3D NEURAL EMBEDDING LIKELIHOOD FOR ROBUST SIM-TO-REAL TRANSFER IN INVERSE GRAPHICS

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ABSTRACT

A central challenge in 3D scene perception via inverse graphics is robustly modeling the gap between 3D graphics and real-world data. We propose a novel 3D Neural Embedding Likelihood (3DNEL) over RGB-D images to address this gap. 3DNEL uses neural embeddings to predict 2D-3D correspondences from RGB and combines this with depth in a principled manner. 3DNEL is trained entirely from synthetic images and generalizes to real-world data. To showcase this capability, we develop a multi-stage inverse graphics pipeline that uses 3DNEL for 6D object pose estimation from real RGB-D images. Our method outperforms the previous state-of-the-art in sim-to-real pose estimation on the YCB-Video dataset, and improves robustness, with significantly fewer large-error predictions. Unlike existing bottom-up, discriminative approaches that are specialized for pose estimation, 3DNEL adopts a probabilistic generative formulation that jointly models multi-object scenes. This generative formulation enables easy extension of 3DNEL to additional tasks like object and camera tracking from video, using principled inference in the same probabilistic model without task specific retraining.

1 INTRODUCTION

There is a widespread need for models that bridge the gap between 3D graphics and real RGB-D data. Accurate simulation environments exist in domains such as autonomous driving, augmented reality, and robotic manipulation, yet robust 3D scene perception remains a central bottleneck. "Inverse graphics" is an appealing approach to 3D scene understanding that treats scene perception as the inverse problem to 3D graphics. However, in practice these methods have been outperformed by more bottom-up, discriminative approaches, especially those using deep learning (LeCun et al., 2015). A key challenge in inverse graphics is modeling the gap between rendered images and observed real-world images. This paper aims to address this gap with 3D Neural Embedding Likelihood (3DNEL), a likelihood model of RGB-D images that is trained entirely from synthetic data and generalizes to real-world data. 3DNEL uses learned neural embeddings to predict dense 2D-3D correspondences from RGB and combines this with 3D information from depth in a principled way.

To showcase 3DNEL's capabilities in sim-to-real transfer, we develop a multi-stage inverse graphics pipeline (MSIGP) that uses 3DNEL for 6D object pose estimation. We demonstrate that 3DNEL can be applied to real RGB-D images without having to train on any real data. Our 3DNEL MSIGP consists of (1) a coarse enumerative procedure that generates pose hypotheses and an initial estimate of the 3D scene and (2) an iterative Markov Chain Monte Carlo (MCMC) process that finetunes the 3D scene. We empirically evaluate 3DNEL MSIGP on the popular YCB-Video (YCB-V) dataset (Xiang et al., 2018). 3DNEL MSIGP outperforms the previous state-of-the-art (SOTA) SurfEMB (Haugaard & Buch, 2022) in sim-to-real 6D pose estimation, albeit at the cost of increased computation. It is also significantly more robust: we show over 50% reduction in high-error pose predictions compared to SurfEMB. Extensive ablation studies illustrate the source of performance improvements.

Existing approaches for 6D pose estimation are predominantly discriminative and bottom-up, and are specialized to the specific task of 6D pose estimation. In contrast, 3DNEL adopts a probabilistic generative formulation which extends beyond just pose estimation. To demonstrate the value of 3DNEL's generative formulation, we present additional experiments on 3DNEL's easy extension to object and camera tracking from video, using principled inference in the same probabilistic model without task specific retraining.

2 RELATED WORK

3D Inverse Graphics Our method follows a long line of work in the "analysis-by-synthesis" paradigm that treats perception as the inverse problem to computer graphics (Kersten & Yuille, 1996; Yuille & Kersten, 2006; Lee & Mumford, 2003; Kersten et al., 2004; Mansinghka et al., 2013; Kulkarni et al., 2015). While elegant and conceptually appealing, robustly modeling the gap between 3D graphics and real-world data, especially using appearance information, remains a central challenge in 3D inverse graphics. Our key observation is that, dense 2D-3D correspondences, widely used in many recent 6D pose estimation methods (Hodan et al., 2020; Li et al., 2019; He et al., 2020; Tremblay et al., 2018; Florence et al., 2018; Haugaard & Buch, 2022), provide a natural way to model appearance information in 3D inverse graphics, and can be combined with depth information into a unified probabilistic generative model to effectively bridge the sim-to-real gap.

Sim-to-Real Transfer Most computer vision systems require annotated real-world data for training (Hodan et al., 2017; Brachmann et al., 2014; Xiang et al., 2018; Krizhevsky et al., 2017) in order to achieve strong performance on real-world data at test time, since without it the "sim-to-real gap" is too difficult to overcome. Practically, it is tedious and expensive to collect and annotate real data. Recent advances in photorealistic rendering and physics-based simulations (Hodan et al., 2018; Tremblay et al., 2018; Denninger et al., 2019) have resulted in strong performance of models trained entirely on synthetic data. SurfEMB (Haugaard & Buch, 2022) is one such model that obtains SOTA performance in 6D pose estimation, outperforming numerous approaches that use real data for training. 3DNEL builds upon SurfEMB's learned dense 2D-3D correspondences, and expands previous work (Gothoskar et al., 2021) on probabilistic modeling of real depth data. It combines RGB and depth information into a unified probabilistic generative model, and improves SurfEMB nontrivially in both accuracy and robustness to achieve new SOTA in sim-to-real 6D pose estimation.

6D Object Pose Estimation 6D object pose estimation aims to infer the rigid $\mathbb{SE}(3)$ transformation (position and orientation) of an object in the camera frame, given an image observation. Discriminatively trained deep learning approaches (Xiang et al., 2018; Li et al., 2018; Deng et al., 2019; He et al., 2020; Sundermeyer et al., 2018) have, in general, outperformed more traditional feature or template matching methods (Besl & McKay, 1992; Rusinkiewicz & Levoy, 2001; Lowe, 1999; Rothganger et al., 2006; Collet et al., 2011). With the advent of RGB-D cameras, there has been a growing interest in leveraging both the appearance information from RGB and the geometric shape information from depth. Existing methods either use depth to post process estimations from RGB (Xiang et al., 2018; Haugaard & Buch, 2022), or fuse learned features from both RGB and real, noisy depth (Wang et al., 2019; He et al., 2021). 3DNEL differs from prior work in two important ways. (1) 3DNEL combines RGB and depth information in a principled probabilistic model, which enables superior sim-to-real transfer. (2) 3DNEL jointly models multi-object scenes using a probabilistic generative formulation, allowing easy extension to tasks beyond 6D pose estimation.

3 3D NEURAL EMBEDDING LIKELIHOOD (3DNEL)

We propose 3DNEL as a unified probabilistic model over RGB-D images. 3DNEL combines RGB and depth information in a principled way to assess how well an observed RGB-D image is explained by a hypothesized latent 3D scene. A naive way to assess this would be to render the latent 3D scene to an RGB-D image and compare the rendered and observed RGB-D images with a probabilistic model that accounts for noise, missing data, and outliers present in real images. Prior work (Gothoskar et al., 2021) demonstrates promising performance of this approach when applied to depth images, but modeling of RGB data is more challenging. Intuitively, the "gap" between rendered and real depth images is mostly noisy spatial displacements and outliers, while the "gap" between rendered and real RGB images comes from additional factors including lighting, shadows, exposure, and focus, which are challenging to account for with a simple noise model. This motivates the use of learning to model RGB in a way that is invariant to these factors.

