Semantic Field of Words Represented as Non-Linear Potential Functions

Anonymous Author(s)
Affiliation
Address
email

Abstract

State-of-the-art word embeddings presume a linear vector space, but this approach does not easily incorporate the nonlinearity that is necessary to represent polysemy. We thus propose a novel semantic FIeld REepresentation, called FIRE, which is a D-dimensional field in which every word is represented as a set of its locations and a nonlinear function covering the field. The strength of a word's relation to another word at a certain location is measured as the function value at that location. With FIRE, compositionality is represented via functional additivity, whereas polysemy is represented via the set of points and the function's multimodality. By implementing FIRE for English and comparing it with previous representation methods via word and sentence similarity tasks, we show that FIRE produces comparable or even better results. In an evaluation of polysemy to predict the number of word senses, FIRE greatly outperformed BERT and Word2vec, providing evidence of how FIRE represents polysemy.

1 Introduction

Word representations have become an important basic technique in NLP. The most important semantic requirement for a word representation,above all, is that similar words are mapped to similar embeddings. Furthermore, the next two most important properties are *compositionality* [25], i.e., the representation of two composed words matches the composition of the two representations, and *polysemy*, the representation of semantic ambiguity. Intuitively, compositionality is a linear quality, whereas polysemy is nonlinear and thus requires a representation via mathematical multimodality, which is difficult to represent within a linear space.

State-of-the-art word representations are based on a linear vector space. The linearity of vectors naturally accommodates compositionality, but the question of how to incorporate polysemy has been an issue. Indeed, recent random-variable representations [27, 21, 23, 16, 8] represent polysemy but at the cost of losing compositionality.

The most important previous work that seems to incorporate polysemy in addition to compositionality could be BERT [6]; however, it is a contextual embedding that represents one vector only under a given context. A polysemous word has multiple meanings even without context. Our idea was thus to devise an entirely new representation that is nonlinear but naturally represents linearity as well.

Hence, FIeld REpresentation, which we call FIRE, is a *non-contextual representation* of words. It is realized in a D-dimensional space in which every point has a quantity, called a semantic *potential*, that is generated by interactions among words. Every word is represented by a pair consisting of a set of the word's locations and a nonlinear potential function representing the word's context, where the field outputs the semantic potential value at a location. Two potential fields can be linearly added, which provides the basis of compositionality. Polysemy is represented through the set of a word's

Submitted to 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Do not distribute.

locations and the multimodality of the field f. Through this representation, the similarity of two words is obtained by adding the potential field values at the word locations.

Our experimental results confirmed that FIRE showed similar or better results in comparison with previous Word2XX representations under settings with the same number of parameters. Furthermore, by representing multimodality over the entire field, FIRE has a much wider expressiveness for polysemy. In an evaluation to predict the number of word senses, FIRE greatly outperformed BERT.

2 Related Works

Vectoral word representations. The most widely used word representations are in vectoral forms, such as *Word2Vec* [13]. [10] showed that Word2Vec is essentially a decomposition of the point-wise mutual information matrix. Other vectoral representation methods extract different information from corpora. Important works include *GloVe* [17], *fastText* [5], *ELMo* [18], and *BERT* [6].

It has been attempted to represent polysemy via multiple vectors per word [24], but such a "mixture" of vectors lost the quality of compositionality.

Random-variable word representations. [27] proposed *Word2Gauss*, which represents each word by a Gaussian distribution. They exploited the covariance matrix to model the semantic ambiguity of words. Asymmetric similarity (via the Kullback-Leibler divergence) was further introduced to represent hypernyms and hyponyms. [21] proposed to exploit existing word vectors by representing a word with a nonparametric probability distribution estimated using its closest word vectors.

Several recent works studied the use of the Wasserstein distance to measure the similarity between words, which are represented by Gaussian distributions [23] or more general elliptical distributions [16]. For elliptical distributions, the Wasserstein distance has a closed form and can be computed efficiently. [8] proposed to use the entropy-regularized Wasserstein distance as the similarity between words that are represented by multinomial distributions and defined on the same set of (64) discrete supports in a low-dimensional space (e.g., $\mathbb{R}^2$). Because the supports form a point cloud, the method is called *Word2Cloud*. The multinomial distribution in Word2Cloud can be viewed as a mixture of Dirac delta functions, a notion that we use in this work. *Word2GM* [2] also uses a mixture of standard distributions, where the random variable follows a mixture of Gaussians.

All these random-variable word representations incorporate polysemy but not compositionality. Furthermore, many of these methods remain word representations (e.g., capturing polysemy and hypernymy) only and have not been extended to sentence representations.

Sentence representations. A typical sentence representation is a vector obtained as a weighted additive composition of vectoral word representations. Various weighting heuristics have been studied, such as the *term frequency–inverse document frequency* (TF-IDF) [12]. [1] proposed the *smooth inverse frequency* (SIF), a simple weighting scheme that works exceptionally well. In this paper, too, we evaluate the capability of the sentence field representation produced by applying the SIF to FIRE word representation.

State-of-the-art vectoral sentence representation methods adopt an adaptive weighting scheme by using a self-attention mechanism [26]; one implementation includes BERT [6]. Such methods can also be applied to our word field representation, which remains a future work.

To the best of our knowledge, [21] proposed the only existing random-variable sentence representation. Representations of the words in a sentence are composed into a Wasserstein barycenter generating a new random variable. Whereas that computation is not additive, FIRE is additive.

3 Semantic Field: FIRE

3.1 Word Representation

FIRE is a semantic field generated by words. In physics, a field is a space in which every point has some physical quantity, of which a common type is the *potential*. FIRE uses the semantic potential generated by words. While particles in a physical field are often identical, the particularity of FIRE

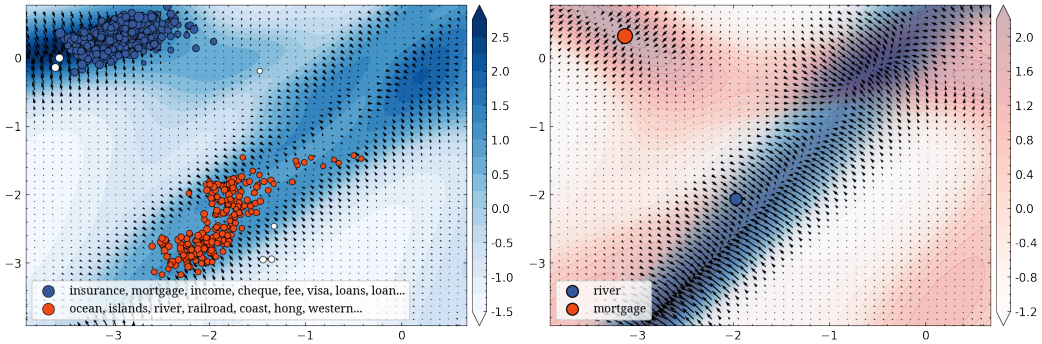

Figure 1: **Left:** Semantic field of *bank* for $D = 2, L = 4, K = 1$, where D is the dimension of this representation space, L is the number of planar transformation layers (see Formula (6)), and K is the number of polysemy locations (Formula (2). The circles represent the locations of 1000 words that were frequent and similar to *bank*. The words were clustered and colored by using the DBSCAN algorithm [7]. Polysemy is visible via two clusters of words, in finance and geography. The white circles represent the locations of words that DBSCAN considered outliers.
Right: Semantic field generated for $D = 2, L = 4, K = 1$ by superposing the potential functions of *river* (blue) and *mortgage* (red). The two points show the words' locations. The color intensities indicate the values of the potential functions at a point, and the arrows indicate the gradients.

is that every word produces a different semantic potential and is thus allocated within the field in association with an individual potential function.

