Dataset Distillation using Neural Feature Regression

Abstract

Dataset distillation aims to learn a small synthetic dataset that preserves most of the information from the original dataset. Dataset distillation can be formulated as a bi-level meta-learning problem where the outer loop optimizes the meta-dataset and the inner loop trains a model on the distilled data. Meta-gradient computation is one of the key challenges in this formulation, as differentiating through the inner loop learning procedure introduces significant computation and memory costs. In this paper, we address these challenges using neural Feature Regression with Pooling (FRePo), achieving the state-of-the-art performance with an order of magnitude less memory requirement and two orders of magnitude faster training than previous methods. The proposed algorithm is analogous to truncated backpropagation through time with a pool of models to alleviate various types of overfitting in dataset distillation. FRePo significantly outperforms the previous methods on CIFAR100, Tiny ImageNet, and ImageNet-1K. Furthermore, we show that high-quality distilled data can greatly improve various downstream applications, such as continual learning and membership inference defense.

1   Introduction

Knowledge distillation [1] is a technique in deep learning to compress knowledge for easy deployment. Most previous works focus on model distillation [2, 3] where the knowledge acquired by a large teacher model is transferred to a small student model. In contrast, dataset distillation [4, 5] aims to learn a small set of synthetic examples preserving most of the information from a large dataset such that a model trained on it can achieve similar test performance as one trained on the original dataset. Distilled data can accelerate model training and reduce the cost of storing and sharing a dataset. Moreover, its highly condensed and synthetic nature can also benefit various applications, such as continual learning [5–8], neural architecture search [5, 7], and privacy-preserving tasks [9, 10].

Dataset distillation was first studied by Maclaurin et al. [11] in the context of gradient-based hyperparameter optimization and subsequently Wang et al. [4] formally proposed dataset distillation as a new task. Dataset distillation can be naturally formulated as a bi-level meta-learning problem. The inner loop optimizes the model parameters on the distilled data (meta-parameters), while the outer loop refines the distilled data with meta-gradient updates.

One key challenge in dataset distillation is computing the meta-gradient. Several methods [4, 11–13] compute it by back-propagating through the unrolled computation graph, but they often suffer from huge compute and memory requirement [14], training instability [15, 16], and truncation bias [17]. To avoid unrolled optimization, surrogate objectives are used to derive the meta-gradient, such as gradient matching [5, 7, 18], feature alignment [8, 19], and training trajectory matching [20]. Nevertheless, a surrogate objective may introduce its own bias [19], and thus, may not accurately reflect the true objective. An alternative is using kernel methods, such as Neural Tangent Kernel (NTK) [21], to approximate the inner optimization [22, 23]. However, computing analytical NTK for modern neural network can be extremely expensive [22, 23].

| (a) CIFAR100 | (b) Tiny ImageNet | (c) ImageNette and ImageWoof |

Figure 1: Example distilled images from 32x32 CIFAR100, 64x64 Tiny ImageNet, and 128x128 ImageNet Subset. The images look real and transfer well to different architectures. They can be used for various downstream applications, such as continual learning and membership inference defense.

Even with an accurate meta-gradient, dataset distillation still suffers from various types of overfitting. For instance, the distilled data can easily overfit to a particular learning algorithm [4, 13, 20], a certain stage of optimization [13, 19], or a certain network architecture [5, 7, 20, 22, 23]. Meanwhile, the model can also overfit the distilled data during training, which is the most common cause of overfitting when we train on a small dataset. All these kinds of overfitting impose difficulties on the training and general-purpose use of the distilled data.

We propose an efficient meta-gradient computation method and a "model pool" to address the overfitting problems. The bottleneck in meta-gradient computation arises due to the complexity of inner optimization, as we need to know how the inner parameters vary with the outer parameters [24]. However, the inner optimization can be pretty simple if we only train the last layer of a neural network to convergence while keeping the feature extractor fixed. In this case, computing the prediction on the real data using the model trained on the distilled data can be expressed as a kernel ridge regression (KRR) with respect to the conjugate kernel [25]. Hence, computing the meta-gradient is simply back-propagating through the kernel and a fixed feature extractor. To alleviate overfitting, we propose to maintain a diverse pool of models instead of periodically training and resetting a single model as in prior work [7, 13, 18]. Intuitively, our algorithm targets the following question: what is the best data to train the linear classifier given the current feature extractor? Due to the diverse feature extractors we use, the distilled data generalize well to a wide range of model distributions.

Summary of Contributions:

- We propose an effective method for dataset distillation. Our method, named neural Feature Regression with Pooling (FRePo), achieves state-of-the-art results on various benchmark datasets with a 100x reduction in training time and a 10x reduction in GPU memory requirement. Our distilled data looks real (Figure 1) and transfers well to different architectures.

- We show that FRePo scales well to datasets with high-resolution images or complex label space. We achieve 7.5% top1 accuracy on ImageNet-1K [26] using only one image per class. The same classifier obtains only 1.1% accuracy from a random subset of real images. The previous methods struggle in this task due to large memory and compute requirements.

- We demonstrate that high-quality distilled data can significantly improve various downstream applications, such as continual learning and membership inference defense.

2 Method

2.1 Dataset Distillation as Bi-level Optimization

Suppose we have a large labeled dataset $\mathcal{T} = \left\{ \left(\mathbf{x}_1, \mathbf{y}_1 \right), \ldots, \left(\mathbf{x}_{|\mathcal{T}|}, \mathbf{y}_{|\mathcal{T}|} \right) \right\}$ with $|\mathcal{T}|$ image and label pairs. Dataset distillation aims to learn a small synthetic dataset $\mathcal{S} = \left\{ \left(\mathbf{x}_1, \mathbf{y}_1 \right), \ldots, \left(\mathbf{x}_{|\mathcal{S}|}, \mathbf{y}_{|\mathcal{S}|} \right) \right\}$ that preserves most of the information in $\mathcal{T}$. We train several neural networks parameterized by

Figure 2: Comparison of FRePo and Unrolled Optimization. S, X_s, Y_s are the distilled dataset, images and labels. $\mathcal{L}$ is the meta-training loss and $\theta^{(k)}$, $g^{(k)}$ are the model parameter and gradient at step k. $f(X)$ is the feature for input X and $K^{\theta}_{X_t X_s}$ is the Gram matrix of X_t and X_s. FRePo is analogous to 1-step TBPTT as it computes the meta-gradient at each step while performing the online model update. However, instead of backpropagating through the inner optimization, FRePo computes the meta-gradient through a kernel and feature extractor.

