Understanding Programmatic Weak Supervision via Source-aware Influence Function

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Abstract

Programmatic Weak Supervision (PWS) aggregates the source votes of multiple weak supervision sources into probabilistic training labels, which are in turn used to train an end model. With its increasing popularity, it is critical to have some tool for users to understand the influence of each component (*e.g.*, the source vote or training data) in the pipeline and interpret the end model behavior. To achieve this, we build on Influence Function (IF) and propose source-aware IF, which leverages the generation process of the probabilistic labels to decompose the end model's training objective and then calculate the influence associated with each (data, source, class) tuple. These primitive influence score can then be used to estimate the influence of individual component of PWS, such as source vote, supervision source, and training data. On datasets of diverse domains, we demonstrate multiple use cases: (1) interpreting incorrect predictions from multiple angles that reveals insights for debugging the PWS pipeline, (2) identifying mislabeling of sources with a gain of 9%-37% over baselines, and (3) improving the end model's generalization performance by removing harmful components in the training objective (13%-24% better than ordinary IF).

1 Introduction

One of the major bottlenecks for deploying modern machine learning models is the need for a substantial amounts of human-labeled training data, often annotated over long periods of time and at great expense. As machine learning models become increasingly powerful but also data hungry, new "data-centric" AI development workflows and systems have emerged, wherein the labeling and development of this training data is positioned and supported as the central development activity. One recent and increasingly popular type of data-centric AI development uses **Programmatic Weak Supervision (PWS)**, wherein users focus on developing a diversity of noisy, programmatic supervision sources [32, 33, 47, 45] to programmatically annotate training data in an efficient way. Specifically, these *weak supervision sources*, *e.g.*, heuristics, knowledge bases, and pre-trained models, are often abstracted as *labeling functions (LFs)* [33], which is a user-defined program that provides potentially noisy labels for some subset of the data. So far, different modeling techniques have been developed to aggregate the noisy votes of LFs to produce training labels (often referred to as a *label model*) [33, 31, 14, 40]. Finally, these training labels are in turn used to train an *end model* for the downstream classification tasks. In this study, we focus on this *two-stage* PWS pipeline [45].

With the increasing popularity of PWS in various applications [13, 36, 40, 19], it is of great importance to provide practitioners with a tool that helps them understand the behavior of a trained end model as an effect of each upstream component (*e.g.*, source vote, training data, LFs, *etc.*) involved in the model's training process. First, by understanding what are the deciding factors that lead to a model's specific prediction, users are able to verify whether such influence is desirable in terms of critical aspects such as model safety and fairness [11, 35, 16]. Secondly, since developing LFs is still

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inevitably a partly manual process in real-world applications and may involve human expert in an iterative process [5, 20, 48], it is beneficial to offer users with feedback of how individual component (like each LF) influences the end model performance on the downstream task. Finally, the rendered understanding of (in)efficacy of the PWS pipeline can be naturally exploited to automatically improve the end model performance.

Towards the goal of developing tools to help understand the model behavior, Influence Function (IF) [22, 15, 8] has been recently applied to interpret the model prediction as influence of the training data for large-scale applications. However, most of existing IF studies are unaware of the generation process of training labels like PWS and only estimate the influence of the training data [22, 3]. Notably, the group IF study [23] has attempted to apply IF on PWS, but it simply estimates the influence of each LF as summation of data influence and is limited to a specified PWS without learnable label model. In this work, we aim to develop a general framework that helps estimate the influence of each PWS's component on the end model and is aware of (yet computationally agnostic to) the choice of label model. To achieve this, we leverage the knowledge of how the probabilistic labels are synthesized from sources and accordingly decompose the training loss of each data into multiple terms, each of which is associated with what we called (i, j, c)-*effect*, *i.e.*, the effect of j-th LF on c-th dimension (corresponding to class c) of the i-th data's probabilistic label. With such a training loss decomposition, we propose two means of calculating our *source-aware IF*, namely, reweighting and weight-moving, based on different perturbations on the training respectively. Each estimated source-aware influence score can be interpreted as the influence of removing the (i, j, c)-effect from the pipeline. By simply aggregating over specific dimension(s), it can be utilized to estimate the influence of different components of PWS including (1) each source vote, (2) each LF and (3) each training data, since the influence score is additive [23].

In experiments, we exploit the source-aware IF in a diversity of 13 classification datasets including tabular, text, and image data, and present multiple applications. First, we show that source-aware IF can help to understand the behavior of the end model in terms of the PWS components; in particular, when interpreting the same incorrect prediction made by two PWS pipelines with different label models, data-level IF might lead to the same influential training data, whereas source-aware IF is capable of revealing the most responsible LF or source vote. Second, we use source-aware IF as a tool to identify mislabeling of each LF and show that it outperforms baselines with a significant margin of 9%-37%. Finally, we demonstrate that the training loss decomposition and source-aware IF enable fine-grained training loss perturbation and consequently lead to better test loss improvement (13%-24% improvement over ordinary IF [22] and group IF [23]).

To summarize, this work makes the following contributions. **First**, we propose source-aware IF tailored for PWS that incorporates our knowledge of the generation process of training labels. **Second**, we develop two means of calculating the source-aware IF based on different training loss perturbations. **Third**, we demonstrate a variety of use cases of the source-aware IF and conduct extensive quantitative and qualitative experiments on real datasets in different domains to verify the efficacy of source-aware IF on understanding and improving PWS pipelines.

2 Preliminary

We denote scalars and generic items as lowercase letters, vectors as lowercase bold letters, and matrices as bold uppercase letters. We use superscripts to index generic items, *e.g.*, the i-th data $x^{(i)}$, and use subscripts to index certain dimensions of a vector, *e.g.*, the i-th component of a vector $\mathbf{v}_i$, or the vector output of a function $\mathbf{f}(\cdot)_i$. All derivation details are deferred to the appendix.

Programmatic Weak Supervision. We target at a C-way classification problem and have a training dataset of size N, *i.e.*, $\mathcal{D} = \{x^{(i)}, y^{(i)}\}_{i\in[N]}$, where $x^{(i)} \in \mathcal{X} = \mathbb{R}^d$ and $y^{(i)} \in \mathcal{Y} = [C]$. The ground truth $y^{(i)}$s are not observed. We have M *labeling functions (LFs)* $\{\lambda^{(j)}(\cdot)\}_{j\in[M]}$. Each $\lambda^{(j)}(\cdot) \in \{-1\} \cup \mathcal{Y}^{(j)}$, where $\mathcal{Y}^{(j)} \subset \mathcal{Y}$. That is, each $\lambda^{(j)}(\cdot)$ either abstains (-1) or outputs a specific label $y \in \mathcal{Y}^{(j)}$ on a given data point. We define the label matrix as $\mathbf{L}_{ij} = \lambda^{(j)}(x^{(i)})$, and with a slight abuse of notation, use $\mathbf{L}(x^{(i)})$ to represent the row of $\mathbf{L}$ corresponding to all of the labels assigned to $x^{(i)}$. The goal of Programmatic Weak Supervision (PWS) is to train an *end classifier* $h_\theta : \mathcal{X} \to \mathcal{Y}$ with $\{x^{(i)}\}_{i\in[N]}$ and $\mathbf{L}$ only, where $h_\theta(x) = \arg\max_{c\in[C]} \mathbf{f}_\theta(x)_c$ and $\mathbf{f}_\theta(x) \in \mathbb{R}^C$ is a vector-valued function parameterized by θ. A typical PWS pipeline involves

a *label model* $\mathbf{g_W}(\mathbf{L}(x))$ parametrized by $\mathbf{W}$, which aims to produce a *weak probabilistic label* $\hat{\mathbf{y}}(x^{(i)}) = \mathbf{g_W}(\mathbf{L}(x^{(i)})) \in \Delta^C$ for each data point $x^{(i)}$ (denoted as $\hat{\mathbf{y}}^{(i)}$ for simplicity). The label model produces a dataset $\hat{\mathcal{D}} = \{x^{(i)}, \hat{\mathbf{y}}^{(i)}\}_{i\in[N]}$, which is then used to train $\mathbf{f}_\theta$ via the *noise-aware loss* [33] $\hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)})) = \mathbb{E}_{y\sim\hat{\mathbf{y}}^{(i)}}[\ell(y, \mathbf{f}_\theta(x^{(i)})] = \sum_{c=1}^{C} \hat{\mathbf{y}}_c^{(i)} \ell(c, \mathbf{f}_\theta(x^{(i)}))$, where $\ell(\cdot,\cdot)$ is the cross-entropy loss, leading to the following training objective:

$$\theta^\star = \arg\min_\theta \hat{\mathcal{L}}(\hat{\mathcal{D}};\theta) = \arg\min_\theta \frac{1}{N}\sum_{i=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)})) = \arg\min_\theta \frac{1}{N}\sum_{i=1}^{N}\sum_{c=1}^{C} -\hat{\mathbf{y}}_c^{(i)} \log\left(\mathbf{f}_\theta(x^{(i)})_c\right), \quad (1)$$

where $\theta^\star$ is the minimizer of the loss $\hat{\mathcal{L}}(\hat{\mathcal{D}};\theta)$ over the training set $\hat{\mathcal{D}}$.

Influence Function. The Influence Function (IF) estimate how the minimizer $\theta^\star$ of the empirical loss ℓ would change if we were to reweight the i-th training example $z^{(i)} = (x^{(i)}, y^{(i)})$ by ϵ_i. The key idea is to make a first-order Taylor approximation of the changes in $\theta^\star$ around $\epsilon_i = 0$. Specifically, if the i-th training sample is upweighted by a small ϵ_i, the perturbed risk minimizer $\theta^\star_{\epsilon_i}$ becomes:

$$\theta^\star_{\epsilon_i} \triangleq \arg\min_{\theta\in\Theta} \frac{1}{N}\sum_{i'=1}^{N} \ell\left(y^{(i')}, \mathbf{f}_\theta(x^{(i')})\right) + \epsilon_i\, \ell\left(y^{(i)}, \mathbf{f}_\theta(x^{(i)})\right). \quad (2)$$

The changes of the model parameters due to the introduction of the weight ϵ_i are:

$$\theta^\star_{\epsilon_i} - \theta^\star \approx \frac{d\theta^\star_{\epsilon_i}}{d\epsilon_i}\Big|_{\epsilon_i=0} \epsilon_i = -\mathbf{H}_{\theta^\star}^{-1} \nabla_\theta \ell\left(y^{(i)}, \mathbf{f}_{\theta^\star}(x^{(i)})\right) \epsilon_i, \quad (3)$$

where $\mathbf{H}_{\theta^\star} = \frac{1}{N}\sum_{i'} \nabla_\theta^2 \ell\left(y^{(i')}, \mathbf{f}_{\theta^\star}(x^{(i')})\right)$ is the Hessian of the objective at $\theta^\star$ and assumed to be positive definite [22]. Similarly, *the change of loss with respect to a single sample* $z' = (x', y')$ is

$$\begin{aligned}\phi_i(z') &= \frac{d\left(\ell(y', \mathbf{f}_{\theta^\star_{\epsilon_i}}(x')) - \ell(y', \mathbf{f}_{\theta^\star}(x'))\right)}{d\epsilon_i} = \frac{d\ell(y', \mathbf{f}_{\theta^\star_{\epsilon_i}}(x'))}{d\epsilon_i}\Big|_{\epsilon_i=0} \\ &= -\nabla_\theta \ell\left(y', \mathbf{f}_{\theta^\star}(x')\right)^\top \mathbf{H}_{\theta^\star}^{-1} \nabla_\theta \ell(y^{(i)}, \mathbf{f}_{\theta^\star}(x^{(i)})).\end{aligned} \quad (4)$$

Then, as shown in previous work [24], *the change of loss over a holdout set*, *e.g.*, the validation set $\mathcal{D}_v$, is simply the sum of the changes over every sample in the set:

$$\phi_i(\mathcal{D}_v) = \mathcal{L}(\mathcal{D}_v;\theta^\star_{\epsilon_i}) - \mathcal{L}(\mathcal{D}_v;\theta^\star) = \frac{1}{|\mathcal{D}_v|}\sum_{z'\in\mathcal{D}_v} \phi_i(z'). \quad (5)$$

The change of loss w.r.t. one sample is often interpreted as its effect on model predictions over the training set [22], while the change of loss over a holdout set can be leveraged to improve the test loss by discarding/downweighting the training examples with negative impact [38, 42, 24]. Considering the extraordinary influence that might be caused by mislabeled examples, the relative influence function (RelatIF) [3] has been proposed to measure the influence of a sample relative to its global effects. The RelatIF can be defined as $\phi_i^{'}(z') = \frac{\phi_i(z')}{\sqrt{\phi_i(z^{(i)})}}$.

3 Methodology

Given an unlabeled training set $\{x^{(i)}\}_{i\in[N]}$ and M labeling functions (LFs) $\{\lambda^{(j)}(\cdot)\}_{j\in[M]}$, we study the scenario where a Programmatic Weak Supervision (PWS) pipeline is employed to synthesize probabilistic labels $\{\hat{\mathbf{y}}^{(i)}\}_{i\in[N]}$ for training an end model $\mathbf{f}_\theta(x)$ for a C-way classification problem. We aim for a general framework for explaining and debugging the PWS pipeline through Influence Function (IF) [22], which estimates the influence of each training data on test loss/prediction for a better understanding of model behavior and is able to improve/debug the training data accordingly. In the PWS pipeline, we have full knowledge of how training labels are synthesized from different sources, *i.e.*, LFs, which opens the opportunity to trace the influence in a source-aware way. In this section, we propose the source-aware IF tailored for PWS and describe its applications.