Let W be a set of words and $\mathcal{S}$ be a D-dimensional real space. Our semantic field is defined on $\mathcal{S}$. A word $w \in W$ is represented as a 2-tuple $[\mu, f(s)]$, where μ represents the word's locations. In a previous linear space model [13], a word was usually located at a single point but could be polysemous, i.e., with multiple locations as in [24]. Let K denote the number of such locations. $f(s) : \mathcal{S} \to \mathbb{R}$ is a continuous real-valued function covering $\mathcal{S}$. $f(s)$ represents the context generated by the surrounding words of w via a neural network, as will be explained in Section 4.1.

Figure 1 (left) shows an example portion of $\mathcal{S}$ computed for $D = 2$ with $\mathcal{S} = [-4, 4]^2$ and $K = 1$, for the term *bank*. There are two clusters related to *bank*, one for a financial bank and the other for a river bank. The circles show the locations μ for 1000 words (among the most frequent 50,000 words) that were frequent and similar to *bank*, with *similarity* as defined in the following section.

3.2 Similarity of Two Words

In our approach, two words $w_i = [\mu_i, f_i(s)]$ and $w_j = [\mu_j, f_j(s)]$ in a field interact. Given the location s_j of w_j, the interaction with w_i is measured by $f_i(s_j)$. As mentioned above, polysemy means that w_j has multiple locations, $s_j^{(k)}, k = 1, \ldots, K$; thus, the $f_i(s_j^{(k)})$ must be added, or more simply, integrated. The location here means that an entity is scattered in a field: it can be multiple and noncontinuous, and it may have different mass at different locations. This notion is mathematically expressed as a *measure*, and an integral with a measure is formulated via the Lebesgue integral.

The simplest measure is the Dirac delta function $\delta(s)$, which indicates that a location is point-wise within a field. For $K = 1$ and $\mu_j \equiv \delta(s_j)$, we have $\int f_i \mathrm{d}\mu_j = f_i(s_j)$. By using the Dirac delta function, we can compute the integral exactly without any approximation.

As we also have the symmetric quantity of w_i's location in the field of $f_j(s)$, the similarity of the two words is naturally expressed as follows:

$$\mathrm{sim}(w_i, w_j) \equiv \int f_i \, \mathrm{d}\mu_j + \int f_j \, \mathrm{d}\mu_i \quad \forall w_i, w_j \in W. \tag{1}$$

Using a measure for a word's location facilitates a simple representation. For $K \geq 1$, μ is a Dirac mixture with weights $m^{(k)}$ as follows:

$$\mu \equiv \sum_{k=1}^{K} m^{(k)} \delta(s^{(k)}). \tag{2}$$

By using this μ for each word, Formula (1) reduces to the following (because $\int f \mathrm{d}\delta(s) = f(s)$):

$$\mathrm{sim}(w_i, w_j) = \sum_{k=1}^{K} m_j^{(k)} f_i(s_j^{(k)}) + \sum_{k=1}^{K} m_i^{(k)} f_j(s_i^{(k)}), \tag{3}$$

which is simply a weighted sum of the two potential functions $f_i(s)$ and $f_j(s)$ at all the locations of w_j and w_i, respectively.

Among various measure possibilities, this paper uses the Dirac delta function, because this formulation enables an extension to define sentence representations in Section 3.3. The similarity in this work between words and sentences via a measure is different from the standard notion of distance in a Euclidean space and is related to metric learning methods [9].

FIRE incorporates polysemy via (1) the multimodality of $f(s)$ and (2) the number K of word locations. Note that the multimodality of $f(s)$ is implemented via a neural network.

3.3 Composition of Words and Similarity of Two Sentences

A set of n words, $\Gamma = [w_1, \ldots, w_n]$, including sentences, can also be represented in $[\mu, f(s)]$ notation when defined as a weighted average of word representations $\{[\mu_1, f_1(s)], \ldots, [\mu_n, f_n(s)]\}$, as follows:

$$\mu = \sum_{i=1}^{n} \gamma_i \mu_i, \quad f(s) = \sum_{i=1}^{n} \gamma_i f_i(s) \quad \forall s \in \mathcal{S}, \tag{4}$$

where γ_i are the weights. In this work, we used the SIF weights [1] as the γ_i. Note that this $f(s)$ is multimodal and thus polysemous, because all the $f_i(s)$ are multimodal.

Figure 1 (right) shows such a superposition of the words *river* (blue) and *mortgage* (red). Here, *river* is at the lower left, with $f_{\text{river}}(s)$ forming a diagonal river-like flow from upper right to lower left, whereas *mortgage* is at the upper left, with $f_{\text{mortgage}}(s)$ a reddish area. The color intensities and arrows represent the potential values and gradients, respectively. The field superposing *river* and *mortgage* is similar to the representation of *bank* on the left, demonstrating the effect of linear composition of FIRE.

The similarity of two sets of words or sentences, $\Gamma = [\mu, f]$ and $\Gamma' = [\mu', f']$, is measured by applying the similarity function (1). Let n and n' denote the respective numbers of words in the two sentences. The words in Γ and Γ' are assigned weights $\gamma = [\gamma_1, \ldots, \gamma_n]^{\mathrm{T}}$ and $\gamma' = [\gamma'_1, \ldots, \gamma'_{n'}]^{\mathrm{T}}$, respectively. Then, as shown in Appendix B, Formula (1) is simply calculated as follows:

$$\mathrm{sim}(\Gamma, \Gamma') = \gamma^{\mathrm{T}} \Sigma \gamma', \tag{5}$$

where Σ is the word similarity matrix, defined as $\Sigma_{ij} = \mathrm{sim}(w_i, w_j)$ with w_i and w_j being words in sentences Γ and Γ', respectively. In other words, the similarity between two sentences can be obtained from the pairwise word similarities between the sentences.

4 Implementation of FIRE

4.1 Potential Functions As Planar Transformations

The simplest way to implement the potential function $f(s)$ would be to construct a neural network NN with a D-dimensional input and 1-dimensional output, i.e., $f(s) = \mathrm{NN}(s; \theta)$, where θ represents the parameters of the neural network. A usual choice for the NN is a *multi-layer perceptron* (MLP).

A more efficient way to produce the potential function would be to use the Jacobian of such an NN: $\mathcal{S} \to \mathcal{S}$. The trace of the Jacobian gives the *divergence* at s of the vector field generated by the NN, which is a scalar in $\mathbb{R}$:

$$f(s) \equiv \mathrm{tr} \frac{\partial \mathrm{NN}(s; \theta)}{\partial s}.$$

As the Jacobian $\partial \mathrm{NN}(s; \theta)/\partial s$ can be decomposed into multiplication of the Jacobians of all the NN's layers, it can be computed efficiently. In fact, it was empirically found to work better than the previous simplest MLP.

For our implementation, we adopt a negative multi-layer planar (MLPlanar) transformation. Each layer is a planar transformation [20] of two vectors v and u with a scalar b, as follows:

$$\text{Planar}(x;\theta) = x + \tanh(v^{\mathrm{T}}x + b)u,$$

where $\theta = \{v, b, u\}$ is the parameter set. The Jacobian of the l-th layer with input $s_{(l)}$ is

$$\frac{\partial \text{Planar}(s_{(l)})}{\partial s_{(l)}} = I + A_{(l)},$$

where $A_{(l)} = \tanh'(v_{(l)}^{\mathrm{T}} s_{(l)} + b_{(l)}) \cdot u_{(l)} v_{(l)}^{\mathrm{T}}$ is a rank-one matrix, with the derivative $\tanh' = 1 - \tanh^2$. Thus,

$$f(s) \equiv \mathrm{tr}\frac{-\partial \text{Planar}(s;\theta)}{\partial s} = -\mathrm{tr}\prod_{l=1}^{L}\big(I + A_{(l)}\big), \tag{6}$$

where $s_{(l)} = \text{Planar}(s_{(l-1)})$ and $s_{(0)} = s$. Formula (6) uses the negative trace, so that words are located at "peaks" as in Figure 1 after training, rather than on the "plain"; this enables FIRE to discover polysemy. Formula (6) expands to a matrix polynomial of degree L, which can represent the multimodality of $f(s)$. For the above case with $D = 2$, evaluating $f(s)$ takes $O(L)$ time.