θ on the dataset S and then compute the validation loss $\mathcal{L}(\mathcal{A}lg\,(\theta, S)\,, \mathcal{T})$ on the real dataset $\mathcal{T}$, where $\mathcal{A}lg\,(\theta, S)$ is the neural network parameters optimized by a learning algorithm $\mathcal{A}lg$ with the model initialization θ and distilled dataset S as its inputs. The validation loss $\mathcal{L}(\mathcal{A}lg\,(\theta, S)\,, \mathcal{T})$ is a noisy objective with the stochasticity coming from random model initialization and inner learning algorithm. Thus, we are interested in minimizing the expected value of this loss, which we denote it as $F(S)$. We formulate the dataset distillation as the following bi-level optimization problem.

$$\overbrace{S^* := \underset{S}{\arg\min}\, F(S)}^{\text{outer-level}}, \quad \text{where } F(S) = \mathbb{E}_{\theta \sim P_\theta}\left[\mathcal{L}\Big(\overbrace{\mathcal{A}lg\,(\theta, S)}^{\text{inner-level}}, \mathcal{T}\Big)\right]. \tag{1}$$

In this bi-level setup, the outer loop optimizes the distilled data to minimize $F(S)$, while the inner loop trains a neural network using the learning algorithm, $\mathcal{A}lg$, to minimize the training loss on the distilled data S. From the meta-learning perspective, the task is defined by the model initialization θ, and we want to learn a meta-parameter S that generalizes well to different models sampled from the model distributions P_θ. During learning, we optimize the meta-parameter S by minimizing the meta-training loss $F(S)$. In contrast, at meta-test time, we train a new model from scratch on S and evaluate the trained model on a held-out real dataset. This meta-test performance reflects the quality of the distilled data.

2.2 Dataset Distillation using Neural Feature Regression with Pooling (FRePo)

The outer-level problem can be solved using gradient-based methods of the form $S \leftarrow S - \alpha\, \nabla_S F(S)$, where α is the learning rate for the distilled data and $\nabla_S F(S)$ is the meta-gradient [27]. For a particular model θ, the meta-gradient can be expressed as $\nabla_S \mathcal{L}(\mathcal{A}lg\,(\theta, S)\,, \mathcal{T})$. Computing this meta-gradient requires differentiating through inner optimization. If $\mathcal{A}lg$ is an iterative algorithm like gradient descent, then backpropagating through the unrolled computation graph [14] can be a solution. However, this type of unrolled optimization introduces significant computation and memory overhead, as the whole training trajectory needs to be stored in memory (Figure 2(b)).

Traditionally, these issues are alleviated with truncated backpropagation through time (TBPTT) [28–30]. Instead of backpropagating through an entire unrolled sequence, TBPTT performs backprop-agation for each subsequence separately. It is efficient because its time and memory complexity scale linearly with respect to the truncation steps. However, truncation may yield highly biased gradients that severely impact training. To mitigate this truncation bias [14], we consider *training only the top layer of a network to convergence*. The key insight is that the data helpful for training the output layer can also help train the whole network. Thus, we decompose the neural network into a feature extractor and a linear classifier. We fix the feature extractor at each meta-gradient computation and train the linear classifier to convergence before updating S. After that, we adjust the feature extractor by training the whole network on the updated distilled data. We note that similar two-phase procedure has been studied in the context of representation learning [31].

Meta-Gradient Computation: If we consider the mean square error loss, then the optimal weights for the linear classifier have a closed-form solution. Moreover, since the feature dimension is typically

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Algorithm 1 Dataset Distillation using Neural Feature Regression with Pooling (FRePo)

Require: $\mathcal{T}$: a labeled dataset; α: the learning rate for the distilled data
Initialization: Initialize a labeled distilled dataset $\mathcal{S} = (X_s, Y_s)$.
Initialization: Initialize a model pool $\mathcal{M}$ with m models $\{\theta_i\}_{i=1}^{m}$ randomly initialized from P_θ.
 1: **while** not converged **do**
 2:    ▷ Sample a model uniformly from the model pool: $\theta_i \sim \mathcal{M}$.
 3:    ▷ Sample a target batch uniformly from the labeled dataset: $(X_t, Y_t) \sim \mathcal{T}$.
 4:    ▷ Compute the meta-training loss $\mathcal{L}$ using Eq. 2
 5:    ▷ Update the distilled data $\mathcal{S}$: $X_s \leftarrow X_s - \alpha\nabla_{X_s}\mathcal{L}$, and $Y_s \leftarrow Y_s - \alpha\nabla_{Y_s}\mathcal{L}$
 6:    ▷ Train the model θ_i on the current distilled data $\mathcal{S}$ for one step.
 7:    ▷ Reinitialize the model $\theta_i \sim P_\theta$ if θ_i has been updated more than K steps.
 8: **end while**
Output: Learned distilled dataset $\mathcal{S} = (X_s, Y_s)$

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larger than the number of distilled data, we can use kernel ridge regression (KRR) with a conjugate kernel [25] rather than solving the weights explicitly . We propose to use the following meta-training loss to train the distilled data:

$$\mathcal{L}\left(\mathcal{A}lg\left(\theta, \mathcal{S}\right), \mathcal{T}\right) = \frac{1}{2}||Y_t - K^{\theta}_{X_t X_s}(K^{\theta}_{X_s X_s} + \lambda I)^{-1}Y_s||_2^2, \tag{2}$$

where (X_t, Y_t) and (X_s, Y_s) are the inputs and labels of the real data $\mathcal{T}$ and distilled data $\mathcal{S}$ respectively. The Gram matrix between real inputs and distilled inputs is denoted as $K^{\theta}_{X_t X_s} \in \mathbb{R}^{|\mathcal{T}|\times|\mathcal{S}|}$, while the Gram matrix between distilled inputs is denoted as $K^{\theta}_{X_s X_s} \in \mathbb{R}^{|\mathcal{S}|\times|\mathcal{S}|}$. λ controls the regularization strength for KRR. Let us denote the neural network feature for a given input X and model parameter θ as $f(X, \theta) \in \mathbb{R}^{N\times d}$, where N is the number of input and d is the feature dimension [1]. The conjugate kernel (CK) is defined by the inner product of the neural network features. Thus, the two Gram matrices are computed as follows:

$$K^{\theta}_{X_t X_s} = f(X_t, \theta)f(X_s, \theta)^{\top}, \quad K^{\theta}_{X_s X_s} = f(X_s, \theta)f(X_s, \theta)^{\top}, \tag{3}$$

Now, computing the meta-gradient $\nabla_{\mathcal{S}}\mathcal{L}\left(\mathcal{A}lg\left(\theta, \mathcal{S}\right), \mathcal{T}\right)$ is just back-propagating through the conjugate kernel and a fixed feature extractor, which is very efficient and takes even fewer operations than computing the gradient for the network's weights. Moreover, we decouple the meta-gradient computation from the model online update. Hence, we can train the online model using any optimizer, and the distilled data will be agnostic to the specific learning algorithm choice. Our proposed method is similar to 1-step TBPTT in that we compute the meta-gradient at each step while performing the online model update. Unlike the conventional 1-step TBPTT, we compute the meta-gradient using a KRR output layer to mitigate truncation bias, illustrated in Figure 2(a).

Model Pool: As discussed in Section 1, there are various types of overfitting in dataset distillation. Several techniques have been proposed to alleviate such problem, such as random initialization [4], periodic reset [13, 5, 7], and dynamic bi-level optimization [19]. These techniques share the same underlying principle: the model diversity matters. Thus, we propose to *maintain a "model pool" filled with diverse set of parameters* obtained from different number of training steps and different random initializations. Unlike the previous methods that periodically training and resetting a single model, FRePo randomly sample a model from the pool at each meta-gradient computation and update it using the current distilled data. However, if a model has been updated more than K steps, we reinitialize it with a new random seed. From the meta-learning perspective, we maintain a diverse set of meta-tasks to sample from and avoid sampling very similar tasks at each consecutive gradient computation to avoid overfitting to a particular setup.

