IDENTIFIABILITY GUARANTEES IN TIME SERIES REPRESENTATION VIA CONTRASTIVE SPARSITY-INDUCING

Anonymous authors
Paper under double-blind review

ABSTRACT

Time series representations learned from high-dimensional data are generally expected to be more robust and better at generalizing to new and potentially out-of-distribution (OOD) scenarios. Yet, this is not always the case, as variations in unseen data or prior assumptions may insufficiently constrain the posterior probability distribution, leading to an unstable model and non-disentangled representations, which in turn less generalization and prediction accuracy. While identifiability and disentangled representations for time series are often said to be beneficial for generalizing downstream tasks, the current empirical and theoretical understanding remains limited. In this work, we provide results on identifiability that guarantee complete disentangled representations via Contrastive Sparsity-inducing Learning, which improves generalization and interpretability. Motivated by this result, we propose the `TimeCSL` framework to learn a disentangled representation that generalizes and maintains compositionality. We conduct a large-scale study on time series source separation, investigating whether sufficiently disentangled representations enhance the ability to generalize to ODD downstream tasks. Our results show that sufficient identifiability in time series representations leads to improved performance under shifted distributions. Our code available at `https://anonymous.4open.science/r/TimeCSL-4320`.

1 INTRODUCTION

Time series representation learning has been proposed as a solution to the lack of robustness, transferability, systematic generalization, and interpretability of current downstream task methods. However, the problem of learning meaningful representation for time series, is still open. This problem is strongly related to learning *disentangled* representations pointed by Bengio et al. (2013). Informally, a representation is considered *disentangled* when its components are in *one-to-one* correspondence with natural and interpretable factors of variations. However, a large body of work have investigated theoretically under which conditions disentanglement is possible through the lens of identifiability originated in works on non-linear independent analysis (ICA) (Comon, 1994; Hyvarinen & Morioka, 2017; Hyvarinen et al., 2019; Khemakhem et al., 2020b), which aims to recover independent latent factors from mixed observations. It has been found in (Locatello et al., 2019; Van der Maaten & Hinton, 2008; Dittadi et al., 2021; Montero et al., 2021; Lachapelle et al., 2022) that without exploiting an appropriate class of assumptions in estimation, the latent variables are not identifiable in the most general case. Existing methods like Generalized Contrastive Learning (GCL) via an auxiliary variable (Hyvarinen et al., 2019), HM-NLICA (Hälvä & Hyvärinen, 2020), PCL (Hyvärinen & Morioka, 2017), and SlowVAE (Klindt et al., 2021) rely on the assumption of mutually independent sources in the data generation process. However, this assumption breaks down for time-lagged or dependent latent variables, distorting identifiability. SlowVAE assumes linear relationships, while TDRL optimizes mutual information between input and latent factors, penalizing static-dynamic interactions, and assumes only time-lagged influences. This requires matching the temporal resolution

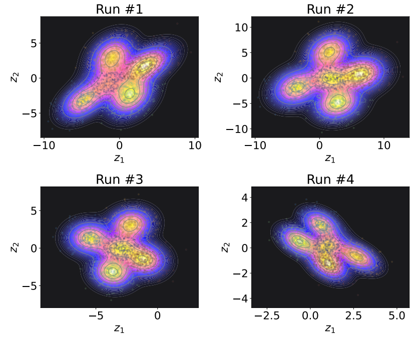

Figure 1: Recovered 5 slots latents for 4 runs of `TimeCSL` on UKDALE dataset.

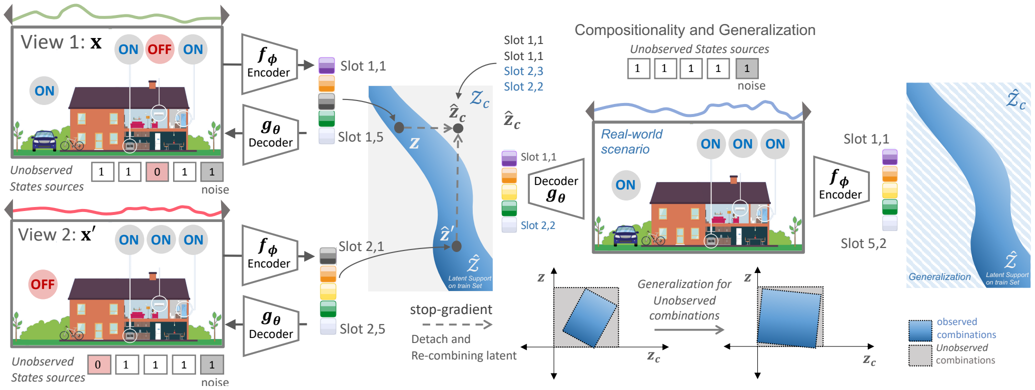

Figure 2: **Multi-view motivating setting for the *energy time series representation*. Left:** We consider $\{1, 2, 3, 4\}$ sources and $\{5\}$ representing measurement **noise** or other irrelevant sources. The mixed observation at different time are: x includes $\{1, 2, 4, 5\}$, and x' includes $\{2, 3, 4, 5\}$. **Center:** Training distribution combinations. **Right:** compositional consistency for ODD based recombining latent slots $(\mathbf{z}, \mathbf{z}')$ allows for generalization, thus improving downstream tasks.

of observations and latent variables (Yao et al., 2022). A more flexible framework is needed to deal with real-world time series (*e.g.*, energy separation), where sources are often dependent, may be correlated in a general nonstationary environments with time-varying relations. Prior work on sparsity through convex optimization with sparsity-inducing norms (Bach et al., 2012) and recent findings in disentanglement using sparse task predictors (Lachapelle et al., 2023a; 2022) show impressive results empirically. A key question is whether these sparsity can guarantee identifiability, and resulting in disentangled representations that capture meaningful features and remain stable under distribution shifts? Indeed, without identifiability, the representation of a model can be unstable and not consistent, in the sense that retraining the same model under small perturbations of the data or hyperparameters may result in wildly different representations. More formally, identifiability means that the parametrization of the model is injective (Roeder et al., 2021; Khemakhem et al., 2020b).

In this work, we establish that achieving Identifiability for time series representation up to affine transformations—essentially, disentanglement—is possible for time series through Contrastive Sparsity-inducing Learning (`TimeCSL`) (see Fig. 1, across 4 runs, the latents are recovered, providing evidence of the latent space recovery up to the affine transformations). Importantly, this can be achieved with commonly adopted weaker assumptions. Specifically, we allow for statistically dependent latent factors, with empirical evidence indicating that relaxing independence improves ODD generalization (Roth et al., 2023; Oublal et al., 2024). Moreover, it doesn't require complete information on auxiliary variables. Two key strengths stand out: first, it accommodates nonlinear task-specific predictors and unknown latent relationships, expanding its applicability to time series. Second, `TimeCSL` reduces reliance on fully labeled data via contrastive learning, offering greater flexibility across time series datasets. Our contributions include:

[1] We rely on the sparsity assumption of time series representation, and provide theoretical insight and empirical arguments on how, and under which condition, identifiability up to affine transformation is preserved. We show that `TimeCSL` outperform an affine transformation *e.g.*, permutation and element-wise transformation.

[2] Unlike many existing identifiability results, we allow for arbitrary dependencies without parametric assumptions, achieving slot latent disentanglement through *Partial Selective Pairing*. This approach is particularly suitable for time series, where obtaining fully labeled data is challenging.

[3] Building on this result, we propose generalization consistency for uncommon OOD correlations. We validate it by showing that `TimeCSL` effectively disentangles latent slots in real-world source separation tasks (*e.g.*, energy disaggregation). Notably, existing architectures (*e.g.*, D3VAE, RNN-VAE) improve by **+11%** `RMSE` in downstream tasks with disentangled representations. We also release over 221 trained models as baselines for future research[1].

Notation Vectors and vector-valued functions are denoted by bold letters. Vectors with factorized dimensionality, such as the latent variable $\mathbf{z} \in \mathbb{R}^{d_{\mathcal{Z}}}$, where the latent space $\mathcal{Z}$ has dimension

---

[1] Pretrained models and usage guidelines: https://anonymous.4open.science/r/TimeCSL-4320

$d_{\mathcal{Z}} = d \times n$, or functions with factorized outputs, like the encoder $\boldsymbol{f}_\phi \colon \mathcal{X} \to \mathbb{R}^{2d_{\mathcal{Z}}}$, where $\boldsymbol{f}_\phi(\mathbf{x}) = \left[\boldsymbol{\mu}_\phi\left(\mathbf{x}\right), \boldsymbol{\sigma}_\phi\left(\mathbf{x}\right)\right]^\top$, are used in this context. We refer to $(\boldsymbol{f}_\phi, \boldsymbol{g}_\theta)$ as the ground truth encoder-decoder, and $(\hat{\boldsymbol{f}}_\phi, \hat{\boldsymbol{g}}_\theta)$ as the learned encoder-decoder, and $\hat{\mathbf{z}} := \{\hat{\boldsymbol{z}}_1, \ldots, \hat{\boldsymbol{z}}_n\}$ is the learned representation of $\mathbf{z} := \{\boldsymbol{z}_1, \ldots, \boldsymbol{z}_n\}$. When indexing with k, we refer to the k-th contiguous sub-vector, such as the learned slot latent $\hat{\boldsymbol{z}}_k := \hat{\boldsymbol{\mu}}_{\phi\,k}\left(\mathbf{x}\right) + \hat{\boldsymbol{\sigma}}_{\phi\,k}\left(\mathbf{x}\right) \odot \boldsymbol{\epsilon}$, where $\boldsymbol{\epsilon} \sim \mathcal{N}(0, \mathbf{I})$, and both $\hat{\boldsymbol{\mu}}_{\phi\,k}\left(\mathbf{x}\right)$, $\hat{\boldsymbol{\sigma}}_{\phi\,k}\left(\mathbf{x}\right) \in \mathbb{R}^d$. Additionally, for a positive integer n, we denote the set $\{1, \ldots, n\}$ as $[n]$.

2 BACKGROUND AND PRELIMINARIES

We formalize our setting for time series representation learning, in which we have a set of high-dimensional time series observations $\mathbf{x}$ as C-variate time series observed at times $t = 1, \ldots, T$. We denote by $\mathbf{x} \in \mathbb{R}^{C \times T}$ resulting matrix with rows denoted by $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_C$. Each row can be seen as a univariate time series in $\mathbb{R}^T$. Without loss of generality, we consider the case where $C = 1$. In the source separation problem, the observed signal $\mathbf{x} \in \mathcal{X}$ is assumed to be a mixture of n sources, denoted as $\mathbf{y} := \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n\} \in \mathcal{Y}$, where each $\boldsymbol{y}_k \in \mathbb{R}^T$, with additive independent noise $\xi \in \mathbb{R}^T$: $\mathbf{x} = \sum_{k=1}^{n} \boldsymbol{y}_k + \xi$. The space $\mathcal{Y}$ representing the individual source signals, satisfies $\mathcal{Y} \subseteq \mathcal{X}$ [2]. Given a data set of N samples, denoted as$\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^N$, the goal is to recover $\mathbf{y}$ from $\mathbf{x}$. Although the observed signal is a sum of sources, the mixing process is inherently *non-linear* due to interactions from multi-state appliances, power distortions, and continuously fluctuating power in NILM (Yue et al., 2020), similar to harmonic distortions and reverberations in audio (Lu et al., 2021).

To formalize this idea, we consider a Euclidean observation space $\mathcal{X}$, and denote by $\mathcal{M}_+^1(\mathcal{X})$ the set of probability measures on $\mathcal{X}$. The standard framework for learning representations typically relies on VAEs (Kingma & Welling, 2014), which consist of two main components: i) the encoder network with parameters ϕ, and ii) the decoder network with parameters θ. The encoder parameterized a distribution $q_\phi(\mathbf{z}|\mathbf{x})$ over the latent space $\mathcal{Z} = \mathbb{R}^{d_{\mathcal{Z}}}$, with $d_{\mathcal{Z}} = d \times n$ representing the dimensionality, serves as a variational approximation of the Bayesian posterior $p_\theta(\mathbf{z}|\mathbf{x})$. The likelihood $p_\theta(\mathbf{x}|\mathbf{z})$ is parameterized by the decoder network. In standard setup, we assume a standard Gaussian prior $p(\mathbf{z}) = \mathcal{N}(\mathbf{0}, \mathbf{I})$ on $\mathcal{Z}$ and Gaussian distributions $q_\phi(\mathbf{z}|\mathbf{x})$. More precisely, for any $\mathbf{x} \in \mathcal{X}$, the distribution $q_\phi(\mathbf{z}|\mathbf{x})$ is a Gaussian distribution with a diagonal covariance matrix $\mathcal{N}(\boldsymbol{\mu}_\phi\left(\mathbf{x}\right), \operatorname{diag}(\boldsymbol{\sigma}_\phi^2\left(\mathbf{x}\right)))$, where $\boldsymbol{\mu}_\phi : \mathcal{X} \to \mathcal{Z}$ and $\boldsymbol{\sigma}_\phi : \mathcal{X} \to \mathcal{Z}_{\geq 0}$. In order to simplify some of the expressions below, it may be useful to express the encoder network as a function $\boldsymbol{f}_\phi : \mathcal{X} \to \mathbb{R}^{2d_{\mathcal{Z}}}, \quad$ where $\boldsymbol{f}_\phi\left(\mathbf{x}\right) = \left[\boldsymbol{\mu}_\phi\left(\mathbf{x}\right), \boldsymbol{\sigma}_\phi\left(\mathbf{x}\right)\right]^\top$ and the decoder is a compositional function $\boldsymbol{g}_\theta : \mathbb{R}^{d_{\mathcal{Z}}} \to \mathbb{R}^{T \times n}$, defined as $\boldsymbol{g}_\theta(\mathbf{z}) = \sum_{k=1}^{n} \boldsymbol{g}_{\theta\,k}(\mathbf{z})$, where each $\boldsymbol{g}_{\theta\,k} : \mathbb{R}^{d_{\mathcal{Z}}} \to \mathbb{R}^{T \times 1}$, mainly, $\boldsymbol{y}_k = \boldsymbol{g}_{\theta\,k}(\mathbf{z})$. The encoder and decoder networks are jointly trained on data set of N samples by minimizing the following objective:

$$\mathcal{L}_{\text{VAE}}(\phi, \theta) = \frac{1}{N} \sum_{i=1}^{N} \left[\mathop{\mathbb{E}}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x}_i)} \left[\log p_\theta(\mathbf{x}_i|\mathbf{z})\right] - \beta \mathrm{KL}(q_\phi(\mathbf{z}|\mathbf{x}_i) \,||\, p(\mathbf{z})) \right], \tag{2.1}$$

where the first part of Eq. (2.1) is the reconstruction loss and the second part is the KL-divergence between the latent distributions (associated to the training samples) and the prior over the latent space, weighted by a hyper-parameter $\beta > 0$ (Higgins et al., 2016). The reconstruction loss measures the similarity between the true source measurements $\mathbf{y} = \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n\}$ and its reconstruction given by a multi-output decoder $\boldsymbol{g}_\theta(\mathbf{z}) =: \{\boldsymbol{g}_{\theta\,1}(\mathbf{z}), \ldots, \boldsymbol{g}_{\theta\,n}(\mathbf{z})\}$, and can be defined in many ways. With a Gaussian likelihood, the reconstruction loss is the squared L_2 norm: $\left\|\sum_{k=1}^{n} \left(\boldsymbol{y}_k - \boldsymbol{g}_{\theta\,k}(\mathbf{z})\right)\right\|^2$, or in an unsupervised fashion, when the label source $\mathbf{y}$, *i.e.,* when the label source $\mathbf{y}$ is absent, the reconstruction loss becomes $\left\|\mathbf{x} - \boldsymbol{g}_\theta(\mathbf{z})\right\|^2$. After training, the VAE defines a generative model using the prior $p(\mathbf{z})$ and the decoder $\boldsymbol{g}_\theta$. The VAE's generated distribution denote by $\boldsymbol{g}_\theta \sharp p(\mathbf{z}) \in \mathcal{M}_+^1(\mathcal{X})$ allows one to generate new samples by first sampling a latent vector from the prior, then passing it through the decoder. We further assume the following:

Assumption 2.1. The decoder $\boldsymbol{g}_\theta$ is a piecewise affine function, such as a multilayer perceptron with ReLU (or leaky ReLU) activations.

A special case of this model is well-studied in theory and applications and in deep generative models literature (Burgess & Kim, 2018; Ahuja et al., 2022). We consider the following generative process:

---

[2] When $\mathbf{x}$ is sparse, it may equal a single source $\boldsymbol{y}_1$, so $\mathcal{Y} \subseteq \mathcal{X}$.

Data-generating process. We assume Asm 2.1, and we consider the following generative model for observations $\mathbf{x}$:

$$\mathbf{x} = \sum_{k=1}^{n} \boldsymbol{g}_{\theta\, k}(\mathbf{z}) + \xi, \quad \mathbf{z} = (\boldsymbol{z}_1, \dots, \boldsymbol{z}_n) \in \mathbb{R}^{d \times n}, \ \mathrm{vec}(\mathbf{z}) \sim \sum_{j=1}^{J} \omega_j \mathcal{N}(\mathrm{vec}(\boldsymbol{\mu}_j), \boldsymbol{\Sigma}_j), \qquad (2.2)$$

where $\xi \in \mathbb{R}^T$, denote independent random noise. Our results include the noiseless case $\xi = 0$ as a special case (*i.e.,* when all sources are well-known). The notation $\mathrm{vec}(\mathbf{z}) \in \mathbb{R}^{d \cdot n}$ denotes the vectorization [3] of $\mathbf{z}$ that follows a a multivariate Gaussian Mixture Model (GMM), and ω_j are the mixture weights (with $\sum_{j=1}^{J} \omega_j = 1$), with mean $\mathrm{vec}(\boldsymbol{\mu}_j) \in \mathbb{R}^{d \cdot n}$ and $\boldsymbol{\Sigma}_j = \boldsymbol{\Sigma}_d \otimes \boldsymbol{\Sigma}_n$ with $\boldsymbol{\Sigma}_d$ being the $d \times d$ covariance and $\boldsymbol{\Sigma}_n$ the $n \times n$ covariance between sub-components *i.e.,* $\boldsymbol{z}_k$. Here, $\otimes$ denotes the Kronecker product. The GMM prior assumption can be generalized to exponential family mixtures (Kivva et al., 2022), provided the prior is analytic and affine-closed. Additionally, GMMs can approximate complex distributions (Nguyen & McLachlan, 2019). This maintains the flexibility and generalization of Eq. (2.2), and we impose no constraints on: 1) ReLU architectures, 2) independence of $\mathbf{z}$, or 3) the complexity of the mixture model or neural network.

Objective. Our goal is to identify the latent variables $\mathbf{z}$ from a set of observations $\mathbf{x}$ that lead to better reconstruction of true sources $\boldsymbol{y}_k = \boldsymbol{g}_{\theta\, k}(\mathbf{z})$, thus $\mathbf{y}$, which meaning recovering $\mathbf{x}$ up to an additive error ξ. In representation learning, we usually cannot recover the exact value of the latent variables, but we can only identify them up to some transformation. Similar notions of identifiability were used in previous works; In classical Linear ICA (Comon, 1994), the observed $\mathbf{x} = \boldsymbol{A}\mathbf{z}$, assumes independent components in $\mathbf{z}$ and a linear mixing matrix $\boldsymbol{A}$. Compared to Eq. (2.2), this simplifies to a linear $\boldsymbol{g}_\theta$ with $\epsilon = 0$, similar to classical nonlinear ICA (Hyvärinen & Pajunen, 1999). Unlike these, our setting via Eq. (2.2) does not require $\boldsymbol{z}_k$ to be independent, recognizing the interdependencies in real-world data, and instead imposes structure on the nonlinear mixing Asm 2.1. Identifiability here ensures a linear mapping between ground truth and learned variables but does not guarantee disentanglement. Following (Lachapelle et al., 2022; Locatello et al., 2020), we extend this to define slot identifiability up to element-wise linear transformations bellow:

Definition 2.2 (Slot Identifiability and Disentangled Representation). An autoencoder $\hat{\boldsymbol{g}}_\theta, \hat{\boldsymbol{f}}_\phi$ *slot-identifies* $\mathbf{z}$ on $\mathcal{Z}$ w.r.t. the true decoder $\boldsymbol{g}_\theta$ if $\hat{\mathbf{z}} = \hat{\boldsymbol{f}}_\phi(\boldsymbol{g}_\theta(\mathbf{z}))$ minimizes the reconstruction loss in Eq. (2.1) (first term), and there exists an invertible transformations $\mathbf{h} := \{\boldsymbol{h}_1, \boldsymbol{h}_2 \dots, \boldsymbol{h}_n\}$, with $\boldsymbol{h}_k \in \mathbb{R}^d$, such that $\hat{\boldsymbol{z}}_k = \boldsymbol{h}_k(\boldsymbol{z}_k) \, \forall k \in [n]$, ensuring a one-to-one mapping. The learned representation $\hat{\mathbf{z}}$ identified *up to permutation, scaling, and element wise linear transformation* $\mathbf{z}$, if there exist a permutation matrix $\boldsymbol{\Pi}$ of $[n]$, an invertible diagonal matrix $\boldsymbol{\Lambda}$ constructed from the scaling factors of $\mathbf{h}$, and an offset $\mathbf{b}$, such that $\hat{\mathbf{z}} = \boldsymbol{\Lambda}\boldsymbol{\Pi}\mathbf{z} + \mathbf{b}$.