4.2 FIRE Training by Skip-Gram with Negative Sampling

For training FIRE, we adopt the *Skip-gram* algorithm with negative sampling (SGNS) [14], an unsupervised method. More advanced training methods (e.g., *GloVe*) can also be applied, which remains a future work.

In SGNS (one negative word per time), a three-word tuple (w_i, w_p, w_n) is sampled from a corpus. Here, w_i is called the *center* word, w_p is a *positive* word that co-occurs with the center word somewhere in the corpus, and w_n is an arbitrary word, called the *negative* word. We use only one negative word per positive, following the suggestion in [10].

The idea of SGNS is to optimize the parameters in word representations such that $\mathrm{sim}(w_i, w_p)$ is high and $\mathrm{sim}(w_i, w_n)$ is low, following the assumption that words co-occurring in the same neighborhood are more similar. For example, the words *bank* and *mortgage* should have a high similarity because they co-occur frequently.

Following the implementation in [13], the optimization function with respect to similarity is:

$$\min \sum_{w_i, w_p, w_n} \sigma(-\mathrm{sim}(w_i, w_p)) + \sigma(\mathrm{sim}(w_i, w_n)), \tag{7}$$

where $\sigma(x) = \log(1 + \exp(x))$ is the *softplus* function. For $D = 2$, evaluation of the similarity in Formula (3) between two words has $O(KL)$ time complexity.

Note that Formula (7) is *not* the *triplet loss* in metric learning. A triplet loss is typically used to evaluate a large neural network shared by all words, whereas in FIRE, each word is represented as an individual small neural network defined by Formula (6).

4.3 Experimental Settings for Training

We obtained FIRE representations on two datasets: the large *Wacky* [3] dataset and the small *text8* dataset, which contain about 3 billion and 17 million tokens, respectively.

For both datasets, we converted the text to lowercase and tokenized it with the NLTK toolkit [4]. We followed the settings used in previous works and replaced infrequent words with `<unk>`. For *Wacky*, infrequent words were those occurring less than 100 times, forming a vocabulary of around 270,000 words; for *text8*, the threshold was 5 times, forming a vocabulary of about 70,000 words.

As mentioned in Section 3.1, we used a 2-dimensional space $\mathcal{S} = [-4, 4]^2$.

For the SGNS hyperparameters, we followed the settings recommended by [13]. We used one negative sample per sample, and the *subsample* rate for w_i was 1e-5 for *Wacky* and 1e-4 for *text8*. The negative sampling probability was adjusted by a power of 0.75. For training FIRE representations,

Table 1: Qualitative comparison of word representation methods. The second column lists the number of parameters, and the remaining columns indicate qualities of the methods.

method	N (# of par.)	Small D	Composi-tionality	Polysemy	Interpretable Polysemy	Complexity $\mathrm{sim}(w_1, w_2)$
			Vectoral representation			
Word2Vec[14]	D	×	√	×	×	$\mathcal{O}(N)$
			Random-variable representations			
Word2Gauss/S[27]	$D+1$	×	×	√	×	$\mathcal{O}(N)$
Word2Gauss/D[27]	$2D$	×	×	√	×	$\mathcal{O}(N)$
Word2GM/S[2]	$(D+2)K$	√	×	√	×	$\mathcal{O}(KN)$
Word2GM/D[2]	$(2D+1)K$	√	×	√	×	$\mathcal{O}(KN)$
Word2Cloud[8]	$K=64$	√	×	√	×	$\mathcal{O}(N^2)$
CMD[21]	D	×	√	√	×	$O(N^2)$
			Contextual vectoral representation			
BERT-large[6]	$D=1024$	×	√	√	×	high
			Our semantic-field representations			
FIRE/m	$(2D+1)L+DK$	√	√	√	√	$\mathcal{O}(LN)$
FIRE	$(2D+1)L+(D+1)K$	√	√	√	√	$\mathcal{O}(LN)$

we adopted the *AdamW* optimizer [11]. The *OneCycle* learning rate scheduler [22] was applied, with the maximum learning rate set to 0.005.

As noted above, the time complexity for training is $O(KL)$. For the *Wacky* dataset, all methods were trained for three epochs, which took about 10 hours for FIRE with 50 parameters on a single GPU. For *text8*, FIRE was trained for 15 epochs.

In similarity benchmarks, conventional vectoral representations use the cosine similarity function, where $\mathrm{sim}(w_i, w_i) = 1\ \forall i$, to provide a natural regularization. As FIRE also requires subtracting regularization terms we used the following for word similarity in our evaluation for fair comparison:

$$\mathrm{sim}(w_i, w_j) \leftarrow \mathrm{sim}(w_i, w_j) - \int f_i \, \mathrm{d}\mu_i - \int f_j \, \mathrm{d}\mu_j. \tag{8}$$

Substituting Σ in Formula (5) with the regularized version of word similarity gives the regularized sentence similarity. Because our similarity function is point-wise, smoothing is also required. Appendix C details this process.

5 Baselines

We evaluated FIRE in comparison with previous semantic representations, as summarized in Table 1. For fair comparison of such different representations, the most important factor is the number of parameters (2nd column). For FIRE (bottom of the table), μ requires K locations with D dimensions, plus the number of weights $m^{(k)}$, i.e., K, giving $(D+1)K$. As for $f(s)$, v and u (Section 4.1) are D-dimensional, are implemented as L layers, and have L intercepts b, giving $(2D+1)L$. The total is thus $(2D+1)L + (D+1)K$ parameters per word. The third column shows whether the method is effective even with small D, where the small value $D = 2$ facilitates visualization.

The 4-5th columns in Table 1 indicate whether a method has the qualities of additive compositionality and polysemy; FIRE is the only non-contextual method that theoretically accommodates both.

For an ablation study, we considered a variant of FIRE, denoted as **FIRE/m**, that fixes $m_i^{(k)} = 1$ in Formulas (2) and (3). **FIRE/m** shares the properties of FIRE, but with fewer parameters per word.

We adopted the baselines listed in Table 1 in this work, as explained in Appendix D. Briefly, Word2Gauss and Word2GM represent words as Gaussian mixtures, whereas Word2Cloud represents words as a multinomial distribution of K discrete points in a low-dimensional space. /S for Word2Gauss and Word2GM indicates a *spherical* covariance matrix rI requiring one parameter (i.e., r), whereas Word2Gauss/D uses a *diagonal* covariance matrix $\mathrm{diag}(r_1, \cdots, r_D)$ requiring D parameters. For Word2Cloud, we refer to the results reported in the original paper, because the original code is not available. As Word2Cloud was only implemented on the relatively small *text8*

Table 2: Spearman's correlation coefficient (%) between the similarity rankings produced by the various word representations and the gold standard. The results are grouped by methods with the same number of parameters. The best and second-best results in each group are in bold and underlined, respectively. The results for the previous methods were obtained by replicating them with the published code, except for Word2Cloud in the bottom block, for which a method without code was taken from the original paper [8].

