A One-Sample Decentralized Proximal Algorithm for Non-Convex Stochastic Composite Optimization

Abstract

We focus on decentralized stochastic non-convex optimization, where n agents work together to optimize a composite objective function which is a sum of a smooth term and a non-smooth convex term. To solve this problem, we propose two single-time scale algorithms: `Prox-DASA` and `Prox-DASA-GT`. These algorithms can find ϵ-stationary points in $\mathcal{O}(n^{-1}\epsilon^{-2})$ iterations using constant batch sizes (i.e., $\mathcal{O}(1)$). Unlike prior work, our algorithms achieve a comparable complexity result without requiring large batch sizes, more complex per-iteration operations (such as double loops), or stronger assumptions. Our theoretical findings are supported by extensive numerical experiments, which demonstrate the superiority of our algorithms over previous approaches.

1 INTRODUCTION

Decentralized optimization is a flexible paradigm for solving complex optimization problems in a distributed manner, and has numerous applications in fields such as machine learning, robotics, and control systems. It has attracted increased attention due to the following benefits: (i) *Robustness*: Decentralized optimization is more robust than centralized optimization because each agent can operate independently, making the system more resilient to failures compared to a centralized system where a coordinator failure or overload can halt the entire system. (ii) *Privacy*: Decentralized optimization can provide greater privacy because each agent only has access to a limited subset of observations, which may help to protect sensitive information. (iii) *Scalability*: Decentralized optimization is highly scalable as it can handle a large datasets in a distributed manner, thereby solving complex optimization problems that are difficult or even impossible to solve in a centralized setting.

Specifically, we consider the following decentralized composite optimization problems in which n agents collaborate to solve

$$\min_{x\in\mathbb{R}^d} \Phi(x) := F(x) + \Psi(x),\ F(x) := \frac{1}{n}\sum_{i=1}^{n} F_i(x), \quad (1)$$

where each function $F_i(x)$ is a smooth function only known to the agent i; $\Psi(x)$ is non-smooth, convex, and shared across all agents; $\Phi(x)$ is bounded below by $\Phi_* > -\infty$. We consider the stochastic setting where the exact function values and derivatives of F_i's are not available. In particular, we assume that $F_i(x) = \mathbb{E}_{\xi_i\sim\mathcal{D}_i}[G_i(x,\xi_i)]$, where ξ_i is a random vector and $\mathcal{D}_i$ is the distribution used to generate samples for agent i. The agents form a connected and undirected network and can communicate with their neighbors to cooperatively solve (1). The communication network can be represented with $G = (V, W)$ where $V = \{v_1, v_2, \ldots, v_n\}$ denotes all devices and $W = [w_{ij}] \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix indicating how two agents are connected.

A majority of the existing decentralized stochastic algorithms for solving (1), require large batch sizes to achieve convergence. The few algorithms that operate with constant batch sizes mainly rely on complicated variance reduction techniques and require stronger assumptions to establish convergence results. To the best of our knowledge, the question of whether it's possible to develop decentralized stochastic optimization algorithms to solve (1) without the above mentioned limitations, remains unresolved. To address this, we propose the two decentralized stochastic proximal algorithms, `Prox-DASA` and `Prox-DASA-GT`, for solving (1) and make the following **contributions**:

- We show that `Prox-DASA` is capable of achieving convergence in both homogenous and bounded heterogeneous settings while `Prox-DASA-GT` works for general decentralized heterogeneous problems.
- We show that both algorithms find an ϵ-stationary point in $\mathcal{O}(n^{-1}\epsilon^{-2})$ iterations using only $\mathcal{O}(1)$ stochastic gradient samples per agent and m communication rounds

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at each iteration, where m can be any positive integer. A topology-independent transient time can be achieved by setting $m = \lceil \frac{1}{\sqrt{1-\rho}} \rceil$.

- Through extensive experiments we demonstrate the superiority of our algorithms over prior works.

A summary of our results and comparison to prior work is provided in Table 1.

Related Works on Decentralized Composite Optimization.

Motivated by wide applications in constrained optimization [Lee and Nedic, 2013, Margellos et al., 2017] and non-smooth problems with a composite structure as (1), arising in signal processing [Ling and Tian, 2010, Mateos et al., 2010, Patterson et al., 2014] and machine learning [Facchinei et al., 2015, Hong et al., 2017], several works have studied the decentralized composite optimization problem in (1), a natural generalization of smooth optimization. For example, Shi et al. [2015], Li et al. [2019], Alghunaim et al. [2019], Ye et al. [2020], Xu et al. [2021], Li et al. [2021], Sun et al. [2022], Wu and Lu [2022] studied (1) in the convex setting. Furthermore, Facchinei et al. [2015], Di Lorenzo and Scutari [2016], Hong et al. [2017], Zeng and Yin [2018], Scutari and Sun [2019] studied (1) in the deterministic setting.

Although there has been a lot of research investigating decentralized composite optimization, the stochastic nonconvex setting, which is more broadly applicable, still lacks a full understanding. Wang et al. [2021] proposes `SPPDM`, which uses a proximal primal-dual approach to achieve $\mathcal{O}(\epsilon^{-2})$ sample complexity. `ProxGT-SA` and `ProxGT-SR-O` [Xin et al., 2021a] incorporate stochastic gradient tracking and multi-consensus update in proximal gradient methods and obtain $\mathcal{O}(n^{-1}\epsilon^{-2})$ and $\mathcal{O}(n^{-1}\epsilon^{-1.5})$ sample complexity respectively, where the latter further uses a `SARAH` type variance reduction method [Pham et al., 2020, Wang et al., 2019]. A recent work [Mancino-Ball et al., 2022] proposes `DEEPSTORM`, which leverages the momentum-based variance reduction technique and gradient tracking to obtain $\mathcal{O}(n^{-1}\epsilon^{-1.5})$ and $\tilde{\mathcal{O}}(\epsilon^{-1.5})$ sample complexity under different stepsize choices. Nevertheless, existing works either require stronger assumptions [Mancino-Ball et al., 2022] or increasing batch sizes [Wang et al., 2021, Xin et al., 2021a].

