W-CTC: A CONNECTIONIST TEMPORAL CLASSIFICATION LOSS WITH WILD CARDS

ABSTRACT

Connectionist Temporal Classification (CTC) loss is commonly used in sequence learning applications. For example, in Automatic Speech Recognition (ASR) task, the training data consists of pairs of audio (input sequence) and text (output label), without temporal alignment information. Standard CTC computes a loss by aggregating over all possible alignment paths, that map the entire sequence to the entire label (full alignment). However, in practice, there are often cases where the label is incomplete. Specifically, we solve the partial alignment problem where the label only matches a middle part of the sequence. This paper proposes the wild-card CTC (W-CTC) to address this issue, by padding wild-cards at both ends of the labels. Consequently, the proposed W-CTC improves the standard CTC via aggregating over even more alignment paths. Evaluations on a number of tasks in speech and vision domains, show that the proposed W-CTC consistently outperforms the standard CTC by a large margin when label is incomplete. The effectiveness of the proposed method is further confirmed in an ablation study.

1 INTRODUCTION

Connectionist Temporal Classification (CTC) was proposed in (Graves et al., 2006) to train end-to-end sequence learning models. It has become the most popular loss function in many sequence learning applications, such as Automatic Speech Recognition (ASR) (Amodei et al., 2016; Battenberg et al., 2017) and Optical Character Recognition (OCR) (Liu et al., 2015; Subramani et al., 2020; Chen et al., 2021). CTC has a number of attractive properties. For example, it comes with a clear interpretation that maximizes the posterior marginal probability summed over all possible alignment paths. In addition, CTC loss can be easily incorporated into neural nets because it is fully differentiable. The gradients can be estimated efficiently by borrowing the well-known forward-backward algorithm from Hidden Markov Models (HMMs) (Rabiner, 1989).

One limitation of standard CTC is that it assumes the label matches the whole input sequence. CTC performs a full alignment between them. However, we often face the partial alignment in practice. For example, a speech transcription often omits the prefacing introduction of the speaker; a beginning of an audio book that tells who read the book, along with various other details, are typically not in the written version; a sign language interpreter often ignore the irrelevant gestures in the beginning and ending stages; etc. The above motivating examples are also referred as the streaming problem (Sakurai et al., 2007). In a streaming speech or video, we only capture a fraction of interested segments to form the labels. It is also common that the label is corrupted or noisy due to variable recording conditions. Partial alignment is useful in semi-supervised or transfer learning as well, which enables us to use a coarse model (from a different domain) to automatically generate labels. We can discard the low-confident outputs and form the incomplete, but more accurate labels, then feed them into the target model to train. In time series domain, streaming Dynamic Time Warping (s-DTW) was proposed (Sakurai et al., 2007) for partial warping, and Cai et al. (2019) extended it to differentiable training in neural nets. Inspired by their work, we propose the wild-card enhanced CTC, dubbed as W-CTC, to perform the end-to-end partial-alignment sequence learning. To the best of the authors' knowledge, this is the first study to apply CTC on incomplete labels.

1.1 PROBLEM FORMULATION

Let $X = [X_0, X_1, \ldots, X_{T-1}]$ be the input sequence of length T, e.g., speech audio frames for an ASR system, or image segments for an OCR system. Let $Y = [Y_0, Y_1, \ldots, Y_{N-1}]$ be the corresponding targets (labels) of length N, e.g., the transcription for ASR, or the recognized string for

OCR. Assume $T \geq N$. Though both X and Y are provided, we don't know how they are aligned. Let the alignment function be $\psi : X \to Y$, which maps a frame $x \in X$ to a target $y \in Y$. Define:

Definition 1 (**Subsequence**). *A **subsequence** of X, denoted as $\hat{X} \subset X$, is a vector of multiple **consecutive** elements in X, i.e. $\hat{X} = [X_i, X_{i+1}, \ldots, X_{i+\tau}]$, where $\tau > 0$.*

Definition 2 (**Full alignment**). *A **full** alignment maps the entire X to the entire Y, i.e. $y = \psi(x) \in Y$, for any $x \in X$, as well as $x = \psi^{-1}(y) \in X$, for any $y \in Y$.*

Definition 3 (**Alignment path**). *An alignment **path**, π, is a vector of the same length as X, i.e. $\pi = [y_0, y_1, \ldots, y_{T-1}]$, where each element $y_i = \psi(X_i) \in Y$, for $0 \leq i < T$.*

Definition 4 (**Valid alignment**). *An alignment path is **valid**, if it satisfies:*

- *Monotonicity: if X_i maps to Y_j, X_{i+t} cannot map to Y_{j-k}, for any $t > 0$ and $k > 0$.*
- *Surjection (many-to-one mapping): $\psi(\cdot)$ is a surjective function, such that a subsequence $\hat{X} \subset X$, could maps to a single $Y_j \in Y$. But each X_i can have only one label $y \in Y$.*

Definition 4 is a common assumption, because we often have a higher sampling rate in audios or images, for a better resolution. In consequence, $T \geq N$, thereafter multiple frames correspond to a single target. In practice, X could be the sequence of frame embedding vectors, i.e. $X_i \in \mathcal{R}^d$, obtained by a neural network. Provided the targets Y, we want to train the neural network by implicitly learning the alignment between X and Y. This is a **full alignment problem** and often handled by CTC loss (Graves et al., 2006) or attention mechanism (Bahdanau et al., 2014).

