# CHARACTERIZING ADVERSARIAL SUBSPACES USING LOCAL INTRINSIC DIMENSIONALITY

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## ABSTRACT

Deep Neural Networks (DNNs) have recently been shown to be vulnerable against adversarial examples, which are carefully crafted instances that can mislead DNNs to make errors during prediction. To better understand such attacks, the properties of subspaces in the neighborhood of adversarial examples need to be characterized. In particular, effective measures are required to discriminate adversarial examples from normal examples in such subspaces. We tackle this challenge by characterizing the expansion dimensionality property of adversarial subspaces, via the use of *Local Intrinsic Dimensionality* (LID). LID assesses the space-filling capability of the subspace surrounding a reference example, based on the distance distribution of the example to its neighbors. We first provide explanations about how adversarial perturbation affects the LID characteristic of adversarial subspaces. Then, we explain how the LID characteristic can be used to discriminate adversarial examples generated using the state-of-the-art attacks. We empirically show that an LID based method can outperform several state-of-the-art detection measures by large margins for five attacks across three benchmark datasets. This suggests that the LID can be a highly effective tool for understanding properties of adversarial subspaces.

## 1 INTRODUCTION

Deep Neural Networks (DNNs) are highly expressive models that have achieved state-of-the-art performance on a wide range of complex problems, such as speech recognition (Hinton et al., 2012) and image classification (Krizhevsky et al., 2012). However, recent studies have found that DNNs can be fooled by adversarial examples (Szegedy et al., 2013; Goodfellow et al., 2014; Nguyen et al., 2015). These are intentionally-perturbed inputs that can fool the network into making incorrect predictions at test time with high confidence, and they are transferable across different networks (Liu et al., 2016; Carlini & Wagner, 2017b; Papernot et al., 2016b). The amount of perturbation required is often small and imperceptible to human observers in the case of images. This undesirable property of deep networks has become a major security concern for applying DNNs to real-world applications such as self-driving cars and identity recognition (Evtimov et al., 2017; Sharif et al., 2016). In this paper, we aim to further understand adversarial attacks by characterizing adversarial subspaces.

An adversarial subspace is the region immediately surrounding an adversarial example. It exists not just in the input space, but also in the activation space of different layers (Szegedy et al., 2013). Developing an understanding of the properties of adversarial subspaces is a key requirement for adversarial defense. Several works have attempted to characterize the properties of adversarial subspaces, but no definitive method yet exists which can discriminate adversarial subspaces from normal data subspaces. Szegedy et al. (2013) argued that adversarial subspaces have low probability (are not naturally occurring), but are densely scattered in the high dimensional representation space of DNNs. However, a linear formulation argues that adversarial subspaces span a multidimensional contiguous space, rather than being scattered randomly in small pockets (Goodfellow et al., 2014; Warde-Farley et al., 2016). Tanay & Griffin (2016) further highlight that adversarial subspaces lie close, yet off the data submanifold. It has also been found that the boundaries of adversarial subspaces are similarly close to legitimate data points in adversarial directions, and the higher the number of orthogonal adversarial directions of these subspaces, the more transferable they are to other models (Tramèr et al., 2017). Generally, we can summarize the known properties of adversarial subspaces as: (1)

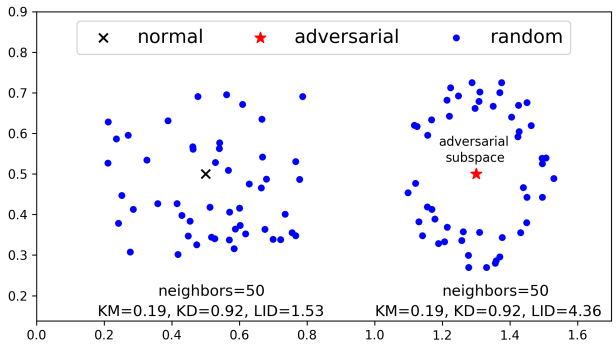


Figure 1: This example shows when density measures fail to characterize the spatial property of adversarial subspace. Gaussian kernel with bandwidth 0.2 is used for KD.

they are of low probability, (2) they span a contiguous multidimensional space, (3) they lie off but are close to the data submanifold, and (4) they have different class distributions from their closest data submanifold.

Some properties of adversarial examples have been analyzed and applied to build up detection methods. For instance, *Kernel Density* (KD) has been proposed as a measure to discriminate adversarial subspaces (Feinman et al., 2017). Carlini & Wagner (2017a) empirically showed that a kernel density based detector is useful, since adversarial subspaces generally have lower density than normal data subspaces, due to their low probability. In addition to kernel density, there are other density type measures, such as number of nearest neighbors and mean distance to $k$ nearest neighbours ($k$-mean distance). However, these have some limitations for characterizing local adversarial subspaces. For example, in Figure 1, the three density based measures fail to differentiate an adversarial example (red star) from a normal example (black cross), as the two examples are locally surrounded by the same number of neighbors (50) and have the same k-mean distance (KM=0.19), as well as the same kernel density (KD=0.92).

