas and parental guidance. *CoRR*, abs/2107.06750, 2021. URL https://arxiv.org/abs/2107.06750.

[28] Wenda Li, Lei Yu, Yuhuai Wu, and Lawrence C. Paulson. Isarstep: a benchmark for high-level mathematical reasoning. In *ICLR*, 2021.

[29] Stanislas Polu and Ilya Sutskever. Generative language modeling for automated theorem proving. *arXiv preprint arXiv:2009.03393*, 2020.

[30] Cezary Kaliszyk, Josef Urban, Henryk Michalewski, and Miroslav Olšák. Reinforcement learning of theorem proving. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, editors, *Advances in Neural Information Processing Systems*, volume 31. Curran Associates, Inc., 2018. URL https://proceedings.neurips.cc/paper/2018/file/55acf8539596d25624059980986aaa78-Paper.pdf.

[31] Markus N. Rabe, Dennis Lee, Kshitij Bansal, and Christian Szegedy. Language modeling for formal mathematics. *CoRR*, abs/2006.04757, 2020. URL https://arxiv.org/abs/2006.04757.

[32] Yuhuai Wu, Markus N. Rabe, Wenda Li, Jimmy Ba, Roger B. Grosse, and Christian Szegedy. LIME: learning inductive bias for primitives of mathematical reasoning. *CoRR*, abs/2101.06223, 2021. URL https://arxiv.org/abs/2101.06223.

[33] Jesse Michael Han, Jason Rute, Yuhuai Wu, Edward Ayers, and Stanislas Polu. Proof artifact co-training for theorem proving with language models. In *International Conference on Learning Representations*, 2022. URL https://openreview.net/forum?id=rpxJc9j04U.

[34] Sean Welleck, Jiacheng Liu, Ronan Le Bras, Hannaneh Hajishirzi, Yejin Choi, and Kyunghyun Cho. Naturalproofs: Mathematical theorem proving in natural language. *CoRR*, abs/2104.01112, 2021. URL https://arxiv.org/abs/2104.01112.

[35] Ari Holtzman, Jan Buys, Li Du, Maxwell Forbes, and Yejin Choi. The curious case of neural text degeneration, 2019. URL https://arxiv.org/abs/1904.09751.

[36] Aaron Meurer, Christopher P. Smith, Mateusz Paprocki, Ondřej Čertík, Sergey B. Kirpichev, Matthew Rocklin, AMiT Kumar, Sergiu Ivanov, Jason K. Moore, Sartaj Singh, Thilina Rathnayake, Sean Vig, Brian E. Granger, Richard P. Muller, Francesco Bonazzi, Harsh Gupta, Shivam Vats, Fredrik Johansson, Fabian Pedregosa, Matthew J. Curry, Andy R. Terrel, Štěpán Roučka, Ashutosh Saboo, Isuru Fernando, Sumith Kulal, Robert Cimrman, and Anthony Scopatz. Sympy: symbolic computing in python. *PeerJ Computer Science*, 3:e103, January 2017. ISSN 2376-5992. doi: 10.7717/peerj-cs.103. URL https://doi.org/10.7717/peerj-cs.103.

[37] Dan Hendrycks, Collin Burns, Steven Basart, Andy Zou, Mantas Mazeika, Dawn Song, and Jacob Steinhardt. Measuring massive multitask language understanding. *CoRR*, abs/2009.03300, 2020. URL https://arxiv.org/abs/2009.03300.

[38] CKE. *Skale centylowe wyników - matura 2021*, 2021. URL https://g.infor.pl/p/_files/37182000/1-matura-2021-centyle-37181904.pdf.

[39] Jordan Hoffmann, Sebastian Borgeaud, Arthur Mensch, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, Tom Hennigan, Eric Noland, Katie Millican, George van den Driessche, Bogdan Damoc, Aurelia Guy, Simon Osindero, Karen Simonyan, Erich Elsen, Jack W. Rae, Oriol Vinyals, and Laurent Sifre. Training compute-optimal large language models, 2022. URL https://arxiv.org/abs/2203.15556.

[40] Trieu H Trinh and Quoc V Le. A simple method for commonsense reasoning. *arXiv preprint arXiv:1806.02847*, 2018.

[41] Alec Radford, Jeffrey Wu, Rewon Child, David Luan, Dario Amodei, Ilya Sutskever, et al. Language models are unsupervised multitask learners. *OpenAI blog*, 1(8):9, 2019.

[42] Nicholas Carlini, Daphne Ippolito, Matthew Jagielski, Katherine Lee, Florian Tramer, and Chiyuan Zhang. Quantifying memorization across neural language models, 2022. URL https://arxiv.org/abs/2202.07646.

[43] Adam Roberts, Hyung Won Chung, Anselm Levskaya, Gaurav Mishra, James Bradbury, Daniel Andor, Sharan Narang, Brian Lester, Colin Gaffney, Afroz Mohiuddin, Curtis Hawthorne, Aitor Lewkowycz, Alex Salcianu, Marc van Zee, Jacob Austin, Sebastian Goodman, Livio Baldini Soares, Haitang Hu, Sasha Tsvyashchenko, Aakanksha Chowdhery, Jasmijn Bastings, Jannis Bulian, Xavier Garcia, Jianmo Ni, Andrew Chen, Kathleen Kenealy, Jonathan H. Clark, Stephan Lee, Dan Garrette, James Lee-Thorp, Colin Raffel, Noam Shazeer, Marvin Ritter, Maarten Bosma, Alexandre Passos, Jeremy Maitin-Shepard, Noah Fiedel, Mark Omernick, Brennan Saeta, Ryan Sepassi, Alexander Spiridonov, Joshua Newlan, and Andrea Gesmundo. Scaling up models and data with `t5x` and `seqio`, 2022. URL https://arxiv.org/abs/2203.17189.

Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] See Section 6
   (c) Did you discuss any potential negative societal impacts of your work? [Yes] See Section 6
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [N/A]
   (b) Did you include complete proofs of all theoretical results? [N/A]
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [No] : The code and models are proprietary.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No] We only run the experiments once. This is the standard approach for work with large language models because of their significant compute requirements.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes]
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes]
   (b) Did you mention the license of the assets? [Yes] : OCWCourses will be released with license . [No] : The mathematical webpages dataset is propietary.
   (c) Did you include any new assets either in the supplemental material or as a URL? [Yes] OCWCourses questions are part of supplementary material
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [Yes] : We share the main instructions given to contractors.
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [No] : We hired a dedicated workforce for this task rather than using a crowdsourcing service. As such, our approach to hiring and compensation is proprietary.