3 RELATED WORK

On the Nonlinear ICA for Time Series Representation Learning. Recent advances in nonlinear ICA has increasingly focused on utilizing temporal structures and nonstationarities for identifiability. (Hyvärinen & Morioka, 2016) introduced Time-Contrastive Learning (TCL), which assumes independent sources and leverages variance differences across data segments. Similarly, Permutation-based Contrastive Learning (PCL) identifies independent sources under the assumption of uniform dependency. i-VAE (Khemakhem et al., 2020a) extended this by using VAEs to approximate joint distributions in nonstationary regimes, relaxing the independence assumption with promising results. Further, (Roth et al., 2023) and (Oublal et al., 2024) explored using contrastive learning for latent space recovery without assuming source independence. LEAPS (Yao et al., 2021) proposed a non-parametric approach for causal discovery in motion, but is limited by assumptions of no instantaneous causal influence and causal constancy. Work by (Lachapelle et al., 2022), and (Klindt et al., 2021) also requires source independence or some intervention (Ahuja et al., 2023) for identifiability. In contrast, our work extends identifiability theory by relaxing the independence assumption. We impose no constraints on $p(\mathbf{z})$ beyond its definition in Eq. (2.2), offering a more flexible framework. Recent studies have explored structural assumptions like orthogonality (Gresele et al., 2021; Zheng et al.,

---

[3] The vectorization of $\mathbf{z}$ (*i.e.,* stacks the columns of $\mathbf{z}$ in a single column vector), following a multivariate Gaussian mixture model, is equivalent to $\mathbf{z}$ following a Matrix Gaussian mixture, as shown in App. A.3.2.

2022) or fixed sparsity (Moran et al., 2022), but our approach generalizes these further. Our intuitive arguments for the reason of that sparsity and contrastive learning may be complementary to each other, and combining them could potentially gain better disentanglement.

Time Series Representation with Out-Of-Distribution. Recent studies in time series representation learning include methods like RNNVAE (Chung et al., 2015), SlowVAE (Klindt et al., 2021), and D3VAE (Li et al., 2023). Other approaches, such as CoTS (Woo et al., 2022), and CDSVAE (Bai et al.) focus on sequential data with contrasive disentnaglement. Transformer-based models, such as Transformer (Zerveas et al., 2021), TimesNet (Wu et al., 2022), Autoformer (Wu et al., 2021), and Informer (Zhou et al., 2021), are designed to capture long-term dependencies but do not focus on identifiability or disentanglement. Understanding whether they preserve disentanglement presentation across runs is crucial for robust representation learning. Inspired by OOD generalization frameworks in object-centric models (Zhao et al., 2022; Netanyahu et al., 2023), this ideas can be extend to time series. OOD generalization has been demonstrated in additive models (Dong & Ma, 2022) and slot-wise functions with nonlinearity (Wiedemer et al., 2023b), assuming identifiability for images. Work by (Lachapelle et al., 2023b) and (Wiedemer et al., 2023a) shows that additivity of the decoder (see § 2) ensures identifiability and decoder generalization under certain assumptions, which we apply to time series for enhanced generalization. Finding generalization with less data has been a major focus, with SSL methods moving beyond negative pairs in CL to alignment with regularization to avoid collapsed representations. Methods like BYOL(Grill et al., 2020) and SimSiam(Chen & He, 2021) achieve this via moving-average updates or stop-gradients, while BarlowTwins (Zbontar et al., 2021) aligns cross-correlation with the identity matrix to reduce redundancy. Extending these to time series, we demonstrate promising generalization by employing a decoder satisfying Asm 2.1.

4 IDENTIFIABILITY GUARANTEES VIA CONTRASTIVE SPARSITY-INDUCING

In this section, we begin with the intuition behind the proposed approach, which leverages sparsity in the mixing process to achieve identifiability. We propose an approach leveraging sparsity in the mixing process to achieve identifiability. Previous methods relying on independence or non-Gaussian priors for identifiability often fail in nonlinear cases, as marginal transformations can preserve independence without revealing true structure (Hyvärinen & Pajunen, 1999; Hyvärinen et al., 2019). We builds on the insight that any alternative solution introducing indeterminacy, beyond permutations or component-wise transformations, would result in a denser structure. Rather than constraining functional forms (Taleb & Jutten, 1999; Ahuja et al., 2023) or relying on auxiliary variables (Khemakhem et al., 2020a), we assume Partial Contrastive Sparsity for time series. This enables learning identifiable and disentangled representations without requiring independence or parametric assumptions on $p(\mathbf{z})$.

① **Partial Contrastive Pairing for Time Series** For instance, in multiview object-centric settings (Bengio et al., 2020) or time series (see Fig. 2), a view $\mathbf{x}$ and its augmentation $\mathbf{x}'$ typically share limited information rather than complete overlap. To address this, we propose a more general case, *Partial Selective Pairing*, which allows pairs to share only a subset of relevant factors, serving as a relaxation of *Selective Pairing* in SSL. Assuming the data process generating Eq. (2.2), we define the shared support indices $\mathcal{S}$ of all sources that actively contribute to $\mathbf{x}$ as $\mathcal{S}(\mathbf{x}) := \{k \mid \boldsymbol{y}_k \neq 0,\ k = 1, 2, \ldots, n\}$. The *Partial Selective Pairing* between observations $\mathbf{x}$ and $\mathbf{x}'$ is based on *shared support* $\mathbf{I}(\mathbf{x}, \mathbf{x}') := \mathcal{S}(\mathbf{x}) \cap \mathcal{S}(\mathbf{x}')$.

Assumption 4.1 (**Sufficient Partial Selective Pairing).** For each factor $k \in [n]$, there exist observations $(\mathbf{x}, \mathbf{x}') \in \mathcal{X}$ such that the union of the shared support indices $\mathbf{i} = \mathbf{I}(\mathbf{x}, \mathbf{x}')$ that do not include k must cover all other factors. Formally:

$$\bigcup_{\mathbf{i} \in \mathcal{I} \mid k \notin \mathbf{i}} \mathbf{i} = [n] \setminus \{k\} \quad , \quad \mathcal{I} := \{\mathbf{i} \subseteq [n] \mid p(\mathbf{i}) > 0\} \tag{4.1}$$

where $\mathcal{I}$ is the set of shared support indices and $p(\mathbf{i}) := \frac{1}{\#\mathcal{X}} \cdot \#\left\{\mathcal{S}(\mathbf{x}) = \mathbf{i},\ \mathbf{x} \in \mathcal{X}\right\}$ gives the probability that the factors indexed by $\mathbf{i}$ are active, with $k \notin \mathbf{i}$ inactive.

In nonlinear ICA, sufficient variability assumes the auxiliary variable diversely affects source distributions (Hyvärinen & Morioka, 2016; Hyvarinen et al., 2019), while (Lachapelle et al., 2023a) adapted this concept for task supports. Similarly, Structural Variability (Ng et al., 2023) ensures each pair of sources influences distinct observed variables. However, overlapping influences often occur in real-world time series, posing practical challenges. Instead, our Partial Selective Pairing assumption Eq. (4.1) permits some overlap, provided the union of shared support indices (excluding the specific source) spans all sources, enabling flexible modeling of source dependencies.

② **Identifiability via Contrastive Sparsity-inducing.** According to Asm 4.1, the sparsity-inducing nature arises from the existence of a source $k \notin \mathbf{i}$. However, this source is still well-defined within the support indicating that existing source k remains inactive in either $\mathbf{x}$ or $\mathbf{x}'$. The use of a sparsity constraint or regularization is inspired by prior work (Ahuja et al., 2023; Lachapelle et al., 2023a) in the context of sparse multitask learning. The loss of zero reconstruction ensures that the encoding $\boldsymbol{f}_\phi(\mathbf{x})$ retains all information, implying that $(\hat{\mathbf{z}}, \hat{\mathbf{z}}')$ achieves sparsity comparable to the ground truth $(\mathbf{z}, \mathbf{z}')$. This sparsity in a latent representation $\hat{\mathbf{z}}$, means only a subset of latent variables are active for a given input $\mathbf{x}$. If $\frac{|\hat{\boldsymbol{\mu}}_{k,\phi}(\mathbf{x})|}{\hat{\boldsymbol{\sigma}}_{k,\phi}(\mathbf{x})}$ is small (e.g., close to zero), it suggests the k-th latent variable is not contributing, thus making it inactive $\boldsymbol{y}_k = 0$. However, $\frac{|\hat{\boldsymbol{\mu}}_{k,\phi}(\mathbf{x})|}{\hat{\boldsymbol{\sigma}}_{k,\phi}(\mathbf{x})}$ is large (e.g., ≥ 1), it implies the source k contribute to $\mathbf{x}$. Bounding this ratio ensures that only the most relevant latent variables remain active, indirectly enforcing sparsity by limiting the number of significant variables. This raises the question of whether minimizing the l_0-norm of the learned latents, with sufficient partial pairing, can identify $\mathbf{z}$ through $\hat{\boldsymbol{g}}_\theta^{-1}(\mathbf{x})$ up to permutation and element-wise linear transformations. While $\boldsymbol{g}_\theta$ is non-linear, sparsity alone only valid for the linear case (Lachapelle et al., 2022) which is a strong assumption and may not be sufficient to resolve the ambiguities introduced by the non-linearity in many real-world cases. Our results shows that, sparsity without additional constraints, does not guarantee identifiability in practice, as for $\mathbf{h} = \hat{\boldsymbol{g}}_\theta^{-1} \circ \boldsymbol{g}_\theta$ can depends on multiple components of $\mathbf{z}$. Building on this insight, we extend the concept of sparsity to contrastive sparsity by assuming Asm 2.1, without requiring bijectivity, and provide conditions under which $\mathbf{z}$ can be identified up to permutation and element-wise transformations.

Theorem 4.2 (Element-wise Identifiability given index support $\mathbf{i}$ for Piecewise Linear $\boldsymbol{g}_\theta$). *Let $\boldsymbol{f}_\phi : \mathbb{R}^{d\times n} \rightarrow \mathbb{R}^{T\times n}$ be a continuous invertible piecewise linear function and $\hat{\boldsymbol{g}}_\theta : \mathbb{R}^{d\times n} \rightarrow \mathbb{R}^{T\times n}$ be a continuous invertible piecewise linear function onto its image. Assume that Asm 4.1, Asm 2.1 holds, and the mixed observations $(\mathbf{x}, \mathbf{x}') \overset{i.i.d.}{\sim} \mathcal{X}$, follows the data-generating process Eq. (2.2). The learnable latent $\hat{\mathbf{z}}$ (resp. $\hat{\mathbf{z}}'$) of $\mathbf{z}$ (resp. $\mathbf{z}'$). If all following conditions hold:*

$$\mathbb{E}\|\hat{\mathbf{z}}\|_0 \leq \mathbb{E}\|\mathbf{z}\|_0 \quad \textit{and} \quad \mathbb{E}\|\hat{\mathbf{z}}'\|_0 \leq \mathbb{E}\|\mathbf{z}'\|_0, \textit{ and,} \tag{4.2}$$

$$\mathcal{R}_{alig}(\hat{\mathbf{z}}, \hat{\mathbf{z}}', \mathbf{i}) := \sum_{i\in\mathbf{i}} \left| \frac{\hat{\boldsymbol{z}}_i'^{\top} \hat{\boldsymbol{z}}_i}{\|\hat{\boldsymbol{z}}_i'\|_2 \|\hat{\boldsymbol{z}}_i\|_2} - 1 \right| = 0. \tag{4.3}$$

then $\mathbf{z}$ is identified by $\mathbf{h} := \hat{\boldsymbol{g}}_\theta^{-1}(\mathbf{x})$, i.e., $\hat{\boldsymbol{g}}_\theta^{-1} \circ \boldsymbol{g}_\theta$ is a permutation composed with element-wise invertible linear transformations (Def. 2.2).

Proof Sketch. The complet proof are given in App. A.2. Intuitively, based result (Kivva et al., 2022) combined with contrastivity between tow latent based on their shared support indices $\mathbf{i}$. This means that for the data that satisfy Asm 4.1, $\boldsymbol{g}_\theta(\mathbf{z})$ and $\hat{\boldsymbol{g}}_\theta(\hat{\mathbf{z}})$ are equally distributed, then there exists an invertible affine transformation such that $\mathbf{h}(\mathbf{z}) = \mathbf{z}'$. Second, we use the strategy of linear identifiability (Lachapelle & Lacoste-Julien, 2022) to obtain element wise identifiaiabiltiy.

This approach is similar to SparseVAE (Moran et al., 2022), which enforces constraints using Spike-and-Slab Lasso. However, our method ensures slot identifiability through Partial Selective Pairing, without requiring strong assumptions or extra constraints on $\mathcal{Z}$. In contrast, SparseVAE uses separate decoders for each feature. Another line of work can dive to constrains the generator $\boldsymbol{g}_\theta$ via its Jacobian $\mathbf{J}\boldsymbol{g}_\theta(\mathbf{z})$, known as compositionality and irreducibility (Von Kügelgen et al., 2021; Brady et al., 2023). Definitions are provided in App. A.1. Within our framework, compositionality means that each high-dimensional source is controlled by only one latent slot $\boldsymbol{z}_k$, enforcing local sparsity. However, minimizing compositionality in $\hat{\boldsymbol{g}}_\theta$ on $\mathcal{Z}$ is computationally infeasible [4].

③ **Invariance for Compositional Generalization Representation** From Thm. 4.2, it follows that $\hat{\boldsymbol{g}}_\theta$ faithfully maps each inferred slot $\boldsymbol{h}_k(\boldsymbol{z}_{\pi(k)})$ to its corresponding source in $\mathbf{x}$ for all possible values of $\boldsymbol{z}_{\pi(k)}$, ensuring identifiability (ID). We extend this to ensuring OOD scenarios by simply composing the latents from the training set and applying a stop gradient to prevent the gradients from

---

[4] For a CNN with 1 million parameters and a batch size of 32, at least 250GB of GPU memory is required.

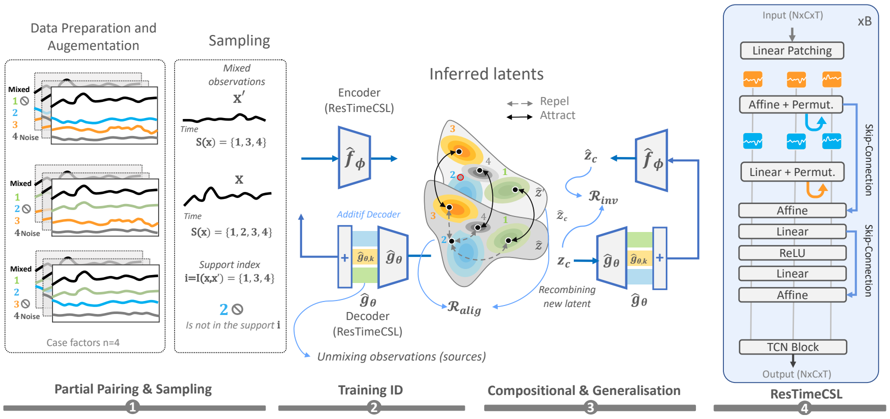

Figure 3: **Overview of `TimeCSL` framework with ResTimeCSL Architecture.** After linearly projecting the time series patches into high dimensional embeddings the ResTimeCSL is affine.

flowing back into the recomposed latent during training (see Fig. 2). During training, simultaneously, we perform ID and OOD, ensuring that the combined latent remains consistent *i.e.,* compositional with the original latent, allowing the model to generalize OOD samples while retaining the ID. Assuming the conditions stated in Thm. 4.2 are satisfied, this implies the existence of transformations $\mathbf{h}$, along with a permutation π, that enable the slot-identification $\mathbf{z}$ for any composition of slots, whether ID or OOD, over $\mathcal{Z}$, as given by

$$\mathbf{z}_c = \boldsymbol{f}_\phi(\boldsymbol{h}_1(\boldsymbol{z}_{\pi(1)}), \ldots, \boldsymbol{h}_n(\boldsymbol{z}_{\pi(n)})), \text{ and } \mathcal{Z}_c = \boldsymbol{f}_\phi(\boldsymbol{h}_1(\mathcal{Z}_{\pi(1)}) \times \cdots \times \boldsymbol{h}_n(\mathcal{Z}_{\pi(n)})). \tag{4.4}$$

The compositional generalization consistency on $\mathcal{Z}'$, holds, *i.e.,* $\hat{\boldsymbol{g}}_\theta^{-1}(\boldsymbol{g}_\theta(\mathbf{z})) = \mathbf{z}_c$ and $\hat{\boldsymbol{g}}_\theta(\mathbf{z}_c) = \boldsymbol{g}_\theta(\mathbf{z})$, if and only if $\mathbf{z}_c$ minimizes the invariance such that,

$$\mathcal{R}_{inv}(\mathbf{z}_c) := \sum_{i \neq k} \left(\frac{\boldsymbol{z}_{c\,i}^\top \boldsymbol{z}_{c\,k}}{\|\boldsymbol{z}_{c\,i}\|_2 \|\boldsymbol{z}_{c\,k}\|_2} \right)^2, \text{ for some } \gamma_{inv} > 0, \gamma_{inv}\mathcal{R}_{inv}(\mathbf{z}_c) = 0. \tag{4.5}$$

The condition in Eq. (4.5) ensure that $\hat{\boldsymbol{f}}_\phi$ inverts $\hat{\boldsymbol{g}}_\theta$ on ID and ODD by re-encoding the latent from inferred ones (see Fig. 3). Implementation details and sampling process of $\mathbf{z}_c$ for this regularization is discussed in § 4.1. To validate Eq. (4.5), we have just to verify the compositional consistency error *i.e.,* $\hat{\boldsymbol{g}}_\theta^{-1}(\hat{\boldsymbol{g}}_\theta(\mathbf{z}_c)) = \mathbf{z}_c$ over $\forall\, \mathbf{z}_c \in \mathcal{Z}_c$. Formally:

$$\mathcal{L}_{cons} := \mathbb{E}_{\mathbf{z}_c \sim q_\phi(\mathbf{z}_c)}[||\hat{\boldsymbol{f}}_\phi(\hat{\boldsymbol{g}}_\theta(\mathbf{z}_c) - \mathbf{z}_c||] = 0, \textit{ where, } \textit{supp}(q_\phi(\mathbf{z}_c)) = \mathcal{Z}' \text{ Eq. (4.4)}. \tag{4.6}$$

4.1 PUTTING IT ALL TOGETHER IN PRACTICE

On the Possibility of Sufficient Partial Pairing In Thm. 4.2, we demonstrated how slot identifiability can be achieved on $\mathcal{Z}$ and ODD $\mathcal{Z}_c$ under the compositionality condition in Eq. (4.6). A key insight is the sufficient partial pairing for contrastive learning (Asm 4.1). This assumption can be relaxed to factor groups when the dataset is complex enough to discern varying features (e.g., in weather time series). For such cases, grouping factors avoids assumption violations. We validated our results on synthetic time series data (assumptions fully satisfied) and energy separation tasks, where used to relax assumptions via grouping factors. Data was prepared in pairs $(\mathbf{x}, \mathbf{x}')$, with additional samples generated as needed to cover all factors.

