Marginalized Importance Sampling for Off-Environment Policy Evaluation

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Abstract: Reinforcement Learning (RL) methods are typically sample-inefficient, making it challenging to train and deploy RL-policies in real world robots. Even a robust policy trained in simulation, requires a real-world deployment to assess their performance. This paper proposes a new approach to evaluate the real-world performance of agent policies without deploying them in the real world. The proposed approach incorporates a simulator along with real-world offline data to evaluate the performance of any policy using the framework of Marginalized Importance Sampling (MIS). Existing MIS methods face two challenges: (1) large density ratios that deviate from a reasonable range and (2) indirect supervision, where the ratio needs to be inferred indirectly, thus exacerbating estimation error. Our approach addresses these challenges by introducing the target policy's occupancy in the simulator as an intermediate variable and learning the density ratio as the product of two terms that can be learned separately. The first term is learned with direct supervision and the second term has a small magnitude, thus making it easier to run. We analyze the sample complexity as well as error propagation of our two step-procedure. Furthermore, we empirically evaluate our approach on Sim2Sim environments such as Cartpole, Reacher and Half-Cheetah. Our results show that our method generalizes well across a variety of Sim2Sim gap, target policies and offline data collection policies. We also demonstrate the performance of our algorithm on a Sim2Real task of validating the performance of a 7 DOF robotic arm using offline data along with a gazebo based arm simulator.

Keywords: Sim2Real, Policy Evaluation, Robot Validation

1 Introduction

Reinforcement Learning (RL) algorithms aim to select actions that maximize the cumulative returns over a finite time horizon. In recent years, RL has shown state-of-the-art performance over a range of complex tasks such as chatbots, [1], games [2], and robotics [3, 4, 5, 6]. However, RL algorithms still require a large number of samples, which can limit their practical use in robotics [7, 8].

A typical approach is to train robust robot policies in simulation and then deploy them on the robot [9, 10]. Such an approach, although useful, does not guarantee optimal performance on the real robot without significant finetuning [11]. In this work, we propose an approach that evaluates the real world performance of a policy using a robot simulator and offline data collected from the real-world [6]. In order to achieve, we employ the framework of off-policy evaluation (OPE) [12].

OPE is the problem of using offline data collected from a possibly unknown *behavior* policy to estimate the performance of a different *target* policy. Classical OPE methods are based on the principle of importance sampling (IS) [13, 14, 15], which reweights each trajectory by its density ratio under the target versus the behavior policies. More recently, significant progress has been made on *marginalized importance sampling* (MIS), which reweights transition tuples using the density

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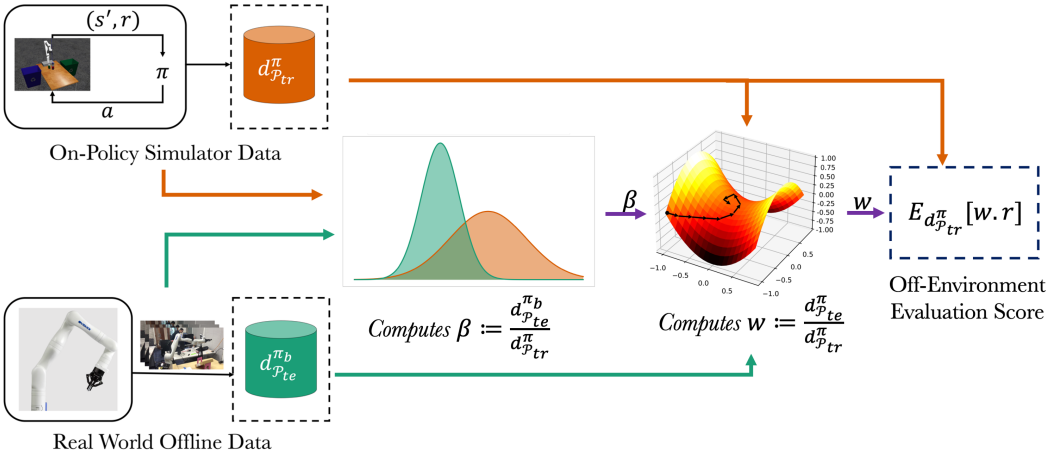


Figure 1: For a given policy π, we first collect on-policy data $d^{\pi}_{\mathcal{P}_{tr}}$ from a simulator environment. Using $d^{\pi}_{\mathcal{P}_{tr}}$ and offline data $d^{\mathcal{D}}_{\mathcal{P}_{te}}$ on the real world, we first calculate the importance sampling factor $\beta = d^{\pi_b}_{\mathcal{P}_{te}}/d^{\pi}_{\mathcal{P}_{tr}}$. This importance sampling factor essentially allows us to re-weight existing off-policy evaluation algorithm to estimate $w = d^{\pi}_{\mathcal{P}_{te}}/d^{\pi}_{\mathcal{P}_{tr}}$ which helps us estimate real-world performance of the agent $J_{\mathcal{P}_{te}}(\pi)$ using on-policy simulator data

ratio (or importance weights) over states instead of trajectories to overcome the so-called curse of horizon [16, 17, 18]. The density ratio is often learned via optimizing minimax loss functions.