We build on SurfEMB that predicts dense 2D-3D correspondences through learned neural embeddings of RGB images, and combine 2D-3D correspondences with depth information to define a 3D likelihood over RGB-D images. 3DNEL jointly models multiple objects in a scene through a mixture model formulation, resulting in a probabilistic generative model of observed RGB-D images.

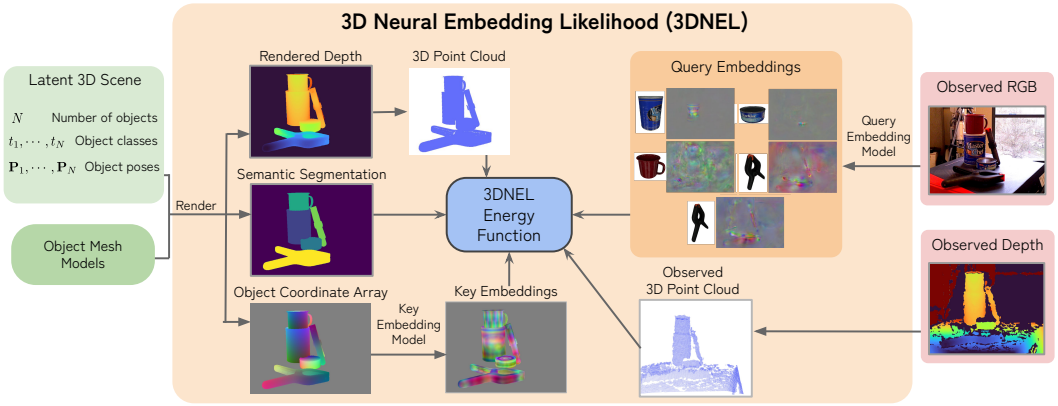

Figure 1: **Computing the 3D Neural Embedding Likelihood** 3DNEL defines the probability of an observed RGB and depth image pair conditioned on a latent 3D scene. The latent 3D scene is first rendered into: (1) a depth image, which is then transformed to a 3D point cloud, (2) a semantic segmentation map, and (3) the object coordinate array (each pixel contains the object frame coordinates of the object surface point from which the pixel originates). The object coordinate array is transformed, via the key models, into key embeddings. The observed RGB image is transformed, via the query models, into query embeddings for each object class. The observed depth is transformed into a 3D point cloud. The 3DNEL Energy Function is computed from the 3D point cloud, semantic segmentation, key embeddings, the observed 3D point cloud, and query embeddings.

3.1 Latent 3D Scene Representation

We represent 3D scenes in terms of the objects that appear in the scene and their poses. Each object is associated with a textured mesh, which captures the 3D shape and appearance information of the object. The poses of the objects in the scene are represented in $\mathbb{SE}(3)$ (rigid transformations in 3D) which define an object's position and orientation in space. The latent 3D scene is represented with the number N of objects in the scene, their classes $t_1, \cdots, t_N \in \{1, \cdots, M\}$, and corresponding poses $\mathbf{P}_1, \cdots, \mathbf{P}_N \in \mathbb{SE}(3)$. We assume uniform prior distributions over object poses (a uniform distribution over a bounded volume for position and a uniform distribution in $\mathbb{SO}(3)$ for orientation).

3.2 Dense 2D-3D correspondence distributions via learned neural embeddings

We build on SurfEMB (Haugaard & Buch, 2022) which learns neural embeddings to establish dense 2D-3D correspondences via contrastive learning. For each object class $t \in \{1, \cdots, M\}$, SurfEMB learns a pair of neural networks: the *query model* $f_t : \{0, \cdots, 255\}^{H \times W \times 3} \mapsto \mathbb{R}^{H \times W \times E}$ and the *key model* $g_t : \mathbb{R}^3 \mapsto \mathbb{R}^E$. The *query model* transforms an input RGB image $\mathbf{I}$ into neural embeddings $\mathbf{Q}^t$ and the *key model* transforms an input 3D model coordinate into an embedding.

For each object class t, we subsample uniformly across the object's surface to get a set Z_t of surface 3D coordinates in object frame. Then, given a pixel with query embedding $\mathbf{q}$, we calculate the probability that this pixel corresponds to a surface point $\tilde{\mathbf{x}} \in Z_t$ on object class t as:

$$\mathbb{P}_{corr}(\tilde{\mathbf{x}}|\mathbf{q}, Z_t, t) = \frac{\exp(\mathbf{q}^T g_t(\tilde{\mathbf{x}}))}{\sum_{\mathbf{x} \in Z_t} \exp(\mathbf{q}^T g_t(\mathbf{x}))} \tag{1}$$

Importantly, these models are trained entirely from synthetic data. The synthetic data is generated using photorealistic rendering of textured mesh models and physics-based simulations. The practical implication is that no annotated real data is required for training, aside from scans of each object instance to construct the textured mesh model, which can be collected quickly without supervision.

3.3 Rendering-based Likelihood

Next, we describe how the learned neural embeddings are used to define a likelihood that computes the probability of an RGB-D image given a hypothesized latent 3D scene. Given a latent 3D scene

(represented as described in Section 3.1), we render: (1) A 3D point cloud $\tilde{\mathbf{C}} \in \mathbb{R}^{H \times W \times 3}$, where $\tilde{\mathbf{C}}_{i,j} \in \mathbb{R}^3$ represents the 3D coordinate of the point (i, j) in camera frame. (2) A semantic segmentation map $\tilde{\mathbf{S}} \in \{0, 1, \cdots, M\}^{H \times W}$ where $\tilde{\mathbf{S}}_{i,j}$ represents the class to which the object point (i, j) belongs. Here 0 represents background. (3) An object coordinate array $\tilde{\mathbf{X}} \in \mathbb{R}^{H \times W \times 3}$ where $\tilde{\mathbf{X}}_{i,j}$ represents the 3D coordinates of the point (i, j) in the object frame of the object of class $\tilde{\mathbf{S}}_{i,j}$.