In this paper, we extend the full alignment problem, to a more general partial alignment case, where the input could be incomplete. That is, the input sequence X, does not necessarily have an end-to-end mapping to the label Y (violate Definition 2). In particular, we solve the missing Y problem, where X is complete. For example, the annotated transcription is incomplete, or corrupted, such that it does not correspond to the full audio speech. It is sometimes referred as the streaming alignment problem, as X can be seen as a long stream signal, while Y only matches part of X.

Definition 5 (**Partial Alignment Problem**). *A problem where the label is a subsequence $\hat{Y} = [Y_k, Y_{k+1}, \ldots, Y_{k+L-1}] \subset Y$, that has a length $L < N$. Both k and L are **unknown**.*

Note that the missing portion are at the beginning and the end of Y. A commonly seen example in ASR system, is the silence/noise at the two sides of X. The transcription often does not include such silence/noise remarks, resulting in a mismatch at the two ends. This is already handled by CTC loss with an additional "blank" label (refer to (Graves et al., 2006)). We tackle the more complex problem, that the missing part is meaningful rather than just noise/silence.

1.2 Brief Introduction of CTC

To introduce CTC, let's first consider the **full alignment problem**. There are many paths leading to the same label Y from input X. Let $\mathcal{Z}$ be the set of all valid paths. The standard CTC loss (Graves et al., 2006) is defined as the negative log-likelihood of the labels given input sequences, i.e.

$$\mathcal{L}_{\text{CTC}}(X, Y) = -\log P(Y|X) = -\log \sum_{\pi \in \mathcal{Z}} P_\pi(Y|X) \tag{1}$$

where $P_\pi(Y|X)$ is the probability of a particular alignment path π. For any path, CTC assumes conditional independence amongst X_i's, $0 \leq i < T$. So the probability of that path, is approximated by a product of the path-determined label's probability at each frame, i.e. $P_\pi(Y|X) \approx \prod_{0 \leq t < T} P_t(\pi[t] \mid X)$. Minimizing the CTC loss is equivalent to maximizing the likelihood of the observed label given the input sequence. CTC is fully differentiable, making it favorable to train an ASR or OCR system, where no exact temporal alignment information is provided. CTC is calculated using Dynamic Programming (DP), which will be further analyzed in Section 2.1.

Our target in this paper is the **partial alignment problem**, which could reduce to the full alignment problem, by trying to do full alignment between the corrupted targets, $\hat{Y}$, and all possible subsequences of X. Let $\hat{X}_{i,\tau} = [X_i, X_{i+1}, \ldots, X_{i+\tau}]$ be one subsequence, starting from i-th frame with a length of $\tau + 1$. The straight-forward way is to loop over all possible (i, τ) pairs, obtain the CTC loss for each subsequence, and summarize them, i.e.,

$$\mathcal{L}'_{\text{CTC}}(X, \hat{Y}) = \sigma\{\mathcal{L}_{\text{CTC}}(\hat{X}_{i,\tau}, \hat{Y}) \mid i + \tau < T\} \tag{2}$$

where $\sigma\{\cdot\}$ means a summary of the set, e.g., summation or max.

This straight-forward approach is too expensive to compute: For each $\mathcal{L}_{\text{CTC}}(\hat{X}_{i,\tau}, \hat{Y})$, a DP is utilized, leading to $O(TN)$ cost. Unfortunately, DP cannot be completely parallelized. Looping through all (i, τ) pairs yields a total complexity of $O(T^3 N)$. A trick is that when i is fixed, we can reuse the DP trellis from $\mathcal{L}_{\text{CTC}}(\hat{X}_{i,\tau}, \hat{Y})$ to obtain $\mathcal{L}_{\text{CTC}}(\hat{X}_{i,\tau+1}, \hat{Y})$, at a cost of $O(N)$ instead of $O(TN)$. Even with this trick, $O(T^2 N)$ is still required as looping over i cannot be avoided. Considering T can be fairly large for audio frames, this approach is hardly deployable in practice.

2 PROPOSED WILD-CARD CTC

2.1 REVIEW OF CTC COMPUTATION

Before presenting W-CTC, we briefly review in detail the Dynamic Programming (DP) based forward algorithm to compute the CTC loss in Equation 1. Taking an English ASR system as an example, let X be a sequence of embedding vectors of T speech frames, and Y be the corresponding transcription of length N. Since there could be silence frames in X, which does not correspond to any English letter, the first step is to augment Y by inserting a "blank" label that could represent the silence. "blank" is inserted between any two characters, as well as the beginning and the end of the transcription. For instance, a transcription of "hello" will be augmented to "-h-e-l-l-o-", where "-" represents the "blank" label. The augmented transcription $\tilde{Y}$ has a length $2N + 1$.

