TRAINABILITY PRESERVING NEURAL PRUNING

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ABSTRACT

Many recent pruning works show trainability plays a critical role in network structured pruning – unattended broken trainability can lead to severe under-performance and unintentionally amplify the effect of finetuning learning rate, resulting in biased (or even misinterpreted) benchmark results. In this paper, we present *trainability preserving pruning* (TPP), a scalable method to preserve network trainability against pruning, aiming for improved pruning performance. TPP regularizes the gram matrix of convolutional filters to decorrelate the pruned filters from the retained filters. In addition to the convolutional layers, per the spirit of preserving the trainability of the whole network, we also propose to regularize the batch normalization parameters. Empirically, TPP performs on par with the ground-truth trainability recovery method on linear MLP networks. On non-linear networks (ResNet56/VGG19 on CIFAR10/100), our TPP outperforms the other counterpart schemes by an obvious margin. Moreover, extensive results on ImageNet with ResNets show TPP consistently performs more favorably against other top-performing structured pruning approaches.

1 INTRODUCTION

Neural network pruning aims to remove the parameters without seriously compromising the performance. It normally consists of three steps (Reed, 1993; Han et al., 2015; 2016b): pretrain a dense model; prune the unnecessary connections or neurons; finetune to regain performance. Pruning is usually categorized into two groups, unstructured pruning (*a.k.a.* element-wise pruning) and structured pruning (*a.k.a.* filter pruning or channel pruning). The former chooses a scalar weight as the basic pruning element; the latter chooses a 3d filter as the basic pruning element. In general, structured pruning is more favored for acceleration because of the regular sparsity; unstructured pruning results in irregular sparsity, which can be considerable without performance degradation but hard to exploit for acceleration without customized hardware (Han et al., 2016a; 2017).

Recent structured pruning works (Renda et al., 2020; Le & Hua, 2021) showed an interesting phenomenon: During finetuning, a larger learning rate (LR) helps achieve a significantly better final performance (*e.g.*, ResNet34 pruned at speedup 1.32$\times$ can be improved by over 1% top-1 accuracy on ImageNet (Deng et al., 2009) using finetuning LR `1e-2` $\boldsymbol{s}$ `1e-3`). The reason behind is argued to be relevant to the trainability of neural networks (Lee et al., 2020; Lubana & Dick, 2021; Wang et al., 2021a). They mainly make two observations to explain the LR effect mystery (Wang et al., 2021a). **(1)** The weights removing operation immediately breaks the trainability or dynamical isometry (Saxe et al., 2014) (the ideal case of trainability) of the trained network. **(2)** The SGD optimization in finetuning can inherently rectify it without extra explicit guidance; a larger LR help recover it faster (and possibly better), thus making the final performance much stronger, especially when the finetuning epochs are insufficient. Although these works (Lee et al., 2020; Lubana & Dick, 2021; Wang et al., 2021a) provide a plausibly sound explanation, a more practical issue is how to recover the broken trainability (or dynamical isometry) or maintain it during pruning. In this regard, (Wang et al., 2021a) proposes to apply weight orthogonalization based on QR decomposition (Trefethen & Bau III, 1997; Mezzadri, 2006) to the pruned model. However, their method is shown to only work for *linear* MLP networks. On modern deep convolutional neural networks (CNNs), how to maintain trainability after pruning is still elusive.

In this paper, we present *trainability preserving pruning* (TPP), a new filter pruning method based on regularization (see Fig. 1 for an overview) that maintains trainability well during pruning. By our observation, the primary cause that pruning breaks dynamical isometry lies in the dependency

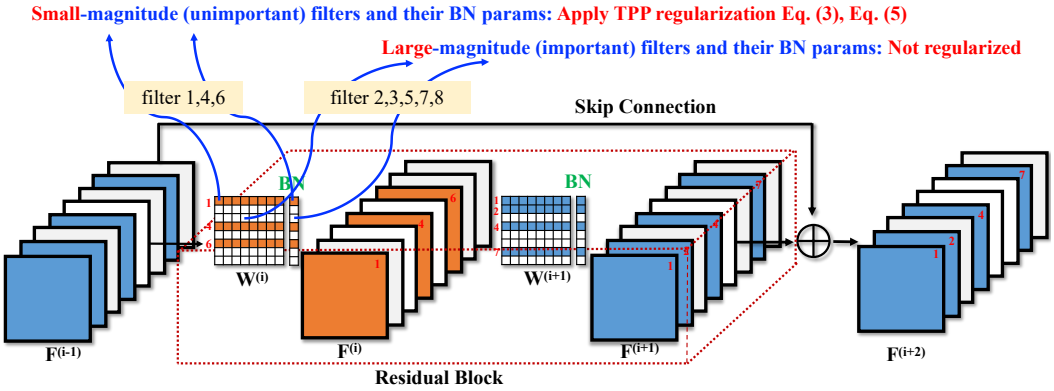


Figure 1: Illustration of the proposed TPP algorithm applied to a typical residual block. Weight parameters are classified into two groups as a typical pruning algorithm does: important (white color) and unimportant (orange or blue color), right from the beginning (before any training starts) based on the L_1-norms of the filters. Then *only* the unimportant parameters are enforced with the proposed TPP regularization terms, which is the key to maintain trainability when the unimportant weights are eventually eliminated from the network. Notably, the critical part of a regularization-based pruning algorithm lies in its *specific regularization term*, *i.e.*, Eqs. (3) and (5), which we will show perform more favorably than other alternatives (see Tabs. 1 and 10).

among parameters. The main idea of our method is thus to *decorrelate* the pruned weights from the kept weights so as to cut off the dependency, so that the following weights removing operation barely hurts the trainability of the network.

Specifically, we propose to regularize the gram matrix of weights: All the entries representing the correlation between the pruned filters (*i.e.*, unimportant filters) and the kept filters (*i.e.*, important filters) are encouraged to diminish to zero. This is the first technical contribution of our method. The second one lies in how to treat the other entries. Conventional dynamical isometry wisdom suggests orthogonality (namely, 1 self-correlation and 0 cross-correlation) even among the kept filters, while we find directly translating the idea here is *unnecessary or even harmful* because the too strong penalty will constrain the optimization, leading to deteriorated local minimum. Rather, we propose to *not impose any regularization* on the correlation entries of kept filters.

Finally, modern deep models are typically equipped with batch normalization (BN) (Ioffe & Szegedy, 2015). However, previous filter pruning papers rarely explicitly take BN into account (except two (Liu et al., 2017; Ye et al., 2018); the differences of our work from theirs will be discussed in Sec. 3.2) to mitigate the side effect when it is removed because its associated filter is removed. Since they are also a part of the whole trainable parameters in the network, unattended removal of them will also lead to severely crippled trainability (especially at large sparsity). Therefore, BN parameters (including both the scale and bias parameters) should also be explicitly taken into account when designing a pruning algorithm. As such, we propose to regularize the learnable parameters of BN to minimize the influence of its absence.

Empirically, the proposed pruning algorithm is rather easy to implement on popular deep learning platforms and robust to hyper-parameter variations. On the ResNet50 ImageNet benchmark, it delivers encouraging results compared to many recent SOTA filter pruning algorithms.

Contributions. **(1)** We present the *first* filter pruning method (trainability preserving pruning) that can effectively maintain trainability during pruning for modern deep networks, through a customized weight gram matrix as regularization target. **(2)** Apart from weight regularization, we also propose to regularize the BN parameters to offset the side-effect of their eventual absence. This has been overlooked by most previous pruning papers, while we show it is pretty important to preserve trainability, especially when the pruning ratio is large. **(3)** Practically, the proposed method is scalable to modern large-scale deep neural networks (*e.g.*, ResNets) and datasets (*e.g.*, ImageNet). It achieves promising pruning performance in the comparison to many state-of-the-art filter pruning methods.

2 RELATED WORK

Neural network pruning. In terms of the sparsity structure induced by pruning, pruning methods mainly fall into structured pruning (filter pruning or channel pruning) (Li et al., 2017; Wen et al., 2016; He et al., 2017; 2018; Wang et al., 2021b) and unstructured pruning (Han et al., 2015; 2016b; LeCun et al., 1990; Hassibi & Stork, 1993; Singh & Alistarh, 2020). Structured pruning results in *regular* sparsity after pruning, easy to be translated to acceleration on commodity hardware. In contrast, unstructured pruning produces *irregular* sparsity, beneficial to compression while hard to leverage for practical acceleration (Wen et al., 2016; Wang et al., 2019) unless with special hardware support (Han et al., 2016a; 2017). For more comprehensive coverage, we recommend surveys (Sze et al., 2017; Cheng et al., 2018a;b; Deng et al., 2020; Wang et al., 2022). This paper targets *structured sparsity (filter pruning)* because it is more imperative to make modern networks (*e.g.*, ResNets (He et al., 2016)) *faster* rather than *smaller* compared to the early single-branch convolutional networks.

