DO DEEP REINFORCEMENT LEARNING ALGORITHMS REALLY LEARN TO NAVIGATE?

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ABSTRACT

Deep reinforcement learning (DRL) algorithms have demonstrated progress in learning to find a goal in challenging environments. As the title of the paper by Mirowski et al. (2016) suggests, one might assume that DRL based algorithms are able to "learn to navigate" and are thus ready to replace classical mapping and path planning algorithms, at least in simulated environments. Yet, from experiments and analysis in this earlier work, it is not clear what strategies these DRL algorithms are learning to navigate the mazes and find the goal. In this paper, we pose and study this question: are DRL algorithms doing some form of mapping and/or path-planning? Our experiments show that the algorithms are not memorizing the maps of mazes at the testing stage but, rather, at the training stage. Hence, the DRL algorithms fall short of qualifying as mapping or path-planning algorithms with any reasonable definition of mapping. We extend the experiments in Mirowski et al. (2016) by separating the set of training and testing maps and by a more systemetic coverage of the space of experiment. Our experiments show that the NavA3C+D_1D_2L algorithm, when trained and tested on the same maps, is able to choose the shorter paths to the goal. However, when tested on unseen maps the algorithms utilize a bug exploration like strategy to find the goal without doing any mapping or path planning.

1 INTRODUCTION

Navigation remains a fundamental problem in mobile robotics and artificial intelligence (Smith & Cheeseman (1986); Elfes (1989)). The problem is classically addressed by separating the eventual task of navigation into two steps, exploration and exploitation. In the exploration stage, the environment is represented as some kind of *map*. In the exploitation stage, the map is used to *plan a path* to a given destination based on some optimality criterion. This classical approach has been quite successful in navigation using a variety of sensors. However, navigation in generic unstructured environments, especially with texture-less Yang et al. (2016), transparent and reflective surfaces **?**, remains a challenge.

More recently, end-to-end navigation methods—methods that attempt to solve the navigation problem without breaking it down into separate parts of mapping and path-planning—have gained traction. With the recent advances of Deep Reinforcement Learning (DRL), these end-to-end navigation methods, like Mnih et al. (2016); Silver et al. (2016); Levine et al. (2017); Mirowski et al. (2016); Oh et al. (2016), forego decisions about the details that are required in the intermediate step of mapping. The potential for simpler yet more capable methods is rich; for example, agents thus trained can potentially optimize the amount of map information that is required to perform the navigation task. One such work, Mirowski et al. (2016), has demonstrated algorithms that are able to explore and find goals in complex environments utilizing only monocular first-person views.

Despite such potential advances, DRL based navigation remains a relatively unexplored field with its own set of limitations. The black-box nature of these methods make them hard to study and the patterns captured by the methods are not well understood. Recent work in analyzing neural networks have shown that such object detection methods can be easily fooled by introducing noise that is imperceptible to humans (Nguyen et al. (2015)). This makes it even more important to analyze these DRL methods across a wide variety of experiments to understand their strengths and weaknesses.

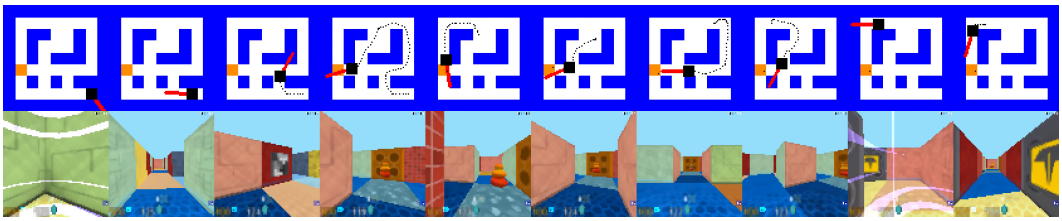

Figure 1: Snapshots of the path taken by the agent while evaluating the model trained on the same map with random goal and random spawn. The first row shows the top view of the robot moving through the maze with the goal location marked in orange. The second row shows the first person view, the only input available to the agent (except reward).

In this work, we build on Mirowski et al. (2016) and analyze their methods across hundreds of maps with increasing difficulty levels. We set up the environment as a randomly generated map, as shown in Fig 1, with an agent and a goal. The agent is provided only with the first-person view and is tasked to find the goal as many times as possible within a fixed amount of time, respawning at the spawn location each time it reaches the goal. We train and evaluate the algorithms with increasingly difficulty. In the easiest stage, we keep the goal location, spawn location and map constant over the training and the testing. We call this set up *static goal, static spawn, and static map*. To increase the difficulty, we incrementally randomize the spawn locations, goal locations and map structures untill all three are random. We discuss the design of experiments in Section 4.1 with more detail.