Conditions on the Network. We proposed ResTimeCSL (see Fig. 3), an efficient architecture for time series modeling that doesn't violate Asm 2.1. It projects time series patches into high-dimensional embeddings and processes them sequentially using a cross-patch linear sublayer and a cross-channel two-layer MLP, similar to the Transformer's FCN sublayer. Each sublayer includes residual connections, two affine element-wise transformations, and uses ReLU or LeakyReLU activations. For training, we leverage a VAE model with a mixture of Gaussians (Jiang et al., 2016) for a fixed latent dimension by n and d, optimizing the objective $\mathcal{L}_{\text{VAE}}$. We sample i.i.d. pairs $(\mathbf{x}, \mathbf{x}') \in \mathcal{X}$. Using a learnable encoder $\hat{\boldsymbol{f}}_\phi$, $\mathbf{x}$ (resp. $\mathbf{x}'$) is encoded into $[\hat{\boldsymbol{\mu}}_{\phi\,k}(\mathbf{x}), \hat{\boldsymbol{\sigma}}_{\phi\,k}(\mathbf{x})]^\top$ (resp. $[\hat{\boldsymbol{\mu}}_{\phi\,k}(\mathbf{x}'), \hat{\boldsymbol{\sigma}}_{\phi\,k}(\mathbf{x}')]^\top$) with reparameterization noise terms (Kingma & Welling, 2022). The inferred latents are $(\hat{\mathbf{z}}, \hat{\mathbf{z}}')$. A learnable decoder $\hat{\boldsymbol{g}}_\theta$ maps $\hat{\mathbf{z}}$ (resp. $\hat{\mathbf{z}}'$) to single-source outputs $\hat{\boldsymbol{y}}_k = \hat{\boldsymbol{g}}_{\theta\,k}(\hat{\mathbf{z}})$

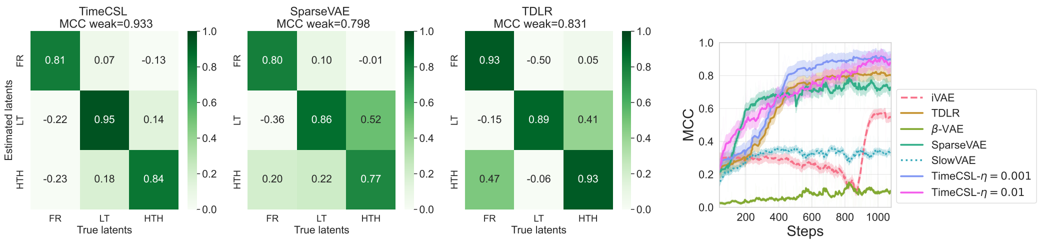

Figure 4: **Identifiability Validation.** `MCC` for factors {FR, LT, HTH} on synthetic data; **Left:** Weak `MCC` for TimeCSL, SparseVAE, and TDRL. **Right:** Baseline comparisons over training steps.

(resp. $\hat{\boldsymbol{y}}'_k = \hat{\boldsymbol{g}}_{\theta k}(\hat{\mathbf{z}}'))$ for $k = 1, \cdots, n$. Summing over these outputs reconstructs the mixed signals $\hat{\mathbf{x}}$ (resp. $\hat{\mathbf{x}}'$). In practice, the sparsity of the ground truth variables $\mathbf{z}$ is unknown, so we instead set a hyperparameters η for the sparsity constraint, furthermore for more stability, instead of $\mathbb{E}\,\|\mathbf{z}\|_0 \leq \eta$ we consider $\|\mathbf{v}\|_{s,\text{norm}} = \frac{1}{d_z}\sum_{i=1}^{d_z}\sum_{j=1}^{n_a+1}|v_{ij}|$. The `TimeCSL` objective serves then as a regularization term for the loss $\mathcal{L}^*_{\text{VAE}}$, that denote the sum $\mathcal{L}_{\text{VAE}}$ computed for both $\mathbf{x}$ and $\mathbf{x}'$. Thus, the final objective can be expressed as follows:

$$\begin{aligned}\mathcal{L}_{\texttt{TimeCSL}}(\phi, \theta; \mathcal{B}) = \mathcal{L}^*_{\text{VAE}}(\phi, \theta; \mathcal{B}) + \mathbb{E}_{\mathcal{B}}[\gamma_{alig}\mathcal{R}_{alig}(\mathbf{z}, \mathbf{z}', \mathbf{i})] + \mathbb{E}_{\mathcal{B}}[\gamma_{inv}\mathcal{R}_{inv}(\mathbf{z}_c, \mathbf{i})] \quad (4.7)\\ + \mathbb{E}_{\mathcal{B}}\|\max(0, \|\hat{\mathbf{z}}\|_s - \eta) + \max(0, \|\hat{\mathbf{z}}'\|_s - \eta)\|,\end{aligned}$$

where $\mathcal{B}$ is a batch of data. The alignment term $\mathcal{R}_{alig}$ penalizes deviations from cosine similarity between corresponding latents, scaled by γ_{alig}. The invariance term $\mathcal{R}_{inv}$, scaled by γ_{inv}, reduce invariance of the composing $\mathbf{z}_c$ from $\hat{\mathbf{z}}, \hat{\mathbf{z}}'$. In our experiments, we use $\eta = 0.01$ or 0.001.

5 EXPERIMENTS

5.1 VALIDATION OF THE THEORY

Datasets and Evaluation Setup. We conducted experiments for time series representation with separation task on three public **Real Datasets:** UK-DALE (Kelly & Knottenbelt, 2015), REDD (Kolter & Johnson, 2011), and REFIT (Murray et al., 2017) providing power measurements from multiple homes. The 70% of the data is used for training (including 10% of data augmentation), while the remaining 40% of real data is evenly divided between validation and testing. Inputs are zero-mean normalized, we consider $T = 256$, $C = 1$ and number factors/sources $n = 5$: Fridge (FR), Dishwasher (DW), Washing Machine (WM), Heater (HTR), and Lighting (LT). The mixed observation may include unlabeled noise factors. **Synthetic Dataset:** we generate a nonlinear mixing observations with $n = 3$, from ground truth available signals of {FR, LT, HTH}from UK-DALE, REDD, and REFIT with adding some Gaussian noise. To generate ODD scenarios Tab. 2 *i.e.,* strong correlation between factors, we adopt the methodology outlined in (Träuble et al., 2021) where $p(y_1, y_2) \propto \exp\left(-||y_1 - \alpha y_2||^2/2\sigma^2\right)$, we modifying the parameter σ to adjust the correlation.

Metrics. To assess slot identifiability, we follow (Locatello et al., 2020) by fitting nonlinear regressors to predict each ground-truth slot $\mathbf{z}_k$ from inferred slots $\hat{\mathbf{z}}_j$, evaluating the fit with the R^2 score. Slot assignments are optimized via the Hungarian algorithm (Kuhn, 1955), and we report the average R^2 over matched slots. Additionally, we use the Mean Correlation Coefficient (`MCC`) metric (Khemakhem et al., 2020a), reporting both *strong* `MCC` (before affine alignment) and *weak* `MCC` (after alignment). All MCCs are computed out-of-sample: the affine map Γ is fitted on one half of the data and evaluated on the other. `RMIG` (Robust Mutual Information GAP) (Do & Tran, 2020), and `DCI` (Disentanglement, Completeness and Informativeness) (Eastwood & Williams, 2018) adapted for time series are used to evaluate the disentanglement of factors *i.e.,* sources. We provide in-depth details of metrics and their implementation in App. B.4.

Contrastive Partial Selective Pairing Pipeline. Four augmentations were sequentially applied to all contrastive methods' pipeline branches. The parameters from the random search are: 1) **Crop and delay:** applied with a 0.5 probability and a minimum size of 50% of the initial sequence. 2) **Cutout or Masking:** time cutout of 5 steps with a 0.8 probability. 3) **Channel Masks powers:** each time series is randomly masked out with a 0.4 probability. 4) **Gaussian noise:** random Gaussian noise is added to window input $\mathbf{x}$ with a standard deviation form 0.1 to 0.3. Further details in **??**.

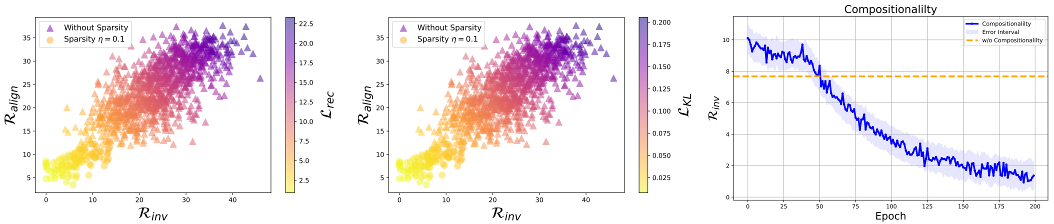

Figure 5: **Experimental validation. Left**: As predicted by Eq. (4.2), inducing sparsity in models that minimize $\mathcal{R}_{alig}$ and $\mathcal{R}_{inv}$ results in representations that are slot-identifiable both in ID and OOD, provided the reconstruction loss $\mathcal{L}_{\text{rec}}$ (as in Eq. (2.1)) is also minimized (see heat-map). A similar trend is observed for the $\mathcal{L}_{\text{KL}}$. **Right**: Compositional error Eq. (4.6) decreases throughout training, indicating that the decoder is implicitly optimized to be compositional, then validating Eq. (4.5).

Table 1: Average performance, considering factors {FR, DW, WM, HTH, LT}with 5 seed on real datasets REFIT and REDD. Metrics reported are: `DCI`, `RMIG` and `RMSE`. Lower values are better for all metrics. (↓ lower is better, ↑ higher is worse Top-1 , Top-2).

Sc.	**Methods**	$\sigma = \infty$			$\sigma = 0.3$			$\sigma = 0.8$		
	Metrics ⇒	**`DCI`** ↓	**`RMIG`** ↓	**`RMSE`** ↓	**`DCI`** ↓	**`RMIG`** ↓	**`RMSE`** ↓	**`DCI`** ↓	**`RMIG`** ↓	**`RMSE`** ↓
REFIT	○ BertNILM	-	-	56.4 ± 2.58	-	-	70.2 ± 1.45	-	-	70.92 ± 1.15
	○ S2S	-	-	54.3 ± 3.12	-	-	69.5 ± 3.56	-	-	69.95 ± 3.26
	○ Autoformer	-	-	49.7 ± 0.81	-	-	50.5 ± 2.15	-	-	52.95 ± 1.63
	○ Informer	-	-	50.3 ± 2.41	-	-	53.5 ± 1.98	-	-	58.95 ± 1.89
	● TimesNet	-	-	49.24 ± 2.87	-	-	51.10 ± 2.64	-	-	54.91 ± 2.31
	● CoST	68.4 ± 2.41	0.94 ± 0.03	47.7 ± 1.35	73.7 ± 2.41	0.98 ± 0.27	53.2 ± 1.02	71.95 ± 1.63	1.00 ± 0.02	58.45 ± 0.82
	○ SlowVAE	78.0 ± 1.09	0.94 ± 0.13	43.2 ± 2.23	81.0 ± 1.82	0.94 ± 0.13	49.2 ± 1.13	79.74 ± 0.84	1.07 ± 0.11	54.65 ± 1.43
	● SlowVAE+HDF	79.8 ± 0.10	0.64 ± 0.05	57.2 ± 2.15	81.1 ± 0.34	0.71 ± 0.14	59.3 ± 1.82	80.37 ± 0.05	0.72 ± 0.03	61.64 ± 1.52
	● TDRL	64.85 ± 1.48	0.42 ± 0.12	28.56 ± 2.15	76.23 ± 1.32	0.48 ± 0.02	26.33 ± 1.97	77.13 ± 1.00	0.58 ± 0.24	31.99 ± 1.64
	○ D3VAE	63.12 ±2.84	0.40 ±0.14	42.28 ±2.13	63.66 ±1.31	0.51 ±0.38	46.11 ±1.58	66.73 ±1.88	0.67 ±0.08	50.10 ±0.74
	○ C-DSVAE	72.42 ± 3.10	0.91 ± 0.15	48.6 ± 2.32	73.12 ± 1.43	0.95 ± 0.41	52.9 ± 1.71	76.29 ± 2.04	1.08 ± 0.09	57.45 ± 0.81
	● C-DSVAE + HDF	67.80 ± 2.91	0.85 ± 0.14	45.45 ± 2.18	68.76 ± 1.34	0.90 ± 0.39	49.69 ± 1.60	71.50 ± 1.92	1.01 ± 0.08	53.85 ± 0.76
	● SparseVAE	61.51 ± 1.31	0.39 ± 0.13	21.01 ± 1.89	67.29 ± 1.17	0.43 ± 0.62	22.71 ± 1.73	68.19 ± 0.88	0.51 ± 0.21	28.91 ± 1.89
	● `TimeCSL`	**59.71 ± 1.27**	**0.36 ± 0.11**	**18.44 ± 1.84**	**65.22 ± 1.13**	**0.41 ± 0.23**	**19.11 ± 1.69**	**66.01 ± 0.86**	**0.48 ± 0.08**	**22.21 ± 1.41**
	Avg.	69.74 ± 1.95	0.80 ± 0.10	47.3 ± 1.92	73.4 ± 1.22	0.90 ± 0.17	52.25 ± 1.47	74.98 ± 1.38	1.00 ± 0.08	54.9 ± 1.25
REDD	○ BertNILM	-	-	61.42 ± 3.47	-	-	67.61 ± 1.95	-	-	69.06 ± 1.43
	○ S2S	-	-	59.08 ± 4.15	-	-	68.60 ± 3.91	-	-	70.68 ± 3.25
	○ Autoformer	-	-	49.87 ± 0.92	-	-	51.53 ± 1.48	-	-	51.88 ± 1.34
	○ Informer	-	-	54.61 ± 1.41	-	-	58.13 ± 0.67	-	-	62.45 ± 1.76
	● TimesNet	-	-	51.37 ± 2.41	-	-	55.35 ± 2.23	-	-	58.47 ± 2.21
	● CoST	62.60 ± 2.20	0.86 ± 0.03	43.53 ± 1.23	67.51 ± 2.11	0.89 ± 0.25	48.71 ± 0.94	65.98 ± 1.50	0.92 ± 0.02	53.32 ± 0.75
	○ SlowVAE	71.14 ± 0.96	0.86 ± 0.12	39.46 ± 2.05	74.34 ± 1.60	0.86 ± 0.12	45.02 ± 1.04	73.19 ± 0.77	0.98 ± 0.10	49.94 ± 1.31
	● SlowVAE+HDF	73.12 ± 0.09	0.59 ± 0.05	52.34 ± 1.97	74.40 ± 0.31	0.65 ± 0.13	54.48 ± 1.67	73.75 ± 0.05	0.66 ± 0.03	56.28 ± 1.40
	● TDRL	59.39 ± 1.31	0.38 ± 0.11	26.12 ± 1.97	69.82 ± 1.19	0.44 ± 0.02	24.10 ± 1.78	70.82 ± 0.91	0.53 ± 0.22	29.27 ± 1.51
	○ D3VAE	59.39 ±2.56	0.74 ±0.13	39.56 ±1.92	59.65 ±1.17	0.78 ±0.34	43.13 ±1.42	62.62 ±1.69	0.89 ±0.07	47.07 ±0.66
	○ C-DSVAE	66.44 ± 2.84	0.83 ± 0.14	44.51 ± 2.13	67.06 ± 1.31	0.87 ± 0.38	48.48 ± 1.58	70.24 ± 1.88	0.99 ± 0.08	52.74 ± 0.74
	● C-DSVAE + HDF	62.20 ± 2.67	0.78 ± 0.13	41.65 ± 2.01	63.23 ± 1.24	0.83 ± 0.36	45.71 ± 1.48	65.73 ± 1.77	0.93 ± 0.07	49.54 ± 0.70
	● SparseVAE	56.39 ± 1.21	0.36 ± 0.12	19.21 ± 1.74	61.60 ± 1.07	0.45 ± 0.57	20.81 ± 1.60	62.65 ± 0.81	0.47 ± 0.19	26.42 ± 1.74
	● `TimeCSL`	**54.74 ± 1.17**	**0.33 ± 0.10**	**16.93 ± 1.70**	**60.10 ± 1.04**	**0.38 ± 0.21**	**17.50 ± 1.56**	**60.31 ± 0.79**	**0.44 ± 0.07**	**20.39 ± 1.30**
	Avg.	69.25 ± 1.87	0.67 ± 0.09	47.4 ± 1.83	74.2 ± 1.36	0.73 ± 0.10	53.16 ± 1.55	75.55 ± 1.23	0.80 ± 0.08	56.31 ± 1.48

Baselines & Implementations. Nonlinear ICA methods are used;β-VAE, iVAE and TCL which leverage nonstationarity establish identifiability but assumes independent factors, and SlowVAE/SlowVAE which exploit temporal constraints but assume independent sources. We provide also variant β-TC/Factor/-VAE such as D3VAE and CDSVAE implemented for time series sequence modeling. We compare `TimeCSL` with downstream task models in energy disaggregation, BertNILM (Yue et al., 2020) and S2S (Chen et al., 2018a) as a baseline, for those models, we keep the same configuration as the original implementation. We run experiments with 5 seeds, reporting average results and standard deviations, using 8 NVIDIA A100 GPUs. Hyperparameters and training details are in App. B.

Results. Fig. 4 shows that standard nonlinear ICA models like β-VAE/C-DSVAE, and SlowVAE struggle with identifiability, while SparseVAE and iVAE perform comparatively better on synthetic data. `TimeCSL` with strong sparsity ($\eta = 0.01$) achieves the best identifiability. Fig. 5 provides convincing probes of the compositional generalization consistency condition Eq. (4.5), where minimizing $\mathcal{R}_{alig}$ and $\mathcal{R}_{inv}$, both with and without sparsity, aligns with the predictions of Thm. 4.2. Slot identifiability improves as reconstruction error decreases, with similar trends observed for $\mathcal{L}_{KL}$. Additionally, Fig. 5 (Left) illustrates a reduction in compositional error as $\mathcal{R}_{inv}$ is minimized, confirming the compositional nature of the decoder as predicted by Eq. (4.5). Empirically, Tab. 7 summarizes the performance of different models as data complexity increases, controlled by correlation levels. The findings show that `TimeCSL` surpasses SparseVAE, demonstrating better disentanglement and reconstruction. However, at higher correlation levels, models without tailored designs for identifiability and disentanglement face challenges, underscoring potential limitations in real-world applications.

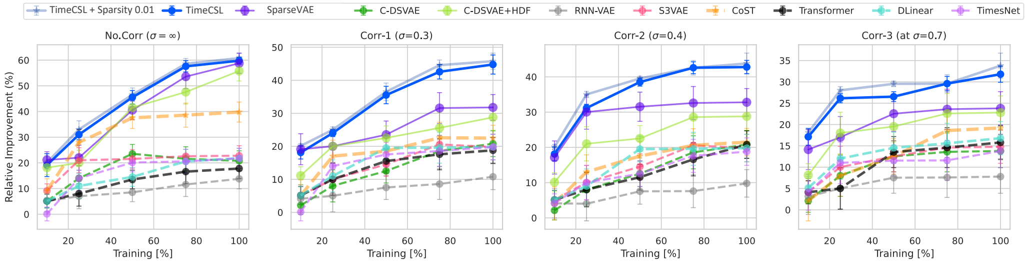

Figure 6: Relative RMSE (%) improvement over baseline BertNILM Yue et al., 2020 for six devices, with the amount of labelled training data as a variable parameter.

