Masked Modeling for Single-cell Clustering of scRNA-seq Data

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Abstract

Single-cell clustering of scRNA-seq data is a typical and challenging problem that predicts cell subtype clusters given gene expression sequences from single-cell RNA data. Previous models utilized classical clustering (*e.g.*, Principal Component Analysis, K-means) on well-annotated data to classify cells. However, they extremely relied on the expected number of clusters as input. To address the problem, in this work, we propose a novel multimodal self-supervised framework with masked expression modeling on single-cell data, namely mask-sc, that can learn compact and discriminative representations by reconstructing masked gene expression for scRNA-seq clustering. Our mask-sc aggregates high-frequency interconnections across multiple groups of expression sequences via a masked expression encoder applied on expression matrices. Then, a sequence-guided decoder is applied to recover sequence-level features of masked expression matrices. Finally, representations extracted from the gene expression encoder can be used for scRNA-seq clustering. We conduct extensive experiments on two scRNA-seq datasets, where empirical results demonstrate the effectiveness of our proposed mask-sc against previous baselines.

1 Introduction

In bioinformatics, single-cell RNA sequencing (scRNA-seq) is a platform to discover cellular subpopulations [1] and transcriptional profiles [2] from individual cells. Through analysis of scRNA-seq, cell biologists are able to understand and interpret intrinsic cell identities. This crucial biological perception intelligence of cells attracts many researchers to analyze the scRNA-seq data.

Early scRNA-seq analysis works leveraged traditional machine learning approaches, such as Principal Component Analysis (PCA) [3], K-means [9], and Gaussian Mixture Models [10], to cluster cellular subtypes. Due to the challenge of high dimensional and significantly sparse sequences in the clustering analysis, the following methods [11, 12, 4, 13, 14, 15, 5, 6, 16, 17] tried to explore different frameworks to address the problem.

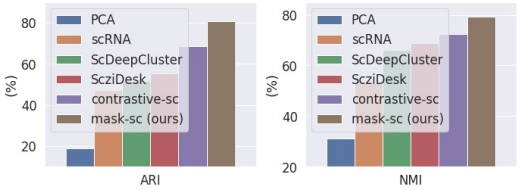

Figure 1: Comparison results (ARI and NMI) of the proposed mask-sc with state-of-the-art methods(PCA [3], scRNA [4], ScDeepCluster [5], SczizDesk [6], and contrastive-sc [7]) on 10 PBMC cells [8] from 10x genomic platform.

Typically, CIDR [11] proposed a hierarchical clustering with an implicit imputation stage before PCA to alleviate the effect of dropouts. scRNA [4] transferred knowledge from a large and well-annotated reference dataset by non-negative matrix factorization for small disease-specific data. With the advance of deep learning, deep neural networks, such as DCA [15] have been widely used to boost clustering performance. ScDeepCluster [5] adopted a clustering layer on the embedding space learned from DCA to enrich representations.

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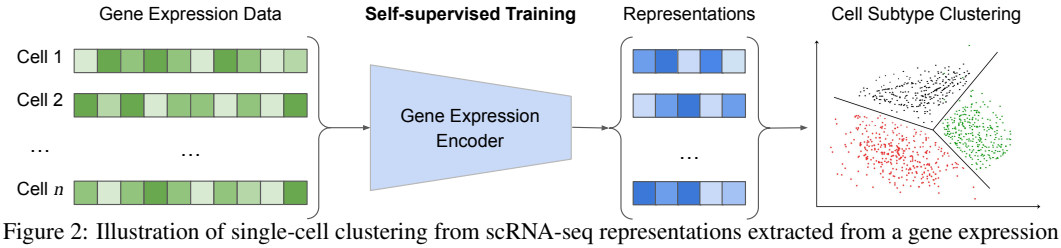

Figure 2: Illustration of single-cell clustering from scRNA-seq representations extracted from a gene expression encoder. The goal of this work is to design a self-supervised training framework to learn such an encoder from gene expression data of multiple cells.

A soft self-training KMeans clustering was introduced in ScziDesk [6] to aggregate similar cells in the same cluster. However, those methods rely on well-annotated data, which limits their generalization to all circumstances. In contrast, we will solve them in our approach by extracting discriminative and compact representations from input expressions via a self-supervised encoder, as shown in Figure 2.

Recently, inspired by the success of self-supervised learning [18, 19, 20, 21, 22, 23] in image and text, contrastive-sc [7] applied a contrastive loss on anchor and augmented sample outputs from an encoder to extract representations for clustering scRNA-seq data. While the state-of-the-art baseline achieved promising performance, they can not fully capture interconnections between expressions of different genes for each cell, as they only took a short fragment of expressions in a cell as an input sample during training. For each anchor sample, the contrastive loss closed the distance between embeddings from the anchor and augmented gene expressions, while pushing away embeddings from different gene expressions. Different from them, we group multiple fragments of expressions as an expression matrix, which is fed into a gene expression encoder to capture interconnections across multiple groups by masked expression modeling. In the meanwhile, we apply a sequence-guided decoder to reconstruct sequence-level features of masked expression matrices and propose a novel multimodal self-supervised training framework for single-cell clustering.