Most existing MIS methods are model-free, relying on data from the real environment to approximate MIS weight function. However, a direct application of MIS methods to robotics carries two main disadvantages. (1) *Large ratios:* MIS method learns distribution mismatch between the behavior and the target policies. When the mis-match between the behavior and the target policy is large, which is often the case, MIS method tend be challenging to learn. (2) *Indirect supervision:* MIS methods requires no samples from target policies. This requires the weight being learned *indirectly* via the Bellman flow equation. In places where coverage of the offline dataset is scarce, MIS methods tend to perform poorly.

In robotics, it is reasonable to assume access to a good but imperfect simulator of the real environment. In this work, we propose an improved MIS estimator that estimates the density ratio mismatch between the real world and the simulator. We further show that such a MIS estimator can be used to evaluate the real-world performance of a robot using just the simulator. As described in figure 1, we estimate the discrepancy between the real world and the simulator by using the target policy's occupancy in the simulator as an intermediate variable. This allows us to calculate the MIS weights as a combination of two factors, which can be learned independently. The first factor has direct supervision, while the second factor has a small magnitude (close to 1), thereby addressing both large ratios and indirect supervision issues mentioned above. We present a straightforward derivation of our method, examine its theoretical properties, and compare it to existing methods (including existing ways of incorporating a simulator in OPE) and baselines through empirical analysis.

We make the following contributions. 1) We derive an MIS estimator for off-environment evaluation (Section 4). 2) We explore the theoretical properties of our off-environment evaluation estimator by proposing a sample-complexity analysis (Section 5) and studying its special cases in linear and kernel settings. 3) We empirically evaluate our estimator on both Sim2Sim as well as Sim2Real environments (Section 7). For the Sim2Sim experiments, we perform a thorough abalation study over different Sim2Sim gap, data-collection policies and target policies, environments (Taxi, Cartpole, Reacher and Half-Cheetah). Furthermore, we demonstrate practicality of our approach on a sim2real task by validating performance of a kinova robotic arm over using offline kinova data along with gazebo based kinova simulator.

2 Preliminaries

Robot learning problems are often modelled as an infinite-horizon discounted Markov Decision Process (MDP). MDP is specified by $(\mathcal{S}, \mathcal{A}, P, R, \gamma, d_0)$. Here, $\mathcal{S}$ and $\mathcal{A}$ are the state and the action spaces, $P : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$ is the transition function ($\Delta(\cdot)$ is the probability simplex). We also define the reward function $R : \mathcal{S} \times \mathcal{A} \to \Delta([0, R_{max}])$, $\gamma \in [0, 1)$ is the discount factor, and $d_0 \in \Delta(\mathcal{S})$ is the initial state distribution. A policy $\pi : \mathcal{S} \to \Delta(\mathcal{A})$ induces a distribution of trajectory: $\forall t \geq 0$, $s_0 \sim d_0, a_t \sim \pi(\cdot|s_t), r_t \sim R(\cdot|s, a), s_{t+1} \sim P(\cdot|s_t, a_t)$. The performance of π is measured by its expected discounted return under the initial state distribution, defined as $J_P(\pi) = (1-\gamma)\mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r_t | \pi, d_0]$; here we use the subscript P in $J_P(\pi)$ to emphasize the dynamics w.r.t. which the return is defined, since we will consider both the true environment and the simulator in the rest of this paper and the subscript will help distinguish between them. $J_P(\pi)$ also has an alternative expression $J_P(\pi) := \mathbb{E}_{(s,a)\sim d_P^\pi, r\sim R(s,a)}[r]$, where

$$d_P^\pi(s, a) = (1-\gamma) \textstyle\sum_{t=0}^{\infty} \gamma^t \mathbb{P}[s_t = s, a_t = a | \pi, d_0] \tag{1}$$

is the discounted state-action occupancy induced by π from d_0. An important quantity associated with a policy is its Q-function Q_P^π, which satisfies the Bellman equation $Q_P^\pi(s, a) = \mathbb{E}_{r\sim R(s,a), s'\sim P(s,a)}[r + \gamma Q_P^\pi(s', \pi)]$, where $f(s', \pi) := \mathbb{E}_{a'\sim\pi(\cdot|s')}[f(s', a')]$. We can also define the state-value function $V_P^\pi(s) = Q_P^\pi(s, \pi)$, and $J(\pi) = (1-\gamma)\mathbb{E}_{s\sim d_0}[V_P^\pi(s)]$.