Mirowski et al. (2016) train and test their algorithms with randomized goals, randomized spawns but constant map structures and show that algorithm is able to exploit the knowledge of the goal location at evaluation time to maximize reward. However, this result is shown to be successful only on one map making the repeatability of the results questionable. It is also unclear whether these results generalize to unseen maps.

Even though separating training and testing sets is standard practice in machine learning, to the best of our knowledge, we are the first work to evaluate any DRL based navigation method on maps with unseen structures. We expand on Mirowski et al. (2016) analysis to address its limitations and ask the question whether DRL based algorithms such as NavA3C+D_1D_2L perform any mapping followed by shortest path planning. Our experiments do not show any evidence of mapping in the cases when algorithms are evaluated on unseen maps and no evidence of optimal path planning even in cases when the map is constant and only the goal is randomized.

We also compute attention-maps for the models to understand what part of image is being used for the navigation. We find that the models discard most of the image information, focusing attention on a small band in the middle of the image except around junctions where the attention is distributed evenly throughout the image.

These findings are the results on training and testing on multiple maps that were randomly chosen from a set of 1100 randomly generated maps. We provide the results of our experiments on ten randomly chosen maps and as well as on a testing set of 100 unseen maps to make sure the results are independent of the choice of map. We will make our code and data available after the blind review process is over.

2 Related Work

Localization and mapping Robotic localization and mapping for navigation is a classic problem in mobile robotics and sensing. Smith & Cheeseman (1986) introduced the idea of propagating spatial uncertainty for robot localization while mapping and Elfes (1989) popularized Occupancy Grids. In the last three decades since these seminal works, the field has exploded with hundreds of algorithms released for multiple kinds of sensors such as cameras, laser scanners, sonars and depth sensors. These algorithms vary in how much detail is captured in the map. For example, topological maps, like Kuipers (1978), aim to capture as little information as possible while occupancy grid maps, Elfes (1989), aim to capture metrically accurate maps in resolutions dependent upon the navigation task.

All these approaches require significant hand-tuning depending on the environment, sensor types and navigation constraints of the hardware. In contrast, end-to-end navigation algorithms optimize the detail of map storage based on the navigation task at hand which makes them worth exploring.

Deep reinforcement learning DRL gained prominence recently when Mnih et al. (2013; 2015) used DRL to create agents that outperformed humans on Atari games while being purely trained on the games visual outputs. Recently, DRL has been applied to end-to-end navigation (Oh et al. (2016); Mirowski et al. (2016); Chaplot et al. (2016)). In fact, it is common for agents to be trained and tested on the same maps with the variation only being applied to the agent's initial spawn point and the map's goal location (Mirowski et al. (2016); Zhu et al. (2017); Kulkarni et al. (2016)).

In contrast, Oh et al. (2016) do test their algorithm on random unseen maps but their agents are trained to choose between multiple possible goal locations based on past observations. The episodes end when the agent collects the goal, so there is no requirement for the algorithm to store map information during their exploration. Thus the decisions made by these agents are to avoid a goal of a particular color and seek other colors rather than remembering the path to the goal. On similar lines, Chaplot et al. (2016) test their method on unseen maps in the VizDoom environment but only vary the maps with unseen textures. Thus their agents are texture invariant but continue to train and test on maps with the same geometric structure.

In this work, we extend the study of these methods in a more comprehensive set of experiments to address the question of whether DRL-based agents remember enough information to obviate mapping algorithms or whether they do in fact need to be augmented with them for future progress.

3 BACKGROUND

Our problem formulation is based upon the work of Mirowski et al. (2016). We summarize the technical setup here for completeness. We refer the reader to Mnih et al. (2016); Mirowski et al. (2016) for more details of the setup.