2 PRELIMINARIES

Notations. $\|\cdot\|$ denotes the ℓ_2-norm for vectors and Frobenius norm for matrices. $\|\cdot\|_2$ denotes the spectral norm for matrices. $\mathbf{1}$ represents the all-one vector, and $\mathbf{I}$ is the identity matrix as a standard practice. We identify vectors at agent i in the subscript and use the superscript for the algorithm step. For example, the optimization variable of agent i at step k is denoted as x_i^k, and z_i^k is the corresponding dual variable. We use uppercase bold letters to represent the matrix that collects all the variables from nodes (corresponding lowercase) as columns. We add an overbar to a letter to denote the average over all nodes. For example, we denote the optimization variables over all nodes at step k as $\mathbf{X}_k = \left[x_1^k, \ldots, x_n^k\right]$. The corresponding average over all notes can be thereby defined as

$$\bar{x}^k = \frac{1}{n}\sum_{i=1}^{n} x_i^k = \frac{1}{n}\mathbf{X}_k\mathbf{1},$$

$$\bar{\mathbf{X}}_k = [\bar{x}^k, \ldots, \bar{x}^k] = \bar{x}^k\mathbf{1}^\top = \frac{1}{n}\mathbf{X}_k\mathbf{1}\mathbf{1}^\top.$$

For an extended valued function $\Psi : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$, its effective domain is written as $\mathrm{dom}(\Psi) = \{x \mid \Psi(x) < +\infty\}$. A function Ψ is said to be proper if $\mathrm{dom}(\Psi)$ is nonempty. For any proper closed convex function Ψ, $x \in \mathbb{R}^d$, and scalar $\gamma > 0$, the proximal operator is defined as

$$\mathbf{prox}_\Psi^\gamma(x) = \arg\min_{y\in\mathbb{R}^d} \left\{\frac{1}{2\gamma}\|y - x\|^2 + \Psi(y)\right\}.$$

For $x, z \in \mathbb{R}^d$ and $\gamma > 0$, the proximal gradient mapping of z at x is defined as

$$\mathcal{G}(x, z, \gamma) = \frac{1}{\gamma}\left(x - \mathbf{prox}_\Psi^\gamma(x - \gamma z)\right).$$

All random objects are properly defined in a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ and write $x \in \mathcal{H}$ if x is $\mathcal{H}$-measurable given a sub-σ-algebra $\mathcal{H} \subseteq \mathscr{F}$ and a random vector x. We use $\sigma(\cdot)$ to denote the σ-algebra generated by all the agument random vectors.

Assumptions. Next, we list and discuss the assumptions made in this work.

Assumption 1. *The weighted adjacency matrix* $\mathbf{W} = (w_{ij}) \in \mathbb{R}^{n\times n}$ *is symmetric and doubly stochastic, i.e.,* $\mathbf{W} = \mathbf{W}^\top, \mathbf{W}\mathbf{1}_n = \mathbf{1}_n, w_{ij} \geq 0, \forall i, j,$ *and its eigenvalues satisfy* $1 = \lambda_1 > \lambda_2 \geq \cdots \geq \lambda_n$ *and* $\rho := \max\{|\lambda_2|, |\lambda_n|\} < 1$.

Assumption 2. *All functions* $\{F_i\}_{1\leq i\leq n}$ *have Lipschitz continuous gradients with Lipschitz constants* $L_{\nabla F_i}$*, respectively. Therefore,* ∇F *is* $L_{\nabla F}$*-Lipchitz continous with* $L_{\nabla F} = \max_{1\leq i\leq n}\{L_{\nabla F_i}\}$.

Assumption 3. *The function* $\Psi : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ *is a closed proper convex function.*

For stochastic oracles, we assume that each node i at every iteration k is able to obtain a local random data vector ξ_i^k. The induced natural filtration is given by $\mathscr{F}_0 = \{\emptyset, \Omega\}$ and

$$\mathscr{F}_k := \sigma\left(\xi_i^t \mid i = 1, \ldots, n,\, t = 1, \ldots, k\right), \forall k \geq 1.$$

We require that the stochastic gradient $\nabla G_i(\cdot, \xi_i^{k+1})$ is unbiased conditioned on the filteration $\mathscr{F}_k$.

Table 1: Comparison of decentralized proximal gradient based algorithms to find an ϵ-stationary solution to stochastic composite optimization in the nonconvex setting. The sample complexity is defined as the number of required samples per agent to obtain an ϵ-stationary point (see Definition 1). We omit a comparison with SPPDM [Wang et al., 2021] as their definition of stationarity differs from ours; see Appendix for further discussions.

Algorithm	Batch Size	Sample Complexity	Communication Complexity	Linear Speedup?	Remark
ProxGT-SA [Xin et al., 2021a]	$\mathcal{O}(\epsilon^{-1})$	$\mathcal{O}(n^{-1}\epsilon^{-2})$	$\mathcal{O}(\log(n)\epsilon^{-1})$	✓	
ProxGT-SR-O [Xin et al., 2021a]	$\mathcal{O}(\epsilon^{-1})$	$\mathcal{O}(n^{-1}\epsilon^{-1.5})$	$\mathcal{O}(\log(n)\epsilon^{-1})$	✓	double-loop; mean-squared smoothness
DEEPSTORM [Mancino-Ball et al., 2022]	$\mathcal{O}(\epsilon^{-0.5})$ then $\mathcal{O}(1)^*$	$\mathcal{O}(n^{-1}\epsilon^{-1.5})$	$\mathcal{O}(n^{-1}\epsilon^{-1.5})$	✓	two time-scale; mean-squared smoothness; double gradient evaluations per iteration
	$\mathcal{O}(1)$	$\mathcal{O}(\epsilon^{-1.5}\lvert\log\epsilon\rvert^{-1.5})$	$\mathcal{O}(\epsilon^{-1.5}\lvert\log\epsilon\rvert^{-1.5})$	✗	
Prox-DASA (Alg. 1)	$\mathcal{O}(1)$	$\mathcal{O}(n^{-1}\epsilon^{-2})$	$\mathcal{O}(n^{-1}\epsilon^{-2})$	✓	bounded heterogeneity
Prox-DASA-GT (Alg. 2)	$\mathcal{O}(1)$	$\mathcal{O}(n^{-1}\epsilon^{-2})$	$\mathcal{O}(n^{-1}\epsilon^{-2})$	✓	

* It requires $\mathcal{O}(\epsilon^{-0.5})$ batch size in the first iteration and then $\mathcal{O}(1)$ for the rest (see m_0 in Algorithm 1 in Mancino-Ball et al. [2022]).