The second step is to build the trellis (DP matrix) M, which has $2N + 1$ rows and T columns, as illustrated in Figure 1a. Each node, $M_{i,j}$, denotes a subproblem's likelihood, i.e. $M_{i,j} = P(\tilde{Y}[0 : i] \mid X[0 : j])$. Since the speech signal could end with the transcription's last English letter, or with additional silence, the final CTC loss is a combination of these two cases. More specifically, it is the negative log of the sum of the two final states, i.e. $\mathcal{L}_{\text{CTC}}(X, \tilde{Y}) = -\log(M_{2N-1,T-1} + M_{2N,T-1})$. These two final-state nodes are colored orange at the right-bottom corner of Figure 1a.

(a) Standard CTC: The two green nodes at top-left corner are initial states for DP recursion. The two orange nodes at bottom-right corner are ending nodes for final loss calculation. The black and blue arrows are allowed transitions based on Equation 3. The red arrows are forbidden.

(b) W-CTC: The first row corresponds to the prepended wild-card "∗" symbol (the blue nodes). There are three initial states. The ending nodes are the entire last two rows, rather than only two right-bottom nodes. These changes enable $\hat{Y}$ to match only a fraction of X.

Figure 1: Illustration of Dynamic Programming based CTC and proposed W-CTC loss calculation.

To fill the trellis, we start from the top-left corner. The value of $M_{0,0}$ is the probability of "blank" given the first frame, i.e. $M_{0,0} = P(\text{-}|X_0)$. Similarly, $M_{1,0} = P(Y_0|X_0)$ is the probability of transcription's first letter. The speech could begin with either silence or the transcription's first English letter, so both $M_{0,0}$ and $M_{1,0}$ are valid initial states (the green nodes at top-left corner in Figure 1a). Note that the probability of a predicted symbol, i.e. $P(\tilde{Y}_i|X_{0:j})$, is often obtained by projecting the embedding vector X_j to a logit vector (length is the augmented vocabulary size), then

applying softmax operation. As the prediction at any time step can be conditioned on the entire sequence of X (e.g., using a bi-directional LSTM model), we write $P(\tilde{Y}_i|X)$ instead of $P(\tilde{Y}_i|X_{0:j})$ to account for the more general cases. Referring to the illustration in fig. 1a, the rules for updating the remaining values $M_{i,j}$ are as follows:

$$M_{i,j} = \begin{cases} P(\tilde{Y}_i|X) \cdot (M_{i,j-1} + M_{i-1,j-1}), & \text{if } \tilde{Y}_i = \text{``-''} \text{ or } \tilde{Y}_i = \tilde{Y}_{i-2} \\ P(\tilde{Y}_i|X) \cdot (M_{i,j-1} + M_{i-1,j-1} + M_{i-2,j-1}), & \text{otherwise} \end{cases} \quad (3)$$

The line 1 in Equation 3 does not have the term $M_{i-2,j-1}$, which means: (1) If $\tilde{Y}_i$ is "blank", the path cannot come from a skip "blank" before (M_{i-2}). (2) If two consecutive letters are the same, e.g., "-l-l-" in "-h-e-l-l-o-", the node corresponding to the second "l" cannot directly have a path from the node corresponding to the first "l". In this case, the "blank" symbol between two "l"s plays a separator role, enabling the decoder to know there are two "l"s instead of one. As a result, the red arrows in Figure 1a are forbidden. In other cases, the node $M_{i,j}$ could have a path from $M_{i-2,j}$, so the blue arrows are allowed in Figure 1a.

Following above DP recursion rules, the trellis M is filled and the values of the two ending nodes (orange nodes at bottom-right corner), are added to obtain the final CTC loss. To achieve numerical stability, the computation is typically done in log scale. This is the forward calculation step. The backward calculation is similar and we omit it due to space limit. Interested readers can refer to (Graves et al., 2006) for details.

2.2 THE PROPOSED W-CTC ALGORITHM

The label Y could be corrupted, e.g., the speech utterance still corresponds to "hello", but the annotated label is only "ell". The original CTC does not work here because it assumes a full alignment between the speech and the transcription (both have to be complete). Inspired by (Sakurai et al., 2007) and (Cai et al., 2019), we modify the original CTC loss computation procedure as follows:

- Not only inserting a "blank" symbol between characters, we also prepend the augmented transcription with a wild-card symbol "∗". For example, "hello" is converted to "∗-h-e-l-l-o-". The wild-card symbol could represent any character in vocabulary including the "blank" symbol, therefore perfectly matches any speech frames. In other words, $P(*|X) = 1$. As a result, the augmented $\tilde{Y}$ now has a length $2N + 2$, and the trellis M has $2N + 2$ rows.

- Instead of two initial-state nodes (the green nodes), we also include the first node that corresponds to the wild-card "∗", leading to a total of 3 starting nodes. This is illustrated in Figure 1b. The top-left 3 green nodes are valid initial states, where $M_{0,0} = P(*|X_0) = 1$.