The problem of navigation is formulated as an interaction between an environment and an agent. At time t the agent takes an action $a_t \in \mathcal{A}$ and observes observation $o_t \in \mathcal{O}$ along with a reward $r_t \in \mathbb{R}$. We assume the environment to be Partially Observable Markov Decision Process (POMDP). In a POMDP, the future state of the environment, $s_{t+1} \in \mathcal{S}$, is conditionally independent of all the past states, $s_{1:t-1}$, given the current state s_t. It is further assumed that o_t and r_t are independent of previous states given current state s_t and last action a_{t-1}. Formally, a POMDP is defined as a six tuple $(\mathcal{O}, C, \mathcal{S}, \mathcal{A}, T, R)$ that is composed of an observation space $\mathcal{O}$, an observation function $C(s_t, a_t) \rightarrow o_t$, a state space $\mathcal{S}$, an action space $\mathcal{A}$, a transition function $T(s_t, a_t) \rightarrow s_{t+1}$ and a reward function $R(s_t, a_t) \rightarrow r_{t+1}$. For our problem setup, the observation space $\mathcal{O}$ is the space of an encoded feature vector that is generated from input image along with previous action and reward. Action space $\mathcal{A}$ contains four actions: rotate left, rotate right, move forward and move backward and reward function R is defined for each experiment so that the reaching the goal leads to high reward with auxilary reward to encourage certain kind of behavior.

For DRL algorithms, the state space $\mathcal{S}$ is not hand tuned, but it is modeled as a semantically meaningless vector of floats. Also, instead of modeling observation function $C(s_t, a_t) \rightarrow o_t$ and $T(s_t, a_t) \rightarrow s_{t+1}$, a combined transition function $T_c(s_t, o_t, a_t, r_t; \theta_T) \rightarrow s_{t+1}$ is modeled such that it estimates the next state s_{t+1} directly considering previous observation as well as reward into account. For policy-based DRL a policy function $\pi(a_{t+1}|s_t, o_t, a_t, r_t; \theta_\pi) \rightarrow \pi_t(a_{t+1}; \theta_\pi)$ and a value function $V(s_t, o_t, a_t, r_t; \theta_V) \rightarrow V_t(\theta_V)$ are also modeled. All three functions T_c, π_t, V_t share most of the parameters in a way such that $\theta_T \subseteq \theta_\pi \cap \theta_V$

Our objective is to estimate unknown weights $\theta = \theta_T \cup \theta_\pi \cup \theta_V$ that maximizes the expected future reward, $R_t = \sum_{k=t}^{t_{end}-t} \gamma^{k-t} r_k$, where γ is the discount factor,

$$\theta^* = \arg\max_\theta \mathbb{E}[R_t] \,. \tag{1}$$

Asynchronous Advantage Actor-Critic In this paper we use the policy-based method called Asynchronous Advantage Actor-Critic (A3C) (Mnih et al. (2016)) that allows weight updates to

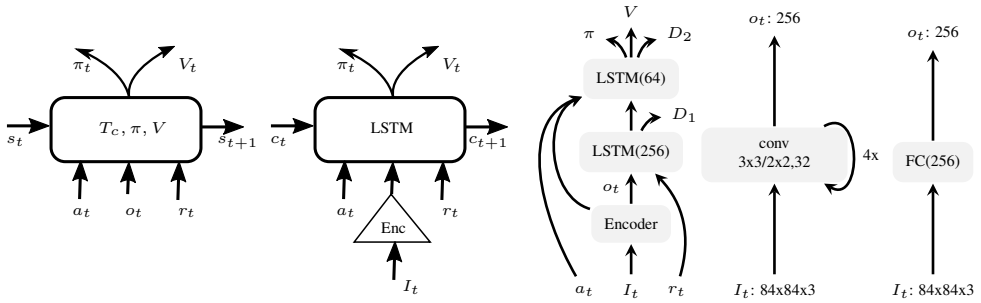

Figure 2: POMDP on the left, neural network implementation on the right. NavA3C+D_1D_2L architecture on left and two possible Encoders: CNN encoder on center, FC encoder on right. We use CNN encoder for the 3D world while FC encoder for the gridworld.

happen asynchronously in a multi-threaded environment. It works by keeping a "shared and slowly changing copy of target network" that is updated every few iterations by accumulated gradients in each of the threads. The gradients are never applied to the local copy of the weights, but the local copy of weights is periodically synced from the shared copy of target weights. The gradient for the weight update is proportional to the product of *advantage*, $R_t - V_t(\theta_V)$, and *characteristic eligibility*, $\nabla_{\theta_\pi} \ln \pi_t(a_{t+1}; \theta_\pi)$ (Williams (1992)), updating the weights according to the following update equations

$$\theta_\pi \leftarrow \theta_\pi + \sum_{t \in \text{episode}} \alpha_\pi \nabla_{\theta_\pi} \ln \pi_t(a_{t+1}; \theta_\pi)(R_t - V_t(\theta_V)) \tag{2}$$

$$\theta_V \leftarrow \theta_V + \sum_{t \in \text{episode}} \alpha_V \frac{\partial (R_t - V_t(\theta_V))^2}{\partial \theta_V}. \tag{3}$$

For more details of the A3C algorithm we refer the reader to Mnih et al. (2016).

