AN EMPIRICAL STUDY OF EXAMPLE FORGETTING DURING DEEP NEURAL NETWORK LEARNING

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Paper under double-blind review

ABSTRACT

Inspired by the phenomenon of catastrophic forgetting, we investigate the learning dynamics of neural networks as they train on single classification tasks. Our goal is to understand whether a related phenomenon occurs when data does not undergo a clear distributional shift. We define a "forgetting event" to have occurred when an individual training example transitions from being classified correctly to incorrectly over the course of learning. Across several benchmark data sets, we find that: (i) certain examples are forgotten with high frequency, and some not at all; (ii) a data set's (un)forgettable examples generalize across neural architectures; and (iii) based on forgetting dynamics, a significant fraction of examples can be omitted from the training data set while still maintaining state-of-the-art generalization performance.

1 INTRODUCTION

Many machine learning models, in particular neural networks, cannot perform *continual learning*. They have a tendency to forget previously learnt information when trained on new tasks, a phenomenon usually called *catastrophic forgetting* (Kirkpatrick et al., 2017; Ritter et al., 2018). One of the hypothesized causes of catastrophic forgetting in neural networks is the shift in the input distribution across different tasks—*i.e.*, a lack of common factors or structure in the inputs of different tasks might lead standard optimization techniques to converge to radically different solutions each time a new task is presented. In this paper, we draw inspiration from this phenomenon and investigate the extent to which a related forgetting process occurs as a model learns examples traditionally considered to belong to *the same task*.

Similarly to the continual learning setting, in stochastic gradient descent (SGD) optimization, each mini-batch can be considered as a mini-"task" presented to the network sequentially. In this context, we are interested in characterizing the learning dynamics of neural networks by analyzing (catastrophic) *example forgetting events*. These occur when examples that have been "learnt" (*i.e.*, correctly classified) at some time t in the optimization process are subsequently misclassified — or in other terms forgotten — at a time $t' > t$. We thus switch the focus from studying interactions between sequentially presented tasks to studying interactions between sequentially presented dataset examples during SGD optimization. Our starting point is to understand whether there exist examples that are consistently forgotten across subsequent training presentations and, conversely, examples that are never forgotten. We will call the latter *unforgettable* examples. We hypothesize that specific examples consistently forgotten between subsequent presentations, if they exist, must not share commonalities with other examples from the same task. We therefore analyze the proportion of forgettable/unforgettable examples for a given task and what effects these examples have on a model's decision boundary and generalization error.

The goal of our investigation is two-fold. First, we attempt to gain insight into the optimization process by analyzing interactions among examples during learning and their influence on the final decision boundary. We are particularly interested in whether we can glean insight on the compressibility of a dataset, and thereby increase data efficiency without compromising generalization accuracy. It is a timely problem that has been the recent focus of few-shot learning approaches via meta-learning (Finn et al., 2017; Ravi & Larochelle, 2017). Second, we aim to characterize whether forgetting statistics can be used to identify "important" samples and detect *outliers* and examples with noisy labels (John, 1995; Brodley & Friedl, 1999; Sukhbaatar et al., 2014; Jiang et al., 2018).

Identifying important, or most informative examples is an important line of work and was extensively studied in the literature. Techniques of note — among others — are predefined curricula of examples (Bengio & LeCun, 2007), *self-paced* learning (Kumar et al., 2010), and more recently meta-learning (Fan et al., 2017). These research directions usually define "hardness" or "commonality" of an example as a function of the loss on that particular example at some point during training (or possibly at convergence). They do not consider whether some examples are consistently forgotten throughout learning. Very recently, Chang et al. (2017) consider re-weighting examples by accounting for the variance of their predictive distribution. This is related to our definition of forgetting events, but the authors provide little analysis of the extent to which the phenomenon occurs in their proposed tasks. Our purpose is to study this phenomenon from an empirical standpoint and characterize its prevalence in different datasets and across different model architectures.

Our experimental findings suggest that: a) there exist a large number of *unforgettable* examples, *i.e.*, examples that are never forgotten once learnt, and those examples are correlated from one neural architecture to another; b) examples with noisy labels are among the most forgotten examples, along with images with "uncommon" features, visually complicated to classify; c) training a neural network on a dataset where a very large fraction of the least forgotten examples have been removed still results in extremely competitive performance on the test set.

2 Related Work

Curriculum Learning and Sample Weighting Curriculum learning is a paradigm that favors learning along a curriculum of examples of increasing difficulty (Bengio et al., 2009). This general idea has found success in a variety of areas since its introduction (Kumar et al., 2010; Lee & Grauman, 2011; Schaul et al., 2015). Kumar et al. (2010) implemented their curriculum by identifying easy examples as those which have a smaller loss. In our experiments, we will empirically validate that unforgettable examples can be safely removed without compromising strong generalization performance. Zhao & Zhang (2015); Katharopoulos & Fleuret (2018) relate sample importance to the norm of the loss gradient computed on the examples with respect to the parameters of the network. More recently, Fan et al. (2017); Kim & Choi (2018); Jiang et al. (2018) aim to learn a curriculum directly from data in order to minimize the task loss. Jiang et al. (2018) also study the robustness of their method in the context of noisy examples. This relates to a rich literature on the detection of outliers and the removal of examples with noisy labels (John, 1995; Brodley & Friedl, 1999; Sukhbaatar et al., 2014; Jiang et al., 2018). We will provide evidence that noisy examples rank higher with respect to the number of forgetting events.