- Instead of only considering two final states $M_{2N-1,T-1}, M_{2N,T-1}$, we consider the entire last two rows (the bottom two rows are all colored orange in Figure 1b). That is, we collect a set of $\mathcal{L}_{\text{CTC}}^{(j)} = -\log(M_{2N-1,j} + M_{2N,j})$, for $j = N - 1$ to $T - 1$, then we summarize them to obtain our W-CTC loss, i.e. $\mathcal{L}_{\text{W-CTC}} = \sigma\{\mathcal{L}_{\text{CTC}}^{(j)} \mid N - 1 \le j < T\}$. We can safely ignore the case of $j < N - 1$ as those are not feasible alignments (the probability $P(Y|X[0:j]) = 0$ if $j < N - 1$), because at least N frames are needed to match N characters in the transcription. So the left nodes in the last two rows in Figure 1b, are not colored orange (not ending nodes). Here the function $\sigma\{\cdot\}$ can be chosen from max, summation, or weighted sum.

The rest of the algorithm is the same as the standard CTC. The DP recursion is the same as Equation 3. Empirically, we found that choosing weighted sum for $\sigma\{\cdot\}$, yields the best results:

$$\mathcal{L}_{\text{W-CTC}} = \sum_{N-1}^{T-1} w_j \mathcal{L}_{\text{CTC}}^{(j)} \text{ , where } [w_{N-1}, \ldots, w_{T-1}] = \text{softmax}\left([-\mathcal{L}_{\text{CTC}}^{(N-1)}, \ldots, -\mathcal{L}_{\text{CTC}}^{(T-1)}]\right) \quad (4)$$

An ablation study on different strategies for $\sigma\{\cdot\}$, is presented in Section 3.4. Note that since an additional wild-card symbol with a constant probability 1 is inserted for each frame, the total probability becomes unnormalized. However, empirically we found that there is no clear difference whether to normalize the total probability. A detailed discussion is provided in Appendix C. Therefore, we use the unnormalized likelihood for simplicity.

Complexity: Since we don't introduce additional cost except for adding a row in the trellis, the time complexity remains the same as the original CTC computation, i.e. $O(TN)$. The DP updates are

the same. It can be speed up by computing the whole column in parallel, resulting in a parallel time complexity of $O(T)$ using $O(N)$ cores. The space complexity is $O(TN)$ as we need to store the graph for backpropogation.

2.3 THE KEY SUMMARY

Compared to the standard CTC, the key changes are: (1) Prepend a "$*$" symbol on Y, resulting in an additional row on top of the trellis M (the blue nodes); (2) There are 3 green nodes to start DP recursion; (3) The orange ending nodes (final states) are the entire last two rows.

The 1st change (prepend a wild-card symbol), ensures that the transcription can start anywhere in the speech frames. Due to $P(*|X) = 1$, there will be no punishment if going from $M_{0,j-1}$ to $M_{0,j}$ (the probability remains the same). The 3rd change (consider last two rows), make the transcription can end anywhere in the frames, rather than matching to the end of the speech. It is equivalent to implicitly adding a wild-card at the end of the transcription. As a result, the explicit front wild-card, and the implicit final wild-card, successfully handle the missing transcription in both ends.

Combining the two changes, we convert the standard CTC built only for **full** alignment, to a very flexible **partial** alignment usage, at negligible cost. The computation is still fully differentiable. Therefore, we can still obtain the gradient and learn the network parameters, even if the transcription only matches the middle part of the speech. These will be validated in the experiment section.

It is worth noting that, the wild-card symbol "$*$", is **NOT** equivalent to the "blank" symbol. The "blank" symbol is within the augmented vocabulary, which has a physical meaning of silence, background noise, or an unknown character. The network still needs to predict the probability of a "blank" symbol at each frame, given the speech. Usually, this probability $P(\text{-}|X)$ is < 1. On the contrary, the wild-card "$*$" is not in the vocabulary. It matches any character in the vocabulary. The probability $P(*|X)$ is always 1. Only the wild-card symbol can let the alignment skip the beginning speech frames ($M_{0,0} \to M_{0,1} \to M_{0,2} \to \ldots$), without any punishment. Just putting the "blank" symbol in the first row won't be able to achieve this.

Limitations: There are a few limitations of the proposed method. (1) It only handles missing transcription (missing Y) problem, but not the case where speech frames are missing (missing X). Since one key assumption is the many-to-one mapping ($\psi(\cdot)$ is surgective), a simple extension of prepending a wild-card in front of X, will not work. (2) The missing part in Y, needs to be at two sides, but not in the middle. Taking the "hello" example, it cannot solve the corrupted transcription "h???o", where the "?" symbol represents the missing part. But it can handle the "?ell?" case. (3) This approach requires the remaining fraction $\hat{Y}$ be a continuous subsequence of Y, e.g., "?ell?" can be solved, but "?e?lo" cannot.

3 EXPERIMENTS

We conduct experiments on speech, OCR and Sign Language tasks in §3.1, §3.2, §3.3, respectively. An ablation study is discussed in §3.4. We implement the forward computing scheme and leverage the auto-grad mechanism to achieve backpropogation. All the codes can be found in the supplementary, as well as the anonymous repository https://github.com/AnonymousRepo1/WCTC.