Pool Diversity: We can increase the regularization strength by increasing the diversity of the model pool by setting a larger K, using data augmentation when training the model on the distilled data, or using models with different architectures. To keep our method simple, we use the same architecture for all models in the pool and do not use any data augmentation when training the model on the distilled data. Thus, our model pool only contains models with different initialization, at different optimization stages, and trained at different time-step of the distilled data.

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[1]In practice, we use all the synthetic data and sample a minibatch from the real dataset to compute the meta-gradient (Algorithm 1).

3   Related Work

Unrolling in Bi-Level Optimization: One way to compute the meta-gradient is to differentiate through the unrolled inner optimization [4, 11–13]. However, this approach inherits several difficulties of the unrolled optimization, such as: 1) large computation and memory cost [14]; 2) truncation bias with short unrolls [17]; 3) exploding or vanishing gradients with long unrolls [15]; 4) chaotic and poorly conditioned loss landscapes with long unrolls [16]. In contrast, our method considers approximating the inner optimization with kernel ridge regression instead of unrolled optimization.

Surrogate Objective: To avoid unrolled optimization, several works turn to surrogate objectives. DC [5], DSA [7], and DCC [18] formulate the dataset distillation as a gradient matching problem between the gradients of neural network weights computed on the real and distilled data. In contrast, DM [8] and CAFE [19] consider the feature distribution alignment between the real and distilled data. Moreover, MTT [20] shows that knowledge from many expert training trajectories can be distilled to a dataset by using a training trajectory matching objective. Nevertheless, surrogate objectives may introduce new biases and thus, may not accurately reflect the true objective. For example, gradient matching approaches [5, 7, 18] only focus on short-range behavior and may easily overfit to a biased set of samples that produce dominant gradients [19, 20].

Closed-form Approximation: An alternative way to cirvumvent unrolled optimization is to find a closed-form approximation to the inner optimization. Based on the correspondence between infinitely-wide neural networks and kernel methods, KIP [22, 23] approximates the inner optimization with NTK [21]. In this case, the meta-gradient can be computed by back-propagating through the NTK. However, computing NTK for modern neural networks is extremely expensive. Thus, using NTK for dataset distillation requires thousands of GPU hours and sophisticated implementation of the distributed kernel computation framework [23].

4   Dataset Distillation

4.1   Implementation Details

We compare our method to four state-of-the-art dataset distillation methods [7, 8, 20, 23] on various benchmark datasets [26, 32–37]. We train the distilled data using Algorithm 1 with the same set of hyperparameters for all experiments except stated otherwise. We preprocess the data in a similar way as in previous work [20, 23] but use a slightly different architecture. We reproduce previous results with and without new changes to account for these differences. We evaluate each distilled data using five random neural networks and report the mean and standard deviation. We report the best of the reported value in the original paper and our reproducing results. For the sake of brevity, we provide more implementation details and ablation studies in the appendix.

4.2   Standard Benchmarks

Distillation Performance: We first evaluate our method on six standard benchmark datasets. We learn 1, 10, and 50 images per class for datasets with only ten classes, while we learn 1 and 10 images per class for CIFAR100 [34] with 100 classes, Tiny ImageNet [35] with 200 classes, and CUB-200 [37] with 200 fine-grained classes. As shown in Table in 1, we achieve the state-of-the-art performance in most settings despite the hyperparameter may be suboptimal. Our method performs exceptionally well on datasets with a complex label space when learning few images per class. For example, we improve the CIFAR100, Tiny ImageNet, and CUB-200 in one image per class setting from 24.3%, 8.8%, and 2.2% to 28.7%, 15.4%, and 12.4%, respectively. Figure 4 shows that our distilled images look real and natural though we do not directly optimize for this objective. We observe a strong correlation between the test accuracy and image quality: the better the image quality, the higher the test accuracy. Our results suggest that a highly condensed dataset does not need to be very different from the real dataset as it may just reflect the most common pattern in a dataset. We also report the KRR predictor's test accuracy using the feature extractor trained on the distilled data. When the dataset is as simple as MNIST [32], the KRR predictor achieves similar performance as the neural network predictor. In contrast, for more complex datasets, the KRR predictor consistently outperforms the neural network predictor, with the most significant gap being 3.7% for Tiny ImageNet in the one image per class setting.

Table 1: Test accuracies of models trained on the distilled data from scratch. † denotes performance better than the original reported performance. KRR preformance is shown in bracket.

	Img/Cls	DSA [7]	DM [8]	KIP [23]	MTT [20]	FRePo
MNIST	1	88.7 ± 0.6	$89.9 \pm 0.8^\dagger$	90.1 ± 0.1	$91.4 \pm 0.9^\dagger$	$\mathbf{93.0 \pm 0.4}$ (92.6 ± 0.4)
	10	$97.9 \pm 0.1^\dagger$	$97.6 \pm 0.1^\dagger$	97.5 ± 0.0	$97.3 \pm 0.1^\dagger$	$\mathbf{98.6 \pm 0.1}$ (98.6 ± 0.1)
	50	$\mathbf{99.2 \pm 0.1}$	98.6 ± 0.1	98.3 ± 0.1	$98.5 \pm 0.1^\dagger$	$\mathbf{99.2 \pm 0.0}$ (99.2 ± 0.1)
F-MNIST	1	70.6 ± 0.6	$71.5 \pm 0.5^\dagger$	73.5 ± 0.5	$75.1 \pm 0.9^\dagger$	$\mathbf{75.6 \pm 0.3}$ (77.1 ± 0.2)
	10	$84.8 \pm 0.3^\dagger$	$83.6 \pm 0.2^\dagger$	86.8 ± 0.1	$\mathbf{87.2 \pm 0.3}^\dagger$	86.2 ± 0.2 (86.8 ± 0.1)
	50	$88.8 \pm 0.2^\dagger$	$88.2 \pm 0.1^\dagger$	88.0 ± 0.1	$88.3 \pm 0.1^\dagger$	$\mathbf{89.6 \pm 0.1}$ (89.9 ± 0.1)
CIFAR10	1	$36.7 \pm 0.8^\dagger$	$31.0 \pm 0.6^\dagger$	$\mathbf{49.9 \pm 0.2}$	46.3 ± 0.8	46.8 ± 0.7 (47.9 ± 0.6)
	10	$53.2 \pm 0.8^\dagger$	$49.2 \pm 0.8^\dagger$	62.7 ± 0.3	65.3 ± 0.7	$\mathbf{65.5 \pm 0.4}$ (68.0 ± 0.2)
	50	$66.8 \pm 0.4^\dagger$	$63.7 \pm 0.5^\dagger$	68.6 ± 0.2	71.6 ± 0.2	$\mathbf{71.7 \pm 0.2}$ (74.4 ± 0.1)
CIFAR100	1	$16.8 \pm 0.2^\dagger$	$12.2 \pm 0.4^\dagger$	15.7 ± 0.2	24.3 ± 0.3	$\mathbf{28.7 \pm 0.1}$ (32.3 ± 0.1)
	10	32.3 ± 0.3	29.7 ± 0.3	28.3 ± 0.1	40.1 ± 0.4	$\mathbf{42.5 \pm 0.2}$ (44.9 ± 0.2)
T-ImageNet	1	$6.6 \pm 0.2^\dagger$	3.9 ± 0.2	$-$	8.8 ± 0.3	$\mathbf{15.4 \pm 0.3}$ (19.1 ± 0.3)
	10	$-$	12.9 ± 0.4	$-$	23.2 ± 0.2	$\mathbf{25.4 \pm 0.2}$ (26.5 ± 0.1)
CUB-200	1	$1.3 \pm 0.1^\dagger$	$1.6 \pm 0.1^\dagger$	$-$	$2.2 \pm 0.1^\dagger$	$\mathbf{12.4 \pm 0.2}$ (13.7 ± 0.2)
	10	$4.5 \pm 0.3^\dagger$	$4.4 \pm 0.2^\dagger$	$-$	$-$	$\mathbf{16.8 \pm 0.1}$ (16.1 ± 0.3)