The main challenge is that gene expression sequences for each cell are high-dimensional. With only a short fragment of expressions in a cell as input, the state-of-the-art self-supervised approach [7] can not learn discriminative and compact global representations by capturing high-frequency interconnections across enough genes in this cell. In the meanwhile, previous machine learning approaches [3, 4, 5, 6] are extremely dependent on the well-annotation of the number of cellular subtypes for training. To address the aforementioned challenges, our key idea is to take as input gene expression matrices composed of grouped expression sequences for self-supervised training, which is different from existing clustering and self-supervised methods. During training, we aim to learn compact representations from input gene expression matrices with high-frequency interconnections across various genes in each cell for discovering potential cellular subtypes.

To this end, we propose a novel multimodal self-supervised training approach based on masked expression modeling for single-cell clustering, termed as masked-sc, that can learn compact representations from high-frequency interconnection across different genes for each cellular subtype. Specifically, our mask-sc leverages gene expression matrix embeddings as input to the gene expression encoder for masked expression modeling to capture interconnections from grouped gene expression matrices in each cell. Then, a sequence-guided decoder is applied to predict sequence-level features of masked expression tokens, where sequence-level feature targets are extracted from a pre-trained sequence-level encoder. Compared to previous scRNA-seq clustering approaches, our method can extract discriminative representations from interconnections across gene expression matrices.

Empirical experiments on two real scRNA-seq datasets comprehensively validate the state-of-the-art performance against the previous scRNA-seq clustering baselines. In addition, qualitative visualizations of clustering results vividly showcase the effectiveness of our mask-sc in learning discriminative representation from Input Matrix Embeddings and Multimodal Encoder-Decoder. Extensive ablation studies also validate the importance of Input Matrix Embeddings for masked expression modeling and Multimodal Encoder-Decoder for sequence-guided reconstruction in learning compact expression representations for single-cell clustering of scRNA-seq data. Meanwhile, qualitative visualizations of learned attention of the gene expression encoder indicate high-frequency interconnections for different cellular subtypes.

Our main contributions can be summarized as follows:

- We present a novel multimodal self-supervised training framework with masked expression modeling for single-cell clustering, namely mask-sc, that can learn discriminative representations from interconnection across different genes for each cell.
- We introduce input expression matrix embeddings for masked modeling on grouped gene expression matrices and multimodal encoder-decoder for sequence-guided reconstruction.
- Extensive experiments on two real scRNA-seq benchmarks comprehensively demonstrate the state-of-the-art superiority of our mask-sc over baselines.

2 Related Work

Self-supervised Representation Learning. Self-supervised representation learning has been addressed in many previous works [19, 24, 21, 18, 20, 25] to learn discriminative representations from internal characteristics of data without any label. Such learnable and transferrable features are beneficial to many downstream tasks, such as image classification [19, 24, 21, 26, 27], object detection [18, 20, 25, 28, 29], semantic textual similarity tasks [22, 23, 30, 31, 32], protein structure prediction [33, 34, 35], and transcription factor binding sites prediction [36, 37, 38]. In this work, our main focus is to leverage a self-supervised training framework to learn compact gene expression representations from scRNA-seq data for identifying potential cell clusters, which is more challenging than those tasks listed above.

Masked Representation Learning. Masked representation learning is one type of unsupervised learning, which aims to learn self-supervised representations by reconstructing desired features of masked data given unmasked parts as clues. In the recent years, masked representation learning has achieved promising results in natural language processing [39, 40, 41, 42, 43] and computer vision [44, 45, 46, 47, 48] community. Typically, BERT [39] randomly masked 15% of word tokens in the sentence and recovered them with unmasked words to learn generalizable textual features via a self-attention transformer [49]. A block-wise masking strategy was proposed in BEiT [44] to reconstruct discrete tokens of masked image patches for pre-training transferrable visual representations. To simplify the masked image encoding framework, MAE [45] directly reconstructed missing pixels of 75% masked patches using vision transformers [50] for self-supervised pre-training. More recently, researchers introduced diverse masking pipelines to show the effectiveness of masked modeling in learning meaningful representations from video [51, 52], audio [53], and MRI/CT scans [54]. However, there is little work exploring the mask modeling methodology on single-cell data, especially on scRNA-seq datasets across various cells. To the best of our knowledge, we are the first to adopt masked gene expression modeling on scRNA-seq data for extracting transferrable representations to do scRNA-seq clustering on multiple cellular subpopulations.