NavA3C+D_1D_2L In this work we use the NavA3C+D_1D_2L architecture as proposed by Mirowski et al. (2016) which builds modifying the network architecture to have two LSTMs and with auxilary outputs of depth predictions along with loop closure predictions. The schematic of the architecture is shown in Fig 2.

4 THE DRL NAVIGATION CHALLENGE

Since deep reinforcement learning algorithms need millions of iterations to train, in the absence of thousands of robotic replicas like Levine et al. (2017), we evaluate the algorithms on a simulated environment. We use the same game engine as Mirowski et al. (2016), called Deepmind Lab (Beattie et al. (2016)). The game is setup such that an agent is placed within a randomly generated maze containing a *goal* at a particular location. On reaching the goal, the agent *respawns* within the same maze while the goal location remains unchanged. Following Mirowski et al. (2016), we scatter the maze with randomly placed smaller apple rewards (+1) to encourage initial explorations and assign the goal a reward of 10. The agent is tasked to find the goal as many times as possible within a fixed amount of time, respawning at the spawn location each time it reaches the goal.

Unlike Mirowski et al. (2016), we include a small wall penalty (-0.2) that pushes the agent away from the wall. The wall penalty is useful to prevent agents from moving along the walls, thereby discarding vision input for exploration. We also use a discrete 4-action space (move forward/backward, rotate left/right)which is different from the 8-action space one used by Mirowski et al. (2016). A smaller action space helps the algorithm train faster while achieving similar reward values.

We generate 1100 random maps using depth first search based maze generation methods. More information on maze generation can be found in the appendix. Of these 1100 maps, 1000 maps are used for training while the remaining 100 are used for testing. Unlike Mirowski et al. (2016) and

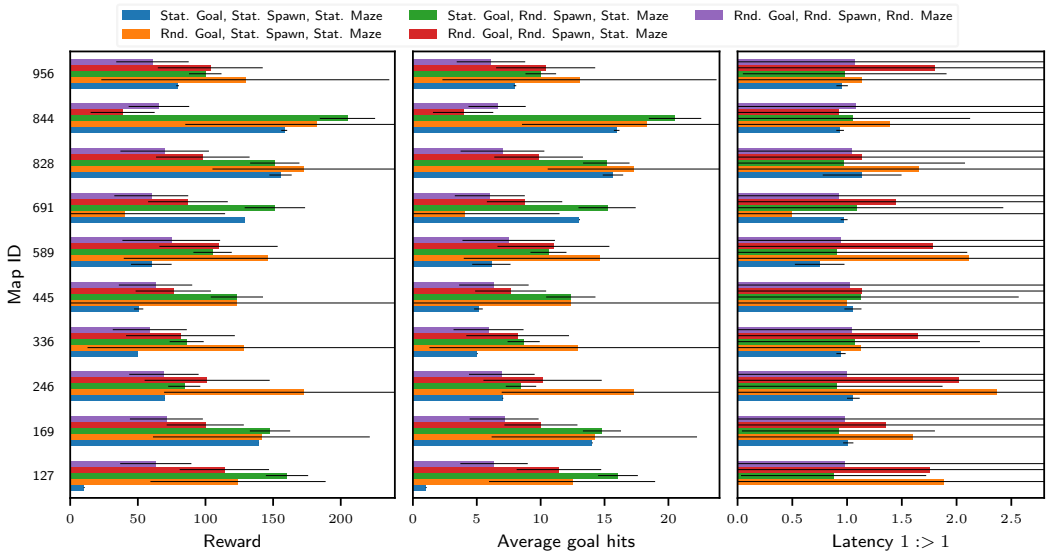

Figure 3: We evaluate the NavA3C+D_1D_2L Mirowski et al. (2016) algorithm on ten randomly chosen maps with increasing difficulty as described in Sec 4.1. Vertical axes is one of the ten maps on which the agent was trained and evaluated. Horizontal axes are different evaluation metrics. We note that when the goal is static then rewards are consistently higher as compared to random goal while static spawn location and random spawn location are roughly close to each other within bounds of uncertainity. As expected, switching each variable from static to random increases the standard deviation on the results. From the Latency $1 :> 1$ results we note that the current state of art algorithms do well when trained and tested on the same map but fail to generalize to new maps when evaluated on ability to exploit the information about goal location. Also note that Latency $1 :> 1$ metric for cases of static goals is expected to be close to one because the location of goal is learned at train time.

