Neural Scene Flow Prior

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Abstract

Before the deep learning revolution, many perception algorithms were based on runtime optimization in conjunction with a strong prior/regularization penalty. A prime example of this in computer vision is optical and scene flow. Supervised learning has largely displaced the need for explicit regularization. Instead, they rely on large amounts of labeled data to capture prior statistics, which are not always readily available for many problems. Although optimization is employed to learn the neural network, at runtime, the weights of this network are frozen. As a result, these learning solutions are domain-specific and do not generalize well to other statistically different scenarios. This paper revisits the scene flow problem that relies predominantly on runtime optimization and strong regularization. A central innovation here is the inclusion of a neural scene flow prior, which utilizes the architecture of neural networks as a new type of implicit regularizer. Unlike learning-based scene flow methods, optimization occurs at runtime, and our approach needs no offline datasets—making it ideal for deployment in new environments such as autonomous driving. We show that an architecture based exclusively on multilayer perceptrons (MLPs) can be used as a scene flow prior. Our method attains competitive—if not better—results on scene flow benchmarks. Also, our neural prior's implicit and continuous scene flow representation allows us to estimate dense long-term correspondences across a sequence of point clouds. The dense motion information is represented by scene flow fields where points can be propagated through time by integrating motion vectors. We demonstrate such a capability by accumulating a sequence of lidar point clouds.

1 Introduction

State-of-the-art results have recently been achieved by learning-based models for the scene flow problem—the task of estimating 3D motion fields from dynamic scenes. However, such models heavily rely on large-scale data to capture prior knowledge, which is not always readily available. Scene flow annotations are expensive, and most methods train on synthetic and unrealistic scenarios to fine-tune on small real datasets.

Poor generalization to unseen, out-of-the-distribution inputs is another problem. Prior information is generally limited to the statistics of the data used for training. Real-world applications such as autonomous driving require robust solutions to low-level vision tasks such as depth, optical, and scene flow estimation that work in any statistically different scenarios. Inspired by recent innovations that make use of coordinate-based networks (*i.e.*, pixels or 3D positions as inputs) [9, 34, 35, 37, 54] for 3D modeling and rendering, we investigate the use of such networks to regularize the scene flow problem without any learning directly from point clouds. Optimization happens at runtime, and instead of learning a prior from data, the network structure itself captures the prior information. It is not limited to the statistics of a specific dataset.

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Optimizing neural networks at execution time is not new. For example, Ulyanov *et al*. [60] showed that a randomly initialized convolutional network could be used as a handcrafted prior for standard inverse problems such as image denoising, super-resolution, and inpainting. Although such deep image priors and coordinate-based networks for neural scene representations have been successfully applied for inverse problems and rendering, none has yet investigated (to the best of our knowledge) the use of network-based priors for regularizing scene flow directly from point clouds.

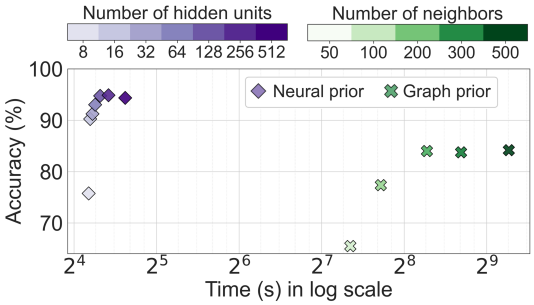

Figure 1: Our neural prior for regularizing scene flow from point clouds achieved greater accuracy and computational time than the recent graph-prior-based method [43]. The evaluation is on the KITTI Scene Flow test set using all available points in each point cloud ($\sim$30k points). We fixed the number of hidden layers in the MLP to 4 and varied the number of hidden units for our method. We varied the number of neighbors to create the graph for the graph prior method. Our proposed method achieved higher accuracy while $\sim$10$\times$ faster, considering the settings with top performance. Accuracy uses the Acc_5 metric, defined in the experiments section.

Our proposed neural prior is based on a simple multilayer perceptron (MLP) architecture, and we show it is powerful enough to implicitly regularize scene flow given two point clouds. The input to the network is 3D points, and the output is a regularized scene flow.

Our neural prior allows for a continuous scene flow representation instead of discrete such as in graph Laplacian-based priors, *e.g.*, [43]. We show how the flow fields captured by our neural prior can be employed to estimate long-term correspondences across a sequence of point clouds. The continuous scene flow allows for better integration/interpolation of motions across time.

Our results are promising and competitive to supervised [28], self-supervised [36, 65], and non-learning methods [1, 43] (see Table 1). Importantly, our method scales to large real-world point clouds while achieving great accuracy and time complexity (see Fig. 1).