scRNA-seq Clustering. scRNA-seq clustering is a challenging problem that predicts cellular subtype clusters from gene expression data of diverse cells. Early methods applied classical Principal Component Analysis (PCA) [3], K-means [9], and Gaussian Mixture Models [10] to cluster cell subpopulations from gene expression data directly. Because of the high dimensionality and sparsity of gene expression sequences, the following work [11, 12, 4, 13, 14, 15, 5, 6, 16, 17] explored diverse pipelines to tackle with this issue. For instance, to classify both pure and transitional cells, SOUP [13] utilized the expression similarity matrix to estimate soft membership for cell-type cluster centers. However, those scRNA-seq clustering baselines mainly rely on well-annotated data, which limits their generalization and violates the fundamental goal of discovering potential cell subtype clusters. More recently, contrastive-sc [7] introduced the same self-supervised framework as SimCLR [19] to extract embeddings of a short gene expression sequence by an InfoNCE-based contrastive loss. Different from them, we develop a novel multimodal self-supervised framework to learn compact and discriminative representations by reconstructing sequence-level features of masked gene expression matrices for scRNA-seq clustering.

3 Method

Given a set of gene expression data from scRNA-seq, our target is to learn a gene expression encoder to extract gene expression embeddings for scRNA-seq clustering. In this work, we propose a novel mask-based self-supervised training framework named mask-sc for extracting compact and discriminative representations from single-cell data, which mainly consists of two modules,

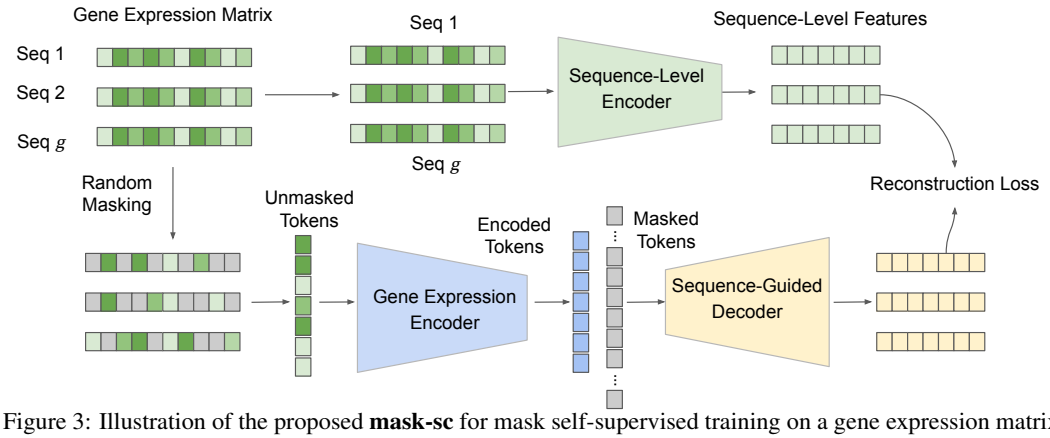

Figure 3: Illustration of the proposed **mask-sc** for mask self-supervised training on a gene expression matrix with g sequences. The gene expression encoder takes as input unmasked tokens generated from random masking on the expression matrix. With encoded expression tokens, a sequence-guided decoder is utilized to predict sequence-level features of masked expression tokens. Note that all targeted sequence-level features are extracted from a pre-trained sequence-level encoder, and this pre-trained encoder is frozen during training. Finally, a reconstruction loss is minimized between the predicted and targeted sequence-level features, which pushes the gene expression encoder to extract discriminative representations from the gene expression matrix.

Input Matrix Embeddings for masked expression matrix modeling in Section 3.2 and Multimodal Encoder-Decoder for sequence-guided reconstruction in Section 3.3.

3.1 Preliminaries

In this section, we first describe the problem setup and notations and then revisit contrastive-sc, the state-of-the-art self-supervised baseline for scRNA-seq clustering.

Problem Setup and Notations. Given a set of gene expression data with m genes from n cells, our goal is to learn a discriminative global gene expression feature from m genes in each cell. Note that m, n denote the number of genes and cells, respectively. D denotes the dimension of embeddings.

Revisit contrastive-sc. To solve the single-cell representation learning problem, contrastive-sc [7] first extracted a sequence-level embedding $\mathbf{s} \in \mathbb{R}^{s \times D}$ with a length of s from a gene expression encoder, and then concatenated all m/s sequences as the whole feature for m genes. During training, they utilized a self-supervised framework similar to SimCLR [19] with an InfoNCE-based contrastive loss to close the distance between the anchor and augmented embeddings of one sequence in the same cell, which is denoted as:

$$\mathcal{L}_{\text{contrastive-sc}} = \frac{1}{B} \sum_{i=1}^{B} -\log \frac{\exp\left(\frac{1}{\tau}\texttt{sim}(\mathbf{s}_i, \hat{\mathbf{s}}_i)\right)}{\sum_{j=1}^{B} \exp\left(\frac{1}{\tau}\texttt{sim}(\mathbf{s}_i, \hat{\mathbf{s}}_j)\right)} \tag{1}$$

where $\mathbf{s}_i, \hat{\mathbf{s}}_i \in \mathbb{R}^{1 \times D}$ denote the anchor and augmented embeddings for ith sample in a mini-batch. B is the batch size. $\texttt{sim}(\mathbf{s}_i, \hat{\mathbf{s}}_i) = \mathbf{s}_i^T \hat{\mathbf{s}}_i / (\|\mathbf{s}_i\| \|\hat{\mathbf{s}}_i\|)$ is the cosine similarity, and τ is the temperature parameter. $B^2 - B$ negative sequences are created within a training batch. By optimizing this loss, they successfully extracted discriminative representations of each short sequence in the same cell.