Deep Generalization The study of the generalization properties of deep neural networks when trained by stochastic gradient descent has been the focus of several recent publications (Zhang et al., 2016; Keskar et al., 2016; Chaudhari et al., 2016; Advani & Saxe, 2017). These studies suggest that the generalization error does not depend solely on the complexity of the hypothesis space. For instance, it has been demonstrated that over-parameterized models with many more parameters than training points can still achieve low test error (Huang et al., 2017; Wang et al., 2018) while being complex enough to fit a dataset with completely random labels (Zhang et al., 2016). A possible explanation for this phenomenon is a form of implicit regularization performed by stochastic gradient descent: deep neural networks trained with sgd have been recently shown to converge to the maximum margin solution in the linearly separable case (Soudry et al., 2017; Xu et al., 2018). In our work, we provide empirical evidence that generalization can be maintained when removing a substantial portion of the training examples and without restricting the complexity of the hypothesis class. This goes along the support vector interpretation provided by Soudry et al. (2017).

3 Defining and Computing Example Forgetting

Our general case study for example forgetting is a standard classification setting. Given a dataset $\mathcal{D} = (\mathbf{x}_i, y_i)_i$ of observation/label pairs, we wish to learn the conditional probability distribution $p(y|\mathbf{x};\theta)$ using a deep neural network with parameters θ. The network is trained to minimize the empirical risk $R = \frac{1}{|\mathcal{D}|}\sum_i L(p(y_i|\mathbf{x}_i;\theta), y_i)$, where L denotes the cross-entropy loss. The minimization is performed using variations of stochastic gradient descent, starting from initial random parameters θ^0, and by sampling examples at random from the dataset $\mathcal{D}$.

Forgetting and learning events We denote $\hat{y}_i^t = \arg\max_k p(y_{ik}|\mathbf{x}_i;\theta^t)$ the predicted label (out of K possible labels) for the example $\mathbf{x}_i$ obtained after t steps of SGD. We also let $\text{acc}_i^t = \mathbb{1}_{\hat{y}_i^t = y_i}$ be a binary variable indicating that the example is correctly classified at time step t. Example i undergoes a *forgetting event* when acc_i^t decreases between two consecutive updates: $\text{acc}_i^t > \text{acc}_i^{t+1}$. In other words, example i is misclassified at step $t+1$ after having been correctly classified at step t. Conversely, a *learning event* has occurred if $\text{acc}_i^t < \text{acc}_i^{t+1}$. Statistics that will be of interest in the next sections include the distribution of forgetting events across examples and the first time a learning event occurs.

Classification margin We will also be interested in analyzing the classification margin. Our predictors have the form $p(y_i|\mathbf{x}_i;\theta) = \sigma(\beta(\mathbf{x}_i))$, where σ is a sigmoid (softmax) activation function in the case of binary (categorical) classification. The classification margin m is defined as the difference between the logit of the correct class and the largest logit among the other classes, *i.e.* $m = \beta_k - \arg\max_{k' \neq k} \beta_{k'}$, where k is the index corresponding to the correct class.

Unforgettable examples We qualify examples as *unforgettable* if they are learnt at some point and experience no forgetting events during the whole course of training: example i is unforgettable if the first time it is learnt t^* verifies $t^* < \infty$ and for all $k \geq t^*$, $\text{acc}_i^k = 1$. Note that, according to this definition, examples that are never learnt during training do not qualify as unforgettable. We refer to examples that have been forgotten at least once as *forgettable*.

3.1 Procedural Description and Experimental Setting

Following the previous definitions, monitoring forgetting events entails computing the prediction for all examples in the dataset at each model update, which would be prohibitively expensive. In practice, for each example, we subsample the full sequence of forgetting events by computing forgetting statistics only when the example is included in the current mini-batch; that is, we compute forgetting across *presentations* of the same example in subsequent mini-batches. This gives a lower bound on the number of forgetting events an example undergoes during training.

We train a classifier on a given dataset and record the forgetting events for each example when they are sampled in the current mini-batch. For the purposes of further analysis, we then sort the dataset's examples based on the number of forgetting events they undergo. Ties are broken at random when sampling from the ordered data. Samples that are never learnt are considered forgotten an infinite number of times for sorting purposes.

Algorithm 1 Computing forgetting statistics.

initialize $\text{prev_acc}_i = 0, i \in \mathcal{D}$
initialize forgetting $T[i] = 0, i \in \mathcal{D}$
while not training done **do**
  $B \sim \mathcal{D}$ # sample a minibatch
  for example $i \in B$ **do**
    compute acc_i
    if $\text{prev_acc}_i > \text{acc}_i$ **then**
      $T[i] = T[i] + 1$
      $\text{prev_acc}_i = \text{acc}_i$
  gradient update classifier on B
return T

We perform our experimental evaluation on three datasets of increasing complexity: *MNIST* (LeCun et al., 1999), a permuted version of *MNIST*, and *CIFAR-10* (Krizhevsky, 2009). We use various model architectures and training schemes that yield test errors comparable with the current state-of-the-art on the respective datasets. In particular, the *MNIST*-based experiments use a network comprised of two convolutional layers followed by a fully connected one, trained using SGD with momentum and dropout. This network achieves 0.8% test error. For *CIFAR-10*, we use a ResNet with cutout (DeVries & Taylor, 2017) trained using SGD and momentum with a particular learning rate schedule. This network achieves a competitive 3.99% test error. For full details of the experimentation, see the Supp. Mat.