3.1 Training loss decomposition

By examining the popular choices of label models $\mathbf{g}_{\mathbf{W}}(\cdot)$ [9, 33, 31], we show that their parameters can be unifiedly represented as a tensor $\mathbf{W} \in \mathbb{R}^{M\times(C+1)\times C}$, where the middle $(C+1)$ dimension size is due to the additional "abstention" label LFs output. The generation process of the probabilistic labels can then be expressed as

$$\forall c \in [C], \qquad \hat{\mathbf{y}}_c^{(i)} = \frac{\sigma(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^{C} \sigma(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k})}, \tag{6}$$

where $\sigma(\cdot)$ could be (1) the identity function $\sigma_{id}(x) = x$ for the classic Majority Voting label model or its variants, or (2) the exponential function $\sigma_{exp}(x) = \exp(x)$ to recover the softmax function used in statistical models such as Dawid-Skene model [9] and Snorkel MeTaL [31]. Intuitively, label models aim to learn a source-specific weight for each LF to aggregate the votes in an unsupervised manner. Details can be found in Appendix A. In both cases, for the i-th data, a single parameter $\mathbf{W}_{j,\mathbf{L}_{ij},c}$ controls the effect of the j-th LF's vote $\mathbf{L}_{ij}$ on the c-th dimension of the probabilistic label, *i.e.*, $\hat{\mathbf{y}}_c^{(i)}$. For convenience, we denote the effect of $\mathbf{L}_{ij}$ on $\hat{\mathbf{y}}_c^{(i)}$ as the (i,j,c)-*effect*, as it uniquely corresponds to the i-th data, the j-th LF and c-th dimension of the aggregated probabilistic label. With such unified formulation of probabilistic label generation, we rewrite the *noise-aware loss* [33] as

$$\hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)})) = -\sum_{c=1}^{C} \frac{\sigma(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^{C} \sigma(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k})} \log\left(\mathbf{f}_\theta(x^{(i)})_c\right). \tag{7}$$

We proceed with the $\sigma(\cdot)$ being the identity function because it enables the loss decomposition introduced below. We will later discuss other choices of $\sigma(\cdot)$. By simple rearrangement, we could decompose the loss over the training set into the *summation* of all $N \times M \times C$ terms:

$$\hat{\mathcal{L}}(\hat{\mathcal{D}};\theta) = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{c=1}^{C} \bar{\ell}_{i,j,c}(\theta), \quad \bar{\ell}_{i,j,c}(\theta) = -\frac{\mathbf{W}_{j,\mathbf{L}_{ij},c}}{\sum_{k=1}^{C}\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k}} \log\left(\mathbf{f}_\theta(x^{(i)})_c\right). \tag{8}$$

Such a decomposition is *source-aware* in the sense that it preserves how the source votes are aggregated via the label model, and each $\bar{\ell}_{i,j,c}(\theta)$ indeed corresponds to the (i,j,c)-effect.

3.2 Source-aware Influence Function

As a consequence of training loss decomposition, we are now able to compute the influence score for each term $\bar{\ell}_{i,j,c}(\theta)$, which is the influence of removing $\bar{\ell}_{i,j,c}(\theta)$ or, in other words, the (i,j,c)-effect from training. We provide two means of calculating the influence score: **reweighting** and **weight-moving**, which correspond to two different ways of perturbing the training loss. Specifically, given an (unseen) instance $z' = (x', y')$ and the cross-entropy loss $\ell(y', \mathbf{f}_\theta(x'))$, we are interested in the influence score defined by the change of loss $\ell(y', \mathbf{f}_\theta(x'))$ caused by the differences in the model parameters θ after perturbing the training loss $\hat{\mathcal{L}}(\hat{\mathcal{D}};\theta)$. We first give the formulation of an ordinary IF for a training data $z^{(i)} = (x^{(i)}, \hat{\mathbf{y}}^{(i)})$ in the context of PWS, which is smilar to Eq. 4:

$$\phi_i(z') = -\nabla_\theta \ell\left(y', \mathbf{f}_{\theta^\star}(x')\right)^\top \mathbf{H}_{\theta^\star}^{-1} \nabla_\theta \hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_{\theta^\star}(x^{(i)})). \tag{9}$$

Here, $\mathbf{H}_{\theta^\star}^{-1} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta^2 \hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_{\theta^\star}(x^{(i)}))$ and $\theta^\star$ is the minimizer of the training loss before the perturbation. We reuse the notation of $\mathbf{H}_{\theta^\star}$ and $\theta^\star$ in this section as their meanings are unchanged.

Reweighting. To study the influence of removing the (i,j,c)-effect, we reweight $\bar{\ell}_{i,j,c}(\theta)$ such that

$$\hat{\mathcal{L}}_{\epsilon_{i,j,c}}^{rw}(\hat{\mathcal{D}};\theta) = \frac{1}{N}\sum_{i'=1}^{N}\sum_{j'=1}^{M}\sum_{c'=1}^{C} \bar{\ell}_{i',j',c'} + \epsilon_{i,j,c}\bar{\ell}_{i,j,c}(\theta). \tag{10}$$

Then, the influence score, *i.e.*, the difference of loss on $z' = (x', y')$ caused by reweighting the loss term $\bar{\ell}_{i,j,c}(\theta)$, is,

$$\bar{\phi}_{i,j,c}^{rw}(z') = -\nabla_\theta \ell\left(y', \mathbf{f}_{\theta^\star}(x')\right)^\top \mathbf{H}_{\theta^\star}^{-1} \nabla_\theta \bar{\ell}_{i,j,c}(\theta^\star). \tag{11}$$

In contrast to ordinary IF that reweights the loss w.r.t. individual training data, the proposed one is finer-grained since it reweights the decomposed loss terms. By reweighting, the resultant probabilistic training label is no longer sum-to-one and the solution (Eq. 11) is only for identity $\sigma(\cdot)$ function.

Table 1: Calculation of influence of each component in PWS using the source-aware IF $\bar{\phi}_{i,j,c}$.

Component of PWS pipeline	Symbol	Calculation
$x^{(i)}$: the i-th data	$\phi_{x^{(i)}}$	$\sum_{j=1}^{M}\sum_{c=1}^{C}\bar{\phi}_{i,j,c}$
$\lambda^{(j)}$: the j-th LF	$\phi_{\lambda^{(j)}}$	$\sum_{i=1}^{N}\sum_{c=1}^{C}\bar{\phi}_{i,j,c}$
$\mathbf{L}_{ij}$: the j-th LF's output on the i-th data	$\phi_{\mathbf{L}_{ij}}$	$\sum_{c=1}^{C}\bar{\phi}_{i,j,c}$
$\mathbf{W}_{j,k,c}$: a individual parameter of label model indexed by (j,k,c)	$\phi_{\mathbf{W}_{j,k,c}}$	$\sum_{i=1}^{N}\bar{\phi}_{i,j,c}\mathbb{1}\{L_{ij}=k\}$

Weight-moving. We also explore an alternative way of perturbing the training loss called weight-moving. Different from reweighting, its computation is agnostic to the $\sigma(\cdot)$ function and the probabilistic training label is still sum-to-one. Specifically, we remove the (i,j,c)-effect in the probabilistic training label $\hat{\mathbf{y}}^{(i)}$ and renormalize it, leading to a new probabilistic training label:

$$\forall c \in [C], \qquad \hat{\mathbf{y}}^{(i)}_{-j'c'} = \frac{\sigma(\sum_{j=1}^{M}\mathbb{1}\{c \neq c' \wedge j \neq j'\}\cdot \mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^{C}\sigma(\sum_{j=1}^{M}\mathbb{1}\{k \neq c' \wedge j \neq j'\}\cdot \mathbf{W}_{j,\mathbf{L}_{ij},k})}. \tag{12}$$

Then, we perturb the training loss by moving the weight of the original loss on the i-th data $\hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)}))$ to a new loss term $\hat{\ell}(\hat{\mathbf{y}}^{(i)}_{-j'c'}, \mathbf{f}_\theta(x^{(i)}))$:

$$\hat{\mathcal{L}}^{wm}_{\epsilon_{i,j,c}}(\hat{\mathcal{D}};\theta) = \frac{1}{N}\sum_{i'=1}^{N}\hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c}\cdot\hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)})) - \epsilon_{i,j,c}\cdot\hat{\ell}(\hat{\mathbf{y}}^{(i)}_{-jc}, \mathbf{f}_\theta(x^{(i)})). \tag{13}$$

When $\epsilon_{i,j,c} = -\frac{1}{N}$, the loss term $\hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)}))$ is indeed replaced by the new loss $\hat{\ell}(\hat{\mathbf{y}}^{(i)}_{-j'c'}, \mathbf{f}_\theta(x^{(i)}))$. We again provide the formulation of corresponding influence score:

$$\bar{\phi}^{wm}_{i,j,c}(z') = -\nabla_\theta \ell\left(y', \mathbf{f}_{\theta^\star}(x')\right)^\top \mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\hat{\ell}(\hat{\mathbf{y}}^{(i)} - \hat{\mathbf{y}}^{(i)}_{-jc}, \mathbf{f}_{\theta^\star}(x^{(i)})). \tag{14}$$

Discussion. Note that the change of loss over a holdout set $\mathcal{D}_v$ can be similarly computed and we denote it as $\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v)$ and $\bar{\phi}^{wm}_{i,j,c}(\mathcal{D}_v)$. We omit the input variable and the superscript and use $\bar{\phi}_{i,j,c}$ to represent the proposed source-aware IF in general. The $\bar{\phi}_{i,j,c}$ corresponds to the influence of a specific perturbation on the training loss: setting the label model parameter $\mathbf{W}_{j,\mathbf{L}_{ij},c}$ to zero for the i-the data so that the (i,j,c)-effect is removed from training. Then, the difference between reweighting and weight-moving reduces to whether renomarlization on the probabilistic labels is performed after the perturbation.

Extension to RelatIF [3]. We additionally demonstrate how to compute the RelatIF [3] in our source-aware IF setting, Specifically, the self-influence is $\bar{\phi}^{self}_{i,j,c} = -\nabla_\theta\bar{\ell}_{i,j,c}(\theta^\star)^\top\mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\bar{\ell}_{i,j,c}(\theta^\star)$. Then, the source-aware RelatIF for both $\bar{\phi}^{rw}_{i,j,c}$ and $\bar{\phi}^{wm}_{i,j,c}$ are $\frac{\bar{\phi}^{rw}_{i,j,c}}{\sqrt{\bar{\phi}^{self}_{i,j,c}}}$ and $\frac{\bar{\phi}^{wm}_{i,j,c}}{\sqrt{\bar{\phi}^{self}_{i,j,c}}}$, respectively.

3.3 Use cases

Case 1: estimating influence of different components. One major benefit of our training loss decomposition and the identity $\sigma(\cdot)$ function is that we could readily reuse the source-aware IF $\bar{\phi}_{i,j,c}$ to estimate different components of the PWS pipeline, *e.g.*, individual data or LF. This is because according to the study of group influence [23], when measuring the change in test prediction/loss, the influence is additive, *i.e.*, the influence of a set is the sum of influences of its constituent points. However, such a nice property only holds true for the reweighting method, while, in theory, the influence calculated by weight-moving does not follow due to the fact that they are not additive. We list the influence of different components of a PWS pipeline as well as how to calculate them using the fine-grain IF $\bar{\phi}_{i,j,c}$ in Table 1. Note that the 1st and 2nd row of Table 1 indeed recover the ordinary IF of a training data [22] and a LF [23], respectively.

Case 2: improving test loss. Another application of IF is to improve the test loss by identifying and then discarding/downweighting the harmful training data [24, 42, 38]. Similarly, the source-aware IF $\bar{\phi}_{i,j,c}$ can be employed to improve the test loss by discarding/downweighting the loss term $\bar{\ell}_{i,j,c}(\theta)$ with negative impact. Specifically, we have the following theorem for reweighting (a similar result for weight-moving can be found in Appendix E.2):

Theorem 1. *Discarding or downweighting the loss terms in* $\mathcal{S}_- = \{\bar{\ell}_{i,j,c}(\cdot) | i \in [N], j \in [M], c \in [C], \bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_t) > 0\}$ *from training could lead to a model with lower loss over a holdout set* $\mathcal{D}_t$*:*

$$\mathcal{L}(\mathcal{D}_t; \theta^\star_{\mathcal{S}_-}) - \mathcal{L}(\mathcal{D}_t; \theta^\star) \approx -\frac{1}{N} \sum_{\bar{\ell}_{i,j,c}(\cdot) \in \mathcal{S}_-} \bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_t) \leq 0$$

where $\theta^\star_{\mathcal{S}_-}$ *is the optimal model parameters obtained after the perturbation.*

In practice, we use the validation set $\mathcal{D}_v$ and $\mathcal{S}_-(\alpha) = \{\bar{\ell}_{i,j,c}(\cdot) | i \in [N], j \in [M], c \in [C], \bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v) > \alpha\}$ to tolerate the estimation error of IF where the hyperparameter α controls the proportion of items to be perturbed [24]. Note that this method of improving test loss relies on the training loss decomposition and $\sigma(\cdot)$ being the identity function.