Motivation To motivate the 3DNEL formulation, we first make the simplifying assumption that in addition to the observed point cloud $\mathbf{C} \in \mathbb{R}^{H \times W \times 3}$, we also observe the semantic segmentation map $\mathbf{S} \in \{0, 1, \cdots, M\}^{H \times W}$, and an object coordinate array $\mathbf{X} \in \mathbb{R}^{H \times W \times 3}$. 3DNEL captures the generative process of $\mathbf{C}, \mathbf{S}$ and $\mathbf{X}$, conditioned on the latent point cloud $\tilde{\mathbf{C}}$, semantic segmentation map $\tilde{\mathbf{S}}$ and object coordinate array $\tilde{\mathbf{X}}$ with a mixture model $\mathbb{P}(\mathbf{C}, \mathbf{S}, \mathbf{X} | \tilde{\mathbf{C}}, \tilde{\mathbf{S}}, \tilde{\mathbf{X}})$:

$$\prod_{i,j} \left(\frac{o}{B} + \frac{1-o}{\sum_{\tilde{i},\tilde{j}} \mathbf{1}[\tilde{\mathbf{S}}_{\tilde{i},\tilde{j}} > 0]} \sum_{(\tilde{i},\tilde{j}):\tilde{\mathbf{S}}_{\tilde{i},\tilde{j}} > 0} \frac{\mathbf{1}[||\mathbf{C}_{i,j} - \tilde{\mathbf{C}}_{\tilde{i},\tilde{j}}||_2 \leq r]}{\frac{4}{3}\pi r^3} \cdot \mathbf{1}[\mathbf{X}_{i,j} = \tilde{\mathbf{X}}_{\tilde{i},\tilde{j}}, \mathbf{S}_{i,j} = \tilde{\mathbf{S}}_{\tilde{i},\tilde{j}}] \right) \quad (2)$$

Intuitively, we assess how well each point (i, j) is explained by a latent point $(\tilde{i}, \tilde{j})$ by combining a probabilistic model $\frac{\mathbf{1}[||\mathbf{C}_{i,j} - \tilde{\mathbf{C}}_{\tilde{i},\tilde{j}}||_2 \leq r]}{\frac{4}{3}\pi r^3}$ on how close the two points are using depth information, and a probabilistic model on the 2D-3D correspondence of the two points $\mathbf{1}[\mathbf{X}_{i,j} = \tilde{\mathbf{X}}_{\tilde{i},\tilde{j}}, \mathbf{S}_{i,j} = \tilde{\mathbf{S}}_{\tilde{i},\tilde{j}}]$ using RGB information. Here r is a hyperparameter that controls the variance of our probabilistic model on depth. To jointly model multiple objects in a scene, we assume each point (i, j) can be explained by multiple latent points, resulting in the mixture model formulation. We assume the observed point cloud $\mathbf{C}$ resides in a bounded region of volume B, and additionally have a uniform distribution on the bounded region as an outlier component with mixture probability o.

Energy-based formulation In practice, while we do observe the point cloud $\mathbf{C}$, we do not directly observe the semantic segmentation map $\mathbf{S}$ or the object coordinate array $\mathbf{X}$. Instead, we observe an RGB image $\mathbf{I}$ from which we can infer dense 2D-3D correspondences using learned query embeddings via Equation 1. We replace the exact match between $\mathbf{X}, \mathbf{S}$ and $\tilde{\mathbf{X}}, \tilde{\mathbf{S}}$ in Equation 2 (i.e. $\mathbf{1}[\mathbf{X}_{i,j} = \tilde{\mathbf{X}}_{\tilde{i},\tilde{j}}, \mathbf{S}_{i,j} = \tilde{\mathbf{S}}_{\tilde{i},\tilde{j}}]$) with Equation 1 to get $\mathbb{P}(\mathbf{I}, \mathbf{C} | \tilde{\mathbf{C}}, \tilde{\mathbf{S}}, \tilde{\mathbf{X}})$ which is proportional to

$$\prod_{i,j} \left(\frac{o}{B} + \frac{1-o}{\sum_{\tilde{i},\tilde{j}} \mathbf{1}[\tilde{\mathbf{S}}_{\tilde{i},\tilde{j}} > 0]} \sum_{(\tilde{i},\tilde{j}):\mathbf{S}_{\tilde{i},\tilde{j}} > 0} \frac{\mathbf{1}[||\mathbf{C}_{i,j} - \tilde{\mathbf{C}}_{\tilde{i},\tilde{j}}||_2 \leq r]}{\frac{4}{3}\pi r^3} \cdot \mathbb{P}_{corr}(\tilde{\mathbf{X}}_{\tilde{i},\tilde{j}} | \mathbf{Q}_{i,j}^{\tilde{\mathbf{S}}_{\tilde{i},\tilde{j}}}, Z_{\tilde{\mathbf{S}}_{\tilde{i},\tilde{j}}}, \tilde{\mathbf{S}}_{\tilde{i},\tilde{j}}) \right) \quad (3)$$

Since the query embedding at a pixel depends on the entire image $\mathbf{I}$ and the mixture components are unnormalized, this results in an energy-based formulation. The normalization constant is well-defined due to the finite number of possible RGB images $\mathbf{I}$ and the bounded region where $\mathbf{C}$ resides.

4 Inferring the latent 3D scene from RGB-D

Figure 2 gives an overview of 3DNEL MSIGP for 6D object pose estimation.

Enumeration-based Pose Hypotheses Generation We develop a novel spherical voting procedure and a heuristic scoring, and employ them in a coarse enumerative procedure to efficiently generate pose hypotheses. Algorithm 1 describes the spherical voting procedure and our heuristic scoring. For each object class, we sample n_k keypoints using farthest point sampling from the object surface, and discretize the camera frame space into a voxel grid of shape (L_x, L_y, L_z). Our spherical voting aggregates evidence for the spatial presence of the object center and the n_k keypoints into corresponding voxel grids. Unlike previous work (He et al., 2020) which casts votes from a pixel to a single spatial location based on predicted keypoint offset, our spherical voting casts votes from a pixel towards a sphere due to the lack of orientation information in dense correspondences. The voxel grids containing aggregated evidence for the spatial presence of object keypoints are then used to heuristically score object poses. Algorithm 2 in the appendix describes our coarse enumerative procedure. We first identify promising positions based on large entries in the object center voxel grid. We then discretize the orientation space into representative orientations, and score all representative orientations for each of the promising positions. We finally use top scoring poses across all promising positions and orientations as our pose hypotheses for the given object class.