4 Characterizing Example Forgetting

4.1 Distribution of forgetting events

Number of forgetting events We estimate the number of forgetting events of all the training examples for the three different datasets (*MNIST*, *permutedMNIST* and *CIFAR-10*) across 5 random seeds. The histograms of forgetting events computed from one seed are shown in Figure 1. Across

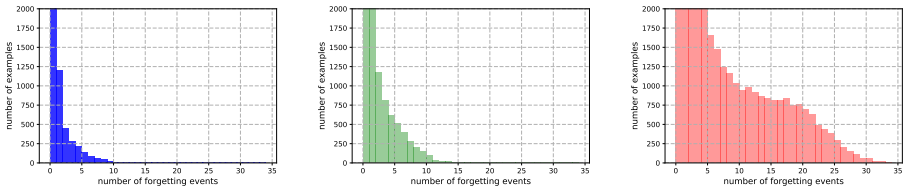


Figure 1: Histograms of forgetting events on (from left to right) *MNIST*, *permutedMNIST* and *CIFAR-10*. Rescaled view where the number of examples have been capped between 0 and 2000. The full view is available in Supplementary Figure 8

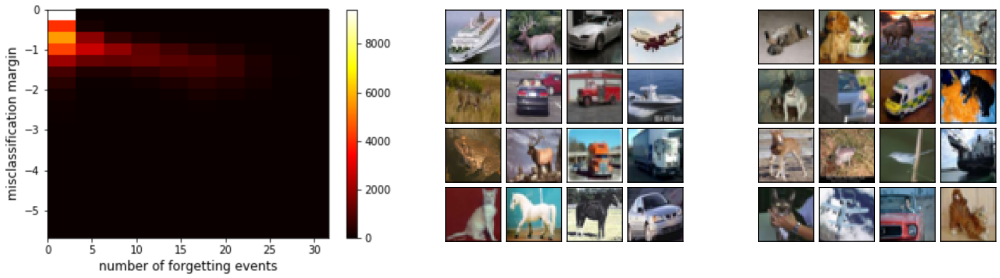


Figure 2: *Left* 2D-histogram of the number of forgetting events and mean misclassification margin across all examples of *CIFAR-10*. There is significant negative correlation (-0.74, Spearman rank correlation) between mean misclassification margin and the number of forgetting events. *Middle and Right* Pictures of unforgettable (*Middle*) and forgettable examples (*Right*). Forgettable examples seem to exhibit peculiar or uncommon features.

5 seeds, there are $55,012$, $45,181$ and $15,628$ unforgettable examples, which represent respectively 91.7%, 75.3%, and 31.3% of the corresponding training sets. Note that datasets with less complexity and diversity of examples, such as *MNIST*, seem to contain significantly more unforgettable examples. *permutedMNIST* exhibits a complexity balanced between *MNIST* (easiest) and *CIFAR-10* (hardest). This finding seems to suggest a correlation between forgetting statistics and the intrinsic dimension of the learning problem, as recently formalized by Li et al. (2018).

First learning events We investigate whether unforgettable and forgettable examples need to be presented different numbers of times in order to be learnt for the first time (*i.e.* for the first learning event to occur, as defined in Sec.3). The distributions of the presentation numbers at which first learning events occur across all datasets can be seen in Supplementary Fig. 9. We observe that, while both unforgettable and forgettable sets contain many examples that are learnt during the first 3-4 presentations, the forgettable examples contain a larger number of examples that are first learnt later in training. The Spearman rank correlation between the first learning event presentations and the number of forgetting events across all training examples is 0.56, indicating a moderate relationship.

Misclassification margin The definition of forgetting events is binary and as such fairly crude compared to more sophisticated estimators of example relevance (Zhao & Zhang, 2015; Chang et al., 2017). In order to qualify its validity, we compute the misclassification margin of forgetting events. The misclassification margin of an example is defined as the mean classification margin (defined in Sec.3) over all its forgetting events, a negative quantity by definition. The Spearman rank correlation between forgetting events and misclassification margin is -0.74 (computed over 5 seeds). Examples which are frequently forgotten have a large misclassification margin.

Visual inspection We visualize some of the unforgettable examples in Figure 2 along with some examples that have been most forgotten in the *CIFAR-10* dataset. Unforgettable samples are easily recognizable and contain the most obvious class attributes or centered objects, *e.g.* a plane on a

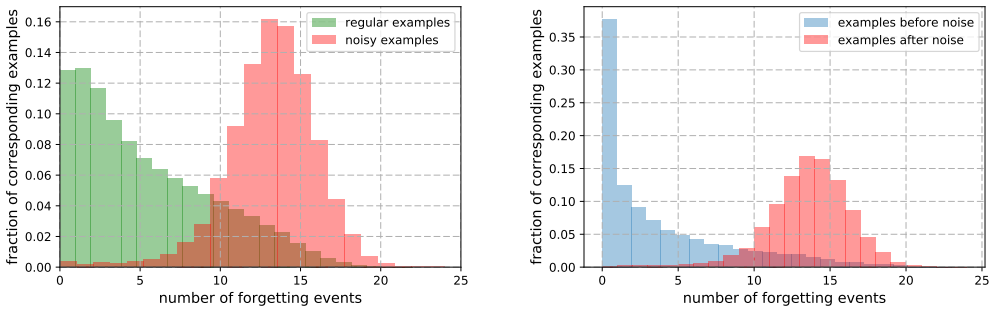


Figure 3: Distributions of forgetting events across training examples in *CIFAR-10* when 20% of labels are randomly changed. *Left.* Comparison of forgetting events between examples with noisy and original labels. The most forgotten examples are those with noisy labels and no noisy examples are unforgettable. *Right.* Comparison of forgetting events between examples with noisy labels and the same examples with their original labels. The majority of examples exhibit more forgetting when their labels are changed.