Experimental Settings: For each dataset, we randomly mask out a portion r of each training label. For example, if $r = 0.5$, all the training labels will have half of their text masked out. The model is expected to learn from the corrupted labels. The same training procedure is used for both the proposed W-CTC as well as the baseline (standard CTC). The **mask-ratio** r ranges from 0 (no corruption) to 0.9 (90 % is missing). After training, we evaluate on the test set. Note that while the labels in the training set are masked, the ground-truth labels in the test set for evaluation, are not corrupted. We perform 3 runs, and plot the averages and error bars (standard deviations).

3.1 AUTOMATIC SPEECH RECOGNITION (ASR) AND PHONEME RECOGNITION (PR)

We use the TIMIT (Garofolo, 1993) dataset in this experiment. The TIMIT dataset includes audios and corresponding transcriptions, at word level and phoneme level. All of the materials are in English. Word transcriptions include 26 English letters plus 10 digits, and phoneme transcriptions use 61 phonemes. There are 630 speakers. Audio is recorded at a sampling rate of 16 kHz. The transcriptions include timestamps specifying which words and phonemes were spoken when.

Our experiments consist of two tasks: (a) Automatic Speech Recognition (ASR) and (b) Phoneme Recognition (PR). The standard word-error-rate (WER) and phoneme-error-rate (PER) are used to evaluate ASR and PR, respectively. Both WER and PER compare a hypothesis sequence to a reference sequence, where the hypothesis is the predicted sequence, and the reference is the gold sequence (the labels). The comparison counts the number of substitutions, S, deletions, D, and insertions, I. Therefore, WER is defined as $\frac{S+D+I}{N}$, where N is the length of the reference (in words). PER is similar to WER but based on phonemes. Note that WER and PER can exceed 100%.

The backbone model is the pretrained Wav2vec-2.0-base (Baevski et al., 2020). We do fine-tuning until convergence. See Appendix A for more details on the training process. See Appendix B for PER results based on a reduced phoneme inventory of 39 phoneme classes (as opposed to 61). These 39 classes are used by a number of other papers, e.g., (Lopes & Perdigao, 2011; Lee & Hon, 1989).

Figure 2: WER / PER vs mask-ratio in ASR and PR tasks, on TIMIT test set.

Figure 2 shows the results on the two tasks: ASR (left panel) and PR (right panel). Both panels plot error rates as a function of mask-ratio r. From the figure, we can see: (1) On both tasks, the models trained using W-CTC achieve better results than standard CTC. The blue curves almost always stay lower than the orange curves. (2) The performance of W-CTC is very stable when a small portion of label is masked. For example, when $r = 0.5$ in ASR experiment, the model trained using W-CTC can still get below 30% WER, while the model using standard CTC has 80% WER (almost not usable in practice). (3) The performance of standard CTC deteriorate nearly linearly against mask-ratio, while W-CTC only deteriorates when more than 50% label is missing. (4) When mask-ratio is higher, e.g., $r > 0.7$, though W-CTC is still slightly better, but as expected, both models struggle and yield more than 90% WER. (5) The only exception is that when $r = 0$ (label is completely clean), the model using standard CTC is slightly better than W-CTC. The reason is the wild-card symbol introduced in W-CTC, brings significantly more alignment paths, therefore increases uncertainty and hurt the performance when data is absolutely clean.

3.2 OPTICAL CHARACTER RECOGNITION (OCR)

The Optical Character Recognition (OCR) task inputs an image containing text, and outputs the recognized text with their locations on the image. OCR systems typically consist of two major components: text detection and text recognition. Text detection creates bounding boxes to segment candidate patches that contain text, and text recognition converts the patches into strings. In the training phase, it cuts the patches (containing text) into "frames", and applies either attention or CTC to do implicit alignment with the text labels. Recent survey papers, e.g., (Chen et al., 2021; Subramani et al., 2020), describe the state-of-the-art OCR systems.

In this paper, we focus on the text recognition block as CTC is widely adopted here. We tested the proposed W-CTC using the well-known CTC-based OCR model, CRNN (Shi et al., 2016). Our implementation is based on (Sun et al., 2020), with a backbone ResNet (He et al., 2016). Testing and training materials were obtained from (Baek et al., 2019). Two standard collections were used for training: (a) MJSynth (MJ, 9 million images) (Jaderberg et al., 2014) and (b) SynthText (ST, 800k images) (Gupta et al., 2016). Each image is 32×100 pixels. We trained for 150k iterations, with a batch size of 500 images. After training, the evaluations are performed on 7 standard benchmarks: IIIT5K_3000 (Mishra et al., 2012), SVT (Wang et al., 2011), IC03_867 (Lucas et al., 2005), IC13_1015 (Karatzas et al., 2013), IC15_2077 (Karatzas et al., 2015), SVTP (Phan et al., 2013) and CUTE80 (Risnumawan et al., 2014). The evaluation metric is the recognition accuracy, ACC, defined as the number of correct images divided by total number of images. Images count as correct if and only if the predicted text matches the ground truth exactly. The higher ACC, the better results.

Figure 3: Test accuracy on 7 standard test sets, as a function of r (mask ratio). The last plot is the average. The proposed W-CTC has generally better accuracy than the standard CTC.