Figure 1 leads us to alternatively consider using the expansion dimensionality, which assesses the *local distance distribution* of a sample of interest to its neighbors, as a potentially effective method of characterizing adversarial examples. This is a family of dimensionality models that provide a local view of the dimensional structure of the data and which has been successfully used in a wide range of applications, such as manifold learning, dimension reduction, similarity search and anomaly detection (Amsaleg et al., 2015; Houle, 2017a). *Local Intrinsic Dimensionality* (LID) is one such expansion model that characterizes the space-filling capability of the local subspace of a reference sample (Houle, 2017a;b). In Figure 1, the LID of the adversarial example (LID=4.36) is much higher than that of the referenced normal data sample (LID=1.53), illustrating how LID can be used to successfully discriminate the adversarial subspace from the normal subspace by capturing the expansion dimensionality properties of adversarial subspaces. In this paper, we aim to study the LID properties of adversarial examples generated using the state-of-the-art attack methods and show that LID can differentiate them from normal examples. In particular, our contributions are:

- We propose the use of LID for characterizing adversarial subspaces of deep networks. We discuss how adversarial perturbation can affect the LID characteristics of an adversarial subspace, and also empirically show that the LID characteristic of an incoming test example can be estimated effectively by a minibatch of training data.
- We found that the LID characteristics of adversarial subspaces are significantly higher than that of normal data subspaces, and such difference is more distinguishable in deeper layers of deep networks.
- We empirically demonstrate that the LID characteristics of adversarial examples generated using five state-of-the-art attack methods can be easily discriminated from that of normal examples and that a classifier using LID features can outperform some existing detection measures on five attacks across three benchmark datasets.

- We show that the LID based detector is robust to the optimization based attack (Carlini & Wagner, 2017a).

## 2 RELATED WORK

In this section, we briefly review two key related areas: adversarial attacks and adversarial defenses.

**Adversarial Attacks:** A wide range of attacks have been proposed to craft adversarial examples to fool deep networks and we mention a selection of such works. The fast Gradient Method (FGM) (Goodfellow et al., 2014) perturbs normal input once by adding a small magnitude of perturbation along the gradient direction, and the basic Iterative Method (BIM) is an iterative version of FGM (Kurakin et al., 2016). A variant of BIM stops immediately at misclassification (BIM-a), and another iterates a fixed number of steps (BIM-b). The Jacobian-based Saliency Map Attack (JSMA) iteratively selects the two most effective pixels to perturb based on the adversarial saliency map until misclassification (Papernot et al., 2016c). The optimization-based Attack (Opt) is arguably the most effective to date and addresses the problem via an optimization framework (Liu et al., 2016; Carlini & Wagner, 2017b).

**Adversarial Defenses:** A number of defense techniques have been introduced, including adversarial training (Goodfellow et al., 2014), distillation (Papernot et al., 2016d), gradient masking (Gu & Rigazio, 2014), and feature squeezing (Xu et al., 2017). However, these defenses can either be evaded by Opt attacks or only provide marginal improvements (Carlini & Wagner, 2017a; He et al., 2017). Given the inherent challenges for adversarial defense, recent works have instead focused on detecting adversarial examples. These works attempt to discriminate adversarial examples (positive class) from both normal and noisy examples (negative class), based on features extracted from different layers of a DNN. Detection subnetworks based on activations (Metzen et al., 2017), cascade detector based on the PCA projection of activations (Li & Li, 2016), augmented neural network detector based on statistical measures, logistic regression detector based on KD, and Bayesian Uncertainty (BU) artifacts (Grosse et al., 2017) are a few such works. However, a recent study by Carlini & Wagner (2017a) has shown that these detection methods can be attacked as well.

## 3 LOCAL INTRINSIC DIMENSIONALITY

In dimensionality theory, expansion dimensionality models (such as expansion dimensionality and local intrinsic dimensionality (Houle, 2017a;b)) measure the rate of growth in the number of data objects encountered as the distance from the reference sample increases. Expansion models work with the distance distributions of a sample to its neighbors. Intuitively, in Euclidean space, the volume of an $m$-dimensional ball grows proportionally to $r^m$, when its size is scaled by a factor of $r$. From this rate of volume growth with distance, the expansion dimension $m$ can be deduced as:

$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^m \Rightarrow m = \frac{\ln(V_2/V_1)}{\ln(r_2/r_1)}. \tag{1}$$

Expansion models of dimensionality provide a *local view* of the dimensional structure of the data, as their estimation is restricted to a neighborhood around the sample of interest. Transferring the concept of expansion dimension in the Euclidean space to a statistical setting leads to the formal definition of the LID (Houle, 2017a).

