ENVISE: ENHANCING VISUAL REPRESENTATIONS FOR FAST AND EFFICIENT OBJECT RECOGNITION

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ABSTRACT

We propose ENVISE, an online distillation framework that ENhances VISual representations for fast and Efficient object recognition. We are motivated from the observation that, in real-world scenarios, a smart vision system should be able to specialize on the subset of frequently occurring classes with high computational efficiency. Our framework comprises of a binary student network (BSN) which is trained using the pseudo-labels generated by the teacher network (TN) on an unlabeled image stream. The BSN must also determine when a rare (or unknown) class is present in the image stream so that the TN can be used in such cases. To achieve this, we propose an attention triplet loss, to ensure the BSN not only learns the labels from the TN but also emphasizes the same semantically relevant regions of the image as the latter. When the prior probabilities of images from the input stream vary, we demonstrate the ability of the BSN to adapt to the TN faster than the real-valued student network. To compute the overall efficiency of ENVISE, we propose GiE (Gain in Efficiency), a new evaluation metric, which measures the overall FLOPs based on the number of times the BSN and TN are used to process the image stream. We benchmark CIFAR-100 and tiny-imagenet datasets by creating meaningful inlier (frequent) and outlier (infrequent) class pairs and show that ENVISE outperforms state-of-the-art (SOTA) outlier detection methods. ENVISE also improves separation between inlier and outlier classes in the feature space and achieves higher gain in computational efficiency than these baselines.

1 INTRODUCTION

Deep CNNs that are widely used for image classification Huang et al. (2017) often require large computing resources, and process each image with high computational complexity (Livni et al. (2014)). In real-world scenarios, the prior probability of occurrence of individual classes in an image stream is often unknown, and varies with the deployed environment. For example, in a zoo, the image stream input to the deep CNN will mostly consist of *animals* while that of *vehicles* would be rare. Other object classes such as *furniture* and *aircraft* would be absent. Therefore, only a subset of the many classes known to a deep CNN may be presented to it for classification during operation.

To process the subset of frequently occurring classes with high computational efficiency and adapt to the varying prior probability of individual classes in the deployed scenario, we propose an online distillation framework - ENVISE. Here, we employ a high capacity general purpose image classifier as the *teacher network* (TN), while the *student network* (SN) is a low capacity network. For greater computational efficiency and faster convergence, we require the coefficients of the SN to be binary and refer to it as the binary student network (BSN). When the BSN is first deployed, it is trained on the unlabeled image stream using the predicted labels of the TN as *pseudo-labels*. Once the BSN converges to the performance of the TN, it is used as the primary classifier to rapidly classify the frequent classes (faster than the TN), thereby increasing the computational efficiency. However, if a rare class appears (i.e. a class absent during online training) in the image stream , the BSN must accurately detect it is as a class it has not yet encountered, which is then processed by the TN. Since the BSN is trained only on the frequent classes, we refer to these classes as *inlier* (IL) and the rare classes as *outlier* (OL) classes . It is important to note, that the OL classes are *outliers* with respect to the BSN only, but are known to the TN. Furthermore, detecting extremely rare classes which are unknown to both the BSN and TN, (i.e. global unknowns) are beyond the scope of this paper. Thus, assuming that the TN knows all possible classes from the deployed environment, our goal is

to increase the overall computational efficiency of the system (without sacrificing performance) by exploiting the higher probability of occurrence of frequent classes in a given scenario.

Our approach for detecting OL classes is motivated from the observation by Ren et al. (2019), where networks incorrectly learn to emphasize the background rather than the semantically important regions of the image leading to poor understanding of the IL classes. We know that *attention maps* highlight the regions of the image responsible for the classifier's prediction (Selvaraju et al. (2017)). In the conventional knowledge distillation process, the attention map of the BSN can focus on the background even when the attention map of the TN emphasizes the semantically meaningful regions of the image. In doing so, the BSN memorizes the labels of the TN, making it difficult to differentiate between the representations of IL and OL classes. To mitigate these issues, we propose an *attention triplet loss* that achieves two key objectives - a) guide the attention map of the correct prediction of the BSN to focus on the semantically meaningful regions, and b) simultaneously ensure that the attention map from the correct and incorrect predictions of the BSN are dissimilar. We show that by focusing on the semantically relevant regions of the image, the BSN will learn to distinguish between the representations of IL and OL classes, thereby improving its ability to detect OL classes.

To assess the overall gain in computational efficiency of ENVISE, we propose a new evaluation metric - *GiE*, based on the number of times the BSN and TN are used to process the image stream. Since the deployed scene is comprised mostly of IC classes with few OD classes, we expect the BSN to be employed most of the time for classifying IC classes. The TN is used rarely (i.e. only when the BSN detects an OD class), thereby improving the overall computational efficiency of the system. To the best of our knowledge, we are the first to propose supervision on attention maps for OD detection, and a new evaluation metric that measures the gain in computational efficiency of an online distillation framework. A summary of our contributions is:

- **Faster convergence of BSN:** We theoretically justify and empirically illustrate that the BSN adapts to the performance of TN faster than the real-valued SN (RvSN). We also demonstrate the faster convergence of BSN for different BSN architectures over its corresponding RvSN.
- **Attention triplet loss (L_{at})**, which guides the BSN to focus on the semantically meaningful regions of the image, thereby improving OD detection.
- **New evaluation metric - GiE** to measure the overall gain in computational efficiency of the online distillation framework, and
- We **benchmark CIFAR-100 and tiny-imagenet datasets** with SOTA outlier detection methods by creating meaningful IC and OD class pairs. ENVISE outperforms these baseline methods for OD detection, improves separation of IC and OD classes in the feature space, and yields the highest gain in computational efficiency.

2 Related Work

Online distillation : Distilling knowledge to train a low-capacity student network from a high-capacity teacher network has been proposed as part of model compression (Hinton et al. (2015)). Wang & Yoon (2020) provide a detailed review of different knowledge distillation methods. Mullapudi et al. (2019) propose an online distillation framework for semantic segmentation in videos, while Abolghasemi et al. (2019) use knowledge distillation to augment a visuomotor policy from visual attention. Lin et al. (2019) propose an ensemble student network that recursively learns from the teacher network in closed loop manner while Kim et al. (2019) use a feature fusion module to distill knowledge. Gao et al. (2019) propose online mutual learning and Cioppa et al. (2019) propose to periodically update weights to train an ensemble of student network. These ensemble based methods require large computational resources and are expensive to train. However, ENVISE involves training a compact model that mimics the performance of the TN with less computation.

Outlier detection : Outlier detection or out-of-distribution detection (OOD) refers to detecting a sample from an unknown class (Hendrycks & Gimpel (2016)). Existing SOTA OOD methods use outlier samples during training or validation. For e.g. Yu & Aizawa (2019) increases the distance between ID and OD samples during training while Vyas et al. (2018); Lee et al. (2018) add perturbations onto test images and train their network to differentiate between ID and OD samples. However, the ENVISE BSN is adaptively trained only on ID class images without any knowledge of outlier class images, which is a more realistic scenario. Without using outlier classes during training, Hendrycks & Gimpel (2016); Papadopoulos et al. (2019); Bendale & Boult (2016); Hendrycks

et al. (2019) employ class probabilities or measure the similarity between ID and outlier features. Yoshihashi et al. (2019) generate discriminative feature space using reconstruction based approaches. However, the likelihood of correct classification by these networks is based on background regions as described in Ren et al. (2019). Hence, these methods fail when applied to real-world ID and outlier class pairs since the confidence of prediction is based on background information. We overcome this drawback by proposing supervision on attention maps where we guide the BSN to focus on the important semantic regions using the attention map of the teacher network as a reference. This enables ENVISE to classify ID images with high confidence while also improving OD detection.

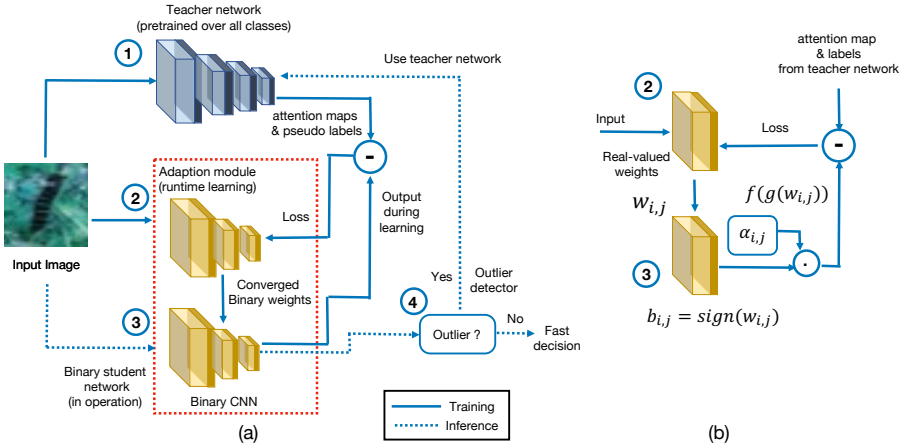

Figure 1: (a) The ENVISE architecture adaptively trains the BSN from predictions of the TN, (b) The process updates real valued weights that minimize the error produced by their binarized version.

3 Online Distillation for Fast and Efficient Object Recognition

The framework of ENVISE is shown in Figure 1 (a). Initially, the TN (indicated by "1" in the figure) classifies the images from the input stream with high accuracy, albeit at a slower rate. The BSN (red dotted box) comprises of an adaptation module ("2") and a binary CNN ("3"). Given an image stream, the adaptation module learns the optimum binary weights to mimic the performance of the TN. Once its accuracy converges on that of the TN, the adaption of the real-valued weights stops but the inference with binary weights continues. The OD detector ("4") uses the softmax output of the BSN as confidence to either generate the final prediction (if the confidence is high), or treats the image as an OD class and redirects it to the TN for classification.

Binarization algorithm for student network: During adaptive training, the BSN does not have access to the labeled data and mimics the behavior of the TN on the ID classes as shown in Figure 1(b). For an input image x, let $f(g(w_{i,j}), x)$ represent the output of the BSN where $w_{i,j}$ are real-valued weights of ith kernel in the jth layer, and $g(.)$ is a binarizing function for these weights. During adaptive training, the error between the output of the BSN and TN is minimized by optimizing $w^*_{i,j} = \arg\min_{g(w_{i,j})} \|L\|$, where L is the overall loss function used for training the BSN. While the intermediate weight $w_{i,j}$ is computed during adaptive learning, the binary versions of these weights $w^*_{i,j} = g(w_{i,j})$ are used for fast computation. *After convergence, the real valued weights are discarded, and only the binary weights are used*. The question becomes what is a good choice for the binarization function $g(.)$? Following Rastegari et al. (2016), we find a binary vector $b_{i,j}$ (whose elements are either $+1$ or -1), and a non-negative scalar $\alpha_{i,j}$ such that $w^*_{i,j} = \alpha_{i,j}.b_{i,j}$ is the best approximation of $w_{i,j}$ in a minimum squared error sense, where $\alpha_{i,j} = \frac{1}{N}|w_{i,j}|_1$, $b_{i,j} = sign(w_{i,j})$. However, unlike Rastegari et al. (2016) where a standalone network is binarized, our BSN learns the $w^*_{i,j}$ during adaptive training in an online distillation framework. In doing so, the BSN compensates for the effects of binarization on the overall loss function.

Our motivation for using the BSN is that it classifies the image stream with high accuracy and utilizes fewer FLOPs as compared to RvSN. This results in an increase in computational efficiency of the overall framework. Furthermore, we demonstrate that the BSN converges to the performance of the TN faster than the RvSN, which we theoretically emphasize in the Lemma below and experimentally validate in Section 4. The proof of the Lemma is presented in the Appendix A.1.

Lemma 3.1. *Let RvSN and BSN represent the real-valued student network and binary student network respectively with the same network architecture. Let $R(.)$ denote the rate of convergence in terms of accuracy when the student network is adaptively trained using the pseudo-labels from the teacher network. Then, $R(BSN) > R(RvSN)$ for the same image stream and number of iterations.*

Knowledge transfer from teacher to student network: Given an unlabeled image stream, the BSN is trained from the predictions of the TN as pseudo labels using cross-entropy loss, as $L_d = -\frac{1}{N}\sum_i y_i log(s_i)$. Here, y and s are the pseudo-labels generated by the TN and the softmax output of the BSN respectively, N is the total images in image stream. Once the BSN converges to the performance of the TN, we employ the BSN as the primary classifier during inference. When the deployed scenario does not change for long periods of time, the BSN classifies the ID classes from the image stream faster than the TN without relying on the latter. This improves the overall computational efficiency of the framework as the TN is now used to classify only the OD classes (number of OD images << number of ID images). However to maintain overall accuracy, the BSN must also accurately determine when the image from the input stream belongs to an OD class so that the TN can be used in such cases.

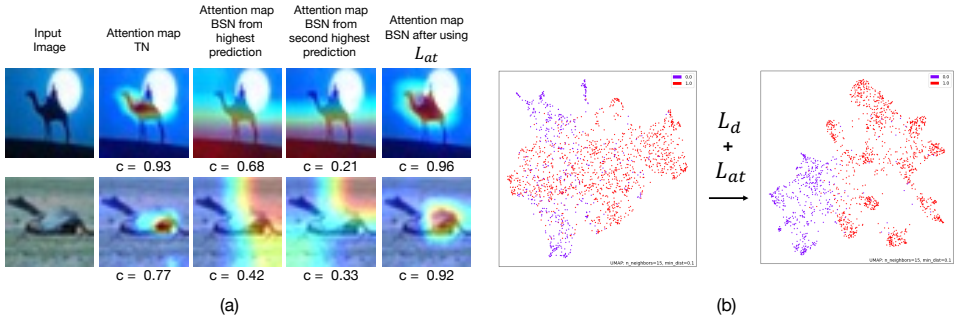

Figure 2: (a) The attention map visualization illustrating that after the BSN is trained using the proposed attention triplet loss, it learns to focus of the semantically meaningful regions in the image. The confidence of prediction (c) also increases. (b) Separation between ID and OD classes in feature space improves with the proposed attention triplet loss. More visualizations in Appendix A.4

We know that attention maps highlight the regions of the image responsible for the classifier's prediction (Li et al. (2018)). Some examples of the attention maps from the correct predictions of the BSN and the TN in Figure 2 (a). The first three columns in Figure 2 (a) show that although the BSN (adaptively trained using L_d) and the TN both correctly classify the images, their attention maps are significantly different. The attention map from the correct prediction of the BSN focuses on the background while that of the TN lies on the semantically meaningful regions of the image. Furthermore, the attention map of the BSN's correct and incorrect prediction are visually similar which causes the BSN to learn incorrect representations of the ID images while correctly classifying them. To better differentiate between ID and OD classes, the BSN should not memorize the predictions of the TN, but learn proper representations of the ID images.