It has been broadly observed (LeCun et al., 1990; Han et al., 2015) and also intuitively straightforward that a naive way of pruning (*e.g.*, random pruning) of a normally-sized (*i.e.*, not intentionally severely over-parameterized) network will lead to significant performance drop. Namely, we need to cleverly choose some "unimportant" parameters to remove to avoid severe performance degradation. Such a criterion for choosing is *pruning criterion*. In the area, there have been two major paradigms to address the pruning criterion problem dating back to the 1990s: regularization-based methods and importance-based methods (Reed, 1993). Specifically, the regularization-based methods select unimportant parameters by adding a sparsity-inducing penalty term jointly optimized with the original objective function (*e.g.*, (Wen et al., 2016; Lebedev & Lempitsky, 2016; Louizos et al., 2018; Liu et al., 2017; Ye et al., 2018; Zhang* et al., 2021; Zhang et al., 2021)). This paradigm can be applied to a random or pretrained network. Importance-based methods choose unimportant parameters by certain established mathematical formula (typically based on the Taylor expansion of the loss objective function) (*e.g.*, (LeCun et al., 1990; Hassibi & Stork, 1993; Han et al., 2015; 2016b; Li et al., 2017; Molchanov et al., 2017; 2019)). This paradigm is majorly applied to a pretrained network. Despite the differences, it is worth noting that these two paradigms are *not* firmly unbridgeable, *i.e.*, we can develop approaches that take advantage of *both* ideas, such as (Ding et al., 2018; Wang et al., 2019; 2021b) – these methods identify unimportant weights per a certain importance criterion; then, they utilize a penalty term to produce sparsity. Our TPP method in this paper is also in this line.

Trainability, dynamical isometry, and orthogonality. Trainability describes the easiness of optimization of a neural network. Dynamical isometry, the perfect case of trainability, is first introduced by (Saxe et al., 2014), stating that singular values of the Jacobian matrix are close to 1. It can be achieved (for linear MLP models) by the orthogonality of weight matrix at initialization. Recent works on this topic mainly focus on how to maintain dynamical isometry *during training* instead of only for initialization (Xie et al., 2017; Huang et al., 2018; Bansal et al., 2018; Huang et al., 2020; Wang et al., 2020). These methods are developed independent of pruning, thus not directly relevant to the proposed method in this work. However, the insights from these works inspire us to our proposed approach (see Sec. 3.2) and possibly more in the future. Several pruning papers study the network trainability issue in the context of network pruning, such as (Lee et al., 2020; Lubana & Dick, 2021; Vysogorets & Kempe, 2021). Our work is different from theirs primarily in that we focus on pruning a pretrained model while they tackle the pruning at initialization problem.

3 METHODOLOGY

3.1 PRELIMINARIES: DYNAMICAL ISOMETRY AND ORTHOGONALITY

The definition of dynamical isometry is that the input-output Jacobian of a network has as many singular values (JSVs) as possible close to 1 (Saxe et al., 2014). With it, the error signal can preserve its norm under propagation without serious amplification or attenuation, which in turn helps the convergence of (very deep) networks. For a single fully-connected layer W, a sufficient and necessary condition to realize dynamical isometry is orthogonality, *i.e.*, $W^T W = I$,

$$\begin{aligned} \mathbf{y} &= W\mathbf{x}, \\ ||\mathbf{y}|| &= \sqrt{\mathbf{y}^T\mathbf{y}} = \sqrt{\mathbf{x}^T W^T W \mathbf{x}} = ||\mathbf{x}||, \quad \textit{iff. } W^T W = I, \end{aligned} \tag{1}$$

where I stands for identity matrix. Orthogonality of a weight matrix can be easily realized by matrix orthogonalization techniques such as QR decomposition (Trefethen & Bau III, 1997; Mez-

zadri, 2006). *Exact* (namely all the Jacobian singular values are exactly 1) dynamical isometry can be achieved for *linear* networks since multiple linear layers essentially reduce to a single 2d weight matrix. In contrast, the convolutional and non-linear cases are much complicated. Previous work (Wang et al., 2021a) has shown that merely considering convolution or ReLU (Nair & Hinton, 2010) renders the weight orthogonalization method much less effective in terms of recovering dynamical isometry after pruning, let alone considering modern deep networks with BN (Ioffe & Szegedy, 2015) and residuals (He et al., 2016). The primary goal of our paper is to bridge this gap.

Following the seminal work of (Saxe et al., 2014), several papers propose to maintain orthogonality *during training* instead of sorely for the initialization. There are primarily two groups of orthogonalization methods for convolutional neural networks: kernel orthogonality (Xie et al., 2017; Huang et al., 2018; 2020) and orthogonal convolution (Wang et al., 2020):

$$\begin{aligned} KK^T = I \Rightarrow \mathcal{L}_{orth} &= KK^T - I, \triangleleft \textbf{ kernel orthogonality} \\ \mathcal{K}\mathcal{K}^T = I \Rightarrow \mathcal{L}_{orth} &= \mathcal{K}\mathcal{K}^T - I, \triangleleft \textbf{ orthogonal convolution} \end{aligned} \tag{2}$$

where clearly the difference lies in the weight matrix K *vs.* $\mathcal{K}$. **(1)** K denotes the original weight matrix in a convolutional layer. Weights of a conv layer make up a 4d tensor $\mathbb{R}^{N\times C\times H\times W}$, where N stands for the number of output channels, C for the number of input channels, H and W for the height and width of conv kernel. Then, K is a reshaped version of the 4d tensor: $K \in \mathbb{R}^{N\times CHW}$ (if $N < CHW$; otherwise, $K \in \mathbb{R}^{CHW\times N}$). **(2)** In contrast, $\mathcal{K} \in \mathbb{R}^{NH_{fo}W_{fo}\times CH_{fi}W_{fi}}$ stands for the doubly block-Toeplitz (DBT) matrix representation of K (H_{fo} stands for the output feature map height, H_{fi} for the input feature map height. W_{fo} and W_{fi} can be inferred the same way for width).

Wang et al. (2020) have shown that orthogonal convolution is more effective than kernel orthogonality (Xie et al., 2017) in that the latter is only a necessary but insufficient condition of the former. In this work, we will evaluate *both* methods to see how effective they are in recovering trainability.

3.2 Trainability Preserving Pruning (TPP)

Our TPP method has two parts. First, we explain how we come up with the proposed scheme and how it intuitively is better than the straight idea of directly applying orthogonality regularization methods (Xie et al., 2017; Wang et al., 2020) here. Second, we propose to regularize batch normalization layers given their prevailing use as a standard component in deep neural networks nowadays.

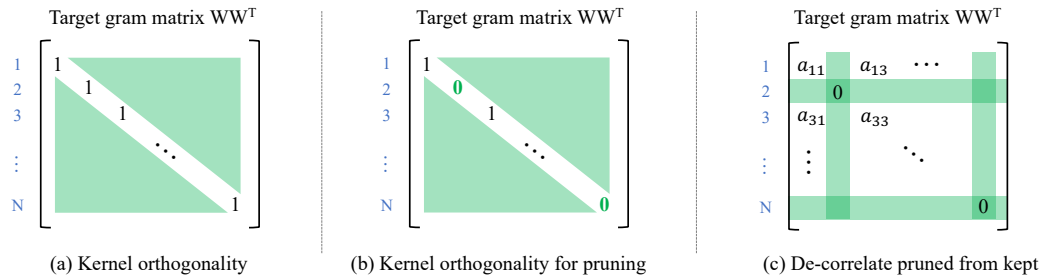
(1) Trainability *vs.* orthogonality. From previous works (Lee et al., 2020; Lubana & Dick, 2021; Wang et al., 2021a), we know recovering the broken trainability (or dynamical isometry) incurred by pruning is very important. Considering orthogonality regularization can encourage dynamical isometry, a pretty straightforward solution is to build upon the existing kernel orthogonality regularization schemes. Specifically, kernel orthogonality regularizes the weight gram matrix to be close to identity matrix (see Fig. 2(a)). In our case, we aim to remove some filters, so naturally we can regularize the weight gram matrix to be close to a *partial identity matrix*, with the diagonal entries at the pruned filters zeroed (see Fig. 2(b)).

The above scheme is simple and straightforward. However, it is not in its best shape by our empirical observation. It imposes too strong *unnecessary* constraint on the remaining weights, which will in turn hurt the optimization. Therefore, we propose to seek a *weaker* constraint, not demanding the perfect trainability (*i.e.*, dynamical isometry realized by orthogonality), but only a *benign* status, which describes a state of the neural network where gradients can flow effectively through the model without being interrupted. Orthogonality requires the Jacobian singular values to be exactly 1; in contrast, a benign trainability only requires them *not to be extremely large or small* so that the network can be trained normally. To this end, we propose to *decorrelate* the kept filters from the pruned ones: in the target gram matrix, all the entries associated with the pruned filters are zero; all the other entries stay *as they are* (see Fig. 2(c)). This scheme will be empirically justified (Tab. 3).