(a) Test Acc   (b) Test Acc (Log Scale)   (c) Time Per Step   (d) GPU Memory Usage

Figure 3: (a,b) Training efficiency comparison when learning 1 Img/Cls on CIFAR100. (c,d) Time per iteration and peak memory usage as we increase the model size.

Label Learning: A similar trend can also be observed for label learning. When the dataset is simple and has only a few classes, label learning may not be necessary. However, it becomes crucial for complex datasets with many labels, such as CIFAR100 and Tiny-ImageNet (See more details in Appendix). Similar to the teacher label in the knowledge distillation [1], we observe that the distilled label also encodes the class similarity. We identify three typical cases in Figure 4d. The first group consists of highly confident labels with a much higher value for one class than other classes (large margin), such as sunflower, bicycle, and chair. In contrast, the distilled labels in the second group are confident but may get confused with some closely-related classes (small margin). For instance, the learned label for "girl" has almost equally high values for the girl, woman, man, boy, and baby, suggesting that these classes are very similar and may be difficult for the model to distinguish them apart. The last group contains distilled labels with low values for all classes, such as bear, beaver, and squirrel. It is often hard for humans to recognize the distilled images in such a group, suggesting that they may be the challenging classes in a dataset.

Training Cost Analysis: Figure 3a, 3b shows that our method is significantly more time-efficient than the previous methods. When learning one image per class on CIFAR100, FRePo reaches a similar test accuracy (23.4%) to the second-best method (24.0%) in 38 seconds, compared to 3805 seconds for MTT, which is roughly two orders of magnitude faster. Moreover, FRePo achieves 92% of its final test accuracy (26.4% out of 28.4%) in only 385 seconds. As shown in Figure 3c, our algorithm takes much less time to perform one gradient step on the distilled data. Thus, we can perform more gradient steps in a fixed time. Furthermore, Figure 3d suggests that our algorithm has much less GPU memory requirement. Therefore, we can potentially use a much larger and more complex model to take advantage of the advancement in neural network architecture.

Table 2: Cross-architecture transfer performance on CIFAR10 with 10 Img/Cls. Despite being trained for a specific architecture, our distilled data transfer well to various architectures unseen during training. Conv is the default evaluation model used for each method. NN, DN, IN, and BN stand for no normalization, default normalization, Instance Normalization, Batch Normalization respectively.

	Train Arch	Evaluation Architecture					
		Conv	Conv-NN	ResNet-DN	ResNet-BN	VGG-BN	AlexNet
DSA [7]	Conv-IN	53.2 ± 0.8	36.4 ± 1.5	42.1 ± 0.7	34.1 ± 1.4	46.3 ± 1.3	34.0 ± 2.3
DM [8]	Conv-IN	49.2 ± 0.8	35.2 ± 0.5	36.8 ± 1.2	35.5 ± 1.3	41.2 ± 1.8	34.9 ± 1.1
MTT [20]	Conv-IN	64.4 ± 0.9	41.6 ± 1.3	49.2 ± 1.1	42.9 ± 1.5	46.6 ± 2.0	34.2 ± 2.6
KIP [23]	Conv-NTK	62.7 ± 0.3	58.2 ± 0.4	49.0 ± 1.2	45.8 ± 1.4	30.1 ± 1.5	57.2 ± 0.4
FRePo	Conv-BN	$\mathbf{65.5 \pm 0.4}$	$\mathbf{65.5 \pm 0.4}$	$\mathbf{58.1 \pm 0.6}$	$\mathbf{57.7 \pm 0.7}$	$\mathbf{59.4 \pm 0.7}$	$\mathbf{61.9 \pm 0.7}$

(a) FRePo   (b) MTT [20]   (c) DSA [7]   (d) Label Visualization

Figure 4: (a,b,c) Distilled 1 img/cls from CIFAR100 using FRePo, MTT, and DSA. High quality images also produce high test accuracy. (d) Three categories of learned labels. (Top) High confidence, large margin; (Middle) High confidence, small margin; (Bottom) Low confidence, small margin.

Cross-Architecture Generalization: We also evaluate the distilled data from CIFAR10 on a wide range of architectures which it has not seen during training, including AlexNet [38], VGG [39], and ResNet [40]. Table 2 shows that our method outperforms previous methods on all unseen architectures. Instance Normalization (IN) [41], as the vital ingredient in several methods (DSA, DM, MTT), seems to hurt the cross-architecture transfer. The performance degrades a lot when no normalization (NN) is applied (Conv-NN, AlexNet) or using a different normalization, like Batch Normalization (BN) [42]. It suggests that the distilled data generated by those methods encode the inductive bias of a particular training architecture. In contrast, our distilled data generalize well to various architectures, including those without normalization (Conv-NN, AlexNet). Note that Figure 1, 4 also indicate that our distilled data encode less architectural bias as the distilled images look natural and authentic. A simple idea to further alleviate the overfitting of a particular architecture is to include more architectures in the model pool. However, the training may not be stable as the meta-gradient computed by different architectures can be very different.

4.3 ImageNet

High Resolution ImageNet Subset To understand how well our method performs on high-resolution images, we evaluate it on ImageNette and ImageWoof datasets [36] with a resolution of 128x128. We learn 1 and 10 images per class on both datasets and report the performance in Table 3 and visualize some distilled images in Figure 1. As shown in Table 3, we outperform MTT on all settings and achieve much better performance when we distill ten images per class on a more difficult dataset ImageWoof. It suggests that our distilled data is better at capturing the discriminative features for each class. Figure 1 shows that our distilled images look real and capture the distinguishable feature of different classes. For the easy dataset (i.e., ImageNette), all images have clear different structures, while for ImageWoof, the texture of each dog seems to be crucial.

Resized ImageNet-1K: We also evaluate our method on a resized version of ILSVRC2012 [26] with a resolution of 64x64 to see how it performs on a complex label space. Surprisingly, we can achieve 7.5% and 9.7% Top1 accuracy using only 1k and 2k training examples, compared to 1.1% and 1.4% using an equally-sized real subset.