dataset # word pairs					MC 30	MEN 3000	RG 65	WS-s 203	WS-r 252	MT 287	MT 771	RW 2034	Verb 143	YP 130	SL 999	SV 3500	Ave.
	#par.	D	L	K													
Dataset: Wacky (3B tokens)																	
Word2Vec	2	2	-	-	1.6	19.4	22.2	16.1	9.3	26.6	17.4	12.6	-2.2	12.0	-0.2	5.5	11.7
Word2Vec	40	40	-	-	67.8	**69.1**	66.1	**70.1**	**52.1**	**65.0**	**60.7**	**27.8**	32.6	**39.6**	18.4	19.3	**49.0**
FIRE/m (ours)		2	4	10	**72.0**	65.2	**71.9**	64.6	47.7	63.5	60.1	25.9	**32.9**	31.8	**20.1**	**19.8**	47.9
Word2Vec	50	50	-	-	**72.7**	**70.7**	69.2	**71.8**	**54.2**	**65.0**	62.1	27.5	30.2	41.4	19.7	**20.3**	**50.4**
GloVe		50	-	-	66.8	68.7	73.4	61.3	43.2	58.6	**62.2**	**28.0**	34.3	42.1	19.4	15.8	47.8
Word2Gauss/D		25	-	-	51.7	50.2	65.5	51.1	48.0	56.9	46.0	23.3	8.8	23.9	26.0	16.4	39.0
Word2Gauss/S		49	-	-	63.7	53.7	69.8	54.3	48.6	57.4	48.9	25.9	19.5	32.8	**26.9**	18.6	43.3
Word2GM/S		23	-	2	58.4	57.7	63.7	58.8	37.4	51.0	53.6	24.1	10.0	28.3	16.0	18.4	39.8
FIRE (ours)		2	4	10	71.8	64.9	**74.2**	67.5	52.1	63.0	59.2	25.8	**34.5**	**45.5**	18.7	19.8	49.8
Word2Vec	100	100	-	-	71.4	**73.6**	75.2	**74.2**	**57.7**	**66.4**	**66.3**	29.1	**39.8**	47.1	21.5	**22.9**	**53.8**
GloVe		100	-	-	69.6	72.3	**77.0**	65.2	49.0	61.9	65.8	30.2	35.0	**49.2**	23.3	18.5	51.4
Word2Gauss/D		50	-	-	68.8	55.8	70.0	55.3	48.3	61.5	50.8	25.6	9.1	37.4	**27.9**	18.5	44.1
Word2Gauss/S		99	-	-	64.4	55.6	71.0	53.2	48.0	60.0	50.3	27.0	15.2	34.8	27.4	19.3	43.9
Word2GM/S		48	-	2	63.1	60.7	68.6	61.0	45.5	56.4	57.5	25.7	23.1	36.8	18.6	20.1	44.8
FIRE (ours)		2	8	20	**75.9**	69.4	76.4	70.4	54.4	62.7	61.1	27.2	33.7	38.9	20.0	21.0	50.9
Word2GM/S[b]	104	50	-	2	79.1	73.6	74.5	75.5	59.5	66.6	60.8	28.6	37.4	45.1	19.7	22.6	53.6
Dataset: text8 (17M tokens)																	
Word2Cloud[a]	64	2	-	-	4.0	25.0	32.0	25.0	5.0	40.0	11.0	6.0	3.0	-	9.0	-	-
FIRE (ours)	50	2	4	10	42.6	43.2	44.8	52.4	38.3	**55.6**	43.0	12.3	36.5	22.8	**14.3**	**9.4**	34.6
FIRE (ours)	64	2	8	8	**56.2**	**46.3**	**50.6**	**56.9**	**44.3**	54.8	**45.7**	**15.8**	**36.8**	**23.5**	14.2	8.7	**37.8**

[a] The results for *Word2Cloud* were taken from the original paper.
[b] The result (#par=104) was acquired with the best pretrained model available at *https://github.com/benathi/word2gm*.

dataset, for comparison, we constructed FIRE models with the same number of parameters and trained them on the *text8* dataset.

6 Evaluation on Basic Word Semantics

First, FIRE was compared with the baselines on 12 word similarity benchmarks to test the basic word semantic representational capability. As these benchmarks are for non-contextual methods, BERT-large was not included. For each benchmark, we computed the similarity of a word to other words and generated a ranking of those words. The gold standard was compared via Spearman's correlation coefficient ρ, with a higher ρ indicating a better method.

The results are listed in Table 2. Each block groups methods with the same number of parameters. Generally speaking, our method outperformed most of the baselines, even the high-dimensional Word2Vec. For either #par=50 or 100, FIRE was the only method whose performance was comparable to that of the vectoral methods. For example, it achieved an average ρ of 47.9% and 49.8%, respectively, for #par=40 and 50; these ρ values were only 1.1% and 0.6% less than those of Word2Vec. These results shows that our potential functions and measures have sufficiently large capacity even with a semantic field as small as $[-4, 4]^2$. In Appendix E, we included further experimental results about K's effect. Briefly, our method is computationally efficient even for a larger K, because the time complexity of FIRE is $O(K)$, rather than $O(K^2)$ as in Word2GM and Word2Cloud.

Another perspective is to specifically compare the low-dimensional representations, i.e., those with small D. Word2Cloud is such a representation and is comparable to FIRE with $D = 2$. Although Word2Cloud was computed on the smaller *text8* dataset, it produced better similarity scores than the 2-dimensional Word2Vec listed in the top row of Table 2. When trained on *text8*, however, FIRE greatly outperformed Word2Cloud, even with a smaller number of parameters.

7 Evaluation of Compositionality via Sentence Similarity Benchmark

FIRE's linear compositional capability was evaluated using a sentence similarity benchmark. A sentence representation is produced by composing words in conventional vectoral word representations, making full use of linear compositionality. Similarly, FIRE represents a sentence by composing word representations as described in Section 3.3. The similarity of two sentences is given by Formula (5) or (8), with respect to the SIF weights [1]. We set $\alpha = 10^{-3}$ for SIF.

Table 3: Performance on the WiC and STS (2012-2016) benchmarks. See Appendix H for the detailed STS results.

	N	total par.	WiC	STS
Word2Vec	50	13M	61.8	59.7
	100	27M	62.0	61.9
FIRE	50	13M	61.9	57.5
	100	27M	62.7	61.1
BERT-large	1024	340M	66.5	63.3

We compared FIRE with vectoral word representations on the Word-in-Context (WiC)[19] and STS (2012-2016) benchmarks. WiC is a binary classification task asking whether a word's meaning differs in two given context sentences. The semantic similarity scores between sentence pairs were computed, with scores below a threshold indicating that the word's meaning differs in the two sentences. STS evaluates sentence similarity by the Pearson correlation coefficient between the predicted similarity and the gold standard.

Table 3 lists the results. Note again that Word2Vec and FIRE are non-contextual, whereas BERT-large is contextual. The second column gives the model's total number of parameters. The Word2Vec and FIRE both had 50 parameters per word. The third gives the accuracy scores.

BERT-large, a contextual method, has 30 times more parameters than the non-contextual Word2Vec and FIRE, and its accuracy was higher because of the context stored in this large memory. Between the non-contextual methods, FIRE performed better than Word2Vec. BERT's accuracy was naturally the best because it has context; more importantly, this result demonstrates that FIRE's compositionality is comparable to Word2Vec.

8 Evaluation of Polysemy

The quality of FIRE lies in its capability to capture polysemy. Different meanings (i.e., senses) of a word are automatically represented as separate modes of a nonlinear function, as seen for *bank* in Figure 1 with finance- and geography-related modes. Appendix A gives more examples. Words are assigned colors according to the clustering results in $\mathcal{S} = [-4, 4]^2$, by the DBSCAN [7] algorithm, which automatically detects the number of clusters via a threshold parameter eps $= 0.5$.

We quantitatively evaluated the polysemy of FIRE via the task of *word sense detection*, where the number of word senses is predicted by each method and compared with a gold standard via *WordNet* [15]. From the *Core WordNet* dataset,[1] with 5000 frequent senses of 3739 different words, we selected words annotated with at least three senses (266 words that are strongly polysemous). From Core WordNet and the whole WordNet, we also selected 276 words that are strongly monosemous. We thus used 542 words, with various numbers of word senses. The selected words are listed in Supplementary Materials.

FIRE was compared with four previous word representations, Word2Vec, Word2Gauss, Word2GM, and BERT. All methods except BERT are non-contextual. With these non-contextual methods including FIRE, for each target word, the most similar 1,000 words were acquired from the most frequent 50,000 words, and their locations were plotted in the 2D plane. For Word2Gauss and Word2GM, we used the mean vectors of the Gaussian distributions. See Appendix I for the details. In Figure 2 (left), we showed the results of three words: "bank" and "rock" are polysemous, while "glacier" is monosemous.