To achieve this, we propose the *attention triplet loss* given in Equation 1 that causes the BSN's attention map to be similar to that of the TN while forcing the attention map from the BSN's correct and incorrect prediction to be different. We use Grad-CAM (Selvaraju et al. (2017)) to compute the attention map A from the one-hot vector of the predicted class. To generate the attention map triplets, we use the attention map from TN's prediction A_t as the *anchor*. The *hard positive* attention map A_{s_p} is obtained from correct prediction of BSN. When the BSN misclassifies an image (prediction of BSN and TN are different), we use the label from the TN to compute A_{s_p}. Motivated by the findings in (Wang et al. (2019)), we observe that the attention maps generated from the correct and incorrect predictions of the BSN are visually similar. Hence, we use the attention map from incorrect prediction as the *hard negative* attention map A_{s_n} to enforce its separability from A_{s_p}. To avoid bad local minima and a collapsed model early on during training, mining suitable hard negatives are important (Schroff et al. (2015)). Hence, we use the attention map from the second most probable class (incorrect prediction) as our hard negative due to its similarity with the hard positive (third and fourth column in Figure 2 (a)). Thus, we formulate the attention triplet loss L_{at} as:

$$L_{at} = \frac{1}{N}\frac{1}{K}\sum_{n}^{N}(\sum_{k}^{K}\left\|A_{t_k} - A_{s_{p_k}}\right\|_2 - \sum_{k}^{K}\left\|A_{t_k} - A_{s_{n_k}}\right\|_2 + \delta) \tag{1}$$

Initially, the hard negative lies within the margin since its squared distance with the hard positive is small. Hence, L_{at} enforces separation between hard positive and hard negative by a distance greater than the value of margin δ, which we empirically set as 1.5. N is the total number of samples and K is the total number of pixels in A. Using L_{at} and L_d, we formulate our final objective function as:

$$L_f = \lambda_1 L_d + \lambda_2 L_{at} \tag{2}$$

where λ_1 and λ_2 are the weights of L_d and L_{at} which we empirically set as 1 and 0.2 respectively. The effect of L_{at} is shown in the last column of Figure 2 (a) where the BSN learns proper representations of the ID image due to which its attention map is very similar to that of the TN (second column). Furthermore, the improvement in feature space separation between the ID and OD classes in Figure 2 (b) shows that the BSN not only forms clusters of the different ID classes (in red), but is also well separated from the OD classes (in purple).

4 Experimental Details

Datasets and implementation: We evaluate ENVISE on CIFAR-100 (Krizhevsky & Hinton (2009)) and tiny-imagenet (TI) (Yao & Miller (2015)) datasets by creating meaningful ID and OD *super-class* pairs that mimic real-world scenarios. On the test set of CIFAR-100 we create 12 super-classes, and on the validation set of the TI dataset we generate 10 super-classes. These super-classes are grouped into different pairs of ID and OD classes as shown in Table 1.

Table 1: Different pairs of ID and OD super-classes on the CIFAR-100 and tiny-ImageNet datasets. The number of individual classes can be obtained by dividing # inlier or # outlier images by 100 for CIFAR-100, and 50 for tiny-imagenet dataset respectively. The classes within each super-class is presented in the Appendix A.4

CIFAR-100 Krizhevsky & Hinton (2009)			tiny-imagenet Yao & Miller (2015)		
ID \ OD classes	# ID images	# OD images	ID \ outlier classes	# ID images	# OD images
aquatic animals \ food container (C_1)	1200	500	animals \ garments (T_1)	1250	900
flora \ electrical items (C_2)	900	500	reptiles \ edible items (T_2)	900	800
fruits & vegetables \ furniture (C_3)	500	500	aquatic animals \ birds (T_3)	600	250
insects \ manmade things (C_4)	800	500	household items \ nature (T_4)	2050	700
animals \ people (C_5)	2500	500	vehicles \ miscellaneous (T_5)	750	1800
outdoor places \ vehicles (C_6)	500	1000	-	-	-

We use DenseNet-201 (Huang et al. (2017)) as the TN, pre-trained on the training set of all classes of CIFAR-100 or the TI dataset individually. The BSN is a VGG16 (Simonyan & Zisserman (2014)) network whose weights, except for the first and last convolution layer, are binarized as described in Section 3. We also compare this with other choices for the BSN including AlexNet, ResNet-18 and ResNet-50. In the Appendix A.3, we illustrate that ENVISE is insensitive to the specific TN and BSN architecture used and achieves high performance gains even with different network architectures.

Training and evaluation: Throughout our experiments, we fix the TN and do not train it. We train the BSN using the pseudo-labels and attention map from the TN using the cost function in eq. 2 only on the ID classes of the super-class from Table 1. This is done using a learning rate of $1e^{-4}$ for 10 epochs using the Adam optimizer (Kingma & Ba (2014)) with a batch size of 10. Once the BSN converges to the performance of the TN, we employ it as our primary classifier during inference. We assume that the distribution of classes will not change rapidly in the deployed scenario, and that the BSN can be used for inference for long durations (e.g days, weeks or months). Thus, the epochs for online distillation are expected to require a small fraction of that time. We also evaluate the RvSN as well as other variants of the SN (e.g. direct quantization of RvSN and pretrained SN) and show that the BSN has the highest accuracy. For each image from the input stream during inference (comprising of ID and OD classes), following Hendrycks & Gimpel (2016), we compute the confidence of prediction from the softmax probability of the predicted class. If the confidence is low, we treat the image as an OD class and transfer it to the TN for classification.

Gradually changing the deployed scenario: When the prior probabilities of the classes change in the deployed scenario, the BSN quickly re-trains to learn the new ID classes and regains efficiency on the new image stream. To illustrate this, Figure 3 depicts the learning behavior of the BSN (green line) when the ID and OD classes are changed. Initially, we assume that the image stream

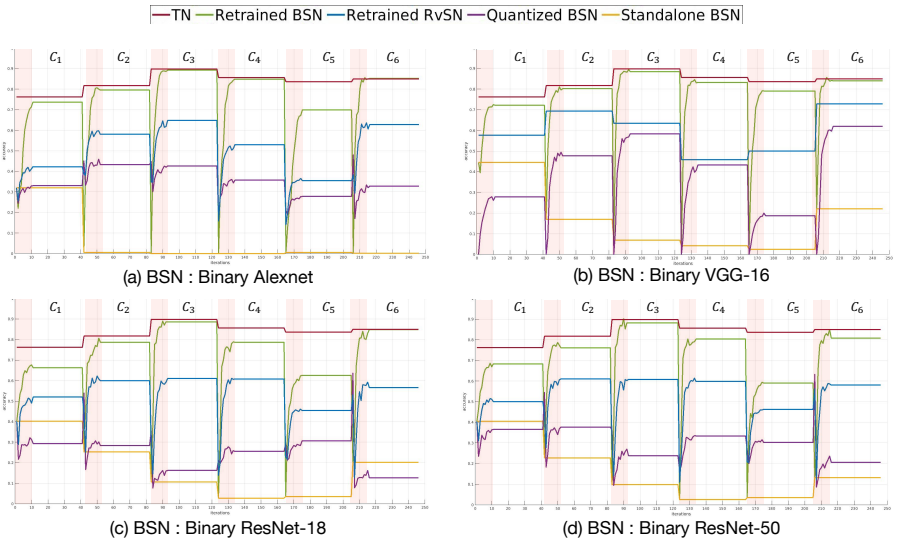

Figure 3: (a) : (d) The BSN converges to the performance of the TN faster than the RvSN and the variants of BSN). The red period illustrates adaptive training and white period illustrates inference. We observe a similar convergence pattern with different binary network architectures.

is comprised of only ID classes from C_1. Once the BSN (in green) converges to the performance of the TN (in red), we stop training the BSN and observe that it achieves an accuracy comparable to the TN's accuracy. The learning behaviour of RvSN (in blue) is slower than the BSN and has poorer accuracy after 10 epochs. Directly binarizing the RvSN (purple line) is also poorer than the BSN, and substantially differs from the binarization algorithm in Section 3 which compensates for the quantization effect. The yellow line shows the performance of the standalone BSN which is not trained during the adaptive training phase.

In Figure 3 we keep the ID/OD superclasses the same for 40 epochs, and then switch the input stream to a different scenario. The 10 epochs shaded in red indicate the adaptive training phase, while the unshaded (white) intervals indicate the 30 epochs for inference. When the scenario changes after 40 epochs (e.g from C_1 to C_2), we observe similar (and consistent) learning behaviour, and the BSN retrains quickly from the TN to regain efficiency. We also observe a similar convergence pattern for different BSN architectures when adaptively trained from the same TN in Figure 3. When comparing the learning convergence of different BSN architectures, we observe that the rate of convergence for Binary AlexNet is the fastest and that of Binary Resnet-50 is the slowest. This illustrates that a smaller binary network would make an ideal BSN in real-world scenarios, since it would utilize fewer epochs during adaptive training.

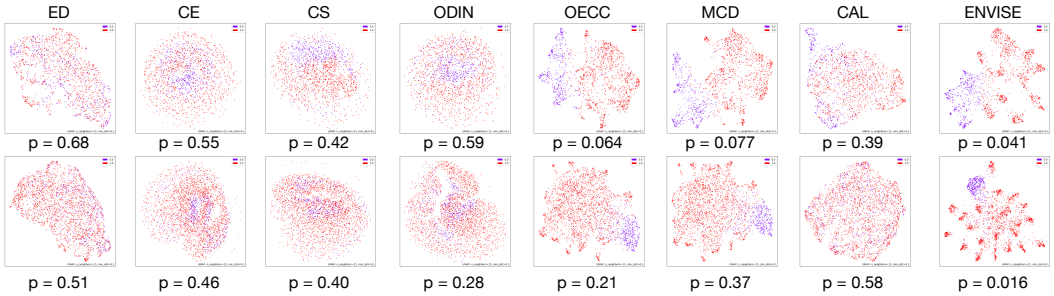

Figure 4: Feature space separation of the ID and OD classes for different baseline methods. $p-$value denotes the overlap between the ID and OD features where, lower value indicates better separation.

Comparison with SOTA outlier detection methods: The ability to accurately distinguish between images of ID and OD classes is key for improving computational efficiency. We therefore benchmark ENVISE on the CIFAR-100 and TI datasets with SOTA OD detection methods like error detection (ED) (Hendrycks & Gimpel (2016)), confidence estimation (CE) (DeVries & Taylor (2018)), confidence scaling (CS) (DeVries & Taylor (2018)), ODIN (Liang et al. (2018)), outlier exposure with confidence control (OECC) (Papadopoulos et al. (2019)), maximum classifier discrepancy (MCD)

(Yu & Aizawa (2019)) and confidence aware learning (CAL) (Moon et al. (2020)). We use the official code of these methods and adaptively train them using their proposed loss functions on our experimental settings. For fair comparisons in terms of network architecture, we use the TN as DenseNet-201 and the SN as our binary VGG-16. Following Hendrycks & Gimpel (2016), we use FPR at 95% True positive rate (TPR), outlier detection error (ODE), area under ROC curve (AuROC), and area under precision-recall curve (AuPR) as our evaluation metric. From Table 2, we observe that ENVISE outperforms the best performing baseline method by achieving the lowest FPR and outlier detection error with high AuROC and AuPR.

We visualize the separation of ID and OD classes in the feature space using UMAP (McInnes et al. (2018)) across the last fully connected layer of the BSN. Figure 4 shows that ENVISE has the best separation between ID and OD classes which is consistent with the quantitative analysis shown in Table 2. To quantify the feature separation between the ID and OD classes, we compute the $p-$value using Wilcoxon's rank sum test (Wilcoxon (1992)) for the null hypothesis that the ID and OD feature distribution are the same i.e they overlap. Ideally for high separation between the ID and OD classes, the *p-value* should be less than 0.05 (rejecting the hypothesis with 95% confidence). We observe from Figure 4 that the *p-value* of ENVISE has the smallest value of 0.041 as compared to the SOTA OD detection methods. This indicates that ENVISE achieves the least overlap between ID and OD classes, thereby outperforming the SOTA OD detection methods in its ability to better detect OD classes. A detailed comparison of ENVISE with each baseline method on different pairs of ID and OD classes and the corresponding feature space visualization is presented in the Appendix A.4. Furthermore, in Appendix A.2, we ablate the effect of the proposed L_{at}, margin (δ) with the BSN and also performance of OD detection with RvSN. We present additional discussions in Appendix A.3.