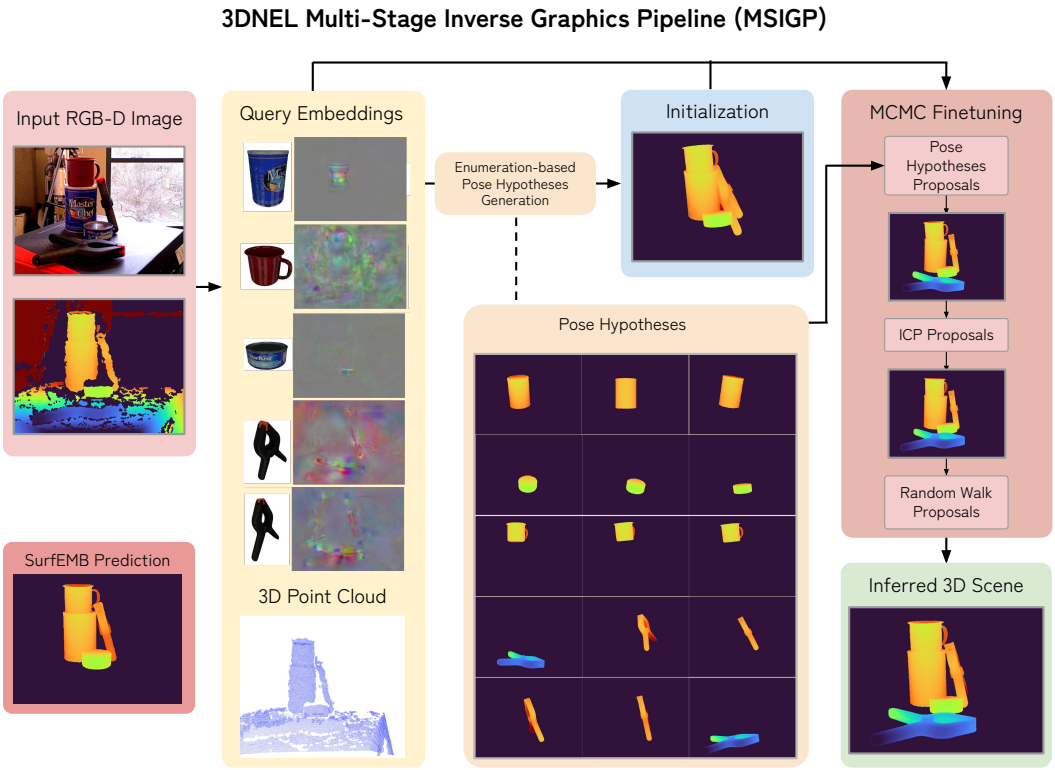

Figure 2: **Using 3DNEL for 3D Scene Parsing** Our 3DNEL MSIGP begins by deriving the query embeddings for each object class from the observed RGB image and the 3D point cloud from the depth image. Then, a fast coarse enumerative procedure produces pose hypotheses for each object, the best of which are used to construct an initial latent 3D scene. Next, we perform MCMC finetuning of the latent 3D scene with 3DNEL using three types of MH proposals (1) pose hypotheses proposals that propose the pose hypotheses generated by the enumerative procedure, (2) ICP proposals that use ICP to align an object to point cloud data, and (3) random walk proposals that propose random pose perturbations. The result is a latent 3D scene that explains the observed RGB-D image. 3DNEL's joint modeling of multiple objects through the mixture model formulation enables robust estimation on this challenging scene with two similar-looking clamps, while SurfEMB (shown in the bottom left) makes per-object predictions and incorrectly predicts both clamps to be in the back.

MCMC Finetuning We initialize the latent 3D scene with top scoring pose hypothesis for each object instance in the scene based on the pose hypotheses for its class, and use the Metropolis-Hastings (MH) algorithm to do MCMC finetuning of the latent 3D scene. At each iteration, MH proposes a new pose for an object from some proposals, and accepts the proposal with the MH acceptance probability. We use three different proposals, all of which sample a pose's position from a spherical normal distribution centered at a desired position and the pose's orientation from a von Mises-Fisher distribution centered at a desired orientation. (1) Pose hypotheses proposals iterate over the generated pose hypotheses twice and make a proposal centered at each. (2) ICP proposals iterate over all objects and use ICP to align each object's 3D model to the observed 3D point cloud with the point cloud from all the other objects masked out, and make a proposal centered at the resulting pose. (3) Finally, random walk proposals that randomly perturb object poses.

Since 3DNEL is a probabilistic likelihood, we can use sampling-based inference via MCMC to approximate the posterior distribution on latent 3D scenes. In contrast, most existing methods for 6D pose estimation produce a single point estimate of the 3D scene. While in this work we are taking a single sample from the approximate posterior to evaluate 6D pose estimation accuracy and robustness, there are domains in which capturing posterior uncertainty is important, for example in settings with noise, missing data, and severe occlusions. In these settings, a single point estimate fails to capture the inherent uncertainty about an object's pose and identity and can be detrimental.

Algorithm 1: Voting-based evidence aggregation and heuristic pose scoring for object class t

Class-specific setup: n_k keypoints with object frame coordinates $x_1^*, \cdots, x_{n_k}^* \in \mathbb{R}^3$, set of surface points Z_t, the dense correspondence distribution $\mathbb{P}_{corr}$.

Spatial discretization: Camera frame coordinates of the center of the boundary voxel $y \in \mathbb{R}^3$, size of the voxel grid (L_x, L_y, L_z), diameter of the voxels $d > 0$.

1 /* Evidence aggregation for x^* with spherical voting */

Input: Query embeddings $\mathbf{Q} \in \mathbb{R}^{H \times W \times E}$, observed point cloud $\mathbf{C}$, keypoint x^*

Output: Voxel grid $\mathbf{V} \in \mathbb{R}^{L_x \times L_y \times L_z}$ with aggregated evidence for the spatial presence of x^*

2 $\mathbf{V} \leftarrow \mathbf{0}$; // Initialize $\mathbf{V} \in \mathbb{R}^{L_x \times L_y \times L_z}$ to an all 0 array

3 **for** $i \leftarrow 1$ **to** H, $j \leftarrow 1$ **to** W **do**

4 $\quad x \leftarrow \arg\max_{\tilde{x}} \mathbb{P}_{corr}(\tilde{x}|\mathbf{Q}_{i,j}, Z_t, t)$

5 $\quad p_{i,j} \leftarrow \max_{\tilde{x}} \mathbb{P}_{corr}(\tilde{x}|\mathbf{Q}_{i,j}, Z_t, t)$

6 $\quad$ **for** $u \leftarrow 1$ **to** L_x, $v \leftarrow 1$ **to** L_y, $w \leftarrow 1$ **to** L_z **do**

7 $\quad\quad c \leftarrow$ GetCenterOfVoxelInCameraFrame (y, d, u, v, w); // Algorithm 2

8 $\quad\quad$ **if** $||\mathbf{C}_{i,j} - c||_2 \approx ||x - x^*||_2$ **then** // See Algorithm 2 for details

9 $\quad\quad\quad \mathbf{V}_{u,v,w} = \mathbf{V}_{u,v,w} + p_{i,j}$

10 /* Heuristic pose scoring */

Input: Object pose $\mathbf{P} \in \mathbb{SE}(3)$, Voxel grids $\mathbf{V}^1, \cdots, \mathbf{V}^{n_k} \in \mathbb{R}^{L_x \times L_y \times L_z}$ containing aggregated evidence for the spatial presence of the n_k keypoints.

Output: A heuristic score $s > 0$ of the object pose $\mathbf{P}$

11 $s \leftarrow 0$;

12 **for** $i \leftarrow 1$ **to** n_k **do**

13 $\quad x \leftarrow \mathbf{P}x_i^*$; // Location of x_i^* in the camera frame for pose $\mathbf{P}$

14 $\quad (u, v, w) \leftarrow$GetIndexOfVoxelOnVoxelGrid (x, y, d); // Algorithm 2

15 $\quad s = s + \mathbf{V}^i_{u,v,w}$;

5 EXPERIMENTS

We conduct extensive experiments on YCB-V, a popular benchmark for 6D object pose estimation. We establish that 3DNEL MSIGP significantly improves the previous SOTA SurfEMB in sim-to-real 6D object pose estimation, and is more robust in terms of reducing large pose estimation errors and overcoming failures of 2D detectors. Detailed ablations illustrate the source of performance improvements, and additional experiments on easy extension of 3DNEL to object and camera tracking from video demonstrates the value of 3DNEL's formulation as a probabilistic generative model.

Evaluation We follow the evaluation protocol of the Benchmark for 6D Object Pose Estimation (BOP) challenge (Hodan et al., 2018). The task is to estimate the 6D poses of objects in a scene, from a single RGB-D image. The BOP challenge assumes knowledge of the number of instances of each object class in the scene. For a predicted pose, we calculate three error metrics with respect to the ground truth pose: Visible Surface Discrepancy (VSD) (Hodaň et al., 2016; Hodan et al., 2018), Maximum Symmetry-Aware Surface Distance (MSSD) (Drost et al., 2017), and Maximum Symmetry-Aware Projection Distance (Brachmann et al., 2016). Average recalls AR_{VSD}, AR_{MSSD}, AR_{MSPD} are computed for each error metric across a range of error thresholds. The aggregate Average Recall (as reported in Table 1) is the average of AR_{VSD}, AR_{MSSD}, and AR_{MSPD}.