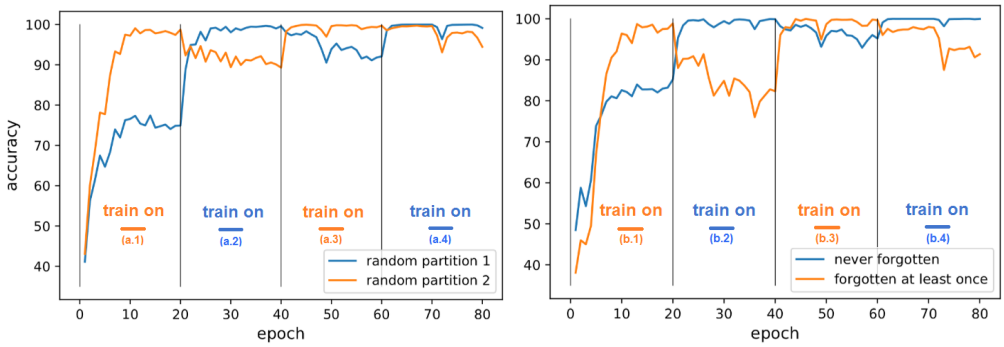
clear sky. On the other hand, the most forgotten examples exhibit more ambiguous characteristics (as in the center image, a truck on a brown background) that may not align with the learning signal common to other examples from the same class.

Detection of noisy examples We further investigate the observation that the most forgettable examples seem to exhibit atypical characteristics. We would expect that if highly forgettable examples have atypical class characteristics, then noisily-labeled examples will undergo more forgetting events. We randomly change the labels of 20% of *CIFAR-10* and record the number of forgetting events of both the noisy and regular examples through training. The distributions of forgetting events across noisy and regular examples are shown in Figure 3. We observe that the most forgotten examples are those with noisy labels and that no noisy examples are unforgettable. We also compare the forgetting events of the noisy examples to that of the same set of examples with original labels and observe a much higher degree of forgetting in the noisy case. The results of these synthetic experiments support the hypothesis that highly forgettable examples exhibit atypical class characteristics.

4.2 Continual Learning Setup

We observed that in harder tasks such as *CIFAR-10*, a significant portion of examples are forgotten at least once during learning. This leads us to believe that catastrophic forgetting may be observed, to some extent, even when considering examples coming from the same task distribution. To test this hypothesis, we perform an experiment inspired by the standard continual learning setup (McCloskey & Cohen, 1989; Kirkpatrick et al., 2017). We create two tasks by randomly sampling 10k examples from the *CIFAR-10* training set and dividing them in two equally-sized partitions (5k examples each). We then train a classifier for 20 epochs on each partition sequentially, while tracking performance on both partitions. The results are reported in Figure 4 (a). We observe some forgetting of the second task when we only train on the first task (panel (a.2)). This is somewhat surprising as the two tasks contain examples from the same underlying distribution.

In Figure 4 (b), we report results obtained by randomly sampling examples based on forgetting statistics. That is, we first compute the forgetting events based on Algorithm 1 and we create our tasks by sampling 5k examples that have zero forgetting events (named f0) and 5k examples that have non-zero forgetting events (named fN). We observe that examples that have been forgotten at least once suffer a more drastic form of forgetting than those included in a random split (compare (a.2) with (b.2)). In panel (b.3), we can observe that examples from task f0 suffer very mild forgetting when training on task fN. This suggests that examples that have been forgotten at least once may be able to "support" those that have never been forgotten.



(a) random partitions

(b) partitioning by forgetting events

Figure 4: The synthetic continual learning setup for *CIFAR-10* highlights that examples that have been forgotten at least once can "support" those that have never been forgotten.

5 Removing Unforgettable Examples

As shown in the previous section, learning on examples that have been forgotten at least once minimally impacts performance on those that are unforgettable. This appears to indicate that unforgettable examples are less informative than others, and, more generally, that the more an example is forgotten during training, the more useful it may be to the classification task. This seems to align with the observations done in Chang et al. (2017), where the authors re-weight training examples by accounting for the variance of their predictive distribution. Here, we test whether it is possible to completely remove a given subset of examples during training.

In Fig. 5 (*Left*), we show the evolution of the generalization performance in *CIFAR-10* when we artificially remove examples from the training dataset. We choose the examples to remove by increasing number of forgetting events. Each point in the figure corresponds to retraining the model from scratch on an increasingly smaller subset of the training data (with the same hyper-parameters as the base model). We observe that when removing a random subset of the dataset, performance rapidly decreases. Comparatively, by removing examples ordered by number of forgetting events, 30% of the dataset can be removed while maintaining comparable generalization performance as the base model trained on the full dataset, and up to 35% can be removed with marginal degradation (less than 0.2%). The results on the other datasets are similar, a large fraction of training examples can be ignored without hurting the final generalization performance of the classifiers (Figure 6).

In Figure 5 (*Right*), we show the evolution of the generalization error when we remove from the dataset $5,000$ examples with increasing forgetting statistics. Each point in the figure corresponds to the generalization error of a model trained on the full dataset minus $5,000$ examples as a function of the average number of forgetting events in those $5,000$ examples. As can be seen, removing the same number of examples with increasingly more forgetting events results in worse generalization for most of the curve. It is interesting to notice the rightmost part of the curve moving up, this suggests that some of the most forgotten examples actually hurt performance. Those can correspond to outliers or mislabeled examples (also see Sec. 4.1). Finding a way to separate those points from very informative ones is an ancient but still active area of research (John, 1995; Jiang et al., 2018).