2 Related work

Non-learning-based scene flow Scene flow is the uplift from the optical flow, which is proposed by Vedula *et al*. [61] as the non-rigid motion field in the 3D space. The authors proposed the optimization-based scene flow estimation using image sequences to infer reconstruction knowledge of the flow in 3D surfaces. Successive RGB/RGB-D image-based work [4, 16–19, 25, 41, 42, 48] used probability-based estimation, coarse-to-fine techniques, 6-DoF parameterization, or object segmentation, *etc*., to improve accuracy and computation time. Although image-based scene flow methods are widely used, direct estimation of the scene flow from the point cloud is still possible through non-rigid registration methods, such as [1, 10, 24, 40]. In this paper, we focus on point cloud-based scene flow estimation.

Learning-based scene flow Image-based learning methods [6, 49, 52, 67] utilize convolution and data supervision to solve scene flow from monocular or RGBD images with the available depth information. Other image-based methods [20, 21, 30, 50, 51] take care of extra occlusion cues in the large-scale autonomous driving scenes. Point-based learning methods become more prevalent with the rapid development of point cloud feature learning [26, 46, 47, 62, 64]. FlowNet3D [28] is a seminal work that estimates scene flow using PointNet++ [47]. Successive work [29, 45, 63] extends point-based learning methods using different feature extraction techniques. One obvious drawback of these supervised learning methods is the demand for sufficient ground truth labels. Besides, supervised methods lack generalizability, while eventually only fitting domain-specific data. Self-supervised methods [23, 36, 58, 65], on the other hand, replaced the loss between the prediction and the ground truth flow with a point distance loss to use the point cloud itself as supervision. Self-supervision can adapt to different datasets and maintain certain generalizability. Nonetheless, massive training data are still required for sufficient learning.

The graph Laplacian method Graph Laplacian [2] is widely used to smooth the surface in mesh processing [5, 13, 55, 56], point cloud denoising [11, 68], *etc*. Here we talk about the recent scene flow estimation using Graph Laplacian [43]. The method explicitly constructed a graph of the point cloud to constrain the non-rigid scene flow as rigid within a specific range. While as a dataless runtime optimization, the method is heavily affected by the hyperparameters of the graph and loses scalability when the point cloud becomes larger, or the neighbors in the graph grow.

Deep neural prior, implicit functions, and neural rendering People seek prior knowledge from huge number of data for recent deep learning methods. Although "enormous" data help in feature representation [57], end-to-end learning still requires high computation capacity and readily available training datasets. Instead, Ulyanov *et al*. [60] proposed a new style of optimization which uses a convolutional neural network to infer prior knowledge from the network architecture. One broader interest is to extend the idea of the network being an image function to the 3D shape modeling, and implicitly represent the continuous shape as level sets of neural networks. By directly mapping the 3D input to binary occupancy nets [9, 34], or signed distance functions [3, 37, 53], it is powerful to model the 3D geometries in a continuous space using the coordinate-based network. The following Scene Representation Networks [54] takes advantage of the coordinate-based network and renders view synthetic images. Mildenhall *et al*. proposed a seminal work NeRF [35], which is a novel way to do neural volume rendering using both point positions and viewing directions based on the coordinate-based network. Dynamic scene synthesis work [12, 15, 27, 38, 44, 59, 66] follows the NeRF framework, and integrates the motions to generate dynamic scenes. Some of these work [15, 24] use scene flow to further constrain or segment dynamic scenes. Nevertheless, we use the coordinate-based network to directly addresses the large-scale, real-world scene flow problem in the autonomous driving scenarios.

3 Approach

Problem definition Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two 3D point clouds sampled from a dynamic scene at time t-1 and t. The number of points in each point cloud, $|\mathcal{S}_1|$ and $|\mathcal{S}_2|$, are typically different and not in correspondence. A 3D point $\mathbf{p} \in \mathcal{S}_1$ moving from time t-1 to time t can be modeled by a translational vector (or flow vector) $\mathbf{f} \in \mathbb{R}^3$, where $\mathbf{p}' = \mathbf{p} + \mathbf{f}$. The collection of flow vectors for all 3D points is the scene flow $\mathcal{F} = \{\mathbf{f}_i\}_{i=1}^{|\mathcal{S}_1|}$.