Specifically, for the l-th layer, we sort the filters by their L_1 norms and select those with the smallest L_1 norms as unimportant filters (denoted as set S_l). Then, the proposed regularization term is,

$$\mathcal{L}_1 = \sum_{l=1}^{L} ||W_l W_l^T \odot (\mathbf{1} - \mathbf{m}\mathbf{m}^T)||_F^2, \ \mathbf{m}_j = 0 \text{ if } j \in S_l, \text{ else } 1, \tag{3}$$

where W denotes the weight matrix; $\mathbf{1}$ represents the matrix full of 1; $\mathbf{m}$ is a 0/1-valued column mask vector; $\odot$ is the Hadamard (element-wise) product; and $||\cdot||_F$ denotes the Frobenius norm.



(a) Kernel orthogonality (b) Kernel orthogonality for pruning (c) De-correlate pruned from kept

Figure 2: Regularization target comparison between the proposed scheme (c) and similar counterparts (a) and (b). Green part stands for zero entries. Index 1 to N denotes the filter indices. In (b, c), filter 2 and N are the unimportant filters to be removed. **(a)** Regularization target of pure kernel orthogonality (an identity matrix), no pruning considered. **(b)** Regularization target of directly applying the weight orthogonality to filter pruning. **(c)** Regularization target of the proposed *weight de-correlation* solution in TPP: only regularize the filters to be removed, leave the others unconstrained. This scheme maintains trainability while imposing the least constraint on the weights.

(2) BN regularization. Per the idea of preserving trainability, BN is not ignorable since BN layers are also trainable. Removing filters will change the internal feature distributions. If the learned BN statistics do not change accordingly, the error will accumulate and result in deteriorated performance (especially for deep networks). Consider the following BN formulation (Ioffe & Szegedy, 2015),

$$f = \gamma \frac{W * X - \mu}{\sqrt{\sigma^2 + \epsilon}} + \beta, \tag{4}$$

where $*$ stands for convolution; μ stands for the running mean, σ^2 for the running variance; ϵ is a small amount for numerical stability; the two learnable parameters are γ and β. Although unimportant weights are enforced with regularization for sparsity, their magnitude can barely be exact zero, making the subsequent removal of filters *biased*. This will modify the feature distribution and render the BN statistics incorrect. Using these biased BN statistics would be improper and damages trainability. To mitigate this influence of BN, we propose to regularize *both* the γ and β of *pruned* feature map channels to zero. The penalty term can be formulated as

$$\mathcal{L}_2 = \sum_{l=1}^{L} \sum_{j \in S_l} \gamma_j^2 + \beta_j^2. \tag{5}$$

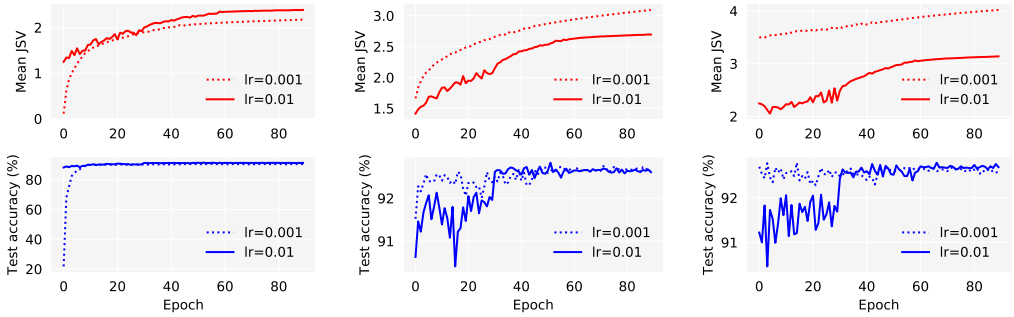
The merits of BN regularization will be justified in our experiments (Tab. 4).

To sum, with the proposed regularization terms, the total error function is

$$\mathcal{E} = \mathcal{L}_{cls} + \frac{\lambda}{2}(\mathcal{L}_1 + \mathcal{L}_2), \tag{6}$$

where $\mathcal{L}_{cls}$ means the original classification loss. The coefficient λ *grows* by a predefined constant Δ per K_u iterations (up to a ceiling τ) during training to ensure the pruned parameters are rather close to zero (inspired by Wang et al. (2019; 2021b)). See Algorithm 1 in Appendix for more details.

Discussion. Prior works (Liu et al., 2017; Ye et al., 2018) also propose to regularize BN for pruning. Our BN regularization method is *starkly different* from theirs. **(1)** In terms of the motivation or goal, Liu et al. (2017); Ye et al. (2018) regularize γ to *learn* unimportant filters, namely, regularizing BN is to indirectly decide which filters are unimportant. In contrast, in our method, unimportant filters are decided by their L_1-norms. We adopt BN regularization for a totally different consideration – to mitigate the side effect of breaking dynamical isometry, which is not mentioned at all in their works. **(2)** In terms of specific technique, Liu et al. (2017); Ye et al. (2018) only regularize the scale factor γ (because it is enough to decide which filters are unimportant) while we regularize *both* the learnable parameters because only regularizing one still impairs dynamical isometry of the network. Besides, we employ different regularization strength for different parameters (by the group of important filters *vs.* unimportant filters), while Liu et al. (2017); Ye et al. (2018) simply use the same penalty strength for *all* parameters – this is another key difference because regularizing all parameters (including those that are meant to be kept) will damage dynamical isometry, which is exactly what we want to avoid. In short, in either general motivation or specific technical details, our proposed BN regularization is *distinct* from previous works (Liu et al., 2017; Ye et al., 2018).



91.36 / 90.54 / 0.0040
(a) L_1

92.79 / 92.77 / 1.0000
(b) L_1+OrthP

92.82 / 92.77 / 3.4875
(c) TPP (ours)

Figure 3: Mean JSV and test accuracy during finetuning with different setups (network: MLP-7-Linear, dataset: MNIST). Below each plot are, in order, the best accuracy of LR `1e-2`, the best accuracy of LR `1e-3`, and the mean JSV right after pruning (*i.e.*, without finetuning). LR `1e-2` and `1e-3` are short for two finetuning LR schedules: {`0:1e-2, 30:1e-3, 60:1e-4, epochs:90`}, {`0:1e-3, 45:1e-4, epochs:90`}. The accuracies are averaged by 5 random runs. For reference, the unpruned model has mean JSV 2.4987, test accuracy 92.77.

4 Experimental Results

Datasets and networks. We first conduct analyses with MLP-7-Linear network on MNIST (LeCun et al., 1998). Then compare our method to other plausible solutions with ResNet56 (He et al., 2016)/VGG19 (Simonyan & Zisserman, 2015) on CIFAR10/100 (Krizhevsky, 2009). Next we evaluate our algorithm on the large-scale ImageNet dataset (Deng et al., 2009) with ResNet34 and 50 (He et al., 2016). Finally, we conduct an ablation study to show the efficacy of two main technical novelties in our approach. On ImageNet, we take the official PyTorch (Paszke et al., 2019) pre-trained models as base models to maintain comparability with other methods. On other datasets, we train our own base models with comparable accuracies to those reported in their original papers. See the Appendix (Tab. 5) for the specific training settings due to limited space here.

Comparison methods. We compare with Wang et al. (2021a), which proposes a method OrthP to recover broken dynamical isometry for pruning *pretrained* models. Besides, since maintaining orthogonality is the key and there are many existing orthogonality regularization papers (Xie et al., 2017; Wang et al., 2020; Huang et al., 2018; 2020; Wang et al., 2020), a straightforward solution is to combine them with L_1-norm pruning (Li et al., 2017) to see if they can help maintain or recover the broken dynamical isometry. There are two possible combination schemes: 1) apply orthogonal regularization methods *before* applying L_1-norm pruning, 2) apply orthogonal regularization methods *after* L_1-norm pruning, namely, in finetuning. Two representative orthogonality regularization methods are selected because of their proved effectiveness: kernel orthogonality (KernOrth) (Xie et al., 2017) and convolutional orthogonality (OrthConv) (Wang et al., 2020), so in total there are 4 combinations: L_1 + KernOrth, L_1 + OrthConv, KernOrth + L_1, OrthConv + L_1.

Comparison metrics. (1) We compare the final test accuracy *after finetuning* with the similar FLOPs budget – this is currently the most prevailing metric to compare different filter pruning methods in classification. Concretely, we compare two settings: a relatively large finetuning LR (`1e-2`) and a small one (`1e-3`). We introduce these settings because previous works (Renda et al., 2020; Le & Hua, 2021; Wang et al., 2021a) showed that finetuning LR has a great impact on the final performance. From this metric, we can see how sensitive different methods are to the finetuning LR. (2) We also compare the test accuracy *before finetuning* – from this metric, we will see how robust different methods are in the face of weight removal.