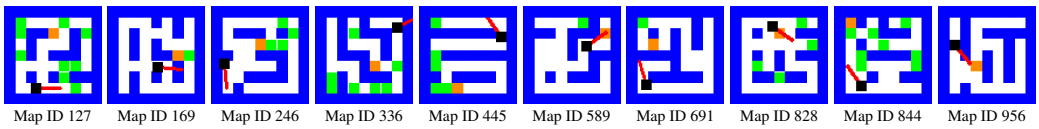

Figure 4: The ten randomly chosen mazes for evaluation. We generate 1100 random mazes and choose ten to evaluate our experiments that require testing and training on the same maps.

similar to Chaplot et al. (2016), we use randomly textured walls in our mazes so that the policies learned are texture-independent.

4.1 Experiments

We evaluate the Nav-A3C algorithm on maps with 5 stages of difficulty. While the Nav-A3C algorithm works smoothly on the easier stages, it does not perform better than bug-exploration methods on the hardest stage. We propose these experiments as a 5-stage benchmark for all end-to-end navigation algorithms.

1. **Static goal, static spawn, and static map** To perform optimally on this experiment, the agent needs to find and learn the shortest path at training time and repeat it during testing.

2. **Static goal, random spawn and static map** This is a textbook version of the reinforcement learning problem, especially in gridworld Sutton & Barto (1998), with the only difference being that the environment is partially observable instead of fully observable. This problem is more difficult than Problem 1 because the agent must find an optimal policy to the goal from each possible starting point in the maze.

3. **Random goal, static spawn, and static map** In this setup, we keep the spawn location and the map fixed during both training and testing but chose a random goal location for each episode. Note that the goal location stays constant throughout an episode. The agent can perform well on this experiment, by remembering the goal location after it has been discovered and exploiting the information to revisit goal faster.

   Following Mirowski et al. (2016), we report the ratio of time taken to hit the goal for the first time (exploration time) vs the average amount of time taken to hit goal subsequently (exploitation time). The metric, called *Latency* $1 :> 1$, is a measure of how efficiently the agent exploits map information to find shorter path once the goal location is known. If this ratio is greater than 1, then we say that the agent is doing better than random exploration and higher values is better.

4. **Random goal, random spawn, and static map** In this version of the experiment both the spawn point and the goal location is randomized. To perform optimally, the agent must localize itself within the map in addition to being able to exploit map-information.

   This is the problem that is addressed by Mirowski et al. (2016) with limited success. They evaluate this case on two maps and report Latency $1 :> 1$ to be greater than 1 in one of the two maps. We evaluate the same metric on ten other maps and provide the tools for application to several more.

5. **Random goal, random spawn, and random map** We believe that any proposed algorithms on end-to-end navigation problems, should be evaluated on unseen maps. To our knowledge, this is the first paper to do so in the case of deep reinforcement learning based navigation methods. We train agents to simultaneously learn to explore 1, 10, 100, 500 and 1000 maps and test them on the same 100 unseen maps. The relevant results can be found in Fig 5 and discussed in Section 5.

The comparative evaluation of the different the stages of this benchmark are shown in Fig 3 and expanded upon in the next section.

5 Results and Analysis

The results for DRL Navigation challenge are shown in Fig 3. We note that when the goal is static then rewards are consistently higher as compared to random goal while static spawn location and random spawn location are roughly close to each other within bounds of uncertainity. As expected, switching each variable from static to random increases the standard deviation on the results. From the Latency $1 :> 1$ results we note that the current state of art algorithms do well when trained and tested on the same map but fail to generalize to new maps when evaluated on ability to exploit the information about goal location. Also note that Latency $1 :> 1$ metric for cases of static goals is expected to be close to one because the location of goal is learned at train time.