Table 2: Performance comparison of ENVISE with the best performing baseline method on different ID and OD class pairs on CIFAR-100 and TI dataset. $\downarrow$ and $\uparrow$ indicate smaller and greater value is better respectively. The detailed comparison with each baseline is in the Appendix A.4

Dataset	Group	Method \ Metric	FPR (95% TPR) $\downarrow$	Detection error $\downarrow$	AuROC $\uparrow$	AuPR (ID) $\uparrow$	AuPR (OD) $\uparrow$
CIFAR-100	C_1	Best Baseline.	0.91	0.43	0.58	0.77	0.34
		ENVISE	**0.89**	**0.40**	**0.62**	**0.80**	**0.39**
	C_2	Best Baseline	0.71	0.32	0.74	0.82	0.64
		ENVISE	**0.40**	**0.14**	**0.93**	**0.96**	**0.87**
	C_3	Best Baseline	0.66	0.25	0.81	0.82	0.80
		ENVISE	**0.18**	**0.18**	**0.89**	**0.91**	**0.87**
	C_4	Best Baseline	0.71	0.31	0.74	0.81	**0.66**
		ENVISE	**0.61**	**0.30**	**0.75**	**0.82**	0.62
	C_5	Best Baseline	0.90	0.43	0.59	0.88	0.21
		ENVISE	**0.76**	**0.28**	**0.79**	**0.95**	**0.42**
	C_6	Best Baseline	0.60	0.23	0.85	0.79	0.91
		ENVISE	**0.49**	**0.16**	**0.91**	**0.87**	**0.94**
Tiny-Imagenet	T_1	Best Baseline.	0.72	0.28	0.84	0.88	0.65
		ENVISE	**0.69**	**0.23**	**0.89**	**0.91**	**0.71**
	T_2	Best Baseline	0.80	0.29	0.75	0.79	0.68
		ENVISE	**0.78**	**0.26**	**0.81**	**0.86**	**0.73**
	T_3	Best Baseline	0.78	0.32	0.68	0.76	0.61
		ENVISE	**0.74**	**0.20**	**0.81**	**0.93**	**0.63**
	T_4	Best Baseline	0.83	0.20	0.69	0.78	**0.51**
		ENVISE	**0.78**	**0.18**	**0.75**	**0.91**	0.44
	T_5	Best Baseline	0.80	0.30	0.90	**0.85**	0.79
		ENVISE	**0.78**	**0.26**	**0.91**	**0.85**.	**0.84**

Gain in Computational Efficiency (GiE): The main focus of our work is to develop an efficient system to classify the image stream with low inference cost and high accuracy. We propose a new evaluation metric - GiE to measure the overall gain in computational efficiency from the number of times the BSN and TN is used individually to classify the image stream. During inference, we require the BSN to accurately classify the ID classes such that it does not rely on the TN thereby reducing the overall computation cost. Furthermore, the BSN should detect an image as OD so that the TN can be used in such cases. However, the BSN should not classify an OD class as an ID class, which results in the drop in efficiency of the overall system.
To process a single image on the CIFAR-100 dataset, the TN uses 9048 MFLOPs and 28.5msec, while the RvSN uses 310 MFLOPs and 3.04 msec. However, the BSN achieves 6 MFLOPs and 0.57

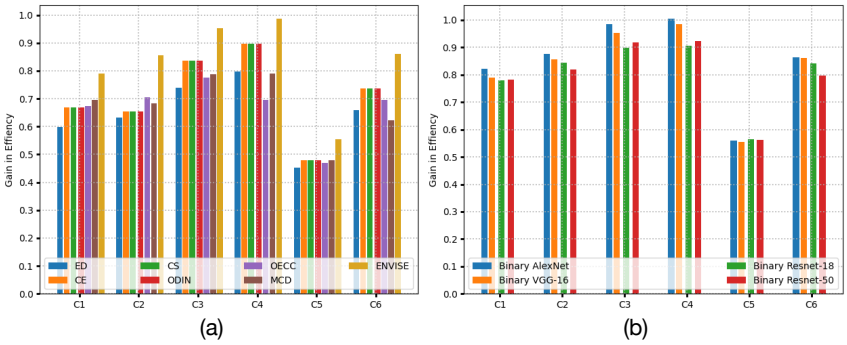

Figure 5: (a) Comparison of ENVISE with SOTA OC detection methods, and (b) Comparison of different BSN architectures in terms of GiE on different super-classes of the CIFAR-100 dataset.

msec; a $\sim 50\times$ reduction in computation and a $\sim 6\times$ speed improvement as compared to RvSN. Furthermore, the BSN occupies $\sim 30\times$ and $\sim 100\times$ less memory than RvSN and TN respectively.

During adaptive training, since each image is processed by both the BSN and TN, the total FLOPs used is $FL_{tr} = (X + Y)$. Once the BSN converges to the performance of the TN, only the BSN is used to process the input image, while the TN is only used when an image is detected as OD. Hence, for ID images the total FLOPs is $FL_{in} = Y + (fp \times X)$, where $(fp \times X)$ indicates the number of times the TN is used when the BSN misclassifies an ID class as OD. Similarly for OD images, the FLOPs is $FL_o = (Y + od \times X)$,where $(od \times X)$ indicates the OD classes correctly detected by the BSN which is then processed by the TN. Assuming that adaptive training and *steady state* inference phase for a given super-class occur for N_{tr} and N_{ss} epochs respectively, the *relative* FLOPS compared to the TN is formulated as

$$RFL = \frac{N_{tr}}{N_T} \times \frac{FL_{tr}}{X} + \frac{N_{ss}}{N_T} \times \frac{(p \times FL_{in} + q \times FL_o)}{X} \tag{3}$$

We want RFL to be as small as possible for maximum efficiency. If the BSN works with perfect accuracy (i.e. $fp = 0$ and $od = 1.0$), then the minimum value of RFL in the steady state (i.e. after convergence of the BSN) is $RFL_{min} = \frac{Y+qX}{X}$. We define GiE as RFL normalized by its minimum value, i.e.

$$GiE = \frac{RFL}{RFL_{min}} = \frac{1}{Y + qX} \left[\frac{N_{tr}}{N_T} \times FL_{tr} + \frac{N_{ss}}{N_T} \times (p \times FL_{in} + q \times FL_o) \right] \tag{4}$$

To numerically compute GiE, we use $X = 9048$ (TN), $Y = 6$ for the BSN while p and q are calculated from Table 1. We analyze the effect of varying ID and OD class probabilities on GiE in Appendix A.3. Figure 5 illustrates that ENVISE outperforms SOTA OD detection methods in terms of GiE for different scenarios on CIFAR-100 dataset. Thus, the L_{at} ensures that the BSN can accurately distinguish between representations of ID and OD classes which enables ENVISE to operate in a computationally efficient manner.

5 Conclusion

We propose ENVISE, an online distillation framework which uses a BSN to adaptively learn relevant knowledge from a TN to quickly classify the frequent classes in the deployed scenario. In doing so, it automatically allocates processing resources (i.e. either the BSN or the TN) to reduce overall computation. To learn proper representations of the ID classes, we propose an attention triplet loss that enables the BSN to learn the semantically relevant regions of the image that are emphasized by the TN. This enables the BSN to distinguish between representations of ID and OD classes thereby improving OD detection. Our experiments show that i) the BSN quickly converges to the TN performance, ii) classifies ID classes more accurately than a RvSN and other variants of the BSN, and iii) distinguishes between ID and OD classes with lower FPR and ODE than SOTA OD methods on CIFAR-100 and tiny-imagenet datasets. We introduce a new metric GiE to assess the overall gain in computational efficiency, and experimentally show that ENVISE can achieve higher GiE than baseline methods. We show that ENVISE is agnostic to the specific TN and BSN and achieves high gains with different BSN and TN architectures.

REFERENCES

Pooya Abolghasemi, Amir Mazaheri, Mubarak Shah, and Ladislau Boloni. Pay attention!-robustifying a deep visuomotor policy through task-focused visual attention. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 4254–4262, 2019.

Abhijit Bendale and Terrance E Boult. Towards open set deep networks. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 1563–1572, 2016.

Anthony Cioppa, Adrien Deliege, Maxime Istasse, Christophe De Vleeschouwer, and Marc Van Droogenbroeck. Arthus: Adaptive real-time human segmentation in sports through online distillation. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition Workshops*, pp. 1–9, 2019.

Terrance DeVries and Graham W Taylor. Learning confidence for out-of-distribution detection in neural networks. *arXiv preprint arXiv:1802.04865*, 2018.

Liang Gao, Xu Lan, Haibo Mi, Dawei Feng, Kele Xu, and Yuxing Peng. Multistructure-based collaborative online distillation. *Entropy*, 21(4):357, 2019.

Dan Hendrycks and Kevin Gimpel. A baseline for detecting misclassified and out-of-distribution examples in neural networks. *arXiv preprint arXiv:1610.02136*, 2016.

Dan Hendrycks, Mantas Mazeika, and Thomas Dietterich. Deep anomaly detection with outlier exposure. *Proceedings of International Conference on Learning Representations*, 2019.

Geoffrey Hinton, Oriol Vinyals, and Jeff Dean. Distilling the knowledge in a neural network. *arXiv preprint arXiv:1503.02531*, 2015.

Gao Huang, Zhuang Liu, Laurens Van Der Maaten, and Kilian Q Weinberger. Densely connected convolutional networks. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 4700–4708, 2017.

Jangho Kim, Minsung Hyun, Inseop Chung, and Nojun Kwak. Feature fusion for online mutual knowledge distillation. *arXiv preprint arXiv:1904.09058*, 2019.

Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.

Alex Krizhevsky and Geoffreyx Hinton. Learning multiple layers of features from tiny images. 2009.

Kimin Lee, Kibok Lee, Honglak Lee, and Jinwoo Shin. A simple unified framework for detecting out-of-distribution samples and adversarial attacks. In *Advances in Neural Information Processing Systems*, pp. 7167–7177, 2018.

Kunpeng Li, Ziyan Wu, Kuan-Chuan Peng, Jan Ernst, and Yun Fu. Tell me where to look: Guided attention inference network. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 9215–9223, 2018.

Shiyu Liang, Yixuan Li, and Rayadurgam Srikant. Enhancing the reliability of out-of-distribution image detection in neural networks. *International Conference on Learning Representations*, 2018.

Rongcheng Lin, Jing Xiao, and Jianping Fan. Mod: A deep mixture model with online knowledge distillation for large scale video temporal concept localization. *arXiv preprint arXiv:1910.12295*, 2019.

Roi Livni, Shai Shalev-Shwartz, and Ohad Shamir. On the computational efficiency of training neural networks. In *Advances in neural information processing systems*, pp. 855–863, 2014.

Leland McInnes, John Healy, Nathaniel Saul, and Lukas Grossberger. Umap: Uniform manifold approximation and projection. *The Journal of Open Source Software*, 3(29):861, 2018.

Jooyoung Moon, Jihyo Kim, Younghak Shin, and Sangheum Hwang. Confidence-aware learning for deep neural networks. In *International Conference on Machine Learning*, 2020.

Ravi Teja Mullapudi, Steven Chen, Keyi Zhang, Deva Ramanan, and Kayvon Fatahalian. Online model distillation for efficient video inference. In *Proceedings of the IEEE International Conference on Computer Vision*, pp. 3573–3582, 2019.

Aristotelis-Angelos Papadopoulos, Mohammad Reza Rajati, Nazim Shaikh, and Jiamian Wang. Outlier exposure with confidence control for out-of-distribution detection. *arXiv preprint arXiv:1906.03509*, 2019.

Mohammad Rastegari, Vicente Ordonez, Joseph Redmon, and Ali Farhadi. Xnor-net: Imagenet classification using binary convolutional neural networks. In *European conference on computer vision*, pp. 525–542. Springer, 2016.

Jie Ren, Peter J Liu, Emily Fertig, Jasper Snoek, Ryan Poplin, Mark Depristo, Joshua Dillon, and Balaji Lakshminarayanan. Likelihood ratios for out-of-distribution detection. In *Advances in Neural Information Processing Systems*, pp. 14680–14691, 2019.

Florian Schroff, Dmitry Kalenichenko, and James Philbin. Facenet: A unified embedding for face recognition and clustering. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 815–823, 2015.

Ramprasaath R Selvaraju, Michael Cogswell, Abhishek Das, Ramakrishna Vedantam, Devi Parikh, and Dhruv Batra. Grad-cam: Visual explanations from deep networks via gradient-based localization. In *Proceedings of the IEEE international conference on computer vision*, pp. 618–626, 2017.

Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*, 2014.

Apoorv Vyas, Nataraj Jammalamadaka, Xia Zhu, Dipankar Das, Bharat Kaul, and Theodore L Willke. Out-of-distribution detection using an ensemble of self supervised leave-out classifiers. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pp. 550–564, 2018.

Lezi Wang, Ziyan Wu, Srikrishna Karanam, Kuan-Chuan Peng, Rajat Vikram Singh, Bo Liu, and Dimitris N Metaxas. Sharpen focus: Learning with attention separability and consistency. In *Proceedings of the IEEE International Conference on Computer Vision*, pp. 512–521, 2019.

Lin Wang and Kuk-Jin Yoon. Knowledge distillation and student-teacher learning for visual intelligence: A review and new outlooks. *arXiv preprint arXiv:2004.05937*, 2020.

Frank Wilcoxon. Individual comparisons by ranking methods. In *Breakthroughs in statistics*, pp. 196–202. Springer, 1992.

Leon Yao and John Miller. Tiny imagenet classification with convolutional neural networks. *CS 231N*, 2(5):8, 2015.

Ryota Yoshihashi, Wen Shao, Rei Kawakami, Shaodi You, Makoto Iida, and Takeshi Naemura. Classification-reconstruction learning for open-set recognition. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 4016–4025, 2019.

Qing Yu and Kiyoharu Aizawa. Unsupervised out-of-distribution detection by maximum classifier discrepancy. In *Proceedings of the IEEE International Conference on Computer Vision*, pp. 9518–9526, 2019.

A Appendix

A.1 Proof of convergence

In Section 3 of the main paper, we describe that, one of the motivation for using the BSN instead of the RvSN is the ability of BSN to adapt to the performance of TN faster than RvSN.

Lemma A.1. *Let RvSN and BSN represent the real-valued student network and binary student network respectively with the same architecture. Let R denote the rate of convergence in terms of accuracy when the student network is adaptively trained using the teacher network's predictions. Then, $R(BSN) > R(RvSN)$ for the same input image stream and number of iterations.*

Proof For the ease of understanding, we assume to have a binary classification problem with 2 classes i.e. $C = 2$, the data points are linearly separable, the weights are initialized to 0 i.e. $w(0) = 0$ and learning rate $\eta = 1$. The initialization value of weights and learning rate do not affect the proof. Let us further assume that we have a stream of input images $x(n)$ where $n = 1, 2, 3...N$, where N is the total number of samples. Since the weights of BSN is derived from RvSN (From Section 3.1 of the main paper), we prove this lemma for RvSN first and then extend it on the BSN. Since, the pretrained performance of the SN is worse than the TN (comparing starting point of blue line with the red line from Figure 3(a) of the main paper), we assume that the SN misclassifies the images from the input stream i.e. $w(n)x(n) < 0$ where $w(n)$ is the weight of the network. The misclassification generates as error through which the networks weights are updated and there exists an optimal weight value w^* such that the network converges i.e. misclassification error $= 0$. The weight update rule using backpropagation is given by:

$$\begin{aligned} w(n+1) &= w(n) + \eta x(n) \\ [\text{ Since } \eta = 1] \quad w(n+1) &= w(n) + x(n) \end{aligned} \tag{5}$$

Expanding eq (1), we have:

$$\begin{aligned} w(1) &= w(0) + x(0) \\ w(2) &= w(1) + x(1) \\ \Rightarrow w(2) &= w(0) + x(0) + x(1) \\ [\text{ Since } w(0) = 0] \quad w(2) &= x(0) + x(1) \end{aligned} \tag{6}$$