**Definition 1** (Local Intrinsic Dimensionality)**.**
*Given a data sample $x \in X$, let $R > 0$ be a random variable denoting the distance from $x$ to other data samples. If the cumulative distribution function $F(r)$ of $R$ is positive and continuously differentiable at distance $r > 0$, the LID of $x$ at distance $r$ is given by:*

$$\mathrm{LID}_F(r) \triangleq \lim_{\epsilon \to 0} \frac{\ln\big(F((1+\epsilon)\cdot r)/F(r)\big)}{\ln(1+\epsilon)} = \frac{r \cdot F'(r)}{F(r)}, \tag{2}$$

*whenever the limit exists.*

$F(r)$ is analogous to the volume $V$ in Equation (1). The last equality of Equation (2) follows by applying L'Hôpital's rule to the limits (Houle, 2017a). $\text{LID}(r)$ can be estimated using the distances of $x$ to its $k$ nearest neighbors (Amsaleg et al., 2015). The local intrinsic dimension at $x$ is in turn defined as the limit, when the radius $r$ tends to zero:

$$\text{LID}_F = \lim_{r \to 0} \text{LID}_F(r). \tag{3}$$

$\text{LID}_F$ describes the relative rate at which its cumulative distance function $F(r)$ increases as the distance $r$ increases from 0. From the dimensionality point of view, $\text{LID}_F$ measures the dimension of the submanifold embedded in $X$ at the locality of $x$. If $\text{LID}_F$ is high, the proportional expansion in the number of data samples in the neighborhood of $x$ is expected to be large. That is, the local submanifold that $x$ lies on has high dimensionality. Conversely, if $\text{LID}_F$ is low, then the dimension of the submanifold embedded at $x$ is low. It should be noted that the expansion dimension defined by LID is different to the physical dimension that is defined as the minimum number of coordinates needed to specify any sample within it. We refer readers to Houle (2017a;b) for more details concerning the LID model.

**Estimation of LID:** To approximate LID and calculate it, here we will discuss the estimator of it. In extreme value theory, the smallest $k$ nearest neighbor distances could be regarded as extreme events associated with the lower tail of the underlying distance distribution. Under very reasonable assumptions, the tails of continuous probability distributions converge to the Generalized Pareto Distribution (GPD), a form of power-law distribution (Coles et al., 2001). Amsaleg et al. (2015) developed several estimators of LID to heuristically approximate the true underlying distance distribution by a transformed GPD; among these, the Maximum Likelihood Estimator (MLE) exhibited a useful trade-off between statistical efficiency and complexity. Given a reference sample $x \sim \mathcal{P}$, where $\mathcal{P}$ represents the data distribution. The MLE estimator of LID makes use of its distances to the first $k$ nearest neighbors as:

$$\widehat{\text{LID}}(x) = -\left( \frac{1}{k} \sum_{i=1}^{k} \log \frac{r_i(x)}{r_k(x)} \right), \tag{4}$$

where $r_i(x)$ is the Euclidean distance between $x$ and its $i$-th nearest neighbor, i.e, $r_1(x)$ is the minimum distance, while $r_k(x)$ is the maximum distance. Here, the $k$ nearest neighbors of $x$ are searched among all available training data samples randomly drawn from $\mathcal{P}$. The LID defined in Equation 3 is the *theoretical* LID and $\widehat{\text{LID}}$ defined in Equation (4) is its *estimate*. In the rest of the paper, we will refer to Equation (4) to calculate the LID estimates.

## 4 Characterizing Adversarial Subspaces

Our aim is to understand adversarial subspaces and therefore derive potential defenses and provide new directions for more efficient attacks. We begin by providing some motivation as to how adversarial perturbation can affect the LID characteristic of adversarial subspaces. We then show how a detector can be designed using LID estimates to discriminate between adversarial and normal examples.

**LID of Adversarial Subspaces:** Consider a sample $x \in X$ lying within a data submanifold $S$, where $X$ is a randomly sample dataset from $\mathcal{P}$ consisting only of normal (unperturbed) examples. Adversarial perturbation of $x$ typically results in a new sample $x'$, whose coordinates differ from those of $x$ by very small amounts. Assuming that $x'$ is indeed a successful adversarial perturbation of $x$, the theoretical LID value associated with $x$ is simply the dimension of $S$, whereas the theoretical LID value associated with $x'$ is the dimension of the adversarial subspace within which it resides.

Since perturbation schemes generally allow the modification of all data coordinates, they exploit the full degrees of freedom afforded by the representational dimension of the data domain. As pointed out by (Goodfellow et al., 2014; Warde-Farley et al., 2016; Tanay & Griffin, 2016), $x'$ is very likely to lie outside $S$ (but very close to $S$ — in a high-dimensional contiguous space). In applications involving high-dimensional data, the representational dimension is typically far larger

than the intrinsic dimension of any given data submanifold, which implies that the theoretical LID of $x'$ is far greater than that of $x$.

In practice, however, the values of LID must be estimated from local data samples. This is typically done by applying an appropriate estimator (such as the Hill estimator shown in Equation (4)) to a $k$-nearest neighborhood of the test samples, for some appropriate fixed choice of $k$. Typically, $k$ is chosen large enough for the estimation to stabilize, but not so large that the sample is no longer local to the test sample. If the dimension of $S$ is reasonably low, one can expect the estimation of the LID of $x$ to be reasonably accurate.

For the adversarial subspace, the samples appearing in the neighborhood of $x'$ can be expected to be drawn from more than one manifold. The proximity of $x'$ to $S$ means that the neighborhood is likely to contain neighbors lying in $S$; however, if the neighborhood were composed mostly of samples drawn from $S$, $x'$ would not likely be an adversarial example. Thus, the neighbors of $x'$ taken together are likely to span a subspace of intrinsic dimensionality much higher than any of these submanifolds considered individually, and the LID estimate computed for $x'$ can be expected to reveal this.