Assumption 4 (Unbiasness)**.** *For any $k \geq 0, x \in \mathscr{F}_k$, and $1 \leq i \leq n$, $\mathbb{E}\left[\nabla G_i(x, \xi_i^{k+1}) \mid \mathscr{F}_k\right] = \nabla F_i(x)$.*

Assumption 5 (Independence)**.** *For any $k \geq 0, 1 \leq i, j \leq n, i \neq j$, ξ_i^{k+1} is independent of $\mathscr{F}_k$, and ξ_i^{k+1} is independent of ξ_j^{k+1}.*

In addition, we consider two standard assumptions on the variance and heterogeneity of stochastic gradients.

Assumption 6 (Bounded variance)**.** *For any $k \geq 0, x \in \mathscr{F}_k$, and $1 \leq i \leq n$, $\mathbb{E}\left[\left\|\nabla G_i(x, \xi_i^{k+1}) - \nabla F_i(x)\right\|^2 \middle| \mathscr{F}_k\right] \leq \sigma_i^2$. Let $\sigma^2 = \frac{1}{n}\sum_{i=1}^n \sigma_i^2$.*

Assumption 7 (Gradient heterogeneity)**.** *There exists a constant $\nu \geq 0$ such that for all $1 \leq i \leq n, x \in \mathbb{R}^d$, $\|\nabla F_i(x) - \nabla F(x)\| \leq \nu$.*

Remark. The above assumption of gradient heterogeneity is standard Lian et al. [2017] and less strict than the bounded second moment assumption on stochastic gradients which implies lipschtizness of functions $\{F_i\}$. However, this assumption is only required for the convergence analysis of Prox-DASA and can be bypassed by employing a gradient tracking step.

3 ALGORITHM

Several algorithms have been developed to solve Problem (1) in the stochastic setting; see Table 1. However, the most recent two types of algorithms that achieve (near)-optimal sample complexities have certain drawbacks: (i) **increasing batch sizes**: ProxGT-SA, Prox-SR-O, and DEEPSTORM with constant step sizes (Theorem 1 in [Mancino-Ball et al., 2022]) require batches of stochastic gradients with batch sizes inverse proportional to tolerance ϵ; (ii) **algorithmic complexities**: ProxGT-SR-O and DEEPSTORM are either double-looped or two-time-scale, and require stochastic gradients evaluated at different parameter values over the same sample, i.e., $\nabla G_i(x, \xi)$ and $\nabla G_i(x', \xi)$. These variance reduction techniques are unfavorable when gradient evaluations are computationally expensive such as forward-backward steps for deep neural networks. (iii) **theoretical weakness**: the convergence analyses of ProxGT-SR-O and DEEPSTORM are established under the *stronger* assumption of mean-squared lipschtizness of stochastic gradients. In addition, Theorem 2 in [Mancino-Ball et al., 2022] fails to provide linear-speedup results for one-sample variant of DEEPSTORM with diminishing stepsizes.

3.1 DECENTRALIZED PROXIMAL AVERAGED STOCHASTIC APPROXIMATION

To address the above limitations, we propose **D**ecentralized **Prox**imal **A**veraged **S**tochastic **A**pproximation (Prox-DASA) which leverages a common averaging technique in stochastic optimation [Ruszczyński, 2008, Mokhtari et al., 2018, Ghadimi et al., 2020] to reduce the error of gradient estimation. In particular, the sequences of dual variables $\mathbf{Z}^k = [z_1^k, \ldots, z_n^k]$ that aim to approximate gradients are defined in the following recursion:

$$\mathbf{Z}^{k+1} = \left\{(1-\alpha_k)\mathbf{Z}^k + \alpha_k \mathbf{V}^{k+1}\right\}\mathbf{W}^m$$
$$\mathbf{V}^{k+1} = [v_1^{k+1}, \ldots, v_n^{k+1}],$$

where each v_i^{k+1} is the local stochastic gradient evaluated at the local variable x_i^k. For complete graphs where each pair of graph vertices is connected by an edge and there is no consensus error for optimization variables, i.e., $\mathbf{W} = \frac{1}{n}\mathbf{1}\mathbf{1}^\top$ and $x_i^k = x_j^k, \forall i, j$, the averaged dual variable over nodes $\bar{z}^k$

Algorithm 1: `Prox-DASA`

Input: $x_i^0 = z_i^0 = \mathbf{0}, \gamma, \{\alpha_k\}_{\geq 0}, m$
for $k = 0, 1, \ldots, K-1$ **do**
  `# Local Update`
  for $i = 1, 2, \ldots, n$ *(in parallel)* **do**
    $y_i^k = \mathbf{prox}_\Psi^\gamma \left(x_i^k - \gamma z_i^k\right)$
    $\tilde{x}_i^{k+1} = (1-\alpha_k)x_i^k + \alpha_k y_i^k$
    `# Compute stochastic gradient`
    $v_i^{k+1} = \nabla G_i(x_i^k, \xi_i^{k+1})$
    $\tilde{z}_i^{k+1} = (1-\alpha_k)z_i^k + \alpha_k v_i^{k+1}$
  end
  `# Communication`
  $[x_1^{k+1}, \ldots, x_n^{k+1}] = [\tilde{x}_1^{k+1}, \ldots, \tilde{x}_n^{k+1}]\mathbf{W}^m$
  $[z_1^{k+1}, \ldots, z_n^{k+1}] = [\tilde{z}_1^{k+1}, \ldots, \tilde{z}_n^{k+1}]\mathbf{W}^m$
end

follows the same averaging rule as in centralized algorithms:

$$\bar{z}^{k+1} = (1-\alpha_k)\bar{z}^k + \alpha_k \bar{v}^{k+1}$$
$$\mathbb{E}[\bar{v}^{k+1}|\mathscr{F}_k] = \nabla F(\bar{x}^k).$$

To further control the consensus errors, we employ a multiple consensus step for both primal and dual iterates $\{x_i^k, z_i^k\}$ which multiply the matrix of variables from all nodes by the weight matrix m times. A pseudo code of `Prox-DASA` is given in Algorithm 1.