Similarly we have $w(3) = x(2) + x(1) + x(0)$ and this recursively continues till $w(n+1)$. Thus,

$$w(n+1) = x(1) + x(2) + x(3) + ...x(n) \tag{7}$$

Ideally, when the network converges, we obtain an optimal weight vector, which is represented as $w^\star$. We define the margin α as the shortest distance of $w^\star$ to the datapoint $x \in x(n)$. The concept of margin is similar to that of support vectors. Hence, the margin α is denoted as

$$\alpha = \underset{x(n) \in C}{\text{argmin}}\, w^\star x(n) \tag{8}$$

We ignore $x(0)$ for ease of calculation. Pre-multiplying both the sides of eq (4) by $w^\star$, we have

$$\begin{aligned} w^\star w &= w^\star x(1) + w^\star x(2) + ...w^\star x(n) \\ [\text{From eq (5)}] \qquad & w^\star w \geq n\alpha \end{aligned} \tag{9}$$

From Cauchy - Schwarz inequality, we have

$$\begin{aligned} \|w^\star\|^2 \|w(n+1)\|^2 &\geq (w^\star w(n+1))^2 \\ \|w^\star\|^2 \|w(n+1)\|^2 &\geq (n\alpha)^2 \\ \boxed{\|w(n+1)\|^2 \geq \frac{n^2\alpha^2}{\|w^\star\|^2}} \end{aligned} \tag{10}$$

We highlight eq. (6) as we will be using this in the later stages of the proof. We re-write eq (2) in a different way as $w(k+1) = x(k) + w(k) \;\; \forall \;\; k \in [0, 1, 2, 3..n]$. Squaring both sides of this equation and expanding it, we have

$$\begin{aligned} \|w(k+1)\|^2 &= \|x(k) + w(k)\|^2 \\ \Rightarrow \|w(k+1)\|^2 &= \|x(k)\|^2 + \|w(k)\|^2 + 2x(k)w(k) \\ [\text{Since in misclassification } \; w(n)x(n) \leq 0] \quad \|w(k+1)\|^2 &\leq \|x(k)\|^2 + \|w(k)\|^2 \\ \Rightarrow \|w(k+1)\|^2 - \|w(k)\|^2 &\leq \|x(k)\|^2 \end{aligned} \tag{11}$$

Iterating eq (7) for all values of $k \in [0, 1, 2, 3..n]$, we have

$$[\text{Since when } k = 0,\ w(k) = 0] \quad \|w(k+1)\|^2 \leq \sum_{k=0}^{n} \|x(k)\|^2 \tag{12}$$

Since $\|x(k)\|^2 > 0$, we define $\beta = \text{argmax}_{x(n) \in C} \|x(k)\|^2$, thus eq (9) can be re-written as

$$\boxed{\|w(n+1)\|^2 \leq n\beta} \tag{13}$$

From eq. (6) and (9), we have

$$\frac{n^2\alpha^2}{\|w^*\|^2} \leq \|w(n+1)\|^2 \leq n\beta \tag{14}$$

From eq (10), we can say that for this inequality to be satisfied, the exists $n^\star$ which denotes the optimal number of samples for which the network converges i.e. we obtain $w^\star$ at $n = n^\star$. This is given as follows:

$$\begin{aligned} \frac{{n^\star}^2\alpha^2}{\|w^\star\|^2} &= n^\star\beta \\ \boxed{n^\star = \frac{\beta\,\|w^\star\|^2}{\alpha^2}} \end{aligned} \tag{15}$$

Thus from eq (11), we can say that the network achieves convergence after being trained on $\frac{\beta\|w^*\|^2}{\alpha^2}$ number of samples. In the case of the BSN, the binary weights $\hat{w}^* = \Delta^* D^*$, where $\Delta^* = \frac{1}{N}|\hat{w}^\star|_1$ and $D^* = sign(w^*)$, Let $n_b^\star$ be the number of samples required for the BSN to converge. Thus, eq (11) for BSN would be given as

$$\begin{aligned} n_b^\star &= \frac{\beta\,\|\hat{w}^\star\|^2}{\alpha^2} \\ n_b^\star &= \frac{\beta\,\|\Delta^* D^*\|^2}{\alpha^2} \\ n_b^\star &= \frac{\beta\,\|w^* D^*\|^2}{N^2\alpha^2} \\ n_b^\star &= \frac{\beta\,\|w^\star\|^2\, D^{*T} D^*}{N^2\alpha^2} \\ [\text{From eq (11) and Rastegari et al. (2016)}] \quad n_b^\star &= \frac{n^\star N}{N^2} \\ \boxed{n_b^\star = \frac{n^\star}{N}} \end{aligned} \tag{16}$$

From eq (11) and (12), we see that $n_b^* < n^*$ i.e. the BSN takes lesser samples than RvSN to converge to the TN's performance given the same network architecture. We also experimentally observe the higher rate of convergence of BSN over RvSN in Figure 3(a) of main paper.

A.2 Ablation Study

All the ablation studies are performed on the CIFAR-100 dataset. We illustrate the effectiveness of the attention triplet loss (L_{at}), the value of margin in the (L_{at} and the effectiveness of using BSN over RvSN for OD detection and overall efficiency of the ENVISE. The qualitative results illustrating the effect on feature space separation between ID and OD classes is shown in Figure **??**. The column IDs refer to column numbers in Table 3.

Effect of attention triplet loss (L_{at}) : To test the effectiveness of the attention triplet loss (L_{at}), we train the BSN without it. Specifically, we use the TN as DenseNet-201, the BSN as Binary VGG-16 and train the BSN using only the distillation loss L_d. Comparing column ID 2 and 3 in Table 3, we

observe that training the BSN using $L_d + L_{at}$ improves the mean FPR by 35%, mean outlier detection error by 37.4%, mean AuROC by 12.7%, mean AuPR of ID class by 8.8% and mean AuPR of OD class by 22.3%. From Figure 6, we observe that the ID (red) and OD features (purple) are barely separable when evaluated on a pre-trained BSN. When the BSN is adaptively trained using only L_d, the features begin to separate but still have some overlap. However training the BSN using $L_d + L_{at}$ significantly ameliorates the feature separation thereby improving OD detection. Furthermore, the *p-value* (noted in Figure 6 below each UMAP visualizations) of BSN when trained using $L_d + L_{at}$ has the smallest value of 0.041. This validates our assertion that L_{at} learns the proper representations of ID classes thereby improving its ability to differentiate between ID and OD classes.

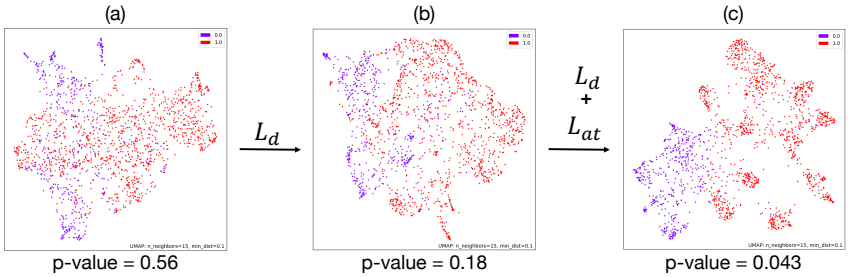

Figure 6: (a):(c) Illustration of the effect of applying L_d and its combination with L_{at} on the separation between the ID (red) and OD (purple) classes in the feature space.

Effect of margin δ : We mention in Section 3 that, in order to avoid a collapsed model early on during training, we select the hard negative attention map such that its squared distance is closest with hard positive attention map. Without using any OD images during training or validation, we empirically observe that L_2 norm of the hard negative attention map with respect to the hard positive attention map ranges between $[0.33, 2.8]$. Hence, we choose δ as 1.5 to ensure separability with respect to hard positive attention map.
To show that ENVISE is insensitive to the specific value of margin, we train the BSN using L_{at} with value of δ as 0.5, 1.0, 1.5, 2.0 and 2.5. From Table 3, comparing column 3 with 4,5,6,7 and 8, we observe that ENVISE is insensitive to the specific value of margin and outperforms the best performing baseline method with different margin values. Furthermore, from Fig. 7, we observe that ENVISE has the best performance when the margin is set to 1.5

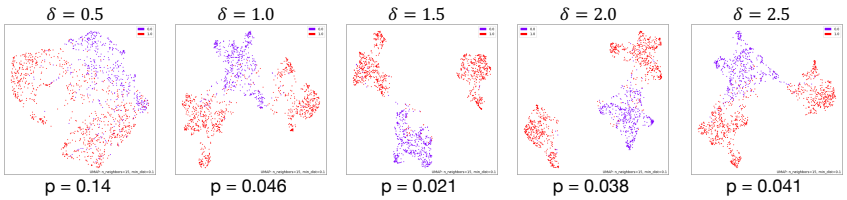

Figure 7: Illustration of the effect of different values of margin on the separation between the ID (red) and OD (purple) classes in the feature space.

Effect of using RvSN instead of BSN To illustrate the effectiveness of using the BSN to improve the overall gain in efficiency of the system, we employ a RvSN instead of the BSN as our SN. We use the TN as DenseNet-201 and real-valued VGG-16 as our SN and adaptively train it using $L_d + L_{at}$. We observe from Table 4 that using BSN instead of RvSN as our SN improves the mean FPR by 4.8%, mean outlier detection error by 22.2%, mean AuROC by 11.2%, mean AuPR of ID class by 21.2% and mean AuPR of OD class by 8.7%. Furthermore, we also observe that mean GiE for the BSN is 0.557 while that of the RvSN is 0.352 which illustrates that ENVISE has 58.2% gain in computational efficiency with BSN than RvSN.

Significance of using hard negative term in L_{at}: As mentioned in Section 3 of the main paper, we observe that the attention map from the correct and incorrect predictions are visually similar. This causes the BSN to learn improper representations from the TN during online distillation. Hence, we choose attention map from the incorrect prediction as hard negative to ensure separability with the attention map from the correct prediction. To illustrate the significance of hard negative attention

map, we train ENVISE without the second term in equation 1 (main paper) along with L_d. The attention L_2 loss is formulated as

$$L_a = \frac{1}{N}\frac{1}{K}\sum_{n}^{N}(\sum_{k}^{K}\left\|A_{t_k} - A_{s_{p_k}}\right\|_2^2) \tag{17}$$

From Figure 8, we observe that the confidence of classifying an image decreases when ENVISE is trained using L_a. Furthermore, the attention map of ENVISE after training using $L_d + L_a$ does not focus on the semantically meaningful regions of the image (fourth column) unlike training it with L_{at} (last column). Furthermore, we also observe training ENVISE with $L_d + L_a$ results a poor OD detection with 0.98 FPR and 0.47 outlier detection error on C_1 as compared to 0.89 and 0.40 FPR and OD detection error with $L_d + L_{at}$. This validates that the hard negative attention map ensures separability with hard positive attention map and also improves the confidence of classification and OD detection.

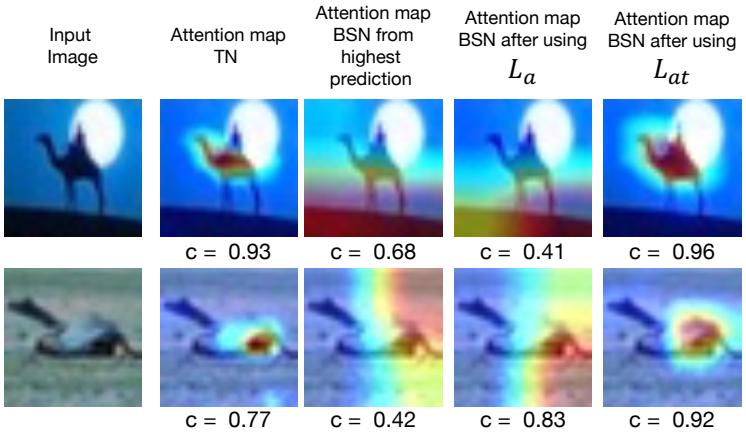

Figure 8: The attention map visualization representing the regions of the image that ENVISE focuses on when trained using L_a and L_{at}. The confidence of classification is reported below each image.

A.3 Discussions

TN does not leak OD class information to BSN during online distillation The BSN is trained using only the pseudo-labels from the TN on the ID classes, we show that during online distillation, there is no information of the local outlier classes (OD) that is transferred to the BSN. To address this , we compute the Shannon entropy $H(.)$ of OD and ID classes of the BSN, before and during the online distillation process (when BSN is trained using $L_d + L_{at}$). Figure 9 illustrates that while the $H(\text{ID}|X)$ classes decreases, the $H(\text{OD}|X)$ class is already high, where X is the image from the input stream. This illustrates that during the online distillation, the BSN learns the information of the ID classes, while information about the OD class is completely absent. Thus, the ability of the BSN to detect OD classes better than the SOTA outlier detection methods, is not due to the OD information leak, but due to the BSN able to accurately differentiate between the representations of the ID and OD classes.

Global vs local outlier detection In Section 4, we illustrate that ENVISE outperforms the SOTA outlier detection methods for detecting OD classes. The OD classes are classes from the image stream which are unknown to the BSN, but are known to the TN. Furthermore, we also show the ability of ENVISE to also detect global outlier OD_g classes i.e. the classes unknown to both the BSN and TN. We pre-train the TN and BSN on the training set of CIFAR-100 dataset, on all the classes except the OD classes of Table 1 (main paper). Specifically, we now treat the OD classes of C_1 as OD_g classes where neither the TN nor the BSN has knowledge of these classes. Table **??** illustrates that ENVISE *tbd*.

Gain with smaller teacher network

Table 3: Ablation study to illustrate the effect of proposed L_{at} loss and the effect of different values of margin δ on the performance of ENVISE for local outlier detection on CIFAR-100 dataset.