4.1 Analysis: MLP-7-Linear+MNIST and ResNet56+CIFAR10

MLP-7-Linear is a seven-layer linear MLP. It is adopted in Wang et al. (2021a) for analysis because linear MLP is the only network that can achieve *exact* dynamical isometry (all JSVs are exactly 1) so far. Their proposed dynamical isometry recovery method OrthP (Wang et al., 2021a) is shown

Table 1: Test accuracy (%) comparison among different dynamical isometry maintenance or recovery methods on ResNet56 + CIFAR10. "Scratch" stands for training from scratch. Each setting is randomly run for 3 times, mean (std) accuracies reported. "KernOrth" means Kernel Orthogonalization (Xie et al., 2017); "OrthConv" means Convolutional Orthogonalization (Wang et al., 2020). Two finetuning LR schedules are evaluated here: initial LR `1e-2` *vs.* `1e-3`. "Acc. diff." refers to the accuracy gap of LR `1e-3` against LR `1e-2`.

ResNet56 + CIFAR10: Baseline accuracy 93.78%, Params: 0.85M, FLOPs: 0.25G					
Pruning ratio r	0.3	0.5	0.7	0.9	0.95
Sparsity/Speedup	31.14%/1.45×	49.82%/1.99×	70.57%/3.59×	90.39%/11.41×	95.19%/19.31×
			`Initial finetuning LR 1e-2`		
Scratch	93.16 (0.16)	92.78 (0.23)	92.11 (0.12)	88.36 (0.20)	84.60 (0.14)
L_1 (Li et al., 2017)	93.79 (0.06)	**93.51** (0.07)	92.26 (0.17)	86.75 (0.31)	83.03 (0.07)
L_1 + OrthP (Wang et al., 2021a)	93.69 (0.02)	93.36 (0.19)	91.96 (0.06)	86.01 (0.34)	82.62 (0.05)
L_1 + KernOrth (Xie et al., 2017)	93.49 (0.04)	93.30 (0.19)	91.71 (0.14)	84.78 (0.34)	80.87 (0.47)
L_1 + OrthConv (Wang et al., 2020)	92.54 (0.09)	92.41 (0.07)	91.02 (0.16)	84.52 (0.27)	80.23 (1.19)
KernOrth (Xie et al., 2017) + L_1	93.49 (0.07)	92.82 (0.10)	90.54 (0.25)	85.47 (0.20)	79.48 (0.81)
OrthConv (Wang et al., 2020) + L_1	93.63 (0.17)	93.28 (0.20)	92.27 (0.13)	86.70 (0.07)	83.21 (0.61)
TPP (ours)	**93.81** (0.11)	93.46 (0.06)	**92.35** (0.12)	**89.63** (0.10)	**85.86** (0.08)
			`Initial finetuning LR 1e-3`		
L_1 (Li et al., 2017)	93.43 (0.06)	93.12 (0.10)	91.77 (0.11)	87.57 (0.09)	83.10 (0.12)
TPP (ours)	**93.54** (0.08)	**93.32** (0.11)	**92.00** (0.08)	**89.09** (0.10)	**85.47** (0.22)
Acc. diff. (L_1)	-0.38	-0.40	-0.50	+0.82	+0.07
Acc. diff. (TPP)	-0.27	-0.14	-0.35	-0.54	-0.39

to achieve exact isometry on linear MLP networks. Since we claim our method TPP can maintain dynamical isometry too, conceivably, our method should play a similar role to OrthP in pruning. To confirm this, we prune the MLP-7-Linear network with our method.

TPP can perform as well as OrthP on linear MLP. Results are presented in Fig. 3. Fig. 3(b) is the one equipped with OrthP, which can *exactly* recover dynamical isometry (note its mean JSV right after pruning is 1.0000), so it works as the oracle here. **(1)** OrthP improves the best accuracy from 91.36/90.54 to 92.79/92.77; using TPP, we obtain 92.81/92.77. Namely, in terms of accuracy, our method is as good as the oracle scheme. **(2)** Note the mean JSV right after pruning – the L_1-norm pruning destroys the mean JSV from 2.4987 to 0.0040, and OrthP brings it back to 1.0000. In comparison, TPP achieves 3.4875, at the same order of magnitude of 1.0000, also as good as OrthP. These demonstrate, in terms of either the final evaluation metric (test accuracy) or the trainability measure (mean JSV), our TPP performs as well as the oracle method OrthP on the linear MLP.

Loss surface analysis with ResNet56+CIFAR10. We further analyze the loss surfaces (Li et al., 2018) of pruned networks (before finetuning) by different methods. Our result (due to limited space, we defer this result to Appendix; see Fig. 4) suggests that the loss surface of our method is *flatter* than other methods, implying the loss landscape is *easier* for optimization.

4.2 RESNET56+CIFAR10 / VGG19+CIFAR100

Here we compare our method to other plausible solutions on the CIFAR10/100 datasets (Krizhevsky, 2009) with non-linear convolutional architectures. The results in Tab. 1 (for CIFAR10) and Tab. 10 (for CIFAR100, deferred to Appendix due to limited space here) show that,

(1) OrthP does not work well – L_1 + OrthP underperforms the original L_1 under all the five pruning ratios for both ResNet56 and VGG19. This further confirms the weight orthogonalization method proposed for linear networks indeed does not generalize to non-linear CNNs.

(2) For KernOrth *vs.* OrthConv, the results look mixed – OrthConv is better when applied *before* the L_1-norm pruning. This is reasonable since OrthConv is shown more effective than KernOrth in enforcing more isometry (Wang et al., 2020), which in turn can stand more damage of pruning.

(3) Of special note is that, none of the above five methods actually outperform the L_1-norm pruning or the simple scratch training. It means that neither enforcing more isometry before pruning nor compensating isometry after pruning can help dynamical isometry recovery. In stark contrast, our proposed TPP method outperforms L_1-norm pruning and scratch *consistently against different pruning ratio* (only one exception is pruning ratio 0.7 on ResNet56, but our method is still the second best and the gap to the best is only marginal: 93.46 *vs.* 93.51). Besides, note that the accuracy trend:

Table 2: Comparison on ImageNet. FLOPs: ResNet34: 3.66G, ResNet50: 4.09G. * implies advanced training recipe (such as cosine LR schedule) is used; we single them out for fair comparison.

Method	Network	Base top-1 (%)	Pruned top-1 (%)	Top-1 drop (%)	Speedup
L_1 (pruned-B) (Li et al., 2017)	ResNet34	73.23	72.17	1.06	**1.32**×
L_1 (pruned-B, reimpl.) (Wang et al., 2021a)		73.31	73.67	-0.36	**1.32**×
Taylor-FO (Molchanov et al., 2019)		73.31	72.83	0.48	1.29×
GReg-2 (Wang et al., 2021b)		73.31	73.61	-0.30	**1.32**×
TPP (ours)		73.31	**73.77**	**-0.46**	**1.32**×
ProvableFP (Liebenwein et al., 2020)	ResNet50	76.13	75.21	0.92	1.43×
MetaPruning (Liu et al., 2019a)		76.6	76.2	0.4	1.37×
GReg-1 (Wang et al., 2021b)		76.13	76.27	-0.14	**1.49**×
TPP (ours)		76.13	**76.44**	**-0.31**	**1.49**×
IncReg (Wang et al., 2019)	ResNet50	75.60	72.47	3.13	2.00×
SFP (He et al., 2018)		76.15	74.61	1.54	1.72×
HRank (Lin et al., 2020)		76.15	74.98	1.17	1.78×
Taylor-FO (Molchanov et al., 2019)		76.18	74.50	1.68	1.82×
Factorized (Li et al., 2019)		76.15	74.55	1.60	**2.33**×
DCP (Zhuang et al., 2018)		76.01	74.95	1.06	2.25×
CCP-AC (Peng et al., 2019)		76.15	75.32	0.83	2.18×
GReg-2 (Wang et al., 2021b)		76.13	75.36	0.77	2.31×
CC (Li et al., 2021)		76.15	75.59	0.56	2.12×
MetaPruning (Liu et al., 2019a)		76.6	75.4	1.2	2.00×
TPP (ours)		76.13	**75.60**	**0.53**	2.31×
LFPC (He et al., 2020)	ResNet50	76.15	74.46	1.69	2.55×
GReg-2 (Wang et al., 2021b)		76.13	74.93	1.20	2.56×
CC (Li et al., 2021)		76.15	74.54	1.61	**2.68**×
TPP (ours)		76.13	**75.12**	**1.01**	2.56×
IncReg (Wang et al., 2019)	ResNet50	75.60	71.07	4.53	3.00×
Taylor-FO (Molchanov et al., 2019)		76.18	71.69	4.49	3.05×
GReg-2 (Wang et al., 2021b)		76.13	73.90	2.23	**3.06**×
TPP (ours)		76.13	**74.51**	**1.62**	**3.06**×

Method	Network	Top-1 (%)	FLOPs (G)
CHEX* (Hou et al., 2022)	ResNet50	77.4	2
CHEX* (Hou et al., 2022)		76.0	1
TPP* (ours)		**77.75**	2
TPP* (ours)		**76.52**	1

in general, with a *larger* pruning ratio, the advantage of TPP over L_1 or Scratch is *more pronounced*. This is because a larger pruning ratio means the dynamical isometry is damaged more, where our method can help more, thus harvest more performance gains. We will see similar trends many times.