**Efficiency through Minibatch Sampling:** Computing neighborhoods with respect to the entirety of the dataset $X$ can be prohibitively expensive, particularly when the (global) intrinsic dimensionality of $X$ is too high to support efficient indexing. For this reason, when $X$ is large, the computational cost can be reduced by estimating the LID of an adversarial example $x'$ from its $k$-nearest neighbor set within a randomly-selected sample (*minibatch*) of the dataset $X$. Since the LID model regards the distances from $x'$ to the members of $X$ as independently-drawn samples from a distribution, any random minibatch induces a smaller sample of distances drawn independently from the same distribution.

Provided that the minibatch is chosen sufficiently large so as to ensure that the $k$-nearest neighbor sets remain in the vicinity of $x'$, estimates of LID computed for $x'$ within the minibatch would resemble those computed within the full dataset $X$. Conversely, as the size of the minibatch is reduced, the variance of the estimates would increase. However, if the gap between the true LID values of $x$ and $x'$ is sufficiently large, even an extremely small minibatch size and / or small neighborhood size could conceivably produce estimates, whose difference is sufficient to reveal the adversarial nature of $x'$. As we shall show in Section 5.1, detection turns out to be possible even for minibatch sizes as small as 100, and for neighborhood sizes as small as 20.

**Using LID to Characterize Adversarial Examples:** We next describe how LID estimates can serve as features to train a detector to distinguish adversarial examples. Our methodology requires that training sets be comprised of three types of examples: adversarial, normal and noisy. This replicates the methodology used in (Feinman et al., 2017; Carlini & Wagner, 2017a), where the rationale for including noisy examples is that DNNs are required to be robust to random input noise (Fawzi et al., 2016) and noisy inputs should not be identified as adversarial attacks. A classifier can be trained by using the training data to construct features for each sample, based on its LID within a normal example minibatch across different layers and where the class label is assigned positive for adversarial examples and assigned negative for normal and noisy examples.

Algorithm 1 describes how the LID features can be extracted for training a LID adversarial detector. Given an initial training set of only normal examples and a pre-trained DNN, the algorithm outputs a detector trained using LID features. The extraction of LID features first begins with generating adversarial and noisy counterparts to normal examples (step 3 and 4) in each minibatch. One minibatch of normal examples ($B_{norm}$) gets used to generate 2 counterpart minibatches of examples: an adversarial ($B_{adv}$) one and a noisy ($B_{noisy}$) one. The adversarial examples are crafted by applying an adversarial attack on normal examples (step 3), whilst noisy examples are crafted by adding the same amount of *random* noise to normal examples as that of adversarial examples (step 4). The LID of each example (either normal, adversarial or noisy) is estimated based on Equation (4) by its $k$ nearest neighbors in the *normal* minibatch (steps 12-14). An implication here is that for any new unknown test example, only a minibatch of normal training examples is used to estimate its LID. For each example and each transformation layer in the DNN, an LID estimate is calculated. The distance function needed for this estimate uses the activation values of the neurons in the given layer as inputs (step 7). We use all transformation layers, including conv2d, max-pooling, dropout, ReLU and softmax, since we expect adversarial subspaces to exist in each layer of the DNN representation

**Algorithm 1** Train a LID based detector

**Input:**
$X$: a dataset of normal examples
$H(x)$: a pre-trained DNN with $L$ transformation layers
$k$: the number of nearest neighbors for LID estimation
**Output:**
Detector(LID) ▷ a detector
1: $\text{LID}_{neg}$=[], $\text{LID}_{pos}$=[]
2: **for** $B_{norm}$ in $X$ **do** ▷ $B_{norm}$: a minibatch of normal examples
3: $B_{adv}$ := adversarial attack $B_{norm}$ ▷ $B_{adv}$: a minibatch of adversarial examples
4: $B_{noisy}$ := add random noise to $B_{norm}$ ▷ $B_{noisy}$: a minibatch of noisy examples
5: $N = |B_{norm}|$ ▷ number of examples in $B_{norm}$
6: $\text{LID}_{norm}$, $\text{LID}_{noisy}$, $\text{LID}_{noisy}$ = zeros$[N, L]$
7: **for** $i$ in $[1, L]$ **do**
8: $A_{norm} = H^i(B_{norm})$ ▷ $i$-th layer activations of $B_{norm}$
9: $A_{adv} = H^i(B_{adv})$ ▷ $i$-th layer activations of $B_{adv}$
10: $A_{noisy} = H^i(B_{noisy})$ ▷ $i$-th layer activations of $B_{noisy}$
11: **for** $j$ in $[1, N]$ **do**
12: $\text{LID}_{norm}[j, i] = -\left(\frac{1}{k}\sum_{i=1}^{k} \log \frac{r_i(A_{norm}[j], A_{norm})}{r_k(A_{norm}[j], A_{norm})}\right)$
13: $\text{LID}_{adv}[j, i] = -\left(\frac{1}{k}\sum_{i=1}^{k} \log \frac{r_i(A_{adv}[j], A_{norm})}{r_k(A_{adv}[j], A_{norm})}\right)$
14: $\text{LID}_{noisy}[j, i] = -\left(\frac{1}{k}\sum_{i=1}^{k} \log \frac{r_i(A_{noisy}[j], A_{norm})}{r_k(A_{noisy}[j], A_{norm})}\right)$
15: ▷ $r_i(A[j], A_{norm})$: the $L_2$ distance of $A_[j]$ to its $i$-th nearest neighbor in $A_{norm}$
16: **end for**
17: **end for**
18: $\text{LID}_{neg}$.append($\text{LID}_{norm}$), $\text{LID}_{neg}$.append($\text{LID}_{noisy}$)
19: $\text{LID}_{pos}$.append($\text{LID}_{adv}$)
20: **end for**
21: Detector(LID) = train a classifier on ($\text{LID}_{neg}$, $\text{LID}_{pos}$)