Metric	super-class	ENVISE L_d only	ENVISE L_d + L_{at}	Best Baseline	ENVISE δ=0.5	ENVISE δ=1.0	ENVISE δ=1.5	ENVISE δ=2.0	ENVISE δ=2.5
column ID		1	2	3	4	5	6	7	8
FPR (95% TPR) ↓	C_1	0.93	**0.89**	0.91	0.95	0.88	0.89	**0.87**	0.93
	C_2	0.81	**0.40**	0.71	0.49	0.61	0.4	**0.35**	0.41
	C_3	0.9	**0.57**	0.66	0.58	0.66	**0.57**	0.65	0.83
	C_4	0.88	**0.61**	0.71	0.84	**0.61**	**0.61**	0.79	0.78
	C_5	0.87	**0.76**	0.90	0.78	0.81	**0.76**	0.83	0.77
	C_6	0.64	**0.49**	0.60	0.54	0.53	**0.49**	0.67	0.57
	mean	0.84	**0.62**	0.75	0.70	0.68	**0.62**	0.69	0.72
Detection error ↓	C_1	**0.39**	0.40	0.43	0.46	**0.33**	0.4	0.38	0.41
	C_2	0.33	**0.14**	0.32	0.17	0.21	**0.14**	0.16	0.16
	C_3	0.35	**0.18**	0.25	0.22	0.25	**0.18**	0.2	0.24
	C_4	0.35	**0.30**	0.31	0.3	0.26	0.3	**0.24**	0.3
	C_5	0.39	**0.28**	0.43	0.29	0.33	**0.28**	0.31	0.3
	C_6	0.21	**0.16**	0.23	**0.16**	0.17	**0.16**	0.25	**0.16**
	mean	0.34	**0.24**	0.33	0.27	0.26	**0.24**	0.26	0.26
AuROC ↑	C_1	**0.64**	0.62	0.58	0.54	**0.71**	0.62	0.66	0.6
	C_2	0.83	**0.93**	0.74	0.9	0.87	0.93	**0.95**	0.91
	C_3	0.69	**0.89**	0.81	0.84	0.83	**0.89**	0.87	0.82
	C_4	0.67	**0.75**	0.74	0.76	0.78	0.75	**0.80**	0.76
	C_5	0.64	**0.79**	0.59	0.76	0.72	**0.79**	0.74	0.76
	C_6	0.87	**0.91**	0.85	0.89	0.89	**0.91**	0.83	**0.91**
	mean	0.72	**0.815**	0.72	0.78	0.80	**0.82**	0.81	0.79
AuPR (inlier) ↑	C_1	**0.81**	0.80	0.77	0.75	**0.85**	0.80	0.82	0.79
	C_2	0.89	**0.96**	0.82	0.94	0.92	**0.96**	0.87	0.95
	C_3	0.73	**0.91**	0.82	0.87	0.84	**0.91**	0.89	0.85
	C_4	0.76	**0.82**	0.81	0.85	**0.88**	0.82	0.82	0.84
	C_5	0.88	**0.95**	0.88	0.94	0.91	**0.95**	0.93	0.94
	C_6	0.81	**0.87**	0.79	0.85	0.84	**0.87**	0.76	0.86
	mean	0.81	**0.89**	0.81	0.87	0.87	**0.89**	0.85	0.87
AuPR (outlier) ↑	C_1	**0.41**	0.39	0.34	0.32	**0.45**	0.39	0.43	0.35
	C_2	0.65	**0.87**	0.64	0.82	0.76	0.87	**0.88**	0.84
	C_3	0.64	**0.87**	0.80	0.78	0.81	**0.87**	0.85	0.77
	C_4	0.52	**0.62**	0.66	0.61	0.62	0.62	**0.68**	0.63
	C_5	0.26	**0.42**	0.21	0.38	0.33	**0.42**	0.34	0.38
	C_6	0.88	**0.94**	0.91	0.92	0.92	**0.94**	0.88	0.94
	mean	0.56	**0.69**	0.59	0.64	0.65	**0.69**	0.68	0.65

Table 4: Ablation study to illustrate the effect of employing the BSN over RvSN on the performance of ENVISE for local outlier detection on CIFAR-100 dataset.

Method	mean FPR (↓)	mean Detection error (↓)	mean AuROC (↑)	mean AuPR (ID) (↑)	mean AuPR (OD (↑)
ENVISE (w/ RvSN)	0.65	0.30	0.73	0.73	0.63
ENVISE (w/ BSN)	**0.62**	**0.24**	**0.82**	**0.89**	**0.69**

GiE when probability of ID class >> probability of OD class: We evaluate GiE of ENVISE for the case when the ID class occurs with a much higher probability than the OD class. Hence for C_2, we consider all 900 images from ID class and 100 images from OD class, making the probability of occurrence of ID class $p = 0.9$ ($\frac{900}{900+100} = 0.9$). In such a setting, we obtain an FPR (f) of 0.26 and outlier detection error (r) of 0.15. From the formulation of Fl_r mentioned in Section 4 of the main paper, we use $X = 9048 \times 1e^6$ and $Y = 6 \times 1e^6$ to obtain GiE $= 0.601$. From Figure 4 in the main paper, we observe that the value of GiE obtained when $p = 0.5$ is 0.40 (C_3). Thus, in the case when the images from ID class occur more frequently than OD class (i.e $p = 0.9$), although outperforming the baseline methods, ENVISE achieves a 50.25% gain in computational efficiency.

Trade-off between accuracy and GiE of ENVISE: Table 5 shows the accuracy of the BSN, the TN and their combined performance (i.e. ENVISE) for all six ID super-class pairs on the CIFAR-100 dataset. Comparing row 2 and row 3 in Table 5, we observe a very minimal loss in accuracy in

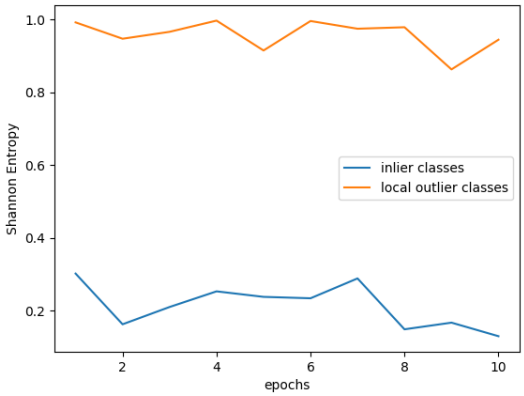

Figure 9: Shannon Entropy of ID and OD of the BSN during online distillation. The entropy of the ID decreases, the entropy of OD is high. This shows that BSN does not learn OD class information.

ENVISE with respect to TN on different super-classes of CIFAR-100 dataset. In ENVISE, during inference, the BSN classifies the ID classes from the image stream. When the BSN misclassifies an ID class as an OD class, the image it then given to the TN for classification. This ensures that overall accuracy of the system is bounded by the performance of the TN. Furthermore, from Figure 5, we observe that ENVISE has the highest GiE which indicates that, along with high computational efficiency, ENVISE also has minimal loss in overall accuracy as compared to a standalone TN.

Table 5: Comparison of accuracy between BSN, TN and ENVISE on the ID classes of CIFAR-100 dataset. The ID images that the BSN misclassifies as OD, are correctly reclassified by the TN

Model	C_1	C_2	C_3	C_4	C_5	C_6
BSN	0.72	0.80	0.83	0.83	0.79	0.84
TN	0.76	0.81	0.84	0.85	0.86	0.90
ENVISE	0.76	0.81	0.83	0.84	0.85	0.89

A.4 Additional Quantitative and Qualitative Analysis

Complete comparison with baseline methods: As mentioned in Section 4 of the main paper under "Comparison with baseline methods", we present the detailed comparison of ENVISE with all the baseline methods on CIFAR-100 and TI datasets in Table 8. We observe that ENVISE achieves lower FPR and detection error as compared to the baseline methods. Specifically, ENVISE outperforms the best performing baseline method (MCD) by 23% and 11% in terms of mean FPR and mean detection error respectively. Furthermore, ENVISE also achieves higher AuROC and AuPR (outlier class) by outperforming MCD by 4.7% and 12.5% in terms of mean AuROC and mean AuPR (outlier class) respectively on CIFAR-100 dataset. Table 8 also shows that ENVISE outperforms the baseline methods achieving lower FPR and detection error and higher AuROC and AuPR (ID and OD class) on TI dataset. Specifically, ENVISE achieve 3.7% and 18.3% lower mean FPR and mean detection error respectively as compared to the best performing baseline method (MCD). ENVISE also outperform MCD by 13.3%, 11% and 3.7% in terms of mean AuROC, mean AuPR (inlier class) and mean AuPR (outlier class) respectively.

Performance of ENVISE is insensitive to specific ID $\setminus$ OD class pairs: As mentioned in Table 1 of the main paper, we evaluate ENVISE on meaningful ID $\setminus$ OD class pairs that mimic real-life scenarios. However, we show that ENVISE is insensitive to the specific ID $\setminus$ OD class pairs created in Table 1 of the main paper. Specifically Table 9, Table 10 and Table 11 show ENVISE outperforms all baseline methods when each ID class is compared to every OD class by achieving lower FPR and detection error and higher AuROC and AuPR on the CIFAR-100 dataset.

Feature space separation: Figure 10 illustrates the comparison of ENVISE with all baselines in terms of the separation between ID and OD images in the feature space for all ID $\setminus$ OD class pairs on CIFAR-100 dataset. To quantify the feature separation between the ID and OD samples, we compute

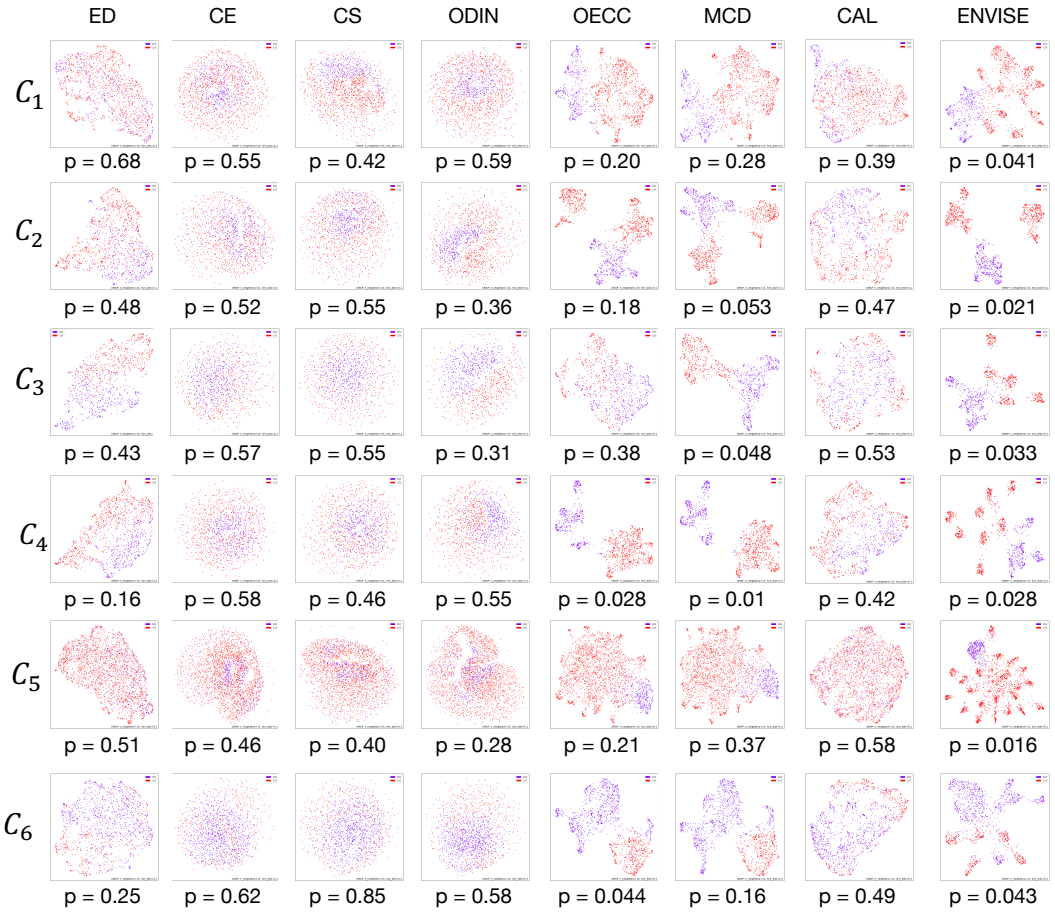

Figure 10: Comparison of ENVISE with baseline methods in terms of separation between ID (in red) and OD (in purple) samples in the feature space on CIFAR-100 dataset. The $p-$value obtained from Wilcoxon's rank sum test is denoted below each visualization where lower value denotes better separation.

the $p-$value using Wilcoxon's rank sum test Wilcoxon (1992) for the null hypothesis that the ID and OD feature distribution are the same. Hence, lower $p-$value indicates better separation. We observe that in most cases ENVISE achieves better separation than baseline methods. Furthermore, we also observe clusters of the ID images (in red) in the feature space.

Sub-classes within each super-class: Table 6 and Table 7 represents the sub-classes within each super-class of the CIFAR-100 and tiny-imagenet (TI) datasets. These sub-classes are the original classes of these datasets respectively. Each super-class is denoted by C_{xi} and C_{xo} where $x = [1, 2, 3, 4, 5, 6]$ representing the inlier and outlier super-class on CIFAR-100 dataset. Furthermore, the super-classes of the tiny-imagenet dataset is denoted as T_{yi} and T_{yo} for inlier and outlier super-class, where $y = [1, 2, 3, 4, 5]$. We use these notations for comparing ENVISE with the SOTA OD detection methods in Table 8, Table 9, Table 10 and Table 11 respectively.

Table 6: Different sub-classes within each *inlier* and *outlier* super-classes on the CIFAR-100 dataset. These sub-classes represent the original classes in the CIFAR-100 dataset.

Super-classes	Sub-classes
water animals (C_{1i})	beaver, dolphin, otter, seal, whale, aquarium fish, flatfish, ray shark, trout, crab, lobster
food containers (C_{1o})	bottles, bowls, caps, cups, plates
flora (C_{2i})	orchids, poppies, roses, sunflower tulips, maple, oak, palm, pine, willow
household electric devices (C_{2o})	clock, keyboard, lamp, telephone, television
fruit and vegetables (C_{3i})	apple, mushrooms, orange, pears, sweet peppers
household furniture (C_{3o})	bed, chair, couch, table, wardrobe
insects (C_{4i})	bee, beetle, butterfly, caterpillar, cockroach, snail, spider, worm
manmade things (C_{4o})	bridge, castle, house, road, skyscrapper
animals (C_{5i})	camel, cattle, chimpanzee, elephant, kangaroo, fox, porcupine, possum racoon,skunk, hamster, mouse, rabbit, shrew, squirel, crocodile dinosaur, lizard, turtle, snake, bear, leopard, tiger, lion, wolf
people (C_{5o})	baby, boy, girl, man, women
outdoor scene (C_{6i})	cloud, forest, mountain, plain, sea
vehicles (C_{6o})	bicycle, bus, motorcycle, pickup truck train, lawn-mover rocket, streetcar tank, tractor

Table 7: Different sub-classes within each *inlier* and *outlier* super-classes on the tiny-imagenet dataset. These sub-classes represent the original classes in the tiny-imagenet dataset.