(4) In Tabs. 1, we also present the results when the initial finetuning LR is `1e-3`. In (Wang et al., 2021a), the authors argue that if the broken dynamical isometry can be well maintained/recovered, the final performance gap between LR `1e-2` and `1e-3` will be diminished. Since we claim our method is able to maintain dynamical isometry, the performance gap should become smaller. This is empirically verified in the table (note the Acc. diff. of TPP *vs.* that of L_1). In general, the accuracy gap between LR `1e-2` and LR `1e-3` of TPP is smaller than that of L_1-norm pruning. Two exceptions are PR 0.9/0.95 on ResNet56 – LR `1e-3` is unusually better than LR `1e-2` for L_1-norm pruning. For now, we do not have a very concrete explanation for this but notably, it does not appear on VGG19. Hence, it probably is related to the residual architecture. We will keep exploring this in the future. Despite that, the general picture from the table is that the accuracy gap between LR `1e-3` and `1e-2` turns smaller with TPP. This is a sign that dynamical isometry is effectively maintained.

4.3 IMAGENET BENCHMARK

We further evaluate TPP on ImageNet (Deng et al., 2009) in comparison to many existing filter pruning algorithms. Results are shown in Tab. 2. Our method is *consistently* better than the others across different speedup ratios. Moreover, with a larger speedup ratio, the advantage of our method is more pronounced. For example, TPP outperforms Taylor-FO (Molchanov et al., 2019) by 1.15% in terms of the top-1 accuracy drop at the 2.31×speedup track; at 3.06×speedup, TPP leads Taylor-FO (Molchanov et al., 2019) by 2.87%. This shows *TPP is more robust to more aggressive pruning*. The reason is easy to see – more aggressive pruning hurts trainability (or dynamical isometry) more (Lee et al., 2019; Wang et al., 2021a), where our method can find more use.

Table 3: Test accuracy (without finetuning) comparison between two plausible schemes "diagonal" *vs.* "decorrelate" in our TPP method.

ResNet56 + CIFAR10: Baseline accuracy 93.78%, Params: 0.85M, FLOPs: 0.25G					
Pruning ratio r	0.3	0.5	0.7	0.9	0.95
TPP (diagonal)	92.67 (0.29)	91.97 (0.02)	**90.21** (0.23)	23.23 (5.19)	14.23 (1.42)
TPP (decorrelate)	**92.74** (0.16)	**92.07** (0.05)	89.95 (0.26)	**30.35** (4.69)	**17.33** (0.50)
Acc. diff.	+0.07	+0.10	-0.26	+7.12	+3.10
VGG19 + CIFAR100: Baseline accuracy 74.02%, Params: 20.08M, FLOPs: 0.80G					
Pruning ratio r	0.1	0.3	0.5	0.7	0.9
TPP (diagonal)	68.70 (0.18)	64.55 (0.14)	55.66 (0.73)	13.76 (0.53)	1.00 (0.00)
TPP (decorrelate)	**72.43** (0.12)	**69.31** (0.11)	**62.59** (0.14)	**18.97** (1.25)	1.00 (0.00)
Acc. diff.	+3.73	+4.76	+6.93	+5.21	+0.00

Table 4: Test accuracy (w/o finetuning) comparison w/ or w/o the proposed BN regularization.

ResNet56 + CIFAR10: Baseline accuracy 93.78%, Params: 0.85M, FLOPs: 0.25G					
Pruning ratio r	0.3	0.5	0.7	0.9	0.95
TPP (w/o BN reg)	92.79 (0.03)	92.23 (0.08)	90.46 (0.21)	44.25 (2.46)	16.52 (0.43)
TPP (w/ BN reg)	**92.94** (0.14)	**92.48** (0.19)	**90.48** (0.09)	**70.53** (1.69)	**23.05** (2.61)
Acc. diff.	+0.15	+0.25	+0.02	+26.28	+6.53
VGG19 + CIFAR100: Baseline accuracy 74.02%, Params: 20.08M, FLOPs: 0.80G					
Pruning ratio r	0.1	0.3	0.5	0.7	0.9
TPP (w/o BN reg)	73.01 (0.13)	71.26 (0.19)	68.67 (0.10)	61.70 (0.46)	1.75 (0.38)
TPP (w/ BN reg)	**73.44** (0.07)	**71.61** (0.12)	**69.28** (0.25)	**65.15** (0.20)	**2.84** (1.13)
Acc. diff.	+0.43	+0.35	+0.51	+3.45	+1.09

We further compare to more strong pruning methods. Notably, DMCP (Guo et al., 2020), LeGR (Chin et al., 2020), EagleEye (Li et al., 2020), and CafeNet (Su et al., 2021) have been shown *outperformed* by CHEX (Hou et al., 2022) (see their Tab. 1) with ResNet50 on ImageNet. Therefore, here we only compare to CHEX. Following CHEX, we employ more advanced training recipe (*e.g.*, cosine LR schedule) referring to TIMM (Wightman et al., 2021). Results in Tab. 2 show that our method *surpasses* CHEX at different FLOPs.

4.4 Ablation Study

Finally, we conduct ablation studies to demonstrate the merits of two key contributions of our method: 1) proposing not to over-penalize the kept weights in orthogonalization (*i.e.*, (c) *vs.* (b) in Fig. 2), 2) proposing to regularize the two learnable parameters in BN.

The results are presented in Tabs. 3 and 4, where we compare the accuracy right after pruning (*i.e.*, without finetuning). We have the following major observations: **(1)** Tab. 3 shows using "decorrelate" (Fig. 2(c)) is better than "diagonal" (Fig. 2(b)) on the whole. Akin to Tabs. 1 and 10, under a larger pruning ratio, the advantage of "decorrelate" is more evident, except for too large sparsity (0.95 for ResNet56, 0.9 for VGG19) because too large sparsity will break the isometry *beyond* repair. **(2)** For BN regularization, in Tab. 4, when it is switched off, the performance degrades. It also poses the similar trend: BN regularization is *more helpful* under the *larger sparsity*.

5 Conclusion

Trainability maintenance is shown critical in neural network structured pruning, while few works have realized it on the modern large-scale non-linear deep networks. To achieve this goal, we present a new filter pruning method named *trainability preserving pruning* (TPP) based on regularization. Specifically, we propose a modified weight gram matrix as regularization target which does not unnecessarily over-penalize the weights. Besides, we propose to regularize the BN parameters to mitigate its damage to trainability. TPP performs as effectively as the ground-truth trainability recovery method and more effective than other counterpart approaches based on weight orthogonality. Furthermore, TPP also delivers promising performance when compared to many recent SOTA filter pruning approaches. To our best knowledge, this is the *first* work that explicitly tackles the trainability preserving problem in structured pruning that scales to the large-scale ImageNet dataset.

REFERENCES

Nitin Bansal, Xiaohan Chen, and Zhangyang Wang. Can we gain more from orthogonality regularizations in training deep networks? In *NeurIPS*, 2018. 3

Jian Cheng, Pei-song Wang, Gang Li, Qing-hao Hu, and Han-qing Lu. Recent advances in efficient computation of deep convolutional neural networks. *Frontiers of Information Technology & Electronic Engineering*, 19(1):64–77, 2018a. 3

Yu Cheng, Duo Wang, Pan Zhou, and Tao Zhang. Model compression and acceleration for deep neural networks: The principles, progress, and challenges. *IEEE Signal Processing Magazine*, 35 (1):126–136, 2018b. 3

Ting-Wu Chin, Ruizhou Ding, Cha Zhang, and Diana Marculescu. Towards efficient model compression via learned global ranking. In *CVPR*, 2020. 9

Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. In *CVPR*, 2009. 1, 6, 8

Lei Deng, Guoqi Li, Song Han, Luping Shi, and Yuan Xie. Model compression and hardware acceleration for neural networks: A comprehensive survey. *Proceedings of the IEEE*, 108(4): 485–532, 2020. 3

Xiaohan Ding, Guiguang Ding, Jungong Han, and Sheng Tang. Auto-balanced filter pruning for efficient convolutional neural networks. In *AAAI*, 2018. 3

Shaopeng Guo, Yujie Wang, Quanquan Li, and Junjie Yan. Dmcp: Differentiable markov channel pruning for neural networks. In *CVPR*, 2020. 9

S. Han, X. Liu, and H. Mao. EIE: efficient inference engine on compressed deep neural network. *ACM Sigarch Computer Architecture News*, 44(3):243–254, 2016a. 1, 3

Song Han, Jeff Pool, John Tran, and William J Dally. Learning both weights and connections for efficient neural network. In *NeurIPS*, 2015. 1, 3