Optimization We want to optimize for a scene flow $\mathcal{F}$ that minimizes the distance between the two point clouds, $\mathcal{S}_1$ and $\mathcal{S}_2$. Given the non-rigidity assumption of the scene, the optimization is inherently unconstrained. Thus, a regularization term C is necessary to constrain the motion field. We therefore solve for scene flow as

$$\mathcal{F}^* = \arg\min_{\mathcal{F}} \sum_{\mathbf{p} \in \mathcal{S}_1} \mathcal{D}\left(\mathbf{p} + \mathbf{f}, \mathcal{S}_2\right) + \lambda \mathcal{C}, \tag{1}$$

where $\mathcal{D}$ is a function to compute the distance from the perturbed point $\mathbf{p}$ by the flow vector $\mathbf{f}$ to its closest neighbor in $\mathcal{S}_2$. λ is a weighting factor for the regularizer. In a classic optimization framework, $\mathcal{C}$ might be a Laplacian regularizer, such as the graph Laplacian prior proposed in [43] to constrain the scene flow to be as-rigid-as-possible. In this paper, we want to investigate using a neural prior to regularize the scene flow.

3.1 Neural scene flow prior

Learning-based scene flow methods learn scene flow priors from a large number of examples. As in Deep Image Prior [60], we want to investigate if the structure of a neural network by itself is sufficient to capture a scene flow prior without any learning.

Here we use a neural network as an implicit regularizer. The parameters are optimized as

$$\mathbf{\Theta}^* = \arg\min_{\mathbf{\Theta}} \sum_{\mathbf{p} \in \mathcal{S}_1} \mathcal{D}\left(\mathbf{p} + g\left(\mathbf{p}; \mathbf{\Theta}\right), \mathcal{S}_2\right), \tag{2}$$

where g is a neural network parameterized by $\mathbf{\Theta}$ to regularize the scene flow $\mathcal{F}$. The input to g is $\mathbf{p}$, which is the point to be disturbed by the flow. The output of g is $\mathcal{F}$ and thus $\mathcal{F}^* = g\left(\mathbf{p};\ \mathbf{\Theta}^*\right)$. For

the distance function $\mathcal{D}$, we define it as

$$\mathcal{D}(\mathbf{p}, \mathcal{S}) = \min_{\mathbf{x} \in \mathcal{S}} \|\mathbf{p} - \mathbf{x}\|_2^2. \tag{3}$$

In practice, we use bidirectional distance function for both point set $\mathcal{S}_1$ and $\mathcal{S}_2$, which is equivalent to Chamfer distance [14].

The objective in Eq. (2), is for the forward scene flow $\mathcal{F}$, which is the one we are interested in. However, it has been shown in [28, 36], that a cycle consistency regularizer encourages better scene flow estimations. The extra regularizer simply enforces the backward flow to be similar to the forward flow, $\mathcal{F}_{bwd} \approx \mathcal{F}$. The optimal backward flow is defined as $\mathcal{F}^*_{bwd} = g\left(\mathbf{p}'; \mathbf{\Theta}^*_{bwd}\right)$, where $\mathbf{p}'$ is the shifted point by the forward flow as $\mathbf{p} + \mathbf{f}$. Note that the network g is the same but with different parameters, $\mathbf{\Theta}_{bwd}$. Using the backward flow as an additional constraint, the optimal network weights are solved as

$$\mathbf{\Theta}^*, \mathbf{\Theta}^*_{bwd} = \underset{\mathbf{\Theta}, \mathbf{\Theta}_{bwd}}{\arg\min} \sum_{\mathbf{p} \in \mathcal{S}_1} \mathcal{D}\left(\mathbf{p} + g\left(\mathbf{p}; \mathbf{\Theta}\right), \mathcal{S}_2\right) + \sum_{\mathbf{p}' \in \mathcal{S}'_1} \mathcal{D}\left(\mathbf{p}' + g\left(\mathbf{p}'; \mathbf{\Theta}_{bwd}\right), \mathcal{S}_1\right), \tag{4}$$

where $\mathcal{S}'_1$ is the shifted $\mathcal{S}_1$ by the forward flow, *i.e.*, $\mathcal{S}'_1 = \mathcal{S}_1 + \mathcal{F}$. For the network g, we use multi-layer perceptrons with ReLU activations. The objective function in Eq. (4) can be optimized by gradient descent techniques using off-the-shelf frameworks with automatic differentiation. We show in the experiments section how the architecture of the neural prior affects performance by varying the number of hidden layers and units.

3.2 Why a neural scene flow prior?

Deep learning relies on massive amounts of data and computational resources to capture prior statistics. Although learning methods have achieved impressive results in most tasks, it still struggles when deployed in environments where the statistics are different from those captured during learning. Using a network architecture as a prior show that we can better regularize low-level vision tasks such as scene flow during runtime operations. Our intuition is that a neural prior act as a strong implicit regularizer able to constrain highly dynamic motion fields to be as smooth as possible—different and simpler than using graph-based priors that apply discrete Laplacian operators to encourage smoothness. A neural scene flow prior scales to large 3D scenes while achieving high-fidelity results at low computational times. Our proposed neural scene flow prior with 8 hidden layers with 128 hidden units has about 116k parameters. FlowNet3D [28], for example, has about 1.2M parameters. Our method has $\sim$10$\times$ fewer parameters than the state-of-the-art supervised methods while achieving competitive, if not better, results. Lastly, our deep scene flow prior captures a continuous flow field that allows us to perform better scene flow interpolation across a sequence of multiple point clouds.