3.4 Beyond the identity function

The aforementioned use cases rely on the additivity of influence score and the training loss decomposition, which are both consequences of $\sigma(\cdot)$ being the identity function. When $\sigma(\cdot)$ is the exponential function, we could approximate the label model $\mathbf{g}_{\mathbf{W}}(\cdot)$ by a new label model $\bar{\mathbf{g}}_{\bar{\mathbf{W}}}(\cdot)$ with parameter $\bar{\mathbf{W}}$ of the same shape but using identity function instead. Obviously, we want the new label model $\bar{\mathbf{g}}_{\bar{\mathbf{W}}}(\cdot)$ to reproduce the same probabilistic labels as those generated by $\mathbf{g}_{\mathbf{W}}(\cdot)$. This motivates us to solve the following optimization problem for $\bar{\mathbf{W}}$:

$$\bar{\mathbf{W}}^\star = \arg\min_{\bar{\mathbf{W}}} \sum_{i=1}^{N} \sum_{c=1}^{C} \Big(\frac{\exp(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^{C} \exp(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k})} - \frac{\sum_{j=1}^{M} \bar{\mathbf{W}}_{j,\mathbf{L}_{ij},c}}{\sum_{k=1}^{C} \sum_{j=1}^{M} \bar{\mathbf{W}}_{j,\mathbf{L}_{ij},k}} \Big)^2. \tag{15}$$

Then, we can replace the label model $\mathbf{g}_{\mathbf{W}}(\cdot)$ with $\bar{\mathbf{g}}_{\bar{\mathbf{W}}^\star}(\cdot)$ everywhere. As a consequence, we are now able to unify various types of label models in a single framework and enjoy the convenience and advantages brought by the linearity of the identity function. Note that for the original label model $\mathbf{g}_{\mathbf{W}}(\cdot)$ with $\sigma(\cdot)$ being the exponential function, we can still derive the influence $\bar{\phi}_{i,j,c}$ (see appendix). We study the effect of replacing original label model with the approximated one on the end model performance in the experiment section.

4 Experiment

4.1 Setup

Datasets. We include the following four classification datasets in WRENCH [47], a collection of benchmarking datasets for PWS: **Census**, **Youtube**, **Yelp**, and **IMDb**. Note that the **Census** dataset is a tabular dataset while the others are textual datasets. We use the labeling functions (LFs) released by WRENCH [47]. We also include the following tabular datasets: **Mushroom** [12], **Spambase** [12], and **PhishingWebsites (PW)** [29], for which we follow the instruction in the WRENCH [47] codebase[1] to generate LFs from a decision tree learned on the labeled data. Finally, we follow [28] to derive LFs for a multiclass image classification task using the DomainNet [30] dataset, which contains 345 classes of images in 6 different domains: real, sketch, quickdraw, painting, infograph and clipart. We construct a classification task for each domain with the 5 classes containing the largest number of instances. We then train classifiers using the selected classes within the remaining five domains as LFs. We name the 6 resultant datasets **DN-real**, **DN-sketch**, **DN-quickdraw**, **DN-painting**, **DN-infograph**, and **DN-clipart** for each domain, respectively. Following [47], we use a pre-trained BERT model [10] to extract features for textual data and use ResNet-18 [17] pre-trained on ImageNet for image data.

Model Training and Evaluation. For label model, we focus on three commonly-used choices: Majority Voting (MV), Dawid-Skene model (DS) [9], and Snorkel [31]. The $\sigma(\cdot)$ function of the latter two label models is the exponential function and we use their approximated variants (as mentioned in Section 3.4) for all experiments. Following [23], we use logistic regression for the end model and defer the experiments on using neural networks to the appendix. We train the end model initialized as zero by gradient descent for 10,000 epochs with learning rate being 0.001, and then calculate

[1] https://github.com/JieyuZ2/wrench/tree/main/datasets/tabular_data

Table 2: The most "responsible" data, LF, and labeling, identified by source-aware IF, for the misclassification of end models (trained with difference label models) on the same test image. We can see that source-aware IF can reveal the cause of misclassification from multiple angles.

Label Model	Misclassified Test Data		Resp. Training Data			Resp. LF	Resp. Labeling		
	Data	Pred. (Prob.)	Data	True Label	Weak Label (Prob.)		LF	Data	Vote
MV		golf club (0.55)		dog	golf club (1.0)	quickdraw	quickdraw		golf club
DS		golf club (0.44)		dog	golf club (0.71)	painting	painting		golf club
Snorkel		golf club (0.66)		dog	golf club (0.94)	clipart	clipart		golf club

the influence score on validation set $\mathcal{D}_v$ throughout the experiments. When using the test loss as evaluation metric, we use $\mathcal{D}_v$ to do model selection and report the test loss of the select model.

For simplicity, we use **IF (R-IF)** to represent ordinary IF (RelatIF) baseline; **RW (R-RW)** and **WM (R-WM)** are the proposed source-aware IF (RelatIF) calculated by the reweighting and weight-moving method respectively. We also use the term **SIF** (**R-SIF**) to represent the source-aware IF (RelatIF) when we do not care whether it is calculated by reweighting or weight-moving. Additional experiments and details are in Appendix G.

4.2 Experimental results

Understand model behavior from multiple angles. First and foremost, source-aware IF can be used to understand the end model behavior from multiple angles, while ordinary IF can only identify influential training data. Here, we take the **DN-real** dataset as an example and use reweighting-based source-aware IF (**RW**) to diagnose misclassified test data. For a single misclassified test data of the end model, we present the most "responsible" (1) training data, (2) LF, and (3) labeling (the vote of a LF on a training data), where being the most "responsible" means having the highest influence score on that data and likely being the primary cause of the misclassification. In Table 2, we show that for the same image misclassified by the end model trained with different label models, we could have totally different explanations and components of the PWS pipeline to blame on. Specifically, for a "*dog*" image misclassified as "*golf club*", we can see that for both the DS and Snorkel label models, the most "responsible" training image are the same but the most "responsible" LF and labeling are different: for DS the LF trained with images of painting domain is likely the primary cause for the misclassification, while for Snorkel we might blame more on the LF trained with clipart images. Without source-aware IF, we can hardly distinguish the cause of misclassification for DS and Snorkel based solely on the training data as the most "responsible" one is identical.

Identify mislabeling of LFs. One unique advantage of source-aware IF over ordinary IF is that it allows us to trace the influence of each LF's labeling on a specific training data ($\phi_{\mathbf{L}_{ij}} = \sum_{c=1}^{C} \bar{\phi}_{i,j,c}$, the 3rd row in Table 1), which is helpful in understanding the expertise of each individual LF and thus provides clues for practitioners to debug LFs, while ordinary IF can only estimate the influence of training data as a whole. We use $\phi_{\mathbf{L}_{ij}}$ as a scoring function of how likely the j-th LF mislabels i-th data, *i.e.*, $\mathbf{L}_{ij} \neq y^{(i)}$, to identify the mislabeling of individual LF. We thus formulate it as a binary classification task and report the average precision (AP), a standard evaluation metric of a scoring function. We use the discrepancy between j-th LF's labeling on i-th data ($\mathbf{L}_{ij}$) and the probabilistic label output by (1) label model (**LM**), (2) end model (**EM**), and (3) a K-nearest neighbor classifier ($K = 10$) trained on validation set (**KNN**) as baseline scoring functions. Specifically, we calculated the discrepancy as one minus the dot product of $\mathbf{L}_{ij}$'s one-hot representation and the probabilistic label; for example, if $\mathbf{L}_{ij}$'s one-hot representation is $[0, 1, 0]^\top$ and the probabilistic label is $[0.5, 0.4, 0.3]^\top$, the discrepancy is $1 - 1 \times 0.4 = 0.6$. Note that **KNN** is a more fair and competitive baseline because it directly leverages the validation set and, as our source-aware IF, does not introduce any parametrized model in addition to what PWS has. We present the results in Table 3; we can see the superiority of source-aware IF in this task since it outperformed baselines in most cases and the averaged gains are about 9%-37% for different label models. Intriguingly, when **LM** and **EM** render unsatisfactory performance, *e.g.*, on **DomainNet**, which indicates the quality of the label or end model is not good, the source-aware IF still performed well. This means it is less sensitive to the quality of PWS, which is critical for debugging the pipeline.

Table 3: Performance comparison results on identifying mislabeling of LFs. We report the average precision (AP) score averaged over LFs for each dataset. The larger the AP is, the better the method identifies mislabeling of LFs.

		MV						DS						Snorkel					
Dataset	**KNN**	**LM**	**EM**	**RW**	**R-RW**	**WM**	**R-WM**	**LM**	**EM**	**RW**	**R-RW**	**WM**	**R-WM**	**LM**	**EM**	**RW**	**R-RW**	**WM**	**R-WM**
Census	0.810	0.809	0.787	0.858	**0.868**	0.834	0.841	0.787	0.787	0.786	0.797	0.796	**0.799**	0.787	0.787	0.800	0.800	0.801	**0.813**
Mushroom	**0.975**	0.923	0.828	0.932	0.923	0.948	**0.949**	0.828	0.828	**0.900**	0.755	0.881	0.865	0.828	0.828	**0.897**	0.837	0.874	0.876
PW	0.822	0.863	0.766	0.891	**0.911**	0.879	0.885	0.766	0.766	**0.892**	0.767	0.888	0.745	0.766	0.766	**0.889**	0.757	0.879	0.756
Spambase	0.782	0.772	0.738	0.916	**0.927**	0.878	0.879	0.738	0.738	0.769	0.774	**0.786**	0.775	0.738	0.738	0.816	0.753	**0.833**	0.750
IMDb	0.702	0.767	0.699	0.740	**0.799**	0.754	0.769	0.699	0.699	**0.785**	0.726	0.764	0.681	0.699	0.699	**0.737**	0.713	0.728	0.711
Yelp	0.752	0.792	0.731	0.888	**0.922**	0.830	0.842	0.731	0.731	**0.844**	0.736	0.784	0.728	0.731	0.731	**0.907**	0.800	0.785	0.755
Youtube	0.831	**0.949**	0.826	0.861	0.917	0.853	0.869	0.826	0.826	**0.908**	0.846	0.877	0.829	0.826	0.826	0.927	**0.933**	0.894	0.889
DN-real	0.711	0.447	0.417	0.959	**0.986**	0.904	0.942	0.417	0.417	**0.580**	0.462	0.550	0.449	0.445	0.417	**0.854**	0.671	0.723	0.476
DN-sketch	0.321	0.339	0.316	0.764	**0.813**	0.669	0.718	0.316	0.316	0.506	**0.515**	0.460	0.432	0.316	0.316	0.497	**0.515**	0.455	0.433
DN-quickdraw	0.362	0.256	0.255	0.840	**0.879**	0.723	0.762	0.255	0.255	**0.435**	0.383	0.384	0.301	0.255	0.255	**0.655**	0.376	0.539	0.293
DN-painting	0.454	0.416	0.360	0.847	**0.912**	0.756	0.817	0.360	0.360	**0.614**	0.537	0.524	0.444	0.360	0.360	**0.715**	0.590	0.630	0.490
DN-infograph	0.361	0.385	0.356	0.639	**0.714**	0.584	0.644	0.356	0.356	**0.516**	0.512	0.495	0.479	0.356	0.356	0.504	**0.526**	0.488	0.464
DN-clipart	0.437	0.487	0.434	0.808	**0.880**	0.784	0.835	0.434	0.434	**0.562**	0.553	0.531	0.520	0.434	0.434	**0.683**	0.608	0.670	0.581
Avg.	0.640	0.631	0.578	0.842	**0.881**	0.800	0.827	0.578	0.578	**0.700**	0.643	0.671	0.619	0.580	0.578	**0.760**	0.683	0.716	0.637

Table 4: Performance comparison results on the test loss of end models.