5.2 ABLATION STUDIES AND DISCUSSION

When and how performing disentanglement? In Tab. 2, we use as `TimeCSL` regularizer and we train models only on (REFIT+REDD) and tested on possible ODD dataset *i.e.,* UKDALE, and we explore its application with alternative structures tailored for time series with analysis impact of nonlieanrtité of the dedcoder (Asm 2.1 not hold), particularly those residual in Difussion based VAE model (D3VAE). We see that the model it generalize after incorporating `TimeCSL` with another method slightly enhances results, as the alternative regularizes assumes independent factorization, potentially compromising the relaxing effect. Secondly, `TimeCSL` shows improved performance as sparsity increases, with R^2 positively correlating with performance. `RMIG` further indicates that integrating attention with `TimeCSL` yields well-disentangled representations. The attention mechanism notably enhances overall model performance, including identifiability metrics, suggesting potential for guaranteeing identifiable disentangled representations.

Table 2: Average R^2, `RMIG` and weaker/strong `MCC` scores on UK-DALE dataset with factors {FR, DW, WM, HTH, LT}. (↓ lower is better, ↑ higher is better Top-1 , Top-2).† indicates implemented.

Method	Activation	$R^2 \uparrow$	`RMIG` ↓	weak `MCC` ↑	strong `MCC` ↑
○ CoST	ReLU	0.165	0.405	0.395	-0.010
○ RNN-VAE (baseline)	LeakyReLU	0.065	0.660	0.340	0.080
○ RNN-VAE+TimeCSL	LeakyReLU	0.169	0.562	0.400	0.038
○ C-DSVAE	ReLU	0.127	0.415	0.685	0.070
○ C-DSVAE+TimeCSL	ReLU	0.167	0.511	0.578	0.167
○ SlowVAE	LeakyReLU	0.263	0.860	0.671	0.082
○ SlowVAE+TimeCSL	LeakyReLU	0.272	0.560	0.387	0.074
○ DIOSC	Softmax	0.280	0.368	0.562	0.194
○ D3VAE (Diffusion)	Softmax	0.271	0.791	0.544	0.188
○ D3VAE+TimeCSL (Diffusion)	Softmax	0.285	0.682	0.573	0.198
○ iVAE	LeakyReLU	0.230	0.408	0.479	0.177
○ TDRL	LeakyReLU	0.223	0.380	0.464	0.172
○ TCL	LeakyReLU	0.115	0.748	0.448	0.165
○ LEAP	LeakyReLU	0.138	0.340	0.538	0.198
● TimeCSL $\eta = 0.001$	ReLU	0.292	0.330	0.629	0.258
● TimeCSL†+self-attention	Softmax	0.205	0.267	0.373	0.106
● TimeCSL † $\eta = 0.01$	ReLU	**0.305**	**0.367**	**0.633**	**0.266**

Is that sparsity enough for Robustness, is donwstream task ? We provide evidence that `TimeCSL` exhibits robustness across different correlation scenarios as illustrated in Fig. 6. In addition, we conduct experiments using different sate of the art architecture for time series representation. The results presented in highlight that `TimeCSL` with sparsity $\eta = 0.1$ consistently than `TimeCSL` with lower sparsity $\eta = 0.01$ outperforms the competing baseline across all three settings, reaffirming its effectiveness and versatility in diverse scenarios where data exibit a strong correlation. Due to limited space, additional results are available in App. B.8.

6 CONCLUSION

In this work, we delved into the effectiveness of contrastive sparsity-inducing techniques in attaining both identifiability and generalization. We showcased that disentangled representations, complemented by sparse-inducing methods through contrastive learning, improve generalization, particularly when the downstream task can be tackled using only a portion of the underlying factors of variation. Looking ahead, future investigations could explore leveraging such meaningful representations for downstream tasks, as evidenced by our primary experiments demonstrating performance enhancement. Furthermore, we posit that such representations could prove efficient in scenarios characterized by limited labeled data for time series representation. We have demonstrated generalization through compositional representations. We built on the literature in generative models and non-linear ICA (Kivva et al., 2022; Hyvarinen et al., 2019; Lachapelle et al., 2022) and made two key assumptions : i) the satisfy partial sufficiency to induce sparsity through contrastive learning, and ii) g_θ it is injective when mixing latents. We suggest that future work should explore relaxing these assumptions.

Limitations & Future Work We recognize that our assumptions regarding time series representation and source separation have potential for extension. The Sufficient Partial Pairing assumption (Asm 4.1) requires sufficient data, and as discussed, it can be relaxed to group factors as well. Our piecewise injectivity assumption (Asm 2.1), although potentially violated in practice, can be revised to take account of structures such as attention mechanisms or instance normalisation.

REFERENCES

K. Ahuja, J. Hartford, and Y. Bengio. Weakly supervised representation learning with sparse perturbations, 2022.

Kartik Ahuja, Divyat Mahajan, Yixin Wang, and Yoshua Bengio. Interventional Causal Representation Learning. In *Proceedings of the 40th International Conference on Machine Learning*, pp. 372–407. PMLR, July 2023. URL https://proceedings.mlr.press/v202/ahuja23a.html. ISSN: 2640-3498.

Rim Assouel, Pau Rodriguez, Perouz Taslakian, David Vazquez, and Yoshua Bengio. Object-centric compositional imagination for visual abstract reasoning. In *ICLR2022 Workshop on the Elements of Reasoning: Objects, Structure and Causality*, 2022.

Francis Bach, Rodolphe Jenatton, Julien Mairal, Guillaume Obozinski, et al. Optimization with sparsity-inducing penalties. *Foundations and Trends® in Machine Learning*, 4(1):1–106, 2012.

Junwen Bai, Weiran Wang, and Carla Gomes. Contrastively Disentangled Sequential Variational Autoencoder.

Y. Bengio, T. Deleu, N. Rahaman, N. R. Ke, S. Lachapelle, O. Bilaniuk, A. Goyal, and C. Pal. A meta-transfer objective for learning to disentangle causal mechanisms. In *International Conference on Learning Representations*, 2020.

Yoshua Bengio, Aaron Courville, and Pascal Vincent. Representation learning: A review and new perspectives. *IEEE transactions on pattern analysis and machine intelligence*, 35(8):1798–1828, 2013.

Jack Brady, Roland S. Zimmermann, Yash Sharma, Bernhard Schölkopf, and Wieland and von Kügelgen, Julius Brendel. Provably learning object-centric representations. In *Proceedings of the 40th International Conference on Machine Learning*, ICML'23, 2023.

Chris Burgess and Hyunjik Kim. 3d shapes dataset. https://github.com/deepmind/3dshapes-dataset/, 2018.

Marc-André Carbonneau, Julian Zaidi, Jonathan Boilard, and Ghyslain Gagnon. Measuring Disentanglement: A Review of Metrics, May 2022. URL http://arxiv.org/abs/2012.09276. arXiv:2012.09276 [cs].

Kunjin Chen, Qin Wang, Ziyu He, Kunlong Chen, Jun Hu, and Jinliang He. Convolutional sequence to sequence non-intrusive load monitoring. *the Journal of Engineering*, 2018(17):1860–1864, 2018a. Publisher: Wiley Online Library.

Ricky T. Q. Chen, Xuechen Li, Roger B Grosse, and David K Duvenaud. Isolating Sources of Disentanglement in Variational Autoencoders. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett (eds.), *Advances in Neural Information Processing Systems*, volume 31. Curran Associates, Inc., 2018b.

Xinlei Chen and Kaiming He. Exploring simple siamese representation learning. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 15750–15758, 2021.

Junyoung Chung, Kyle Kastner, Laurent Dinh, Kratarth Goel, Aaron C Courville, and Yoshua Bengio. A Recurrent Latent Variable Model for Sequential Data. In *Advances in Neural Information Processing Systems*, volume 28. Curran Associates, Inc., 2015. URL https://proceedings.neurips.cc/paper_files/paper/2015/hash/b618c3210e934362ac261db280128c22-Abstract.html.

Pierre Comon. Independent component analysis, a new concept? *Signal processing*, 36(3):287–314, 1994.

A. Dittadi, F. Träuble, F. Locatello, M. Wuthrich, V. Agrawal, O. Winther, S. Bauer, and B. Schölkopf. On the transfer of disentangled representations in realistic settings. In *International Conference on Learning Representations*, 2021.

Kien Do and Truyen Tran. THEORY AND EVALUATION METRICS FOR LEARNING DISENTANGLED REPRESENTATIONS. 2020.

Kefan Dong and Tengyu Ma. First Steps Toward Understanding the Extrapolation of Nonlinear Models to Unseen Domains. In *The Eleventh International Conference on Learning Representations*, September 2022.

Cian Eastwood and Christopher KI Williams. A framework for the quantitative evaluation of disentangled representations. In *International Conference on Learning Representations*, 2018.

Luigi Gresele, Julius von Kügelgen, Vincent Stimper, Bernhard Schölkopf, and Michel Besserve. Independent mechanism analysis, a new concept? *Advances in neural information processing systems*, 34:28233–28248, 2021.

Jean-Bastien Grill, Florian Strub, Florent Altché, Corentin Tallec, Pierre Richemond, Elena Buchatskaya, Carl Doersch, Bernardo Avila Pires, Zhaohan Guo, Mohammad Gheshlaghi Azar, et al. Bootstrap your own latent-a new approach to self-supervised learning. *Advances in neural information processing systems*, 33:21271–21284, 2020.

Hermanni Hälvä and Aapo Hyvärinen. Hidden markov nonlinear ICA: unsupervised learning from nonstationary time series. In *Proceedings of the Thirty-Sixth Conference on Uncertainty in Artificial Intelligence, UAI 2020, virtual online, August 3-6, 2020*, volume 124 of *Proceedings of Machine Learning Research*, pp. 939–948, 2020.

Irina Higgins, Loic Matthey, Arka Pal, Christopher Burgess, Xavier Glorot, Matthew Botvinick, Shakir Mohamed, and Alexander Lerchner. beta-vae: Learning basic visual concepts with a constrained variational framework. In *International conference on learning representations*, 2016.

Aapo Hyvarinen and Hiroshi Morioka. Unsupervised feature extraction by time-contrastive learning and nonlinear ica. *Advances in neural information processing systems*, 29, 2016.

Aapo Hyvarinen and Hiroshi Morioka. Nonlinear ica of temporally dependent stationary sources. In *Artificial Intelligence and Statistics*, pp. 460–469. PMLR, 2017.

Aapo Hyvarinen, Hiroaki Sasaki, and Richard Turner. Nonlinear ica using auxiliary variables and generalized contrastive learning. In *The 22nd International Conference on Artificial Intelligence and Statistics*, pp. 859–868. PMLR, 2019.

A. Hyvärinen and H. Morioka. Unsupervised feature extraction by time-contrastive learning and nonlinear ica. In *Advances in Neural Information Processing Systems*, 2016.

A. Hyvärinen and H. Morioka. Nonlinear ICA of Temporally Dependent Stationary Sources. In *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics*, 2017.

A. Hyvärinen and P. Pajunen. Nonlinear independent component analysis: Existence and uniqueness results. *Neural Networks*, 1999.

A. Hyvärinen, H. Sasaki, and R. E. Turner. Nonlinear ica using auxiliary variables and generalized contrastive learning. In *AISTATS*. PMLR, 2019.

E. Jang, S. Gu, and B. Poole. Categorical reparameterization with gumbel-softmax. *Proceedings of the 34th International Conference on Machine Learning*, 2017.

Zhuxi Jiang, Yin Zheng, Huachun Tan, Bangsheng Tang, and Hanning Zhou. Variational deep embedding: An unsupervised and generative approach to clustering. *arXiv preprint arXiv:1611.05148*, 2016.

Jack Kelly and William Knottenbelt. The UK-DALE dataset, domestic appliance-level electricity demand and whole-house demand from five UK homes. *Scientific data*, 2, 2015. Publisher: Nature Publishing Group.

I. Khemakhem, D. Kingma, R. Monti, and A. Hyvärinen. Variational autoencoders and nonlinear ica: A unifying framework. In *Proceedings of the Twenty Third International Conference on Artificial Intelligence and Statistics*, 2020a.

Ilyes Khemakhem, Diederik Kingma, Ricardo Monti, and Aapo Hyvarinen. Variational autoencoders and nonlinear ica: A unifying framework. In *International Conference on Artificial Intelligence and Statistics*, pp. 2207–2217. PMLR, 2020b.

Hyunjik Kim and Andriy Mnih. Disentangling by Factorising, July 2019. URL `http://arxiv.org/abs/1802.05983`. arXiv:1802.05983 [cs, stat].

D. P. Kingma and M. Welling. Auto-encoding variational bayes. In *2nd International Conference on Learning Representations*, 2014.

Diederik P. Kingma and Max Welling. Auto-Encoding Variational Bayes, December 2022. URL `http://arxiv.org/abs/1312.6114`. arXiv:1312.6114 [cs, stat].

Bohdan Kivva, Goutham Rajendran, Pradeep Ravikumar, and Bryon Aragam. Identifiability of deep generative models without auxiliary information. *Advances in Neural Information Processing Systems*, 35:15687–15701, 2022.

David A. Klindt, Lukas Schott, Yash Sharma, Ivan Ustyuzhaninov, Wieland Brendel, Matthias Bethge, and Dylan M. Paiton. Towards nonlinear disentanglement in natural data with temporal sparse coding. In *ICLR*, 2021.

J Zico Kolter and Matthew J Johnson. REDD: A public data set for energy disaggregation research. In *Workshop on data mining applications in sustainability (SIGKDD), San Diego, CA*, volume 25, 2011. Issue: Citeseer.

H. W. Kuhn. The Hungarian method for the assignment problem. *Naval Research Logistics Quarterly*, 2(1-2):83–97, March 1955. ISSN 00281441, 19319193. doi: 10.1002/nav.3800020109.

Zeb Kurth-Nelson, Timothy Edward John Behrens, Greg Wayne, Kevin J. Miller, Lennart Luettgau, Raymond Dolan, Yunzhe Liu, and Philipp Schwartenbeck. Replay and compositional computation. *Neuron*, 111:454–469, 2022.

S. Lachapelle and S. Lacoste-Julien. Partial disentanglement via mechanism sparsity. In *UAI 2022 Workshop on Causal Representation Learning*, 2022.

S. Lachapelle, P. Rodriguez Lopez, Y. Sharma, K. E. Everett, R. Le Priol, A. Lacoste, and S. Lacoste-Julien. Disentanglement via mechanism sparsity regularization: A new principle for nonlinear ICA. In *First Conference on Causal Learning and Reasoning*, 2022.

Sébastien Lachapelle, Tristan Deleu, Divyat Mahajan, Ioannis Mitliagkas, Yoshua Bengio, Simon Lacoste-Julien, and Quentin Bertrand. Synergies between disentanglement and sparsity: Generalization and identifiability in multi-task learning. In *International Conference on Machine Learning*, pp. 18171–18206. PMLR, 2023a.

Sébastien Lachapelle, Divyat Mahajan, Ioannis Mitliagkas, and Simon Lacoste-Julien. Additive decoders for latent variables identification and cartesian-product extrapolation. *arXiv preprint arXiv:2307.02598*, 2023b.

Yan Li, Xinjiang Lu, Yaqing Wang, and Dejing Dou. Generative Time Series Forecasting with Diffusion, Denoise, and Disentanglement, January 2023. URL `http://arxiv.org/abs/2301.03028`. arXiv:2301.03028 [cs].

Francesco Locatello, Stefan Bauer, Mario Lucic, Gunnar Raetsch, Sylvain Gelly, Bernhard Schölkopf, and Olivier Bachem. Challenging common assumptions in the unsupervised learning of disentangled representations. In *international conference on machine learning*, pp. 4114–4124. PMLR, 2019.

Francesco Locatello, Dirk Weissenborn, Thomas Unterthiner, Aravindh Mahendran, Georg Heigold, Jakob Uszkoreit, Alexey Dosovitskiy, and Thomas Kipf. Object-Centric Learning with Slot Attention. In *Advances in Neural Information Processing Systems*, volume 33, pp. 11525–11538. Curran Associates, Inc., 2020.

Chaochao Lu, Yuhuai Wu, Jośe Miguel Hernández-Lobato, and Bernhard Schölkopf. Nonlinear invariant risk minimization: A causal approach, 2021.

M. L. Montero, C. JH Ludwig, R. P. Costa, G. Malhotra, and J. Bowers. The role of disentanglement in generalisation. In *International Conference on Learning Representations*, 2021.

G. E. Moran, D. Sridhar, Y. Wang, and D. Blei. Identifiable deep generative models via sparse decoding. *Transactions on Machine Learning Research*, 2022.

David Murray, Lina Stankovic, and Vladimir Stankovic. An electrical load measurements dataset of united kingdom households from a two-year longitudinal study. *Scientific data*, 4(1):1–12, 2017.

Aviv Netanyahu, Abhishek Gupta, Max Simchowitz, Kaiqing Zhang, and Pulkit Agrawal. Learning to Extrapolate: A Transductive Approach. In *The Eleventh International Conference on Learning Representations*, February 2023.

Ignavier Ng, Yujia Zheng, Xinshuai Dong, and Kun Zhang. On the identifiability of sparse ica without assuming non-gaussianity. *Advances in Neural Information Processing Systems*, 36:47960–47990, 2023.

Hien D Nguyen and Geoffrey McLachlan. On approximations via convolution-defined mixture models. *Communications in Statistics-Theory and Methods*, 48(16):3945–3955, 2019.

Khalid Oublal, Said Ladjal, David Benhaiem, Emmanuel LE BORGNE, and François Roueff. Disentangling time series representations via contrastive independence-of-support on l-variational inference. In *The Twelfth International Conference on Learning Representations*, 2024. URL `https://openreview.net/forum?id=iI7hZSczxE`.

G. Roeder, L. Metz, and D. P. Kingma. On linear identifiability of learned representations. In *Proceedings of the 38th International Conference on Machine Learning*, 2021.

Karsten Roth, Mark Ibrahim, Zeynep Akata, Pascal Vincent, and Diane Bouchacourt. Disentanglement of Correlated Factors via Hausdorff Factorized Support, February 2023. URL `http://arxiv.org/abs/2210.07347`. arXiv:2210.07347 [cs, stat].

P. Sorrenson, C. Rother, and U. Köthe. Disentanglement by nonlinear ica with general incompressible-flow networks (gin). In *International Conference on Learning Representations*, 2020.

A. Taleb and C. Jutten. Source separation in post-nonlinear mixtures. *IEEE Transactions on Signal Processing*, 1999.

Frederik Träuble, Elliot Creager, Niki Kilbertus, Francesco Locatello, Andrea Dittadi, Anirudh Goyal, Bernhard Schölkopf, and Stefan Bauer. On disentangled representations learned from correlated data. In *International Conference on Machine Learning*, pp. 10401–10412. PMLR, 2021.

Laurens Van der Maaten and Geoffrey Hinton. Visualizing data using t-sne. *Journal of machine learning research*, 9(11), 2008.

J. Von Kügelgen, Y. Sharma, L. Gresele, W. Brendel, B. Schölkopf, M. Besserve, and F. Locatello. Self-supervised learning with data augmentations provably isolates content from style. In *Thirty-Fifth Conference on Neural Information Processing Systems*, 2021.

Thaddäus Wiedemer, Jack Brady, Alexander Panfilov, Attila Juhos, Matthias Bethge, and Wieland Brendel. Provable compositional generalization for object-centric learning. *arXiv preprint arXiv:2310.05327*, 2023a.

Thaddäus Wiedemer, Prasanna Mayilvahanan, Matthias Bethge, and Wieland Brendel. Compositional generalization from first principles. *arXiv preprint arXiv:2307.05596*, 2023b.

Gerald Woo, Chenghao Liu, Doyen Sahoo, Akshat Kumar, and Steven Hoi. Cost: Contrastive learning of disentangled seasonal-trend representations for time series forecasting. *arXiv preprint arXiv:2202.01575*, 2022.

Haixu Wu, Jiehui Xu, Jianmin Wang, and Mingsheng Long. Autoformer: Decomposition transformers with auto-correlation for long-term series forecasting. *Advances in neural information processing systems*, 34:22419–22430, 2021.

Haixu Wu, Tengge Hu, Yong Liu, Hang Zhou, Jianmin Wang, and Mingsheng Long. Timesnet: Temporal 2d-variation modeling for general time series analysis. In *The eleventh international conference on learning representations*, 2022.