Training To allow easy comparison with baselines, we use publicly released pretrained SurfEMB models for our experiments. These include query models, key models, mask predictors for different object classes, and an additional 2D detector from CosyPose (Labbé et al., 2020), all trained entirely from synthetic data. While sharing the same underlying models, our 3DNEL MSIGP is significantly different from SurfEMB's PnP+RANSAC procedure, and demonstrates nontrivial improvements in overall accuracy as well as robustness. We expect additional training tailored towards the 3DNEL formulation can further improve performance, but leave this to future work.

3DNEL MSIGP We follow He et al. (2021) and use Ku et al. (2018) to fill in missing depth. Recent strong pose estimation methods commonly rely on 2D detection and mask prediction. Since our spherical voting procedure can robustly generate pose hypotheses from entire query embedding

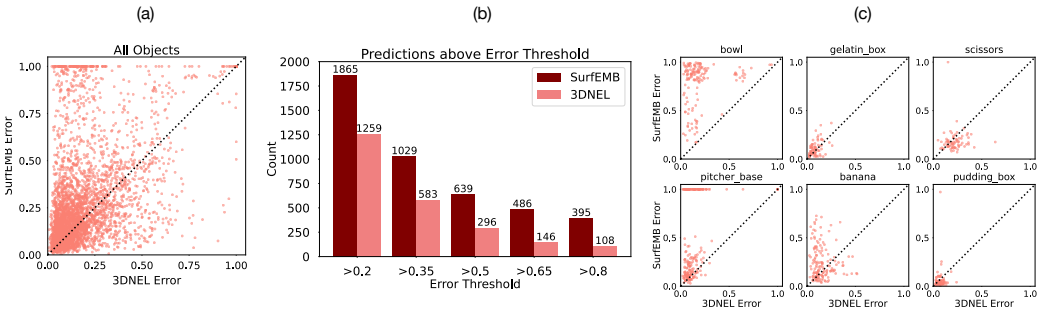

Figure 3: **3DNEL MSIGP yields greater robustness than SurfEMB** (a) Comparison of prediction error (measured by VSD) between SurfEMB and 3DNEL MSIGP across 4123 object instances in YCB-V. Each instance is represented as a point on the scatter plot. Points above the dashed line represent instances for which 3DNEL MSIGP has lower prediction error. (b) The number of instances with prediction error above a certain error threshold, across multiple thresholds. 3DNEL MSIGP makes significantly less high-error pose predictions than SurfEMB (over 50% less above 0.5). (c) Scatter plots per object classes for 6 representative object classes out of the 21 object classes.

Category	Method	Average Recall
Sim-to-Real	3DNEL MSIGP (Ours)	**83.68%**
	SurfEMB (Haugaard & Buch, 2022)	80.00%
	FFB6D (He et al., 2021)	75.80%
Ablations	3DNEL MSIGP (No RGB in Likelihood)	61.57%
	3DNEL MSIGP (No Depth in Likelihood)	50.85%
	3DNEL MSIGP (SurfEMB initialization + MCMC)	82.73%
	3DNEL MSIGP (Initialization only)	62.36%
	3DNEL MSIGP (No 2D Detection)	76.53%
Real and Synthetic Training	FFB6D ((He et al., 2021)	85.50%
	CosyPose (Labbé et al., 2020) (4 Views)	84.00%
	SurfEMB (Haugaard & Buch, 2022)	82.40%
	Pix2Pose (Park et al., 2019)	78.00%
	W-PoseNet (Xu et al., 2019)	77.90%
	Koenig-Hybrid (König & Drost, 2020)	70.10%

Table 1: **3DNEL MSIGP yields greater accuracy than baselines and ablations** We report Average Recall on the YCB-Video dataset. 3DNEL MSIGP is the top-performing sim-to-real method for 6D pose estimation. Results are averaged over 5 runs. Standard deviation is below 0.2% for all setups.

images, our 3DNEL MSIGP can work well even without separate 2D detection and mask prediction (Table 1, *3DNEL MSIGP (No 2D Detection)*). However, empirically we find that leveraging additional information from 2D detection and mask prediction can further improve performance. In our experiments, we use the same 2D detection and mask prediction from SurfEMB to obtain query embeddings at the right scale, and focus the query embedding extraction, spherical voting and 3DNEL evaluation on the relevant regions. We pick hyperparameters by visually inspecting inference results on a small number of real training images outside the test set. See Appendix A.2 for details.

Our superior performance comes at the cost of increased computation. Empirically we observe our current (highly unoptimized) implementation can be 6x slower than SurfEMB. In Section 6 we discuss this limitation and propose some potential solutions.

5.1 Accuracy

In Table 1, we report the Average Recall for 3DNEL MSIGP, its various ablations and representative bottom-up baselines. In the first section we compare with two baselines trained entirely on synthetic data: the current SOTA SurfEMB (Haugaard & Buch, 2022) which uses depth to heuristically refine pose estimates with RGB information, and FFB6D (He et al., 2021) which is a bottom-up approach that fuses appearance information from RGB with geometry information from depth. 3DNEL formulates a probabilistic 3D likelihood over depth data and appearance information from RGB in the form of 2D-3D correspondence distributions, and significantly outperforms SurfEMB, FFB6D.

	Scene ID											
	48	49	50	51	52	53	54	55	56	57	58	59
SurfEMB Single Frame	77.6%	67.0%	83.7%	91.3%	80.0%	59.8%	88.4%	76.7%	70.5%	77.3%	92.4%	84.1%
3DNEL MSIGP Single Frame	71.9%	77.5%	83.1%	87.7%	87.5%	84.1%	88.4%	80.4%	82.8%	85.3%	94.3%	86.4%
3DNEL Object Tracking	73.9%	78.5%	84.6%	90.0%	91.3%	92.0%	90.7%	83.7%	91.1%	87.9%	95.6%	64.7%
3DNEL Camera Tracking	**81.5%**	**94.7%**	**97.5%**	**97.0%**	**97.0%**	**97.0%**	**97.2%**	**97.5%**	**96.8%**	**92.2%**	**98.0%**	**97.0%**

Table 2: **Extending 3DNEL to object and camera tracking improves performance compared to SurfEMB and ablations** We apply 3DNEL to object and camera tracking from video. Our results show that we can further improve pose estimation accuracy over the single frame setting.