Support vectors Various explanations of the *implicit generalization* of deep neural networks (Zhang et al., 2016) have been offered: flat minima generalize better and stochastic gradient descent converges towards them (Hochreiter & Schmidhuber, 1997; Kleinberg et al., 2018), gradient descent protects against overfitting (Advani & Saxe, 2017; Tachet et al., 2018), deep networks' structure biases learning towards simple functions (Neyshabur et al., 2014; Perez et al., 2018). But it remains a poorly understood phenomenon. An interesting direction of research is to study the convergence properties of gradient descent in terms of maximum margin classifiers. It has been shown recently (Soudry et al., 2017) that on separable data, a linear network will learn such a maximum margin classifier. This supports the idea that stochastic gradient descent implicitly converges to solutions that maximally separate the dataset, and additionally, that some data points are more relevant than

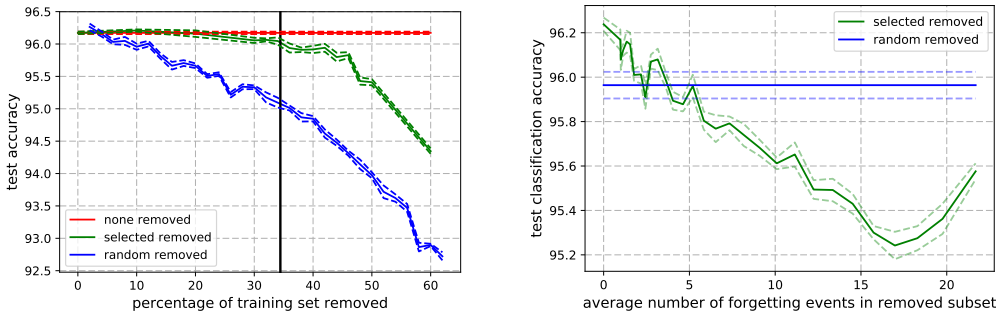


Figure 5: *Left* Generalization performance on *CIFAR-10* of ResNet18 where increasingly larger subsets of the training set are removed (mean +/- std error of 5 seeds). When the removed examples are selected at random, performance drops very fast. Selecting the examples according to our ordering can reduce the training set significantly without affecting generalization. The vertical line indicates the point at which all unforgettable examples are removed from the training set. *Right* Difference in generalization performance when contiguous chunks of 5000 increasingly forgotten examples are removed from the training set. Most important examples tend to be those that are forgotten the most.

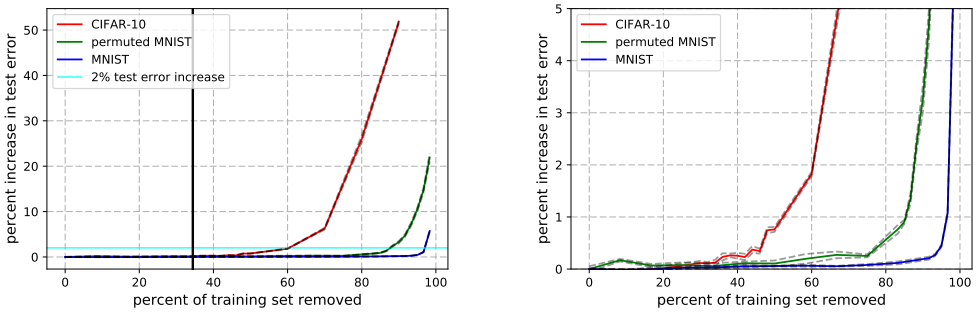


Figure 6: Decrease in generalization performance when fractions of the training sets are removed. When the subsets are selected appropriately, performance is maintained after removing up to 30% of *CIFAR-10*, 50% of *permutedMNIST*, and 80% of *MNIST*. Vertical black line indicates the point at which all unforgettable examples are removed. *Right* is a zoomed in version of *Left*.

others to the decision boundary learnt by the classifier. Those points play a part equivalent to support vectors in the *support vector machine* paradigm. Our results confirm that a significant portion of training data points have little to no influence on the generalization performance when the decision function is learnt with SGD. Forgettable training points may be considered as analogs to *support vectors*, important for the generalization performance of the model. The number of forgetting events of an example is a relevant metric to detect such support vectors. It also correlates well with the misclassification margin (see Sec.4.1) which is a proxy for the distance to the decision boundary.

Intrinsic dataset dimension As mentioned above, the datasets we study have various fractions of unforgettable events (91.7% for *MNIST*, 75.3% for *permutedMNIST* and 31.3% for *CIFAR-10*). We also see in Figure 6 that performance on those datasets starts to degrade at different fractions of removed examples: the number of support vectors varies from one dataset to the other, based on the complexity of the underlying data distribution. This last statement can be put in perspective with the intrinsic dataset dimension defined by Li et al. (2018) as the codimension in the parameter space of the solution set: for a given architecture, the higher the intrinsic dataset dimension, the larger the number of support vectors, and the fewer the number of unforgettable examples.

6 TRANSFERABLE FORGETTING EVENTS

Our definition of forgetting events is based on training a given architecture, with a given optimizer for a given number of time steps. We investigate to what extent the forgetting statistics of examples depend on those factors.

Across seeds To test the stability of our metric with respect to the variance generated by stochastic gradient descent, we compute the number of forgetting events per example for 10 different random seeds and measure their correlation. From one seed to another, the average Pearson correlation is 89.2%. When randomly splitting the 10 different seeds into two sets of 5, the cumulated number of forgetting events within those two sets shows a high correlation of 97.6%.

Throughout training We compute the Spearman rank correlation between the ordering obtained at the end of training (200 epochs) and the ordering after various number of epochs. As seen in Fig. 7 (*Left*), the ordering is very stable after 75 epochs, and we found a reasonable number of epochs to get a good correlation to be 25 (see the Supp. Mat. for precision-recall plots).

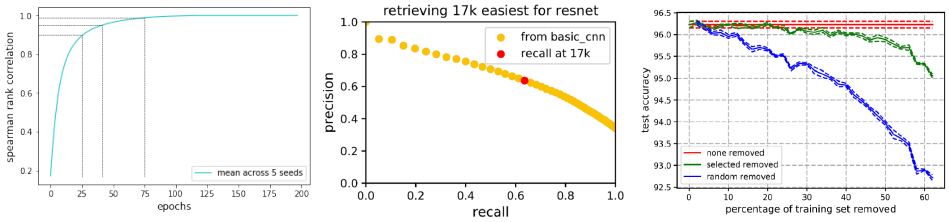


Figure 7: *Left.* Ranking of examples by forgotten events stabilizes after 75 epochs in *CIFAR-10*. *Middle.* Precision and recall of retrieving the unforgettable examples of ResNet18, using the example ordering of a simpler convolutional neural network. *Right.* Generalization performance on *CIFAR-10* of a WideResNet using the example ordering of ResNet18.