3.2 GRADIENT TRACKING

The constant ν defined in Assumption 7 measures the heterogeneity between local gradients and global gradients, and hence the variance of datasets of different agents. To remove ν in the complexity bound, Tang et al. [2018] proposed the D^2 algorithm, which modifies the x update in D-PSGD [Lian et al., 2017]. However, it requires one additional assumption on the eigenvalues of the mixing matrix $\mathbf{W}$. Here we adopt the gradient tracking technique, which was first introduced to deterministic distributed optimization to improve the convergence rate [Xu et al., 2015, Di Lorenzo and Scutari, 2016, Nedic et al., 2017, Qu and Li, 2017], and was later proved to be useful in removing the data variance (i.e., ν) dependency in the stochastic case [Zhang and You, 2019, Lu et al., 2019, Pu and Nedić, 2021, Koloskova et al., 2021]. In the convergence analysis of `Prox-DASA`, an essential step is to control the heterogeneity of stochastic gradients, i.e., $\mathbb{E}[\left\|\mathbf{V}^{k+1} - \bar{\mathbf{V}}^{k+1}\right\|^2]$, which requires bounded heterogeneity of local gradients (Assumption 7). To pypass this assumption, we employ a gradient tracking step by replacing $\mathbf{V}^{k+1}$ with pseudo stochastic gradients $\mathbf{U}^{k+1} = [u_1^{k+1}, \ldots, u_n^{k+1}]$, which is updated as follows:

$$\mathbf{U}^{k+1} = \left(\mathbf{U}^k + \mathbf{V}^{k+1} - \mathbf{V}^k\right)\mathbf{W}^m.$$

Provided that $\mathbf{U}^0 = \mathbf{V}^0$ and $\mathbf{W}\mathbf{1} = \mathbf{1}$, one can show that $\bar{u}^k = \bar{v}^k$ at each step k. In addition, with the consensus procedure over $\mathbf{U}^k$, the heterogeneity of pseudo stochastic gradients $\mathbb{E}[\left\|\mathbf{U}^{k+1} - \bar{\mathbf{U}}^{k+1}\right\|^2]$ can be bounded above. The proposed algorithm, which we name as `Prox-DASA` with Gradient Tracking (`Prox-DASA-GT`), is presented in Algorithm 2.

Algorithm 2: `Prox-DASA-GT`

Input: $x_i^0 = z_i^0 = \mathbf{0}, \gamma, \{\alpha_k\}_{\geq 0}, m$
for $k = 0, 1, \ldots, K$ **do**
  `# Local Update`
  for $i = 1, 2, \ldots, n$ *(in parallel)* **do**
    $y_i^k = \mathbf{prox}_\Psi^\gamma \left(x_i^k - \gamma z_i^k\right)$
    $\tilde{x}_i^{k+1} = (1-\alpha_k)x_i^k + \alpha_k y_i^k$
    `# Compute stochastic gradient`
    $v_i^{k+1} = \nabla G_i(x_i^k, \xi_i^{k+1})$
    $\tilde{u}_i^{k+1} = u_i^k + v_i^{k+1} - v_i^k$
    $\tilde{z}_i^{k+1} = (1-\alpha_k)z_i^k + \alpha_k u_i^k$
  end
  `# Communication`
  $[x_1^{k+1}, \ldots, x_n^{k+1}] = [\tilde{x}_1^{k+1}, \ldots, \tilde{x}_n^{k+1}]\mathbf{W}^m$
  $[u_1^{k+1}, \ldots, u_n^{k+1}] = [\tilde{u}_1^{k+1}, \ldots, \tilde{u}_n^{k+1}]\mathbf{W}^m$
  $[z_1^{k+1}, \ldots, z_n^{k+1}] = [\tilde{z}_1^{k+1}, \ldots, \tilde{z}_n^{k+1}]\mathbf{W}^m$
end

3.3 CONSENSUS ALGORITHM

In practice, we can leverage accelerated consensus algorithms, e.g., Liu and Morse [2011], Olshevsky [2017], to speed up the multiple consensus step $\mathbf{W}^m$ to achieve improved communication complexities when $m > 1$. Specifically, we can replace $\mathbf{W}^m$ by a Chebyshev-type polynomial of $\mathbf{W}$, which can improve the ρ-dependency of the communication complexity from a factor of $\frac{1}{1-\rho}$ to $\frac{1}{\sqrt{1-\rho}}$.

4 CONVERGENCE ANALYSIS

4.1 NOTION OF STATIONARITY

For centralized optimization problems with non-convex objective function $F(x)$, a standard measure of non-stationarity of a point $\bar{x}$ is the squared norm of proximal gradient mapping of $\nabla F(\bar{x})$ at $\bar{x}$, i.e.,

$$\|\mathcal{G}(\bar{x}, \nabla F(\bar{x}), \gamma)\|^2 = \left\|\frac{1}{\gamma}\left(x - \mathbf{prox}_\Psi^\gamma(\bar{x} - \gamma\nabla F(\bar{x}))\right)\right\|^2.$$

For the smooth case where $\Psi(x) \equiv 0$, the above measure is reduced to $\|\nabla F(\bar{x})\|^2$.

However, in the decentralized setting with a connected network G, we solve the following equivalent reformulated

consensus optimization problem:

$$\begin{aligned} \min_{x_1,\ldots,x_n\in\mathbb{R}^d} \quad & \frac{1}{n}\sum_{i=1}^{n}\{F_i(x_i)+\Psi(x_i)\} \\ \text{s.t.} \quad & x_i = x_j,\ \forall (i,j). \end{aligned} \tag{2}$$

To measure the non-stationarity in Problem (2), one should not only consider the stationarity violation at each node but also the consensus errors over the network. Therefore, Xin et al. [2021a] and Mancino-Ball et al. [2022] define an ϵ-stationary point $\mathbf{X} = [x_1, \ldots, x_n]$ of Problem 2 as

$$\mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}\left\{\|\mathcal{G}(x_i,\nabla F(x_i),\gamma)\|^2 + L_{\nabla F}^2\|x_i-\bar{x}\|^2\right\}\right] \le \epsilon. \tag{3}$$

In this work, we use a general measure as follows.

Definition 1. *Let $\mathbf{X} = [x_1, \ldots, x_n]$ be random vectors generated by a decentralized algorithm to solve Problem 2 and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$. We say that $\mathbf{X}$ is an ϵ-stationary point of Problem 2 if*

$$\begin{aligned} &\textit{(stationarity violation)} \quad \mathbb{E}\left[\|\mathcal{G}(\bar{x},\nabla F(\bar{x}),\gamma)\|^2\right] \le \epsilon, \\ &\textit{(consensus error)} \quad \mathbb{E}\left[\tfrac{L_{\nabla F}^2}{n}\left\|\mathbf{X}-\bar{\mathbf{X}}\right\|^2\right] \le \epsilon. \end{aligned}$$

The next inequality characterizes the difference between the gradient mapping at $\bar{x}$ and x_i, which relates our definition to (3). Noting that by non-expansiveness of the proximal operator, we have $\|\mathcal{G}(x_i,\nabla F(x_i),\gamma) - \mathcal{G}(\bar{x},\nabla F(\bar{x}),\gamma)\| \le \frac{2+\gamma L_{\nabla F}}{\gamma}\|x_i-\bar{x}\|$, implying $\frac{1}{n}\sum_{i=1}^{n}\|\mathcal{G}(x_i,\nabla F(x_i),\gamma)\|^2$

$$\lesssim \|\mathcal{G}(\bar{x},\nabla F(\bar{x}),\gamma)\|^2 + \tfrac{1}{\gamma^2 n}\left\|\mathbf{X}-\bar{\mathbf{X}}\right\|^2.$$

4.2 MAIN RESULTS

We present the complexity results of our algorithms below.