Weiran Yao, Yuewen Sun, Alex Ho, Changyin Sun, and Kun Zhang. Learning temporally causal latent processes from general temporal data. *arXiv preprint arXiv:2110.05428*, 2021.

Weiran Yao, Guangyi Chen, and Kun Zhang. Temporally disentangled representation learning. *Advances in Neural Information Processing Systems*, 35:26492–26503, 2022.

Zhenrui Yue, Camilo Requena Witzig, Daniel Jorde, and Hans-Arno Jacobsen. BERT4NILM: A Bidirectional Transformer Model for Non-Intrusive Load Monitoring. In *Proceedings of the 5th International Workshop on Non-Intrusive Load Monitoring*, NILM'20, pp. 89–93, New York, NY, USA, November 2020. Association for Computing Machinery. ISBN 978-1-4503-8191-8. doi: 10.1145/3427771.3429390. URL https://dl.acm.org/doi/10.1145/3427771.3429390.

Jure Zbontar, Li Jing, Ishan Misra, Yann LeCun, and Stéphane Deny. Barlow Twins: Self-Supervised Learning via Redundancy Reduction, June 2021. URL http://arxiv.org/abs/2103.03230. arXiv:2103.03230 [cs, q-bio].

George Zerveas, Srideepika Jayaraman, Dhaval Patel, Anuradha Bhamidipaty, and Carsten Eickhoff. A transformer-based framework for multivariate time series representation learning. In *Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining*, KDD '21, pp. 2114–2124, New York, NY, USA, 2021. Association for Computing Machinery. ISBN 9781450383325. doi: 10.1145/3447548.3467401. URL https://doi.org/10.1145/3447548.3467401.

Linfeng Zhao, Lingzhi Kong, Robin Walters, and Lawson LS Wong. Toward compositional generalization in object-oriented world modeling. In *International Conference on Machine Learning*, pp. 26841–26864. PMLR, 2022.

Y. Zheng, I. Ng, and K. Zhang. On the identifiability of nonlinear ICA: Sparsity and beyond. In *Advances in Neural Information Processing Systems*, 2022.

Haoyi Zhou, Shanghang Zhang, Jieqi Peng, Shuai Zhang, Jianxin Li, Hui Xiong, and Wancai Zhang. Informer: Beyond efficient transformer for long sequence time-series forecasting. In *Proceedings of AAAI*, 2021.

Supplementary Material:

To ensure a comprehensive understanding of our paper and to support reproducibility and reliability, we present additional results and provide complete proofs for the theorems articulated in the main paper. This supplementary material is meticulously organized as follows:

Table of Contents

A Definitions, Propositions, and Proofs Analysis **16**
  A.1 Generalization, Compositionality and irreducibility assumptions 16
  A.2 Element-wise Identifiability given index support i for Piecewise Linear 17
  A.3 The Generative Process and The ELBO for Multivariates Mixture Gaussian . . . 19
    A.3.1 Variational Lower Bound for `TimeCSL` 21
    A.3.2 The Equivalence Between Matrix Normal and Multivariate Normal Distributions 22
  A.4 Assumption Structural Variability and Partial 23

B Experiments and Implementation Settings **24**
  B.1 Implementation source. (TimeCSL-Lib) 24
  B.2 Datasets. 24
  B.3 Contrastive Partial Selective Pairing - Data Augmentations 24
  B.4 Implementation of Metrics and study case 25
    B.4.1 Alignment prior to measuring Weak `MCC` 25
    B.4.2 Measuring Identifiability strong-MCC and weak-MCC 25
    B.4.3 Measuring disentanglement of the learned representation 25
  B.5 Pipeline Correlated samples. 27
  B.6 Impact of GELU vs ReLU/LeakyReLU on Decoder Behavior 27
  B.7 Validation of results on synthetic Data Generation 28
  B.8 that sparsity enough for Robustness, is donwstream task ? 28

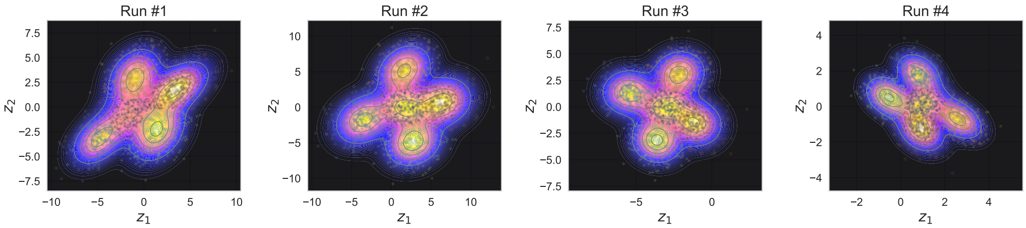

Figure 7: Recovered latent spaces for 4 runs of `TimeCSL` on REDD dataset #1 with 5 latents ($n = 5, d = 16$).

A DEFINITIONS, PROPOSITIONS, AND PROOFS ANALYSIS

In this section, we detail the contributions of the paper, including all the details. Although there is no change in their contents, the formulation of some definitions and theorems are slightly altered here to be more precise and cover edge cases omitted in the main text. Hence, the numbering of the restated elements is reminiscent of that used in the main text.

A.1 GENERALIZATION, COMPOSITIONALITY AND IRREDUCIBILITY ASSUMPTIONS

Compositional contrast In recent work on compositionality (Assouel et al., 2022; Zhao et al., 2022; Kurth-Nelson et al., 2022) and its importance in learning models that can generalize well to novel situations, the concept of *compositional contrast* has emerged as a powerful tool for evaluating how well a model separates information into independent, non-interacting components.

Table 3: Related work in nonlinear ICA for time series. A blue check denotes that a method has an attribute, whereas a red cross denotes the opposite. † indicates an approach we implemented.

Approach	**Temporal Data**	**Dependent Factors**	**Nonparametric Expression**	**Stationary Process**
TCL (Hyvarinen & Morioka, 2016)	✓	✗	✗	✗
PCL (Hyvarinen & Morioka, 2017)	✓	✗	✓	✓
GCL (Hyvarinen et al., 2019)	✓	✗	✓	✗
iVAE (Khemakhem et al., 2020b)	✗	✗	✗	✗
GIN (Sorrenson et al., 2020)	✗	✗	✗	✗
HM-NLICA (Hälvä & Hyvärinen, 2020)	✓	✗	✓	✗
SlowVAE (Klindt et al., 2021)	✓	✗	✗	✓
(Yao et al., 2021) **LEAP (Theorem 1)**	✓	✓	✓	✗
(Yao et al., 2021) **LEAP (Theorem 2)**	✓	✓	✗	✓
TimeCSL (our)† **TimeCSL (Theorem 1)**	✓	✓	✓	✓+✗

This concept is particularly relevant in the context of time series analysis or image generation, where the model's ability to decompose an input into distinct parts, or "slots," can significantly impact the quality of predictions and interpretability. Compositionality ensures that each slot, or latent variable, corresponds to a specific factor or component of the data. In highly compositional models, these components do not interact with each other—each one affects a distinct aspect of the output. In contrast, non-compositional models tend to mix these components, making it harder to disentangle the factors and interpret the model's output. Evaluating how well a model adheres to compositionality principles can be challenging, as it requires quantifying how independent the slots are in their contribution to the final output. To address this, Brady et al. (2023) introduced the notion of *compositional contrast*, which measures the extent to which the model's latent variables (slots) interact when producing the final output. This measure is particularly useful in determining whether a decoder is truly compositional—that is, whether each slot contributes independently of the others, or if there are unwanted interactions between them. Before we introduce the formal definition of compositional contrast, it is important to understand the underlying principle. The intuition behind the compositional contrast is that if a model is fully compositional, each slot should affect only a specific subset of the output (e.g., one region of an image or one time series variable) and have no influence on other components. Conversely, if the model is not compositional, changes in one slot will influence multiple components of the output simultaneously, indicating that the slots are not independent. The compositional contrast function captures this idea by calculating how much the gradients of each slot (with respect to the model's output) overlap. If the gradients of different slots with respect to the same output component are non-zero, this suggests interaction between the slots, indicating a lack of compositionality. The function sums these interactions across all slots and output components, providing a single value that quantifies the degree of interaction. A lower compositional contrast value suggests higher compositionality, while a higher value indicates more interaction between slots. Formally, the compositional contrast is defined as follows:

Definition A.1 (Compositional Contrast)**.** Let $\boldsymbol{g}_\theta : \mathcal{Z} \to \mathcal{X}$ be differentiable. The *compositional contrast* of $\boldsymbol{g}_\theta$ at $\mathbf{z}$ is

$$C_{\text{comp}}(\boldsymbol{g}_\theta, \mathbf{z}) = \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{j=k+1}^{K} \left\| \frac{\partial \boldsymbol{g}_{\theta n}}{\partial \mathbf{z}_k}(\mathbf{z}) \right\| \left\| \frac{\partial \boldsymbol{g}_{\theta n}}{\partial \mathbf{z}_j}(\mathbf{z}) \right\| . \tag{A.1}$$

This contrast function was proven to be zero if and only if $\boldsymbol{g}_\theta$ is compositional according to Eq. (4.5). The function can be understood as computing each pairwise product of the (L2) norms for each pixel's gradients with respect to any two distinct slots $k \neq j$ and taking the sum. This quantity is non-negative and will only be zero if each pixel is affected by at most one slot, ensuring that $\boldsymbol{g}_\theta$ satisfies Eq. (4.5). We can use this function to measure the compositional of a decoder in our experiments (see § 4), where it serves as a key indicator of how effectively the model decomposes its inputs into independent components. More empirical and theoretical details on the function can be found in Brady et al. (2023).

A.2 ELEMENT-WISE IDENTIFIABILITY GIVEN INDEX SUPPORT I FOR PIECEWISE LINEAR

In this section, we present the proof of Thm. 4.2. To establish a solid foundation for the argument, we first restate Asm 4.1, which plays a pivotal role in the proof.

Assumption 4.1 (Sufficient Partial Selective Pairing). For each factor $k \in [n]$, there exist observations $(\mathbf{x}, \mathbf{x}') \in \mathcal{X}$ such that the union of the shared support indices $\mathbf{i} = \mathbf{I}(\mathbf{x}, \mathbf{x}')$ that do not include k

must cover all other factors. Formally:

$$\bigcup_{\mathbf{i}\in\mathcal{I}|k\notin\mathbf{i}} \mathbf{i} = [n]\setminus\{k\} \quad , \quad \mathcal{I} := \{\mathbf{i}\subseteq[n] \mid p(\mathbf{i})>0\} \tag{4.1}$$

where $\mathcal{I}$ is the set of shared support indices and $p(\mathbf{i}) := \frac{1}{\#\mathcal{X}} \cdot \#\left\{\mathcal{S}(\mathbf{x}) = \mathbf{i},\, \mathbf{x}\in\mathcal{X}\right\}$ gives the probability that the factors indexed by $\mathbf{i}$ are active, with $k\notin\mathbf{i}$ inactive.

Additionally, we introduce some notation. For $\mathbf{i}\in\mathcal{I}$, we assume that the probability measure $\mathbb{P}_{\mathbf{z}_\mathbf{i}}$ admits a density with respect to the Lebesgue measure on $\mathbb{R}^{|\mathbf{i}|}$. We let $\equiv$ denote equality in the distribution.

Theorem 4.2 (Element-wise Identifiability given index support $\mathbf{i}$ for Piecewise Linear $\boldsymbol{g}_\theta$)**.** *Let $\boldsymbol{f}_\phi : \mathbb{R}^{d\times n} \to \mathbb{R}^{T\times n}$ be a continuous invertible piecewise linear function and $\hat{\boldsymbol{g}}_\theta : \mathbb{R}^{d\times n} \to \mathbb{R}^{T\times n}$ be a continuous invertible piecewise linear function onto its image. Assume that Asm 4.1, Asm 2.1 holds, and the mixed observations $(\mathbf{x},\mathbf{x}') \overset{i.i.d.}{\sim} \mathcal{X}$, follows the data-generating process Eq. (2.2). The learnable latent $\hat{\mathbf{z}}$ (resp. $\hat{\mathbf{z}}'$) of $\mathbf{z}$ (resp. $\mathbf{z}'$). If all following conditions hold:*

$$\mathbb{E}\|\hat{\mathbf{z}}\|_0 \leq \mathbb{E}\|\mathbf{z}\|_0 \quad \textit{and} \quad \mathbb{E}\|\hat{\mathbf{z}}'\|_0 \leq \mathbb{E}\|\mathbf{z}'\|_0, \textit{ and,} \tag{4.2}$$

$$\mathcal{R}_{alig}(\hat{\mathbf{z}},\hat{\mathbf{z}}',\mathbf{i}) := \sum_{i\in\mathbf{i}}\left|\frac{\hat{\boldsymbol{z}}_i'^{\top}\hat{\boldsymbol{z}}_i}{\|\hat{\boldsymbol{z}}_i'\|_2\|\hat{\boldsymbol{z}}_i\|_2} - 1\right| = 0. \tag{4.3}$$

then $\mathbf{z}$ is identified by $\mathbf{h} := \hat{\boldsymbol{g}}_\theta^{-1}(\mathbf{x})$, i.e., $\hat{\boldsymbol{g}}_\theta^{-1}\circ\boldsymbol{g}_\theta$ is a permutation composed with element-wise invertible linear transformations (Def. 2.2).

Proof. The proving strategy has three steps: Intuitively, based result (Kivva et al., 2022) combined with contrastivity beteween tow latent based their shared support indices $\mathbf{i}$. This means that for the data that satisfy Asm 4.1, $\boldsymbol{g}_\theta(\mathbf{z})$ and $\hat{\boldsymbol{g}}_\theta(\hat{\mathbf{z}})$ are equally distributed, then there exists an invertible affine transformation such that $\mathbf{h}(\mathbf{z}) = \mathbf{z}'$. Second, we use the strategy of linear identifiability (Lachapelle & Lacoste-Julien, 2022) to obtain element wise identifiaiabiltiy:

Step 1) Contrastive Sparsity and Linear Identifiability given pairs i We begin by recalling the result from Kivva et al. (2022) on the existing of an invertible function affine transformation $\boldsymbol{h}_k$, we adapt this for the case where if the reconstruction objective is minizzed and alignment. The theorem on identifiability of MVNs states:

Theorem A.2. *Let $\boldsymbol{g}_\theta, \boldsymbol{g}'_\theta : \mathbb{R}^{d\times n} \to \mathbb{R}^{C\times T}$ be piecewise affine functions satisfying 2.1. Let $\mathbf{z} \sim \sum_{i=1}^{J}\omega_i\mathcal{N}(\boldsymbol{\mu}_i,\boldsymbol{\Sigma}_i)$ and $\mathbf{z}' \sim \sum_{j=1}^{J'}\omega_j'\mathcal{N}(\boldsymbol{\mu}_j',\boldsymbol{\Sigma}_j')$ be a pair of GMMs (in reduced form). Suppose that $\boldsymbol{g}_\theta(\mathbf{z})$ and $\boldsymbol{g}'_\theta(\mathbf{z}')$ are equally distributed. Then there exists an invertible affine transformation $\mathbf{h} : \mathbb{R}^{d\times n} \to \mathbb{R}^{d\times n}$ such that $\mathbf{h}(\mathbf{z}) \equiv \mathbf{z}'$, i.e., $J = J'$ and for some permutation π we have $\omega_i = \omega'_{\pi(k)}$ and $\mathbf{h}\sharp\mathcal{N}(\boldsymbol{\mu}_i,\boldsymbol{\Sigma}_i) = \mathcal{N}(\boldsymbol{\mu}'_{\pi(i)},\boldsymbol{\Sigma}'_{\pi(i)})$.*

We recall that the transformation and the number of components can be unknown and arbitrary, and that no assumption of separation or independence is necessary for the distribution.

By Theorem C.2 (Kivva et al., 2022), since contrastive learning involves the minimisation of a contrastive loss which ensures that similar data points (positive pairs) are moved closer together and dissimilar data points (negative pairs) are moved further apart. Let the inferred latent representation $(\mathbf{z},\mathbf{z}')$ be handled by the exact same function $\boldsymbol{f}_\phi$, and we consider the zero reconstruction under $\mathcal{R}_{aling} = 0$ for all slot indices in $\mathbf{i}$. Alongside this, contrastive loss minimization induces the distributions of $\boldsymbol{g}_\theta(\mathbf{z})$ and $\boldsymbol{g}_\theta(\mathbf{z}')$ to become indistinguishable on $i\in\mathbf{i}$ to be well-aligned, apart from for $k\notin\mathbf{i}$, but as we consider the **??** on the sufficient partial pairing that will cover this factor k in another pairing sample of the pair $(\mathbf{x},\mathbf{x}')$. Thus, according to Theorem C.2 (Kivva et al., 2022), there must exist an invertible affine transformation $\mathbf{h}$ such that $\mathbf{h}(\mathbf{z}) \equiv \mathbf{z}'\mathbf{z})$. It is more likely to observe that :

$$\sum_{j=1}^{J}\omega_k\boldsymbol{g}_\theta\sharp\mathcal{N}(\mu_k,\sigma_k) \sim \boldsymbol{g}_\theta\sharp\boldsymbol{f}_\phi(\sum_{j=1}^{J}\omega_k\mathcal{N}(\mu_k,\sigma_k)\Big). \tag{A.2}$$

In other words, minimizing to hold (i) and zeros error construction, implies a mixture model whose components are piecewise affine transformations identifiable.

Step 2) Sparsity Pattern of an Invertible Matrix with an element-wise linear transformation Since $\mathbf{x} = \boldsymbol{g}_\theta(\mathbf{z})$, we can rewrite perfect reconstruction as:

$$\mathbb{E}\|\boldsymbol{g}_\theta(\mathbf{z}) - \hat{\boldsymbol{g}}_\theta(\boldsymbol{f}_\phi(\boldsymbol{g}_\theta(\mathbf{z})))\|_2^2 = 0 \tag{10}$$

This means $\boldsymbol{g}_\theta$ and $\hat{\boldsymbol{g}}_\theta \circ \boldsymbol{f}_\phi \circ \boldsymbol{g}_\theta$ are equal $\mathbb{P}_\mathbf{z}$-almost everywhere. Both of these functions are continuous, $\boldsymbol{g}_\theta$ by Asm 2.1, and $\hat{\boldsymbol{g}}_\theta \circ \boldsymbol{f}_\phi \circ \boldsymbol{g}_\theta$ because $\hat{\boldsymbol{g}}_\theta$ is continuous, and $\boldsymbol{g}_\theta$, $\boldsymbol{f}_\phi$ are linear. Since they are continuous and equal $\mathbb{P}_\mathbf{z}$-almost everywhere $\mathcal{Z}$, this means that they must be equal over the support of $\mathcal{Z}$, i.e.,

$$\boldsymbol{g}_\theta(\mathbf{z}) = \hat{\boldsymbol{g}}_\theta \circ \boldsymbol{f}_\phi \circ \boldsymbol{g}_\theta(\mathbf{z}), \quad \forall \mathbf{z} \in \mathcal{Z}. \tag{11}$$

This can be easily shown by contradiction considering any slot latent $\mathbf{z}' \in \mathcal{Z}$ on which $\boldsymbol{g}_\theta$ and $\hat{\boldsymbol{g}}_\theta \circ \boldsymbol{f}_\phi \circ \boldsymbol{g}_\theta$ are different, i.e., $\hat{\boldsymbol{g}}_\theta \circ \boldsymbol{f}_\phi \circ \hat{\boldsymbol{g}}_\theta(\mathbf{z}') \neq \boldsymbol{g}_\theta(\mathbf{z}')$. This would imply that $(\boldsymbol{g}_\theta - \hat{\boldsymbol{g}}_\theta \circ \boldsymbol{f}_\phi \circ \boldsymbol{g}_\theta)$, which is also a continuous function, is non-zero at $\mathbf{z}'$ and in its neighborhood, which contradict the assumption that $\boldsymbol{g}_\theta$ and $\hat{\boldsymbol{g}}_\theta \circ \boldsymbol{f}_\phi \circ \boldsymbol{g}_\theta$ are the same $\mathbb{P}_\mathbf{z}$-almost everywhere. We can now apply the inverse of $\hat{\boldsymbol{g}}_\theta$ on both sides to obtain

$$\hat{\boldsymbol{g}}_\theta^{-1} \circ \boldsymbol{g}_\theta(\mathbf{z}) = \boldsymbol{f}_\phi \circ \boldsymbol{g}_\theta(\mathbf{z}) = \mathbf{h}(\mathbf{z}), \quad \forall \mathbf{z} \in \mathcal{Z}. \tag{12}$$

Since both $\boldsymbol{g}_\theta$ and $\boldsymbol{f}_\phi$ are invertible linear functions, given the fisrt part of the proof (**Step** 1-App. A.2) $\mathbf{h}$ is also an invertible linear function. We now show that $\mathbf{h}$ is a permutation composed with an element-wise linear transformation. To do this, we leverage the sparsity constraint:

$$\mathbb{E}\|\hat{\mathbf{z}}\|_0 \leq \mathbb{E}\|\mathbf{z}\|_0 \tag{A.3}$$

$$\mathbb{E}\|\boldsymbol{f}_\phi(\boldsymbol{g}_\theta(\mathbf{z}))\|_0 \leq \mathbb{E}\|\mathbf{z}\|_0 \tag{A.4}$$

$$\mathbb{E}\|\mathbf{h}(\mathbf{z})\|_0 \leq \mathbb{E}\|\mathbf{z}\|_0 \tag{A.5}$$

$$\tag{A.6}$$

Since $\boldsymbol{h}_k$ is invertible linear transformation, we have that $\boldsymbol{h}_k(\mathbf{z}) = \mathbf{w}_k \cdot \mathbf{z}$ and its determinant is non-zero, i.e.,

$$\det(\mathbf{h}) := \sum_{\pi \in \mathcal{P}} \operatorname{sign}(\pi) \prod_{k=1}^{n} \mathbf{h}_{k,\pi(k)} \neq 0, \tag{A.7}$$

where $\mathcal{P}$ denotes the set of all n-permutations. This expression implies that at least one term in the sum is non-zero, meaning there exists a permutation $\pi \in \mathcal{P}$ such that for every $k \in [n]$, $\frac{\partial \boldsymbol{h}_k}{\partial \mathbf{z}_{\pi(k)}} \neq 0$.