4 Experiments

We evaluated the performance of our neural prior for scene flow on synthetic and real-world datasets. We focused on the accuracy of the estimated scene flow and the computational cost of our method. We performed experiments on different neural network settings and analyzed the performance of the neural prior to regularising scene flow. Remarkably, we show that a simple MLP-based prior to regularize scene flow is enough to achieve competitive results to the state-of-the-art scene flow methods.

Datasets We used four scene flow datasets: **1) FlyingThings3D** [31] which is an extensive collection of randomly moving synthetic objects. We used the preprocessed data from [28]; **2) KITTI Scene Flow** [32, 33] which has real self-driving scenes collected using lidar. We used the subset released by [28]; **3) Argoverse** [8] which is a large-scale autonomous driving dataset with challenging scenes. However, no official scene flow annotations are available yet. We followed the data processing method proposed in [43] to collect pseudo-ground-truth scene flow; and **4) nuScenes** [7] which is also a large-scale, real-world autonomous driving dataset. nuScenes also lacks scene flow annotations. We used the same preprocessing method as in [43] to obtain ground truth.

Metrics We employed the widely used metrics as in [28, 36, 43, 65] to evaluate our method, which are: $\mathcal{E}$ to denote the end-point error (EPE), which is the mean absolute distance of two point clouds;

Acc_5 to denote the accuracy or percentage of estimated flows when $\mathcal{E} < 0.05$m or $\mathcal{E}' < 5\%$, where $\mathcal{E}'$ is the relative error; Acc_{10} denotes the percentage of estimated flows where $\mathcal{E} < 0.1$m or $\mathcal{E}' < 10\%$; and θ_ϵ which is the mean angle error between the estimated and ground-truth scene flows.

Implementation details We defined our neural prior for scene flow as a simple coordinate-based MLP architecture with 8 hidden layers, a fixed length of 128 for the hidden units, Rectified Linear Unit (ReLU) activation and shared weights across points. The network input is the 3D point cloud $\mathbf{P}_{t\text{-}1}$, and the output is the scene flow $\mathbf{F}$. We used PyTorch [39] for the implementation and optimized the objective function with Adam [22]. The weights were randomly initialized. We set a fixed learning rate of $8\text{e}{-3}$ and run the optimization for 5k iterations with early stopping on the loss. For our settings and datasets, we found the optimization to mostly converge in less than 1k iterations. All experiments were run on a machine with an NVIDIA Quadro P5000 GPU and a 16 Intel(R) Xeon(R) W-2145 CPU @ 3.70GHz.

Training setup for the learning-based methods We used the publicly available implementation of the learning-based methods to perform our experiments. The training settings for each method are: **FlowNet3D:** trained on FlyingThings3D with supervision; and **Just Go with the Flow** and PointPWC-Net: trained on FlyingThings3D with supervision and fine-tuned on domain-matched datasets with self-supervision (*i.e.*, fine-tuned and tested on statistically similar data). Note that FlowNet3D was only trained on the synthetic FlyingThings3D to demonstrate the poor generalizability of learning-based methods to other domains. Just Go with the Flow and PointPWC-Net are self-supervised methods, and although they do not require ground-truth annotations, they still require large-scale datasets for training to achieve competitive performance.

Optimization setup for the non-learning methods Non-rigid ICP [1] was originally proposed for mesh registration. We adapted it for point cloud registration for fair comparison reasons. A graph Laplacian prior was recently proposed in [43] to optimize scene flow from point clouds. We implemented the method using the hyperparameters defined by the authors. The weight for the graph prior term is set to 10, the number of neighbors k to build the k-NN graph, if not explicitly specified, is set to 50, the learning rate to 0.1, and the number of iterations to 1.5k.

4.1 Choosing the neural prior architecture

Fig. 2 shows how the performance of our method is affected when varying the neural prior MLP architecture: number of hidden layers and hidden units. The experiments were performed on the KITTI test set and all points included (*i.e.*, without point sampling). The average number of points for the KITTI dataset is about 30k. We ran our method five times with different random seeds to include the uncertainty levels in the plot.

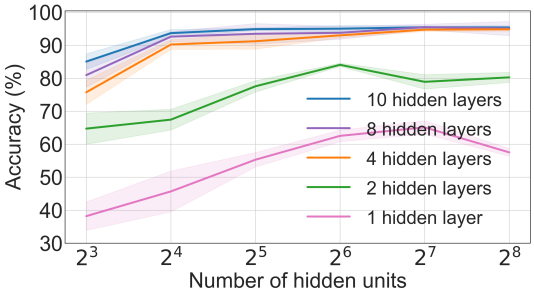

Figure 2: We analyzed the performance of our method on the KITTI test set when varying the number of hidden layers and hidden units of the MLP architecture. Accuracy is the Acc_5 metric.