In contrast, BERT is a contextual model requiring context to count the number of senses. For each target word, 1,000 context sentences were collected from the Wacky dataset, and corresponding BERT vectors for the target word were generated. For a target word that was splitted into subwords by the BERT's default tokenizer, we took average of the subword vectors to represent the target word. The rightmost column of Figure 2 (left) shows BERT's results for the four sample words.

[1] `http://wordnetcode.princeton.edu/standoff-files/core-wordnet.txt`

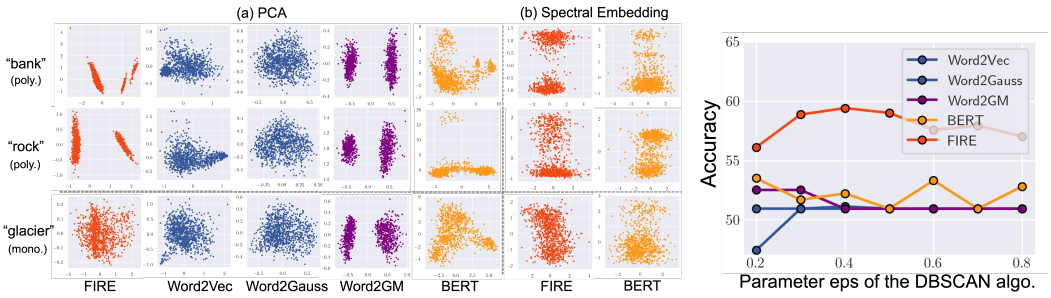

Figure 2: (This figure was redesigned following the reviewers' comments.) **Left:** Geometric distributions of word embeddings computed with five methods for the polysemous words *bank* (top row) and *rock* (middle row), and the monosemous *glacier* (bottom row). **Right:** Accuracy of polysemous/monosemous detection with respect to a threshold parameter.

In plotting the Word2X methods, FIRE, and BERT on the 2D plane, we tested two dimension reduction methods of Principal Component Analysis and Spectral Embeddings to acquire 2D points from the vectors for BERT (1024-d) and the other methods. For the details, see Appendix I.

Figure 2 clearly shows how well FIRE captured the different senses. The term "bank" produced three senses, with those to the right corresponding to a river bank and "a large pile" as listed in many dictionaries. On the other hand, such clusters for Word2Vec and BERT are difficult to see. As Word2Vec is monosemous, it is not expected to produce any clusters, whereas clear clusters are barely visible for BERT. Although BERT performed very well in contextual processing, this shows how context and polysemy are different. Word2GM uses two Gaussian components for each word and is biased to show two clusters, even for the monosemous word "glacier."

For all 542 words, the number of clusters t was counted with DBSCAN with different threshold *eps*: it was applied to to PCA-reduced 2-d planes, which produced a better result for BERT than applying DBSCAN directly to the original high-dimensional spaces. The number of clusters is compared with the gold standard. As the gold standard consisted of 49% (=266/542) polysemous and 51% monosemous words, the result was judged accurate by $t = 1$ for monosemous words and $t > 1$ for polysemous words.

Figure 2 (right) shows the results for the five methods, with the horizontal and vertical axes indicating *eps* and the accuracy, respectively. Word2Vec, Word2Gauss, and Word2GM performed at chance level for any *eps* value. BERT was slightly better but could not detect the number of senses. In contrast, FIRE showed a clear capability to identify polysemy. Appendix F gives a further experimental result on detecting the number of word senses.

9 Conclusion

We proposed FIRE, a semantic field representation that is defined on a D-dimensional space, with a field of word interactions. A word is represented by a pair that consists of a set of multiple locations for the word and a function representing the word's context, where the latter is implemented as a semantic potential function covering the field. The interaction of words is measured via a similarity function, which is naturally defined as the sum of the potential functions at the word locations. The multiple possible word locations and the potential function's multimodality account for the quality of polysemy, and the functional addition formally guarantees the quality of additive compositionality.

FIRE is implemented via a planar transformation of a neural network. Multiple images of a 2D field were shown to demonstrate the capability of polysemy representation. FIRE was also evaluated through a large-scale analysis of word and sentence similarity benchmarks, in which it showed comparable or better results than conventional Word2XX representations. Moreover, FIRE greatly outperformed BERT and Word2Vec in evaluating polysemy by predicting the number of word senses. These results provide evidence of FIRE's capability to incorporate polysemy as a nonlinear quality with compositionality as a linear quality.

Our future work will entail extension of this framework to a more general representation, which will further improve the capability of FIRE.

References

[1] Sanjeev Arora, Yingyu Liang, and Tengyu Ma. A simple but tough-to-beat baseline for sentence embeddings. In *5th International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Conference Track Proceedings*. OpenReview.net, 2017.

[2] Ben Athiwaratkun and Andrew Wilson. Multimodal word distributions. In *Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 1645–1656, Vancouver, Canada, July 2017. Association for Computational Linguistics.

[3] Marco Baroni, Silvia Bernardini, Adriano Ferraresi, and Eros Zanchetta. The wacky wide web: a collection of very large linguistically processed web-crawled corpora. *Language resources and evaluation*, 43(3):209–226, 2009.

[4] Steven Bird, Ewan Klein, and Edward Loper. *Natural language processing with Python: analyzing text with the natural language toolkit*. " O'Reilly Media, Inc.", 2009.

[5] Piotr Bojanowski, Edouard Grave, Armand Joulin, and Tomas Mikolov. Enriching word vectors with subword information. *arXiv preprint arXiv:1607.04606*, 2016.

[6] Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. BERT: Pre-training of deep bidirectional transformers for language understanding. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*, pages 4171–4186, Minneapolis, Minnesota, June 2019. Association for Computational Linguistics.

[7] Martin Ester, Hans-Peter Kriegel, Jörg Sander, Xiaowei Xu, et al. A density-based algorithm for discovering clusters in large spatial databases with noise. In *kdd*, volume 96, pages 226–231, 1996.

[8] Charlie Frogner, Farzaneh Mirzazadeh, and Justin Solomon. Learning embeddings into entropic wasserstein spaces. *arXiv preprint arXiv:1905.03329*, 2019.

[9] Mahmut Kaya and Hasan Şakir Bilge. Deep metric learning: A survey. *Symmetry*, 11(9):1066, 2019.

[10] Omer Levy and Yoav Goldberg. Neural word embedding as implicit matrix factorization. *Advances in neural information processing systems*, 27:2177–2185, 2014.

[11] Ilya Loshchilov and Frank Hutter. Decoupled weight decay regularization. In *International Conference on Learning Representations*, 2018.

[12] Christopher Manning and Hinrich Schutze. *Foundations of statistical natural language processing*. MIT press, 1999.

[13] Tomás Mikolov, Kai Chen, Greg Corrado, and Jeffrey Dean. Efficient estimation of word representations in vector space. In Yoshua Bengio and Yann LeCun, editors, *1st International Conference on Learning Representations, ICLR 2013, Scottsdale, Arizona, USA, May 2-4, 2013, Workshop Track Proceedings*, 2013.

[14] Tomás Mikolov, Ilya Sutskever, Kai Chen, Gregory S. Corrado, and Jeffrey Dean. Distributed representations of words and phrases and their compositionality. In Christopher J. C. Burges, Léon Bottou, Zoubin Ghahramani, and Kilian Q. Weinberger, editors, *Advances in Neural Information Processing Systems 26: 27th Annual Conference on Neural Information Processing Systems 2013. Proceedings of a meeting held December 5-8, 2013, Lake Tahoe, Nevada, United States*, pages 3111–3119, 2013.

[15] George A Miller. Wordnet: a lexical database for english. *Communications of the ACM*, 38(11):39–41, 1995.

[16] Boris Muzellec and Marco Cuturi. Generalizing point embeddings using the wasserstein space of elliptical distributions. In *Proceedings of the 32nd International Conference on Neural Information Processing Systems*, pages 10258–10269, 2018.