Following the steps outlined in Theorem B.4 by (Lachapelle et al., 2022), and under the assumption of Asm 4.1, we extend the disentanglement analysis to our setting. This leads to the conclusion that $\mathbf{h}$ can be expressed as a permutation composed with an element-wise invertible linear transformation, based on the shared support indices $\mathbf{i}$ of the latent slot within the subspace $\mathcal{Z}_\mathbf{i}$. Specifically, there exists a permutation π on $[n]$ such that, for each latent slot k, the corresponding permutation is given by $\pi(k)$. Since $\mathcal{I}$ is a finite set, which allows us to order its elements as $\{\mathbf{i}_1, \ldots, \mathbf{i}_{|\mathcal{I}|}\}$. Therefore, we can express $\mathcal{Z}$ as the union $\mathcal{Z} = \bigcup_{i=1}^{|\mathcal{I}|} \mathcal{Z}^{(\mathbf{i}_i)}$. While we have already shown that $\mathbf{h}$ is affine on each $\mathcal{Z}_\mathbf{i}$, we now demonstrate that $\mathbf{h}$ is linear on $\mathcal{Z}$, i.e., $\mathbf{h}(\mathbf{z})$ is a linear function on the entire set $\mathcal{Z} = \bigcup_{\mathbf{i} \in \mathcal{I}} \mathcal{Z}_\mathbf{i}$. This completes the proof. ☐

A.3 THE GENERATIVE PROCESS AND THE ELBO FOR MULTIVARIATES MIXTURE GAUSSIAN

We in this subsection how `TimeCSL` is trained based an a VAE process does similar to (Kivva et al., 2022; Jang et al., 2017), whcih more kind of unsupervised generative approach for clustering that performance well, we herein first describe the generative process of `TimeCSL`. Specifically, suppose there are n slots latents each has a dimension d, an observed sample $\mathbf{x} \sim \mathcal{X}$ is generated by the following process:

Algorithm 1 Generative Process

```
Input: Prior probabilities $\boldsymbol{w}$, neural network parameters $\boldsymbol{\theta}$
for $j = 1, 2, \dots, N$ do
    Sample slot $k \sim \text{Cat}(\boldsymbol{w})$
    Sample latent vector $\mathbf{z}^{(j)} \sim \mathcal{N}(\boldsymbol{\mu}_k^{(j)}, \boldsymbol{\sigma}_k^{(j)2}\mathbf{I})$
    Compute $[\boldsymbol{\mu}_\phi\left(\mathbf{x}^{(j)}\right); \log \boldsymbol{\sigma}_\phi\left(\mathbf{x}^{(j)}\right)^2] = \boldsymbol{g}_\theta(\mathbf{z}^{(j)})$
    Sample observation $\mathbf{x}_j \sim \mathcal{N}(\boldsymbol{\mu}_\theta\left(\mathbf{x}^{(j)}\right), \boldsymbol{\sigma}_\theta\left(\mathbf{x}^{(j)}\right)^2\mathbf{I})$ or $\text{Ber}(\boldsymbol{\mu}_\theta\left(\mathbf{x}^{(j)}\right))$
end for
return $\{\mathbf{x}^{(j)}, \mathbf{z}^{(j)}, k\}_{j=1}^{N}$
```

Lemma A.3. *Given two multivariate Gaussian distributions* $q(\mathbf{z}) = \mathcal{N}(\mathbf{z}; \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\sigma}}^2\mathbf{I})$ *and* $p(\mathbf{z}) = \mathcal{N}(\mathbf{z}; \boldsymbol{\mu}, \boldsymbol{\sigma}^2\mathbf{I})$*, we have:*

$$\int q(\mathbf{z}) \log p(\mathbf{z})\, d\mathbf{z} = \sum_{j=1}^{J} -\frac{1}{2}\log\left(2\pi\sigma_j^2\right) - \frac{\hat{\sigma}_j^2}{2\sigma_j^2} - \frac{(\hat{\mu}_j - \mu_j)^2}{2\sigma_j^2} \tag{A.8}$$

where μ_j*,* σ_j*,* $\hat{\mu}_j$ *and* $\hat{\sigma}_j$ *simply denote the* j^{th} *element of* $\boldsymbol{\mu}$*,* $\boldsymbol{\sigma}$*,* $\hat{\boldsymbol{\mu}}$ *and* $\hat{\boldsymbol{\sigma}}$*, respectively, and* $J = d \times n$ *is the dimensionality of* $\mathbf{z}$*.*

Proof.

$$\int q(\mathbf{z}) \log p(\mathbf{z})\, d\mathbf{z} = \int \mathcal{N}(\mathbf{z}; \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\sigma}}^2\mathbf{I}) \log \mathcal{N}(\mathbf{z}; \boldsymbol{\mu}, \boldsymbol{\sigma}^2\mathbf{I})\, d\mathbf{z}$$

$$= \int \prod_{j=1}^{J} \frac{1}{\sqrt{2\pi\hat{\sigma}_j^2}} \exp\left(-\frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\right) \log \left[\prod_{j=1}^{J} \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(z_j - \mu_j)^2}{2\sigma_j^2}\right)\right] d\mathbf{z}$$

$$= \sum_{j=1}^{J} \int \frac{1}{\sqrt{2\pi\hat{\sigma}_j^2}} \exp\left(-\frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\right) \log \left[\frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(z_j - \mu_j)^2}{2\sigma_j^2}\right)\right] dz_j$$

$$= \sum_{j=1}^{J} \int \frac{1}{\sqrt{2\pi\hat{\sigma}_j^2}} \exp\left(-\frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\right) \left[-\frac{1}{2}\log(2\pi\sigma_j^2)\right] dz_j - \int \frac{1}{\sqrt{2\pi\hat{\sigma}_j^2}} \exp\left(-\frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\right) \frac{(z_j - \mu_j)^2}{2\sigma_j^2}\, dz_j$$

$$= \sum_{j=1}^{J} -\frac{1}{2}\log(2\pi\sigma_j^2) - \int \frac{1}{\sqrt{2\pi\hat{\sigma}_j^2}} \exp\left(-\frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\right) \frac{(z_j - \hat{\mu}_j)^2 + 2(z_j - \hat{\mu}_j)(\hat{\mu}_j - \mu_j) + (\hat{\mu}_j - \mu_j)^2}{2\hat{\sigma}_j^2} \frac{\hat{\sigma}_j^2}{\sigma_j^2}\, dz_j$$

$$= \boldsymbol{b} - \frac{\hat{\sigma}_j^2}{\sigma_j^2} \int \frac{1}{\sqrt{2\pi\hat{\sigma}_j^2}} \exp\left(-\frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\right) \frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\, dz_j - \int \frac{1}{\sqrt{2\pi\hat{\sigma}_j^2}} \exp\left(-\frac{(z_j - \hat{\mu}_j)^2}{2\hat{\sigma}_j^2}\right) \frac{(\hat{\mu}_j - \mu_j)^2}{2\sigma_j^2}\, dz_j$$

$$= \boldsymbol{b} - \frac{\hat{\sigma}_j^2}{\sigma_j^2} \int \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_j^2}{2}\right) \frac{x_j^2}{2}\, dx_j - \frac{(\hat{\mu}_j - \mu_j)^2}{2\sigma_j^2}$$

$$= \boldsymbol{b} - \frac{\hat{\sigma}_j^2}{\sigma_j^2} \int \frac{1}{\sqrt{2\pi}} \left(-\frac{x_j}{2}\right) d\left(\exp\left(-\frac{x_j^2}{2}\right)\right) - \frac{(\hat{\mu}_j - \mu_j)^2}{2\sigma_j^2}$$

$$= \boldsymbol{b} - \frac{\hat{\sigma}_j^2}{\sigma_j^2} \left[\frac{1}{\sqrt{2\pi}} \left(-\frac{x_j}{2}\right) \exp\left(-\frac{x_j^2}{2}\right)\Big|_{-\infty}^{\infty} - \int \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_j^2}{2}\right) d\left(-\frac{x_j}{2}\right)\right] - \frac{(\hat{\mu}_j - \mu_j)^2}{2\sigma_j^2}$$

$$= \sum_{j=1}^{J} -\frac{1}{2}\log\left(2\pi\sigma_j^2\right) - \frac{\hat{\sigma}_j^2}{2\sigma_j^2} - \frac{(\hat{\mu}_j - \mu_j)^2}{2\sigma_j^2}$$

where $\boldsymbol{b}$ denotes $\sum_{j=1}^{J} -\frac{1}{2}\log(2\pi\sigma_j^2)$ for simplicity.

□

A.3.1 VARIATIONAL LOWER BOUND FOR `TimeCSL`

A `TimeCSL` instance is tuned to maximize the likelihood of the given data points. Given the generative process in Section A.3, by using Jensen's inequality, the log-likelihood of `TimeCSL` can be written as:

$$\begin{aligned}\log p(\mathbf{x}) &= \log \int_{\mathbf{z}} \sum_{c} p(\mathbf{x}, \mathbf{z}, c) d\mathbf{z} \\ &\geq E_{q(\mathbf{z},c|\mathbf{x})}[\log \frac{p(\mathbf{x}, \mathbf{z}, c)}{q(\mathbf{z}, c|\mathbf{x})}] = \mathcal{L}_{\text{ELBO}}(\mathbf{x}) \end{aligned} \tag{A.9}$$

where $\mathcal{L}_{\text{ELBO}}$ is the evidence lower bound (ELBO), $q(\mathbf{z}, c|\mathbf{x})$ is the variational posterior to approximate the true posterior $p(\mathbf{z}, c|\mathbf{x})$. In `TimeCSL`, we assume $q(\mathbf{z}, c|\mathbf{x})$ to be a mean-field distribution and can be factorized as:

$$q(\mathbf{z}, c|\mathbf{x}) = q(\mathbf{z}|\mathbf{x})q(k|\mathbf{x}). \tag{A.10}$$

Then, according to Equation A.10, the $\mathcal{L}_{\text{ELBO}}(\mathbf{x})$ in Equation A.9 can be rewritten as:

$$\begin{aligned}\mathcal{L}_{\text{ELBO}}(\mathbf{x}) &= E_{q(\mathbf{z},c|\mathbf{x})}\left[\log \frac{p(\mathbf{x}, \mathbf{z}, c)}{q(\mathbf{z}, c|\mathbf{x})}\right] \\ &= E_{q(\mathbf{z},c|\mathbf{x})}\left[\log p(\mathbf{x}, \mathbf{z}, c) - \log q(\mathbf{z}, c|\mathbf{x})\right] \\ &= E_{q(\mathbf{z},c|\mathbf{x})}[\log p(\mathbf{x}|\mathbf{z}) + \log p(\mathbf{z}|c) \\ &\quad + \log p(k) - \log q(\mathbf{z}|\mathbf{x}) - \log q(k|\mathbf{x})] \end{aligned} \tag{A.11}$$

In `TimeCSL`, similar to VAE, we use a neural network g to model $q(\mathbf{z}|\mathbf{x})$:

$$[\hat{\boldsymbol{\mu}}; \log \hat{\boldsymbol{\sigma}}^2] = \boldsymbol{f}_{\phi}(\mathbf{x}; \boldsymbol{\phi}) \tag{A.12}$$

$$q(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mathbf{z}; \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\sigma}}^2 \mathbf{I}) \tag{A.13}$$

where $\boldsymbol{\phi}$ is the parameter of network g.

By substituting the terms in Equation A.11 and using the SGVB estimator and the *reparameterization* trick, the $\mathcal{L}_{\text{ELBO}}(\mathbf{x})$ can be rewritten as: [5]

$$\begin{aligned}\mathcal{L}_{\text{ELBO}}(\mathbf{x}) =& \frac{1}{N} \sum_{l=1}^{N} \sum_{i=1}^{D} x_i \log \boldsymbol{\mu}_x^{(l)}|_i + (1 - x_i) \boldsymbol{f}_{\phi}(1 - \boldsymbol{\mu}_x^{(l)}|_i) \\ & - \frac{1}{2} \sum_{k=1}^{K} \gamma_k \sum_{j=1}^{J} (\log \boldsymbol{\sigma}_k^2|_j + \frac{\hat{\boldsymbol{\sigma}}^2|_j}{\boldsymbol{\sigma}_k^2|_j} + \frac{(\hat{\boldsymbol{\mu}}|_j - \boldsymbol{\mu}_k|_j)^2}{\boldsymbol{\sigma}_k^2|_j}) \\ & + \sum_{k=1}^{n} \gamma_k \log \frac{w_k}{\gamma_k} + \frac{1}{2} \sum_{j=1}^{J} (1 + \log \hat{\boldsymbol{\sigma}}^2|_j) \end{aligned} \tag{A.14}$$

where N is the number of Monte Carlo samples in the SGVB estimator, D is the dimensionality of $\mathbf{x}$ and $\boldsymbol{\mu}_x^{(l)}$, x_i is the i^{th} element of $\mathbf{x}$, J is the dimensionality of $\boldsymbol{\mu}_k$, $\boldsymbol{\sigma}_k^2$, $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\sigma}}^2$, and $*|_j$ denotes the j^{th} element of $*$, n is the number of slots, w_k is the prior probability of slot k, and γ_k denotes $q(k|\mathbf{x})$ for simplicity.

In Equation A.14, we compute $\boldsymbol{\mu}_x^{(l)}$ as

$$\boldsymbol{\mu}_x^{(l)} = f_{\phi}(\mathbf{z}^{(l)}; \theta), \tag{A.15}$$

where $\mathbf{z}^{(l)}$ is the l^{th} sample from $q(\mathbf{z}|\mathbf{x})$ by Equation A.13 to produce the Monte Carlo samples. According to the *reparameterization* trick, $\mathbf{z}^{(l)}$ is obtained by

$$\mathbf{z}^{(l)} = \hat{\boldsymbol{\mu}} + \hat{\boldsymbol{\sigma}} \circ \boldsymbol{\epsilon}^{(l)}, \tag{A.16}$$

---

[5] This is the case when the observation $\mathbf{x}$ is binary. For the real-valued situation, the ELBO can be obtained in a similar way.

where $\boldsymbol{\epsilon}^{(l)} \sim \mathcal{N}(0, \mathbf{I})$, $\circ$ is element-wise multiplication, and $\hat{\boldsymbol{\mu}}$, $\hat{\boldsymbol{\sigma}}$ are derived by Equation A.12.

We now describe how to formulate $\gamma_c \triangleq q(k|\mathbf{x})$ in Equation A.14 to maximize the ELBO. Specifically, $\mathcal{L}_{\text{ELBO}}(\mathbf{x})$ can be rewritten as:

$$
\begin{aligned}
\mathcal{L}_{\text{ELBO}}(\mathbf{x}) &= E_{q(\mathbf{z},c|\mathbf{x})}\left[\log \frac{p(\mathbf{x},\mathbf{z},c)}{q(\mathbf{z},c|\mathbf{x})}\right] \\
&= \int_{\mathbf{z}} \sum_{c} q(k|\mathbf{x}) q(\mathbf{z}|\mathbf{x}) \left[\log \frac{p(\mathbf{x}|\mathbf{z})p(\mathbf{z})}{q(\mathbf{z}|\mathbf{x})} + \log \frac{p(k|\mathbf{z})}{q(k|\mathbf{x})}\right] d\mathbf{z} \\
&= \int_{\mathbf{z}} q(\mathbf{z}|\mathbf{x}) \log \frac{p(\mathbf{x}|\mathbf{z})p(\mathbf{z})}{q(\mathbf{z}|\mathbf{x})} d\mathbf{z} - \int_{\mathbf{z}} q(\mathbf{z}|\mathbf{x}) D_{KL}(q(k|\mathbf{x})||p(k|\mathbf{z})) d\mathbf{z}
\end{aligned}
\tag{A.17}
$$

Once the training is done by maximizing the ELBO w.r.t the parameters of $\{\boldsymbol{\pi}, \boldsymbol{\mu}_k, \boldsymbol{\sigma}_k, \boldsymbol{\theta}, \boldsymbol{\phi}\}$, $k \in \{1, \cdots, K\}$, a latent representation $\mathbf{z}$ can be extracted for each observed sample $\mathbf{x}$ by Equation A.12 and Equation A.13.

A.3.2 THE EQUIVALENCE BETWEEN MATRIX NORMAL AND MULTIVARIATE NORMAL DISTRIBUTIONS

In our formulation, we use a vectorization of the matrix $\mathbf{z} \in \mathbb{R}^{d\times n}$, which follows a multivariate Gaussian model. We now show that this can also be interpreted as a Matrix Normal distribution. The equivalence between the Matrix Normal and the Multivariate Normal density functions can be established using properties of the trace and the Kronecker product.

Proof. Let $\mathbf{z}$ be modeled as a mixture of J Matrix Normal distributions. Each component of this mixture is characterized by a mean matrix $\boldsymbol{\mu}_j \in \mathbb{R}^{d\times n}$ and a covariance matrix $\boldsymbol{\Sigma}_j = \boldsymbol{\Sigma}_n \otimes \Sigma_n \in \mathbb{R}^{d\times d} \otimes \mathbb{R}^{n\times n}$, where $\boldsymbol{\Sigma}_n$ and $\boldsymbol{\Sigma}_n$ are the row and column covariance matrices, respectively. The probability density function of $\mathbf{z}$ is thus given by

$$
f_{\mathbf{z}}(\mathbf{z}) = \sum_{j=1}^{J} \omega_j \mathcal{N}(\mathbf{z} \mid \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j),
$$

where ω_j are the mixing weights such that $\omega_j > 0$ and $\sum_{j=1}^{J} \omega_j = 1$.

The Matrix Normal distribution is defined as

$$
\mathcal{N}(\mathbf{z} \mid \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j) = \frac{1}{(2\pi)^{\frac{dn}{2}} |\boldsymbol{\Sigma}_j|^{\frac{n+d}{2}}} \exp\left(-\frac{1}{2} \operatorname{tr}\left[\boldsymbol{\Sigma}_d^{-1}(\mathbf{z}-\boldsymbol{\mu}_j)^T \boldsymbol{\Sigma}_n^{-1}(\mathbf{z}-\boldsymbol{\mu}_j)\right]\right),
$$

where $\mathbf{z}$ is a $d \times n$ matrix, and the covariance matrix $\boldsymbol{\Sigma}_j$ is the Kronecker product $\Sigma_n \otimes \boldsymbol{\Sigma}_n$, with Σ_n and $\boldsymbol{\Sigma}_n$ being the covariance matrices of the rows and columns of $\mathbf{z}$, respectively.