	MV								DS							Snorkel						
Dataset	**ERM**	**IF**	**GIF**	**R-IF**	**RW**	**R-RW**	**WM**	**R-WM**	**ERM**	**IF**	**R-IF**	**RW**	**R-RW**	**WM**	**R-WM**	**ERM**	**IF**	**R-IF**	**RW**	**R-RW**	**WM**	**R-WM**
Census	0.433	0.369	0.359	0.372	**0.353**	0.361	0.362	0.362	0.660	**0.563**	0.603	0.574	0.577	0.611	0.618	0.601	0.441	0.448	0.401	**0.382**	0.391	0.385
Mushroom	0.238	0.195	0.202	0.194	**0.156**	0.186	0.162	0.168	0.389	0.270	0.299	0.276	**0.215**	0.333	0.350	0.353	0.198	0.241	0.188	**0.180**	0.186	0.182
PW	0.416	**0.301**	0.328	0.308	0.312	0.308	0.305	0.304	0.499	0.338	0.338	0.333	**0.329**	0.331	0.331	0.496	0.350	0.351	0.306	0.305	0.307	**0.305**
Spambase	0.469	0.312	0.330	0.317	**0.295**	0.312	0.309	0.309	0.513	0.310	0.321	0.306	0.341	0.301	**0.293**	0.406	0.284	0.289	**0.258**	0.278	0.262	0.279
IMDb	0.501	0.501	0.582	0.501	0.494	**0.488**	0.496	0.492	0.640	0.628	0.618	0.618	**0.607**	0.626	0.629	0.584	0.577	0.583	0.570	**0.561**	0.578	0.571
Yelp	0.502	0.498	0.425	0.502	**0.398**	0.411	0.420	0.420	0.465	0.462	0.465	0.382	**0.370**	0.454	0.440	0.693	0.516	0.693	0.387	**0.385**	0.516	0.507
Youtube	0.351	0.318	**0.278**	0.295	0.284	0.284	0.301	0.283	0.332	**0.240**	0.272	0.263	0.261	0.325	0.338	0.332	0.320	0.332	0.264	**0.229**	0.296	0.302
DN-real	0.990	0.627	0.581	0.713	0.162	**0.117**	0.162	0.125	0.868	0.536	0.543	0.140	0.137	0.160	**0.115**	0.878	0.566	0.584	0.149	0.136	0.131	**0.103**
DN-sketch	1.270	1.056	1.262	1.064	**0.785**	0.816	0.827	0.841	1.389	1.073	1.139	**0.990**	0.999	1.072	1.070	1.309	1.069	1.076	0.915	0.910	0.915	**0.900**
DN-quickdraw	1.443	1.032	1.362	1.123	0.661	**0.642**	0.811	0.810	1.337	0.938	0.953	0.719	**0.664**	0.783	0.792	1.467	0.993	1.041	0.855	**0.849**	1.099	1.061
DN-painting	1.197	0.948	0.966	0.957	0.419	**0.386**	0.497	0.427	1.024	0.769	0.840	**0.469**	0.473	0.493	0.471	1.139	0.910	0.904	0.453	0.395	0.531	**0.389**
DN-infograp	1.270	1.398h	**1.191**	1.373	1.288	1.246	1.247	1.214	1.208	1.307	1.371	1.284	1.303	**1.206**	1.243	1.220	1.324	1.319	1.211	**1.146**	1.236	1.185
DN-clipart	1.041	1.022	**0.872**	0.974	0.971	1.082	1.172	0.948	1.015	0.993	**0.793**	0.988	1.007	0.950	1.117	0.921	0.704	0.790	**0.650**	0.735	0.814	0.893
Avg.	0.779	0.660	0.672	0.669	**0.506**	0.511	0.544	0.516	0.795	0.648	0.658	0.565	**0.560**	0.588	0.601	0.800	0.635	0.666	0.508	**0.499**	0.559	0.543

Improve test loss. We compared the proposed source-aware IF as well as its RelatIF variant to ordinary IF/RelatIF in terms of the capability of improving the test loss. For source-aware IF/RelatIF, we discarded loss terms in $\mathcal{S}_-(\alpha)$ and then re-trained the end model (as described in Section 3.3), while for IF/RelatIF, we likewise discarded training data in $\mathcal{D}_-(\alpha) = \{z^{(i)} | i \in [N], \phi_i(\mathcal{D}_v) > \alpha\}$. For these methods, we tuned the hyperparameter α on validation set for best validation loss. We additionally included the group influence (**GIF**) [23] that estimates the influence of each LF and is only applicable to **MV** as in [23]; we discarded k LFs with highest negative impact and tune k on validation set. The results are in Table 4, where **ERM (Emperical Risk Minimization)** is the ordinary training without any perturbation. From the results, we conclude that although ordinary IF/RelatIF can already improve the test loss over ordinary training (**ERM**) by a large margin (>0.1 test loss on average), the source-aware IF/RelatIF can further boost the performance by a similar margin (>0.1 averaged test loss improvement over IF/RelatIF). This shows the benefit of employing IF in a source-aware manner and provides insights for practitioners to improve the PWS pipeline.

Discussion. One may develop sophisticated methods to identify mislabeling of LFs or to improve test loss and achieve better performance than source-aware IF, but source-aware IF is designed as a tool of interpreting the end model prediction and when used in these applications, it does not introduce additional parameterized model. Also, we believe that source-aware IF can inspire advanced methods in the future and be leveraged by them to further boost the performance on the task of identifying mislabeling of LFs or improving test loss.

4.3 Additional studies

Estimate ordinary data-level IF via source-aware IF. As described in Section 3.3, we can use the source-aware IF calculated by the reweighting method to estimate the data-level influence score ($\phi_{x^{(i)}} = \sum_{j=1}^{M} \sum_{c=1}^{C} \bar{\phi}_{i,j,c}$, the 1st row in Table 1), which coincides with the influence computed by ordinary IF in theory. Thus, we are curious about how well the source-aware IF can be used to estimate the ordinary data-level IF. Because IF is known to be noisy in the value but good in their ranking [4, 23], we instead study the ranking correlation between $\phi_{x^{(i)}}$ and the ordinary IF. In Table 5

we report the Spearman's ranking correlation coefficient (≤ 1), where larger coefficients indicate better estimation of the ordinary IF based on $\phi_{x^{(i)}}$ in terms of the ranking. From the results, we can see that the $\phi_{x^{(i)}}$ actually preserves the ranking of ordinary IF quite well as Spearman's coefficients are from 0.71 to 0.99, which further demonstrates the efficacy of source-aware IF.

Table 5: Spearman's ranking correlation coefficient (≤ 1) between influence calculated by ordinary data-level IF and that estimated from source-aware IF. All results pass the significance test ($\rho < 0.05$).

Dataset	Census	Mushroom	PW	Spambase	IMDb	Yelp	Youtube	DN-real	DN-sketch	DN-quickdraw	DN-painting	DN-infograph	DN-clipart	Avg.
MV	0.987	0.981	0.955	0.989	0.827	0.898	0.926	0.890	0.764	0.877	0.873	0.760	0.777	0.885
DS	0.980	0.975	0.990	0.992	0.904	0.640	0.769	0.893	0.777	0.867	0.854	0.730	0.821	0.861
Snorkel	0.969	0.981	0.988	0.987	0.878	0.959	0.716	0.891	0.756	0.876	0.861	0.711	0.754	0.871

Efficacy of label model approximation. Since we bypassed the non-linearity of exponential function in DS and Snorkel with their approximated variants (Section 3.4), it is natural to ask how well the approximated label model can perform and whether or not the approximation is well-haved. To answer these questions, in Figure 1 we show that although the averaged test loss of ordinary training (**ERM**) and best of IF/R-IF (**Best IF/R-IF**) are slightly better than their counterpart of approximated label model (**Approx. ERM** and **Approx. Best IF/R-IF**), the best averaged test loss obtained by SIF/R-SIF with the approximated label model (**Approx. Best SIF/R-SIF**) is much lower. Such an observation indicates that with the goal of improving test loss, it is beneficial to choose $\sigma(\cdot)$ to be the identity function so that the training loss can be decomposed and perturbed based on source-aware IF.

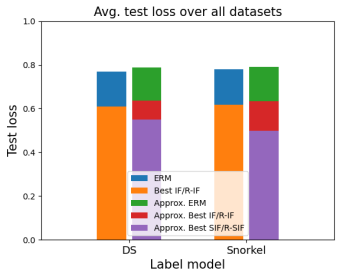

Figure 1: Avg. test loss of the end model with/without label model approximation.

5 Related work

Programmatic Weak Supervision. In the Programmatic Weak Supervision (PWS) literature [33, 45], popular choices of labeling functions (LFs) include user-crafted heuristics [32, 39, 44], pretrained models [2, 28], external knowledge bases [18, 26], and crowd-sourced labels [9, 25]. Recently, researchers have developed various label models [33, 31, 14, 40, 46, 43, 37]. In the PWS pipeline, the behavior of the end model is directly dictated by the soft labels, wherein the soft labels themselves are also intermediate products of the label model and the LFs' votes. Thus, to interpret the trained end model and potentially debug the PWS workflow, it is critical to understand the influence of each PWS component on the end model's performance, which has yet received adequate research attention.

Influence Function. Among various ways to understand a model's behavior through the training data points, Influence Function (IF) is an effective technique with a wide range of applications [22, 23, 16, 7]. In PWS, although one can directly apply IF to understand a model's behavior through the weakly labeled training data, more insights may be unveiled by further investigations of how each PWS component, e.g., LFs and label models, influences the end model. In similar spirit to us, [6] developed multi-stage IF to trace a fine-tuned model's behavior back to the pretraining data. Perhaps most similarly to our work, [23] conducted initial exploration on using group influence to study how removing an LF from the PWS pipeline would affect the end model's performance. However, the group influence method is only applicable to a specific simplified setting of PWS.

6 Conclusion

In this work, by leveraging the knowledge of how probabilistic training labels are aggregated from different sources in the PWS pipeline, we have proposed source-aware IF that enables users to estimate the influence of different components (*e.g.*, source vote, training data, LFs, *etc.*) in the pipeline on the end model's behavior, and unlocked various practical use cases. Specifically, we have demonstrated how the proposed source-aware IF can be used to (1) unveil the most influential training data point, LF, or a specific vote to a misclassification made by a model, (2) identify mislabeling of LFs on certain data points, and (3) improve the end model's test performance.

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Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] See appendix.
   (c) Did you discuss any potential negative societal impacts of your work? [Yes] See appendix.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] See appendix.
   (b) Did you include complete proofs of all theoretical results? [Yes] See appendix.
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] See supplemental materials.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No] For experiments with randomness, we report averaged results over 5 runs.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes] See appendix.
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes]
   (b) Did you mention the license of the assets? [Yes]
   (c) Did you include any new assets either in the supplemental material or as a URL? [Yes]
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [Yes]
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [Yes]
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]

(b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]

(c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]

A Label model reparametarization and illustrations

A.1 Majority Voting

The Majority Voting (MV) is the most intuitive algorithm for aggregate LFs' annotations. The MV methods can be formalize as

$$\hat{\mathbf{y}}_c^{(i)} = \frac{\sum_{j=1}^{M} \mathbb{1}\{\mathbf{L}_{ij} = c\}}{\sum_{k=1}^{C} \sum_{j=1}^{M} \mathbb{1}\{\mathbf{L}_{ij} = k\}} \tag{16}$$

By letting $\mathbf{W}_{j,k,c} = \mathbb{1}\{k = c\}$, we re-formulate the above as

$$\forall c \in [C], \qquad \hat{\mathbf{y}}_c^{(i)} = \frac{\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},c}}{\sum_{k=1}^{C} \sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k}}. \tag{17}$$

A.2 Dawid-Skene model

The parameters of the Dawid-Skene(DS) [9] model are given by

$$\pi_{c,l}^{(j)} = \frac{\text{number of times the } j\text{-th LF votes for } l \text{ when true label is } c}{\text{number of data voted by } j\text{-th LF whose true label is } c}, \tag{18}$$

and p_c, which is prior of $p(y = c)$. Let $n_{i,l}^{(j)}$ be the number of times j-th LF assigns l for data $x^{(i)}$ and $T_{i,c}$ be the indicator variables: if the true label of data $x^{(i)}$ is c, then $T_{i,c} = 1$, otherwise 0. When the true label for all data are available, the likelihood is given by

$$\prod_{i=1}^{N} \prod_{c=1}^{C} \left\{ p_c \prod_{j=1}^{M} \prod_{l=1}^{C+1} \left(\pi_{c,l}^{(j)}\right)^{n_{i,l}^{(j)}} \right\}^{T_{i,c}} \tag{19}$$

By the Bayes' theorem:

$$\hat{\mathbf{y}}_c^{(i)} = p(y^{(i)} = c \mid \boldsymbol{L}_i) = p(T_{i,c} = 1 \mid \boldsymbol{L}_i) = \frac{\prod_{j=1}^{M} \prod_{l=1}^{C+1} p_c \left(\pi_{c,l}^{(j)}\right)^{n_{i,l}^{(j)}}}{\sum_{k=1}^{C} \left(\prod_{j=1}^{M} \prod_{l=1}^{C+1} p_k \left(\pi_{k,l}^{(j)}\right)^{n_{i,l}^{(j)}} \right)}. \tag{20}$$

We can re-write Eq. 21 as

$$\hat{\mathbf{y}}_c^{(i)} = \frac{\sigma_{exp}\left(\sum_{j=1}^{M} \sum_{l=1}^{C+1} \left(n_{i,l}^{(j)} \log \pi_{c,l}^{(j)} \right) + \log p_c \right)}{\sum_{k=1}^{C} \left(\sigma_{exp}\left(\sum_{j=1}^{M} \sum_{l=1}^{C+1} \left(n_{i,l}^{(j)} \log \pi_{k,l}^{(j)} \right) + \log p_k \right) \right)}, \tag{21}$$

By omitting the last $\log p_c$ term and letting $\mathbf{W}_{j,\mathbf{L}_{ij},c} = \sum_{l=1}^{C+1} \left(n_{i,l}^{(j)} \log \pi_{c,l}^{(j)} \right)$, we have

$$\forall c \in [C], \quad \hat{\mathbf{y}}_c^{(i)} = \frac{\sigma_{exp}(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^{C} \sigma_{exp}(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k})}. \tag{22}$$

The additional $\log p_c$ term can be absorbed in the summation by introducing an additional LF whose label space consists of only one element and the corresponding parameter in $\mathbf{W}$ is $\mathbf{p} = [\log(p(y = 1)), \log(p(y = 2)), \cdots, \log(p(y = C))]^\top$. We omit this case for simplicity.