Table 3: Distillation performance on higher resolution (128x128) dataset (i.e. ImageNette, Image-Woof) and medium resolution (64x64) dataset with a complex label space (i.e. ImageNet-1K). FRePo scales to high-resolution images and learns the discriminate feature of complex datasets.

Img/Cls	ImageNette (128x128)		ImageWoof (128x128)		ImageNet (64x64)	
	1	10	1	10	1	2
Random Subset	23.5 ± 4.8	47.7 ± 2.4	14.2 ± 0.9	27.0 ± 1.9	1.1 ± 0.1	1.4 ± 0.1
MTT [20]	47.7 ± 0.9	63.0 ± 1.3	28.6 ± 0.8	35.8 ± 1.8	$-$	$-$
FRePo	$\mathbf{48.1 \pm 0.7}$	$\mathbf{66.5 \pm 0.8}$	$\mathbf{29.7 \pm 0.6}$	$\mathbf{42.2 \pm 0.9}$	$\mathbf{7.5 \pm 0.3}$	$\mathbf{9.7 \pm 0.2}$

5 Application

5.1 Continual Learning

Continual learning (CL) [43] aims to address the catastrophic forgetting problem [43–45] when a model learns sequentially from a stream of tasks. A commonly used strategy to recall past knowledge is based on a replay buffer, which stores representative samples from previous tasks [46–49]. Since sample selection is an important component of constructing an effective buffer [48–51], we believe distilled data can be a key ingredient for a continual learning algorithm due to its highly condensed nature. Several works [6–8, 52] have successfully applied the dataset distillation to the continual learning scenario. Our work shows that we can achieve much better results by using a better dataset distillation technique.

We follow Zhao and Bilen [8] that sets up the baseline based on GDumb [49] which greedily stores class-balanced training examples in memory and train model from scratch on the latest memory only. In that case, the continual learning performance only depends on the quality of the replay buffer. We perform 5 and 10 step class-incremental learning [53] on CIFAR100 with an increasing buffer size of 20 images per class. Specifically, we distill 400 and 200 images at each step and put them into the replay buffer. We follow the same class split as Zhao and Bilen [8] and compare our method to random [49], herding [54, 55], DSA [7], and DM [8]. We use the default data preprocessing and default model for each method in this experiment as we find it gives the best performance for each method. We use the test accuracy on all observed classes as the performance measure [8, 48].

Figure 5 shows that our method performs significantly better than all previous methods. The final test accuracy for all classes for our method (FRePo) and the second-best method (DM) are 41.6%, 33.9% in 5-step learning, and 38.0%, 34.0% in 10-step learning. However, we notice that for FRePo, distilling 2000 images in a continual learning setup achieves a similar test accuracy (41.6%) as distilling only 1000 images from the whole dataset (41.3% from Table 1). In addition, performance drops as we perform more steps. It suggests that FRePo considers all available classes to derive the most condensed dataset. Splitting the data into multiple groups and performing independent distillation may generate redundant information or fail to capture the distinguishable features.

5.2 Membership Inference Defense

Membership inference attacks (MIA) aim to infer whether a given data point has been used to train the model or not [56–58]. Ideally, we want a model to learn from the data but not memorize it to preserve privacy. However, deep neural networks are well-known for their ability to memorize all the training examples, even on large and randomly labeled datasets [59]. Several methods have been proposed to defend against such attacks by either modifying the training procedure [60] or changing the inference workflow [61]. This section shows that the distilled data contain little information regarding sample presence in the original dataset. Thus, instead of training on the original datasets, training on distilled data allows privacy preservation while retaining model performance.

We consider three distilled data generated by DSA [7], DM [8] and FRePo. We perform five popular "black box" MIA provided by Tensorflow Privacy [62] on models trained on the real data or the data distilled from it. The attack methods include a threshold attack and four model-based attacks using logistic regression (LR), multi-layer perceptron (MLP), random forest (RF) and K-nearest neighbor (KNN). The inputs to those attack methods are ground-truth labels, model predictions, and losses. To measure the privacy vulnerability of the trained model, we compute the area under the ROC curve (AUC) of an attack classifier. Following prior work, [56, 63], we keep a balanced set of training examples (member) and test examples (non-member) with 10K each to maximize the uncertainty of

Table 4: AUC of five attackers on models trained on the real and distilled MNIST data. The model trained on the real data is vulnerable to MIAs, while the model trained on the distilled data is robust to MIAs. Training on distilled data allows privacy preservation while retaining model performance.

	Test Acc (%)	Attack AUC				
		Threshold	LR	MLP	RF	KNN
Real	**99.2 ± 0.1**	0.99 ± 0.01	0.99 ± 0.00	1.00 ± 0.00	1.00 ± 0.00	0.97 ± 0.00
Subset	96.8 ± 0.2	0.52 ± 0.00	**0.50 ± 0.01**	**0.53 ± 0.01**	0.55 ± 0.00	0.54 ± 0.00
DSA	98.5 ± 0.1	**0.50 ± 0.00**	0.51 ± 0.00	0.54 ± 0.00	0.54 ± 0.01	0.54 ± 0.01
DM	98.3 ± 0.0	**0.50 ± 0.00**	0.51 ± 0.01	0.54 ± 0.01	0.54 ± 0.01	0.53 ± 0.01
FRePo	98.5 ± 0.1	0.52 ± 0.00	0.51 ± 0.00	**0.53 ± 0.01**	**0.52 ± 0.01**	**0.51 ± 0.01**

(a) 5-step CL   (b) 10-step CL   (c) MIA: ACC   (d) MIA: AUC

Figure 5: (a,b) Multi-class accuracies across all classes observed up to a certain time point. We perform significantly better than other methods in both 5 and 10 step class-incremental continual learning. (c,d) Test accuracy and attack AUC as we increase the number of training steps. AUC keeps increasing when training a model on the real data for more steps. In contrast, AUC keeps low when training on distilled data.

MIA. Thus, the random guessing strategy results in a 50% MIA accuracy. We conduct experiments on MNIST and FashionMNIST with a distillation size of 500. For space reasons, we provide more implementation details and results in appendix.

As shown in Table 4, all models trained on the distilled data preserve privacy as their attack AUCs are closed to random guessing. However, we observe a small drop in test accuracy compared to the model trained on the full dataset, which is expected as we only distill 500 examples instead of 10,000 examples. Compared to the model trained on an equally sized subset of the original data, the model trained on distilled data results in much better test performance. Figure 5c, 5d demonstrate the trade-off between test accuracy and attack effectiveness as measured by ROC AUC. It shows that early stopping can be an effective technique to preserve privacy. However, we will still be under high MIA risk if we perform early stopping by monitoring the validation loss. In contrast, training a model on the distilled data does not have this problem as the attack AUCs keep at a very low level regardless of training steps.

6 Conclusion

We propose neural Feature Regression with Pooling (FRePo) to overcome two challenges in dataset distillation: meta-gradient computation and various types of overfitting in dataset distillation. We obtain state-of-the-art performance on various datasets with a 100x reduction in training time and a 10x reduction in GPU memory requirement. The distilled data generated by FRePo looks real and natural and generalizes well to a wide range of architectures. Furthermore, we demonstrate two applications that take advantage of the high-quality distilled data, namely, continual learning and membership inference defense.