Overall, the performance of our method improved as we increased the number of hidden layers and hidden units. For small numbers of hidden layers (*e.g.*, 1 and 2), the performance deteriorated when the number of hidden units is large, around 128 and 256 (or 2^7 and 2^8). We chose the MLP architecture with the best performance with relatively low computation time for our following experiments: 8 hidden layers and 128 (2^7) hidden units.

4.2 Comparing to other methods

Table 1 shows how our method stands against other state-of-the-art methods on different datasets and metrics. We set the number of points to 2,048, which was randomly sampled from all points, to be consistent with already established scene flow benchmarks. We ran the experiments 5 times to report uncertainties for our method and the graph prior method [43] since both are runtime

Table 1: Performance of our method and others across different datasets and metrics. ● are supervised methods, ● are self-supervised methods and ● are non-learning methods. All experiments were run with 2,048 points. ↑ means larger values are better while ↓ means smaller values are better. We did no report standard deviations smaller than 1e−2.

	FlyingThings3D [31] *Train: 19,967 samples, Test: 2,000 samples*				nuScenes Scene Flow [7] *Train: 1,513 samples, Test: 310 samples*			
	$\mathcal{E}(m)\downarrow$	$Acc_5(\%)\uparrow$	$Acc_{10}(\%)\uparrow$	$\theta_\epsilon(rad)\downarrow$	$\mathcal{E}(m)\downarrow$	$Acc_5(\%)\uparrow$	$Acc_{10}(\%)\uparrow$	$\theta_\epsilon(rad)\downarrow$
● FlowNet3D [28]	**0.134**	**22.64**	**54.17**	**0.305**	0.505	2.12	10.81	0.620
● Just Go with the Flow [36]	—				0.625	6.09	0.139	0.432
● PointPWC-Net [65]	—				0.431	6.87	22.42	0.406
● Non-rigid ICP [1]	0.339	14.05	35.68	0.480	0.402	6.99	21.01	0.492
● Graph prior [43]	0.255	16.56±0.02	42.05±0.02	0.362	0.289	20.12±0.01	43.54±0.02	0.337
● Ours	0.234	19.16±0.23	46.74±0.46	0.341	**0.175±0.01**	**35.18±1.32**	**63.45±0.46**	**0.279±0.04**
	KITTI Scene Flow [32, 33] *Train: 100 samples, Test: 50 samples*				Argoverse Scene Flow [8] *Train: 2,691 samples, Test: 212 samples*			
	$\mathcal{E}(m)\downarrow$	$Acc_5(\%)\uparrow$	$Acc_{10}(\%)\uparrow$	$\theta_\epsilon(rad)\downarrow$	$\mathcal{E}(m)\downarrow$	$Acc_5(\%)\uparrow$	$Acc_{10}(\%)\uparrow$	$\theta_\epsilon(rad)\downarrow$
● FlowNet3D [28]	0.199	10.44	38.89	0.386	0.455	1.34	6.12	0.736
● Just Go with the Flow [36]	0.218	10.17	34.38	0.254	0.542	8.80	20.28	0.715
● PointPWC-Net [65]	0.177	13.29	42.15	0.272	0.409	9.79	29.31	0.643
● Non-rigid ICP [1]	0.338	22.06	43.03	0.460	0.461	4.27	13.90	0.741
● Graph prior [43]	0.099	63.60±0.09	81.18±0.08	0.176	0.257	25.24±0.04	47.60±0.02	0.467
● Ours	**0.050±0.01**	**81.68±2.00**	**93.19±1.30**	**0.133±0.01**	**0.159±0.01**	**38.43±0.48**	**63.08±0.59**	**0.374±0.01**

optimization methods with uncertainties for each run. The learning-based methods and non-rigid ICP are deterministic during runtime.

Our method achieved better performance in most datasets and metrics. We considered the well-known Non-rigid ICP as a baseline for the non-learning methods (●). Our method outperformed the recent graph prior method by a large margin. The supervised FlowNet3D method (●) had better performance on FlyingThings3D because it was trained on it with supervision. If the dataset is out-of-the-distribution, the method produced unreliable results. The self-supervised methods (●), Just Go with the Flow and PointPWC-Net, despite not being exposed to ground-truth labels during training, still generated better results than supervised methods—showing that self-supervision is an important direction for scene flow estimation. Still, it is remarkable that with a simple MLP regularizer and an off-the-shelf optimization framework, we can robustly estimate scene flow from point clouds with great accuracy.