A.3 Snorkel MeTaL

The parameters μ of Snorkel MeTaL [31] are given by

$$\mu_{j,c,m^{(j)}} = p(y = c, \lambda^{(j)} = m^{(j)}) = \mathbb{E}\left[T_{j,c,m^{(j)}}\right], \tag{23}$$

where $T_{j,c,m^{(j)}}$ is the indicator, $T_{j,c,m^{(j)}} = \mathbb{1}\{y = c, \lambda^{(j)} = m^{(j)}\}$. Given the prior of $p(y)$, by the Bayes' theorem we have:

$$p_\mu(y = c, \boldsymbol{\lambda} = \boldsymbol{m}) = p_\mu(\boldsymbol{\lambda} = \boldsymbol{m} \mid y = c)p(y = c) = \frac{\prod_{j=1}^{M} p_\mu(y = c, \lambda^{(j)} = m^{(j)})}{p(y = c)^{M-1}}. \tag{24}$$

We can further infer that

$$\begin{aligned} p_\mu(y = c \mid \boldsymbol{\lambda} = \boldsymbol{m}) &= \frac{p_\mu(y = c, \boldsymbol{\lambda} = \boldsymbol{m})}{\sum_{k=1}^{C} p_\mu(y = k, \boldsymbol{\lambda} = \boldsymbol{m})} \\ &= \frac{\prod_{j=1}^{M} p_\mu(y = c, \lambda^{(j)} = m^{(j)})}{p(y = c)^{M-1}} \Big/ \sum_{k=1}^{C} \frac{\prod_{j=1}^{M} p_\mu(y = k, \lambda^{(j)} = m^{(j)})}{p(y = k)^{M-1}} \\ &= \frac{p_c \prod_{j=1}^{M} (\frac{\mu_{j,c,m^{(j)}}}{p_c})}{\sum_{k=1}^{C} p_k \prod_{j=1}^{M} (\frac{\mu_{j,k,m^{(j)}}}{p_k})}. \end{aligned} \tag{25}$$

By letting $\mathbf{W}_{j,k,c} = \log \frac{\mu_{j,c,k}}{p_c}$ and, again, omitting the $\log p_c$ term similar to the above derivation for the DS model, we re-formulate the above as

$$\forall c \in [C], \qquad \hat{\mathbf{y}}_c^{(i)} = \frac{\sigma_{exp}\left(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},c}\right)}{\sum_{k=1}^{C} \sigma_{exp}\left(\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k}\right)}. \tag{26}$$

A.4 Illustration

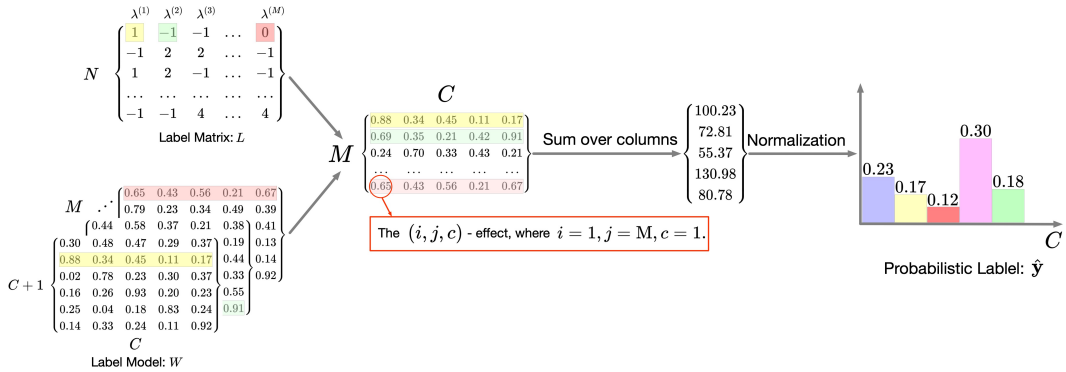

Figure 2: Illustrations of the generation process of the probabilistic training label and the (i, j, c)-effect. We demonstrate how the probabilistic training label for the first data point is generated from the label matrix (source votes, in other words) and the label model.

For a better understanding, we visualize how a probabilistic training label is generated in Figure 2.

A.5 More complicated label model

For more complicated label models that we cannot formulate the inferred label as Eq. 6, we can still use the approximation described in Section 3.4. Consider a label model $\mathbf{g}(\mathbf{L}(x), x) \in \mathcal{F}$ in arbitrary functional class $\mathcal{F}$, *e.g.*, neural network, and having additional dependency on data feature x[2], we can still approximate such complicated function with identity function-based label model $\bar{\mathbf{g}}_{\bar{\mathbf{W}}(x)}(\mathbf{L}(x))$ similar to the aforementioned one except that $\bar{\mathbf{W}}(x) : \mathcal{X} \to \mathbb{R}^{M \times (C+1) \times C}$ is a similarly complicated function, *e.g.*, neural network, that maps each data $x \in \mathcal{X}$ to a unique label model parameter $\bar{\mathbf{W}}(x)$. We leave the exploration of more complicated form of label models into future work.

[2]Several recent neural network-based label models roughly follow this functional form [34, 21].

B Influence Function derivation: the reweighting method

Fellow the derivation of [22], we can directly compute the influence of fine-grained sample $(x^{(i)}, \frac{1}{s_i}\mathbf{W}_{j,\mathbf{L}_{ij},c})$ through up-weighting its corresponding fine-grained level loss with $\epsilon_{i,j,c}$. In the following, we explicitly define the reweighted loss function with different $\sigma(\cdot)$ function.

B.1 Case 1: identity function

We define the loss with reweighted sample as,

$$\hat{\mathcal{L}}_{\epsilon_{i,j,c}}(\hat{\mathcal{D}};\theta) = \frac{1}{N}\sum_{i'=1}^{N}\sum_{j'=1}^{M}\sum_{c'=1}^{C}\bar{\ell}_{i',j',c'}(\theta) + \epsilon_{i,j,c}\cdot\bar{\ell}_{i,j,c}(\theta), \tag{27}$$

and the corresponding risk minimizer as $\theta^{\star}_{\epsilon_{i,j,c}}$. According to the Equation (4), the influence on the loss of one test sample $(x_k, y_k) \in \mathcal{D}_v$ is $-\nabla_\theta \hat{\ell}\left(\mathbf{y}_k, \mathbf{f}_{\theta^\star}(x_k)\right)^\top \mathbf{H}_{\theta^\star}^{-1}\nabla_\theta \bar{\ell}_{i,j,c}(\theta^\star)\cdot\epsilon_{i,j,c}$, where $\mathbf{H}_{\theta^\star}^{-1} = \frac{1}{N}\sum_{i,j,c}\nabla_\theta^2\bar{\ell}_{i,j,c}(\theta^\star)$.

The influence score on the loss of $z' = (x', y')$ by discarding the loss term $\bar{\ell}_{i,j,c}(\theta)$ is,

$$\bar{\phi}^{rw}_{i,j,c}(z') = \nabla_\theta \hat{\ell}\left(\mathbf{y}_k, \mathbf{f}_{\theta^\star}(x_k)\right)^\top \mathbf{H}_{\theta^\star}^{-1}\nabla_\theta \bar{\ell}_{i,j,c}(\theta^\star). \tag{28}$$

And the influence on the loss on the set $\mathcal{D}_v$ can be defined as,

$$\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v) = -\frac{1}{|\mathcal{D}_v|}\sum_{z'\in\mathcal{D}_v}\bar{\phi}^{rw}_{i,j,c}(z'). \tag{29}$$

B.2 Case 2: exponential function

With exponential function, the label for sample $x^{(i)}$ can be presented as,

$$\hat{\mathbf{y}}_c^{(i)} = \frac{\exp(\sum_{j=1}^{M}\mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^{C}\exp(\sum_{j=1}^{M}\mathbf{W}_{j,\mathbf{L}_{ij},k})},$$

where $c \in [C]$. For the sample $x^{(i)}$, if the value of j-th label function on c-th class is upweighted with $\epsilon_{i,j,c}$, then the corresponding label can be written as,

$$\begin{aligned}\hat{\mathbf{y}}^{(i)}_{c,\epsilon_{i,j,c}} &= \frac{\exp(\sum_{j'=1}^{M}\mathbf{W}_{j',\mathbf{L}_{ij'},c} + \epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k'=1}^{C}\exp(\sum_{j'=1}^{M}\mathbf{W}_{j',\mathbf{L}_{ij'},k'} + \epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c})}\\ &= \frac{\exp(\sum_{j'=1}^{M}\mathbf{W}_{j',\mathbf{L}_{ij'},c} + \epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k'=1}^{C}\exp(\sum_{j'=1}^{M}\mathbf{W}_{j',\mathbf{L}_{ij'},k'}) + \exp(\mathbf{W}_{j,\mathbf{L}_{ij},c})\cdot(\exp(\epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c}) - 1)}.\end{aligned}$$

We assume that $\sum_{k'=1}^{C}\exp(\sum_{j'=1}^{M}\mathbf{W}_{j',\mathbf{L}_{ij'},k'}) \gg \exp(\mathbf{W}_{j,\mathbf{L}_{ij},c})\cdot(\exp(\epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c}) - 1)$. Note, this assumption holds in general, because $\epsilon_{i,j,c}$ is close to zero as we discussed in the Section 2. Then we have,

$$\begin{aligned}\hat{\mathbf{y}}^{(i)}_{c,\epsilon_{i,j,c}} &= \frac{\exp(\sum_{j'=1}^{M}\mathbf{W}_{j',\mathbf{L}_{ij'},c} + \epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k'=1}^{C}\exp(\sum_{j'=1}^{M}\mathbf{W}_{j',\mathbf{L}_{ij'},k'})}\\ &= \hat{\mathbf{y}}_c^{(i)} + \exp(\epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c} - 1)\cdot\hat{\mathbf{y}}_c^{(i)}.\end{aligned}$$

Then, the reweighted risk over the whole training set becomes

$$\hat{\mathcal{L}}_{\epsilon_{i,j,c}}(\hat{\mathcal{D}};\theta) = -\frac{1}{N}\left(\sum_{i'=1}^{N}\sum_{c'=1}^{C}\hat{\mathbf{y}}_{c'}^{(i')}\log\left(\mathbf{f}_\theta(x^{(i')})_{c'}\right) + \exp(\epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c} - 1)\cdot\hat{\mathbf{y}}_c^{(i)}\log\left(\mathbf{f}_\theta(x^{(i)})_c\right)\right). \tag{30}$$

We denote the minimizer of the reweighted risk $\hat{\mathcal{L}}_{\epsilon_{i,j,c}}(\hat{\mathcal{D}};\theta)$ as $\theta^{rw^\star}_{\epsilon_{i,j,c}}$. Then the change of parameters can be presented as,

$$\theta^{rw^\star}_{\epsilon_{i,j,c}} - \theta^\star = -\mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\big(\hat{\mathbf{y}}^{(i)}_c \log\left(\mathbf{f}_{\theta^\star}(x^{(i)})_c\right)\big) \cdot \exp(\epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c} - 1). \tag{31}$$

And the change with respect to $\epsilon_{i,j,c}$ is,

$$\frac{d(\theta^{rw^\star}_{\epsilon_{i,j,c}} - \theta^\star)}{d\epsilon_{i,j,c}} = \frac{d\theta^{rw^\star}_{\epsilon_{i,j,c}}}{d\epsilon_{i,j,c}} = -\mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\big(\hat{\mathbf{y}}^{(i)}_c \log\left(\mathbf{f}_{\theta^\star}(x^{(i)})_c\right)\big) \cdot \frac{d\exp(\epsilon_{i,j,c}\mathbf{W}_{j,\mathbf{L}_{ij},c} - 1)}{d\epsilon_{i,j,c}} \tag{32}$$

According to the Talyor expansion, we have $\exp(x) = 1 + x + O(x)$. Then, we can obtain,

$$\frac{d(\theta^{rw^\star}_{\epsilon_{i,j,c}} - \theta^\star)}{d\epsilon_{i,j,c}} = \frac{d\theta^{rw^\star}_{\epsilon_{i,j,c}}}{d\epsilon_{i,j,c}} = -\mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\big(\hat{\mathbf{y}}^{(i)}_c \log\left(\mathbf{f}_{\theta^\star}(x^{(i)})_c\right)\big) \cdot \mathbf{W}_{j,\mathbf{L}_{ij},c}. \tag{33}$$

The influence score on the loss of $z' = (x', y')$ by discarding the loss term $\bar{\ell}_{i,j,c}(\theta)$ is,

$$\bar{\phi}^{rw}_{i,j,c}(z') = \nabla_\theta\hat{\ell}\left(\mathbf{y}_k, \mathbf{f}_{\theta^\star}(x_k)\right)^\top \mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\big(\hat{\mathbf{y}}^{(i)}_c \log\left(\mathbf{f}_{\theta^\star}(x^{(i)})_c\right)\big) \cdot \mathbf{W}_{j,\mathbf{L}_{ij},c}. \tag{34}$$

C Influence Function derivation: the weight-moving method

Instead of employing the decomposing loss function, we introduce a more general influence estimation method - weight-moving Influence, which get ride of the loss decomposition and approximation and is agnostic to the selection of $\sigma(\cdot)$ function. As we introduced previously, the label of sample $x^{(i)}$ for each class c can be defined as:

$$\forall c \in [C], \qquad \hat{\mathbf{y}}^{(i)}_c = \frac{\sigma(\sum_{j=1}^M \mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^C \sigma(\sum_{j=1}^M \mathbf{W}_{j,\mathbf{L}_{ij},k})}. \tag{35}$$