However, this sequence-level contrastive learning framework can not fully capture the interconnections between expressions of different genes for each cell, as the length of each sequence s is much less than the total number of genes m in the same cell, *i.e.*, $s \ll m$. To address this issue, we propose a novel mask-based single-cell self-supervised training framework that can aggregate the inter-connection across multiple groups of gene expressions from a gene expression matrix with grouping g sequences, as shown in Figure 3.

3.2 Input Matrix Embeddings for Masked Expression Modeling

In order to explicitly learn the interconnections from the grouped gene expression matrix for each cell, we introduce learnable matrix patch embeddings that are extracted from raw expression input via a 2D convolutional layer, *i.e.*, $\mathbf{x} \in \mathbb{R}^{G \times D}$, where G denotes the total number of patches for each

gene matrix. Assume the patch resolution of each matrix is P, the patch-wise raw input for the gene matrix is formally denoted as $\mathbf{g} \in \mathbb{R}^{G \times P \times P}$. Note that $G = g/P \times s/P$.

With local-patch expression representations $\{\mathbf{x}_i\}_{i=1}^{G}$ for the gene expression matrix, we first apply a self-attention transformer encoder $\phi(\cdot)$ to aggregate the patch-level features from the raw input matrix embeddings and align the features with the global token embedding $\mathbf{c} \in \mathbb{R}^{1 \times D}$ as:

$$\begin{aligned} \hat{\mathbf{c}}, \{\hat{\mathbf{x}}_i\}_{i=1}^{G} &= \{\phi(\mathbf{x}_j, \mathbf{X}, \mathbf{X})\}_{j=1}^{1+G} \\ \mathbf{X} = \{\mathbf{x}_j\}_{j=1}^{G} &= [\mathbf{c}; \{\mathbf{x}_i\}_{i=1}^{G}] \end{aligned} \tag{2}$$

where $[\ ;\ ]$ denotes the concatenation operator. $\hat{\mathbf{c}}, \hat{\mathbf{x}}_i \in \mathbb{R}^{1 \times D}$, and D is the dimension size of embeddings. The self-attention operator is formulated as

$$\phi(\mathbf{x}_j, \mathbf{X}, \mathbf{X}) = \text{Softmax}(\frac{\mathbf{x}_j \mathbf{X}^\top}{\sqrt{D}})\mathbf{X} \tag{3}$$

Then, to capture the interaction across each patch of expression, we exploit a random masking mechanism and a decoder to predict the masked patch tokens given encoded tokens as clues. Specifically, a random mask with M entries as zero is applied along the number dimension of patch-level expression embeddings $\mathbf{x} \in \mathbb{R}^{G \times D}$. Let the number of unmasked patches be U, then we apply the self-attention transformer encoder $\phi(\cdot)$ to aggregate unmasked patch-level embeddings with the global token embeddings as:

$$\begin{aligned} \hat{\mathbf{c}}_u, \{\hat{\mathbf{x}}_i^u\}_{i=1}^{U} &= \{\phi(\mathbf{x}_j^u, \mathbf{X}^u, \mathbf{X}^u)\}_{j=1}^{1+U} \\ \mathbf{X}^u = \{\mathbf{x}_j^u\}_{j=1}^{U} &= [\mathbf{c}; \{\mathbf{x}_i^u\}_{i=1}^{U}] \end{aligned} \tag{4}$$

where $\hat{\mathbf{c}}_u, \hat{\mathbf{x}}_i^u \in \mathbb{R}^{1 \times D}$ denote the global and patch-level tokens encoded from unmasked input expression embeddings. Finally, those encoded expression tokens $\{\hat{\mathbf{x}}_i^u\}_{i=1}^{U}$ are passed into the decoder to generate the targeted features of masked patch tokens. With masked expression modeling, the encoder is optimized to capture the interconnections across different genes in each cell. Furthermore, the pre-trained encoder can learn discriminative representations from input gene expression matrix embeddings for discovering potential cellular subtypes. It is worth noting that, to the best of our knowledge, we are the first to apply masked self-supervised training on expression matrices for scRNA-seq clustering.