[17] Jeffrey Pennington, Richard Socher, and Christopher D. Manning. Glove: Global vectors for word representation. In Alessandro Moschitti, Bo Pang, and Walter Daelemans, editors, *Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing, EMNLP 2014, October 25-29, 2014, Doha, Qatar, A meeting of SIGDAT, a Special Interest Group of the ACL*, pages 1532–1543. ACL, 2014.

[18] Matthew E. Peters, Mark Neumann, Mohit Iyyer, Matt Gardner, Christopher Clark, Kenton Lee, and Luke Zettlemoyer. Deep contextualized word representations. In *Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long Papers)*, pages 2227–2237, New Orleans, Louisiana, June 2018. Association for Computational Linguistics.

[19] Mohammad Taher Pilehvar and Jose Camacho-Collados. Wic: the word-in-context dataset for evaluating context-sensitive meaning representations. *arXiv preprint arXiv:1808.09121*, 2018.

[20] Danilo Rezende and Shakir Mohamed. Variational inference with normalizing flows. In *International conference on machine learning*, pages 1530–1538. PMLR, 2015.

[21] Sidak Pal Singh, Andreas Hug, Aymeric Dieuleveut, and Martin Jaggi. Context mover's distance & barycenters: Optimal transport of contexts for building representations. In *International Conference on Artificial Intelligence and Statistics*, pages 3437–3449. PMLR, 2020.

[22] Leslie N Smith and Nicholay Topin. Super-convergence: Very fast training of neural networks using large learning rates. In *Artificial Intelligence and Machine Learning for Multi-Domain Operations Applications*, volume 11006, page 1100612. International Society for Optics and Photonics, 2019.

[23] Chi Sun, Hang Yan, Xipeng Qiu, and Xuanjing Huang. Gaussian word embedding with a wasserstein distance loss. *ArXiv*, abs/1808.07016, 2018.

[24] Fei Tian, Hanjun Dai, Jiang Bian, Bin Gao, Rui Zhang, Enhong Chen, and Tie-Yan Liu. A probabilistic model for learning multi-prototype word embeddings. In *Proceedings of COLING 2014, the 25th International Conference on Computational Linguistics: Technical Papers*, pages 151–160, 2014.

[25] Ran Tian, Naoaki Okazaki, and Kentaro Inui. The mechanism of additive composition. *Machine Learning*, 106(7):1083–1130, 2017.

[26] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. In *Advances in neural information processing systems*, pages 5998–6008, 2017.

[27] Luke Vilnis and Andrew McCallum. Word representations via gaussian embedding. *arXiv preprint arXiv:1412.6623*, 2014.

A Further Examples of Polysemy Discovery

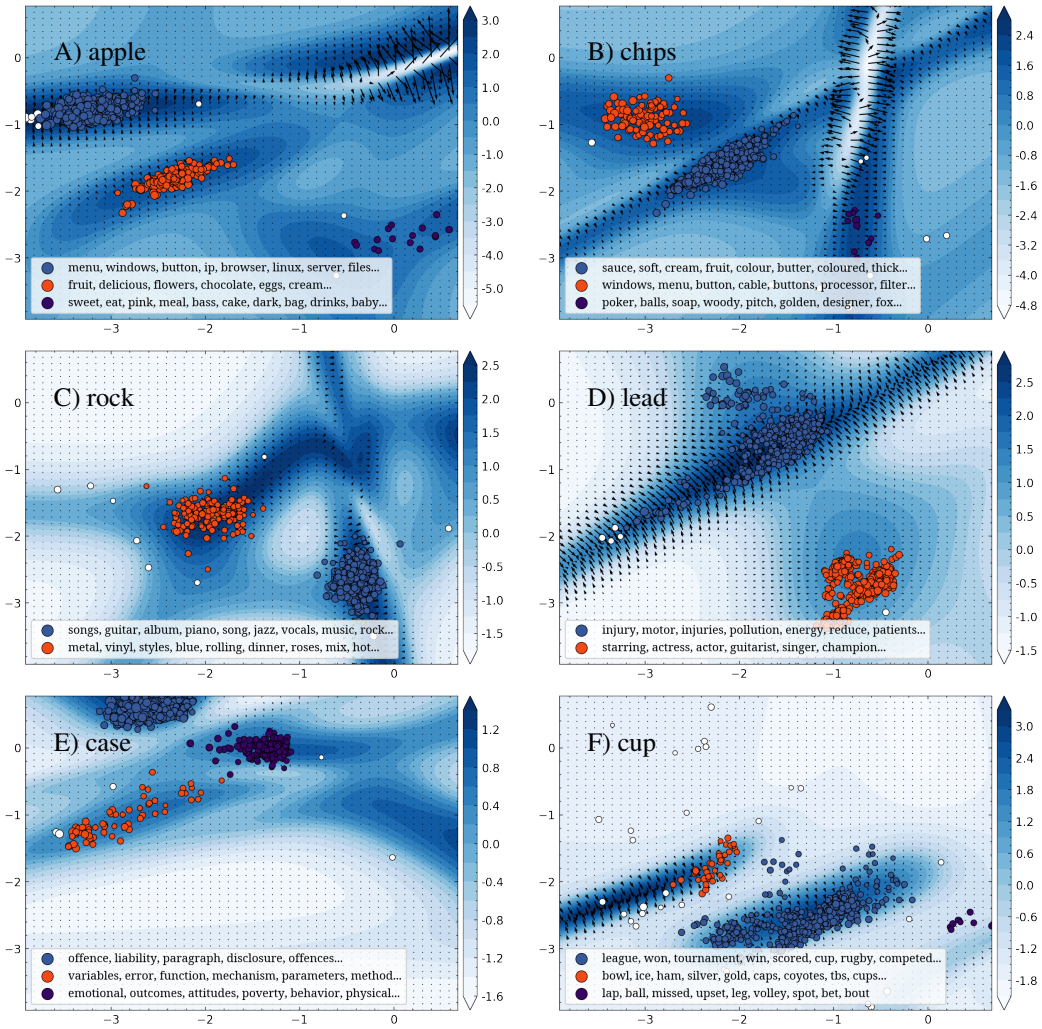

Figure 3: Semantic fields of (A) *apple*, (B) *chips*, (C) *rock*, (D) *lead*, (E) *case*, and (F) *cup*, with the most similar words shown at the bottom of each graph. The words were clustered, with corresponding colors for each group, by using the DBSCAN algorithm [7], which is based on the Euclidean distance. The white circles represent words that the algorithm evaluated as outliers.

B Derivation of Sentence Similarity

Formula (5) is derived as follows:

$$
\begin{aligned}
\mathrm{sim}(\Gamma, \Gamma') &= \int \Big(\sum_{i=1}^{n} \gamma_i f_i\Big)\, \mathrm{d}\Big(\sum_{j=1}^{n'} \gamma'_j \mu'_j\Big) + \int \Big(\sum_{j=1}^{n'} \gamma'_j f'_j\Big)\, \mathrm{d}\Big(\sum_{i=1}^{n} \gamma_i \mu_i\Big) \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n'} \gamma_i \gamma'_j \left(\int f_i \, \mathrm{d}\mu'_j + \int f'_j \, \mathrm{d}\mu_i \right) \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n'} \gamma_i \gamma'_j \mathrm{sim}(w_i, w_j) \qquad (9) \\
&= \gamma^{\mathrm{T}} \Sigma \gamma'. \qquad (10)
\end{aligned}
$$

C Smoothing of Similarity Scores by Standardization

One way to improve our framework is to standardize the similarity function. Formally, for a word w_i, we apply the following transformation:

$$\overline{\mathrm{sim}}(w_i, w_j) \equiv \frac{\mathrm{sim}(w_i, w_j) - \left(\mathrm{sim}(w_i, \bar{w}) + \mathrm{sim}(\bar{w}, w_j)\right)/2}{\left(\mathrm{Var}\big[\mathrm{sim}(w_i, w)\big]\,\mathrm{Var}\big[\mathrm{sim}(w, w_j)\big]\right)^{0.25}}, \tag{11}$$

where the mean is defined as

$$\bar{w} = \frac{1}{|W|}\sum_{w \in W} w = \left[\frac{1}{W}\sum_{w \in W}\mu_w, \frac{1}{|W|}\sum_{w \in W} f_w\right], \tag{12}$$

the variance is defined as

$$\mathrm{Var}\big[\mathrm{sim}(w_i, w)\big] = \frac{1}{|W|-1}\sum_{w \in W}\mathrm{sim}(w_i, w - \bar{w})^2, \tag{13}$$

and $\mathrm{Var}\big[\mathrm{sim}(w, w_j)\big]$ is defined similarly.