To connect the Matrix Mixture Normal distribution with the Mixture of Multivariate Normal distributions, we vectorize the matrix $\mathbf{z}$. The vectorization of a matrix $\mathbf{z} \in \mathbb{R}^{d\times n}$ is given by

$$
\operatorname{vec}(\mathbf{z}) = \begin{bmatrix} z_{11} & z_{21} & \cdots & z_{d1} & z_{12} & \cdots & z_{dn} \end{bmatrix}^T \in \mathbb{R}^{1\times(d\cdot n)}
$$

where $\mathbf{z}_i$ denotes the i-th column of $\mathbf{z}$, and the resulting vector $\operatorname{vec}(\mathbf{z})$ is a $d \cdot n$-dimensional vector.

Now, substituting the vectorized form of $\mathbf{z}$ into the Matrix Normal distribution, we have

$$
\mathcal{N}(\operatorname{vec}(\mathbf{z}) \mid \operatorname{vec}(\boldsymbol{\mu}_j), \boldsymbol{\Sigma}_j) = \frac{1}{(2\pi)^{\frac{dn}{2}} |\boldsymbol{\Sigma}_j|^{\frac{d+n}{2}}} \exp\left(-\frac{1}{2} \bar{\mathbf{z}}^T \boldsymbol{\Sigma}_j^{-1} \bar{\mathbf{z}}\right), \tag{A.18}
$$

where $\bar{\mathbf{z}} = \operatorname{vec}(\mathbf{z}) - \operatorname{vec}(\boldsymbol{\mu}_j)$. Next, observe that the mixture model for $\mathbf{z}$ in the original form becomes

$$f_{\mathbf{z}}(\mathbf{z}) = \sum_{j=1}^{J} \omega_j \mathcal{N}(\text{vec}(\mathbf{z}) \mid \text{vec}(\boldsymbol{\mu}_j), \boldsymbol{\Sigma}_n \otimes \Sigma_n), \tag{A.19}$$

which is a mixture of multivariate normal distributions in the vectorized space $\mathbb{R}^{d \cdot n}$. This shows that the Matrix Mixture Normal distribution is equivalent to a Mixture of Multivariate Normal distributions upon vectorization. To complete the proof, we use the determinant property of the Kronecker product:

$$|\boldsymbol{\Sigma}_n \otimes \boldsymbol{\Sigma}_n| = |\boldsymbol{\Sigma}_n|^n |\boldsymbol{\Sigma}_n|^d. \tag{A.20}$$

Thus, the determinant of the covariance matrix $\boldsymbol{\Sigma}_n \otimes \boldsymbol{\Sigma}_n$ can be written as the product of the determinants of $\boldsymbol{\Sigma}_n$ and $\boldsymbol{\Sigma}_n$, raised to the appropriate powers. This confirms that the matrix mixture normal distribution is indeed equivalent to the mixture of multivariate normal distributions. ☐

A.4 ASSUMPTION STRUCTURAL VARIABILITY AND PARTIAL

Assumption-1 (Ng et al., 2023) stipulates that for any pair of sources k and ℓ, the supports of the corresponding columns in the mixing matrix $\boldsymbol{A}$ (denoted $\mathbf{a}_k$ and $\mathbf{a}_\ell$) must differ in at least two observed variables, i.e.,

$$|\text{supp}(\mathbf{a}_k) \cup \text{supp}(\mathbf{a}_\ell)| - |\text{supp}(\mathbf{a}_k) \cap \text{supp}(\mathbf{a}_\ell)| > 1.$$

Example.1 (Assumption-1 fails) This ensures distinct influences across observed variables. If the supports are nearly identical, Assumption-1 fails. For example, consider the mixing matrix $\boldsymbol{A}$:

$$\begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \\ \mathbf{x}_3(t) \\ \mathbf{x}_4(t) \end{bmatrix} = \begin{bmatrix} 1 & 0.5 & 0 & 0.2 \\ 0.3 & 1 & 0.4 & 0 \\ 0 & 0.2 & 1 & 0.5 \\ 0.1 & 0 & 0.6 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_1(t) \\ \boldsymbol{y}_2(t) \\ \boldsymbol{y}_3(t) \\ \boldsymbol{y}_4(t) \end{bmatrix} + \epsilon$$

with supports $\text{supp}(\mathbf{a}_1) = \{1, 2, 4\}$, $\text{supp}(\mathbf{a}_2) = \{1, 2, 3\}$, $\text{supp}(\mathbf{a}_3) = \{2, 3, 4\}$, and $\text{supp}(\mathbf{a}_4) = \{1, 3, 4\}$. For $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$, the difference in support is 2 (validating Assumption-1), as is the case for $\boldsymbol{y}_3$ and $\boldsymbol{y}_4$. However, the significant overlap in the observed variables they influence ($\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ both affect $\mathbf{x}_1(t), \mathbf{x}_2(t)$, and $\boldsymbol{y}_3$ and $\boldsymbol{y}_4$ affect $\mathbf{x}_3(t), \mathbf{x}_4(t)$) limits the ability to uniquely identify each source, pointing to a practical challenge in real-world data.

B EXPERIMENTS AND IMPLEMENTATION SETTINGS

B.1 IMPLEMENTATION SOURCE. (TIMECSL-LIB)

We have implemented the ResTimeCSL architecture from scratch, and our code is available at `https://anonymous.4open.science/r/TimeCSL-4320`. Some components of our code are inspired by the following works:

- The GMM-based VAE sampling is inspired by VaDE (Jiang et al., 2016), and we adapted the implementation from `https://github.com/mperezcarrasco/Pytorch-VaDE`.
- For the Diffusion model D3VAE (Li et al., 2023), we utilized the authors' implementation from `https://github.com/PaddlePaddle/PaddleSpatial/tree/main/research/D3VAE`.
- Regarding the methods listed in Tab. 3, the TCL model was adapted from `https://github.com/hmorioka/TCL/tree/master/tcl`, while the other models are derived from `https://github.com/rpatrik96/nl-causal`.
- For iVAE (Khemakhem et al., 2020b), we used the implementation available at `https://github.com/MatthewWilletts/algostability`.

Our experiments were conducted with 5 different random seeds, and we report the average results along with standard deviations. The experiments were run using 8 NVIDIA A100 GPUs.

B.2 DATASETS.

In this section, we provide details about the datasets used for our experiments. We consider both real-world and synthetic datasets, each with specific characteristics relevant to the study. The table below summarizes the key properties of these datasets, including the number of samples, input dimensions, the number of sources/factors, and the names of the factors. The real-world datasets include REDD, REFIT, and UKDALE, which are commonly used in energy consumption modeling. Additionally, we employ synthetic datasets (Synthetic-1, Synthetic-2, and Synthetic-3) to simulate various scenarios with varying factors and input sizes. These datasets allow for comprehensive testing of our proposed method across different contexts.

Table 4: Synthetic Dataset and Real world datasets

Dataset	# Samples	Input Dim	# Sources/Factors	Factors name
REDD	5400	256	3	{FR, DW, WM, HTH, LT}
REFIT	1299	256	5	{FR, DW, WM, HTH, LT}
UKDALE	1300	256	5	{FR, DW, WM, HTH, LT}
Synthetic-1	12000	24	3	{FR, LT, HTH}
Synthetic-2	11000	96	5	{FR, LT, HTH}
Synthetic-3	11000	64	3	{FR, LT, HTH}

B.3 CONTRASTIVE PARTIAL SELECTIVE PAIRING - DATA AUGMENTATIONS

Four augmentations were sequentially applied to all contrastive methods' pipeline branches. The parameters from the random search are: 1) **Crop and delay:** applied with a 0.5 probability and a minimum size of 50% of the initial sequence. 2) **Cutout or Masking:** time cutout of 5 steps with a 0.8 probability. 3) **Channel Masks powers:** each time series is randomly masked out with a 0.4 probability. 4) **Gaussian noise:** random Gaussian noise is added to window input $\mathbf{x}$ with a standard deviation form 0.1 to 0.3. Further details in **??**. Also in our experiments, we utilize a composition of three data augmentations, applied in the following order - scaling, shifting, and jittering, activating with a probability of 0.3 to 0.5.

Scaling The time-series is scaled by a single random scalar value, obtained by sampling $\epsilon \sim \mathcal{N}(0, 0.5)$, and each time step is $\mathbf{x}'_t = \epsilon x_t$.

Shifting The time-series is shifted by a single random scalar value, obtained by sampling $\epsilon \sim \mathcal{N}(0, 0.5)$ and each time step is $\mathbf{x}'_t = x_t + \epsilon$.

Jittering I.I.D. Gaussian noise is added to each time step, from a distribution $\epsilon_t \sim \mathcal{N}(0, 0.5)$, where each time step is now $\mathbf{x}'_t = x_t + \epsilon_t$.

B.4 IMPLEMENTATION OF METRICS AND STUDY CASE

Previous work has relied on the Mean Correlation Coefficient (MCC) as a metric to quantify identifiability. For consistency with previous work, we report this metric, but also propose a new metric to quantify identifiability up to an affine transformation. There are two challenges in designing such a metric: Firstly, for two Gaussian mixtures, standard distance metrices such as TV-distance or KL-divergence do not have a closed form. Secondly, we need to find an affine map A that best aligns a pair of Gaussian mixtures. Therefore, developing a metric to quantify identifiability up to an affine transformation has natural challenges. We propose $\boldsymbol{d}_{\text{aff},L2}$, defined below, as an additional metric in this setting.

B.4.1 ALIGNMENT PRIOR TO MEASURING WEAK MCC

We seek an affine map Γ to align two GMMs using two methods. One approach, used in previous works on MCC, is Canonical Correlation Analysis (CCA). Alternatively, we explore a different method. For two GMMs, we iterate over all permutations of the components, and for each permutation, we compute the optimal map Γ that aligns the components. While ideally Γ would align both the means and the covariance matrices, solving this as an optimization problem is challenging. Thus, we focus on aligning the means of the first GMM to those of the second GMM. The map Γ is found by solving the least-squares problem:

$$\min_{\Gamma} \sum_i \|\mu_1^{(i)} - \Gamma \mu_2^{(i)}\|^2 \tag{B.1}$$

This can be efficiently solved using Singular Value Decomposition (SVD). Empirically, aligning the means provides good results.

B.4.2 MEASURING IDENTIFIABILITY STRONG-MCC AND WEAK-MCC

The other metric we consider is the Mean Correlation Coefficient (MCC) metric which had been used in prior works (Khemakhem et al., 2020a). There are two versions of MCC that have been used:

1. The *weak* MCC is defined to be the MCC after alignment via the affine map Γ transformation see App. B.4.1.
2. The *strong* MCC is defined to be the MCC before alignment.

Furthermore, in this work, we consider two different metrics. For a pair of distributions p_1, p_2, we define $\boldsymbol{d}_{\text{aff},L2}$ loss as

$$\boldsymbol{d}_{\text{aff},L2}(p_1, p_2) = \min_{\substack{A:\mathbb{R}^m \to \mathbb{R}^m, \\ \text{affine}}} \Delta_{L_2}(\Gamma_\sharp p_1, p_2), \quad \text{where} \quad \Delta_{L_2}(p_1, p_2) = \frac{\|p_1 - p_2\|_{L_2}}{\|p_1\|_{L_2}^{1/2} \, \|p_2\|_{L_2}^{1/2}} \tag{B.2}$$

In our experiments, we report both the strong `MCC` and weak `MCC`. Moreover, all reported `MCC` s are out-of-sample, i.e. the optimal affine map Γ is computed over half the dataset and then reused for the other half of the dataset.

B.4.3 MEASURING DISENTANGLEMENT OF THE LEARNED REPRESENTATION

In implementing the disentanglement metrics, we adhere to the methodology outlined in (Locatello et al., 2019), expanding it to accommodate time series data. For the computation of DCI metrics, we employ a gradient boosted tree from the scikit-learn package.

β-VAE, Metric Disentanglement is then measured as the accuracy of a linear classifier that predicts the index of the fixed factor based on the coordinate-wise sum of absolute differences between the representation vectors in the two mini-batches. (Higgins et al., 2016) suggest fixing a random attributes of variation in the underlying generative model and sampling two mini-batches of observations $\mathbf{x}$. We sample two batches of 256 points with a random factor fixed to a randomly sampled value across the two batches, and the others varying randomly. We compute the mean representations for these points and take the absolute difference between pairs from the two batches. We then average these 64 values to form the features of a training (or testing) point.

FactorVAE Metric (Kim & Mnih, 2019) (Kim & Mnih, 2019) address several issues with this metric by using a majority vote classifier that predicts the index of the fixed ground-truth attribute based on the index of the representation vector with the least variance. First, we estimate the variance of each latent dimension by embedding $10k$ random samples from the data set, excluding collapsed dimensions with variance smaller than .05. Second, we generate the votes for the majority vote classifier by sampling a batch of 64 points, all with a factor fixed to the same random value. Third, we compute the variance of each dimension of their latent representation and divide it by the variance of that dimension computed on the data without interventions. The training point for the majority vote classifier consists of the index of the dimension with the smallest normalized variance. We train on $10k$ points and evaluate on $5k$ points.

Mutual Information Gap Metric (Chen et al., 2018b) β-VAE metric and the FactorVAE metric are neither general nor unbiased as they depend on some hyperparameters (Chen et al., 2018b). They compute the mutual information between each ground-truth factor and each dimension in the computed representation $r(\mathbf{x})$. For each ground-truth factor $\boldsymbol{z}_k$, they then consider the two dimensions in $r(\mathbf{x})$ that have the highest and second highest mutual information with $\boldsymbol{z}_k$. The Mutual Information Gap (MIG) is then defined as the average, normalized difference between the highest and second highest mutual information of each factor with the dimensions of the representation. The original metric was proposed evaluating the sampled representation. Instead, we consider the mean representation, in order to be consistent with the other metrics. We estimate the mutual information by binning each dimension of the representations. Then, the score is computed as follows:

$$MIG = \frac{1}{K}\sum_{k=1}^{K}\left[I(v_{jk}, z_k) - \max I(v_j, z_k)\right]$$

Where $\boldsymbol{z}_k$ is a factor of variation, $\boldsymbol{v}_i$ is a dimension of the latent representation.

MIG [8] computes the MI between each code and factor I(vi , zj). Then the code dimension with maximum MI is identified I(vi , z) for each factor. Next, the second highest MI, I(vi , z○), is subtracted from this maximal value. This difference constitutes the gap. The gap is then normalized by the entropy of the factor: MIG = I(vi , z) I(vi , z○) H(vi) (3) The MIG score of all factors are averaged to report one score. Robust MIG (RMIG) was proposed in [9]. It is identical to MIG in essence, but proposes a more robust formulation when MI is computed from the input space, which does not apply in our context. For the remainder of the paper we will refer to both MIG and RMIG as MIG-RMIG because our results apply to both in the same way.

Disentanglement, Completeness and Informativeness (DCI) In (Carbonneau et al., 2022), a framework is proposed to evaluate disentangled representations using metrics for modularity, compactness, and explicitness, referred to as disentanglement, completeness, and informativeness. Regressors predict factors from codes, with modularity and compactness estimated by importance weights R_{ij}. These weights are computed using a lasso regressor or random forests. The compactness for factor $\mathbf{v}_i$ is defined as:

$$C_i = 1 + \sum_{j=1}^{d} p_{ij}\log_d p_{ij}, \quad p_{ij} = \frac{R_{ij}}{\sum_{k=1}^{d} R_{ik}}.$$

Compactness for the entire representation is the average over all factors. The modularity for code dimension $\mathbf{z}_j$ is:

$$D_j = 1 + \sum_{i=1}^{M} p_{ij}\log_M p_{ij}, \quad p_{ij} = \frac{R_{ij}}{\sum_{k=1}^{M} R_{kj}}.$$

The modularity score is the weighted average over all code dimensions, with weights ρ_j reflecting their importance in predicting factors. Explicitness is defined by the MSE of the regressor, normalized between 0 and 1:

$$\text{Explicitness} = 1 - 6 \cdot \text{MSE}, \quad \text{MSE} = E[(\mathbf{x} - \mathbf{y})^2] = \frac{1}{6}.$$

.

Time Disentanglement Score TDS Time series data often exhibit variations that may not always align with conventional metrics, especially when considering the presence or absence of underlying attributes. To address this challenge, (Oublal et al., 2024) introduce the Time Disentanglement Score (TDS), a metric designed to assess the disentanglement of attributes in time series data. The foundation of TDS lies in an Information Gain perspective, which measures the reduction in entropy when an attribute is present compared to when it's absent.

$$TDS = \frac{1}{dim(\mathbf{z})} \sum_{n \neq m} \sum_{k} \frac{||z_m - z^+_{n,k}||^2}{\text{Var}[z_m]}, \tag{B.3}$$

In the context of TDS, we augment factor m in a time series window $\mathbf{x}$ with a specific objective: to maintain stable entropy when the factor is present and reduce entropy when it's absent. This augmentation aims to capture the essence of attribute-related information within the data.

B.5 PIPELINE CORRELATED SAMPLES.

Robustness of the model to correlations between data is assessed by examining different pairs. We focus mainly on linear correlations between two different devices and on the case where one device correlates with two others. To do this, we parameterize the correlations by sampling a dataset from the common distribution. We build on the correlation time series framework by introducing a pairwise correlation between the attributes y_m and y_n as follows: $p(y_m, y_n) \propto \exp\left(-||y_m - \alpha y_n||^2/2\sigma^2\right)$, where α is a scaling factor. A high value of σ indicates a lower correlation between the normalised attributes y_m and y_n (No.Corr, $\sigma = \infty$). We also extend this framework to cover correlations between several attributes in the time window T. Therefore, we consider correlation pair scenarios such as : *No correlation*; *Pair:1* washer-dryer; *Pair:2* dryer-oven and, finally, a *Random pair:* approach with randomly selected appliances.

- **No Corr.:** No Correlation during training. (default evaluation setting)
- **Pair: 1** washer-dryer.
- **Pair: 2** dryer-oven.
- **Pair: 3** lighting and television.
- **Pair: 4** microwave and oven.
- **Pair: 5** washer-dishwasher.
- **Random Pairs**,

B.6 IMPACT OF GELU VS RELU/LEAKYRELU ON DECODER BEHAVIOR

In this study, we evaluate the impact of different activation functions on the decoder's behavior to satifies Asm 2.1. Specifically, we compare the use of ReLU (a piecewise affine activation) and GELU (a smooth, nonlinear activation) within an MLP decoder. The results suggest that the choice of activation function has a significant impact on the latent representation produced by the model.

ReLU Activation: The decoder becomes piecewise affine, meaning that it can be broken down into affine transformations over different regions of the input space. This causes the decoder to create latent representations that reflect distinct linear transformations in various regions of the input. As a result, the learned latent space is structured around these distinct affine regions, potentially making the model more sensitive to certain regions of the data space and leading to more discrete or sharply defined latent representations.

Table 5: Average performance, considering factors {FR, DW, WM, HTH, LT}with 5 seed on real datasets REFIT and REDD. Metrics reported are: `DCI`, `RMIG` and `RMSE`. Lower values are better for all metrics. (↓ lower is better, ↑ higher is worse Top-1 , Top-2).