Broader Impact "Synthetic data", in the broader sense of artificial data created by generative models, can help researchers understand how an otherwise opaque learning machine "sees" the world. There have been concerns regarding the risk of fake data. This paper explores a new research direction in generating synthetic data only for downstream classification tasks. We believe this work can provide additional interpretability and potentially address the common concerns in machine learning regarding training data privacy.

References

[1] Geoffrey E. Hinton, Oriol Vinyals, and Jeffrey Dean. Distilling the knowledge in a neural network. *CoRR*, abs/1503.02531, 2015. URL http://arxiv.org/abs/1503.02531.

[2] Takashi Fukuda, Masayuki Suzuki, Gakuto Kurata, Samuel Thomas, Jia Cui, and Bhuvana Ramabhadran. Efficient knowledge distillation from an ensemble of teachers. In Francisco Lacerda, editor, *Interspeech 2017, 18th Annual Conference of the International Speech Communication Association, Stockholm, Sweden, August 20-24, 2017*, pages 3697–3701. ISCA, 2017. URL http://www.isca-speech.org/archive/Interspeech_2017/abstracts/0614.html.

[3] Antonio Polino, Razvan Pascanu, and Dan Alistarh. Model compression via distillation and quantization. In *6th International Conference on Learning Representations, ICLR 2018, Vancouver, BC, Canada, April 30 - May 3, 2018, Conference Track Proceedings*. OpenReview.net, 2018. URL https://openreview.net/forum?id=S1XolQbRW.

[4] Tongzhou Wang, Jun-Yan Zhu, Antonio Torralba, and Alexei A. Efros. Dataset distillation. *CoRR*, abs/1811.10959, 2018. URL http://arxiv.org/abs/1811.10959.

[5] Bo Zhao, Konda Reddy Mopuri, and Hakan Bilen. Dataset condensation with gradient matching. In *9th International Conference on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*. OpenReview.net, 2021. URL https://openreview.net/forum?id=mSAKhLYLSsl.

[6] Andrea Rosasco, Antonio Carta, Andrea Cossu, Vincenzo Lomonaco, and Davide Bacciu. Distilled replay: Overcoming forgetting through synthetic samples. *CoRR*, abs/2103.15851, 2021. URL https://arxiv.org/abs/2103.15851.

[7] Bo Zhao and Hakan Bilen. Dataset condensation with differentiable siamese augmentation. In Marina Meila and Tong Zhang, editors, *Proceedings of the 38th International Conference on Machine Learning, ICML 2021, 18-24 July 2021, Virtual Event*, volume 139 of *Proceedings of Machine Learning Research*, pages 12674–12685. PMLR, 2021. URL http://proceedings.mlr.press/v139/zhao21a.html.

[8] Bo Zhao and Hakan Bilen. Dataset condensation with distribution matching. *CoRR*, abs/2110.04181, 2021. URL https://arxiv.org/abs/2110.04181.

[9] Guang Li, Ren Togo, Takahiro Ogawa, and Miki Haseyama. Soft-label anonymous gastric x-ray image distillation. *CoRR*, abs/2104.02857, 2021. URL https://arxiv.org/abs/2104.02857.

[10] Jack Goetz and Ambuj Tewari. Federated learning via synthetic data. *CoRR*, abs/2008.04489, 2020. URL https://arxiv.org/abs/2008.04489.

[11] Dougal Maclaurin, David Duvenaud, and Ryan P. Adams. Gradient-based hyperparameter optimization through reversible learning. In Francis R. Bach and David M. Blei, editors, *Proceedings of the 32nd International Conference on Machine Learning, ICML 2015, Lille, France, 6-11 July 2015*, volume 37 of *JMLR Workshop and Conference Proceedings*, pages 2113–2122. JMLR.org, 2015. URL http://proceedings.mlr.press/v37/maclaurin15.html.

[12] Ondrej Bohdal, Yongxin Yang, and Timothy M. Hospedales. Flexible dataset distillation: Learn labels instead of images. *CoRR*, abs/2006.08572, 2020. URL https://arxiv.org/abs/2006.08572.

[13] Ilia Sucholutsky and Matthias Schonlau. Improving dataset distillation. *CoRR*, abs/1910.02551, 2019. URL http://arxiv.org/abs/1910.02551.

[14] Paul Vicol, Luke Metz, and Jascha Sohl-Dickstein. Unbiased gradient estimation in unrolled computation graphs with persistent evolution strategies. In Marina Meila and Tong Zhang, editors, *Proceedings of the 38th International Conference on Machine Learning, ICML 2021, 18-24 July 2021, Virtual Event*, volume 139 of *Proceedings of Machine Learning Research*, pages 10553–10563. PMLR, 2021. URL http://proceedings.mlr.press/v139/vicol21a.html.

[15] Razvan Pascanu, Tomás Mikolov, and Yoshua Bengio. On the difficulty of training recurrent neural networks. In *Proceedings of the 30th International Conference on Machine Learning, ICML 2013, Atlanta, GA, USA, 16-21 June 2013*, volume 28 of *JMLR Workshop and Conference Proceedings*, pages 1310–1318. JMLR.org, 2013. URL `http://proceedings.mlr.press/v28/pascanu13.html`.

[16] Luke Metz, Niru Maheswaranathan, Jeremy Nixon, C. Daniel Freeman, and Jascha Sohl-Dickstein. Understanding and correcting pathologies in the training of learned optimizers. In Kamalika Chaudhuri and Ruslan Salakhutdinov, editors, *Proceedings of the 36th International Conference on Machine Learning, ICML 2019, 9-15 June 2019, Long Beach, California, USA*, volume 97 of *Proceedings of Machine Learning Research*, pages 4556–4565. PMLR, 2019. URL `http://proceedings.mlr.press/v97/metz19a.html`.

[17] Yuhuai Wu, Mengye Ren, Renjie Liao, and Roger B. Grosse. Understanding short-horizon bias in stochastic meta-optimization. In *6th International Conference on Learning Representations, ICLR 2018, Vancouver, BC, Canada, April 30 - May 3, 2018, Conference Track Proceedings*. OpenReview.net, 2018. URL `https://openreview.net/forum?id=H1MczcgR-`.

[18] Saehyung Lee, Sanghyuk Chun, Sangwon Jung, Sangdoo Yun, and Sungroh Yoon. Dataset condensation with contrastive signals. *CoRR*, abs/2202.02916, 2022. URL `https://arxiv.org/abs/2202.02916`.

[19] Kai Wang, Bo Zhao, Xiangyu Peng, Zheng Zhu, Shuo Yang, Shuo Wang, Guan Huang, Hakan Bilen, Xinchao Wang, and Yang You. CAFE: learning to condense dataset by aligning features. *CoRR*, abs/2203.01531, 2022. doi: 10.48550/arXiv.2203.01531. URL `https://doi.org/10.48550/arXiv.2203.01531`.

[20] George Cazenavette, Tongzhou Wang, Antonio Torralba, Alexei A. Efros, and Jun-Yan Zhu. Dataset distillation by matching training trajectories. *CoRR*, abs/2203.11932, 2022. doi: 10.48550/arXiv.2203.11932. URL `https://doi.org/10.48550/arXiv.2203.11932`.