Super-classes	Sub-classes
animals (T_{1i})	Labrador retriever, hog, Chihuahua, orangutan, chimpanzee, koala, lesser panda golden retriever, lion, baboon, African elephant, Yorkshire terrier, bison, standard poodle, cougar, gazelle, ox, Egyptian cat Persian cat, German shepherd, guinea pig, bighorn
birds (T_{1o})	black stork, goose, king penguin, albatross, crane
reptiles / insects (T_{2i})	European fire salamander, bee, scorpion, fly, black widow, ladybug, slug, cockroach, grasshopper, centipede, mantis, sulphur butterfly dragonfly, snail, boa constrictor, spider web, trilobite, tarantula
aquatic animals (T_{2o})	brain coral, American lobster, American alligator, tailed frog, spiny lobster sea slug, dugong, coral reef, bullfrog, sea cucumber, jellyfish, goldfish
edible items (T_{3i})	orange, pizza, mushroom, banana, ice cream, pomegranate, pretzel, mashed potato potpie, cauliflower, meat loaf, bell pepper, guacamole, lemon, ice lolly, confectionery
garments (T_{3o})	swimming trunks, apron, poncho, academic gown, military uniform, neck brace vestment, kimono, sombrero, fur coat, cardigan, bikini, miniskirt, bow tie sunglasses, sandal, sock, Christmas stocking
Household items (T_{4i})	picket fence, candle, chain, rocking chair, torch, iPod frying pan, dumbbell, water jug, teddy, plate, walking stick, computer keyboard bucket, comic book, sewing machine, remote control, teapot, barn, volleyball lawn mower, hourglass, desk, lampshade, bathtub, wok, CD player, rugby ball stopwatch, magnetic compass, space heater, plunger, backpack, wooden spoon broom, dining table, basketball, punching bag, refrigerator, umbrella
places / manmade things (T_{4o})	seashore, water tower, triumphal arch, cliff dwelling, suspension bridge dam, steel arch bridge, cliff, monarch, obelisk, lakeside, fountain, altar, alp
Vehicles (T_{5i})	school bus, lifeboat, jinrikisha, beach wagon, tractor, moving van, trolleybus police van, go kart, freight car, bullet train, sports car, gondola, limousine, convertible
Miscellaneous (T_{5o})	beaker, parking meter, drumstick, reel, potter's wheel, beacon, acorn, gasmask, projectile cannon, cash machine, maypole, pay phone, flagpole, bannister, thatch, pill, bottle, pop bottle barbershop, birdhouse, binoculars, organ, abacus, nail turnstile, beer bottle, oboe viaduct, scoreboard, barrel, pole, syringe, chest, butcher shop, espresso, snorkel, brass

Table 8: Performance comparison of ENVISE with the baseline methods on different ID and OD classes on CIFAR-100 and TI datasets as presented in Table 2. The ↓ indicates smaller value is better and ↑ indicates greater value is better.

Metric	inlier \ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE	inlier \ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE
FPR (95% TPR) ↓	$C_{1i} \setminus C_{1o}$	0.94	0.91	0.91	0.91	0.93	0.94	0.95	**0.89**	$T_{1i} \setminus T_{1o}$	0.72	0.99	0.98	0.98	0.81	0.87	0.93	**0.69**
	$C_{2i} \setminus C_{2o}$	0.88	0.72	0.71	0.72	0.86	0.85	0.91	**0.40**	$T_{2i} \setminus T_{2o}$	0.87	0.98	1.0	0.87	0.84	0.80	0.87	**0.78**
	$C_{3i} \setminus C_{3o}$	0.82	0.68	0.66	0.67	0.90	0.89	0.90	**0.57**	$T_{3i} \setminus T_{3o}$	0.78	0.89	0.82	0.82	0.82	0.81	0.85	**0.74**
	$C_{4i} \setminus C_{4o}$	0.82	0.75	0.71	0.76	0.85	0.90	0.91	**0.61**	$T_{4i} \setminus T_{4o}$	0.94	1.0	1.0	0.83	0.87	0.84	0.92	**0.78**
	$C_{5i} \setminus C_{5o}$	0.94	0.92	0.92	0.92	0.90	0.92	0.95	**0.76**	$T_{5i} \setminus T_{5o}$	0.88	0.88	0.87	0.82	0.90	0.80	0.83	**0.78**
	$C_{6i} \setminus C_{6o}$	0.82	0.66	0.60	0.67	0.89	0.95	0.93	**0.49**	-	-	-	-	-	-	-	-	
	mean	0.87	0.77	0.75	0.77	0.89	0.91	0.92	**0.62**	-	0.84	0.95	0.93	0.86	0.85	0.82	0.88	**0.75**
Detection error ↓	$C_{1i} \setminus C_{1o}$	0.45	0.47	0.47	0.47	0.43	0.43	0.43	**0.40**	$T_{1i} \setminus T_{1o}$	0.39	0.48	0.44	0.43	0.28	0.38	0.39	**0.23**
	$C_{2i} \setminus C_{2o}$	0.40	0.32	0.33	0.32	0.37	0.38	0.38	**0.14**	$T_{2i} \setminus T_{2o}$	0.40	0.43	0.42	0.31	0.29	0.31	0.34	**0.26**
	$C_{3i} \setminus C_{3o}$	0.38	0.25	0.26	0.26	0.36	0.33	0.41	**0.18**	$T_{3i} \setminus T_{3o}$	0.45	0.39	0.41	0.32	0.37	0.32	0.43	**0.20**
	$C_{4i} \setminus C_{4o}$	0.38	0.32	0.31	0.32	0.39	0.42	0.38	**0.30**	$T_{4i} \setminus T_{4o}$	0.29	0.31	0.33	0.38	0.51	0.20	0.32	**0.18**
	$C_{5i} \setminus C_{5o}$	0.49	0.43	0.43	0.43	0.43	0.46	0.47	**0.28**	$T_{5i} \setminus T_{5o}$	0.34	0.37	0.34	0.34	0.36	0.30	0.36	**0.26**
	$C_{6i} \setminus C_{6o}$	0.35	0.23	0.23	0.23	0.35	0.49	0.39	**0.16**	-	-	-	-	-	-	-	-	
	mean	0.41	0.34	0.34	0.34	0.39	0.42	0.41	**0.24**	-	0.37	0.40	0.39	0.36	0.36	0.30	0.37	**0.23**
AuROC ↑	$C_{1i} \setminus C_{1o}$	0.55	0.54	0.54	0.54	0.58	0.58	0.58	**0.62**	$T_{1i} \setminus T_{1o}$	0.72	0.56	0.55	0.58	0.55	0.84	0.79	**0.89**
	$C_{2i} \setminus C_{2o}$	0.62	0.74	0.73	0.74	0.67	0.68	0.66	**0.93**	$T_{2i} \setminus T_{2o}$	0.75	0.55	0.55	0.65	0.66	0.71	0.72	**0.81**
	$C_{3i} \setminus C_{3o}$	0.67	0.81	0.81	0.81	0.67	0.69	0.62	**0.89**	$T_{3i} \setminus T_{3o}$	0.65	0.51	0.50	0.58	0.64	0.68	0.65	**0.81**
	$C_{4i} \setminus C_{4o}$	0.66	0.74	0.74	0.73	0.65	0.60	0.64	**0.75**	$T_{4i} \setminus T_{4o}$	0.69	0.54	0.53	0.64	0.64	0.69	0.69	**0.75**
	$C_{5i} \setminus C_{5o}$	0.47	0.59	0.59	0.59	0.58	0.55	0.53	**0.79**	$T_{5i} \setminus T_{5o}$	0.90	0.55	0.54	0.69	0.61	0.73	0.90	**0.91**
	$C_{6i} \setminus C_{6o}$	0.70	0.85	0.85	0.85	0.68	0.38	0.63	**0.91**	-	-	-	-	-	-	-	-	
	mean	0.61	0.71	0.71	0.71	0.64	0.58	0.61	**0.82**	-	0.74	0.54	0.53	0.63	0.62	0.73	0.75	**0.77**
AuPR (inlier) ↑	$C_{1i} \setminus C_{1o}$	0.76	0.74	0.74	0.74	0.77	0.76	0.77	**0.80**	$T_{1i} \setminus T_{1o}$	0.78	0.61	0.61	0.66	0.63	0.88	0.86	**0.91**
	$C_{2i} \setminus C_{2o}$	0.75	0.82	0.81	0.82	0.79	0.80	0.78	**0.96**	$T_{2i} \setminus T_{2o}$	0.79	0.56	0.56	0.66	0.58	0.66	0.78	**0.86**
	$C_{3i} \setminus C_{3o}$	0.68	0.82	0.82	0.82	0.71	0.71	0.63	**0.91**	$T_{3i} \setminus T_{3o}$	0.63	0.71	0.71	0.69	0.75	0.76	0.76	**0.93**
	$C_{4i} \setminus C_{4o}$	0.74	0.81	0.81	0.81	0.72	0.69	0.75	**0.82**	$T_{4i} \setminus T_{4o}$	0.68	0.75	0.75	0.74	0.77	0.78	0.77	**0.91**
	$C_{5i} \setminus C_{5o}$	0.82	0.88	0.88	0.88	0.87	0.85	0.85	**0.95**	$T_{5i} \setminus T_{5o}$	**0.85**	0.32	0.31	0.87	0.69	0.84	**0.85**	**0.85**
	$C_{6i} \setminus C_{6o}$	0.56	0.78	0.79	0.78	0.58	0.26	0.46	**0.87**	-	-	-	-	-	-	-	-	
	mean	0.72	0.81	0.81	0.81	0.74	0.68	0.71	**0.89**	-	0.75	0.59	0.59	0.72	0.68	0.78	0.80	**0.89**
AuPR (outlier) ↑	$C_{1i} \setminus C_{1o}$	0.32	0.33	0.33	0.34	0.34	0.34	0.34	**0.39**	$T_{1i} \setminus T_{1o}$	0.38	0.48	0.47	0.48	0.65	0.65	0.65	**0.71**
	$C_{2i} \setminus C_{2o}$	0.47	0.64	0.64	0.64	0.52	0.53	0.49	**0.87**	$T_{2i} \setminus T_{2o}$	0.50	0.52	0.51	0.64	0.59	0.68	0.67	**0.73**
	$C_{3i} \setminus C_{3o}$	0.67	0.79	0.79	0.80	0.63	0.64	0.60	**0.87**	$T_{3i} \setminus T_{3o}$	0.47	0.29	0.29	0.47	0.44	0.61	0.59	**0.63**
	$C_{4i} \setminus C_{4o}$	0.55	0.65	**0.66**	0.64	0.54	0.49	0.49	0.62	$T_{4i} \setminus T_{4o}$	0.29	0.29	0.28	0.45	0.43	**0.51**	**0.51**	0.44
	$C_{5i} \setminus C_{5o}$	0.16	0.21	0.21	0.21	0.21	0.20	0.18	**0.42**	$T_{5i} \setminus T_{5o}$	0.79	0.73	0.74	0.79	0.57	0.78	0.78	**0.84**
	$C_{6i} \setminus C_{6o}$	0.81	0.90	0.91	0.90	0.78	0.63	0.74	**0.94**	-	-	-	-	-	-	-	-	
	mean	0.50	0.59	0.59	0.59	0.50	0.47	0.47	**0.69**	-	0.49	0.46	0.46	0.57	0.54	0.65	0.64	**0.67**

Table 9: Comparison of ENVISE with the baseline methods on ID class pairs C_1 and C_2 with every OD class on CIFAR-100. The $\downarrow$ indicates smaller value is better and $\uparrow$ indicates greater value is better.