Theorem 2. *Suppose Assumption 1, 2, 3, 4, 5, 6 hold and the total number of iterations $K \ge K_0$, where K_0 is a constant that only depends on constants $(n, L_{\nabla F}, \varrho(m), \gamma)$, where $\varrho(m) = \frac{(1+\rho^{2m})\rho^{2m}}{(1-\rho^{2m})^2}$. Let C_0 be some initialization-dependent constant and R be a random integer uniformly distributed over $\{1, 2, \ldots, K\}$. If we set $\alpha_k \asymp \sqrt{\frac{n}{K}}, \gamma \asymp \frac{1}{L_{\nabla F}}$, then the followings hold.*

(Prox-DASA) *Suppose Assumption 7 also holds. In Algorithm 1 we have*

$$\begin{aligned} &\mathbb{E}\left[\left\|\mathcal{G}(\bar{x}^R,\nabla F(\bar{x}^R),\gamma)\right\|^2\right] \\ &\quad\lesssim \frac{\gamma^{-1}C_0+\sigma^2}{\sqrt{nK}} + \frac{n(\sigma^2+\gamma^{-2}\nu^2)\varrho(m)}{K}, \\ &\mathbb{E}\left[\frac{L_{\nabla F}^2}{n}\left\|\mathbf{X}_R-\bar{\mathbf{X}}_R\right\|^2\right] \lesssim \frac{n(\sigma^2+\gamma^{-2}\nu^2)\varrho(m)}{K}. \end{aligned}$$

(Prox-DASA-GT) *In Algorithm 2 we have*

$$\begin{aligned} &\mathbb{E}\left[\left\|\mathcal{G}(\bar{x}^R,\nabla F(\bar{x}^R),\gamma)\right\|^2\right] \lesssim \frac{\gamma^{-1}C_0+\sigma^2}{\sqrt{nK}} + \frac{n\sigma^2\varrho(m)}{K}, \\ &\mathbb{E}\left[\frac{L_{\nabla F}^2}{n}\left\|\mathbf{X}_R-\bar{\mathbf{X}}_R\right\|^2\right] \lesssim \frac{n\sigma^2\varrho(m)}{K}. \end{aligned}$$

In Theorem 2 for simplicity we assume $\gamma \asymp \frac{1}{L_{\nabla F}}$, which can be relaxed to $\gamma > 0$. We have the following corollary characterizing the complexity of Algorithm 1 and 2 for finding ϵ-stationary points. The proof is immediate.

Corollary 3. *Under the same conditions of Theorem 2, provided that $K \gtrsim n^3\varrho(m)$, for any $\epsilon > 0$ the sample complexity per agent for finding ϵ-stationary points in Algorithm 1 and 2 are $\mathcal{O}(\max\{n^{-1}\epsilon^{-2}, K_T\})$ where the transient time $K_T \asymp \max\{K_0, n^3\varrho(m)\}$.*

Remark (Sample Complexity). For a sufficiently small $\epsilon > 0$, Corrollary 3 implies that the sample complexity of Algorithm 1 and 2 matches the optimal lower bound $\mathcal{O}(n^{-1}\epsilon^{-2})$ in decentralized stochastic non-convex optimization [Lu and De Sa, 2021].

Remark (Transient Time and Communication Complexity). Our algorithms are able to achieve convergence with a single communication round per iteration, i.e., $m = 1$, leading to a topology-independent $\mathcal{O}(n^{-1}\epsilon^{-2})$ communication complexity. In this case, however, the transient time K_T still depends on ρ, as is also the case for smooth optimization problems [Xin et al., 2021b]. If we consider multiple consensus steps per iteration with the communication complexity being $\mathcal{O}(mn^{-1}\epsilon^{-2})$, setting $m \asymp \lceil\frac{1}{1-\rho}\rceil$ (or $m \asymp \lceil\frac{1}{\sqrt{1-\rho}}\rceil$ for accelerated consensus algorithms) results in a topology-independent transient time given that $\varrho(m) \asymp 1$.

4.3 PROOF SKETCH

Here, we present a sketch of our convergence analyses and defer details to Appendix. Our proof relies on the merit function below:

$$\begin{aligned} W(\bar{x}^k,\bar{z}^k) = &\underbrace{\Phi(\bar{x}^k)-\Phi_*}_{\text{function value gap}} + \underbrace{\Psi(\bar{x}^k)-\eta(\bar{x}^k,\bar{z}^k)}_{\text{primal convergence}} \\ &+ \lambda\underbrace{\left\|\nabla F(\bar{x}^k)-\bar{z}^k\right\|^2}_{\text{dual convergence}}, \end{aligned}$$

where $\eta(x,z) = \min_{y\in\mathbb{R}^d}\left\{\langle z, y-x\rangle + \frac{1}{2\gamma}\|y-x\|^2 + \Psi(y)\right\}$. Let $y_+^k := \mathbf{prox}_\Psi^\gamma\left(\bar{x}^k - \gamma\bar{z}^k\right)$. Then, the proximal gradient mapping of $\bar{z}^k$ at $\bar{x}^k$ is $\mathcal{G}(\bar{x}^k,\bar{z}^k,\gamma) = \frac{1}{\gamma}(\bar{x}^k - y_+^k)$. Since y_+^k is the minimizer of a $1/\gamma$-strongly convex function, we have

$$\begin{aligned} \left\langle \bar{z}^k, y_+^k - \bar{x}^k\right\rangle + &\frac{1}{2\gamma}\|y_+^k-\bar{x}^k\|^2 + \Psi(y_+^k) \\ &\le \Psi(\bar{x}^k) - \frac{1}{2\gamma}\|y_+^k-\bar{x}^k\|^2, \end{aligned}$$

which implies $\Psi(\bar{x}^k) - \eta(\bar{x}^k, \bar{z}^k) \geq \frac{\gamma}{2}\left\|\mathcal{G}(\bar{x}^k, \bar{z}^k, \gamma)\right\|^2$.