[21] Jaehoon Lee, Lechao Xiao, Samuel S. Schoenholz, Yasaman Bahri, Roman Novak, Jascha Sohl-Dickstein, and Jeffrey Pennington. Wide neural networks of any depth evolve as linear models under gradient descent. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d'Alché-Buc, Emily B. Fox, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 8570–8581, 2019. URL `https://proceedings.neurips.cc/paper/2019/hash/0d1a9651497a38d8b1c3871c84528bd4-Abstract.html`.

[22] Timothy Nguyen, Zhourong Chen, and Jaehoon Lee. Dataset meta-learning from kernel ridge-regression. In *9th International Conference on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*. OpenReview.net, 2021. URL `https://openreview.net/forum?id=l-PrrQrK0QR`.

[23] Timothy Nguyen, Roman Novak, Lechao Xiao, and Jaehoon Lee. Dataset distillation with infinitely wide convolutional networks. In Marc'Aurelio Ranzato, Alina Beygelzimer, Yann N. Dauphin, Percy Liang, and Jennifer Wortman Vaughan, editors, *Advances in Neural Information Processing Systems 34: Annual Conference on Neural Information Processing Systems 2021, NeurIPS 2021, December 6-14, 2021, virtual*, pages 5186–5198, 2021. URL `https://proceedings.neurips.cc/paper/2021/hash/299a23a2291e2126b91d54f3601ec162-Abstract.html`.

[24] Jonathan Lorraine, Paul Vicol, and David Duvenaud. Optimizing millions of hyperparameters by implicit differentiation. In Silvia Chiappa and Roberto Calandra, editors, *The 23rd International Conference on Artificial Intelligence and Statistics, AISTATS 2020, 26-28 August 2020, Online [Palermo, Sicily, Italy]*, volume 108 of *Proceedings of Machine Learning Research*, pages 1540–1552. PMLR, 2020. URL `http://proceedings.mlr.press/v108/lorraine20a.html`.

[25] Radford M. Neal. Bayesian learning for neural networks. 1995.

[26] Olga Russakovsky, Jia Deng, Hao Su, Jonathan Krause, Sanjeev Satheesh, Sean Ma, Zhiheng Huang, Andrej Karpathy, Aditya Khosla, Michael Bernstein, Alexander C. Berg, and Li Fei-Fei. ImageNet Large Scale Visual Recognition Challenge. *International Journal of Computer Vision (IJCV)*, 115(3):211–252, 2015. doi: 10.1007/s11263-015-0816-y.

[27] Aravind Rajeswaran, Chelsea Finn, Sham M. Kakade, and Sergey Levine. Meta-learning with implicit gradients. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d'Alché-Buc, Emily B. Fox, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 113–124, 2019. URL `https://proceedings.neurips.cc/paper/2019/hash/072b030ba126b2f4b2374f342be9ed44-Abstract.html`.

[28] P.J. Werbos. Backpropagation through time: what it does and how to do it. *Proceedings of the IEEE*, 78(10):1550–1560, 1990. doi: 10.1109/5.58337.

[29] Ilya Sutskever. *Training recurrent neural networks*. University of Toronto Toronto, ON, Canada, 2013.

[30] Corentin Tallec and Yann Ollivier. Unbiasing truncated backpropagation through time. *CoRR*, abs/1705.08209, 2017. URL `http://arxiv.org/abs/1705.08209`.

[31] Jimmy Ba, Murat A Erdogdu, Taiji Suzuki, Zhichao Wang, Denny Wu, and Greg Yang. High-dimensional asymptotics of feature learning: How one gradient step improves the representation. *arXiv preprint arXiv:2205.01445*, 2022.

[32] Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. *Proc. IEEE*, 86(11):2278–2324, 1998. doi: 10.1109/5.726791. URL `https://doi.org/10.1109/5.726791`.

[33] Han Xiao, Kashif Rasul, and Roland Vollgraf. Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms. *CoRR*, abs/1708.07747, 2017. URL `http://arxiv.org/abs/1708.07747`.

[34] Alex Krizhevsky. Learning multiple layers of features from tiny images. Technical report, 2009.

[35] Ya Le and Xuan S. Yang. Tiny imagenet visual recognition challenge. 2015.

[36] Jeremy Howard. A smaller subset of 10 easily classified classes from imagenet, and a little more french. URL `https://github.com/fastai/imagenette/`.

[37] Technical report.

[38] Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton. Imagenet classification with deep convolutional neural networks. In Peter L. Bartlett, Fernando C. N. Pereira, Christopher J. C. Burges, Léon Bottou, and Kilian Q. Weinberger, editors, *Advances in Neural Information Processing Systems 25: 26th Annual Conference on Neural Information Processing Systems 2012. Proceedings of a meeting held December 3-6, 2012, Lake Tahoe, Nevada, United States*, pages 1106–1114, 2012. URL `https://proceedings.neurips.cc/paper/2012/hash/c399862d3b9d6b76c8436e924a68c45b-Abstract.html`.

[39] Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. In Yoshua Bengio and Yann LeCun, editors, *3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings*, 2015. URL `http://arxiv.org/abs/1409.1556`.

[40] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *2016 IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2016, Las Vegas, NV, USA, June 27-30, 2016*, pages 770–778. IEEE Computer Society, 2016. doi: 10.1109/CVPR.2016.90. URL `https://doi.org/10.1109/CVPR.2016.90`.

[41] Dmitry Ulyanov, Andrea Vedaldi, and Victor S. Lempitsky. Instance normalization: The missing ingredient for fast stylization. *CoRR*, abs/1607.08022, 2016. URL `http://arxiv.org/abs/1607.08022`.

[42] Sergey Ioffe and Christian Szegedy. Batch normalization: Accelerating deep network training by reducing internal covariate shift. In Francis R. Bach and David M. Blei, editors, *Proceedings of the 32nd International Conference on Machine Learning, ICML 2015, Lille, France, 6-11 July 2015*, volume 37 of *JMLR Workshop and Conference Proceedings*, pages 448–456. JMLR.org, 2015. URL http://proceedings.mlr.press/v37/ioffe15.html.

[43] James Kirkpatrick, Razvan Pascanu, Neil C. Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A. Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, Demis Hassabis, Claudia Clopath, Dharshan Kumaran, and Raia Hadsell. Overcoming catastrophic forgetting in neural networks. *CoRR*, abs/1612.00796, 2016. URL http://arxiv.org/abs/1612.00796.

[44] Robert French. Catastrophic forgetting in connectionist networks. *Trends in cognitive sciences*, 3:128–135, 05 1999. doi: 10.1016/S1364-6613(99)01294-2.

[45] Anthony V. Robins. Catastrophic forgetting, rehearsal and pseudorehearsal. *Connect. Sci.*, 7 (2):123–146, 1995. doi: 10.1080/09540099550039318. URL https://doi.org/10.1080/09540099550039318.

[46] Pietro Buzzega, Matteo Boschini, Angelo Porrello, and Simone Calderara. Rethinking experience replay: a bag of tricks for continual learning. In *25th International Conference on Pattern Recognition, ICPR 2020, Virtual Event / Milan, Italy, January 10-15, 2021*, pages 2180–2187. IEEE, 2020. doi: 10.1109/ICPR48806.2021.9412614. URL https://doi.org/10.1109/ICPR48806.2021.9412614.