space, as mentioned in Section 5.1. The LID values for the example are then used as feature values (one feature for each transformation layer). Finally, a classifier such as logistic regression classifier is trained using the LID features. At test time, the LID features of an unknown test example can be computed based on its $k$ nearest neighbors found in a minibatch of normal examples randomly selected from the training set. The test example can then be classified by the LID based detector to either the positive (adversarial) or negative (non-adversarial) class based on its LID feature values.

## 5 USING LID CHARACTERISTICS TO PREDICT ATTACKS

In this section, we evaluate the subspace characterization power of LID by the LID detector for 5 current adversarial attacks. In our experiments, The attacks were selected based on their effectiveness and diversity: FGM, BIM-a, BIM-b, JSMA, Opt as introduced in Section 2. LID is compared with state-of-the-art detection measures KD and BU as mentioned in Section 2, on three benchmark datasets: MNIST, CIFAR-10 and SVHN. We first introduce the experimental setup and then report and discuss the results.

**Experimental Setup:** For each dataset, a classification DNN was independently pretrained on 83% of the images as pre-training (*pre-train*) and 17% of the data was set aside for test data (*pre-test*). Any *pre-test* images not correctly classified were discarded and the remainder of the *pre-test* images were sub-divided into *train* (80%) and *test* (20%). Each of these sets was randomly partitioned into minibatches of size 100. The LID, KD and BU based detectors were trained separately on the *train* set using the scheme in Algorithm 1 with the calculation of LID replaced by KD and BU in the cases of KD and BU detectors. All detectors were then evaluated against an equal number of normal, noisy and adversarial images crafted using steps 2-4 in Algorithm 1, based on images from the test set (*test*). The LID, KD and BU characteristics of those test images were extracted using steps 1-18 in Algorithm 1. It should be noted that the test images (*test*) were never seen during any training

process, avoiding cross contamination. The adversarial examples for both training and test were generated by applying one of the five selected attacks. The noisy examples for the JSMA attack were crafted by randomly clipping an equal number of perturbed pixels, while noisy images for other attacks were crafted by adding the same amount of $L_2$ Gaussian noise, following the setting in Feinman et al. (2017). We used the logistic regression classifier as detector, and report its AUC score as the metric for performance, as suggested by Feinman et al. (2017); Carlini & Wagner (2017a).

**Deep Neural Networks:** The pretrained DNN used for MNIST was a 5-layer ConvNet with max-pooling and dropout. It achieved 99.29% classification accuracy on (normal) *pre-test* images. For CIFAR-10, a 12-layer ConvNet with max-pooling and dropout was used. This model reported an accuracy of 84.56% on (normal) *pre-test* images For SVHN, we trained a 6-layer ConvNet with max-pooling and dropout. It achieved 92.18% accuracy on (normal) *pre-test* images. We deliberately did not tune the DNNs, as their performance was close to the state-of-the-art and could thus be considered sufficient for use in an adversarial study (Feinman et al., 2017).

**Parameter Tuning:** We tuned the bandwidth ($\sigma$) parameter for KD and the number of nearest neighbors ($k$) for LID based on nested cross validation within the training set (*train*) . We did not tune the number of prediction times ($T$) (50 in our setting) for BU as it is not sensitive provided $T > 20$ (Carlini & Wagner, 2017a). The bandwidth was tuned by a grid search over $[0, 10)$ in log-space and the $k$ nearest neighbors were tuned by a grid search over $[10, 100)$ with respect to a minibatch size of 100, based on their detection AUC. For a given dataset, the parameter setting selected was the one with highest AUC averaged across all attacks. The optimal bandwidths chosen for MNIST, CIFAR-10 and SVHN were 3.79, 0.26, 1.0 respectively, while the $k$ for LID was set to 20 for MNIST and CIFAR-10, and 30 for SVHN.

Our implementation is based on the detection framework by Feinman et al. (2017). For FGM, JSMA, BIM-a, and BIM-b attacks, we used the *cleverhans* library (Papernot et al., 2016a) and used the author's implementation for Opt attack (Carlini & Wagner, 2017b). We scaled all images to $[0, 1]$. Our code is available at: `https://www.dropbox.com/s/cc2vpqya0il78j6/LID_adv_detection.zip`.