Following standard practices in optimization, we set $\gamma = \frac{1}{L_{\nabla F}}$ below for simplicity. However, our algorithms do not require any restriction on the choice of γ.

Step 1: Leveraging the merit function with $\lambda \asymp \gamma$, we can first obtain an essential lemma (Lemma 11 in Appendix) in our analyses, which says that for sequences $\{x_i^k, z_i^k\}_{1\leq i\leq n, k\geq 0}$ generated by `Prox-DASA(-GT)` (Algorithm 1 or 2) with $\alpha_k \lesssim \min\{1, (1+\gamma)^{-2}, \gamma^2(1+\gamma)^{-2}\}$, we have

$$W(\bar{x}^{k+1}, \bar{z}^{k+1}) - W(\bar{x}^k, \bar{z}^k) \leq -\alpha_k\left\{\Theta^k + \Upsilon^k + \alpha_k\Lambda^k + r^{k+1}\right\},$$

where $\mathbb{E}[r^{k+1} \mid \mathscr{F}_k] = 0$, $\Lambda^k \asymp \gamma\left\|\bar{\Delta}^{k+1}\right\|^2$,

$$\Theta^k \asymp \frac{1}{\gamma}\|\bar{x}^k - \bar{y}^k\|^2 + \gamma\left\|\nabla F(\bar{x}^k) - \bar{z}^k\right\|^2,$$

$$\Upsilon^k \asymp \frac{\gamma}{n}\left\|\mathbf{Z}_k - \bar{\mathbf{Z}}_k\right\|^2 + \frac{1}{n\gamma}\left\|\mathbf{X}_k - \bar{\mathbf{X}}_k\right\|^2,$$

and $\bar{\Delta}^{k+1} = \bar{v}^{k+1} - \frac{1}{n}\sum_{i=1}^n \nabla F_i(x_i^k) = \bar{u}^{k+1} - \frac{1}{n}\sum_{i=1}^n \nabla F_i(x_i^k)$ (for `Prox-DASA-GT`). Thus, by telescoping and taking expectation with respect to $\mathscr{F}_0$, we have

$$\begin{aligned}&\sum_{k=0}^K \alpha_k \mathbb{E}\left[\left\|\bar{x}^k - \bar{y}^k\right\|^2 + \gamma^2\left\|\nabla F(\bar{x}^k) - \bar{z}^k\right\|^2\right] \\ &\lesssim \gamma W(\bar{x}^0, \bar{z}^0) + \gamma^2\sigma^2\boxed{\sum_{k=0}^K \frac{\alpha_k^2}{n}} \\ &\quad + \sum_{k=0}^K \frac{\alpha_k\left\{\mathbb{E}\left[\left\|\mathbf{X}_k - \bar{\mathbf{X}}_k\right\|^2 + \gamma^2\left\|\mathbf{Z}_k - \bar{\mathbf{Z}}_k\right\|^2\right]\right\}}{n}.\end{aligned} \tag{4}$$

Step 2: We then analyze the consensus errors. Without loss of generality, we consider $\mathbf{X}_0 = \bar{\mathbf{X}}_0 = \mathbf{0}$, i.e., all nodes have the same initialization at $\mathbf{0}$. For $m \in \mathbb{N}_+$, define

$$\varrho(m) = \frac{(1+\rho^{2m})\rho^{2m}}{(1-\rho^{2m})^2}.$$

Then, we have the following fact:

- $\varrho(m)$ is monotonically decreasing with the maximum value being $\varrho(1) = \frac{(1+\rho^2)\rho^2}{(1-\rho^2)^2} := \varrho_1$;
- $\varrho(m) \leq 1$ if and only if $\rho^{2m} \leq \frac{1}{3}$.

With the definition of $\varrho(m)$ and assuming $0 < \alpha_{k+1} \leq \alpha_k \leq 1$, we can show the consensus errors have the following upper bounds.

`Prox-DASA`: Let $\alpha_k \lesssim \varrho(m)^{-\frac{1}{2}}$, we have

$$\begin{aligned}\sum_{k=0}^K \frac{\alpha_k}{n}\mathbb{E}\left[\left\|\mathbf{X}_k - \bar{\mathbf{X}}_k\right\|^2\right] &\leq \sum_{k=0}^K \frac{\gamma^2\alpha_k}{n}\mathbb{E}\left[\left\|\mathbf{Z}_k - \bar{\mathbf{Z}}_k\right\|^2\right] \\ &\lesssim (\gamma^2\sigma^2 + \nu^2)\varrho(m)\boxed{\sum_{k=0}^K \alpha_k^3}.\end{aligned} \tag{5}$$

`Prox-DASA-GT`: Let $\alpha_k \lesssim \min\{\varrho(m)^{-1}, \varrho(m)^{-\frac{1}{2}}\}$, we have

$$\begin{aligned}\sum_{k=0}^K \frac{\alpha_k}{n}\mathbb{E}\left[\|\mathbf{X}_k - \bar{\mathbf{X}}_k\|^2\right] &\leq \sum_{k=0}^K \frac{\gamma^2\alpha_k}{n}\mathbb{E}\left[\|\mathbf{Z}_k - \bar{\mathbf{Z}}_k\|^2\right] \\ &\lesssim \varrho(m)^2\boxed{\sum_{k=0}^K \alpha_k^3}\left\{\gamma^2\sigma^2 + \alpha_k^2\mathbb{E}\left[\|\bar{x}^k - \bar{y}^k\|^2\right]\right\}.\end{aligned} \tag{6}$$

We can also see that to obtain a topology-independent iteration complexity, the number of communication rounds can be set as $m = \lceil\frac{\log 3}{2(1-\rho)}\rceil$, which implies $\varrho(m) \leq 1$.