Evaluation Accuracy: Figure 3 shows a consistent results over different testsets. For standard CTC (red curves), when $r > 0.1$, the performance drops significantly. On the contrary, the model trained using W-CTC (green curves), performs very stable even until 70% of label masked. This indicates that the model can successfully learn the image embeddings, as well as the mappings to characters, from partial labels using W-CTC. Similarly, when data is nearly clean ($r \leq 0.1$), there is a small gap between W-CTC and standard CTC, due to the uncertainty introduced by the wild-card symbol.

(a) Model A, standard CTC.   (b) Model B, standard CTC.   (c) Model C, W-CTC.

Figure 4: The alignment paths (in the trellis M) from CTC computation. Model A and C are trained on corrupted labels $r = 0.7$; model B is trained on clean data $r = 0$. Model A and B use standard CTC; model C uses W-CTC. Model A fails to produce correct alignment. Model B has correct alignment but with confusions. Model C provides clear path though trained on corrupted labels.

Alignment Path: To better illustrate how the partial alignment is done using the W-CTC loss, we provide a sample image with the corrupted label, and demonstrate how the DP trellis M are different between using standard CTC and W-CTC. Recall that in Equation 3, M provides a soft alignment path that generates the likelihood of $P(Y|X)$.

The sample image contains 11 characters, "beautifully". However, 70% of the label is masked and only 4 characters "eaut" are left to form the corrupted label. We feed the image and the corrupted label into three models: (1) Model A, trained using standard CTC, on the training data with labels 70% masked ($r = 0.7$). According to Figure 3, when $r = 0.7$, model A would learn nearly nothing from the data. (2) Model B, trained using standard CTC, but on the clean training data ($r = 0$). This model successfully learn the representations and should be able to predict the characters correctly. But it will face the difficulty dealing with the partial label here. (3) Model C, trained using W-CTC, on the training data with label 70% masked ($r = 0.7$). Figure 3 shows that the model using W-CTC can still learn from the heavily corrupted ($r = 0.7$) labels. In Figure 4, we show the trellis M for all three models. Here model A and B are using standard CTC's trellis, while model C continues to use

the W-CTC's. So the augmented label of model A and B, is "-e-a-u-t-" ("-" is blank), while model C has the label prepended by a wild-card, i.e. "∗-e-a-u-t-".

In Figure 4, the lighter the color, the higher probability values. The nodes with higher values, form a soft alignment path that maps the input image to the corresponding label. As expected, model A (in Figure 4a) does not make reasonable predictions, therefore the path does not reflect the alignments between the partial label and the input image. In contrast, since model B is trained using clean data, it is able to get a reasonable path aligning 'eaut' to the image (shown in Figure 4b). Model B also maps the front of the image (containing letter "b", which is not in the corrupted label), to the "blank" symbol (the 1st and 3rd rows). However, it clearly struggles in the beginning (the lighter nodes in the left), and does not have a strong confidence (the path's color is darker). The final model C, although trained on the corrupted data, still generates a very reasonable alignment path in Figure 4c. The partial label is well aligned with the middle of the image, where the first row corresponds to the wild-card symbol that has a constant probability 1. Unlike model B, there are no confusions of other paths in Figure 4c. Considering that model C is trained purely on heavily corrupted data and only see 30% of training labels, the proposed W-CTC is very effective.

3.3 CONTINUOUS SIGN LANGUAGE RECOGNITION (CSLR)

Sign Language is a visual language composed of hand shape, hand movements, facial expression, mouth, head movements, etc. It is the most important communication medium for people having listening disabilities. In computer vision domain, researchers focus on recognizing a sequence of sign glosses from continuous sign language videos. Sign glosses refer to the spoken language words that represent the signs' meanings. This is called the Continuous Sign Language Recognition (CSLR) task, which has attracted lots of efforts, e.g., (Koller et al., 2019; Niu & Mak, 2020). Typically, CSLR models use CTC loss because no temporal alignment is provided in the data.

In this experiment, we leverage the framework proposed by Camgoz et al. (2020), and perform the CSLR (sign to gloss) task, with standard CTC and the W-CTC loss during training. The model is the encoder of Sign Language Recognition Transformer (SLRT) (Afouras et al., 2018). The dataset is PHOENIX14T (Camgoz et al., 2018), which provides both spoken language translations and gloss level annotations for weather broadcasts. Word-error-rate (WER) is used to measure the recognition quality on the test set. The results are presented in Table 1.

Table 1: WER on CSLR task. W-CTC significantly outperforms CTC when label corrupted.

mask ratio r	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
CTC	**0.286**	0.422	0.537	0.663	0.735	0.814	0.896	0.945	0.989	0.981
W-CTC	0.297	**0.328**	**0.346**	**0.395**	**0.418**	**0.392**	**0.440**	**0.568**	**0.926**	**0.916**

From Table 1, we can see similar results as before. Only when data is absolutely clean ($r = 0$), standard CTC is slightly better (0.286 vs 0.297 WER). Starting from $r = 0.1$, the WER of the standard CTC model, deteriors quickly. On the contrary, W-CTC model can still achieve good WER up to $r = 0.6$. That means, even only trained on heavily corrupted labels, the model can still learn the representation of videos very well using the W-CTC loss.