OPE and Marginalized Importance Sampling: In off-policy evaluation (OPE), we want to evaluate a target policy π using data collected from a different policy in the real environment, denoted by its dynamics P. As a standard simplification, we assume data is generated i.i.d. as $(s, a) \sim \mu, r \sim R(s, a), s' \sim P(s, a)$, and the sample size is n. When the data is generated from some behavior policy π_b, μ can correspond to its occupancy $d_P^{\pi_b}$. We will use $\mathbb{E}_\mu[\cdot]$ as a shorthand for taking expectation over (s, a, r, s') generated from such a data distribution in the real environment.

The key idea in marginalized importance sampling (MIS) is to learn the weight function $w_P^{\pi/\mu}(s, a) := \frac{d_P^\pi(s,a)}{\mu(s,a)}$;[1] once this function is known, $J(\pi)$ can be estimated as $J(\pi) = \mathbb{E}_{(s,a)\sim d_P^\pi, r\sim R(s,a)}[r] = \mathbb{E}_\mu[w_P^{\pi/\mu}(s, a) \cdot r]$. Note that $\mathbb{E}_\mu[\cdot]$ can be empirically approximated by the dataset sampled from the real environment. The real challenge in MIS is how to learn $w_P^{\pi/\mu}$. Existing works often do so by optimizing minimax loss functions using Q-functions as discriminators, and is subject to both difficulties (large ratios and indirect supervision) mentioned in the introduction. We refer the readers to [19] for a summary of some typical MIS methods.

Learning Density Ratios from Direct Supervision: Given two distributions p and q over the same space $\mathcal{X}$, the density ratio $p(x)/q(x)$ can be learned *directly* if we have access to samples from both p and q, using the method proposed by [20]:

$$\frac{p(x)}{q(x)} = \arg \max_{f:\mathcal{X}\to\mathbb{R}_{>0}} \mathbb{E}_{x\sim p}[\ln f(x)] - \mathbb{E}_{x\sim q}[f(x)] + 1. \tag{2}$$

To guarantee generalization over a finite sample when we approximate the expectations empirically, we will need to restrict the space of f that we search over to function classes of limited capacities, such as RKHS or neural nets, and $p(x)/q(x)$ can still be well approximated as long as it can be represented in the chosen function class (i.e., *realizable*). More concretely, if we have n samples $x_1, \ldots, x_n$ from p and m samples $\tilde{x}_1, \ldots, \tilde{x}_m$ from q, and use $\mathcal{F}$ to approximate $p(x)/q(x)$, the learned density ratio can be made generalizable by adding a regularization term $I(f)$ to improve the statistical and computational stability of learning.

$$\arg\max_{f\in\mathcal{F}} \frac{1}{n} \sum_i \ln f(x_i) - \frac{1}{m} \sum_j f(\tilde{x}_j) + \frac{\lambda}{2} I(f)^2, \tag{3}$$

[1] Whenever we refer to a densitiy ratio of form $p(x)/q(x)$ in this paper, we are implicitly making the coverage assumption that $q(x) > 0\ \forall x$ where $p(x) > 0$.

3 Related Work

We review related works in this section, focusing on comparing to existing OPE methods that can leverage the side information provided by an imperfect simulator.

Marginalized Importance Sampling (MIS): MIS methods tend to assume the framework of data collection and policy evaluation being on the same environment. To that end, there are both model-free [21, 22, 23] and model-based variants of MIS methods and face the aforementioned two challenges (large magnitude of weights and indirection supervision) simultaneously. Model based variants of MIS sometimes tend to be doubly robust (DR) in nature [18, 24] and can benefit from Q-functions as control variates, which can be supplied by the simulator. However, the DR version of MIS is a meta estimator, and the weight $d^{\pi}_{P_{te}}/\mu$ still needs to be estimated via a "base" MIS procedure. Therefore, the incorporation of the simulator information does not directly address the challenges we are concerned with, and there is also opportunity to further combine our estimator into the DR form of MIS.

Model-based methods: Model-based estimators first approximate the transition dynamics of the target environment [25], which is further used to generate rollouts and evaluate performance for any target policy. One way of incorporating a given imperfect simulator in this approach is to use the simulator as "base predictions", and only learn an additive correction term, often known as residual dynamics [26]. This approach combines the two sources of information (simulator and data from target environment) in a very different way compared to ours, and are more vulnerable to misspecification errors than model-free methods.