In the second section, we evaluate various ablations of 3DNEL MSIGP. We first investigate the importance of principled combination of RGB and depth. We evaluate *3DNEL MSIGP (No RGB in Likelihood)*, where we drop $\mathbb{P}_{corr}$ in Equation 3, and *3DNEL MSIGP (No Depth in Likelihood)* where we replace the 3D ball $\mathbf{1}[||\mathbf{C}_{i,j} - \tilde{\mathbf{C}}_{\tilde{i},\tilde{j}}||_2 \leq r]$ in Equation 3 with a 3×3 2D patch. Both are substantially worse. We next evaluate the importance of different components in 3DNEL MSIGP. We evaluate *3DNEL MSIGP (SurfEMB initialization + MCMC)*, which replaces the enumeration-based pose hypotheses generation with SurfEMB predictions and does MCMC finetuning with 3DNEL. This leads to 2.73% improvements over SurfEMB but does not outperform 3DNEL MSIGP, illustrating the effectiveness of MCMC finetuning and the value of having multiple pose hypotheses. We evaluate *3DNEL MSIGP (Initialization only)*, which uses initialiations from the coarse enumerative procedure as pose estimates. This is substantially worse, demonstrating the importance of MCMC finetuning. We finally evaluate *3DNEL MSIGP (No 2D Detection)*, which uses spherical voting to aggregate evidence from entire query embedding images to generate pose hypotheses. This uses much less information than SurfEMB (no 2D detection and mask prediction), yet results are not far off, demonstrating the effectiveness of our enumeration-based pose hypotheses generation.

In the last section, we contextualize 3DNEL MSIGP's performance in relation to methods that depend on real data. 3DNEL MSIGP outperforms several methods, despite no exposure to real data before test time. We also observe that previous bottom-up approaches that combine RGB and depth information rely heavily on annotated real data. FFB6D, for example, has average recall of 85.5% but this drops to 75.8% without real data. We hypothesize this is because these methods essentially overfit to the characteristics of noisy depth and struggle to generalize to real RGB-D images without having seen them at training. In contrast, as a result of our principled probabilistic modeling of depth information, we can transfer from simulated data to real data without sacrificing performance.

5.2 Robustness

Figure 3 contains a more detailed comparison between SurfEMB and 3DNEL MSIGP. Across all YCB-V test images, there are 4123 object instances. Each point on the scatter plots corresponds to an object instance, and the point's x and y coordinates are the pose prediction error of 3DNEL MSIGP and SurfEMB, respectively. Points above the dashed line correspond to object instances for which 3DNEL MSIGP had a lower prediction error than SurfEMB. Figure 3(a) shows the scatter plot for all 4123 predictions and Figure 3(c) shows the scatter plots for 6 representative object classes. Figure 3(b) shows, across a range of error thresholds, the number of pose predictions with error above that threshold. 3DNEL MSIGP significantly and consistently reduces the the number of high-error pose predictions, highlighting 3DNEL MSIGP's robustness compared to bottom-up approaches.

We find many instances, especially concentrated within a few object classes, for which SurfEMB is systematically making bad pose predictions. This is primarily a result of the noisy neural embeddings, which SurfEMB's PnP+RANSAC procedure does not handle well. 3DNEL MSIGP is able to correctly estimate the object's pose as a result of enumeration-based pose hypotheses generation, which can robustly handle noisy neural embeddings and identify a variety of pose hypotheses. 3DNEL MSIGP also improves over SurfEMB on test images where the 2D object detector that SurfEMB relies on fails. SurfEMB uses a neural 2D object detector and only makes pose predictions within the predicted bounding box. In contrast, 3DNEL MSIGP considers the full image using spherical voting and therefore can make reasonable pose estimates in spite of missing or wrong 2D detections. Out of the 4123 object instances, there are 174 object instances for which there is no 2D detection and therefore no prediction from SurfEMB. Figure 4 and Figure A2 in the appendix visualize 3DNEL MSIGP and SurfEMB's pose predictions for representative YCB-V test images.

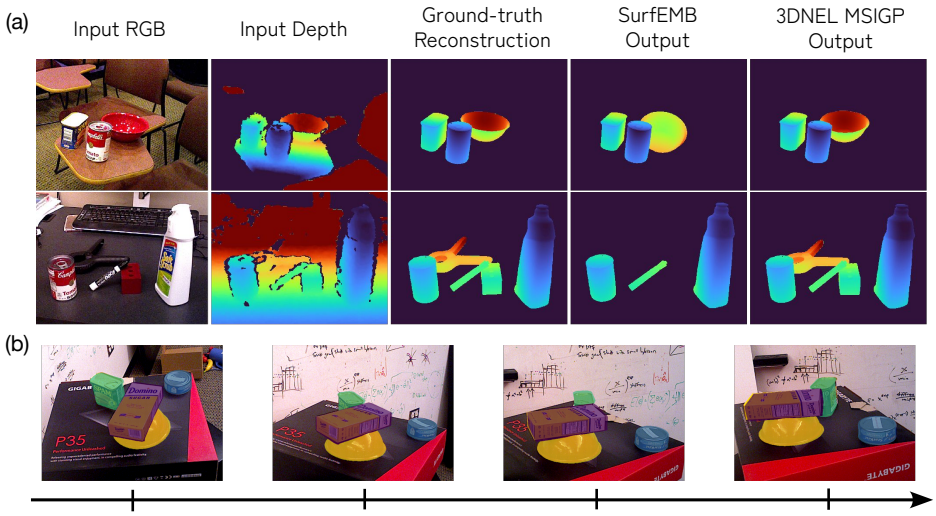

Figure 4: **Qualitative results of 3DNEL used for pose estimation and tracking** (a) Results of 3DNEL MSIGP for 6D pose estimation (b) Results of camera tracking from video using 3DNEL. Predicted object poses are overlayed on each video frame.

5.3 Extension to Object and Camera Tracking from Video

We demonstrate that, due to its probabilistic generative formulation, we can easily extend 3DNEL to support object and camera tracking from video using principled probabilistic inference in the same model without task specific retraining. We extend the our single-frame 3DNEL to the multi-timestep setup by introducing a simple dynamics prior that samples the object pose at time $t + 1$ uniformly from the space of poses with position at most 3cm away from the object pose at time t.

For object tracking, we initialize the object poses at the first frame using our single-frame prediction, and apply MCMC finetuning with ICP and random walk proposals for subsequent frames, taking into account the dynamics prior. For camera tracking, we initialize object poses at the first frame to ground truth annotations to avoid introducing systematic errors. We again apply MCMC finetuning with ICP and random walk proposals. However, we further assume we know the scene is static and only the camera moves, which translates into jointly updating the poses of all objects in a scene.

Results in Table 2 show that the same inference procedure can readily handle such extensions, taking into account the dynamics prior for object tracking and the knowledge of a static scene for camera tracking within the same probabilistic generative model. We observe comprehensive improvements over single frame predictions, except for object tracking in scene 59 where we make large errors in the first frame with our single frame predictions and are not able to recover in later frames.

6 Discussion

This paper presents 3DNEL, a 3D likelihood for RGB-D images based on neural embeddings, and demonstrated SOTA performance in sim-to-real 6D object pose estimation on YCB-V. However, the current work has several limitations. Since 3DNEL is formulated to explain the whole scene, inference with 3DNEL sometimes moves highly occluded objects to explain visible backgrounds. In addition, certain objects are difficult to capture with depth cameras because of their reflective surfaces, which can lead to degraded pose estimation due to the importance of depth cues in 3DNEL. We expect a background model and an outlier model that can properly fall back to use only RGB data when depth is missing will help address these issues. Finally, while 3DNEL improves pose estimation accuracy and robustness, this comes at the cost of increased computation. This is due in part to the lack of performance engineering efforts, and to the non-standard nature of our inference pipeline, which lacks highly optimized software implementations. In future work, we plan to additionally leverage batched GPU rendering and compilation, which will enable a large number of MCMC proposals to be evaluated in parallel and therefore significantly speed up our pipeline.