Further, we define the label after removing the output value of j'-th label function on the c'-th class for sample i as:

$$\forall c \in [C], \qquad \hat{\mathbf{y}}^{(i)}_{-j'c',c} = \frac{\sigma(\sum_{j=1}^M \mathbb{1}[c \neq c' \vee j \neq j'] \cdot \mathbf{W}_{j,\mathbf{L}_{ij},c})}{\sum_{k=1}^C \sigma(\sum_{j=1}^M \mathbb{1}[k \neq c' \wedge j \neq j'] \cdot \mathbf{W}_{j,\mathbf{L}_{ij},k})}. \tag{36}$$

Similarly, we can also define the label vector $\hat{\mathbf{y}}^{(i)}_{-j'c'}$ for the sample $x^{(i)}$. Then, we define the minimizer of the weight-moving loss as:

$$\theta^{wm^\star}_{\epsilon_{i,j,c}} = \arg\min \frac{1}{N}\sum_{i'=1}^N \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c} \cdot \hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)})) - \epsilon_{i,j,c} \cdot \hat{\ell}(\hat{\mathbf{y}}^{(i)}_{-jc}, \mathbf{f}_\theta(x^{(i)})). \tag{37}$$

The change of parameters with respect to $\epsilon_{i,j,c}$ can be written as,

$$\frac{d(\theta^{wm^\star}_{\epsilon_{ijc}} - \theta^\star)}{d\epsilon_{i,j,c}} = \frac{d\theta^{wm^\star}_{\epsilon_{ijc}}}{d\epsilon_{i,j,c}} \tag{38}$$

$$= -\mathbf{H}^{-1}_{\theta^\star}\Big[\nabla_\theta \sum_{c'=1}^C \hat{\mathbf{y}}^{(i)}_{c'} \log\left(\mathbf{f}_{\theta^\star}(x^{(i)})\right) - \nabla_\theta \sum_{c'=1}^C \hat{\mathbf{y}}^{(i)}_{-jc,c'} \log\left(\mathbf{f}_{\theta^\star}(x^{(i)})\right)\Big] \tag{39}$$

$$= -\mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\hat{\ell}(\hat{\mathbf{y}}^{(i)} - \hat{\mathbf{y}}^{(i)}_{-jc}, \mathbf{f}_{\theta^\star}(x^{(i)})) \tag{40}$$

With $\epsilon_{i,j,c} = -\frac{1}{N}$, the weight for the sample $(x^{(i)}, \hat{\mathbf{y}}^{(i)})$ is moved to the the sample $(x^{(i)}, \hat{\mathbf{y}}^{(i)}_{-jc})$. Then, the influence of the loss on sample $z' = (x', y')$ by removing the output value of j-th label function on the c-th class for sample i is,

$$\bar{\phi}^{wm}_{i,j,c}(z') = -\nabla_\theta\ell\left(y', \mathbf{f}_{\theta^\star}(x')\right)^\top \mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\hat{\ell}(\hat{\mathbf{y}}^{(i)} - \hat{\mathbf{y}}^{(i)}_{-jc}, \mathbf{f}_{\theta^\star}(x^{(i)})). \tag{41}$$

We then define the weight-moving influence over the set $\mathcal{D}_v$ as,

$$\bar{\phi}^{wm}_{i,j,c}(\mathcal{D}_v) = \frac{1}{|\mathcal{D}_v|}\sum_{z' \in \mathcal{D}_v} \bar{\phi}^{wm}_{i,j,c}(z') \tag{42}$$

D Connection between weight-moving and reweighting method

To show the connection between weight-moving influence ($\bar{\phi}^{wm}_{i,j,c}$) and reweighting influence ($\bar{\phi}^{rw}_{i,j,c}$), we firstly decompose the weight-moving loss (37). For the identity function, we have:

$$\mathcal{L}^{wm} = \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c}\cdot \hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)})) - \epsilon_{i,j,c}\cdot \hat{\ell}(\hat{\mathbf{y}}^{(i)}_{-jc}, \mathbf{f}_\theta(x^{(i)})) \tag{43}$$

$$= \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c}\Big[-\sum_{c'=1}^{C} \frac{\sum_{j=1}^{M}\mathbf{W}_{j,\mathbf{L}_{ij},c'}}{\sum_{k=1}^{C}\sum_{j=1}^{M}\mathbf{W}_{j,\mathbf{L}_{ij},k}} \log\left(\mathbf{f}_\theta(x^{(i)})_{c'}\right)\Big] \tag{44}$$

$$- \epsilon_{i,j,c}\Big[-\sum_{c'=1}^{C} \frac{\sum_{j'=1}^{M}\mathbb{1}[c\neq c' \vee j\neq j']\cdot\mathbf{W}_{j',\mathbf{L}_{ij'},c'}}{\sum_{k=1}^{C}\sum_{j'=1}^{M}\mathbb{1}[k\neq c' \wedge j\neq j']\cdot\mathbf{W}_{j',\mathbf{L}_{ij'},k}} . \log\left(\mathbf{f}_\theta(x^{(i)})_{c'}\right)\Big] \tag{45}$$

For simplicity, we denote $C_i = \sum_{k=1}^{C}\sum_{j=1}^{M}\mathbf{W}_{j,\mathbf{L}_{ij},k}$ and $C'_i = \sum_{k=1}^{C}\sum_{j'=1}^{M}\mathbb{1}\{k\neq c', j\neq j'\}\cdot\mathbf{W}_{j',\mathbf{L}_{ij'},k}$. Note $\frac{C'_i}{C_i}\to 1$, with $j\to\infty$ or $C\to\infty$. Further, we assume $\frac{C'_i}{C_i}\approx 1$. Then, we have,

$$\mathcal{L}^{wm} = \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c}\Big[-\frac{\mathbf{W}_{j,\mathbf{L}_{ij},c}}{C_i}\log\left(\mathbf{f}_\theta(x^{(i)})_c\right)\Big] \tag{46}$$

$$\approx \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c}\cdot\bar{\ell}_{i,j,c}(\theta). \tag{47}$$

Note, the above loss function is same as the loss we defined in the Equation (27). Therefore, the weight-moving influence, $\bar{\phi}^{wm}_{i,j,c}(\mathcal{D}_v)$, is an approximation of the reweighting influence, $\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v)$, especially in the setting with large number of labeling function M and large number of class C.

E Proofs for theoretical analysis

We first show the correctness of Theorem 1, in which we employ the reweighting influence as a proxy to downweight or discard data samples. Then, we show that the similar conclusion can be obtained with weight-moving influence as the proxy. We summarized the conclusion for weight-moving influence into the Theorem 2.

E.1 Proofs for Theorem 1

Theorem 1. *Discarding or downweighting the loss terms in $\mathcal{S}_- = \{\bar{\ell}_{i,j,c}(\cdot)|i\in[N], j\in[M], c\in[C], \bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_t)>0\}$ from training could lead to a model with lower loss over a holdout set $\mathcal{D}_t$:*

$$\mathcal{L}(\mathcal{D}_t;\theta^\star_{\mathcal{S}_-}) - \mathcal{L}(\mathcal{D}_t;\theta^\star) \approx -\frac{1}{N}\sum_{\bar{\ell}_{i,j,c}(\cdot)\in\mathcal{S}_-}\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_t) \leq 0$$

where $\theta^\star_{\mathcal{S}_-}$ is the optimal model parameters obtained after the perturbation.

As we derived in Appendix B.1, the reweighting influence represent that the change of loss over the set $\mathcal{D}_t$ through upweighting the fine-grained sample $(x^{(i)}, \frac{1}{s_i}\mathbf{W}_{j,\mathbf{L}_{ij},c})$ by $\epsilon_{i,j,c}$,

$$\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v) = \mathcal{L}(\mathcal{D}_t;\theta^\star_{\mathcal{S}_-}) - \mathcal{L}(\mathcal{D}_t;\theta^\star) = -\sum_{(x_k,y_k)\in\mathcal{D}_t}\nabla_\theta\hat{\ell}\left(\mathbf{y}_k, \mathbf{f}_{\theta^\star}(x_k)\right)^\top \mathbf{H}^{-1}_{\theta^\star}\nabla_\theta\bar{\ell}_{i,j,c}(\theta^\star).$$

Through downweighting or discarding (note, discarding a sample is equal to downweighting it with $\epsilon_{i,j,c} = -1/N$) the fine-grained samples with $\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v) > 0$, the change of the loss is equal to $\epsilon_{i,j,c}\cdot\bar{\phi}^{rw}_{i,j,c} < 0$. As commonly assumed in the previous works [42, 24], the influence caused by reweighting different samples is independent. Then, with $\epsilon_{i,j,c} = -1/N$, we can get the conclusion that $\mathcal{L}(\mathcal{D}_t;\theta^\star_{\mathcal{S}_-}) - \mathcal{L}(\mathcal{D}_t;\theta^\star) \approx -\frac{1}{N}\sum_{\bar{\ell}_{i,j,c}(\cdot)\in\mathcal{S}_-}\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_t) \leq 0$.

E.2 Proofs for Theorem 2

Theorem 2. *Discarding or downweighting the training loss term in* $\mathcal{S}_- = \{\bar{\ell}_{i,j,c}(\theta) | i \in [N], j \in [M], c \in [C], \bar{\phi}^{wm}_{i,j,c}(\mathcal{D}_v) > \alpha\}$ *from training could lead to a model with lower loss over a test set* $\mathcal{D}_t$*:*

$$\mathcal{L}(\mathcal{D}_t; \theta^\star_{\mathcal{S}_-}) - \mathcal{L}(\mathcal{D}_t; \theta^\star) \leq 0$$

where $\alpha \in \mathbb{R}^+$ *is a positive number close to* 0*, and* $\theta^\star_{\mathcal{S}_-}$ *is the optimal model parameters obtained after the perturbation.*

We first decompose the weight-moving loss in the following form,

$$\begin{aligned}
\mathcal{L}^{wm} &= \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c} \cdot \hat{\ell}(\hat{\mathbf{y}}^{(i)}, \mathbf{f}_\theta(x^{(i)})) - \epsilon_{i,j,c} \cdot \hat{\ell}(\hat{\mathbf{y}}^{(i)}_{-jc}, \mathbf{f}_\theta(x^{(i)})) \\
&= \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c}\Big[- \sum_{c'=1}^{C} \frac{\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},c'}}{\sum_{k=1}^{C}\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k}} \log\left(\mathbf{f}_\theta(x^{(i)})_{c'}\right)\Big] \\
&- \epsilon_{i,j,c}\Big[- \sum_{c'=1}^{C} \frac{\sum_{j'=1}^{M} \mathbb{1}[c \neq c' \vee j \neq j'] \cdot \mathbf{W}_{j',\mathbf{L}_{ij'},c'}}{\sum_{k=1}^{C}\sum_{j'=1}^{M} \mathbb{1}[k \neq c' \wedge j \neq j'] \cdot \mathbf{W}_{j',\mathbf{L}_{ij'},k}} . \log\left(\mathbf{f}_\theta(x^{(i)})_{c'}\right)\Big] \\
&= \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c}\Big[- \frac{\mathbf{W}_{j,\mathbf{L}_{ij},c}}{C_i} \log\left(\mathbf{f}_\theta(x^{(i)})_{c}\right)\Big] \\
&- \epsilon_{i,j,c}(\frac{1}{c'_i} - \frac{1}{c_i}) \sum_{j'=1}^{M}\sum_{c'=1}^{C} \mathbb{1}[c \neq c' \vee j \neq j'] \big(- \mathbf{W}_{j',\mathbf{L}_{ij'},c'} \log\left(\mathbf{f}_\theta(x^{(i)})_{c'}\right)\big) \\
&= \frac{1}{N}\sum_{i'=1}^{N} \hat{\ell}(\hat{\mathbf{y}}^{(i')}, \mathbf{f}_\theta(x^{(i')})) + \epsilon_{i,j,c} \cdot \bar{\ell}_{i,j,c}(\theta) - \epsilon_{i,j,c} \cdot \tilde{L}_{i,j,c}
\end{aligned}$$

where, we denote $\tilde{L}_{i,j,c} = (\frac{1}{c'_i} - \frac{1}{c_i}) \sum_{j'=1}^{M}\sum_{c'=1}^{C} \mathbb{1}[c \neq c' \vee j \neq j'] \big(- \mathbf{W}_{j',\mathbf{L}_{ij'},c'} \log\left(\mathbf{f}_\theta(x^{(i)})_{c'}\right)\big)$ and $C_i = \sum_{k=1}^{C}\sum_{j=1}^{M} \mathbf{W}_{j,\mathbf{L}_{ij},k}$ and $C'_i = \sum_{k=1}^{C}\sum_{j'=1}^{M} \mathbb{1}[k \neq c', j \neq j'] \cdot \mathbf{W}_{j',\mathbf{L}_{ij'},k}$. Note, as we discussed in the Appendix D, $\frac{1}{c'_i} - \frac{1}{c_i}$ is a small positive value and close to 0. Therefore, $\tilde{L}_{i,j,c} \ll \bar{\ell}_{i,j,c}$.

$$\begin{aligned}
\bar{\phi}^{wm}_{i,j,c}(\mathcal{D}_v) &= \mathcal{L}^{wm}(\mathcal{D}_t; \theta^\star_{\epsilon_{i,j,c}}) - \mathcal{L}^{wm}(\mathcal{D}_t; \theta^\star) \\
&= \bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v) + \sum_{(x_k,y_k)\in\mathcal{D}_t} \nabla_\theta \hat{\ell}\left(\mathbf{y}_k, \mathbf{f}_{\theta^\star}(x_k)\right)^\top \mathbf{H}^{-1}_{\theta^\star} \nabla_\theta \tilde{L}_{i,j,c}.
\end{aligned}$$

We assume $\sum_{(x_k,y_k)\in\mathcal{D}_t} \nabla_\theta \hat{\ell}\left(\mathbf{y}_k, \mathbf{f}_{\theta^\star}(x_k)\right)^\top \mathbf{H}^{-1}_{\theta^\star} \nabla_\theta \tilde{L}_{i,j,c} \cdot \epsilon_{i,j,c} \leq \alpha$, where $\alpha > 0$. Then, through selecting the samples with $\bar{\phi}^{wm}_{i,j,c}(\mathcal{D}_v) \geq \alpha$, the reweighting influence term $\bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v) \geq 0$. Then we can get the conclusion that by reweighting the training loss term in $\mathcal{S}_- = \{\bar{\ell}_{i,j,c}(\theta) | i \in [N], j \in [M], c \in [C], \bar{\phi}^{rw}_{i,j,c}(\mathcal{D}_v) > \alpha\}$ from training, we have $\mathcal{L}(\mathcal{D}_t; \theta^\star_\epsilon) - \mathcal{L}(\mathcal{D}_t; \theta^\star) \leq 0$.