In addition, we have the following fact that relates the consensus error of $\mathbf{Y}$ to the consensus errors of $\mathbf{X}$ and $\mathbf{Z}$:

$$\begin{aligned}\left\|y_+^k - \bar{y}^k\right\|^2 + \frac{1}{n}\left\|\mathbf{Y}_k - \bar{\mathbf{Y}}_k\right\|^2 &= \frac{1}{n}\sum_{i=1}^n\left\|y_i^k - y_+^k\right\|^2 \\ &\leq \frac{2}{n}\left\{\|\mathbf{X}_k - \bar{\mathbf{X}}_k\|^2 + \gamma^2\|\mathbf{Z}_k - \bar{\mathbf{Z}}_k\|^2\right\}.\end{aligned} \tag{7}$$

Step 3: Let R be a random integer with

$$\Pr(R = k) = \frac{\alpha_k}{\sum_{k=1}^K \alpha_k}, \quad k = 1, 2, \ldots, K,$$

and dividing both sides of (5) by $\sum_{k=1}^K \alpha_k$, we can obtain that for `Prox-DASA`, the consensus error of $\mathbf{X}_R$ satisfies

$$\mathbb{E}\left[\frac{1}{n}\left\|\mathbf{X}_R - \bar{\mathbf{X}}_R\right\|^2\right] \lesssim (\gamma^2\sigma^2 + \nu^2)\varrho(m)\frac{\sum_{k=0}^K \alpha_k^3}{\sum_{k=1}^K \alpha_k}.$$

Moreover, noting that

$$\begin{aligned}\|\mathcal{G}(\bar{x}, \nabla F(\bar{x}), \gamma)\|^2 \lesssim &\frac{1}{\gamma^2}\left\{\left\|\bar{x}^k - \bar{y}^k\right\|^2 + \left\|y_+^k - \bar{y}^k\right\|^2\right\} \\ &+ \left\|\nabla F(\bar{x}^k) - \bar{z}^k\right\|^2,\end{aligned}$$

and combining (4) with (5), we can get

$$\begin{aligned}\mathbb{E}\left[\left\|\mathcal{G}(\bar{x}^R, \nabla F(\bar{x}^R), \gamma)\right\|^2\right] \lesssim &\underbrace{\boxed{\frac{W(\bar{x}^0, \bar{z}^0)}{\gamma\sum_{k=1}^K \alpha_k}}}_{\text{initialization-related term}} \\ &+ \underbrace{\boxed{\sigma^2\frac{\sum_{k=0}^K \alpha_k^2}{n\sum_{k=1}^K \alpha_k}}}_{\text{variance-related term}} + \underbrace{\boxed{(\sigma^2 + \gamma^{-2}\nu^2)\varrho(m)\frac{\sum_{k=0}^K \alpha_k^3}{\sum_{k=1}^K \alpha_k}}}_{\text{consensus error}}.\end{aligned}$$

Thus, setting $\alpha_k \asymp \sqrt{\frac{n}{K}}$, we obtain the convergence results of `Prox-DASA`:

$$\begin{aligned}&\mathbb{E}\left[\left\|\mathcal{G}(\bar{x}^R, \nabla F(\bar{x}^R), \gamma)\right\|^2\right] \\ &\quad\lesssim \frac{\gamma^{-1}W(\bar{x}^0, \bar{z}^0) + \sigma^2}{\sqrt{nK}} + \frac{n(\sigma^2 + \gamma^{-2}\nu^2)\varrho(m)}{K}, \\ &\mathbb{E}\left[\frac{1}{\gamma^2 n}\left\|\mathbf{X}_R - \bar{\mathbf{X}}_R\right\|^2\right] \lesssim \frac{n(\sigma^2 + \gamma^{-2}\nu^2)\varrho(m)}{K}.\end{aligned}$$

For `Prox-DASA-GT`, we can complete the proof with similar arguments by combining (6) with (4) and noting that $\varrho(m)^2\alpha_k^4 \lesssim 1$.

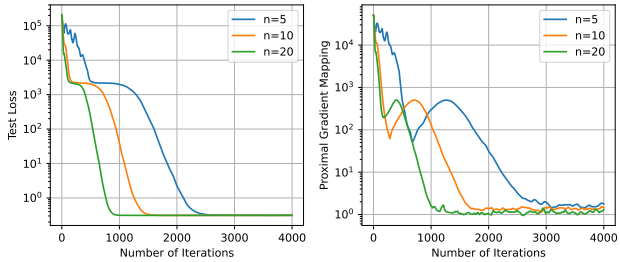

Figure 1: Linear-speedup performance of `Prox-DASA` for decentralized online sparse phase retrievel problems. (`Prox-DASA-GT` has relatively the same plots)

5 EXPERIMENTS

5.1 SYNTHETIC DATA

To demonstrate the effectiveness of our algorithms, we first evaluate our algorithms using synthetic data for solving sparse single index models [Alquier and Biau, 2013] in the decentralized setting. We consider the homogeneous setting where the data sample at each node $\xi = (X, Y)$ is generated from the same single index model $Y = g(X^\top \theta_*) + \varepsilon$, where $X, \theta \in \mathbb{R}^d$ and $\mathbb{E}[\varepsilon|X] = 0$. In this case, we solve the following L_1-regularized least square problems:

$$\min_{\theta \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} \mathop{\mathbb{E}}_{(X,Y)\sim\mathcal{D}} \left[(Y - g(X^\top \theta))^2 \right] + \lambda \|\theta\|_1$$

In particular, we set $\theta_* \in \mathbb{R}^{100}$ to be a sparse vector and $g(\cdot) = (\cdot)^2$ which corresponds to the sparse phase retrieval problem [Jaganathan et al., 2016]. We simulate streaming data samples with batch size $= 1$ for training and 10,000 data samples per node for evaluations, where X and ϵ are sampled independently from two gaussian distributions. We employ a ring topology for the network where self-weighting and neighbor weights are set to be $1/3$. We set the penalty parameter $\lambda = 0.01$, the total number of iterations $K = 10,000$, $\alpha_k = \sqrt{n/K}$, $\gamma = 0.01$, and the number of communication rounds per iteration $m = \lceil \frac{1}{1-\rho} \rceil$. We plot the test loss and the norm of proximal gradient mapping in the log scale against the number of iterations in Figure 1, which shows that our decentralized algorithms have an additional linear speed-up with respect to n. In other words, the algorithms become faster as more agents are added to the network.