4 Weight Estimator

Recall that our goal is to incorporate a given simulator into MIS. We will assume that the simulator shares the same $\mathcal{S}, \mathcal{A}, \gamma, d_0$ with the real environment, but has its own transition function P_{tr} which can be different from P_{te}. As we will see, extension to the case where the reward function is unknown and must be inferred from sample rewards in the data is straightforward, and for simplicity we will only consider difference in dynamics for most of the paper.

Split the weight: The key idea in our approach is to split the weight $w^{\pi/\mu}_{P}$ into two parts by introducing $d^{\pi}_{P_{tr}}$ as an intermediate variable:

$$w^{\pi/\mu}_{P}(s,a) = \frac{d^{\pi}_{P_{te}}(s,a)}{\mu(s,a)} = \underbrace{\frac{d^{\pi}_{P_{tr}}(s,a)}{\mu(s,a)}}_{\substack{:=\beta \\ \text{(direct supervision)}}} \cdot \underbrace{\frac{d^{\pi}_{P_{te}}(s,a)}{d^{\pi}_{P_{tr}}(s,a)}}_{:=w^{\pi}_{P_{te}/P_{tr}}\text{(magnitude} \simeq 1)}$$

Note that $d^{\pi}_{P_{tr}}$ is the occupancy of π in the simulator, which we have free access to. The advantage of our approach is that by estimating β and $w^{\pi}_{P_{te}/P_{tr}}$ separately, we avoid the situation of running into the two challenges mentioned before simultaneously, and instead address one at each time: $\beta = d^{\pi}_{P_{tr}}/\mu$ has large magnitude but can be learned directly via [20] without the difficult minimax optimization typically required by MIS, and we expect $w^{\pi}_{P_{te}/P_{tr}} = d^{\pi}_{P_{te}}/d^{\pi}_{P_{tr}}$ to be close to 1 when $P_{te} \approx P_{tr}$ (and thus easier to learn).

Estimate $w^{\pi}_{P_{te}/P_{tr}}$: Since β is handled by the method of [20], the key remaining challenge is how to estimate $w^{\pi}_{P_{te}/P_{tr}}$. (Interestingly, β also plays a key role in estimating $w^{\pi}_{P_{te}/P_{tr}}$, as will be shown below.) Note that once we have approximated $w^{\pi}_{P_{te}/P_{tr}}$ with some w, we can directly reweight the state-action pairs from the simulator (i.e., $d^{\pi}_{P_{tr}}$) if the reward function is known (this is only assumed for the purpose of derivation), i.e.,

$$J_{P_{te}}(\pi) \approx \mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}, r\sim R(s,a)}[w \cdot r],$$

and this becomes an identity if $w = w^{\pi}_{P_{te}/P_{tr}}$. Following the derivation in [18, 19], we now reason about the error of the above estimator for an arbitrary w to derive an upper bound as our loss for

learning w:

$$
\begin{aligned}
&|\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}, r\sim R(s,a)}[w\cdot r] - J_{P_{te}}(\pi)| \\
=& |\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}, s'\sim P(s,a)}[w\cdot (Q^{\pi}_{P_{te}}(s,a) - \gamma Q^{\pi}_{P_{te}}(s',\pi))] - (1-\gamma)\mathbb{E}_{s\sim d_0}[Q^{\pi}_{P_{te}}(s,\pi)]| \\
\leq& \sup_{q\in\mathcal{Q}} |\mathbb{E}_{d^{\pi}_{P_{tr}}\times P_{te}}[w\cdot (q(s,a) - \gamma q(s',\pi))] - (1-\gamma)\mathbb{E}_{s\sim d_0}[q(s,\pi)]|. \qquad (4)
\end{aligned}
$$

Here $d^{\pi}_{P_{tr}} \times P_{te}$ is a shorthand for $(s,a) \sim d^{\pi}_{P_{tr}}, s' \sim P(s,a)$. In the last step, we handle the unknown Q^{π}_{P} by a relaxation similar to [16, 18, 19], which takes an upper bound of the error over $q \in \mathcal{Q}$ for some function class $\mathcal{Q} \subset \mathbb{R}^{\mathcal{S}\times\mathcal{A}}$, and the inequality holds as long as $Q^{\pi}_{P} \in conv(\mathcal{Q})$ with $conv(\cdot)$ being the convex hull.