Song Han, Huizi Mao, and William J Dally. Deep compression: Compressing deep neural networks with pruning, trained quantization and huffman coding. In *ICLR*, 2016b. 1, 3

Song Han, Junlong Kang, Huizi Mao, Yiming Hu, Xin Li, Yubin Li, Dongliang Xie, Hong Luo, Song Yao, Yu Wang, et al. ESE: Efficient speech recognition engine with sparse lstm on fpga. In *ACM/SIGDA International Symposium on Field-Programmable Gate Arrays*, 2017. 1, 3

B. Hassibi and D. G. Stork. Second order derivatives for network pruning: Optimal brain surgeon. In *NeurIPS*, 1993. 3

K. He, X. Zhang, S. Ren, and J. Sun. Deep residual learning for image recognition. In *CVPR*, 2016. 3, 4, 6

Yang He, Guoliang Kang, Xuanyi Dong, Yanwei Fu, and Yi Yang. Soft filter pruning for accelerating deep convolutional neural networks. In *IJCAI*, 2018. 3, 8

Yang He, Yuhang Ding, Ping Liu, Linchao Zhu, Hanwang Zhang, and Yi Yang. Learning filter pruning criteria for deep convolutional neural networks acceleration. In *CVPR*, 2020. 8

Yihui He, Xiangyu Zhang, and Jian Sun. Channel pruning for accelerating very deep neural networks. In *ICCV*, 2017. 3

Zejiang Hou, Minghai Qin, Fei Sun, Xiaolong Ma, Kun Yuan, Yi Xu, Yen-Kuang Chen, Rong Jin, Yuan Xie, and Sun-Yuan Kung. Chex: Channel exploration for cnn model compression. In *CVPR*, 2022. 8, 9

Lei Huang, Xianglong Liu, Bo Lang, Adams Yu, Yongliang Wang, and Bo Li. Orthogonal weight normalization: Solution to optimization over multiple dependent stiefel manifolds in deep neural networks. In *AAAI*, 2018. 3, 4, 6

Lei Huang, Li Liu, Fan Zhu, Diwen Wan, Zehuan Yuan, Bo Li, and Ling Shao. Controllable orthogonalization in training dnns. In *CVPR*, 2020. 3, 4, 6

Sergey Ioffe and Christian Szegedy. Batch normalization: Accelerating deep network training by reducing internal covariate shift. In *ICML*, 2015. 2, 4, 5

Alex Krizhevsky. Learning multiple layers of features from tiny images. Technical report, Citeseer, 2009. 6, 7

Duong H Le and Binh-Son Hua. Network pruning that matters: A case study on retraining variants. In *ICLR*, 2021. 1, 6

V. Lebedev and V. Lempitsky. Fast convnets using group-wise brain damage. In *CVPR*, 2016. 3

Y. LeCun, J. S. Denker, and S. A. Solla. Optimal brain damage. In *NeurIPS*, 1990. 3

Yann LeCun, Léon Bottou, Yoshua Bengio, Patrick Haffner, et al. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11):2278–2324, 1998. 6

Namhoon Lee, Thalaiyasingam Ajanthan, and Philip Torr. Snip: Single-shot network pruning based on connection sensitivity. In *ICLR*, 2019. 8

Namhoon Lee, Thalaiyasingam Ajanthan, Stephen Gould, and Philip HS Torr. A signal propagation perspective for pruning neural networks at initialization. In *ICLR*, 2020. 1, 3, 4

Bailin Li, Bowen Wu, Jiang Su, and Guangrun Wang. Eagleeye: Fast sub-net evaluation for efficient neural network pruning. In *ECCV*, 2020. 9

Hao Li, Asim Kadav, Igor Durdanovic, Hanan Samet, and Hans Peter Graf. Pruning filters for efficient convnets. In *ICLR*, 2017. 3, 6, 7, 8, 14, 15, 16

Hao Li, Zheng Xu, Gavin Taylor, Christoph Studer, and Tom Goldstein. Visualizing the loss landscape of neural nets. In *NeurIPS*, 2018. 7

Tuanhui Li, Baoyuan Wu, Yujiu Yang, Yanbo Fan, Yong Zhang, and Wei Liu. Compressing convolutional neural networks via factorized convolutional filters. In *CVPR*, 2019. 8

Yuchao Li, Shaohui Lin, Jianzhuang Liu, Qixiang Ye, Mengdi Wang, Fei Chao, Fan Yang, Jincheng Ma, Qi Tian, and Rongrong Ji. Towards compact cnns via collaborative compression. In *CVPR*, 2021. 8

Lucas Liebenwein, Cenk Baykal, Harry Lang, Dan Feldman, and Daniela Rus. Provable filter pruning for efficient neural networks. In *ICLR*, 2020. 8

Mingbao Lin, Rongrong Ji, Yan Wang, Yichen Zhang, Baochang Zhang, Yonghong Tian, and Ling Shao. Hrank: Filter pruning using high-rank feature map. In *CVPR*, 2020. 8

Zechun Liu, Haoyuan Mu, Xiangyu Zhang, Zichao Guo, Xin Yang, Kwang-Ting Cheng, and Jian Sun. Metapruning: Meta learning for automatic neural network channel pruning. In *ICCV*, 2019a. 8

Zhuang Liu, Jianguo Li, Zhiqiang Shen, Gao Huang, Shoumeng Yan, and Changshui Zhang. Learning efficient convolutional networks through network slimming. In *ICCV*, 2017. 2, 3, 5

Zhuang Liu, Mingjie Sun, Tinghui Zhou, Gao Huang, and Trevor Darrell. Rethinking the value of network pruning. In *ICLR*, 2019b. 13, 14

Christos Louizos, Max Welling, and Diederik P Kingma. Learning sparse neural networks through l_0 regularization. In *ICLR*, 2018. 3

Ekdeep Singh Lubana and Robert P Dick. A gradient flow framework for analyzing network pruning. In *ICLR*, 2021. 1, 3, 4

Francesco Mezzadri. How to generate random matrices from the classical compact groups. *arXiv preprint math-ph/0609050*, 2006. 1, 3

P. Molchanov, S. Tyree, and T. Karras. Pruning convolutional neural networks for resource efficient inference. In *ICLR*, 2017. 3

Pavlo Molchanov, Arun Mallya, Stephen Tyree, Iuri Frosio, and Jan Kautz. Importance estimation for neural network pruning. In *CVPR*, 2019. 3, 8

Vinod Nair and Geoffrey E Hinton. Rectified linear units improve restricted boltzmann machines. In *ICML*, 2010. 4

Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, et al. Pytorch: An imperative style, high-performance deep learning library. In *NeurIPS*, 2019. 6, 13

Hanyu Peng, Jiaxiang Wu, Shifeng Chen, and Junzhou Huang. Collaborative channel pruning for deep networks. In *ICML*, 2019. 8

R. Reed. Pruning algorithms – a survey. *IEEE Transactions on Neural Networks*, 4(5):740–747, 1993. 1, 3

Alex Renda, Jonathan Frankle, and Michael Carbin. Comparing rewinding and fine-tuning in neural network pruning. In *ICLR*, 2020. 1, 6

Andrew M Saxe, James L McClelland, and Surya Ganguli. Exact solutions to the nonlinear dynamics of learning in deep linear neural networks. In *ICLR*, 2014. 1, 3, 4

Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. In *ICLR*, 2015. 6

Sidak Pal Singh and Dan Alistarh. Woodfisher: Efficient second-order approximations for model compression. In *NeurIPS*, 2020. 3

Xiu Su, Shan You, Tao Huang, Fei Wang, Chen Qian, Changshui Zhang, and Chang Xu. Locally free weight sharing for network width search. In *ICLR*, 2021. 9

Vivienne Sze, Yu-Hsin Chen, Tien-Ju Yang, and Joel S Emer. Efficient processing of deep neural networks: A tutorial and survey. *Proceedings of the IEEE*, 105(12):2295–2329, 2017. 3

Lloyd N Trefethen and David Bau III. *Numerical linear algebra*, volume 50. Siam, 1997. 1, 3

Artem Vysogorets and Julia Kempe. Connectivity matters: Neural network pruning through the lens of effective sparsity. *arXiv preprint arXiv:2107.02306*, 2021. 3

Huan Wang, Xinyi Hu, Qiming Zhang, Yuehai Wang, Lu Yu, and Haoji Hu. Structured pruning for efficient convolutional neural networks via incremental regularization. *IEEE Journal of Selected Topics in Signal Processing (JSTSP)*, 14(4):775–788, 2019. 3, 5, 8

Huan Wang, Can Qin, Yue Bai, and Yun Fu. Dynamical isometry: The missing ingredient for neural network pruning. *arXiv preprint arXiv:2021.05916*, 2021a. 1, 4, 6, 7, 8, 16

Huan Wang, Can Qin, Yulun Zhang, and Yun Fu. Neural pruning via growing regularization. In *ICLR*, 2021b. 3, 5, 8, 13, 14, 15