Between architectures A limitation of our method is that it requires computing the ordering from a previous run. An interesting question is whether that ordering could be obtained from a simpler architecture than residual networks. We train a network with two convolutional layers followed by two fully connected ones (see Supp. Mat. for the full architecture) and compare the resulting ordering with the one obtained with the ResNet18. Figure 7 (*Middle*) shows a precision-recall plot of the unforgettable examples computed with the residual network. We see a reasonably strong agreement between the unforgettable examples of the convolutional neural network and the ones of the ResNet18. Finally, we train a WideResNet (Zagoruyko & Komodakis, 2016) on truncated data sets (using the example ordering from ResNet18) and plot its generalization performance in Figure 7 (*Right*): the network still performs near optimally with 30% of the dataset removed. This opens up promising avenues where forgetting statistics could be computed by smaller architectures.

7 CONCLUSION AND FUTURE WORK

In this paper, inspired by the phenomenon of catastrophic forgetting, we investigate the learning dynamics of neural networks when training on single classification tasks. We show that catastrophic forgetting can occur in the context of what is usually considered to be a single task. Inspired by this result, we find that some examples within a task are more prone to being forgotten, while others are consistently unforgettable. We also find that forgetting statistics seem to be fairly stable with respect to the various characteristics of training, hinting that they actually uncover intrinsic properties of the data rather than idiosyncrasies of the training schemes. Furthermore, these unforgettable examples seem to play little part in the final performance of the classifier as they can be removed from the training set without hurting generalization. This supports recent research interpreting deep neural networks as max margin classifiers in the linear case.

Future work involves understanding forgetting events better from a theoretical perspective, exploring potential applications to other areas of supervised learning, such as speech or text and to reinforcement learning where forgetting is prevalent due to the continual shift of the underlying distribution.

REFERENCES

Madhu S. Advani and Andrew M. Saxe. High-dimensional dynamics of generalization error in neural networks. *CoRR*, abs/1710.03667, 2017.

Yoshua Bengio and Yann LeCun. Scaling learning algorithms towards AI. In *Large Scale Kernel Machines*. MIT Press, 2007.

Yoshua Bengio, Jérôme Louradour, Ronan Collobert, and Jason Weston. Curriculum learning. In *Proceedings of the 26th annual international conference on machine learning*, pp. 41–48. ACM, 2009.

Carla E Brodley and Mark A Friedl. Identifying mislabeled training data. *Journal of artificial intelligence research*, 11:131–167, 1999.

Haw-Shiuan Chang, Erik Learned-Miller, and Andrew McCallum. Active Bias: Training More Accurate Neural Networks by Emphasizing High Variance Samples. In *Advances in Neural Information Processing Systems*, pp. 1002–1012, 2017.

Pratik Chaudhari, Anna Choromanska, Stefano Soatto, Yann LeCun, Carlo Baldassi, Christian Borgs, Jennifer Chayes, Levent Sagun, and Riccardo Zecchina. Entropy-SGD: Biasing Gradient Descent Into Wide Valleys. *ICLR '17*, 2016.

Terrance DeVries and Graham W Taylor. Improved regularization of convolutional neural networks with cutout. *arXiv preprint arXiv:1708.04552*, 2017.

Yang Fan, Fei Tian, Tao Qin, and Jiang Bian. Learning What Data to Learn. 2017.

Chelsea Finn, Pieter Abbeel, and Sergey Levine. Model-agnostic meta-learning for fast adaptation of deep networks. *In Proc. of ICML*, 2017.

S. Hochreiter and J. Schmidhuber. Flat minima. *Neural Computation*, 9(1):1–42, 1997.

Gao Huang, Zhuang Liu, Laurens van der Maaten, and Kilian Q Weinberger. Densely Connected Convolutional Networks. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 4700–4708, 2017.

Lu Jiang, Zhengyuan Zhou, Thomas Leung, Li-Jia Li, and Li Fei-Fei. MentorNet: Learning data-driven curriculum for very deep neural networks on corrupted labels. In *Proceedings of the 35th International Conference on Machine Learning*. PMLR, 2018.

George H John. Robust decision trees: removing outliers from databases. In *Proceedings of the First International Conference on Knowledge Discovery and Data Mining*, pp. 174–179. AAAI Press, 1995.

Angelos Katharopoulos and Franois Fleuret. Not all samples are created equal: Deep learning with importance sampling. In Jennifer G. Dy and Andreas Krause (eds.), *ICML*, volume 80 of *JMLR Workshop and Conference Proceedings*, pp. 2530–2539. JMLR.org, 2018. URL http://dblp.uni-trier.de/db/conf/icml/icml2018.html#KatharopoulosF18.

Nitish Shirish Keskar, Dheevatsa Mudigere, Jorge Nocedal, Mikhail Smelyanskiy, and Ping Tak Peter Tang. On large-batch training for deep learning: Generalization gap and sharp minima. *arXiv preprint arXiv:1609.04836*, 2016.

Tae-Hoon Kim and Jonghyun Choi. Screenernet: Learning curriculum for neural networks. *CoRR*, abs/1801.00904, 2018. URL http://dblp.uni-trier.de/db/journals/corr/corr1801.html#abs-1801-00904.

Diederik Kingma and Jimmy Ba. Adam: A method for stochastic optimization, 2014. URL http://arxiv.org/abs/1412.6980. cite arxiv:1412.6980Comment: Published as a conference paper at the 3rd International Conference for Learning Representations, San Diego, 2015.