F Computation detail of inverse Hessian

The estimation of influence score requires the computation of the inverse hessian. The size of hessian matrix is propotional to the number of model parameters, thus directly computing the inverse hessian $\mathbf{H}^{-1}_{\theta^\star}$ is very expensive. For the $\mathbf{H}^{-1}_{\theta^\star}$ involved in the reweighting influence, Equation(11), and the weight-moving influence Equation (14), we employed the LiSSA (Linear time Stochastic Second-Order Algorithm) method [1], which provide an unbiased estimation of the Hessian-vector product through implicitly computing it with a mini-batch of samples. As demonstrated in the previous works [4, 41], the stochastic method is efficient and relatively accurate for sample-wise influence estimation. The algorithm can be summarized as:

- Step 1. Let $v := \sum_{(x',y')\in\mathcal{D}_v} \nabla_\theta \ell\left(y', \mathbf{f}_{\theta^\star}(x')\right)$ $\big(\sum_{(x',y')\in\mathcal{D}_v} \nabla_\theta \ell\left(y', \mathbf{f}_{\theta^\star}(x')\right)$ for weight-moving method $\big)$, and initialize the inverse HVP estimation $\mathbf{H}^{-1}_{0,\theta^\star} v = v$.

- Step 2. For $i \in \{1, 2, \ldots, J\}$, recursively compute the inverse HVP estimation using a batch size B of randomly sampled a data point $(x^{i'}, y^{i'})$, $\mathbf{H}_{i,\theta^\star}^{-1} v = v + \left(I - \nabla_\theta^2 \ell(y^{(i')}, \mathbf{f}_{\theta^\star}(x^{(i')})\right) \mathbf{H}_{i-1,\theta^\star}^{-1} v$, where J is a sufficiently large integer so that the above quantity converges.
- Step 3. Repeat Step 1-2 T times independently, and return the averaged inverse HVP estimations.

For the computation of self-influence, the influence estimation is required for each training sample. For real-world dataset, estimating each sample separately using the LiSSA method is intolerable. Instead of applying the stochastic method for each training sample, we leverage the relation between Hessian matrix and Fisher matrix, and use the K-FAC method [27] directly compute the inverse Hessian matrix. We refer interested readers to Barshan et al. [3] for details regarding the K-FAC approximation for the computation of inverse Hessian.

G Experimental details and additional results

G.1 Dataset statistics and implementation details

We summarize the dataset statistics in Table 6. All of the involved datasets are either publicly available or will be released upon the acceptance of this paper.

Table 6: Dataset statistics.

Domain (↓)	Dataset (↓)	#Label	#LF	#Train Data	#Valid Data	#Test Data
Tabular	Census	2	83	10,083	5,561	16,281
	Mushroom	2	20	6481	812	813
	PW	2	15	8654	1105	1106
	spambase	2	15	3595	460	461
Text	IMDb	2	5	20,000	2,500	2,500
	Yelp	2	8	30,400	3,800	3,800
	Youtube	2	10	1,586	120	250
Image	DN-real	5	5	2,587	323	324
	DN-sketch	5	5	1,777	222	223
	DN-quickdraw	5	5	2,000	250	250
	DN-painting	5	5	2,462	308	308
	DN-infograph	5	5	1,213	152	152
	DN-clipart	5	5	773	97	97

All experiments ran on a machine with an Intel(R) Xeon(R) CPU E5-2678 v3 with a 126G memory and a GeForce GTX 1080Ti-11GB GPU.

All the code was implemented in Python and largely based on the WRENCH [47] codebase.

G.2 Estimate actual effect of LF via source-aware IF

As described in Section 3.3, we can use the source-aware IF calculated by the reweighting method to estimate the influence score of each LF ($\phi_{\lambda^{(j)}} = \sum_{i=1}^{N} \sum_{c=1}^{C} \bar{\phi}_{i,j,c}$, the 2nd row in Table 1). Here, we study how well these influence scores reflect the actual effect of each LF. In Table 7, we again report the Spearman's ranking correlation coefficient (≤ 1). Please note that the estimated influence of each LF is more likely to be prone to noise since it involves more terms in summation than that of a training data. Also note that here we compare the estimated influence against the actual effect, which is calculated by removing the loss terms associated with each LF and retraining the end model. From the results, we can see that although there do exists cases where the results do not pass the significance test, the averaged ranking correlations are good (from 0.675-0.766). Such observations indicate that source-aware IF could be useful when estimating the influence of each IF.

G.3 Additional experiments on neural network

Although when the end model is neural network, the theory of IF breaks down [22], we are curious about whether the proposed source-aware IF is still effective. Thus, we conduct the experiments of

Table 7: Spearman's ranking correlation coefficient (≤ 1) between actual effect of each LF and that estimated from source-aware IF. We highlight results that do not pass the significance test in underline.

Dataset	Census	Mushroom	PW	Spambase	IMDb	Yelp	Youtube	DN-real	DN-sketch	DN-quickdraw	DN-painting	DN-infograph	DN-clipart	Avg.
MV	0.968	0.883	0.929	0.939	-0.700	0.262	0.842	1.000	0.300	0.900	1.000	1.000	1.000	0.717
DS	0.801	0.534	0.950	0.939	1.000	0.810	0.830	0.800	0.400	0.400	1.000	0.700	0.800	0.766
Snorkel	0.884	0.974	0.907	1.000	1.000	0.833	0.370	0.900	1.000	0.600	1.000	-0.600	-0.100	0.675

Table 8: Performance comparison results on identifying mislabeling of LFs. We report the average precision (AP) score averaged over LFs for each dataset. The larger the AP is, the better the method identify mislabeling of LFs.

		MV				DS				Snorkel			
Dataset	**KNN**	**LM**	**EM**	**RW**	**WM**	**LM**	**EM**	**RW**	**WM**	**LM**	**EM**	**RW**	**WM**
Census	0.810	0.809	0.787	**0.854**	0.824	0.787	0.787	**0.789**	0.788	0.787	0.787	**0.805**	0.803
Mushroom	**0.975**	0.923	0.828	**0.956**	0.954	0.828	0.828	**0.908**	0.893	0.828	0.828	**0.895**	0.861
PW	0.822	0.863	0.766	**0.887**	0.865	0.766	0.766	**0.887**	0.884	0.766	0.766	**0.880**	0.873
Spambase	0.782	0.772	0.738	**0.871**	0.801	0.738	0.738	0.784	**0.789**	0.738	0.738	0.799	**0.809**
IMDb	0.702	0.767	0.699	**0.786**	0.773	0.699	0.699	**0.771**	0.761	0.699	0.699	0.732	**0.732**
Yelp	0.752	0.792	0.731	**0.836**	0.813	0.731	0.731	0.761	**0.775**	0.731	0.731	0.836	**0.839**
Youtube	0.831	**0.949**	0.826	0.859	0.872	0.826	0.826	**0.889**	0.885	0.826	0.826	**0.909**	0.905
DN-real	0.711	0.447	0.417	**0.906**	0.878	0.417	0.417	**0.573**	0.536	0.445	0.417	**0.746**	0.651
DN-sketch	0.321	0.339	0.316	**0.730**	0.665	0.316	0.316	**0.490**	0.465	0.316	0.316	**0.424**	0.421
DN-quickdraw	0.362	0.256	0.255	**0.713**	0.675	0.255	0.255	**0.437**	0.389	0.255	0.255	**0.507**	0.494
DN-painting	0.454	0.416	0.360	**0.736**	0.697	0.360	0.360	**0.615**	0.557	0.360	0.360	**0.650**	0.598
DN-infograph	0.361	0.385	0.356	**0.621**	0.606	0.356	0.356	**0.538**	0.516	0.356	0.356	0.416	**0.418**
DN-clipart	0.437	0.487	0.434	**0.844**	0.822	0.434	0.434	**0.556**	0.545	0.434	0.434	0.630	**0.644**
Avg.	0.640	0.631	0.578	**0.815**	0.788	0.578	0.578	**0.692**	0.676	0.580	0.578	**0.710**	0.696

identifying mislabeling of LFs (see Table 8) and improving test loss (see Table 9) using two-layer neural network with the ReLU activation function as end model in this section. For simplicity, we do not include the methods based on RelatIF. From the results, we can see that, similar to the experimental results in the main body of this paper, source-aware IF outperforms baselines with a large margin. This indicates that source-aware IF is still effective even when the theory does not hold.

Table 9: Performance comparison results on the test loss of end models.

	MV					DS				Snorkel			
Dataset	**ERM**	**IF**	**GIF**	**RW**	**WM**	**ERM**	**IF**	**RW**	**WM**	**ERM**	**IF**	**RW**	**WM**
Census	0.484	0.382	0.368	**0.368**	0.381	0.663	0.432	**0.414**	0.432	0.580	**0.391**	0.407	0.453
Mushroom	0.220	0.161	0.168	**0.144**	0.152	0.370	0.216	**0.212**	0.242	0.336	0.190	**0.168**	0.284
PW	0.394	**0.309**	0.372	0.334	0.338	0.477	0.318	**0.316**	0.322	0.487	**0.333**	0.358	0.333
Spambase	0.529	0.336	0.370	**0.307**	0.349	0.663	0.345	**0.336**	0.364	0.411	**0.283**	0.299	0.295
IMDb	0.496	**0.481**	0.494	0.498	0.498	0.652	0.605	0.613	**0.586**	0.585	**0.576**	0.582	0.580
Yelp	0.524	0.352	0.463	0.394	**0.344**	0.460	**0.373**	0.447	0.452	0.513	**0.406**	0.497	0.490
Youtube	0.332	**0.215**	0.230	0.291	0.272	0.337	**0.265**	0.302	0.312	0.326	**0.269**	0.288	0.291
DN-real	1.053	0.618	0.547	**0.395**	0.469	0.860	0.482	0.214	**0.190**	0.918	0.545	0.431	**0.409**
DN-sketch	1.263	0.926	1.245	**0.869**	0.894	1.579	**1.058**	1.093	1.065	1.502	1.089	0.993	**0.989**
DN-quickdraw	1.626	1.106	1.492	**0.757**	0.768	1.345	0.840	**0.683**	0.769	1.620	1.444	1.188	**1.158**
DN-painting	1.242	0.876	0.947	**0.755**	0.797	1.043	0.680	**0.615**	0.654	1.210	0.872	0.915	**0.857**
DN-infograp	1.477	1.251	1.169	1.238	**1.180**	1.207	**1.198**	1.282	1.269	1.504	1.490	1.418	**1.336**
DN-clipart	1.084	0.913	0.830	**0.741**	0.779	1.000	**0.901**	1.042	0.993	0.944	0.676	0.729	**0.624**
Avg.	0.825	0.610	0.669	**0.545**	0.555	0.820	0.593	**0.582**	0.589	0.841	0.659	0.636	**0.623**

G.4 The study of label model approximation

In this study, we take a close look at the label model approximation. Specifically, we present the Mean Squared Error (MSE) and expected disagreement (DE) between the output of label model **DS/Snorkel** and their approximated version in Table 10. The MSE is evaluated against the predicted label posterior of models, and the ED is the expectation of $Y_1 \neq Y_2$ over the training samples, where Y_1 and Y_2 is the predicted label of the label model and its approximated version. From the results, we can see that both metrics have quite low value across datasets for binary classification, while they are relatively larger for multi-class classification, which is because label model for binary classification is much easier to approximate. Even for multi-class classification, all the EDs are still less then 22%, which indicates the approximated label model could replicate most of the predicted labels of original label model.

Table 10: .