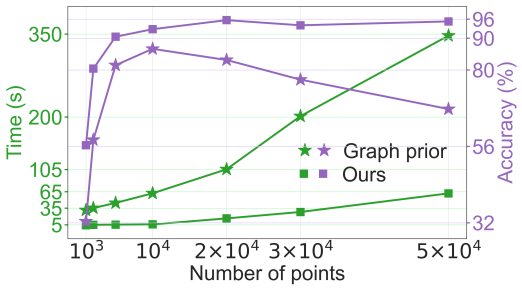

Figure 3: Performance of our neural prior and the graph prior [43] when varying the number of points. Our method achieved higher accuracy (Acc_5) and better time complexity. Results were averaged over the KITTI Scene Flow dataset.

Table 2: Performance of our neural prior and the graph prior [43] when using all points available. Our method achieved better performance on all metrics by a margin while being ∼5× faster if k=50 and ∼10× faster if k=200 (for KITTI).

	KITTI Scene Flow *Average number of points: ∼30k*				
	$\mathcal{E}\downarrow$ (m)	$Acc_5\uparrow$ $(\%)$	$Acc_{10}\uparrow$ $(\%)$	$\theta_\epsilon\downarrow$ (rad)	Time ↓ (s)
Graph prior (k=50)	0.225	65.50	70.32	0.277	162.97
Graph prior (k=200)	0.082	84.00	88.45	0.141	310.12
Ours	**0.025**	**95.68**	**98.00**	**0.085**	**38.33**
	Argoverse Scene Flow *Average number of points: ∼50k*				
	$\mathcal{E}\downarrow$ (m)	$Acc_5\uparrow$ $(\%)$	$Acc_{10}\uparrow$ $(\%)$	$\theta_\epsilon\downarrow$ (rad)	Time ↓ (s)
Graph prior (k=50)	0.249	46.92	61.72	0.494	410.21
Ours	**0.043**	**86.04**	**94.07**	**0.244**	**84.46**

4.3 Estimating scene flow from large point clouds

Real-world point clouds collected from depth sensors such as lidar typically have tens of thousands of points. We evaluated the performance of our method on large point clouds and compared it against the graph prior method. The KITTI and Argoverse Scene Flow datasets were used, given that both have large point clouds.

Figure 4: Qualitative example of a scene flow estimation using our proposed method. The complex and highly dynamic driving scene is from the Argoverse Scene Flow dataset. The scene flow estimated by our method is close to the ground truth. We also show a prediction using the supervised FlowNet3D method trained on FlyingThings3D and fine-tuned on the KITTI Scene Flow dataset. Note how the scene flow deviated from the ground truth when the inference was performed on an out-of-the-distribution sample. The scene flow color encodes the magnitude (color intensity) and direction (angle) of the flow vectors. For example, the purplish vehicles are heading northeast.

Fig. 3 shows the performance on the KITTI Scene Flow dataset, in terms of accuracy and computational time, of our method and the graph prior method when varying the number of points. Our method's accuracy (Acc_5) increased as the number of points grew until around 20k points and then saturated, while the computational time slowly increased. In contrast, the graph prior achieved lower accuracy and dramatic growth in computation. The computational complexity of our MLP-based prior grows linearly in the number of points, $\mathcal{O}(n)$, while the graph prior grows quadratically in the number of points, $\mathcal{O}(n^2)$. The graph prior relies on the construction of a graph Laplacian matrix to use as a regularizer (*i.e.*, $\mathbf{L} \in \mathbb{R}^{n \times n}$, where n is the number of points).

The accuracy of the graph prior method degraded after 10k points. The graph regularizer might need more than 5k iterations for higher density point clouds to converge to a reasonable solution or carefully tuned schedulers to accelerate its convergence. Moreover, the k-NN graph is built with 50 neighbors, and for higher density point clouds, larger graphs might be necessary for a better regularization. Here, we were interested in the trade-offs between accuracy and computational time. Our proposed neural prior clearly shows that we can scale it to larger point clouds with great accuracy but still maintaining good time efficiency.

Table 2 shows a quantitative comparison between our method and the graph prior method on the KITTI and Argoverse Scene Flow datasets when using all points. Our method achieved better performance on all metrics. We also reported results for the graph prior method when setting the number of neighbors, k, to 200. According to our results in Fig. 1, the scene flow accuracy saturated after k=200. Fig. 4 shows a qualitative example of a scene flow estimation using our method.