James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, and Others. Overcoming catastrophic forgetting in neural networks. *Proceedings of the national academy of sciences*, pp. 201611835, 2017.

Robert Kleinberg, Yuanzhi Li, and Yang Yuan. An alternative view: When does sgd escape local minima? *CoRR*, abs/1802.06175, 2018. URL http://dblp.uni-trier.de/db/journals/corr/corr1802.html#abs-1802-06175.

Alex Krizhevsky. Learning multiple layers of features from tiny images. 2009. URL https://www.cs.toronto.edu/~kriz/learning-features-2009-TR.pdf.

M Pawan Kumar, Benjamin Packer, and Daphne Koller. Self-Paced Learning for Latent Variable Models. In *Proc. of NIPS*, pp. 1–9, 2010.

Y. LeCun, C. Cortes C., and C. Burges. The mnist database of handwritten digits. 1999. URL http://yann.lecun.com/exdb/mnist/.

Yong Jae Lee and Kristen Grauman. Learning the easy things first: Self-paced visual category discovery. In *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*, pp. 1721–1728. IEEE, 2011.

Chunyuan Li, Heerad Farkhoor, Rosanne Liu, and Jason Yosinski. Measuring the intrinsic dimension of objective landscapes. *CoRR*, abs/1804.08838, 2018. URL http://dblp.uni-trier.de/db/journals/corr/corr1804.html#abs-1804-08838.

Michael McCloskey and Neal J Cohen. Catastrophic interference in connectionist networks: The sequential learning problem. In *Psychology of learning and motivation*, volume 24, pp. 109–165. Elsevier, 1989.

Behnam Neyshabur, Ryota Tomioka, and Nathan Srebro. In search of the real inductive bias: On the role of implicit regularization in deep learning. *CoRR*, abs/1412.6614, 2014. URL http://dblp.uni-trier.de/db/journals/corr/corr1412.html#NeyshaburTS14.

Guillermo Valle Perez, Chico Q. Camargo, and Ard A. Louis. Deep learning generalizes because the parameter-function map is biased towards simple functions. *CoRR*, abs/1805.08522, 2018. URL http://dblp.uni-trier.de/db/journals/corr/corr1805.html#abs-1805-08522.

Sachin Ravi and Hugo Larochelle. Optimization as a model for few-shot learning. *In Proc. of ICLR*, 2017.

Hippolyt Ritter, Aleksandar Botev, and David Barber. Online Structured Laplace Approximations For Overcoming Catastrophic Forgetting. 2018. URL http://arxiv.org/abs/1805.07810.

Tom Schaul, John Quan, Ioannis Antonoglou, and David Silver. Prioritized experience replay. *arXiv preprint arXiv:1511.05952*, 2015.

Daniel Soudry, Elad Hoffer, Mor Shpigel Nacson, Suriya Gunasekar, and Nathan Srebro. The Implicit Bias of Gradient Descent on Separable Data. 2017. URL http://arxiv.org/abs/1710.10345.

Sainbayar Sukhbaatar, Joan Bruna, Manohar Paluri, Lubomir Bourdev, and Rob Fergus. Training convolutional networks with noisy labels. *arXiv preprint arXiv:1406.2080*, 2014.

R. Tachet, M. Pezeshki, S. Shabanian, A. Courville, and Y. Bengio. On the learning dynamics of deep neural networks. 2018. doi: arXiv:1809.06848v1. URL https://arxiv.org/abs/1809.06848.

Huan Wang, Nitish Shirish Keskar, Caiming Xiong, and Richard Socher. Identifying Generalization Properties in Neural Networks. pp. 1–23, 2018. doi: arXiv:1809.07402v1. URL http://arxiv.org/abs/1809.07402.

Tengyu Xu, Yi Zhou, Kaiyi Ji, and Yingbin Liang. Convergence of sgd in learning relu models with separable data. *CoRR*, abs/1806.04339, 2018. URL http://dblp.uni-trier.de/db/journals/corr/corr1806.html#abs-1806-04339.

Sergey Zagoruyko and Nikos Komodakis. Wide residual networks, 2016. URL http://arxiv.org/abs/1605.07146. cite arxiv:1605.07146.

Chiyuan Zhang, Samy Bengio, Moritz Hardt, Benjamin Recht, and Oriol Vinyals. Understanding deep learning requires rethinking generalization. *arXiv preprint arXiv:1611.03530*, 2016.

Peilin Zhao and Tong Zhang. Stochastic Optimization with Importance Sampling for Regularized Loss Minimization. In *Proc. of ICML*, 2015.

8 EXPERIMENTATION DETAILS

Detailed distributions

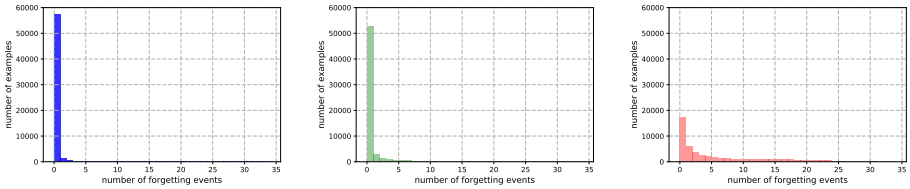


Figure 8: Full view of forgetting event counts on (from left to right) *MNIST*, *permutedMNIST* and *CIFAR-10*.