5.2 REAL-WORLD DATA

Following Mancino-Ball et al. [2022], we consider solving the classification problem

$$\min_{\theta \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{|\mathcal{D}_i|} \sum_{(x,y)\in\mathcal{D}_i} \ell_i(f(\theta, x), y) + \lambda \|\theta\|_1, \quad (8)$$

on a9a and MNIST datasets[1]. Here, ℓ_i denotes the cross entropy loss and f represents a neural network parameterized by θ with x being its input. $\mathcal{D}_i$ is the training set only available to agent i. The L_1 regularization term is used to impose a sparsity structure on the neural network. We use the code in Mancino-Ball et al. [2022] for `SPPDM`, `ProxGT-SR-O/E`, `DEEPSTORM`, and then implement `Prox-DASA` and `Prox-DASA-GT` under their framework, which mainly utilizes PyTorch [Paszke et al., 2019] and mpi4py [Dalcin and Fang, 2021]. We use a 2-layer perception model on a9a and the LeNet architecture [LeCun et al., 2015] for the MNIST dataset. We have 8 agents which connect in the form of a ring for a9a and a random graph for MNIST. To demonstrate the performance of our algorithms in the constant batch size setting, the batch size is chosen to be 4 for a9a and 32 for MNIST for all algorithms. The number of communication rounds per iteration m is set to be 1 for all algorithms. We evaluate the model performance periodically during training, and then plot the results in Figure 2, from which we observe that both `Prox-DASA` and `Prox-DASA-GT` have considerably good performance with small variance in terms of test accuracy, training loss, and stationarity. In particular, it should be noted that although `DEEPSTORM` achieves better stationarity in Figure 2(l) and 2(i), training a neural network by using `DEEPSTORM` takes longer time than `Prox-DASA` and `Prox-DASA-GT` since it uses the momentum-based variance reduction technique, which requires **two forward-backward passes** (see, e.g., Eq. (10) and Algorithm 1 in Mancino-Ball et al. [2022]) to compute the gradients in one iteration per agent while ours only require **one**, which saves a large amount of time (see Table 1 in Appendix). We include further details of our experiments in the Appendix.

6 CONCLUSION

In this work, we propose and analyze a class of single time-scale decentralized proximal algorithms (`Prox-DASA-(GT)`) for non-convex stochastic composite optimization in the form of (1). We show that our algorithms achieve linear speed-up with respect to the number of agents using an $\mathcal{O}(1)$ batch size per iteration under mild assumptions. Furthermore, we demonstrate the efficiency and effectiveness of our algorithms through extensive experiments, in which our algorithms achieve relatively better results with less training time using a small batch size comparing to existing methods.

[1]Available at https://www.openml.org.

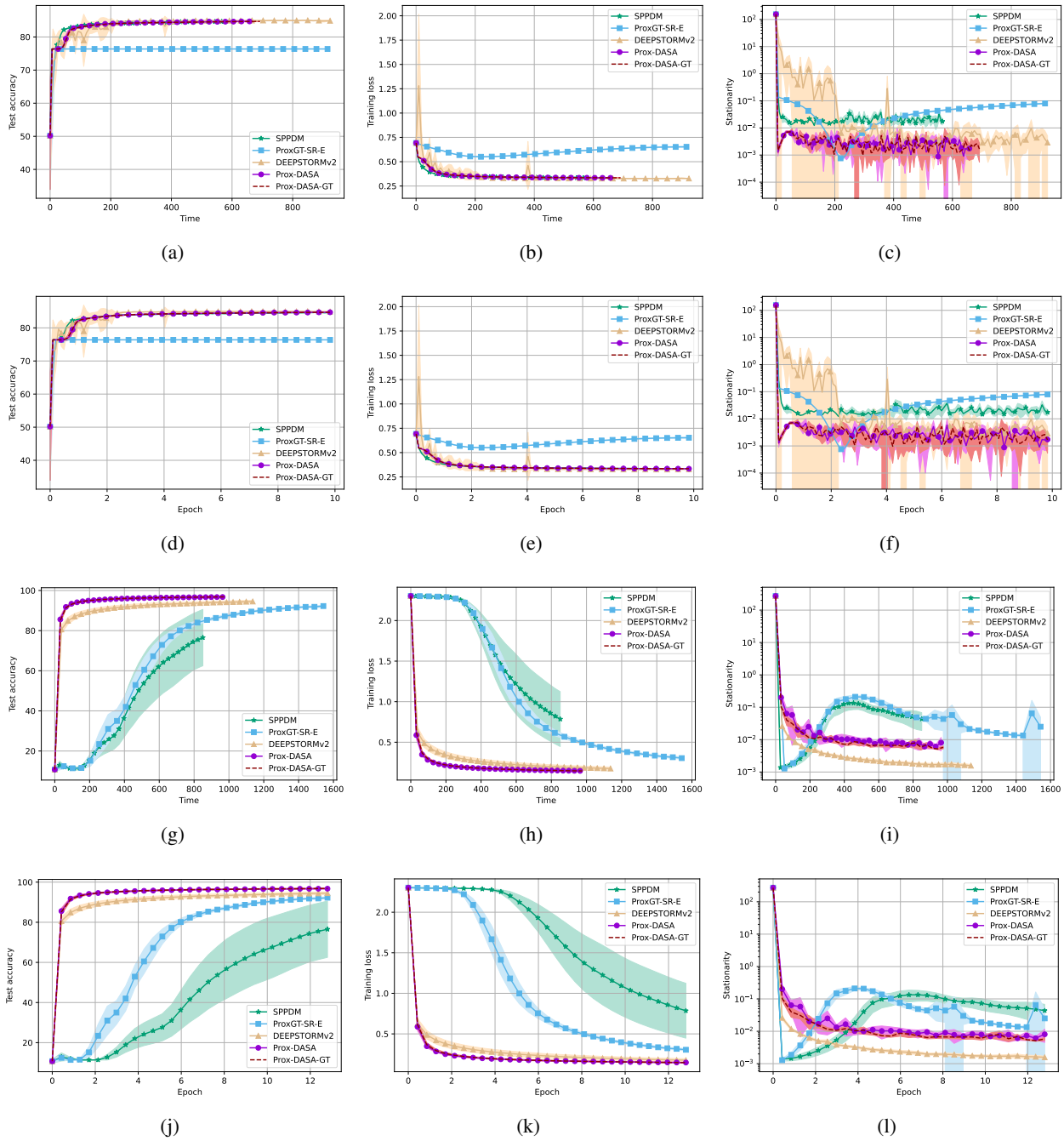

Figure 2: Comparisons between SPPDM [Wang et al., 2021], ProxGT−SR−E [Xin et al., 2021a], DEEPSTORM [Mancino-Ball et al., 2022], Prox−DASA 1, and Prox−DASA−GT 2. The first two rows correspond to a9a and the last two rows correspond to MNIST. The results are averaged over 10 trials, and the shaded regions represent confidence intervals. The vertical axes in the third column are log-scale. It should be noted that ProxGT−SR−E maintains another hyperparameter q (see, e.g., Algorithm 4 and Theorem 3 in [Xin et al., 2021a]) and computes gradients using a full batch every q iterations. For simplicity, we do not include that amount of epochs when we plot this figure. In other words, the real number of epochs required to obtain a point on ProxGT−SR is larger than plotted in the figures in the second and fourth rows. We include the plots that take q into account in Appendix.

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