[47] Yaoyao Liu, Bernt Schiele, and Qianru Sun. Adaptive aggregation networks for class-incremental learning. In *IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2021, virtual, June 19-25, 2021*, pages 2544–2553. Computer Vision Foundation / IEEE, 2021. URL https://openaccess.thecvf.com/content/CVPR2021/html/Liu_Adaptive_Aggregation_Networks_for_Class-Incremental_Learning_CVPR_2021_paper.html.

[48] Sylvestre-Alvise Rebuffi, Alexander Kolesnikov, and Christoph H. Lampert. icarl: Incremental classifier and representation learning. *CoRR*, abs/1611.07725, 2016. URL http://arxiv.org/abs/1611.07725.

[49] Ameya Prabhu, Philip H. S. Torr, and Puneet K. Dokania. Gdumb: A simple approach that questions our progress in continual learning. In Andrea Vedaldi, Horst Bischof, Thomas Brox, and Jan-Michael Frahm, editors, *Computer Vision - ECCV 2020 - 16th European Conference, Glasgow, UK, August 23-28, 2020, Proceedings, Part II*, volume 12347 of *Lecture Notes in Computer Science*, pages 524–540. Springer, 2020. doi: 10.1007/978-3-030-58536-5_31. URL https://doi.org/10.1007/978-3-030-58536-5_31.

[50] Rahaf Aljundi, Min Lin, Baptiste Goujaud, and Yoshua Bengio. Gradient based sample selection for online continual learning. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d'Alché-Buc, Emily B. Fox, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 11816–11825, 2019. URL https://proceedings.neurips.cc/paper/2019/hash/e562cd9c0768d5464b64cf61da7fc6bb-Abstract.html.

[51] Rahaf Aljundi, Eugene Belilovsky, Tinne Tuytelaars, Laurent Charlin, Massimo Caccia, Min Lin, and Lucas Page-Caccia. Online continual learning with maximal interfered retrieval. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d'Alché-Buc, Emily B. Fox, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 11849–11860, 2019. URL https://proceedings.neurips.cc/paper/2019/hash/15825aee15eb335cc13f9b559f166ee8-Abstract.html.

[52] Yaoyao Liu, Yuting Su, An-An Liu, Bernt Schiele, and Qianru Sun. Mnemonics training: Multi-class incremental learning without forgetting. In *2020 IEEE/CVF Conference on Computer*

Vision and Pattern Recognition, CVPR 2020, Seattle, WA, USA, June 13-19, 2020, pages 12242–12251. Computer Vision Foundation / IEEE, 2020. doi: 10.1109/CVPR42600.2020.01226. URL `https://openaccess.thecvf.com/content_CVPR_2020/html/Liu_Mnemonics_Training_Multi-Class_Incremental_Learning_Without_Forgetting_CVPR_2020_paper.html`.

[53] Gido M. van de Ven and Andreas S. Tolias. Three scenarios for continual learning. *CoRR*, abs/1904.07734, 2019. URL `http://arxiv.org/abs/1904.07734`.

[54] Francisco M. Castro, Manuel J. Marín-Jiménez, Nicolás Guil, Cordelia Schmid, and Karteek Alahari. End-to-end incremental learning. In Vittorio Ferrari, Martial Hebert, Cristian Sminchisescu, and Yair Weiss, editors, *Computer Vision - ECCV 2018 - 15th European Conference, Munich, Germany, September 8-14, 2018, Proceedings, Part XII*, volume 11216 of *Lecture Notes in Computer Science*, pages 241–257. Springer, 2018. doi: 10.1007/978-3-030-01258-8_15. URL `https://doi.org/10.1007/978-3-030-01258-8_15`.

[55] Yutian Chen, Max Welling, and Alexander J. Smola. Super-samples from kernel herding. In Peter Grünwald and Peter Spirtes, editors, *UAI 2010, Proceedings of the Twenty-Sixth Conference on Uncertainty in Artificial Intelligence, Catalina Island, CA, USA, July 8-11, 2010*, pages 109–116. AUAI Press, 2010. URL `https://dslpitt.org/uai/displayArticleDetails.jsp?mmnu=1&smnu=2&article_id=2148&proceeding_id=26`.

[56] Reza Shokri, Marco Stronati, Congzheng Song, and Vitaly Shmatikov. Membership inference attacks against machine learning models. In *2017 IEEE Symposium on Security and Privacy, SP 2017, San Jose, CA, USA, May 22-26, 2017*, pages 3–18. IEEE Computer Society, 2017. doi: 10.1109/SP.2017.41. URL `https://doi.org/10.1109/SP.2017.41`.

[57] Yunhui Long, Vincent Bindschaedler, Lei Wang, Diyue Bu, Xiaofeng Wang, Haixu Tang, Carl A. Gunter, and Kai Chen. Understanding membership inferences on well-generalized learning models. *CoRR*, abs/1802.04889, 2018. URL `http://arxiv.org/abs/1802.04889`.

[58] Ahmed Salem, Yang Zhang, Mathias Humbert, Pascal Berrang, Mario Fritz, and Michael Backes. Ml-leaks: Model and data independent membership inference attacks and defenses on machine learning models. In *26th Annual Network and Distributed System Security Symposium, NDSS 2019, San Diego, California, USA, February 24-27, 2019*. The Internet Society, 2019.

[59] Chiyuan Zhang, Samy Bengio, Moritz Hardt, Benjamin Recht, and Oriol Vinyals. Understanding deep learning requires rethinking generalization. In *5th International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Conference Track Proceedings*. OpenReview.net, 2017. URL `https://openreview.net/forum?id=Sy8gdB9xx`.

[60] Milad Nasr, Reza Shokri, and Amir Houmansadr. Machine learning with membership privacy using adversarial regularization. In David Lie, Mohammad Mannan, Michael Backes, and XiaoFeng Wang, editors, *Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security, CCS 2018, Toronto, ON, Canada, October 15-19, 2018*, pages 634–646. ACM, 2018. doi: 10.1145/3243734.3243855. URL `https://doi.org/10.1145/3243734.3243855`.

[61] Jinyuan Jia, Ahmed Salem, Michael Backes, Yang Zhang, and Neil Zhenqiang Gong. Memguard: Defending against black-box membership inference attacks via adversarial examples. In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, *Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security, CCS 2019, London, UK, November 11-15, 2019*, pages 259–274. ACM, 2019. doi: 10.1145/3319535.3363201. URL `https://doi.org/10.1145/3319535.3363201`.

[62] tensorflow/privacy: library for training machine learning models with privacy for training data, 2022. URL `https://github.com/tensorflow/privacy`.

[63] Samuel Yeom, Irene Giacomelli, Matt Fredrikson, and Somesh Jha. Privacy risk in machine learning: Analyzing the connection to overfitting. In *31st IEEE Computer Security Foundations Symposium, CSF 2018, Oxford, United Kingdom, July 9-12, 2018*, pages 268–282. IEEE Computer Society, 2018. doi: 10.1109/CSF.2018.00027. URL `https://doi.org/10.1109/CSF.2018.00027`.