### 5.1 LID Characteristics of Current Attacks

We provide empirical results showing the LID characteristics of adversarial examples generated by the most effective attack to date – the Opt attack. The left subfigure in Figure 2 shows the LIDs (for the pre-softmax layer) of 100 randomly selected normal, noisy and adversarial (Opt attack) CIFAR-10 examples. We observe that the LIDs of adversarial examples are significantly higher than those of normal or noisy examples (these lines overlap each other). This supports our expectation that adversarial subspaces have higher intrinsic dimensionality than normal data subspaces as discussed in Section 4. It also suggests that the transit from normal example to adversarial example may follow directions where the complexity of local data submanifold significantly increases (increasing LID).

We further show in the right subfigure of Figure 2 that the LIDs of adversarial examples are more easily discriminated from other examples at deeper layers of the network. The 12-layer ConvNet we used for CIFAR-10 consists of 26 transformation layers: the input layer ($L_0$), conv2d/max-pooling ($L_{1-17}$), dense/dropout ($L_{18-24}$) and the softmax layer ($L_{25}$). As can be seen, the LID characteristics of adversarial examples become distinguishable (detection AUC$> 0.5$) from the dense layers ($L_{18-24}$) and significantly different after the softmax layer ($L_{25}$). This suggests that the fully-connected and softmax transformations are more vulnerable to adversarial perturbations than convolutional transformations. Some further LID characteristics for the MNIST and SVHN datasets are provided in the Appendix A.2.

In regard to stability of performance based on parameter variation ($k$ for LID, or bandwidth for KD), we can observe in Figure 3 that LID is more stable than KD, exhibiting less variation in AUC as the parameter varies. From this Figure, we also see that KD requires significantly different optimal settings for different types of data. For simpler datasets such as MNIST and SVHN, it requires quite high bandwidths for the best performance. However, large bandwidth may cause inaccurate density estimation (Jones et al., 1996). The reason behind this and tradeoffs merit future investigation.

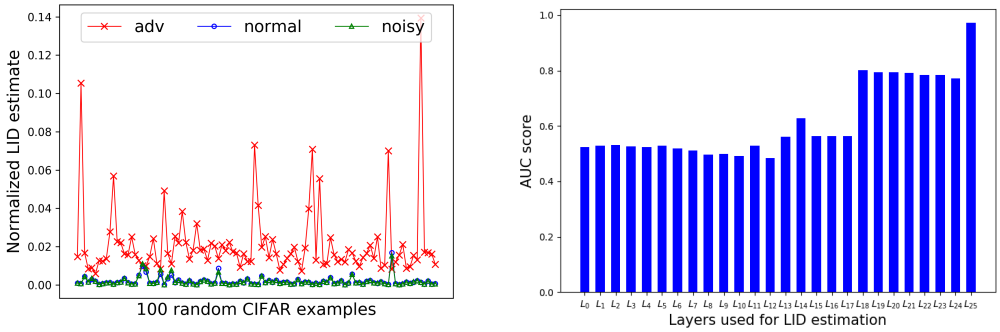


Figure 2: Left figure shows the normalized LID (pre-softmax layer) of 100 normal (blue), noisy (green) and Opt attack (red x-cros) CIFAR-10 examples. The noisy and adversarial examples were generated from the normal example. Right figure shows the detection performance (AUC) based on LIDs of different layers. $L_i$ denotes $i$-th transformation layers.

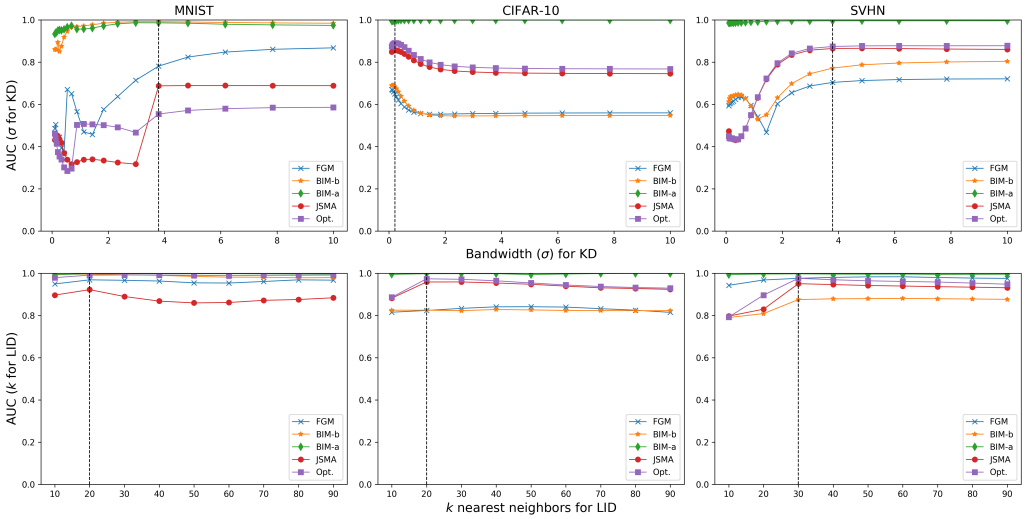


Figure 3: Top row: tuning bandwidth $\sigma$ for KD by a grid search over $[0, 10)$ in log-space, separately for each dataset. Bottom row: tuning $k$ for LID by a grid search over $[10, 100)$ for minibatch size 100, separately for each dataset. The vertical dashed lines denote the selected parameter choice.