Dataset	Metric	Census	Mushroom	PW	Spambase	IMDb	Yelp	Youtube	DN-real	DN-sketch	DN-quickdraw	DN-painting	DN-infograph	DN-clipart	Avg.
DS	MSE	0.00041	0.00000	0.00015	0.00003	0.00033	0.00004	0.00094	0.02127	0.02910	0.01689	0.01620	0.01735	0.02620	0.00992
	ED	0.00130	0.00000	0.00150	0.00000	0.00000	0.00000	0.00398	0.14766	0.21159	0.18400	0.14866	0.12861	0.15912	0.07588
Snorkel	MSE	0.00097	0.00003	0.00111	0.00011	0.00005	0.00050	0.00024	0.00995	0.00729	0.00370	0.01250	0.00749	0.00992	0.00414
	ED	0.00400	0.00031	0.00300	0.00056	0.00011	0.00322	0.00066	0.13993	0.10692	0.09100	0.19659	0.04369	0.12807	0.05524

G.5 How many mislabelings should be removed to have positive impact on test loss?

Here, we would like to answer the question of how many mislabeling to remove to have positive impact on the test loss. Concretely, this could be reflected by a quantity β we call maximal removing portion (MRP). That is, if we remove more than $\beta\%$ top-ranked negatively influential labelings, the resultant test loss after re-training will be larger than original test loss without any removing. In other words, removing any portion of labelings between 0% and $\beta\%$ has a positive impact on the test loss. We take label model MV and our RW method as an example for this study. We found that the MRP β is closely correlated to the accuracy of training labels produced by label model (see Figure 3). This is quite intuitive- for high-quality training set, the MRP would be low since we do not have to remove many labelings when training labels are already accurate, while for low-accuracy cases, the MRP would be high since most of the training labels are incorrect so we could remove a large portion of labelings but still be able to improve the test loss.

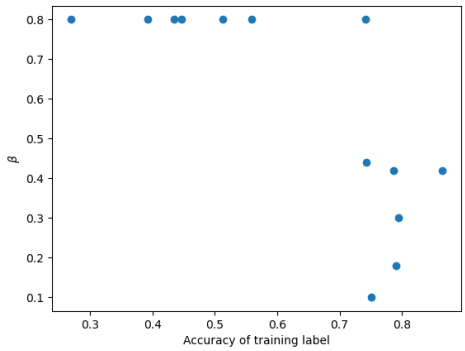

Figure 3: The correlation between maximal removing portion β and the accuracy of synthesized training labels.

G.6 Additional results for model behavior understanding

We include additional results of model behavior understanding for a textual dataset, Yelp. As below, T1-T5 are involved text samples, and the results are presented in Table 11. From the results, we

observed similar phenomenon as Table 2 in the main body of the paper, *i.e.*, our method provides explanations of model behavior from multiple angles.

T1: Goodness gracious. Put your big pants on. Parking costs $5. Er-ma-gawd. (Apparently that was an issue with other reviewers.) Carpool if it helps.
Ready now?
Prepared?
I had the delight of giggling like a kid tonight. We came in a large group and I was up front for a section. (The people including spoilers in their reviews clearly dropped their ice cream.)
While coming as a large group is fun it's even better to be broken into smaller groups. This way the mystery of what is around a bend is... A mystery.
I gotta tell you:
There was screaming.
Sure everyone has their strengths. (Apparently I'm not afraid of the dark. Who knew?) Put me near someone in costume who looks like a doll? One that follows you like a shadow?
I SCREAM.
No shame there.
(I swear she was eight.)
I've never been a fan of scary movies. I cover my eyes the entire time. This was a different kind of fear. A mixture of the unknown group dynamics sound and beer. (They serve that outside!)
Fantastic way to kick off Halloween!
p.s. Photos are *not* allowed.
(Outside by the bar and stage are ok.)
Label: Positive

T2: New to Pittsburgh and the first thing I had to do was scope out all the theaters! This weekend was a visit to the Manor! So let's start:
Location: Lovely Squirrel Hill and it just fits in perfectly with the charming neighborhood and wide variety of dining options from across the world.
Ambiance: Loved the classic look and feel of the venue. Walking in I could just imagine this place being used decades ago as the premiere spot in Pittsburgh for movie premieres with the long red carpet rolled out.
Food: Largest menu I've ever seen at a theater! Not only was the variety outstanding but their popcorn might be the cheapest in town! Didn't have a chance to try out the food but looking forward to making it a meal next time!
Drinks: Alcohol... and plenty (along with daily specials!). Automatic win in my books.
Screening Rooms: Here's where the great and bad come into play. I was ecstatic to see digital projection in room #4. The image was super crisp from the get-go and the colours were vibrant (unlike the case at many other theatres). The digital projection greatly enhanced the experience. The seats were also super comfy and reclined just perfectly! The bad? I have to admit I love Stadium Seating. I was lucky in that the place wasn't crowded but your view is automatically blocked (if only just a little) the second someone sits in front of you.
I love this place and can't wait to go back for their amazing selection of hard-to-find film and great movie-going experience!
Label: Positive

T3: For those who wants to see how this MGM Grand buffet looks like I have recorded a full slow and steady tour of this entire place in HD. if you guys want to see what they serve for Dinner before you guys dine there click on the YouTube link to see the a steady tour of MGM Grand Buffet in HD: or go onto YouTube and type ""Tour of Mgm Grand Buffet in Las Vegas in HD"" [a link]
There are a lot of food to choose from. I would rather eat here than at Bellagio buffet because the last time I ate at Bellagio it doesn't seem to offer that much food.
There are abundant amount of desserts but it's located at the far end of the buffet.
Overrall this place is really good. Plenty of jumbo cocktail shrimps and cold snow crab legs. I wish it was steam through.
It's best to go at 3:30 and pay the lunch price and get dinner food once it strikes 4pm.
As I said before if you want to see how this MGM buffet looks like watch my video below: [a link]
Label: Positive

T4: Blue 32 is my favorite place in town to watch a game (other than my living room). It's the best sports

Table 11: The most "responsible" data, LF, and labeling, identified by source-aware IF, for the misclassification of end models (trained with difference label models) on the same text sample (the Yelp dataset). We can see that source-aware IF can reveal the cause of misclassification from multiple angles. The T1-T6 are text samples, which can be found in Section G.6. The L1, L3, and L5 correspond to the 1st, 3rd, and 5th LF of the Yelp dataset, and the details of the LFs can be found in https://github.com/weakrules/Denoise-multi-weak-sources/blob/master/rules-noisy-labels/Yelp/yelp_rules.py.

Label Model	Misclassified Test Data		Resp. Training Data		Resp. LF	Resp. Labeling		
	Data	Pred. (Prob.)	Data	Weak Label (Prob.)		LF	Data	Vote
MV	T1	Negative (0.65)	T2	Negative (1.0)	L3	L5	T3	Negative
DS		Negative (0.81)	T4	Negative (0.99)	L1	L3	T5	Negative
Snorkel		Negative (0.64)	T6	Negative (0.97)	L1	L5	T3	Negative

bar food I've had in the area the booths are really comfortable and there are a ton of high quality HD TVs. I've been there for Oregon football games which are always fun (except when we play an Arizona school) as well as Arizona basketball games and the NCAA tournament. I've also been there randomly when I'm craving their boneless wings...
One of the best memories I have in AZ was at Blue 32 watching the Sweet 16 game this year between Arizona and Duke. Seeing the roof blown off by all the U of A fans was pretty special...It reminded me of the fanaticism that I see in Oregon for Duck football. That's what's so great about Blue 32. It's not 100% crazy but there's always a good energy going with the occasional freakout moment when Derrick Williams throws down on what seems like the entire Duke starting 5...
As far as food goes I love their boneless buffalo wings. They also have good onion rings and fries and their burgers sandwiches and salads are also pretty good. They have a great happy hour with cheap appetizer options (cheap boneless wings!!!) and good beer specials.
There's ample parking and outdoor seating as well. There isn't a bad seat in the house because they have so many TVs. Regardless of where you're sitting you'll be able to watch what you came to watch. If it's really busy service gets pretty slow so keep that in mind if you end up here during a big game. Overall Blue 32 is a great place to come for good food and drinks while you're watching your favorite team.
Label: Positive

T5: Seriously great ball park.We had great seats 3 rows back halfway between home & 1st but i dont think there is a bad seat in this park. Amazing view over the river & the downtown area of Pittsburgh..... & of course the Red Sox won!!
Label: Positive

T6: This is our 3rd time here and the prev 2times were good experiences with awesome overpriced desserts-but hey it's Vegas. We called ahead for a table for about 20 as we were celebrating my sons birthday with his baseball team. Service was painfully slow at 1100pm we were reminded constantly about their closing time of 1200 and the food was cold!!!! Not how to celebrate my sons 15th birthday at all—all the boys just wanted to leave without ordering the Treasure Chest. We were approached by the manager Shawn who could not have been more rude–he questioned why we cancelled the treasure chest ice dessert as they were making it—first off never ordered it just asked questions about it told him the service was very slow with cold food and our boys were falling asleep. He looked like he wanted to say more to us as he was very upset but then he walked away. Then we finally hot the bill!!!!!!!!! So overpriced for horrible tasting food!!!! Even drinks were $5 each. We started to settled up the bill and my friend took out her Rewards card and it was denied!!!! Really–we were told the bill was already complete and they would add the points but not take off the 10%!!! By then it was after midnight and thus manager just wanted us out—wrong move Shawn!!!! My friend and I got our 10% off and he cont to have smart ass comments for us.
I told him thanks for ruining m sons Birthday and he said –" like I ruined it ". I will be contacting the general manager and corporate.
Funny as we went for dessert on Friday night around 1000ish had great service and dessert which is why my son wanted to come back with his team!!!
Label: Negative

G.7 Additional experiments on improving test accuracy / F1

In the main body of the paper, we leverage the proposed method to improve test loss; in this study, we instead present the improved model performance in terms of standard classification metrics, *i.e.*, F1 score and accuracy. Specifically, for binary classification, we adopt F1 score, while for multi-class classification, we use accuracy. For simplicity, we do not include the methods based on RelatIF. From the results, we can draw similar conclusion as Table 4 in the main body, *i.e.*, our methods outperform baselines in most of cases and achieve better averaged performance.

Table 12: Performance comparison results on the classification metrics of end models.

	MV					DS				Snorkel			
Dataset	**ERM**	**IF**	**GIF**	**RW**	**WM**	**ERM**	**IF**	**RW**	**WM**	**ERM**	**IF**	**RW**	**WM**
Census	0.579	0.642	**0.653**	0.649	0.648	0.516	**0.605**	0.596	0.590	0.554	0.610	0.624	**0.630**
Mushroom	0.893	0.913	0.952	0.952	**0.958**	0.853	0.896	**0.899**	0.850	0.863	0.929	**0.936**	0.928
PW	0.844	0.876	0.873	0.877	**0.878**	0.799	0.866	0.863	**0.870**	0.807	0.867	0.867	**0.875**
Spambase	0.783	0.881	0.867	0.872	**0.883**	0.690	0.867	**0.870**	0.865	0.842	0.870	**0.901**	0.901
IMDb	0.789	0.789	0.740	0.790	**0.793**	**0.626**	0.612	0.626	0.626	**0.786**	0.786	0.786	0.786
Yelp	0.839	0.839	**0.847**	0.842	0.833	0.853	0.853	**0.862**	0.850	0.850	0.850	**0.852**	0.845
Youtube	0.790	0.810	**0.887**	0.861	0.870	0.824	**0.888**	0.858	0.821	0.858	0.898	**0.899**	0.883
DN-real	0.892	0.944	0.917	**0.966**	0.957	0.685	0.920	**0.966**	0.948	0.849	0.954	0.960	**0.966**
DN-sketch	0.552	0.632	0.538	**0.682**	0.664	0.484	0.507	**0.673**	0.578	0.538	0.659	**0.664**	0.628
DN-quickdraw	0.420	0.764	0.400	**0.780**	0.724	0.544	0.700	**0.740**	0.736	0.360	**0.720**	0.668	0.560
DN-painting	0.656	0.821	0.763	0.818	**0.860**	0.695	**0.847**	0.831	0.847	0.614	0.815	**0.854**	0.834
DN-infograp	**0.612**	0.586	0.566	0.526	0.599	0.553	0.559	**0.579**	0.546	0.539	0.520	0.546	**0.579**
DN-clipart	0.691	0.711	0.691	**0.742**	0.732	0.639	0.691	**0.711**	0.670	0.701	**0.804**	0.794	0.784
Avg.	0.718	0.785	0.746	0.797	**0.800**	0.674	0.755	**0.775**	0.754	0.705	0.791	**0.796**	0.785

H Limitation and social impacts

Limitations. Although we study several popular label models and show how to use approximated label model in case that users need to use a label model that is not included in this work, user may still encounter difficulties in real applications when using complicated label model. Also, this work focuses on the two-stage PWS pipeline, while very recently there are some one-stage methods, to which our framework may not be applicable. In the main body of the paper, we use logistic regression as end model in our experiments, while in appendix we present experimental results of a simple two-layer neural network. But we do not include experiments on complicated deep learning models like Convolutional Neural Network.

Social impacts. This work aims at help people understanding the Programmatic Weak Supervision and has the potential to resolve the bias in the generation process of training label, which might have positive social impact. We do not foresee any form of negative social impact induced by our work.