Approximate $d^{\pi}_{P_{tr}} \times P_{te}$: The remaining difficulty is that we will need samples from $d^{\pi}_{P_{tr}} \times P_{te}$, i.e., (s,a) sampled from π's occupancy in the P_{tr} *simulator*, and the next s' generated in the P_{te} *real environment*. While there is no natural dataset for such a distribution, we can take the data from the real environment, $(s,a,s') \sim \mu \times P_{te}$, and reweight it using $\beta = d^{\pi}_{P_{tr}}/\mu$ to approximate expectation w.r.t. $d^{\pi}_{P_{tr}} \times P_{te}$, i.e.,

$$
(s,a,s') \sim d^{\pi}_{P_{tr}} \times P_{te} \quad \Longleftrightarrow (s,a,s') \sim \mu \times P_{te} \text{ reweighted with } \beta := d^{\pi}_{P_{tr}}/\mu.
$$

Based on such an observation, we can further upper-bound $|\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}, r\sim R(s,a)}[w\cdot r] - J_{P_{te}}(\pi)|$ from end of Equation 4 with:

$$
\sup_{q\in\mathcal{Q}} L_w(w,\beta,q) := |\mathbb{E}_{\mu}[w\cdot\beta\cdot (q(s,a) - \gamma q(s',\pi))] - (1-\gamma)\mathbb{E}_{s\sim d_0}[q(s,\pi)]|. \qquad (5)
$$

As our derivation has shown, this is a valid upper bound of the error as long as $conv(\mathcal{Q})$ can represent $Q^{\pi}_{P_{te}}$. We also need to show that the upper bound is non-trivial, i.e., when $w = w^{\pi}_{P_{te}/P_{tr}}$, the upper bound should be 0. This is actually easy to see, as for any q:

$$
\begin{aligned}
&L(w^{\pi}_{P_{te}/P_{tr}}, \beta, q) := |\mathbb{E}_{d^{\pi}_{P_{tr}}\times P_{te}}[w^{\pi}_{P_{te}/P_{tr}}\cdot (q(s,a) - \gamma q(s',\pi))] - (1-\gamma)\mathbb{E}_{s\sim d_0}[q(s,\pi)]| \qquad (6) \\
&= |\mathbb{E}_{d^{\pi}_{P_{te}}\times P}[q(s,a) - \gamma q(s',\pi)] - (1-\gamma)\mathbb{E}_{s\sim d_0}[q(s,\pi)]| = 0. \qquad (7)
\end{aligned}
$$

The last step directly follows from the fact that $d^{\pi}_{P_{te}}$ is a valid discounted occupancy and obeys the Bellman flow equation. Therefore, it makes sense to search for w over a function class $\mathcal{W} \subset \mathbb{R}^{\mathcal{S}\times\mathcal{A}}$ to minimize the loss $\sup_{q\in\mathcal{Q}} L(w,\beta,q)$.

Final estimator: To summarize our estimation procedure, we will first use [20] to estimate $\hat{\beta} \approx d^{\pi}_{P'}/\mu$ with a function class $\mathcal{F}$, and plug the solution into our loss for estimating $w^{\pi}_{P_{te}/P_{tr}}$, i.e.,

$$
\hat{w} = \arg\min_{w\in\mathcal{W}} \sup_{q\in\mathcal{Q}} L_w(w,\hat{\beta},q). \qquad (8)
$$

As mentioned above, if the reward function is known, we can use $\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}, r\sim R(s,a)}[\hat{w}\cdot r]$ as our estimation of $J_{P_{te}}(\pi)$. We can also demonstrate interesting properties of our optimization like the effect of a linear function class and RKHS function class. This discussion is deferred to the supplementary materials section 10.1.

5 Sample Complexity Guarantee

In this section, we provide sample complexity guarantee for our estimators. Note that our method involves two different estimation procedures. We first estimate β, and plug the estimated β into the minimax optimization. It is therefore important to understand how the error in the first stage propagates and combines with the errors in the second stage. To address this question, we provide a "end-to-end" theoretical analysis of our estimator in this section, taking into consideration that both β and $w^{\pi}_{P_{te}/P_{tr}}$ needs to be estimated from data. We first make the following assumptions.

Assumption 1 (Boundedness)**.** *We assume* $\forall f \in \mathcal{F}$, $0 < C_{\mathcal{F},\min} \leq f \leq C_{\mathcal{F},\max}$. *Define* $C_{\mathcal{F}} := C_{\mathcal{F},\max} + \max(\log C_{\mathcal{F},\max}, -\log C_{\mathcal{F},\min})$. *Similarly,* $\forall w \in \mathcal{W}$, $w \in [0, C_{\mathcal{W}}]$, *and* $\forall q \in \mathcal{Q}$, $q \in [0, C_{\mathcal{Q}}]$.