### 5.2 Discrimination Power Analysis

**LID Outperforms KD and BU:** We compare the detection performance of LID with detectors trained with features of KD and BU, respectively, as well as the one trained with the combination of them (KD+BU). As shown in Table 1, LID outperforms the KD and BU measures (both individually and combined) by large margins on all attacks across all datasets. For the most effective attack to date, i.e., the Opt attack, the LID based detector achieved AUCs of $99.24\%$, $98.93\%$ and $97.60\%$ on MNIST, CIFAR-10 and SVHN, respectively, compared to AUCs of $95.35\%$, $93.77\%$ and $90.66\%$ for the combination of KD and BU. This strong performance suggests that LID is a highly promising characteristic to discriminate the properties of adversarial subspaces. We also observe that KD was not effective for some attacks, such as the FGM, JSMA and BIM-a attacks, whist the BU measure failed to detect the FGM and BIM-b attacks on MNIST dataset.

**Effect of Larger Minibatch Size for LID:** In regard to minibatch size, a default value of 100 was used, with a view to ensuring efficiency and this can deliver state of the art performance (the results in Table 1). However, it is also interesting to question whether use of larger minibatch size could further improve detection performance AUC of the LID method. Figure 5 in the Appendix (A.3)

Table 1: A comparison of the discrimination power (AUC score of a logistic regression classifier) between LID, KD and BU. The AUC score is computed for each attack on each dataset, and the best results are highlighted in **bold**.

| Dataset | Feature | FGM | BIM-a | BIM-b | JSMA | Opt |
|---|---|---|---|---|---|---|
| MNIST | KD | 78.12% | 99.14% | 98.61% | 68.77% | 95.15% |
| | BU | 32.37% | 91.55% | 25.46% | 88.74% | 71.29% |
| | KD+BU | 82.43% | 99.20% | 98.81% | 90.12% | 95.35% |
| | LID | **96.89%** | **99.60%** | **99.83%** | **92.24%** | **99.24%** |
| CIFAR-10 | KD | 64.92% | 68.38% | 98.70% | 85.77% | 91.35% |
| | BU | 70.53% | 81.60% | 97.32% | 87.36% | 91.39% |
| | KD+BU | 70.40% | 81.33% | 98.90% | 88.91% | 93.77% |
| | LID | **82.38%** | **82.51%** | **99.78%** | **95.87%** | **98.93%** |
| SVHN | KD | 70.39% | 77.18% | 99.57% | 86.46% | 87.41% |
| | BU | 86.78% | 84.07% | 86.93% | 91.33% | 87.13% |
| | KD+BU | 86.86% | 83.63% | 99.52% | 93.19% | 90.66% |
| | LID | **97.61%** | **87.55%** | **99.72%** | **95.07%** | **97.60%** |

Table 2: The failure rate of an adaptive attack targeting the LID based detector.

| | MNIST | CIFAR-10 | SVHN |
|---|---|---|---|
| Attack Failure Rate (one-layer) | 100% | 95.7% | 97.2% |
| Attack Failure Rate (all-layer) | 100% | 100% | 100% |

shows the effect of using minibatch size of 1000 for different choices of $k$. It illustrates that a larger batch size can improve detection AUC even further and we conjecture this is because it facilitates a more accurate estimation of LID (recall discussion in Section 4). A comprehensive investigation of minibatch size is an interesting area for future work.

**Robustness to Adaptive Attack:** To further evaluate the robustness of the LID based detector, we applied an adapted Opt attack in a white-box setting to attack our LID detector. Similar to the one used to attack the KD based detector in Carlini & Wagner (2017a), we adopted the Opt $L_2$ attack with an adapted adversarial objective:

$$\text{minimize } \|x - x_{adv}\|_2^2 + \alpha \cdot \big(\ell(x_{adv}) + \ell(\text{LID}(x_{adv}))\big), \tag{5}$$

where $\alpha$ is a constant balancing the amount of perturbation and the adversarial strength. The LID features used to detect this adapted attack is the pre-softmax layer LID, in contrast to all transformation layers as used in our LID based detector, since the Opt attack is known to attack the pre-softmax layer (logits), and here we follow the similar setting for a fair comparison (Carlini & Wagner, 2017b;a). The optimal constant $\alpha$ is chosen via an internal binary search for $\alpha \in [10^{-3}, 10^{6}]$. The rationale for the minimization of LID characteristic is that adversarial examples have higher LID than normal examples, as we demonstrated in Section 5.1.

We applied the above adapted attack on 1000 normal images randomly chosen from the detection test set. The deep network used was the same unsecured ConvNets as used in the previous experiments. Instead of AUC as mesured in the previous sections, we report accuracy here as a measurement to evaluate the adaptive attack performance as suggested by Carlini & Wagner (2017a). As shown in Table 2, the adapted attack failed to find a valid adversarial example $100\%$, $95.7\%$ and $97.2\%$ of the time on MNIST, CIFAR-10 and SVHN, respectively. In addition, when apply the LID based detector trained on all transformation layers, it can still correctly detect all these attacks $100\%$ of the time.