Such standardization works because of our measure's point-wise nature, such that the similarity function in Formula (1) is optimized only at a point. In other words, f is optimized very locally, rather than globally in $\mathcal{S}$. Standardization of the similarity function serves to provide regularization against overfitting. It is executed after word representations are obtained, and thus, it does not affect the training process.

A typical use of this standardization is for a word similarity benchmark, as in this work. Such a benchmark usually contains hundreds of word pairs, which form a set of words. We thus define W as this set, and we apply the above standardization to the word representations with respect to W.

D Summary of Baseline Methods

Word2Vec[13] [2] A word is represented by a vector, trained via the Skip-gram algorithm.

GloVe[17] [3] This is a vectoral method similar to Word2Vec, but it uses a different training objective.

Word2Gauss [27][4] A word is represented by a Gaussian random variable, parameterized by a mean vector and a covariance matrix. There are two variants of Word2Gauss: Word2Gauss/S uses a *spherical* covariance matrix rI that requires one parameter (i.e., r), whereas Word2Gauss/D uses a *diagonal* covariance matrix $\mathrm{diag}(r_1, \cdots, r_D)$ that requires D parameters.

Word2GM [2][5] A word is represented as a weighted mixture of K Gaussian distributions, each of which is parameterized as in **Word2Gauss**. Word2GM also has two variants: Word2GM/S uses a spherical covariance matrix, and Word2GM/D uses a diagonal covariance matrix.

The original paper recommended setting $K = 2$ and using a spherical covariance matrix. Accordingly, we compared that approach with FIRE here.

Word2Cloud [8]. A word is represented by a multinomial distribution of K discrete points in a low-dimensional space (e.g., $\mathbb{R}^2$). Word2Cloud also has K components. The similarity between two words is evaluated via the Wasserstein distance, which has at least $O(K^2)$ time complexity even with a fast approximation algorithm. The original paper used $K = 64$.

BERT-large [6] The *large* version of BERT is composed of 24 Transformer layers that are stacked on vectoral word representations. In each Transformer layer, words interact with other words to produce new vectors that contain contextual information. The vectoral output for a word is called a contextualized word vector, which was tested in Section 8.

[2] `https://github.com/dav/word2vec`.

[3] `https://github.com/stanfordnlp/GloVe`.

[4] We used the code for Word2GM and limited it to $K = 1$, because this produced better results than the original code for Word2Gauss at `https://github.com/seomoz/word2gauss`.

[5] `https://github.com/benathi/word2gm`.

E Supplementary Evaluation Results for Word Similarity

In FIRE, K is the number of locations for each word and is an important factor in polysemy. In comparison, Word2GM also implements polysemy, via mixtures. As seen in Table 1 from the rows for Word2GM and FIRE, the mixture-based approach performed well even with small D.

From this perspective, we compared Word2GM and FIRE/m in terms of K, under the constraint of $D = 2$. Here, we used FIRE/m for fair comparison, so that we could construct models with a similar number of parameters to Word2GM. The number of NN layers, L, was also set so that both models would have roughly equal numbers of parameters, as listed in the 2nd to 5th columns of Table 4.

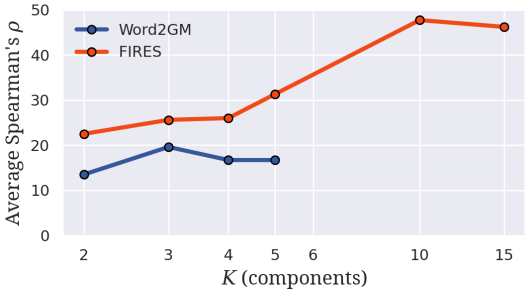

Figure 4: Average Spearman's ρ for Word2GM and FIRE with respect to different K for $D = 2$, with the two methods constrained to have (roughly) the same numbers of parameters for each K. Because of its high computational cost, we limited Word2GM to $K \leq 5$.

Table 4: Spearman's correlation coefficient (%) between the similarity rankings produced by the word representations and the gold standard.

task # word pairs					MC 30	MEN 3000	RG 65	WS-s 203	WS-r 252	MT 287	MT 771	RW 2034	Verb 143	YP 130	SL 999	SV 3500	Ave.
	D	K	#par.	L													
Word2GM/S	2	2	8	-	16.2	16.4	34.9	7.7	0.6	9.4	20.0	**14.6**	-3.9	**12.5**	**24.4**	9.3	13.5
FIRE/m			9	1	**23.1**	**30.0**	**45.0**	**38.8**	**20.0**	**36.2**	**30.5**	14.5	**11.6**	-2.5	12.5	**11.4**	**22.5**
Word2GM/S	2	3	12	-	**34.2**	22.3	37.7	18.5	7.9	19.3	24.6	**16.1**	6.3	12.0	**24.4**	11.3	19.6
FIRE/m			11	1	21.0	**36.6**	**40.0**	**38.8**	**23.3**	**36.3**	**32.5**	14.0	**24.0**	**14.1**	14.7	**12.3**	**25.6**
Word2GM/S	2	4	16	-	17.8	22.0	26.7	16.8	15.3	20.3	23.5	**18.4**	-7.2	**19.6**	**18.3**	8.5	16.7
FIRE/m			13	1	**28.0**	**35.6**	**52.2**	**38.6**	**19.7**	**38.3**	**34.5**	13.8	**16.0**	9.0	14.7	**12.1**	**26.0**
Word2GM/S	2	5	20	-	23.6	23.1	34.6	12.9	12.7	9.5	23.2	12.8	8.5	7.9	**21.2**	10.2	16.7
FIRE/m			20	2	**25.1**	**48.1**	**49.9**	**40.1**	**28.4**	**44.1**	**42.5**	**17.1**	**32.9**	**16.7**	18.1	**12.9**	**31.3**
Word2GM/S	2	10	40	-	(took more than a week)												
FIRE/m			40	4	71.2	66.0	71.7	63.2	53.6	64.1	59.7	23.0	30.4	29.6	21.7	18.1	47.7
FIRE/m	2	15	50	4	57.1	66.6	67.4	61.9	53.3	65.5	58.4	22.7	24.2	38.6	20.1	18.3	46.2

As mentioned in Section 4, our method is computationally efficient even for a larger K: the time complexities of Word2GM and Word2Cloud grow by at least $O(K^2)$, whereas FIRE has $O(K)$ complexity. For example, the Word2GM training took around 24 hours for $K = 2$, but the time grew to around 9 days for $K = 6$. Accordingly, we only tested $K = 2, 3, 4, 5$ for Word2GM, whereas for FIRE, we tested K up to 15.

Figure 4 shows the average Spearman's ρ across the tasks of the word similarity benchmark. The values are listed in Table 4 (rightmost column). For Word2GM, an increase in K did not improve the performance for $K \geq 4$. In contrast, FIRE enabled implementation of large K (e.g., 10), for which the performance greatly improved. Thus, FIRE has a larger potential to improve the performance on word similarity benchmarks, by increasing the parameter K.

F Supplementary Evaluation of Detection of Word Senses

For all 542 words, the number of clusters was counted above a threshold "eps" in the clustering algorithm of DBSCAN. The output number and the Pearson correlation with the gold standard were then computed. Figure 5 shows the results for five methods, with the horizontal axis indicating the parameter "eps" and the vertical axis indicating the Pearson correlation.