Note that the function class $\mathcal{F}$ needs to be lower bounded by $C_{\mathcal{F},\min}$, and the negative log of this lower bound will appear in the sample-complexity guarantee. This is because the method of [20] explicitly calculates $\log f$ and is numerically sensitive when f is close to 0. We then need to assume that the function classes we use are well-specified, a.k.a. realizable. These assumptions are directly comparable to those made in [18]. Also, handling small amount of misspecification is routine and we omit such an extension for simplicity.

Assumption 2 (Realizability of $\mathcal{F}$)**.** $d^{\pi}_{P'}/\mu \in \mathcal{F}$.

Theorem 5.1. *Let $\hat{\beta}$ be our approximation of $d^{\pi}_{P'}/\mu$ which we found using [20]. We utilize this $\hat{\beta}$ to further optimize for $\hat{w}_n$ (equation 8) using n samples. In both cases, $\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}}[\cdot]$ is also approximated with n samples from the simulator P_{tr}. Then, under Assumptions 1 and 2 along with the additional assumption that $Q^{\pi}_{P_{te}} \in C(\mathcal{Q})$ with probability at least $1-\delta$, the Off Environment Evaluation error can be bounded as*

$$
\begin{aligned}
&|\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}, r\sim R(s,a)}[\hat{w}_n \cdot r] - J_{P_{te}}(\pi)| \leq \min_{w\in\mathcal{W}} \max_{q\in\mathcal{Q}} |L_w(w,\beta,q)| \\
&+ 2C_{\mathcal{W}} \cdot C_{\mathcal{Q}} \cdot \tilde{O}\left(\sqrt{\left\|\frac{d^{\pi}_{P'}}{\mu}\right\|_{\infty} \cdot \left(4\mathbb{E}\mathcal{R}_n(\mathcal{F}) + C_{\mathcal{F}}\sqrt{\frac{2\log(\frac{2}{\delta})}{n}}\right)}\right) \\
&+ 2\mathcal{R}_n(\mathcal{W},\mathcal{Q}) + C_{\mathcal{Q}} C_{\mathcal{W}} \sqrt{\frac{\log(\frac{2}{\delta})}{2n}}
\end{aligned}
\tag{9}
$$

where $\mathcal{R}_n(\mathcal{F}), \mathcal{R}_n(\mathcal{W},\mathcal{Q})$ are the Radamacher complexities of function classes $\{(x,y) \to f(x) - \log(f(y)) : f \in \mathcal{F}\}$ and $\{(s,a,s') \to (w(s,a) \cdot \frac{d^{\pi}_{P'}(s,a)}{\mu(s,a)} \cdot (q(s,a) - \gamma q(s',\pi)) : w \in \mathcal{W}, q \in \mathcal{Q}\}$, respectively, $\|d^{\pi}_{P'}/\mu\|_{\infty} := \max_{s,a} d^{\pi}_{P'}(s,a)/\mu(s,a)$ measures the distribution shift between $d^{\pi}_{P'}$ and μ, and $\tilde{O}(\cdot)$ is the big-Oh notation suppressing logarithmic factors.

Note that we do not make realizability assumption for $\mathcal{W}$ in the theorem above. Realizability assumption is reflected in the $\inf_{w\in\mathcal{W}} \sup_{q\in\mathcal{Q}} |L(w,\beta,q)|$, which equals 0 when $d^{\pi}_{P_{te}}/d^{\pi}_{P_{tr}} \in \mathcal{W}$. The remaining terms vanishes to 0 at an $O(1/\sqrt{n})$ rate when $n \to \infty$.

6 Generalizing Off-Environment Policy Evaluation

One key advantage of our two-step approach is that we can improve many existing off-policy evaluation algorithm with a similar two-step process. We now demonstrate one such derivation for Dual-DICE [21]. Consider the following optimization to estimate: $w^{\pi}_{P_{te}/P_{tr}}$

$$
\frac{d^{\pi}_{P_{te}}}{d^{\pi}_{P_{tr}}} := \arg\min_{x:S\times A\to\mathbb{R}} J(x) = \frac{1}{2}\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}}[x(s,a)^2] - \mathbb{E}_{(s,a)\sim d^{\pi}_{P_{te}}}[x(s,a)]
$$

With a change of variable $x(s,a) = \nu(s,a) - \gamma\mathbb{E}_{s'\sim P_{te}(\cdot|s,a), a'\sim\pi(\cdot|s')}[\nu(s',a')] = \mathcal{B}^{\pi}_{P_{te}} v(s,a)$, this becomes