Metric	inlier \ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE	inlier \ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE
FPR (95% TPR) $\downarrow$	$C_{1i} \setminus C_{1o}$	0.94	0.91	0.91	0.91	0.93	0.94	0.95	**0.89**	$C_{2i} \setminus C_{1o}$	0.92	0.82	0.89	0.87	0.79	0.86	0.85	**0.69**
	$C_{1i} \setminus C_{2o}$	0.95	0.97	0.96	0.81	0.90	0.97	0.86	**0.68**	$C_{2i} \setminus C_{2o}$	0.88	0.72	0.71	0.72	0.86	0.85	0.91	**0.40**
	$C_{1i} \setminus C_{3o}$	0.94	1.0	1.0	0.81	0.93	0.98	0.76	**0.68**	$C_{2i} \setminus C_{3o}$	0.93	0.89	0.88	0.89	0.86	0.89	0.81	**0.70**
	$C_{1i} \setminus C_{4o}$	0.94	0.99	0.84	0.83	0.94	0.89	0.83	**0.76**	$C_{2i} \setminus C_{4o}$	0.95	0.92	0.93	0.73	0.87	0.86	0.78	**0.68**
	$C_{1i} \setminus C_{5o}$	0.94	0.99	0.89	0.78	0.83	0.88	0.77	**0.42**	$C_{2i} \setminus C_{5o}$	0.92	0.89	0.88	0.96	0.73	0.86	0.76	**0.45**
	$C_{1i} \setminus C_{6o}$	0.94	0.80	0.90	0.71	0.92	0.81	0.83	**0.79**	$C_{2i} \setminus C_{6o}$	0.92	0.86	0.86	0.83	0.83	0.87	0.88	**0.69**
	mean	0.94	0.94	0.92	0.81	0.91	0.91	0.83	**0.70**	-	0.92	0.85	0.86	0.83	0.82	0.87	0.83	**0.60**
Detection error $\downarrow$	$C_{1i} \setminus C_{1o}$	0.45	0.47	0.47	0.47	0.43	0.43	0.43	**0.40**	$C_{2i} \setminus C_{1o}$	0.41	0.41	0.41	0.33	0.33	0.38	0.41	**0.31**
	$C_{1i} \setminus C_{2o}$	0.50	0.49	0.49	0.35	0.40	0.40	0.38	**0.31**	$C_{2i} \setminus C_{2o}$	0.40	0.32	0.33	0.32	0.37	0.38	0.38	**0.14**
	$C_{1i} \setminus C_{3o}$	0.49	0.50	0.50	0.38	0.44	0.48	0.36	**0.32**	$C_{2i} \setminus C_{3o}$	0.44	0.45	0.45	0.41	0.40	0.38	0.37	**0.36**
	$C_{1i} \setminus C_{4o}$	0.46	0.50	0.42	0.41	0.45	0.37	0.35	**0.32**	$C_{2i} \setminus C_{4o}$	0.47	0.45	0.45	0.39	0.39	0.43	0.38	**0.34**
	$C_{1i} \setminus C_{5o}$	0.49	0.50	0.50	0.36	0.27	0.34	0.29	**0.18**	$C_{2i} \setminus C_{5o}$	0.39	0.43	0.45	0.32	0.24	0.31	0.28	**0.21**
	$C_{1i} \setminus C_{6o}$	0.49	0.43	0.40	0.35	0.41	0.37	0.38	**0.35**	$C_{2i} \setminus C_{6o}$	0.43	0.47	0.46	**0.34**	0.36	0.40	0.39	0.35
	mean	0.48	0.48	0.46	0.39	0.40	0.40	0.37	**0.31**	-	0.42	0.42	0.43	0.35	0.35	0.38	0.37	**0.23**
AuROC $\uparrow$	$C_{1i} \setminus C_{1o}$	0.55	0.54	0.54	0.54	0.58	0.58	0.58	**0.62**	$C_{2i} \setminus C_{1o}$	0.61	0.53	0.57	0.68	0.72	0.69	0.68	**0.74**
	$C_{1i} \setminus C_{2o}$	0.46	0.47	0.49	0.74	0.62	0.64	0.71	**0.76**	$C_{2i} \setminus C_{2o}$	0.62	0.74	0.73	0.74	0.67	0.68	0.66	**0.93**
	$C_{1i} \setminus C_{3o}$	0.41	0.31	0.39	0.63	0.56	**0.68**	0.67	0.64	$C_{2i} \setminus C_{3o}$	0.54	0.60	0.59	0.61	0.62	0.60	0.71	**0.74**
	$C_{1i} \setminus C_{4o}$	0.52	0.50	0.45	0.71	0.55	0.63	0.70	**0.79**	$C_{2i} \setminus C_{4o}$	0.54	0.53	0.60	0.59	**0.64**	0.57	0.61	0.62
	$C_{1i} \setminus C_{5o}$	0.47	0.23	0.64	0.71	0.77	0.79	0.83	**0.89**	$C_{2i} \setminus C_{5o}$	0.61	0.64	0.64	0.77	0.83	0.77	0.76	**0.85**
	$C_{1i} \setminus C_{6o}$	0.49	0.67	0.67	0.63	0.58	0.69	0.69	**0.73**	$C_{2i} \setminus C_{6o}$	0.58	0.51	0.50	0.62	0.68	0.64	0.63	**0.70**
	mean	0.48	0.45	0.53	0.66	0.61	0.67	0.70	**0.74**	-	0.58	0.59	0.61	0.68	0.69	0.66	0.68	**0.71**
AuPR (inlier) $\uparrow$	$C_{1i} \setminus C_{1o}$	0.76	0.74	0.74	0.74	0.77	0.76	0.77	**0.80**	$C_{2i} \setminus C_{1o}$	0.64	0.63	0.63	0.61	0.61	0.63	0.69	0.77
	$C_{1i} \setminus C_{2o}$	0.67	0.68	0.69	0.81	0.77	0.70	0.75	**0.84**	$C_{2i} \setminus C_{2o}$	0.75	0.82	0.81	0.82	0.79	0.80	0.78	**0.96**
	$C_{1i} \setminus C_{3o}$	0.64	0.60	0.63	0.71	0.71	0.57	0.68	**0.73**	$C_{2i} \setminus C_{3o}$	0.61	0.51	0.60	0.71	0.71	0.67	0.69	**0.73**
	$C_{1i} \setminus C_{4o}$	**0.74**	0.68	0.68	0.78	0.73	0.65	**0.74**	0.71	$C_{2i} \setminus C_{4o}$	0.62	0.57	0.57	0.63	0.61	0.66	0.58	**0.67**
	$C_{1i} \setminus C_{5o}$	0.68	0.58	0.58	0.76	0.88	0.82	0.83	**0.94**	$C_{2i} \setminus C_{5o}$	0.75	0.54	0.59	0.77	0.80	0.74	0.75	**0.89**
	$C_{1i} \setminus C_{6o}$	0.54	0.66	0.69	0.60	0.60	0.61	0.63	**0.69**	$C_{2i} \setminus C_{6o}$	0.56	0.66	0.66	0.61	0.62	0.63	0.63	**0.68**
	mean	0.66	0.66	0.67	0.73	0.74	0.69	0.73	**0.79**	-	0.66	0.62	0.64	0.69	0.69	0.69	0.69	**0.78**
AuPR (outlier) $\uparrow$	$C_{1i} \setminus C_{1o}$	0.32	0.33	0.33	0.34	0.34	0.34	0.34	**0.39**	$C_{2i} \setminus C_{1o}$	0.44	0.41	0.41	0.60	0.59	0.55	0.53	**0.62**
	$C_{1i} \setminus C_{2o}$	0.28	0.28	0.29	0.63	0.40	0.33	0.58	**0.64**	$C_{2i} \setminus C_{2o}$	0.47	0.64	0.64	0.64	0.52	0.53	0.49	**0.87**
	$C_{1i} \setminus C_{3o}$	0.27	0.21	0.42	0.64	0.35	0.42	0.58	**0.68**	$C_{2i} \setminus C_{3o}$	0.41	0.26	0.29	0.61	0.48	0.51	0.53	**0.65**
	$C_{1i} \setminus C_{4o}$	0.31	0.26	0.39	0.61	0.43	0.50	0.58	**0.65**	$C_{2i} \setminus C_{4o}$	0.38	0.50	0.49	0.61	0.49	0.53	0.58	**0.69**
	$C_{1i} \setminus C_{5o}$	0.29	0.21	0.41	0.73	0.52	0.73	0.75	**0.78**	$C_{2i} \setminus C_{5o}$	0.43	0.48	0.47	0.75	0.70	0.71	**0.80**	**0.80**
	$C_{1i} \setminus C_{6o}$	0.46	0.57	0.52	0.66	0.53	0.55	0.68	0.72	$C_{2i} \setminus C_{6o}$	0.59	0.54	0.54	0.66	0.70	0.69	0.68	**0.71**
	mean	0.32	0.31	0.39	0.60	0.43	0.48	0.59	**0.64**	-	0.45	0.47	0.47	0.65	0.58	0.57	0.60	**0.72**

Table 10: Comparison of ENVISE with the baseline methods on ID class pairs C_3 and C_4 with every OD class on CIFAR-100. The $\downarrow$ indicates smaller value is better and $\uparrow$ indicates greater value is better.

Metric	inlier $\setminus$ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE	inlier $\setminus$ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE
FPR (95% TPR) $\downarrow$	$C_{3i} \setminus C_{1o}$	0.96	0.89	0.83	0.86	0.74	0.78	0.68	**0.47**	$C_{4i} \setminus C_{1o}$	0.94	1.0	1.0	0.71	0.86	0.85	0.77	**0.51**
	$C_{3i} \setminus C_{2o}$	0.92	0.87	0.87	0.76	0.80	0.82	0.79	**0.57**	$C_{4i} \setminus C_{2o}$	0.93	0.96	0.96	0.73	0.89	0.87	0.67	**0.59**
	$C_{3i} \setminus C_{3o}$	0.82	0.75	0.71	0.76	0.85	0.90	0.91	**0.61**	$C_{4i} \setminus C_{3o}$	0.95	0.90	0.90	0.80	0.91	0.92	0.69	**0.62**
	$C_{3i} \setminus C_{4o}$	0.94	0.92	0.94	0.77	0.92	0.90	0.77	**0.73**	$C_{4i} \setminus C_{4o}$	0.94	0.92	0.92	0.92	0.90	0.92	0.95	**0.76**
	$C_{3i} \setminus C_{5o}$	0.95	0.95	0.95	0.80	0.68	0.70	0.48	**0.29**	$C_{4i} \setminus C_{5o}$	0.93	0.89	0.90	0.80	0.79	0.82	0.62	**0.33**
	$C_{3i} \setminus C_{6o}$	0.92	0.85	0.85	0.87	0.86	0.87	0.69	**0.65**	$C_{4i} \setminus C_{6o}$	0.92	0.87	0.87	0.74	0.92	0.90	0.70	**0.61**
	mean	0.92	0.87	0.86	0.80	0.81	0.83	0.72	**0.55**	-	0.94	0.93	0.93	0.78	0.88	0.88	0.73	**0.57**
Detection error $\downarrow$	$C_{3i} \setminus C_{1o}$	0.44	0.47	0.48	0.41	0.26	0.34	0.28	**0.22**	$C_{4i} \setminus C_{1o}$	0.42	0.5	0.5	0.41	0.32	0.31	0.29	**0.26**
	$C_{3i} \setminus C_{2o}$	0.42	0.44	0.43	0.40	0.33	0.33	0.31	**0.30**	$C_{4i} \setminus C_{2o}$	0.45	0.5	0.5	0.44	0.36	0.34	0.39	**0.30**
	$C_{3i} \setminus C_{3o}$	0.38	0.25	0.26	0.26	0.36	0.33	0.41	**0.18**	$C_{4i} \setminus C_{3o}$	0.49	0.46	0.48	0.37	0.39	0.38	0.41	**0.31**
	$C_{3i} \setminus C_{4o}$	0.38	0.48	0.48	0.36	0.41	0.38	**0.29**	0.32	$C_{4i} \setminus C_{4o}$	0.38	0.32	0.31	0.32	0.39	0.42	0.38	**0.30**
	$C_{3i} \setminus C_{5o}$	0.50	0.49	0.49	0.32	0.20	0.29	0.19	**0.14**	$C_{4i} \setminus C_{5o}$	0.47	0.47	0.47	0.41	0.22	0.21	0.27	**0.16**
	$C_{3i} \setminus C_{6o}$	0.38	0.41	0.40	0.33	0.35	0.36	0.37	**0.32**	$C_{4i} \setminus C_{6o}$	0.42	0.41	0.41	0.40	0.37	0.34	0.33	**0.31**
	mean	0.42	0.42	0.42	0.35	0.32	0.34	0.31	**0.25**	-	0.44	0.44	0.45	0.39	0.34	0.33	0.35	**0.27**
AuROC $\uparrow$	$C_{3i} \setminus C_{1o}$	0.57	0.60	0.59	0.74	0.79	0.81	0.79	**0.83**	$C_{4i} \setminus C_{1o}$	0.58	0.29	0.26	0.70	0.72	0.74	0.73	**0.75**
	$C_{3i} \setminus C_{2o}$	0.59	0.46	0.47	0.74	0.72	0.73	0.69	**0.77**	$C_{4i} \setminus C_{2o}$	0.55	0.27	0.38	0.60	0.65	0.70	0.73	**0.78**
	$C_{3i} \setminus C_{3o}$	0.67	0.81	0.81	0.81	0.67	0.69	0.62	**0.89**	$C_{4i} \setminus C_{3o}$	0.50	0.39	0.40	0.56	0.61	0.65	0.66	**0.69**
	$C_{3i} \setminus C_{4o}$	0.59	0.41	0.42	0.79	0.60	0.64	0.75	**0.84**	$C_{4i} \setminus C_{4o}$	0.66	0.74	0.74	0.73	0.65	0.60	0.64	**0.75**
	$C_{3i} \setminus C_{5o}$	0.49	0.39	0.38	0.72	0.86	0.87	0.86	**0.93**	$C_{4i} \setminus C_{5o}$	0.52	0.41	0.40	0.62	0.82	0.84	0.88	**0.90**
	$C_{3i} \setminus C_{6o}$	0.64	0.59	0.59	0.73	0.68	0.67	0.69	**0.75**	$C_{4i} \setminus C_{6o}$	0.59	0.53	0.53	0.78	0.66	0.69	0.75	**0.78**
	mean	0.54	0.54	0.54	0.76	0.72	0.74	0.73	**0.84**	-	0.57	0.44	0.45	0.67	0.68	0.70	0.73	**0.78**
AuPR (inlier) $\uparrow$	$C_{3i} \setminus C_{1o}$	0.55	0.42	0.40	0.73	0.70	0.69	0.66	**0.74**	$C_{4i} \setminus C_{1o}$	0.68	0.43	0.44	0.71	0.69	0.69	0.68	**0.73**
	$C_{3i} \setminus C_{2o}$	0.57	0.41	0.41	0.75	0.69	0.70	0.73	0.78	$C_{4i} \setminus C_{2o}$	0.65	0.44	0.45	0.64	0.63	0.67	0.67	**0.68**
	$C_{3i} \setminus C_{3o}$	0.68	0.82	0.82	0.82	0.71	0.71	0.63	**0.91**	$C_{4i} \setminus C_{3o}$	0.62	0.45	0.44	0.62	**0.69**	0.63	0.66	**0.69**
	$C_{3i} \setminus C_{4o}$	0.57	0.43	0.43	0.76	0.57	0.70	0.71	**0.77**	$C_{4i} \setminus C_{4o}$	0.74	0.81	0.81	0.81	0.72	0.69	0.75	**0.82**
	$C_{3i} \setminus C_{5o}$	0.50	0.42	0.41	0.69	0.87	0.89	0.88	**0.91**	$C_{4i} \setminus C_{5o}$	0.62	0.53	0.53	0.74	0.88	0.81	0.83	**0.92**
	$C_{3i} \setminus C_{6o}$	0.45	0.37	0.37	0.64	0.49	0.67	0.66	**0.68**	$C_{4i} \setminus C_{6o}$	0.53	0.43	0.45	0.72	0.59	0.63	0.72	**0.83**
	mean	0.55	0.48	0.62	0.73	0.67	0.73	0.71	**0.80**	-	0.64	0.52	0.52	0.71	0.59	0.69	0.72	**0.78**
AuPR (outlier) $\uparrow$	$C_{3i} \setminus C_{1o}$	0.54	0.49	0.48	0.74	0.77	0.77	0.69	**0.84**	$C_{4i} \setminus C_{1o}$	0.45	0.24	0.26	0.67	0.56	0.59	0.66	**0.71**
	$C_{3i} \setminus C_{2o}$	0.57	0.43	0.43	0.73	0.69	0.70	0.68	**0.76**	$C_{4i} \setminus C_{2o}$	0.42	0.25	0.24	0.60	0.50	0.66	**0.67**	**0.67**
	$C_{3i} \setminus C_{3o}$	0.67	0.79	0.79	0.80	0.63	0.64	0.60	**0.87**	$C_{4i} \setminus C_{3o}$	0.39	0.25	0.25	0.56	0.47	0.59	0.62	**0.68**
	$C_{3i} \setminus C_{4o}$	0.56	0.49	0.48	0.72	0.56	0.69	0.69	**0.74**	$C_{4i} \setminus C_{4o}$	0.55	0.65	**0.66**	0.64	0.54	0.49	0.49	0.62
	$C_{3i} \setminus C_{5o}$	0.50	0.45	0.44	0.72	0.83	0.82	0.86	**0.92**	$C_{4i} \setminus C_{5o}$	0.41	0.38	0.37	0.69	0.66	0.69	0.77	**0.86**
	$C_{3i} \setminus C_{6o}$	0.75	0.77	0.77	0.77	0.78	0.77	0.72	**0.82**	$C_{4i} \setminus C_{6o}$	0.61	0.44	0.44	0.73	0.66	0.69	0.71	**0.78**
	mean	0.60	0.57	0.57	0.75	0.71	0.73	0.66	**0.83**	-	0.47	0.37	0.37	0.65	0.57	0.62	0.65	**0.72**

Table 11: Comparison of ENVISE with the baseline methods on ID class pairs C_5 and C_6 with every OD class on CIFAR-100. The $\downarrow$ indicates smaller value is better and $\uparrow$ indicates greater value is better.