Huan Wang, Can Qin, Yulun Zhang, and Yun Fu. Recent advances on neural network pruning at initialization. In *IJCAI*, 2022. 3

Jiayun Wang, Yubei Chen, Rudrasis Chakraborty, and Stella X Yu. Orthogonal convolutional neural networks. In *CVPR*, 2020. 3, 4, 6, 7, 13, 16

Wei Wen, Chunpeng Wu, Yandan Wang, Yiran Chen, and Hai Li. Learning structured sparsity in deep neural networks. In *NeurIPS*, 2016. 3

Ross Wightman, Hugo Touvron, and Hervé Jégou. Resnet strikes back: An improved training procedure in timm. *arXiv preprint arXiv:2110.00476*, 2021. 9

Di Xie, Jiang Xiong, and Shiliang Pu. All you need is beyond a good init: Exploring better solution for training extremely deep convolutional neural networks with orthonormality and modulation. In *CVPR*, 2017. 3, 4, 6, 7, 16

Jianbo Ye, Xin Lu, Zhe Lin, and James Z Wang. Rethinking the smaller-norm-less-informative assumption in channel pruning of convolution layers. In *ICLR*, 2018. 2, 3, 5

Yulun Zhang*, Huan Wang*, Can Qin, and Yun Fu. Aligned structured sparsity learning for efficient image super-resolution. In *NeurIPS*, 2021. 3

Zhenyu Zhang, Xuxi Chen, Tianlong Chen, and Zhangyang Wang. Efficient lottery ticket finding: Less data is more. In *ICML*, 2021. 3

Zhuangwei Zhuang, Mingkui Tan, Bohan Zhuang, Jing Liu, Yong Guo, Qingyao Wu, Junzhou Huang, and Jinhui Zhu. Discrimination-aware channel pruning for deep neural networks. In *NeurIPS*, 2018. 8

A Appendix

A.1 Implementation Details

Code reference. We mainly refer to the following code implementations in this work. They are all open-licensed.

- Official PyTorch ImageNet example[1];
- GReg-1/GReg-2 (Wang et al., 2021b)[2];
- OrthConv (Wang et al., 2020)[3];
- Rethinking the value of network pruning (Liu et al., 2019b)[4].

Data split. All the datasets in this paper are public datasets with standard APIs in PyTorch (Paszke et al., 2019). We employs these standard APIs for the train/test data split to keep fair comparison with other methods.

Training settings. The specific training settings are summarized in Tab. 5. For the hyper-parameters that are introduced in our TPP method: regularization granularity Δ, regularization ceiling τ and regularization update interval K_u, we summarize them in Tab. 6. These hyper-parameters mainly refer to the official implementation of GReg-1 (Wang et al., 2021b) since we tap into a similar growing regularization scheme as it does.

Table 5: Training setting summary. For the SGD solver, in the parentheses are the momentum and weight decay. For LR schedule, the first number is initial LR; the second (in brackets) is the epochs when LR is decayed by factor 1/10; and #epochs stands for the total number of epochs.

Dataset	MNIST	CIFAR10/100	ImageNet
Solver	SGD (0.9, 1e-4)	SGD (0.9, 5e-4)	SGD (0.9, 1e-4)
Batch size	100	256	
LR schedule (scratch)	1e-2, [30,60], #epochs:90	1e-1, [100,150], #epochs:200	1e-1, [30,60], #epochs:90
LR schedule (prune)	Fixed (1e-3)		
LR schedule (finetune)	1e-2,[30,60], #epochs:90	1e-2, [60,90], #epochs:120	1e-2, [30,60,75], #epochs:90

Hardware and running time. We conduct all our experiments using 4 NVIDIA V100 GPUs (16GB memory per GPU). It takes roughly 41 hrs to prune ResNet50 on ImageNet using our TPP method (pruning and 90-epoch finetuning both included). Among them, 12 hrs (namely, close to 30%) are spent on pruning and 29 hrs are spent on finetuning (about 20 mins per epoch).

[1]https://github.com/pytorch/examples/tree/master/imagenet
[2]https://github.com/MingSun-Tse/Regularization-Pruning
[3]https://github.com/samaonline/Orthogonal-Convolutional-Neural-Networks
[4]https://github.com/Eric-mingjie/rethinking-network-pruning/tree/master/imagenet/l1-norm-pruning

Table 6: Hyper-parameters of our methods.

Dataset	MNIST	CIFAR10/100	ImageNet
Regularization granularity Δ	1e-4		
Regularization ceiling τ	1		
Regularization update interval K_u	10 iterations		5 iterations

Layer-wise pruning ratios. The layer-wise pruning ratios are pre-specified in this paper. For the ImageNet benchmark, we *exactly* follow GReg-1/GReg-2 (Wang et al., 2021b) for the layer-wise pruning ratios to keep fair comparison to it. The specific numbers are summarized in Tab. 7. Each number is the pruning ratio shared by all the layers of a stage in ResNet34/50.

Table 7: Summary of layer-wise pruning ratios.

Dataset	Network	Speedup	Pruned top-1 accuracy (%)	Pruning ratio
ImageNet	ResNet34	1.32×	73.77	[0, 0.50, 0.60, 0.40, 0, 0]*
ImageNet	ResNet50	1.49×	76.44	[0, 0.30, 0.30, 0.30, 0.14, 0]
ImageNet	ResNet50	2.31×	75.60	[0, 0.60, 0.60, 0.60, 0.21, 0]
ImageNet	ResNet50	2.56×	75.12	[0, 0.74, 0.74, 0.60, 0.21, 0]
ImageNet	ResNet50	3.06×	74.51	[0, 0.68, 0.68, 0.68, 0.50, 0]

* As Wang et al. (2021b) does, in addition to the pruning ratios, we skip several layers, following the setting of L_1 (pruned-B) (Li et al., 2017). Concretely, we refer to the implementation of Liu et al. (2019b) at https://github.com/Eric-mingjie/rethinking-network-pruning/tree/master/imagenet/l1-norm-pruning.

A.2 Sensitivity Analysis of Hyper-parameters

Among the three hyper-parameters in Tab. 6, regularization ceiling τ works as a termination condition. We only requires it to be large enough to ensure the weights are compressed to a very small amount. It does not have to be 1. The final performance is also less sensitive to it. The pruned performance seems to be more sensitive to the other two hyper-parameters, so here we conduct hyper-parameter sensitivity analysis to check their robustness.

Results are presented in Tab. 8 and Tab. 9. Pruning ratio 0.7 (for ResNet56) and 0.5 (for VGG19) are chosen here because the resulted sparsity is the most representative (*i.e.*, not too large or small). (1) For K_u, in general, a larger K_u tends to deliver a better result. This is no surprise since a larger K_u implies more time for the network to adapt and recover when undergoing the penalty. (2) For Δ, we do not see a clear pattern here: either a small or large Δ can achieve the best result (for different networks). On the whole, when varying the hyper-parameters within a reasonable range, the performance is pretty robust, no catastrophic failures. Moreover, note that the default setting is actually not the best for both K_u and Δ. This is because we did *not* hard-tune our hyper-parameters, however, they still achieve encouraging performance compared to the counterpart methods, as we have shown in the main text.

Table 8: Sensitivity analysis of K_u on CIFAR10/100 datasets with the proposed TPP algorithm. $K_u = 10$ is the default setting. Pruning ratio 0.7 (ResNet56) and 0.5 (VGG19) are explored here. Each setting is randomly run for 3 times, mean and standard deviation accuracy reported. **The best** is highlighted with **bold** and the worst is highlighted with blue color.

K_u	1	5	10	15	20
Acc. (%, ResNet56)	$92.24_{\pm 0.03}$	$92.42_{\pm 0.14}$	$92.35_{\pm 0.12}$	$\mathbf{92.50_{\pm 0.10}}$	$92.31_{\pm 0.16}$
Acc. (%, VGG19)	$71.33_{\pm 0.06}$	$71.45_{\pm 0.21}$	$71.61_{\pm 0.08}$	$71.43_{\pm 0.21}$	$\mathbf{71.68_{\pm 0.18}}$

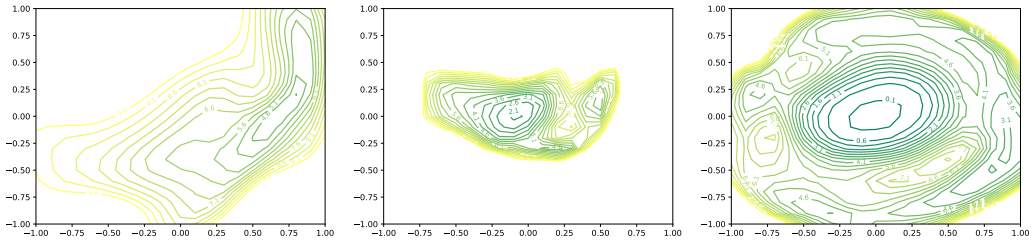
A.3 More Results

Loss surface visualization. The loss surface visualization of ResNet56 on CIFAR10 is presented in Fig. 4.