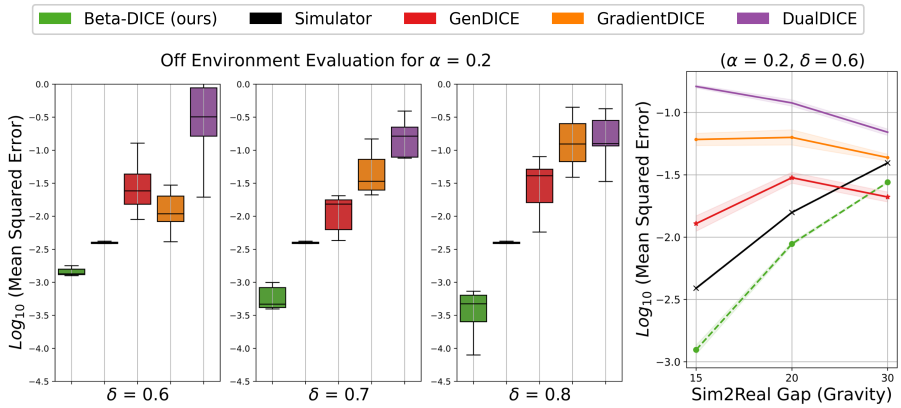
$$
\begin{aligned}
&\frac{1}{2}\mathbb{E}_{(s,a)\sim d^{\pi}_{P_{tr}}}[(\mathcal{B}^{\pi}_{P_{te}} v(s,a))^2] - (1-\gamma)\mathbb{E}_{s_0\sim d_0, a_0\sim\pi(\cdot|s_0)}[\nu(s_0,a_0)] \\
&\frac{1}{2}\mathbb{E}_{(s,a)\sim d_{\mu}}[\beta(s,a) \cdot \mathcal{B}^{\pi}_{P_{te}} v(s,a)] - (1-\gamma)\mathbb{E}_{s_0\sim d_0, a_0\sim\pi(\cdot|s_0)}[\nu(s,a)]
\end{aligned}
$$

Where the second line comes from re-weighting the expectation with $\beta(s,a) = \frac{d^{\pi}_{P_{tr}}}{\mu}$. This allows us to use samples from μ. We finish the derivation by exploiting Fenchel Duality [27], we get the following derivation for $w^{\pi}_{P_{te}/P_{tr}}$

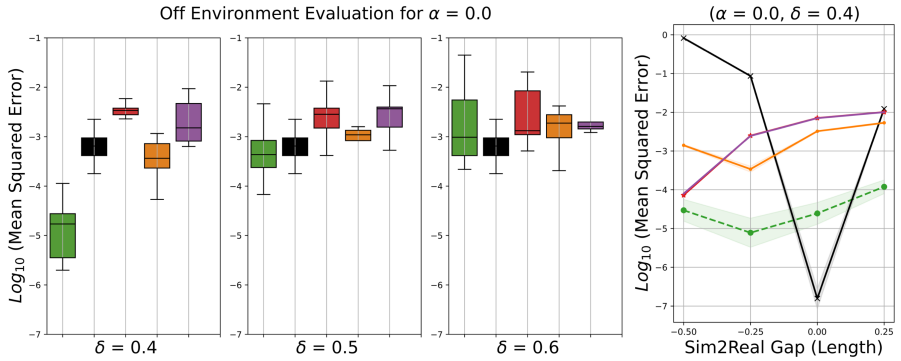
$$
\begin{aligned}
&\frac{d^{\pi}_{P_{te}}}{d^{\pi}_{P_{tr}}} := \arg\min_{\nu:S\times A\to\mathbb{R}} \max_{\zeta:S\times A\to\mathbb{R}} J(\nu,\zeta) \\
&= \mathbb{E}_{\substack{(s,a,s')\sim\mu \\ a'\sim\pi(\cdot|s')}}[\beta(s,a) \cdot \mathcal{B}^{\pi}_{P_{te}} v(s,a) \cdot \zeta(s,a) - \frac{\zeta(s,a)^2}{2})] + (1-\gamma)\mathbb{E}_{s_0\sim d_0, a_0\sim\pi(\cdot|s_0)}[\nu(s_0,a_0)].
\end{aligned}
$$

Table 1: Log mean squared error between the performance predicted by our β-DICE algorithm and the real world performance of the robot. We observe that our method is able to outperform DICE based baselines by a comfortable margin.