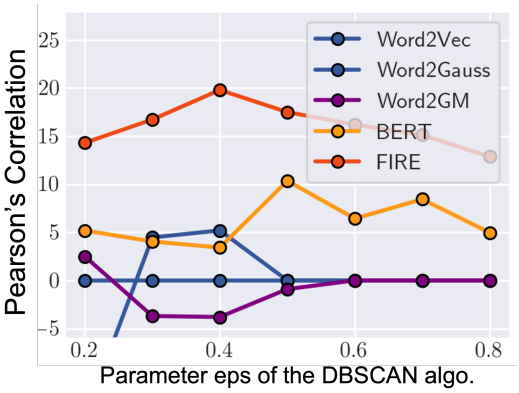

Figure 5: (This figure was redesigned following the reviewers' comments.) Pearson correlation between the detected number of senses and the gold standard for five methods. The horizontal axis indicates the threshold parameter "eps" of DBSCAN.

For all the tested choices of "eps," FIRE achieved the highest Pearson correlation coefficients with the gold standard for the number of word senses, reaching 19.8% for eps $= 0.4$. BERT showed moderate performance, with correlation coefficients between 4% and 11%, while the Word2Vec, Word2Gauss, and Word2GM methods failed for most choices of "eps."

G Dimensionalities Other Than 2

Table 5: Average scores on the 12 word similarity benchmarks for FIRE defined on $[-4, 4]^D$ with different D. Upper: We set $L = 4$ and $K = 10$ while varying D. Lower: We used several parameter sets for $D = 1, 2, 3, 4$, requiring around 50 parameters to represent each word.

D	L	K	# parameters	Average word similarity score
1	4	10	32	39.0
2	4	10	50	49.8
5	4	10	104	50.9
			# parameters ≈ 50	
1	10	10	50	39.4
2	4	10	50	49.8
3	3	7	49	46.0
4	3	5	52	44.9

H Results for Each of 24 STS Datasets

The STS benchmark comprises 24 datasets released during 2012-2016. Each dataset is a list of sentence pairs. For each sentence pair, a human-annotated, gold-standard similarity score is provided. We tested Word2Vec, BERT, and our FIRE on the 24 datasets. Following the conventional setting, a model's performance was evaluated by the Pearson correlation coefficients between the predicted similarity scores and the gold standard. Table 6 lists the results. Each row is a dataset, and each column is a sentence-embedding model. The bottom row gives the average scores, which are presented in Table 3 in the main text.

Table 6: Pearson correlation coefficients on each of the STS datasets (rows) for Word2Vec, BERT, and our proposed FIRE (columns). Each block contains datasets released in the same year.

N (# parameters)	50 Word2Vec	50 FIRE	100 Word2Vec	100 FIRE	1024 BERT-large
12/MSRphr	38.3	28.1	39.1	33.9	38.7
12/MSRvid	76.0	73.9	78.8	76.3	70.6
12/SMTeuroparl	46.2	33.7	47.7	36.9	50.6
12/WordNet	66.7	65.0	69.3	67.7	69.0
12/SMTnews	54.7	35.8	51.6	48.3	53.3
Average	56.4	47.3	57.3	52.6	56.4
13/FNWN	39.3	37.8	42.9	42.4	43.2
13/Headlines	62.2	62.7	64.2	66.7	69.8
13/WordNet	71.2	73.8	73.5	75.7	58.8
Average	57.6	58.1	60.2	61.6	57.3
14/Forum	32.6	27.3	34.9	31.9	39.9
14/News	67.8	60.0	70.1	68.5	73.8
14/Headlines	58.6	58.7	60.3	62.5	66.1
14/Images	72.8	70.8	75.9	74.9	70.8
14/WordNet	74.9	76.3	77.2	78.7	68.4
14/Twitter	60.8	59.2	63.8	60.6	66.1
Average	61.3	58.7	63.7	62.8	64.2
15/Forums	49.8	52.8	54.0	54.6	54.9
15/Students	64.6	64.9	66.7	66.0	72.6
15/Belief	54.4	58.8	57.2	63.1	64.1
15/Headlines	66.4	66.3	69.1	69.8	71.9
15/Images	74.1	74.1	77.4	76.9	77.8
Average	61.8	63.4	64.9	66.1	68.3
16/Answer	37.1	42.5	38.0	41.4	53.0
16/Headlines	63.6	66.4	65.9	68.7	71.3
16/Plagiarism	73.2	69.7	76.4	69.8	77.3
16/Postediting	67.0	67.6	69.3	71.8	81.9
16/Question	59.3	54.6	62.6	58.7	55.6
Average	60.0	60.2	62.4	62.1	67.8
Average of all	59.7	57.5	61.9	61.1	63.3

I Plotting on 2D Plane

In Figure 2 (left), we showed qualitative results for FIRE and the other methods, where the 1000 most similar words out of the most frequent 50000 words were shown on a 2D plane. Here, we describe the two dimension reduction methods that we used.

I.1 PCA

For Figure 2 (a), we applied principal component analysis to the word "vectors" and reduced the dimensionality to 2 to make the vectors visualizable on a 2D plane. As some methods such as Word2Gauss are not vectoral, we used slightly different ways to extract representative vectors for these methods, as follows:

FIRE ($D = 2, L = 4, K = 1$) We used the word location $s \in \mathbb{R}^2$ as the vector. Because we used $K = 1$, there was only one such vector for each of the 1000 words. For FIRE, PCA is equivalent to performing rotation and normalization.

Word2Vec The word vector in $\mathbb{R}^{50}$ was used.

Word2Gauss/S The mean vector of the Gaussian distribution in $\mathbb{R}^{49}$ was used.

Word2GM/S ($K = 2$) Each word was represented by two independent Gaussian components, which represented different senses of the word. For a target word such as "bank," we considered 500 components that were similar to the first component of "bank" and 500 that were similar to the second. As with Word2Gauss, we used the mean vector to represent a component. We used the Word2GM/S model listed in Table 2, which assigns 50 parameters to each word; thus, the mean vectors were in $\mathbb{R}^{23}$.

BERT We collected 1000 sentences for each target word from the Wacky dataset. A sentence was input to the BERT model, and a contextualized word vector of the target word was acquired, with 1024 dimensions.

I.2 Spectral Embedding of Word Similarity Matrix

Another way to visualize the most similar 1000 words was to use the similarity matrix between words. For FIRE, the similarity was computed with Formulas (1) and (8). For BERT, we used the cosine similarity between the contextualized word vectors.

A spectral embedding was acquired by applying SVD decomposition to the similarity matrix and taking only the first two eigenvectors. We used the implementation in the *sklearn.manifold* package.

Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes] All claims are supported by experimental results.
   (b) Did you describe the limitations of your work? [Yes] The longer training time ($\sim$10 hours) could be a limitation, which we have described in Section 4.3.
   (c) Did you discuss any potential negative societal impacts of your work? [N/A] We did not recognize any negative societal impacts of this work.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes] I have.
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [N/A]
   (b) Did you include complete proofs of all theoretical results? [N/A]
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] Code, data, and pretrained models are in the supplementary materials.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See Section 4.3.
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [N/A] We did not report error bars because it would be too computationally expensive.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes] See Section 4.3.
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes] The datasets *wacky* and *WordNet* were cited. As for *text8*, we were unable to find a proper way to cite the creators.
   (b) Did you mention the license of the assets? [Yes] BERT-large & Word2Vec: Apache License 2.0; Word2Gauss: MIT; Word2GM: BSD 3; Wacky: CC Attribution-Noncommercial-Share Alike 4.0 International; text8: unavailable; WordNet 3.0 license.
   (c) Did you include any new assets either in the supplemental material or as a URL? [Yes] We included a subset of the Core WordNet dataset, which we used to evaluate polysemy.
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A] The new dataset was a subset of the WordNet that is open.
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A] The datasets do not contain personally identifiable information.
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]