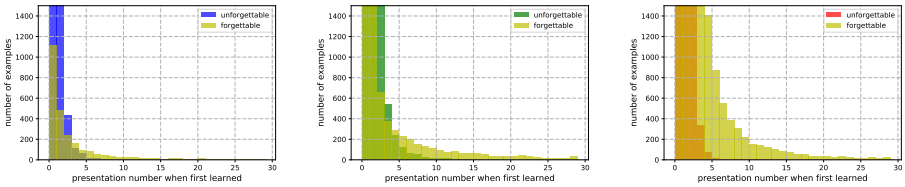


Figure 9: From left to right, distributions of the first presentation at which each unforgettable and forgettable example was learned in *MNIST*, *permutedMNIST* and *CIFAR-10* respectively. Rescaled view where the number of examples have been capped between 0 and 1500 for visualization purposes. Unforgettable examples are generally learnt early during training, thus may be considered as "easy" in the sense of Kumar et al. (2010), i.e. may have a low loss during most of the training.

permutedMNIST The *permutedMNIST* data set is obtained by applying a fixed random permutation of the pixels to all the images of the standard *MNIST* data set. This typically makes the data set harder to learn for convolutional neural networks as local patterns, *e.g.* the horizontal bar of the 7, get shuffled. This statement is supported by the two following facts:

- The number of unforgettable examples for *permutedMNIST* is 45181 versus 55012 for *MNIST*.
- The intrinsic data set dimension (Li et al., 2018) of *permutedMNIST* is 1400 compared to 290 for the untouched data set.

Network Architectures

We use a variety of different architectures in the main text. Below are their specifications.

The architecture for the *MNIST* and *permutedMNIST* experiments is the following:

1. a first convolutional layer with 5 by 5 filters and 10 feature maps,
2. a second convolutional layer with 5 by 5 filters and 20 feature maps,
3. a fully connected layer with 50 hidden units
4. the output layer, with 10 logits, one for each class.

We apply ReLU nonlinearities to the feature maps and to the hidden layer. The last layer is passed through a softmax to output probabilities for each class of the data set.

The ResNet18 architecture used for *CIFAR-10* is described thoroughly in DeVries & Taylor (2017), its implementation can be found at `https://github.com/uoguelph-mlrg/Cutout`.

The second one is a WideResNet (Zagoruyko & Komodakis, 2016), with a depth of 28 and a widen factor of 10. We used the implementation found at `https://github.com/meliketoy/wide-resnet.pytorch`.

The convolutional architecture used in Section 6 is the following:

1. a first convolutional layer with 5 by 5 filters and 6 feature maps,
2. a 2 by 2 max pooling layer
3. a second convolutional layer with 5 by 5 filters and 16 feature maps,
4. a first fully connected layer with 120 hidden units
5. a second fully connected layer with 84 hidden units
6. the output layer, with 10 logits, one for each class.

Optimization

The *MNIST* networks are trained to minimize the cross-entropy loss using stochastic gradient descent with a learning rate of 0.01 and a momentum of 0.5.

The ResNet18 is trained using cutout, data augmentation and stochastic gradient descent with a 0.9 Nesterov momentum and a learning rate starting at 0.1 and divided by 5 at epochs 60, 120 and 160.

The WideResNet is trained using Adam (Kingma & Ba, 2014) and a learning rate of 0.001.

9 *Noising* THE DATA SETS

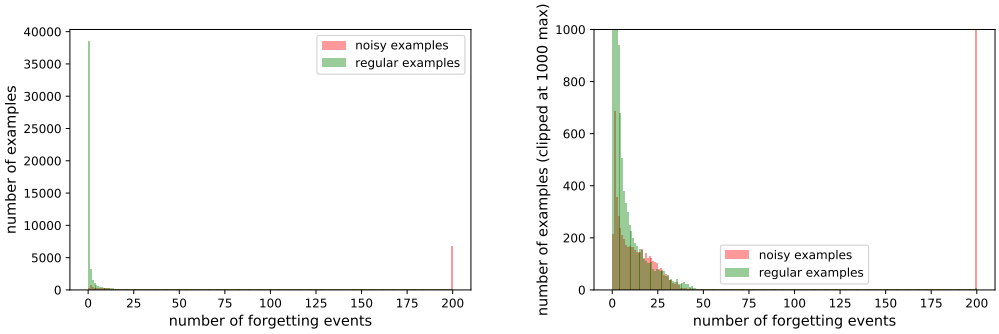


Figure 10: Distribution of forgetting events across all training examples in MNIST when 20% of labels are randomly changed. *Right.* The number of examples are clipped to a maximum of 1000 for the plot so the distribution is visible. Half of the noisy examples are never learned (have 200 forgetting events). 2% of noisy examples are unforgettable.

10 STABILITY OF THE FORGETTING EVENTS

In this section, we plot precision-recall diagrams for the unforgettable and most forgotten examples of *CIFAR-10* obtained on ResNet18 after 200 epochs and various prior time steps. We see in particular that at 75 epochs, the examples on both side of the spectrum can be retrieved with very high precision and recall.

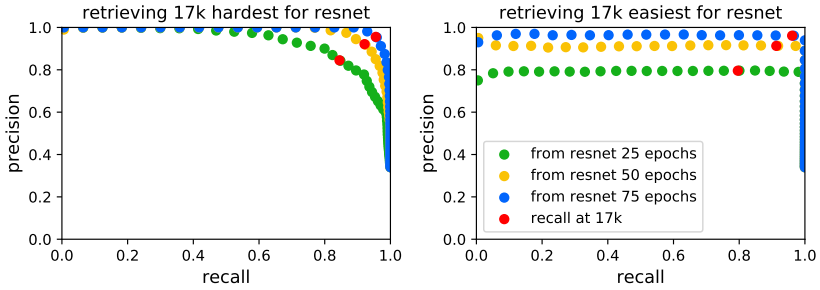


Figure 11: Right: precision and recall of retrieving the unforgettable examples from a full run of ResNet18 (200 epochs), using the example ordering after 25, 50, and 75 epochs. The unforgettable examples are retrieved with high precision and recall after 50 epochs.