	Log_{10} Mean Squared Error ($\downarrow$)				
Algorithm	Kinova (Sim2Real)	Taxi	Cartpole	Reacher	Half-Cheetah
Simulator	-3.96	-0.19	-2.58	-1.09	1.18
β-DICE (Ours)	**-4.38**	**-1.60**	**-4.19**	**-4.08**	**-3.42**
GenDICE	-3.48	-0.13	-2.84	-2.61	-2.96
GradientDICE	-3.49	-0.59	-1.45	-3.17	-2.16
DualDICE	-3.48	-0.48	-0.99	-2.88	-2.12



(a) Target Policy $\alpha = 0.2$



(b) Target Policy $\alpha = 0.0$

Figure 2: We demonstrate the effectiveness of β-DICE on Cartpole (a) and Reacher (b) Sim2Sim environment. For Cartpole environment we demonstrate the performance of β-DICE over for a Sim2Sim pair of $\{10, 15\}m/s^2$. Similarly for Reacher the Sim2Sim pair is (0.1m, 0.75m) for the length of the link. On left hand side, we demonstrate the effect of β-DICE with different data collection policies while keeping the target policies fixed. On the right hand side, we demonstrate the impact of β-DICE with increasing Sim2Sim gap keeping the offline data collection policy the same. We observe that our β-DICE algorithm comfortably outperforms closest DICE based baselines.

Note that such a derivation is not limited to DualDICE but can be extended to other DICE estimators as well. For example, GradientDICE [22]—which is an empirically state-of-the-art estimator in the DICE family—can also be similarly adapted as below, which we use in our experiments. Detailed derivation for the same can be found in the supplementary materials 10.3.

7 Experiments

7.1 Sim2Sim Validation of β-DICE

Experimental Setting: For the Sim2Sim experiments, we vary a key parameter which defines the transition dynamics of these environments. We call this modified environment the training environment (simulator) and the default setting as the test environment. We choose modified parameter so that the performance predicted by the training environment is off by at-least 20-40% of the test environment performance. Our Sim2Sim study also exhibits the performance of β-DICE with varying data-collection policies and varying target policies and varying Sim2Sim gap. Both data collection and target policies are parameterized as a mixture of a pre-trained optimal policy and noisy policy, with the mixing factor being δ and α respectively. Larger the value of δ, α, noisier the policy. Typically the target policies are less noisier than the offline data collection policy, thus making the problem of evaluation specially harder. Detailed experimental details along with the setup has been detailed in Appendix 10.10.

Results and Observations: In figures 2a and 2b we present the performance of β-DICE over varying data-collection policies and sim2sim gap, while keeping the target policy fixed. Additionally Sim2Sim experiments detailing the performance of β-DICE over different environments can be found in Supplementary materials 10.10. A performance summary of all of these sim2sim experiments in table 1. We observe that β-DICE is comfortably able to outperform several DICE-based baselines across a range of target policies, data-collection policies and sim2sim gap. However, as the Sim2Sim gap increases the performs drops significantly.

7.2 Real world performance validation on Kinova Robotic Arm

Experimental Setting: We demonstrate the effectiveness of our β-DICE algorithm for a sim2real validation task on a Kinova robotic arm. We first collect offline data by asking users to move the arm from one-position to another via RC controllers. Our data collection ensures sufficient coverage of the robotic arm's task space. We then use this offline data along with our in-house gazebo based simulator to experimentally validate the real-world performance of a PID controller using β-DICE that moves our robot from a given initial location to any desired location.

Results and Observations: Our results along with different baselines are averaged over 10 different locations are tabulated in Table 1. We observe that β-DICE is able to outperform state-of-the-art showing an improvement of 60% over the nearest baseline. There are two key conclusions from all of our experiments. One, although β-DICE outperforms state-of-the art baselines in off-policy evaluation. We observe that the performance drops when the gap between the target policy and behavior policy increases. Two, prediction error decreases with an increasing gap between training and test environment. This is because information that can be transferred between both the environments decreases.

8 Limitations

There are a couple of limitations of our work that we wish to address in future work. (1) Our algorithm expects high quality data with sufficient coverage of the state-action space. Identifying the confidence interval of our estimator w will not only ensure sample efficient evaluation, but also help us in designing robust offline reinforcement learning algorithm. (2) Similar to DICE class of min-max optimization, our algorithm too suffers from high variance in their performance. Efforts are required to reduce this variance.

9 Conclusion and Future Work

We derive a novel MIS estimator that is able to evaluate real world performance of a robot using offline data and an imperfect robot simulator. We develop sample complexity bounds, and empirically validate our approach on diverse Sim2Sim environments like Cartpole, HalfCheetah and Sim2Real environment like KinovaGen3 robot. For future work, we wish to utilize this framwork of off-environment evaluation to learn optimal robot policies using simulation and a limited amount of real-world offline data.

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