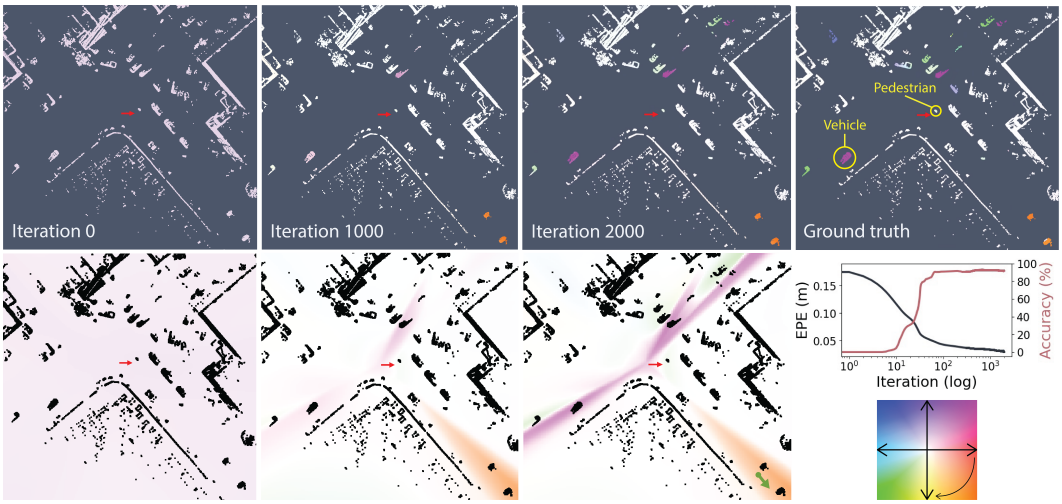

Figure 5: Example showing how the estimated scene flow and the continuous flow field (bottom) given by our neural prior change as the optimization converges to a solution. We show a top-view dynamic driving scene from Argoverse Scene Flow. The scene flow color encodes the magnitude (color intensity) and direction (angle) of the flow vectors. For example, the purplish vehicles are heading northeast. The red arrow shows the position and direction of travel of the autonomous vehicle, which is stopped, waiting for a pedestrian to cross the street. Note how the predicted scene flow is close to the ground truth at iteration 2k. At iteration 0, the scene flow is random, given the random initialization of the neural prior. Thus having very small magnitudes for the random directions. As the optimization went on, the flow fields became better constrained. A simple way to interpret the flow fields is to imagine sampling a point at any location in the continuous scene flow field to recover an estimated flow vector. For example, imagine sampling a point around the orange region in the flow field at iteration 2k (green arrow in the bottom right). The direction of the flow vector will be pointing southeast at a specific magnitude, similar to the vehicles in the orange region.

4.4 A continuous scene flow field

Our neural prior implicitly regularizes the scene flow through a simple MLP network. Thus, our method allows for a continuous scene flow representation instead of a discrete representation such as in graph Laplacian-based priors, *e.g.*, [43].

An advantage of a continuous scene flow representation is that we can reuse the optimal network weights to estimate dense long-term correspondences across a sequence of point clouds, as we demonstrated in Section 4.6. Fig. 5 shows how the estimated scene flow and the continuous flow field change as the optimization converges to a solution.

4.5 Failure case

Fig. 6 shows a failure case. Few corresponding points due to missing parts in the scene might lead to incorrect flow estimations. This is a limitation due to our nearest-neighbor-based distance loss.

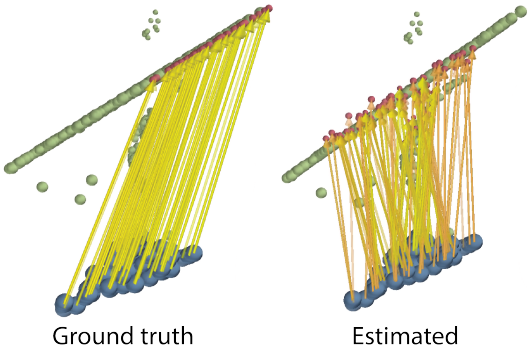

Figure 6: Example of a failure case. Partial scene from FlyingThings3D. Our nearest-neighbor-based distance loss might fail when handling large missing parts, occlusions, and bad correspondences. Green points are the target, and red points are the shifted blue points by the estimated scene flow (yellowish arrows).