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A Appendix

A.1 Details on Enumeration-based pose hypotheses generation

In Algorithm 2 we present a detailed description of our enumeration-based pose hypotheses generation process that is introduced in Section 4. In practice we also optionally apply non-max suppression in the `TopPositions` function in the algorithm to make sure the promising positions we identify are well spread out to cover different parts of the image and better represent uncertainty.

A.2 Details on inference pipeline implementation

We use OpenGL for rendering, and use Taichi (Hu et al., 2019) for both spherical voting and 3DNEL evaluation. A Python implementation reproducing the results reported in the paper will be released upon acceptance.

As we describe in the main text, we pick hyperparameters by visually inspecting detection results on a small number of real training images that are outside the test set.

We select $n_k = 8$ keypoints from the surface of each object class, and use $y = (-350.0, -210.0, 530.0), L_x = 129, L_y = 87, L_z = 168$ and $d = 5.0$ to make the voxel grids large enough to cover all the keypoints that can be present in the camera frame. Here the units for the values in y and d are mm.

We discretize $\mathbb{SO}(3)$ into 6400 representative orientations. We generate these orientations by first picking 200 points roughly uniformly on the unit sphere using the Fibonacci sphere. The 6400 representative orientations are genearted by first rotating the axis $(0, 0, 1.0)$ to point to one of the 200 points, followed by one of 32 in-plane rotations around the axis.

In the best performing setup, we leverage the same 2D detector used in SurfEMB as part of the pose hypotheses generation process. For each object class in the scene, there can be multiple 2D detections. For each 2D detection, we do spherical voting just within the detector crop and generate 80 pose hypotheses per detector crop. When there is a missing 2D detection, we obtain the query embeddings at 1.5x the original scale, and do spherical voting on the whole image. In such cases, when we identify top positions, we additionally do non-max suppression with a filter size of 10 to spread the promising positions out. For each such promising position we identify top 2 orientation, and we consider all promising positions and generate in total 30 pose hypotheses.

Different 2D detector crops correspond to looking at the image at different scales. Empirically we find that scale is important in getting high quality query embeddings using SurfEMB's pretrained

Algorithm 2: Enumeration-based Pose Hypotheses Generation

/* Basic setups */

Object-specific: A set of surface points Z_t for object class t, query model f_t, key model g_t, n_k keypoints with object frame coordinates $x_1^*, \cdots, x_{n_k}^* \in \mathbb{R}^3$.

Spatial discretization: Camera frame coordinates of the center of the boundary voxel $y \in \mathbb{R}^3$, size of the voxel grid (L_x, L_y, L_z), diameter of the voxels $d > 0$.

Orientation discretization: n_r representative orientations $\mathbf{R}_1, \cdots, \mathbf{R}_{n_r} \in \mathbb{SO}(3)$.

Parameters: Number of pose hypotheses to generate n_p, number of top positions n_t.

/* Enumeration-based pose hypotheses generation */

Input: RGB image $\mathbf{I} \in \mathbb{R}^{H \times W \times 3}$, observed point cloud $\mathbf{C} \in \mathbb{R}^{H \times W \times 3}$.

Output: Top scoring pose hypotheses $\mathbf{P}_1^t, \cdots, \mathbf{P}_{n_p}^t \in \mathbb{SE}(3)$.

$\mathbf{Q} \leftarrow f_t(\mathbf{I})$; // Get query embeddings $\mathbf{Q} \in \mathbb{R}^{H \times W \times E}$ from the RGB image $\mathbf{I}$

$\mathbf{V}^0 \leftarrow$ Voting $(\mathbf{Q}, \mathbf{C}, (0,0,0))$; // Aggregation for object center $(0,0,0)$

for $i \leftarrow 1$ **to** n_k **do** // Aggregation for n_k keypoints.

  $\mathbf{V}^i \leftarrow$ Voting $(\mathbf{Q}, \mathbf{C}, x_i^*)$; // $\mathbf{V}^i \in \mathbb{R}^{L_x \times L_y \times L_z}$

/* Identify top positions based on $\mathbf{V}^0$'s largest entires. */

$l_1, \cdots, l_{n_t} \leftarrow$ TopPositions $(\mathbf{V}^0)$; // $l_1, \cdots, l_{n_t} \in \mathbb{R}^3$

for $i \leftarrow 1$ **to** n_t, $j \leftarrow 1$ **to** n_r **do**

  $s_{i,j} \leftarrow$ Scoring $(l_i, \mathbf{R}_j, \mathbf{V}^1, \cdots, \mathbf{V}^{n_k})$; // Heuristic pose scoring

$\mathbf{P}_1^t, \cdots, \mathbf{P}_{n_p}^t \leftarrow$ RankByScore $(l_1, \cdots, l_{n_t}, s_{i,j}, i = 1, \cdots, n_t, j = 1, \cdots, n_r)$;

return $\mathbf{P}_1^t, \cdots, \mathbf{P}_{n_p}^t$;

/* Voting and heuristic scoring */

def Voting $(\mathbf{Q}, \mathbf{C}, x^*)$ // Voting-based evidence aggregation

  $\mathbf{V} \leftarrow \mathbf{0}$; // Initialize $V \in \mathbb{R}^{L_x \times L_y \times L_z}$ to all 0 array

  for $i \leftarrow 1$ **to** H, $j \leftarrow 1$ **to** W **do**

    $x \leftarrow \arg\max_{\tilde{x}} \mathbb{P}_{corr}(\tilde{x}|Q_{i,j}, Z_t, t)$, $p_{i,j} \leftarrow \max_{\tilde{x}} \mathbb{P}_{corr}(\tilde{x}|Q_{i,j}, Z_t, t)$;

    for $u \leftarrow 1$ **to** L_x, $v \leftarrow 1$ **to** L_y, $w \leftarrow 1$ **to** L_z **do**

      $c \leftarrow (y_1 + (u-1)d, y_2 + (v-1)d, y_3 + (w-1)d)$;

      if $||\mathbf{C}_{i,j} - c||_2 \approx ||x - x^*||_2$ **then**

        $\mathbf{V}_{u,v,w} = \mathbf{V}_{u,v,w} + p_{i,j}$

  return $\mathbf{V}$

def Scoring $(l, \mathbf{R}, \mathbf{V}^1, \cdots, \mathbf{V}^{n_k})$ // Heuristic pose scoring

  $s \leftarrow 0$; // Initialize score to 0

  for $i \leftarrow 1$ **to** n_k **do**

    $x \leftarrow \mathbf{R}x_i^* + l$; // Location of x_i^* in world frame for pose $l, \mathbf{R}$

    $(u, v, w) \leftarrow Round[(x - y)/d]$; // Identify corresponding voxel of x

    $s = s + \mathbf{V}_{u,v,w}^i$; // Heuristic scoring

  return s

query models. When there are multiple 2D detections for an object class, we score the corresponding scales with the mean heuristic score of the top 5 pose hypotheses from that detector crop, and use the highest scoring scale to obtain the query embeddings for the object class. When there is no 2D detections we fall back to a default scale of 1.5.