Sc.	**Methods**	$\sigma = \infty$			$\sigma = 0.3$			$\sigma = 0.8$		
	Metrics ⇒	**DCI ↓**	**RMIG ↓**	**RMSE ↓**	**DCI ↓**	**RMIG ↓**	**RMSE ↓**	**DCI ↓**	**RMIG ↓**	**RMSE ↓**
REFIT	○ BertNILM	-	-	36.86 ± 1.68	-	-	45.84 ± 1.00	-	-	46.29 ± 0.76
	○ S2S	-	-	35.46 ± 2.04	-	-	45.36 ± 2.47	-	-	45.76 ± 2.26
	○ Autoformer	-	-	32.45 ± 0.56	-	-	33.02 ± 1.49	-	-	34.68 ± 1.13
	○ Informer	-	-	32.92 ± 1.67	-	-	35.03 ± 1.71	-	-	38.47 ± 1.54
	● TimesNet	-	-	32.12 ± 1.99	-	-	33.38 ± 1.83	-	-	35.84 ± 1.61
	● CoST	44.68 ± 1.57	0.61 ± 0.02	31.14 ± 0.93	48.01 ± 1.57	0.64 ± 0.09	34.81 ± 0.71	46.98 ± 1.13	0.65 ± 0.01	38.14 ± 0.57
	○ SlowVAE	50.96 ± 0.71	0.61 ± 0.09	28.26 ± 1.54	53.04 ± 1.26	0.61 ± 0.09	32.15 ± 0.78	52.14 ± 0.58	0.70 ± 0.08	35.74 ± 1.03
	● SlowVAE+HDF	52.17 ± 0.07	0.42 ± 0.02	37.35 ± 1.49	53.00 ± 0.12	0.46 ± 0.05	38.86 ± 1.26	52.53 ± 0.03	0.47 ± 0.01	40.22 ± 1.06
	● TDRL	42.34 ± 1.02	0.28 ± 0.04	18.64 ± 1.41	49.75 ± 0.87	0.31 ± 0.01	17.18 ± 1.36	50.43 ± 0.69	0.38 ± 0.08	20.91 ± 1.07
	○ D3VAE	41.30 ±1.97	0.26 ±0.05	27.64 ±1.40	41.55 ±0.91	0.33 ±0.26	30.11 ±1.10	43.47 ±1.31	0.44 ±0.03	32.77 ±0.51
	○ C-DSVAE	47.35 ± 2.14	0.59 ± 0.05	31.78 ± 1.61	47.79 ± 0.99	0.62 ± 0.26	34.55 ± 1.18	50.02 ± 1.42	0.71 ± 0.03	37.57 ± 0.53
	● C-DSVAE + HDF	44.31 ± 1.93	0.56 ± 0.05	29.68 ± 1.51	45.01 ± 0.92	0.59 ± 0.25	32.42 ± 1.04	46.68 ± 1.33	0.66 ± 0.03	35.12 ± 0.50
	● SparseVAE	40.15 ± 0.86	0.25 ± 0.09	13.72 ± 1.30	43.98 ± 0.81	0.28 ± 0.21	14.81 ± 1.20	44.53 ± 0.58	0.31 ± 0.07	18.89 ± 1.30
	● `TimeCSL`	**39.02 ± 0.87**	**0.23 ± 0.07**	**12.03 ± 1.26**	42.51 ± 0.74	**0.27 ± 0.15**	**12.72 ± 1.16**	42.91 ± 0.59	**0.31 ± 0.05**	**14.76 ± 0.92**
	Avg.	45.62 ± 1.27	0.52 ± 0.07	31.02 ± 1.26	48.02 ± 0.85	0.58 ± 0.12	34.08 ± 1.04	48.92 ± 1.18	0.64 ± 0.06	35.67 ± 0.91
REDD	○ BertNILM	-	-	40.06 ± 2.41	-	-	44.14 ± 1.22	-	-	45.04 ± 0.99
	○ S2S	-	-	38.48 ± 2.87	-	-	45.07 ± 2.71	-	-	46.22 ± 2.26
	○ Autoformer	-	-	33.56 ± 0.79	-	-	34.13 ± 2.07	-	-	37.51 ± 1.81
	○ Informer	-	-	36.02 ± 2.37	-	-	37.61 ± 1.98	-	-	38.81 ± 2.36
	● TimesNet	-	-	36.69 ± 2.08	-	-	39.08 ± 2.71	-	-	42.55 ± 2.35
	● CoST	50.87 ± 1.13	0.58 ± 0.06	28.93 ± 1.81	53.10 ± 1.23	0.61 ± 0.14	30.72 ± 1.31	52.63 ± 1.19	0.67 ± 0.14	33.15 ± 1.12
	○ SlowVAE	48.11 ± 1.06	0.45 ± 0.05	31.73 ± 2.19	50.15 ± 1.35	0.47 ± 0.06	34.12 ± 1.57	50.97 ± 0.78	0.55 ± 0.02	35.27 ± 1.06
	● SlowVAE + HDF	51.09 ± 1.64	0.34 ± 0.04	32.85 ± 2.40	51.97 ± 1.07	0.39 ± 0.05	35.72 ± 2.17	51.85 ± 1.58	0.43 ± 0.06	37.38 ± 2.51
	● TDRL	45.12 ± 2.15	0.39 ± 0.05	22.87 ± 1.36	50.61 ± 1.53	0.44 ± 0.03	23.98 ± 1.41	51.18 ± 0.90	0.49 ± 0.08	27.13 ± 2.30
	○ D3VAE	43.77 ± 1.31	0.36 ± 0.06	28.43 ± 1.61	46.17 ± 0.86	0.39 ± 0.04	30.14 ± 1.35	48.02 ± 1.23	0.44 ± 0.06	32.46 ± 1.10
	○ C-DSVAE	49.68 ± 2.12	0.55 ± 0.07	31.03 ± 2.15	49.92 ± 1.05	0.58 ± 0.08	33.60 ± 1.77	51.51 ± 1.76	0.61 ± 0.03	35.38 ± 1.42
	● C-DSVAE + HDF	47.38 ± 1.19	0.53 ± 0.05	30.76 ± 2.13	48.85 ± 1.62	0.56 ± 0.03	32.89 ± 2.04	49.98 ± 1.34	0.60 ± 0.05	34.25 ± 1.22
	● SparseVAE	46.56 ± 2.49	0.44 ± 0.08	19.88 ± 2.06	50.49 ± 1.07	0.47 ± 0.06	21.42 ± 2.53	50.83 ± 1.73	0.53 ± 0.05	23.59 ± 2.17
	● `TimeCSL`	**43.45 ± 1.12**	**0.33 ± 0.02**	**16.32 ± 2.16**	**47.33 ± 1.29**	**0.35 ± 0.04**	**17.22 ± 2.01**	**48.09 ± 0.81**	**0.39 ± 0.06**	**18.95 ± 2.08**
	Avg.	47.02 ± 1.56	0.45 ± 0.06	28.04 ± 1.84	50.43 ± 1.19	0.48 ± 0.09	30.32 ± 1.56	50.95 ± 1.26	0.54 ± 0.07	32.83 ± 1.57

LeakyReLU Activation: In contrast, the GELU activation is smooth and nonlinear across the entire input space. This means that the decoder no longer operates piecewise affine, and the latent space learned by the model is more continuous and smooth. Since GELU smoothly transforms the input, it enables the decoder to create more nuanced, continuous latent representations. The absence of piecewise linear behavior allows for better modeling of complex, smooth relationships in the data, which may improve generalization to unseen data or tasks that require such smooth transformations.

B.7 VALIDATION OF RESULTS ON SYNTHETIC DATA GENERATION

We first construct two trend patterns. The first trend patterns follows a nonlinear, saturating pattern, $y_t = \frac{1}{1+\exp \beta_1 (t-\beta_1)} + \epsilon_t$ for $\beta_1 = 0.2, \beta_1 = 60, \epsilon_t \sim \mathcal{N}(0, 0.3)$. The second pattern is a mixture of ARMA process, $\mathrm{ARMA}(2,2) + \mathrm{ARMA}(3,3) + \mathrm{ARMA}(4,4)$, where the AR and MA parameters are as follows, ({.9, -.1}, {.2, -.5}), ({.1, .2, .3}, {.1, .65, -.45}), ({.3, .5, -.5, -.3}, {.1, .1, -.2, -.3}). Next, we construct three seasonal patterns, consisting of sine waves with the following period, phase, and amplitudes, {20, 0, 3}, {50, .2, 3}, {100, .5, 3}. The final time series are constructed as follows, generate a trend pattern $\boldsymbol{f}_\phi(t)$ and seasonal pattern $s(t)$, the final time series is $y(t) = \boldsymbol{f}_\phi(t) + s(t), t = 0, \ldots, 999$. We do this for all pairs of trend and seasonal patterns, constructing a total of 6 time series.

B.8 THAT SPARSITY ENOUGH FOR ROBUSTNESS, IS DONWSTREAM TASK ?

Table 6: Performance of different methods on synthetic datasets for various test cases. Metrics reported are `DCI`, `RMIG` and `RMSE`. Lower values are better for all metrics. **Bold** indicates the best performance, while **second** and **first** indicate the second and first best performance respectively.

Sc.	**Methods**	$\sigma = \infty$			$\sigma = 0.3$			$\sigma = 0.8$		
	Metrics ⇒	**DCI ↓**	**RMIG ↓**	**RMSE ↓**	**DCI ↓**	**RMIG ↓**	**RMSE ↓**	**DCI ↓**	**RMIG ↓**	**RMSE ↓**
REFIT	BertNILM	-	-	52.81 ± 25.41	-	-	75.78 ± 7.76	-	-	66.50 ± 6.69
	S2S	-	-	47.99 ± 24.45	-	-	63.64 ± 20.56	-	-	67.93 ± 15.57
	Autoformer	-	-	61.52 ± 7.66	-	-	52.23 ± 11.25	-	-	48.45 ± 9.31
	Informer	-	-	48.59 ± 10.89	-	-	59.29 ± 11.36	-	-	63.45 ± 10.52
	FEDformer	-	-	48.29 ± 10.88	-	-	58.74 ± 11.23	-	-	64.55 ± 9.92
	TimesNet	-	-	63.57 ± 10.61	-	-	67.02 ± 9.10	-	-	69.93 ± 9.89
	C-DSVAE	72.83 ± 11.71	1.08 ± 0.45	40.50 ± 6.45	71.76 ± 9.74	1.08 ± 0.44	51.67 ± 7.88	72.64 ± 10.89	1.23 ± 0.51	55.26 ± 7.80
	SlowVAE	82.31 ± 11.96	1.08 ± 0.47	43.46 ± 7.93	81.65 ± 10.75	1.08 ± 0.46	54.81 ± 5.93	84.09 ± 6.93	1.27 ± 0.49	53.65 ± 7.48
	CoST	79.86 ± 10.86	1.16 ± 0.23	50.14 ± 6.77	79.16 ± 10.49	1.15 ± 0.22	55.91 ± 5.72	80.16 ± 9.68	1.25 ± 0.20	58.76 ± 5.51
	SlowVAE+HDF	88.69 ± 1.11	1.11 ± 0.24	65.87 ± 8.13	85.99 ± 1.34	0.97 ± 0.21	69.94 ± 7.29	89.47 ± 0.58	1.14 ± 0.24	72.21 ± 7.47
	C-DSVAE + HDF	76.94 ± 6.38	0.89 ± 0.37	33.61 ± 5.80	75.66 ± 6.53	0.84 ± 0.33	37.92 ± 5.88	74.45 ± 5.78	0.89 ± 0.40	42.58 ± 6.49
	SparseVAE	71.35 ± 8.48	0.67 ± 0.25	26.46 ± 5.68	72.67 ± 8.54	0.68 ± 0.27	31.07 ± 5.34	73.98 ± 8.23	0.74 ± 0.29	32.56 ± 5.16
	`TimeCSL`	75.44 ± 6.93	0.59 ± 0.17	25.53 ± 6.69	74.50 ± 6.29	0.61 ± 0.19	29.23 ± 6.57	76.66 ± 5.70	0.74 ± 0.16	33.76 ± 6.73
REFIT	BertNILM	-	-	60.83 ± 5.80	-	-	72.63 ± 2.25	-	-	71.02 ± 2.55
	S2S	-	-	53.73 ± 5.84	-	-	65.57 ± 5.35	-	-	69.21 ± 4.06
	Autoformer	-	-	54.60 ± 1.70	-	-	50.48 ± 2.82	-	-	50.39 ± 2.26
	Informer	-	-	45.92 ± 3.03	-	-	53.77 ± 2.86	-	-	61.08 ± 2.51
	FEDformer	-	-	45.55 ± 3.00	-	-	53.62 ± 2.93	-	-	60.26 ± 2.51
	TimesNet	-	-	54.68 ± 3.68	-	-	55.28 ± 3.02	-	-	59.24 ± 3.41
	C-DSVAE	74.83 ± 5.72	1.12 ± 0.23	47.04 ± 3.14	73.42 ± 2.40	1.10 ± 0.21	53.02 ± 3.49	75.29 ± 3.34	1.21 ± 0.14	54.81 ± 3.46
	SlowVAE	80.92 ± 2.73	1.10 ± 0.20	44.58 ± 3.11	79.95 ± 2.64	1.09 ± 0.18	51.92 ± 2.58	81.45 ± 1.57	1.21 ± 0.14	50.69 ± 2.99
	CoST	71.18 ± 3.83	1.04 ± 0.06	47.10 ± 1.66	71.01 ± 3.86	1.05 ± 0.05	53.58 ± 1.39	70.56 ± 3.50	1.14 ± 0.04	55.29 ± 1.22
	SlowVAE+HDF	81.13 ± 0.17	0.85 ± 0.08	60.50 ± 3.01	80.21 ± 0.19	0.79 ± 0.07	62.72 ± 2.77	81.68 ± 0.10	0.89 ± 0.05	64.03 ± 2.99
	C-DSVAE + HDF	74.77 ± 1.56	0.78 ± 0.05	35.62 ± 2.52	74.39 ± 1.51	0.75 ± 0.05	38.40 ± 1.83	74.88 ± 0.98	0.79 ± 0.07	39.95 ± 1.62
	SparseVAE	69.84 ± 4.10	0.62 ± 0.06	27.28 ± 2.59	69.95 ± 4.15	0.60 ± 0.05	29.61 ± 1.67	72.52 ± 3.77	0.65 ± 0.07	30.35 ± 1.45
	`TimeCSL`	71.72 ± 3.23	0.46 ± 0.04	25.02 ± 2.77	71.21 ± 2.58	0.51 ± 0.03	25.91 ± 2.62	72.68 ± 2.33	0.61 ± 0.02	28.82 ± 2.83

Table 7: Average performance, considering factors {FR, DW, WM, HTH, LT}with 5 seed on real datasets REFIT and REDD. Metrics reported are `DCI`, `RMIG` and `RMSE`. Lower values are better for all metrics. (↓ lower is better, ↑ higher is worse Top-1 , Top-2).

Sc.	**Methods**	$\sigma = \infty$			$\sigma = 0.3$			$\sigma = 0.8$		
	Metrics ⇒	**DCI ↓**	**RMIG ↓**	**RMSE ↓**	**DCI ↓**	**RMIG ↓**	**RMSE ↓**	**DCI ↓**	**RMIG ↓**	**RMSE ↓**
REFIT	○ BertNILM	-	-	56.4 ± 2.58	-	-	70.2 ± 1.45	-	-	70.92 ± 1.15
	○ S2S	-	-	54.3 ± 3.12	-	-	69.5 ± 3.56	-	-	69.95 ± 3.26
	○ Autoformer	-	-	49.7 ± 0.81	-	-	50.5 ± 2.15	-	-	52.95 ± 1.63
	○ Informer	-	-	50.3 ± 2.41	-	-	53.5 ± 1.98	-	-	58.95 ± 1.89
	○ FEDformer	-	-	50.3 ± 2.12	-	-	52.5 ± 2.45	-	-	59.01 ± 1.76
	● TimesNet	-	-	49.24 ± 2.87	-	-	51.10 ± 2.64	-	-	54.91 ± 2.31
	○ C-DSVAE	72.42 ± 3.10	0.96 ± .15	48.6 ± 2.32	73.12 ± 1.43	0.95 ± .15	52.9 ± 2.31	74.29 ± 2.04	1.08 ± .09	52.99 ± 1.91
	○ SlowVAE	78.0 ± 1.09	0.94 ± .13	43.2 ± 2.23	78.0 ± 1.09	0.94 ± .13	49.2 ± 1.13	79.74 ± 0.84	1.07 ± .11	49.65 ± 1.43
	● CoST	68.4 ± 2.41	0.97 ± .03	47.7 ± 1.35	68.4 ± 2.41	0.97 ± .03	53.2 ± 1.02	69.95 ± 1.63	1.00 ± .02	53.45 ± 0.82
	● SlowVAE+HDF	79.8 ± .10	0.64 ± .05	57.2 ± 2.15	79.8 ± .10	0.64 ± .05	61.3 ± 1.82	80.37 ± .05	0.72 ± .03	61.64 ± 1.52
	● C-DSVAE + HDF	73.1 ± 1.01	0.69 ± .02	34.4 ± 1.89	73.1 ± 1.01	0.69 ± .02	38.1 ± 1.34	74.25 ± 0.59	0.73 ± .05	38.48 ± 1.04
	● SparseVAE	67.2 ± 2.01	0.52 ± .02	24.3 ± 1.81	67.2 ± 2.01	0.52 ± .02	27.4 ± 1.13	71.79 ± 1.27	0.58 ± .04	27.77 ± 0.83
	● `TimeCSL`	**63.5 ± 1.35**	**0.38 ± .02**	**19.6 ± 1.95**	**69.3 ± 1.2**	**0.44 ± .02**	**20.3 ± 1.79**	**70.12 ± 0.91**	**0.51 ± .01**	**23.63 ± 1.49**
REDD	○ BertNILM	-	-	61.42 ± 3.47	-	-	67.61 ± 1.95	-	-	69.06 ± 1.43
	○ S2S	-	-	59.08 ± 4.15	-	-	68.60 ± 3.91	-	-	70.68 ± 3.25
	○ Autoformer	-	-	49.87 ± 0.92	-	-	51.53 ± 1.48	-	-	51.88 ± 1.34
	○ Informer	-	-	54.23 ± 1.78	-	-	57.70 ± 1.78	-	-	62.51 ± 1.55
	○ FEDformer	-	-	52.84 ± 1.69	-	-	55.83 ± 1.82	-	-	61.92 ± 1.57
	● TimesNet	-	-	51.37 ± 2.41	-	-	55.35 ± 2.23	-	-	58.47 ± 2.21
	○ C-DSVAE	72.97 ± 3.44	1.04 ± 0.16	47.17 ± 2.11	73.60 ± 1.82	0.98 ± 0.14	52.16 ± 1.89	73.96 ± 2.46	1.11 ± 0.12	53.73 ± 1.79
	○ SlowVAE	77.41 ± 1.67	0.94 ± 0.15	46.61 ± 1.91	77.80 ± 1.63	0.95 ± 0.14	49.82 ± 1.71	79.47 ± 1.26	1.04 ± 0.13	50.88 ± 1.58
	● CoST	70.75 ± 2.01	0.96 ± 0.09	48.92 ± 1.62	70.87 ± 2.04	0.96 ± 0.09	52.73 ± 1.34	71.93 ± 1.84	0.98 ± 0.09	54.46 ± 1.19
	● SlowVAE+HDF	79.97 ± 0.14	0.72 ± 0.05	56.96 ± 2.34	79.77 ± 0.14	0.72 ± 0.05	59.75 ± 2.21	80.22 ± 0.07	0.75 ± 0.03	60.77 ± 2.22
	● C-DSVAE + HDF	73.85 ± 0.85	0.69 ± 0.05	34.19 ± 1.47	73.71 ± 0.85	0.69 ± 0.05	37.53 ± 1.21	74.34 ± 0.56	0.71 ± 0.04	39.35 ± 1.06
	TDRL	70.86 ± 0.816	0.57 ± 0.041	32.80 ± 1.41	70.75 ± 0.816	0.57 ± 0.041	36.04 ± 1.16	71.94 ± 0.54	0.58 ± 0.033	37.83 ± 1.02
	● SparseVAE	70.13 ± 1.44	0.61 ± 0.04	25.46 ± 1.10	70.13 ± 1.44	0.61 ± 0.04	28.99 ± 1.22	71.44 ± 1.30	0.63 ± 0.05	29.47 ± 1.10
	● `TimeCSL`	66.14 ± 1.66	0.40 ± 0.04	19.81 ± 1.29	69.00 ± 1.41	0.44 ± 0.04	20.46 ± 1.45	70.41 ± 1.22	0.48 ± 0.03	22.08 ± 1.36