4.6 Scene flow integration

Here we demonstrate how to use our method to perform scene flow integration. Given a

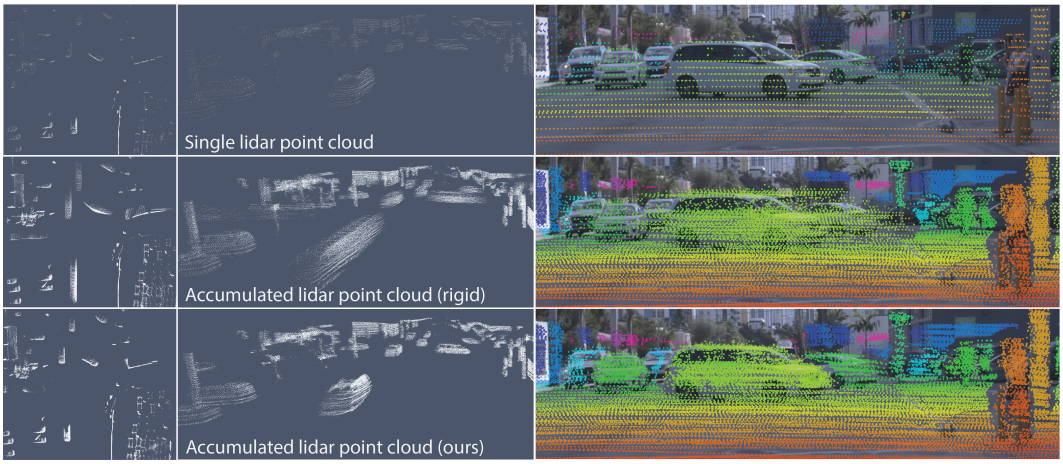

Figure 7: Example of a scene flow integration to densify an Argoverse lidar point cloud. The left and middle columns are a top and front view of the point cloud, respectively. The rightmost column shows the accumulated point cloud projected onto the image. Note the smearing effect on the dynamic objects when rigidly accumulating the point clouds (middle row). Our accumulation method nicely produced a denser point cloud while taking care of all dynamic objects in the scene.

temporal sequence of point sets, $\{\mathcal{S}_0, \mathcal{S}_1, \mathcal{S}_2, \cdots, \mathcal{S}_M\}$, we first optimize for the pairwise scene flows, $\{\mathcal{F}_{0\to1}, \mathcal{F}_{1\to2}, \cdots, \mathcal{F}_{M\text{-}1\to M}\}$, using our proposed method with the optimal neural prior parameters, $\{\mathbf{\Theta}^*_{0\to1}, \mathbf{\Theta}^*_{1\to2}, \cdots, \mathbf{\Theta}^*_{M\text{-}1\to M}\}$, saved. Then, starting from $\mathbf{f}_{0\to1}$, we can integrate long-term flows using the classic Forward Euler method recursively for m=1:M-1 iterations as

$$\mathbf{f}_{0\to m+1} = \mathbf{f}_{0\to m} + g(\mathbf{p}_0 + \mathbf{f}_{0\to m}; \mathbf{\Theta}^*_{m\to m+1}). \tag{5}$$

This gives the long-term scene flow $\mathcal{F}_{0\to M} = \{(\mathbf{f}_{0\to M})_i\}_{i=0}^{|\mathcal{S}_0|}$, from $\mathcal{S}_0 \to \mathcal{S}_M$. Note that we are not relying on discrete nearest-neighbor-based interpolations. Our neural scene flow prior is a continuous representation that naturally provides continuous scene flow estimations. Fig. 7 shows an example of an Argoverse scene where we applied such a technique to integrate 10 point clouds into a single frame to densify the point cloud.

5 Conclusion

We revisited the runtime optimization for scene flow estimation with a deep neural network to infer prior information. Our neural prior gets rid of the need for massive annotated data, while being scalable with large point clouds. Additionally, since we infer prior knowledge from the network architectures instead of from data, our approach can generalize to any out-of-the-distribution scenarios as compared to learning-based methods. We show that how a hand-designed coordinate-based network architecture can serve as a new type of implicit regularizer for the scene flow problem. We believe this paper shows a promising direction for large-scale, real-world scene flow estimation without data supervision.

Broader impact Our proposed neural scene flow prior allows for estimating 3D motion fields from large-scale dynamic scenes without annotating massive data but preserving generalizability—making it useful for scenarios where motion prediction is required. Especially in computer vision and robotics communities, robust scene flow estimations is crucial. For example, autonomous vehicles need to predict future distribution of the surrounding objects to avoid catastrophe in dynamic environments.

Our work also encourages further exploration in the combination of the innovative coordinate-based networks and classical runtime optimization algorithm. This dataless approach offers an affordable solution for many industrial problems without adequate supervision (*e.g.*, autonomous driving). Although as an AI-based research, it could be misused, we still want to motivate the community to take full advantage of the innovations of this work to benefit society.

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Checklist

The checklist follows the references. Please read the checklist guidelines carefully for information on how to answer these questions. For each question, change the default **[TODO]** to [Yes] , [No] , or [N/A] . You are strongly encouraged to include a **justification to your answer**, either by referencing the appropriate section of your paper or providing a brief inline description. For example:

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1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
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   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]

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(c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

A Appendix

Optionally include extra information (complete proofs, additional experiments and plots) in the appendix. This section will often be part of the supplemental material.