We use $r = 5.0$ in evaluating 3DNEl for all our experiments. We implement 3DNEL with Taichi to leverage powerful GPUs. When identifying points in the latent point cloud $\tilde{\mathbf{C}}$ (organized as an $H \times W \times 3$ image) that is within distance r from a point (i, j) in the observed point cloud $\mathbf{C}$, to further speed up 3DNEL evaluation, we only look at the points from the patch of size $(10, 10)$ centered at (i, j).

For MCMC finetuning, we go through all pose hypotheses twice in the pose hypotheses proposals. We go through all objects 5 times in the ICP proposals. We go through all objects 50 times in the random walk proposals. Each time we go through an object in the random walk proposals, we propose 7 types of random pose perturbations.

A.3 ADDITIONAL ROBUSTNESS RESULTS

In Figure A1 we include additional robustness results.

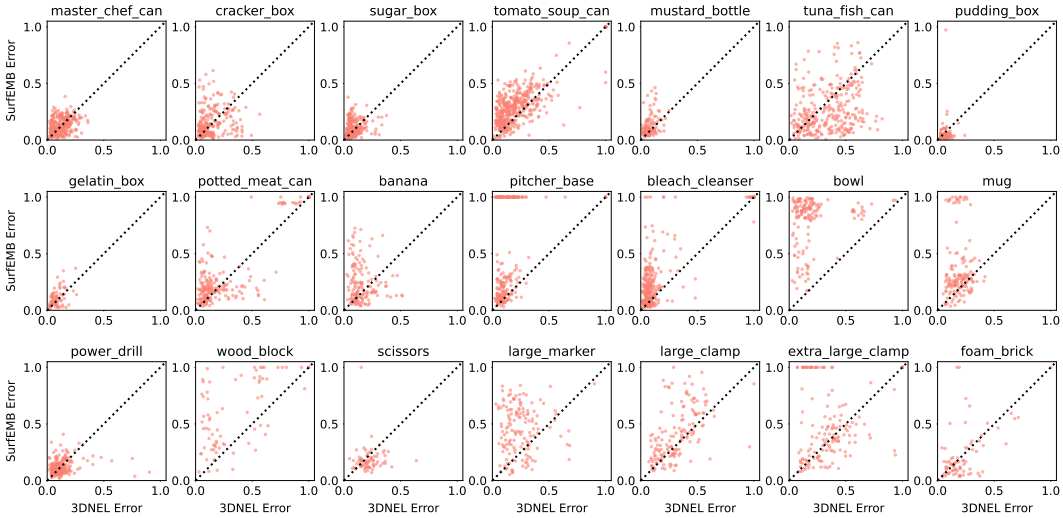

Figure A1: We compare the prediction error (using the VSD error metric) of SurfEMB and 3DNEL MSIGP across all 4123 object instances in the YCB-V test dataset, with each instance represented as a point on this scatter plot. In the main text Figure 3(a), we show the scatter plot across all objects. Here, we show the results per object.

A.4 ADDITIONAL VISUALIZATIONS OF SURFEMB AND 3DNEL MSIGP PREDICTIONS ON YCB-V TEST IMAGES

In Figure A2 we include additional visualizations of SurfEMB and 3DNEL MSIGP predictions on YCB-V test images.

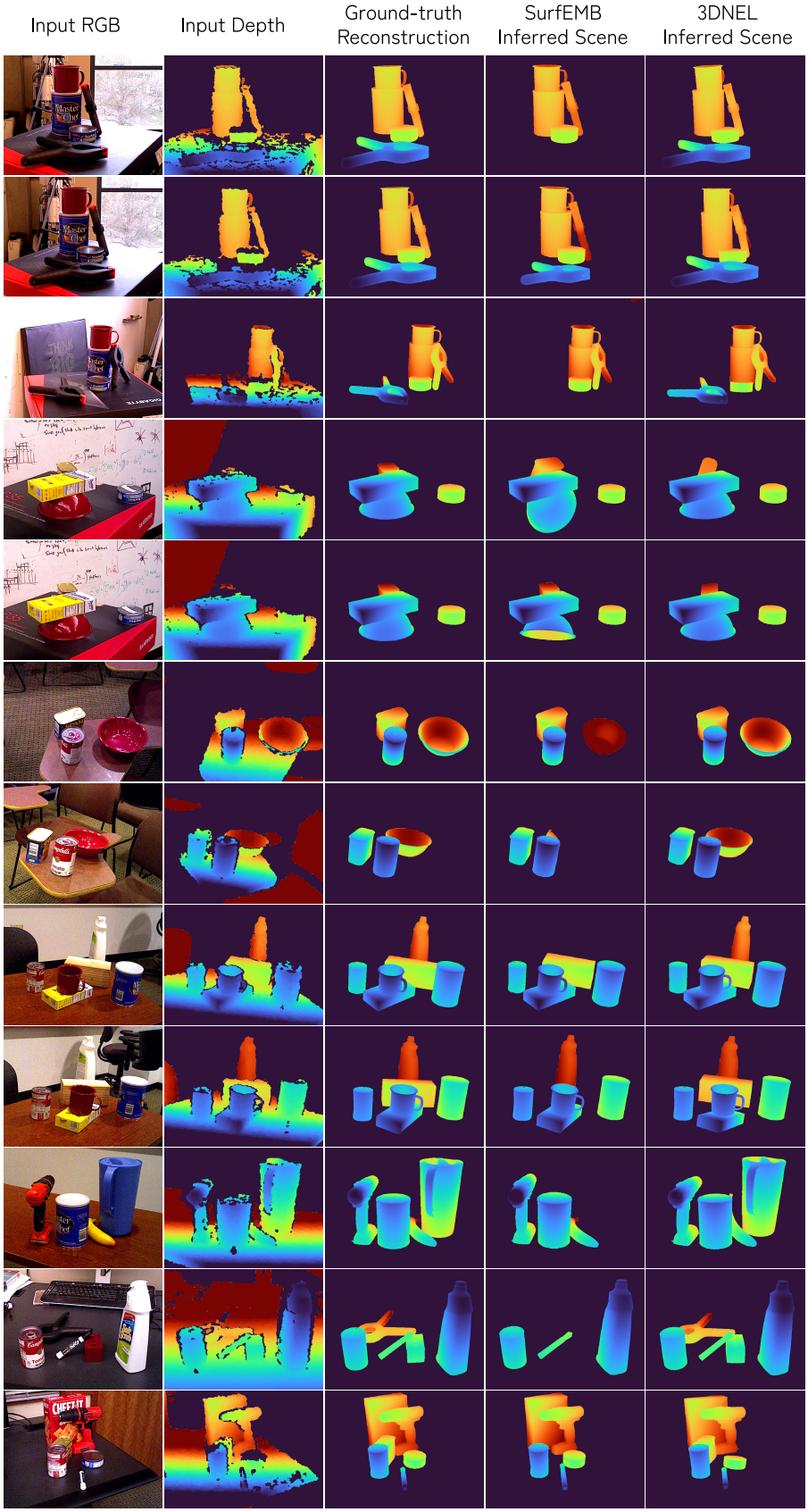

Figure A2: Additional visualizations of SurfEMB and 3DNEL MSIGP predictions on YCB-V test images