3.4 ABLATION STUDY ON CHOICE OF $\sigma\{\mathcal{L}_{\mathrm{CTC}}^{(j)}\}$

In this section, we conduct ablation study on how to combine the last two rows in the trellis of W-CTC computation. In other words, the choice of $\sigma\{\mathcal{L}_{\mathrm{CTC}}^{(j)}\}$. We use the weighted sum, illustrated in Equation 4, in previous experiments. The other options could be taking the maximum or summing the probabilities. We formally define these three choices as:

$$\sigma\{\mathcal{L}_{CTC}^{(j)}\} = \begin{cases} \text{sum-prob (log-sum-exp): } -\log\left(\sum_j e^{(-\mathcal{L}_{\mathrm{CTC}}^{(j)})}\right) = -\log\left(\sum_j P(Y|X[0:j])\right) \\ \text{max-prob (equivalent to } \min_j \mathcal{L}_{\mathrm{CTC}}^{(j)}): \min_j \mathcal{L}_{\mathrm{CTC}}^{(j)} = -\log\left(\max_j P(Y|X[0:j])\right) \\ \text{weighted-sum (Equation 4): } \sum_j w_j \mathcal{L}_{\mathrm{CTC}}^{(j)}, \text{ where } w_j = \text{softmax}\{-\mathcal{L}_{\mathrm{CTC}}^{(j)}\}[j] \end{cases}$$

$$(5)$$

where $N - 1 \le j \le T - 1$. Note that the 'sum-prob' means summing (over j) the probabilities of alignments between subsequence $X[0:j]$ and label Y, i.e. $P(Y|X[0:j])$. And 'max-prob' means selecting the j-th final node with the maximum probability among those alignments. So 'max-prob'

is also equivalent to taking the minimum of $\mathcal{L}_{\text{CTC}}^{(j)}$. We perform 3 runs for each choice of $\sigma\{\cdot\}$, and present the average results on ASR, PR and CSLR tasks.

Figure 5: Ablation study on choices of $\sigma\{\cdot\}$. The 'max-prob' yields the worst performance in all three experiments. The 'weighted-sum' is slightly better than 'sum-prob' in general.

From Figure 5, we can tell that taking the max (orange curves) always yields worst performance (higher than other curves). Especially in the CSLR task, when mask-ratio $r < 0.5$, 'max' leads to a word-error-rate much higher than 1. The 'sum-prob' and 'weighted-sum' are similar but 'weighted-sum' wins by a small margin. For example, in ASR task, 'weighted-sum' performs better when $r < 0.7$; In PR task, all three curves are very close but 'sum-prob' is better only when $r = 0.5$; In CSLR task, 'weightes-sum' generally performs better across different r. Therefore, choosing 'weighted-sum' for $\sigma\{\cdot\}$ provides gains over other options.

4 RELATED WORK

Connectionist Temporal Classification (CTC) (Graves et al., 2006) was proposed as a loss function to train end-to-end sequence learning models. It is widely used in speech recognition, e.g., (Amodei et al., 2016; Battenberg et al., 2017); scene text recognition, e.g., (Graves et al., 2008; Shi et al., 2016); video action labeling, e.g. (Huang et al., 2016; Lin et al., 2017; Rouast & Adam, 2020); sign language translation, e.g., (Camgoz et al., 2020; Cui et al., 2017); and many other tasks.

To improve CTC, a variety of modified versions of CTC are proposed from different aspects. In (Higuchi et al., 2020), the authors borrowed the idea from masked language model, and propose the Mask-CTC for non-autoregressive end-to-end ASR training. The work in (Lee & Watanabe, 2021) used CTC for intermediate layers as a regularization to the CTC training. Talnikar et al. (2021) proposed a joint training framework by taking advantages of both the unsupervised contrastive loss and the supervised CTC loss, for ASR. Entropy regularized CTC (Liu et al., 2018) was proposed to mitigate a problem that CTC often produces a peaky output distribution (Miao et al., 2015). A more recent work, Zeyer et al. (2021), provides a deeper numerical analysis on the role of the "blank" label and its resulting peaky behavior. Although CTC has not been applied to the partial-label training problem, a few research (Sakurai et al., 2007; Cai et al., 2019) in the time series domain, have studied a similar partial alignment problem. This paper is inspired by these works.

Attention (Bahdanau et al., 2014) is another popular method that could achieve sequence alignment. For example, in (Zheng et al., 2021), the authors concatenate speech and text embeddings, and use self-attention to find how they are mapped. Unlike CTC, attention does not enforce monotonicity, which may yield incorrect alignments. Researchers has combined attention with CTC, e.g., (Kim et al., 2017; Nakatani, 2019), to take advantage of the flexibility and power of attention, as well as the strong and clear sequence aligning capability from CTC.