3.3 Multimodal Encoder-Decoder for Sequence-guided Reconstruction

With the benefit of masked expression modeling from input matrix embeddings mentioned above, we propose a novel and explicit sequence-guided decoder to reconstruct the sequence-level features of masked tokens. Based on encoded tokens $\{\hat{\mathbf{x}}_i^u\}_{i=1}^{U}$ and learnable masked tokens $\{\mathbf{m}_i\}_{i=1}^{M}$, a light self-attention transformer decoder $\phi(\cdot)$ is applied to generate sequence-level embeddings $\{\hat{\mathbf{s}}_i^m\}_{i=1}^{M}$ as:

$$\begin{aligned} \hat{\mathbf{c}}^d, \{\hat{\mathbf{s}}_i^u\}_{i=1}^{U}, \{\hat{\mathbf{s}}_i^m\}_{i=1}^{M} &= \{\phi(\mathbf{x}_j^d, \mathbf{X}^d, \mathbf{X}^d)\}_{j=1}^{1+U} \\ \mathbf{X}^d = \{\mathbf{x}_j^d\}_{j=1}^{1+U+M} &= [\hat{\mathbf{c}}; \{\hat{\mathbf{x}}_i^u\}_{i=1}^{U}; \{\mathbf{m}_i\}_{i=1}^{M}] \end{aligned} \tag{5}$$

where $\hat{\mathbf{c}}^d, \{\hat{\mathbf{s}}_i^u\}_{i=1}^{U}, \{\hat{\mathbf{s}}_i^m\}_{i=1}^{M}$ denote the global, unmasked, masked prediction from the decoder. To recover the sequence-level features, we apply the pre-trained contrastive-sc [7] as the sequence-level encoder to extract the target features $\{\mathbf{s}_i^m\}_{i=1}^{M}$ for masked patches of gene expression. Note that this sequence-level encoder is frozen for stabilizing training. The overall model with a multimodal encoder-decoder architecture is simply optimized to reconstruct the sequence-level features of masked expression tokens as:

$$\mathcal{L} = \frac{1}{M} \sum_{i=1}^{M} ||\mathbf{s}_i^m - \hat{\mathbf{s}}_i^m||_2^2 \tag{6}$$

Optimizing the loss will promote the model to capture discriminatively sequence-level features by learning expression interconnections across different genes for each cell. During inference, we use the pre-trained self-attention encoder to extract patch-level features $\{\hat{\mathbf{x}}_i\}_{i=1}^{G}$ from input gene expression matrices. Mean pooling is finally applied along the number dimension of the encoded patch-level representations to generate the global feature for each cell.

Table 1: Comparison results (%) on 10 PBMC cells and worm neuron cells datasets. ↑ denotes that a large value is better.

Method	10 PBMC cells			worm neuron cells		
	ARI (↑)	NMI (↑)	Silhouette (↑)	ARI (↑)	NMI (↑)	Silhouette (↑)
PCA [3]	19.00	31.21	40.24	4.65	17.56	17.69
scRNA [4]	47.15	52.91	23.27	24.58	41.52	10.85
ScDeepCluster [5]	51.54	65.98	48.52	42.40	59.23	23.63
ScziDesk [6]	55.20	68.67	14.46	4.95	22.02	19.78
contrastive-sc [7]	68.63	72.54	58.73	45.03	58.80	27.83
mask-sc (ours)	**80.64** (+12.01)	**79.23** (+6.69)	**69.41** (+10.68)	**57.77** (+12.74)	**71.90** (+12.67)	**48.12** (+20.29)

4 Experiments

4.1 Experimental Setup

Datasets. Following scDeepCluster [5], we apply 10 PBMC cells [8] from 10x genomic platform and worm neuronal cells [55] from sci-RNA-seq for experiments. We use the same split in [7] for training and testing, where the number of cells varies from 870 to 9552, and 4-16 cellular subtype clusters are annotated for evaluation.

Evaluation Metrics. Following previous work [7, 5, 6], we apply Adjusted Rand Index (ARI) [56], Normalized Mutual Information (NMI) [57], Silhouette [58] score for evaluation. ARI score calculates the ratio of sample pairs assigned to the correct cluster labels. NMI score measures the agreement of the ground truth and predicted cluster assignments. A larger value of ARI and NMI is better, which means that the predicted cluster matches the ground-truth cluster. Silhouette score measures the compactness of the generated clusters, and a higher score means that the predicted clusters are denser and better separated.

Implementation. Our implementation is based on PyTorch [59] framework. For each gene expression matrix with a total of 4096 expression entries, we apply a patch size of 4×4 to generate 256 patch-level expression embeddings, *i.e.*, $g \times s = 4096, P = 4, G = 256$. The mask ratio is 75% in our experiments, *i.e.*, $U = 64, M = 192$. For the gene expression encoder, the depth of self-attention transformers is 12, and the number of attention heads is 12. For the sequence-guided decoder, the depth of self-attention transformers is 8, and the number of attention heads is 16. The dimension size of embeddings is 768, *i.e.*, $D = 768$. The model is trained with the AdamW [60] optimizer with a learning rate of 1e-4, and hyper-parameters β_1 = 0.9, β_2 = 0.95. The model is trained for 1600 epochs with a batch size of 256. After self-supervised training, we use the K-means algorithm [9] in the scikit-learn package for evaluation to cluster gene expression representations.