VGG19 + CIFAR100. The results of VGG19 on CIFAR100 is shown in Tab. 10.

Table 9: Sensitivity analysis of Δ on CIFAR10/100 datasets with the proposed TPP algorithm. $\Delta = 1e-4$ is the default setting. Pruning ratio 0.7 (ResNet56) and 0.5 (VGG19) are explored here. Each setting is randomly run for 3 times, mean and standard deviation accuracy reported. **The best** is highlighted with **bold** and the worst is highlighted with blue color.

Δ	1e-5	5e-5	1e-4	5e-4	1e-3
Acc. (%, ResNet56)	$92.37_{\pm 0.12}$	$92.29_{\pm 0.10}$	$92.35_{\pm 0.12}$	$92.40_{\pm 0.15}$	$\mathbf{92.44_{\pm 0.13}}$
Acc. (%, VGG19)	$71.39_{\pm 0.19}$	$71.37_{\pm 0.14}$	$\mathbf{71.61_{\pm 0.08}}$	$71.58_{\pm 0.31}$	$71.25_{\pm 0.31}$



(a) L_1 (Li et al., 2017) (b) GReg-2 (Wang et al., 2021b) (c) TPP (ours)

Figure 4: Loss surface visualization of pruned models by different methods (w/o finetuning). ResNet56 on CIFAR10. Pruning ratio: 0.9 (*zoom in to examine the details*).

Examination of the early finetuning phase. To further understand how pruning hurts trainability and how our TPP method maintains it, in Tab. 11, we list the mean JSVs of the first 10-epoch finetuning (at pruning ratio 0.9). Note the obvious mean JSV gap between LR 0.01 and LR 0.001 *without OrthP*: LR 0.01 can reach mean JSV 0.65 just after 1 epoch of finetuning while LR 0.001 takes over 8 epochs. When OrthP is used, this gap greatly shrinks. We also list the test accuracy of the first 10-epoch finetuning in Tab. 12. Particularly note that the *test accuracy correlates well with the mean JSV trend* under each setting, implying that the damaged trainability primarily answers for the under-performance of LR 0.001 after pruning.

Then, when TPP is used in place of OrthP, we can see after 1-epoch finetuning, the model can achieve mean JSV above 1 and test accuracy over 90%, which are particularly similar to the effect of using OrthP. These re-iterate that the proposed TPP method can work just as effectively as the ground-truth trainability recovery method OrthP on this toy setup.

A.4 Algorithm Details

The details of our TPP method is summarized in Algorithm 1.

Algorithm 1 Trainability Preserving Pruning (TPP)

Input: Pretrained model $\mathbf{\Theta}$, layer-wise pruning ratio r_l of l-th layer, for $l \in \{1, 2, \cdots, L\}$.
Input: Regularization ceiling τ, penalty coefficient update interval K_u, penalty granularity Δ.
Init: Iteration $i = 0$. $\lambda_j = 0$ for all filter j. Set pruned filter indices S_l by L_1-norm sorting.
while $\lambda_j \leq \tau$, for $j \in S_l$ **do**
    if $i \,\%\, K_u \;= 0$ **then**
        $\lambda_j = \lambda_j + \Delta$ for $j \in S_l$. ◁ Update regularization co-efficient in Eq. (6)
    end if
    Network forward, loss (Eq. (6)) backward, parameter update by stochastic gradient descent.
    Update iteration: $i = i + 1$.
end while
Remove filters in S_l to obtain a smaller model $\mathbf{\Theta}'$.
Finetune $\mathbf{\Theta}'$ to regain accuracy.
Output: Finetuned model $\mathbf{\Theta}'$.

Table 10: Test accuracy (%) comparison among different dynamical isometry maintenance or recovery methods on VGG19 + CIFAR100. "Scratch" stands for training from scratch. Each setting is randomly run for 3 times, mean (std) accuracies reported. "KernOrth" means Kernel Orthogonalization (Xie et al., 2017); "OrthConv" means Convolutional Orthogonalization (Wang et al., 2020). Two finetuning LR schedules are evaluated here: initial LR `1e-2` *vs.* `1e-3`. "Acc. diff." refers to the accuracy gap of LR `1e-3` against LR `1e-2`.

VGG19 + CIFAR100: Baseline accuracy 74.02%, Params: 20.08M, FLOPs: 0.80G					
Pruning ratio r	0.1	0.3	0.5	0.7	0.9
Sparsity/Speedup	19.24%/1.23×	51.01%/1.97×	74.87%/3.60×	90.98%/8.84×	98.96%/44.22×
			`Initial finetuning LR 1e-2`		
Scratch	72.84 (0.25)	71.88 (0.14)	70.79 (0.08)	66.51 (0.11)	54.37 (0.40)
L_1 (Li et al., 2017)	74.01 (0.18)	73.01 (0.22)	71.49 (0.14)	66.05 (0.04)	51.36 (0.11)
L_1 + OrthP (Wang et al., 2021a)	74.00 (0.04)	72.30 (0.49)	68.09 (0.24)	62.22 (0.15)	48.07 (0.31)
L_1 + KernOrth (Xie et al., 2017)	73.72 (0.26)	72.53 (0.09)	71.23 (0.10)	65.90 (0.14)	50.75 (0.30)
L_1 + OrthConv (Wang et al., 2020)	73.18 (0.10)	72.25 (0.31)	70.82 (0.11)	64.51 (0.43)	48.31 (0.18)
KernOrth (Xie et al., 2017) + L_1	73.73 (0.23)	72.41 (0.12)	70.31 (0.12)	64.10 (0.19)	50.72 (0.87)
OrthConv (Wang et al., 2020) + L_1	73.55 (0.18)	72.67 (0.09)	71.24 (0.23)	65.66 (0.10)	50.53 (0.46)
TPP (ours)	**74.02** (0.24)	**73.19** (0.07)	**71.61** (0.08)	**67.78** (0.31)	**57.70** (0.37)
			`Initial finetuning LR 1e-3`		
L_1 (Li et al., 2017)	73.67 (0.05)	72.04 (0.12)	70.21 (0.02)	64.72 (0.17)	48.43 (0.44)
TPP (ours)	**73.83** (0.02)	**72.29** (0.07)	**71.16** (0.12)	**67.47** (0.17)	**56.73** (0.34)
Acc. diff. (L_1)	-0.34	-0.97	-1.28	-1.33	-2.93
Acc. diff. (TPP)	-0.19	-0.90	-0.45	-0.31	-0.97

Table 11: Mean JSV of the first 10 epochs under different finetuning settings. Epoch 0 refers to the model just pruned, before any finetuning. Pruning ratio is 0.9. Note, with OrthP, the mean JSV is 1 because OrthP can achieve exact isometry

Epoch	0	1	2	3	4	5	6	7	8	9	10
LR=10^{-2}, w/o OrthP	0.00	0.65	0.89	1.01	0.97	1.10	1.14	1.27	1.33	1.29	1.42
LR=10^{-3}, w/o OrthP	0.00	0.00	0.00	0.00	0.10	0.25	0.35	0.42	0.51	0.74	0.86
LR=10^{-2}, w/ OrthP	1.00	1.23	1.42	1.43	1.40	1.47	1.51	1.56	1.60	1.65	1.68
LR=10^{-3}, w/ OrthP	1.00	1.50	1.64	1.73	1.84	1.87	1.93	1.96	1.99	2.00	2.04
LR=10^{-2}, w/ TPP (ours)	2.98	2.15	2.01	1.67	1.97	2.10	1.97	2.08	2.05	2.06	2.06
LR=10^{-3}, w/ TPP (ours)	2.98	2.96	2.95	2.99	2.99	3.01	3.21	3.19	3.05	3.04	3.02

Table 12: Test accuracy (%) of the first 10 epochs *corresponding to Tab. 11* under different finetuning settings. Epoch 0 refers to the model just pruned, before any finetuning. Pruning ratio is 0.9

Epoch	0	1	2	3	4	5	6	7	8	9	10
LR=10^{-2}, w/o OrthP	9.74	64.34	80.01	80.23	81.77	85.80	85.82	86.21	86.35	86.60	86.15
LR=10^{-3}, w/o OrthP	9.74	9.74	9.74	11.89	21.34	27.75	32.96	35.38	49.66	64.89	68.97
LR=10^{-2}, w/ OrthP	9.74	91.05	91.39	91.33	91.37	91.74	91.69	90.74	91.39	91.58	91.44
LR=10^{-3}, w/ OrthP	9.74	90.81	91.59	91.77	91.85	92.04	92.12	92.22	92.12	92.33	92.25
LR=10^{-2}, w/ TPP (ours)	89.21	91.54	91.01	91.45	91.83	91.56	90.89	91.33	90.68	91.54	91.21
LR=10^{-3}, w/ TPP (ours)	89.21	92.12	91.82	92.09	92.15	91.95	92.00	92.02	92.09	92.08	92.08