## 6 Conclusion

In this paper, we have addressed the challenge of understanding the properties of adversarial subspaces, particularly with a view to detecting adversarial examples. We characterized the expansion dimensionality property of adversarial subspaces via the use of Local Intrinsic Dimensionality (LID), and showed how these could be used as features in an adversarial example detection process.

Our empirical results suggest that LID is a highly promising method for characterizing adversarial examples and can be used to deliver state of the art discrimination performance. From a theoretical perspective, we have provided initial intuition about why LID features are effective, but deeper investigation in this direction is a very promising area for further work and may open up enriched understanding of new methods for both adversarial example attack and defence.

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# A Appendix

## A.1 Statistics of Adversarial Attacks

Table 3: The $L_2$ mean perturbation and model accuracy on adversarial examples.

| | MNIST | | CIFAR | | SVHN | |
|---|---|---|---|---|---|---|
| | $L_2$ | Acc. | $L_2$ | Acc. | $L_2$ | Acc. |
| FGM | 7.26 | 11.09% | 2.74 | 3.15% | 7.09 | 6.17% |
| BIM-a | 2.30 | 10.43% | 0.48 | 0.00% | 0.83 | 0.13% |
| BIM-b | 3.42 | 10.42% | 2.39 | 0.00% | 5.53 | 0.13% |
| JSMA | 5.40 | 10.00% | 3.64 | 0.04% | 3.09 | 0.16% |
| Opt | 4.21 | 3.92% | 0.37 | 0.01% | 0.59 | 0.26% |

### A.2 LID CHARACTERISTICS OF CURRENT ATTACKS

Figure 4 illustrates LID characteristics of the most effective attack to date, i.e., Opt attack on MNIST and SVHN datasets. On both datasets, the LIDs of adversarial examples are significantly higher than that of normal or noisy examples. In the right subfigure, the LIDs of normal examples and its noisy counterparts were overlapped.

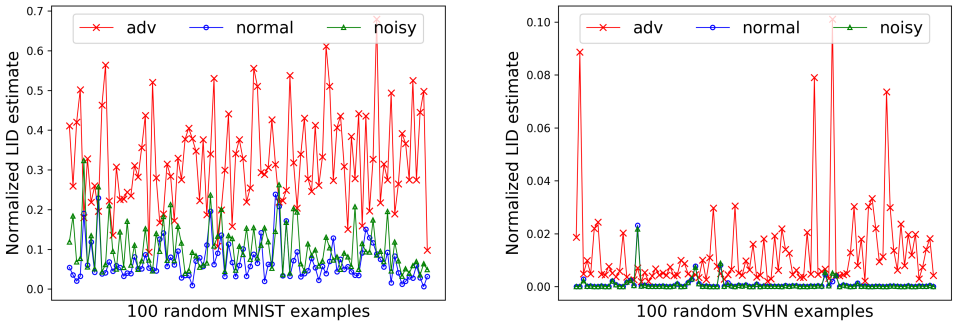


Figure 4: The plots show the normalized LIDs of 100 randomly selected normal (blue), noisy (green) and Opt attack (red x-cross) examples. The noisy and adversarial examples were generated from the normal example. Left figure shows LIDs (pre-softmax) of MNIST examples while right figure shows LIDs (after-softmax) of SVHN examples. Normal and noisy example curves are superimposed in the right figure.

### A.3 DISCRIMINATION POWER UNDER DIFFERENT PARAMETER SETTINGS

Figure 5 shows the discrimination power (detection AUC) of LID characteristics estimated using two different minibatch sizes (the default setting of 100, and a larger size of 1000). Horizontal axis represents different $k$ nearest neighbors used from $10\%$ to $90\%$ percent to the batch size. We observe that the peak AUC is higher for the larger minibatch size.

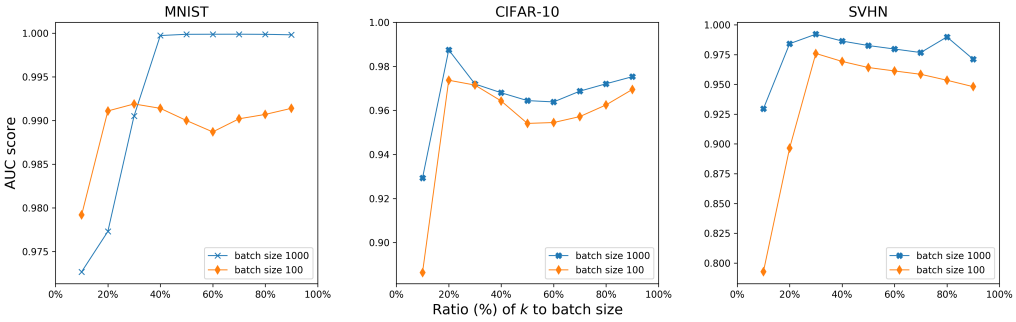


Figure 5: The detection AUC score of LIDs estimated using different $k$ with a large batch size of 1000. The results are shown for detecting Opt attack on MNIST, CIFAR-10 and SVHN datasets.