4.2 Comparison to Prior Work

In this work, we propose a novel and effective self-supervised training framework for scRNA-seq clustering. To demonstrate the effectiveness of the proposed mask-sc, we comprehensively compare it to previous scRNA-seq clustering baselines: 1) PCA [3]: a traditional machine learning approach with raw gene expression sequence as input to extract principal components; 2) scRNA [4]: a baseline based on non-negative matrix factorization by using transferred knowledge from large and well-annotated data for reference; 3) ScDeepCluster [5]: an improved deep count autoencoder by adding a clustering layer to the embedding space; 4) ScziDesk [6]: a soft KMeans clustering method to aggregate similar cells from the same cellular subtype; 5) contrastive-sc [7]: the state-of-the-art self-supervised framework with InfoNCE-based contrastive loss to extract embeddings from sequences only for scRAN-seq clustering.

Table 1 reports the quantitative comparison results on 10 PBMC cells and worm neuron cells datasets. As can be seen, we achieve the best performance in terms of all metrics compared to previous scRAN-seq clustering baselines on 10 PBMC cells benchmark. In particular, the proposed mask-sc significantly outperforms PCA [3], the traditional machine learning approach, by 61.64 ARI, 48.02 NMI, and 29.17 Silhouette. Moreover, we achieve superior performance gains of 33.49 ARI, 26.32 NMI, and 46.14 Silhouette compared to scRNA [4], which indicates the importance of masked self-supervised training for learning discriminative representations from interconnections across different genes in each cell. Meanwhile, our mask-sc outperforms contrastive-sc, the current state-of-the-art self-supervised approach for scRNA-seq clustering, where we achieve the performance gains of 12.01

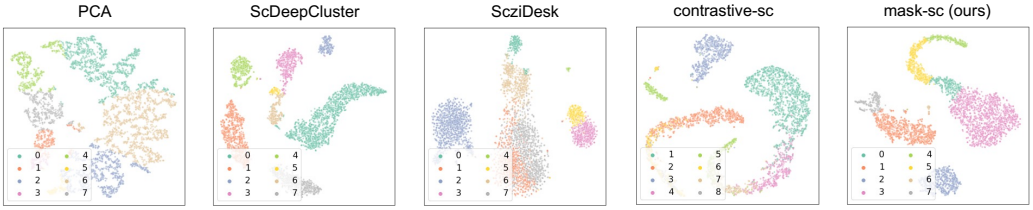

Figure 4: Qualitative comparisons with PCA [3], ScDeepCluster [5], ScziDesk [6], contrastive-sc [7] on 10 PBMC cells dataset. The proposed mask-sc generates much more intra-subtype compact and inter-subtype separable representations for single-cell clustering. Note that each spot denotes the feature of one cell, and each color refers to one cellular subtype, such as "subtype 5" in yellow and "subtype 7" in gray.

Table 2: Ablation studies on Input Matrix Embeddings (IME) and Multimodal Encoder-Decoder (MED).

IME	MED	ARI (↑)	NMI (↑)	Silhouette (↑)
✗	✗	68.63	72.54	58.73
✓	✗	72.98 (**+4.35**)	75.80 (**+3.26**)	60.39 (**+1.66**)
✗	✓	74.15 (**+5.52**)	77.56 (**+5.02**)	62.37 (**+3.64**)
✓	✓	80.64 (**+12.01**)	79.23 (**+6.69**)	69.41 (**+10.68**)

ARI, 6.69 NMI, and 10.68 Silhouette. These significant improvements demonstrate the superiority of our method in learning compact embeddings from gene expression for clustering.

In addition, significant gains in worm neuron cells benchmark can be observed in Table 1. Compared to ScziDesk [6], the recent baseline based on soft KMeans clustering, we achieve the results gains of 52.82 ARI, 49.88 NMI, and 28.34 Silhouette. Furthermore, when evaluated on this challenging benchmark with more cellular subtypes, the proposed approach still outperforms contrastive-sc [7] by 12.74 ARI, 13.10 NMI, and 20.29 Silhouette. We also achieve highly better results against ScDeepCluster [5], the improved clustering baseline by adding a clustering layer to the embedding space in deep count autoencoder. These results validate the effectiveness of our approach in learning discriminative features with patch-level interconnections from gene expression matrices for each cellular subtype.