5 CONCLUSIONS

This paper studies the partial alignment (streaming) problem in sequence learning, where the label only matches a middle portion of the input sequence. To address this missing Y problem, we propose the W-CTC loss, which improves the standard CTC loss with wild-cards. The wild-cards are prepended and also appended (implicitly done by considering the entire last two rows in trellis), to the original labels. The W-CTC is efficiently computed using the proposed scheme. Comprehensive experiments on a variety of speech and vision tasks, demonstrate that the W-CTC significantly outperforms standard CTC loss, when label has mismatches. Ablation study is conducted to verify our approach. Future work can extend this method to the missing X (input sequence) problem.

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A   APPENDIX: EXPERIMENTS DETAILS

Here we list the literatures and their code repositories that our implementation is based on, in Table 2. We also list the key training hyper-parameters in Table 3.

Table 2: List of code bases and their references that we use in the experiments.

W-CTC implementation	NA	https://github.com/vadimkantorov/ctc
ASR and PR experiments	Baevski et al. (2020)	https://github.com/huggingface/transformers
OCR experiment	Sun et al. (2020)	https://github.com/Media-Smart/vedastr
CSLR experiment	Camgoz et al. (2020)	https://github.com/neccam/slt

Table 3: List of key training hyper-parameters.

	Batch-size	Optimizer	LR	Steps
ASR	32	AdamW	1e-4	7k (50 epochs)
PR	32	AdamW	1e-4	7k (50 epochs)
OCR	500	AdaDelta	1	150k
CSLR	32	Adam	1e-3	Stop if no better for 800 steps

B   APPENDIX: PR ON REDUCED PHONEME SET

It is common in literature, e.g., (Lopes & Perdigao, 2011), (Lee & Hon, 1989), (Baevski et al., 2020), to use the reduced phoneme set to test the Phoneme Recognition performance. The reduced set maps the original 61 phonemes into 39 classes. The mapping table in (Lopes & Perdigao, 2011) is provided below. The original phoneme "q" is removed in the reduced set.

Table 4: Mapping table from 61 phonemes to 39 phonemes to form the reduced set.

aa,ao	ah,ax,ax-h	er,axr	hh,hv	ih,ix	l,el	m,em	n,en,nx	ng,eng	sh,zh	uw,ux	pcl,tcl,kcl,bcl,dcl,gcl,h#,pau,epi
aa	ah	er	hh	ih	l	m	n	ng	sh	uw	sil

We train the model using original 61 phonemes, and evaluate on the reduced 39 phonemes set, by following the mapping in Table 4. The average performance over 3 runs, are reported in Figure 6 below. The results are similar as the experiments on the full 61 phone set, which is shown in Figure 2's right panel. With standard CTC loss, the trained model's performance drops quickly as the mask-ratio r increases. In contrast, the W-CTC helps the model learn from corrupted labels and yield consistent performance until $r > 0.6$.

Figure 6: PER vs mask-ratio, on reduced phoneme set (39 phonemes) of TIMIT test set.

C APPENDIX: NORMALIZATION

Note that since an additional wild-card symbol "$*$", with a constant probability 1, is added in each frame, the total probability accumulating T frames is no longer normalized. Specifically, $Z = 2^T$ is the normalization factor. Therefore, to make $P(Y|X)$ a proper likelihood, it should be normalized by Z, i.e. $P(Y|X)/Z$. Otherwise, the longer input sequence (larger T), the higher $P(Y|X)$ values it can get. Due to the fact $\mathcal{L}_{\text{CTC}} = -\log P(Y|X)$, the loss on longer sequence would be smaller than the shorter ones. In other words, it punishes more on shorter X, regardless of the prediction quality. Clearly, this is not what we want. A mitigation to this, is to normalize the probability and obtain the normalized W-CTC loss:

$$\mathcal{L}_{\text{norm}}^{(j)} = -\log\left(\frac{P(Y|X)}{2^j}\right) \approx \mathcal{L}_{\text{CTC}}^{(j)} + 0.3j \quad \Rightarrow \quad \mathcal{L}'_{\text{norm}} = \sigma\{\mathcal{L}_{\text{CTC}}^{(j)} + 0.3j\} \tag{6}$$

From Equation 6, we can see that the longer sequence (larger j), the more punishment ($\log_e(2^j) \approx 0.3j$) is added to the original CTC loss.

Unfortunately this scheme does not converge in our experiments, so we slightly modify the normalization method. Rather than individually change each $\mathcal{L}_{\text{CTC}}^{(j)}$ for each j, we put a single normalization factor after obtaining $\mathcal{L}_{\text{W-CTC}} = \sigma\{\mathcal{L}_{\text{CTC}}^{(j)}\}$, by assuming the total probability for $\mathcal{L}_{\text{W-CTC}}$ is 2^T. Therefore we got the normalized version:

$$\mathcal{L}'_{\text{W-CTC}} = \mathcal{L}_{\text{W-CTC}} - 0.3T \tag{7}$$

Now the modified version can converge. However, we didn't observe a clear advantage to perform normalization in this way. In the following Figure 7, we compare the performance of unnormalized and normalized version of W-CTC. This is obtained from the average of 3 runs. It is very hard to observe a clear trend that the normalized version outperforms the unnormalized version. So in the experiments, we just use the unnormalized W-CTC for simplicity.

Figure 7