Metric	inlier \ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE	inlier \ outlier class	ED	CE	CS	ODIN	OECC	MCD	CAL	ENVISE
FPR (95% TPR) $\downarrow$	$C_{5i} \setminus C_{1o}$	0.95	0.88	0.89	0.77	0.89	0.92	0.73	**0.62**	$C_{6i} \setminus C_{1o}$	0.93	0.72	0.73	0.73	0.82	0.79	0.71	**0.63**
	$C_{5i} \setminus C_{2o}$	0.94	1.0	1.0	0.76	0.84	0.87	0.81	**0.72**	$C_{6i} \setminus C_{2o}$	0.93	0.79	0.79	0.72	0.85	0.80	0.66	**0.55**
	$C_{5i} \setminus C_{3o}$	0.93	0.94	0.94	0.74	0.90	0.91	0.72	**0.69**	$C_{6i} \setminus C_{3o}$	0.95	0.81	0.82	0.81	0.87	0.86	0.64	**0.58**
	$C_{5i} \setminus C_{4o}$	0.95	1.0	1.0	**0.78**	0.94	0.94	0.81	0.92	$C_{6i} \setminus C_{4o}$	0.95	0.87	0.87	0.77	0.87	0.86	0.69	**0.51**
	$C_{5i} \setminus C_{5o}$	0.94	0.92	0.92	0.92	0.90	0.92	0.95	**0.76**	$C_{6i} \setminus C_{5o}$	0.95	0.73	0.74	0.74	0.76	0.73	0.53	**0.37**
	$C_{5i} \setminus C_{6o}$	0.94	1.0	1.0	0.74	0.92	0.93	0.88	**0.59**	$C_{6i} \setminus C_{6o}$	0.82	0.66	0.60	0.67	0.89	0.95	0.93	**0.49**
	mean	0.94	0.96	0.96	0.79	0.90	0.92	0.82	**0.72**	-	0.92	0.76	0.76	0.75	0.84	0.83	0.69	**0.52**
Detection error $\downarrow$	$C_{5i} \setminus C_{1o}$	0.42	0.43	0.44	0.42	0.34	0.34	0.33	**0.30**	$C_{6i} \setminus C_{1o}$	0.43	0.36	0.36	0.41	0.37	0.39	**0.32**	**0.32**
	$C_{5i} \setminus C_{2o}$	0.45	0.50	0.50	0.39	0.35	0.34	0.41	**0.33**	$C_{6i} \setminus C_{2o}$	0.44	0.42	0.42	0.40	0.30	0.32	0.30	**0.29**
	$C_{5i} \setminus C_{3o}$	0.40	0.43	0.44	0.38	0.38	0.37	0.41	**0.36**	$C_{6i} \setminus C_{3o}$	0.46	0.41	0.40	0.37	0.36	0.36	0.32	**0.30**
	$C_{5i} \setminus C_{4o}$	0.43	0.50	0.50	**0.39**	0.43	0.44	**0.39**	0.43	$C_{6i} \setminus C_{4o}$	0.49	0.45	0.44	0.39	0.30	0.32	0.28	**0.26**
	$C_{5i} \setminus C_{5o}$	0.49	0.43	0.43	0.43	0.43	0.46	0.47	**0.28**	$C_{6i} \setminus C_{5o}$	0.43	0.37	0.37	0.39	0.19	0.21	0.28	**0.18**
	$C_{5i} \setminus C_{6o}$	0.40	0.50	0.50	0.40	0.38	0.38	0.34	**0.26**	$C_{6i} \setminus C_{6o}$	0.35	0.23	0.23	0.23	0.35	0.49	0.39	**0.16**
	mean	0.43	0.47	0.47	0.40	0.39	0.39	0.39	**0.33**	-	0.43	0.37	0.37	0.37	0.31	0.35	0.32	**0.25**
AuROC $\uparrow$	$C_{5i} \setminus C_{1o}$	0.59	0.42	0.43	0.64	0.68	0.68	0.58	**0.69**	$C_{6i} \setminus C_{1o}$	0.58	0.65	0.65	0.65	0.77	0.76	0.79	**0.83**
	$C_{5i} \setminus C_{2o}$	0.56	0.40	0.41	0.69	0.68	0.69	0.72	**0.76**	$C_{6i} \setminus C_{2o}$	0.56	0.53	0.54	0.65	0.64	0.63	0.66	**0.71**
	$C_{5i} \setminus C_{3o}$	0.61	0.36	0.36	0.68	0.63	0.64	0.62	**0.71**	$C_{6i} \setminus C_{3o}$	0.55	0.60	0.60	0.67	0.68	0.67	0.70	**0.75**
	$C_{5i} \setminus C_{4o}$	0.58	0.36	0.35	0.57	0.57	0.56	0.63	**0.69**	$C_{6i} \setminus C_{4o}$	0.50	0.53	0.53	0.62	0.75	0.72	0.69	**0.76**
	$C_{5i} \setminus C_{5o}$	0.47	0.59	0.59	0.59	0.58	0.55	0.53	**0.79**	$C_{6i} \setminus C_{5o}$	0.59	0.66	0.66	0.66	0.86	0.86	**0.88**	**0.88**
	$C_{5i} \setminus C_{6o}$	0.59	0.47	0.47	**0.77**	0.63	0.64	0.69	0.76	$C_{6i} \setminus C_{6o}$	0.70	0.85	0.85	0.85	0.68	0.38	0.63	**0.91**
	mean	0.56	0.43	0.44	0.66	0.63	0.63	0.64	**0.73**	-	0.58	0.64	0.64	0.68	0.73	0.67	0.73	**0.81**
AuPR (inlier) $\uparrow$	$C_{5i} \setminus C_{1o}$	0.87	0.71	0.71	0.75	0.70	0.70	0.79	**0.88**	$C_{6i} \setminus C_{1o}$	0.58	0.57	0.57	0.65	0.76	0.73	0.71	**0.77**
	$C_{5i} \setminus C_{2o}$	0.86	0.70	0.71	0.69	0.69	0.70	0.66	**0.87**	$C_{6i} \setminus C_{2o}$	0.56	0.49	0.50	0.66	0.72	0.70	0.73	**0.79**
	$C_{5i} \setminus C_{3o}$	0.88	0.76	0.77	0.78	0.68	0.79	0.80	**0.85**	$C_{6i} \setminus C_{3o}$	0.55	0.54	0.55	0.67	0.64	0.62	0.68	**0.74**
	$C_{5i} \setminus C_{4o}$	**0.87**	0.69	0.69	0.72	0.75	0.75	0.71	0.80	$C_{6i} \setminus C_{4o}$	0.49	0.50	0.50	0.61	0.65	0.60	**0.66**	**0.66**
	$C_{5i} \setminus C_{5o}$	0.82	0.88	0.88	0.88	0.87	0.85	0.85	**0.95**	$C_{6i} \setminus C_{5o}$	0.58	0.60	0.60	0.66	0.79	0.82	0.79	**0.84**
	$C_{5i} \setminus C_{6o}$	0.78	0.59	0.59	0.63	0.69	0.77	0.80	**0.85**	$C_{6i} \setminus C_{6o}$	0.56	0.78	0.79	0.78	0.58	0.26	0.46	**0.87**
	mean	0.85	0.72	0.73	0.74	0.73	0.76	0.77	**0.87**	-	0.55	0.58	0.59	0.67	0.69	0.57	0.56	**0.77**
AuPR (outlier) $\uparrow$	$C_{5i} \setminus C_{1o}$	0.19	0.20	0.21	0.50	0.56	0.56	0.62	**0.63**	$C_{6i} \setminus C_{1o}$	0.55	0.70	0.71	0.64	0.72	0.73	0.69	**0.74**
	$C_{5i} \setminus C_{2o}$	0.19	0.19	0.20	**0.77**	0.40	0.60	0.68	0.62	$C_{6i} \setminus C_{2o}$	0.54	0.62	0.62	0.64	0.71	0.71	0.69	**0.78**
	$C_{5i} \setminus C_{3o}$	0.22	0.22	0.23	0.37	0.43	0.45	0.49	**0.56**	$C_{6i} \setminus C_{3o}$	0.52	0.62	0.64	0.66	0.66	0.65	0.71	**0.75**
	$C_{5i} \setminus C_{4o}$	0.19	0.30	0.30	0.43	0.31	**0.49**	**0.49**	**0.49**	$C_{6i} \setminus C_{4o}$	0.49	0.57	0.57	0.66	0.70	0.69	0.73	**0.81**
	$C_{5i} \setminus C_{5o}$	0.16	0.21	0.21	0.21	0.21	0.20	0.18	**0.42**	$C_{6i} \setminus C_{5o}$	0.53	0.69	0.69	0.65	0.81	0.81	0.80	**0.90**
	$C_{5i} \setminus C_{6o}$	0.34	0.29	0.29	0.41	0.47	0.48	0.44	**0.50**	$C_{6i} \setminus C_{6o}$	0.81	0.90	0.91	0.90	0.78	0.63	0.74	**0.94**
	mean	0.22	0.24	0.24	0.45	0.40	0.37	0.48	**0.54**	-	0.57	0.68	0.69	0.69	0.73	0.70	0.73	**0.82**

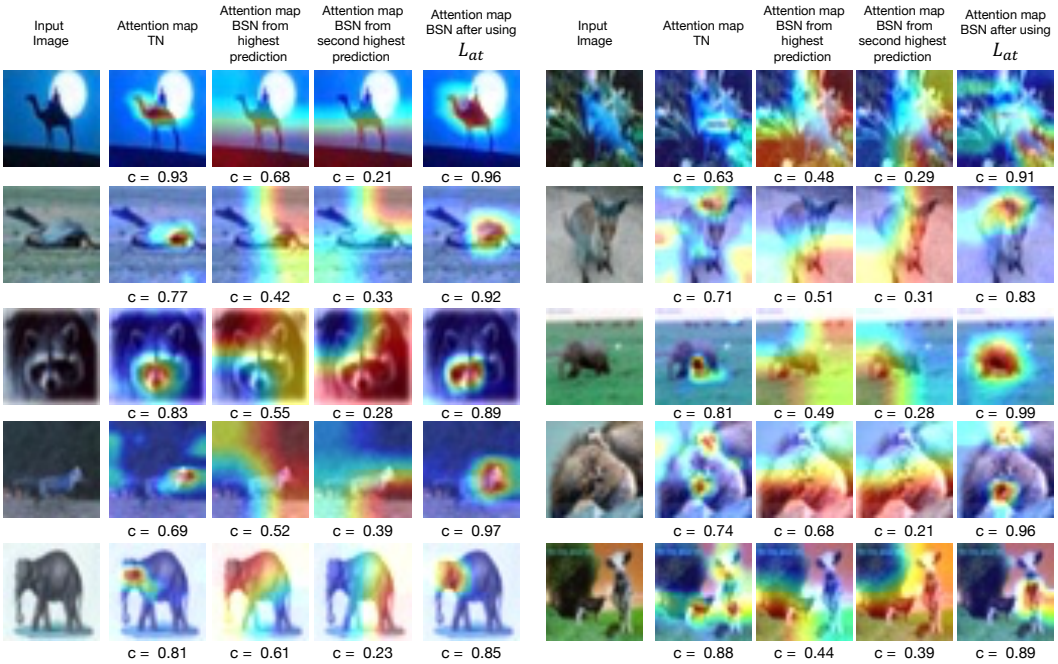

Figure 11: The attention map visualization representing the regions of the image that the network focuses on for classification. The confidence of correct classification is reported below each image

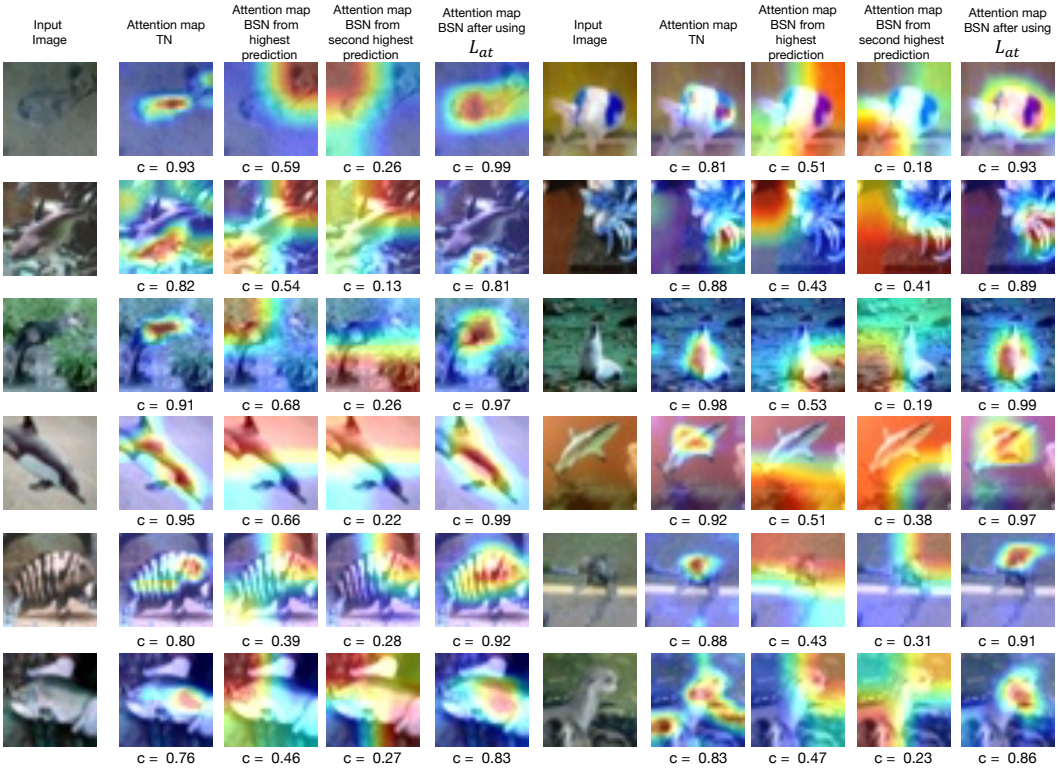

Figure 12: The attention map visualization representing the regions of the image that the network focuses on for classification. The confidence of correct classification is reported below each image