To qualitatively evaluate scRNA-seq clustering, we compare the proposed mask-sc with PCA [3], ScDeepCluster [5], ScziDesk [6], and contrastive-sc [7] on the 10 PBMC cells dataset with 8 cellular subtypes in Figure 4. To better evaluate the quality, we visualize the learned representations of each cellular subtype by t-SNE [61]. Note that each spot denotes the feature of one cell, and each color refers to one cellular subtype, such as "subtype 5" in yellow and "subtype 7" in gray. From comparisons, three main observations can be derived: 1) with the raw gene expression sequence as input, PCA [3], the traditional machine learning approach, fails to predict some cellular subtypes, including "subtype 0" and "subtype 4"; 2) the quality of cellular subtypes generated by our method is much better than the strong self-supervised baseline, contrastive-sc [7]. 3) the proposed mask-sc achieves competitive even better results on clustering results against ScDeepCluster [5], the improved deep count autoencoder with a clustering layer added to the embedding space. Furthermore, as can be observed in the last column, gene expression representations extracted by the proposed mask-sc are both intra-subtype compact and inter-subtype separable. In contrast to our compact embeddings in the cellular subtype semantic space, there still exists mixtures of multiple cellular subtypes among features learned by ScziDesk [6]. These meaningful visualizations further showcase the superiority of our mask-sc with input matrix embedding and multi-modal encoder-decoder for masked expression modeling in extracting compact gene expression representations for scRNA-seq clustering.

4.3 Experimental Analysis

In this section, we performed ablation studies to validate the benefit of introducing the Input Matrix Embeddings for masked expression modeling and Multimodal Encoder-Decoder for sequence-guided reconstruction. We also conducted extensive experiments to explore the effect of input expression matrix size and mask ratio on scRNA-seq clustering. Furthermore, we visualized learned attention of self-attention layers in the gene expression encoder after self-supervised training.

Input Matrix Embeddings & Multimodal Encoder-Decoder. To demonstrate the effectiveness of the introduced Input Matrix Embeddings for masked expression modeling (IME) and Multimodal Encoder-Decoder for sequence-guided reconstruction, we ablate the necessity of each module and

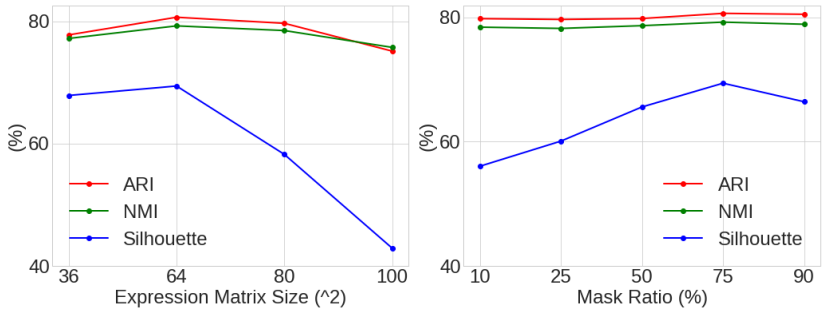

Figure 5: Effect of gene expression matrix size and mask ratio on the final performance of scRNA-seq clustering. The proposed mask-sc achieves the best clustering performance in terms of all metrics when the expression matrix size is 64×64 and the mask ratio is 75%.

report the quantitative comparison results in Table 2. We can observe that adding IME to the vanilla baseline highly raises the results of scRNA-seq clustering by 4.35 ARI, 3.26 NMI, and 1.66 Silhouette, which validates the benefit of Input Matrix Embeddings in learning discriminative expression representations for discovering accurate cellular subtypes. Meanwhile, introducing only MED in the baseline also increases the clustering performance in terms of all metrics, which indicates the importance of reconstructing sequence-level features for scRNA-seq clustering. More importantly, incorporating IME for masked expression modeling and MED for sequence-guided reconstruction together into the baseline significantly raises the performance by 12.01 ARI, 6.69 NMI, and 10.68 Silhouette. These improving results demonstrate the importance of IME for masked expression modeling and MED for sequence-guided reconstruction in learning discriminative representations with interconnections across different genes for each cell.

Effect of expression matrix size and mask ratio. The expression matrix size and mask ratio used in the proposed masked expression modeling approach affect the extracted expression representations for scRNA-seq clustering. To explore such effects more comprehensively, we ablated the expression matrix size from $\{36 \times 36, 64 \times 64, 80 \times 80, 100 \times 100\}$ and varied the mask ratio from $\{10\%, 25\%, 50\%, 75\%, 90\%\}$. The comparison results of scRNA-seq clustering are shown in Figure 5. When the expression matrix size is 64×64 and the mask ratio is 75%, we achieve the best clustering performance in terms of all metrics. With the increase of expression matrix size from 36×36 to 64×64, the proposed mask-sc consistently raises results, which shows the importance of utilizing input matrix embeddings for masked expression modeling to learning compact representation from interconnections of more genes in each cell. However, increasing the expression matrix size from 64×64 to 80×80 and 100×100 will not continually improve the results of ARI and NMI. In particular, a drastic drop can be observed in the Silhouette score, which means that the generated clusters are not dense and well-separable. This is might be caused by the high sparsity of non-zero values in the gene expression matrix. In this case, masked expression modeling without discriminating this sparsity will deteriorate the quality of expression representations pre-trained from many zero entries in the input matrix.

In terms of mask ratio, the performance of the proposed mask-sc climbs with the increase of the mask ratio from 10% to 75%. Compared to the Silhouette score, there are no significant changes in ARI and NMI. This interesting trend could be due to the self-property of these metrics. Higher ARI and NMI indicate that the predicted cluster assignment matches the ground-truth cluster assignment, while a larger value of the Silhouette score refers to denser and better-separated clusters. The former metrics can not measure the quality of expression embeddings extracted from the pre-trained gene expression encoder, but the latter is a strict metric for measuring the compactness of learned expression representations for scRNA-seq clustering. Meanwhile, when the mask ratio is increased to 90%, the Silhouette score drops significantly but ARI and NMI decrease at an insignificant range. This decreasing trend is also observed in the highly influential masked image modeling approach [45]. For masked expression modeling, we are the first to observe such an effect on input matrix embeddings for extracting discriminative representations from gene expression matrices to conduct clustering on single-cell data.

Learned attention showcases high-frequency interconnections. High-frequency interconnections across different genes are essential for us to learn compact and discriminative representations of scRNA-seq clustering. To better understand how the model captures interconnections across genes during masked expression modeling, we visualized input gene expression matrices with learned attention from the last self-attention layer of the gene expression encoder in Figure 6. Specifically, we computed the mean attention value of the last self-attention layer with 12 heads in the gene expression encoder. As can be seen, the pre-trained encoder can successfully learn high-frequency interconnections across different genes for each input gene expression matrix. In addition, similar interconnection patterns can be observed in the same cellular subtypes. For instance, the gene expression matrix at the location (Row 2, Column 4) has a similar interconnection pattern as the ones at the location (Row 5, Column 2) and location (Row 6, Column 6). Meanwhile, these three gene expression matrices are indeed from the same cellular subtype. Another similar case can be seen in gene expression matrices at location (Row 5, Column 1), location (Row 3, Column 1), and location (Row 5, Column 5).

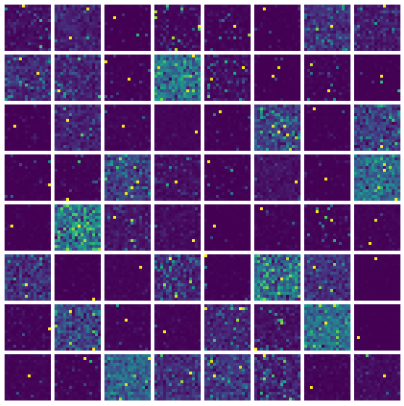

Figure 6: Visualization of input gene expression matrices with attention from the last attention layer of the self-supervised gene expression encoder trained on 10 PBMC cells dataset. We can observe that the pre-trained encoder can learn diverse interconnections across genes for each input expression matrix. Similar cellular subtypes have similar interconnection patterns, such as expression matrices at the locations (Row 2, Column 4), (Row 5, Column 2), and (Row 6, Column 6).

Overall, these meaningful visualizations further demonstrate the effectiveness of the proposed mask-sc with input matrix embeddings for masked expression modeling and multimodal encoder-decoder for sequence-guided reconstruction in learning discriminative representations from interconnects across various genes for scRNA-seq clustering. Regarding future applications, attention-based interconnection patterns potentially provide a discriminative input to replace both raw expression sequence and sequence features for discovering cellular subtypes. The benefit of using attention-based interconnections for scRNA-seq clustering will not depend on well-annotated data from human experts. Instead, these interconnections can still be derived from unsupervised pre-training from large-scale gene expression data without high-quality annotations, which is much cheaper than labeling cellular subtypes from high-dimensional gene expression data.

5 Conclusion

In this work, we present mask-sc, a novel multimodal self-supervised framework with masked expression modeling on single-cell data, that can learn discriminative gene expression representations by reconstructing sequence-level features of masked expressions for scRNA-seq clustering. Our mask-sc potentially aggregates interconnections across grouped expression sequences through a self-supervised expression encoder with random masked gene expression matrices as input. Then, we leverage a sequence-guided decoder is leveraged to reconstruct sequence-level features of masked expression matrices. Extensive experiments on two real scRNA-seq datasets demonstrate the state-of-the-art performance of our proposed method.

Limitation. Although the proposed mask-sc achieves superior results on single-cell clustering of scRNA-seq data, the performance gains of our approach on human organs are not significant. One possible reason is that our model easily overfits the pretext task of masked expression modeling during pre-training, and the solution is to incorporate dropout and momentum encoders together for masked expression modeling. Meanwhile, we observe that our model performs worse on the "Quake Smart seq2 Limb Muscle" dataset with a smaller training size. The future work could add more training data or incorporate contrastive learning with masked expression modeling to increase the compactness of generated clusters.

Broader Impact. The proposed approach successfully learns discriminative representations of interconnections across different genes for each cell from manually-collected datasets, which might cause the model to learn internal biases in the data. For instance, the model could fail to discover unseen but crucial cellular subtypes. Therefore, these issues should be addressed for the deployment of real applications in open-world problems.

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