5.1 Evaluation on unseen maps

The results for training on N maps, where $N \in \{10, 100, 500, 1000\}$, and testing on 100 unseen maps are shown in Fig 5. We observe that their is significant jump of average reward and averge goal hits when number of training maps is increased from 10 to 100 but no significant increase when number of training maps are increased from 100 to 500 to 1000. This is due to the fact that the bug exploration strategy learnt by the algorithm, is learnt with enough variation in 100 maps and training on additional maps does not add to the learnt strategy.

5.2 Effect of apples and texture

We evaluate the effect of apples and texture during evaluation time in Fig 5. We train the algorithm on randomly chosen training maps with random texture and evaluate them no maps with and without random texture and also on maps with and without apples. When the apples are present, we place the apples with probability 0.25 in each block of the map. We find that the algorithm, being trained on random textures and random placement of apples, is robust to presence or absence of textures and apples.

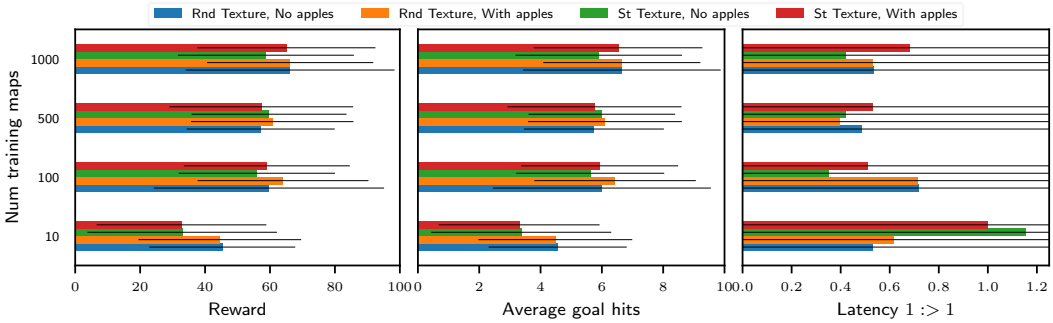

Figure 5: Plots showing the effect of number of training maps with random texture (Rnd Texture) and presence of apples (With apples), when evaluated on unseen maps. We note that although the difference between mean metrics is negligible as compared to standard deviation of the metrics. Hence we say that the effect of apples or textures can be ignored. The only clear trend is apparent Latency $1 :> 1$ metric which suggest that random texture along without apples is advantageous in exploiting goal location while finding the goal second time onwards.

5.3 Qualitative evaluation on simple maps

To evaluate what strategies that algorithm is employing to reach the goal we evaluate the algorithm on very simple maps where there are only two paths to reach the goal. The qualitative results for the evaluation are shown in Fig 6.

Square map A Square map (Fig 6) is the simplest possible map with two paths to the goal. We evaluate the algorithm trained on 1000 random maps on square map. We observe that the agent greedily moves in the direction of initialization. This may be because of the initial learning which is motivated by small rewards of getting apples. We compute the percentage of times the agent takes the shortest path over a trial of 100 episodes. We find the agent takes the shortest path only 50.4% ($\pm 12.8\%$) of the times, no better than random.

Wrench map To eliminate the dependency on initial orientation, we evaluate the algorithm on Wrench map as shown in Fig 6. We fix in the spawn point at the bottom of the tail so that shortest path is independent of the spawn orientation. The decision about the shortest path is made at the junction where the agent can either chose to go left or right. We find that the agent is taking shortest path only 32.9% ($\pm 25.1\%$) of the times which is again to better than random.

Goal map Similarly to the wrench map, the goal map (Fig 6) provides decision point independent of the initial orientation, but it penalizes wrong path more than the wrench map 42.6% ($\pm 35.1\%$) of the times which is again no better than random.

5.4 Computing Attention Maps

We use the normalized sum of absolute gradient of the loss with respect to the input image as a proxy for attention in the image. The gradients are normalized for each image so that the maximum gradient is one. The attention values are then used as a soft mask on the image to create the visualization as shown in Fig 7

We observe that the attention is uniformly distributed on the image when the agent spawns. The attention narrows down to a few pixels in the center when the agent is navigating through the corridor. It spreads to the entire image around turns and junctions. The attention also pays close attention to important objects like goal, apples and unique decals.

6 Conclusion

In this work we show that state of art Deep reinforcement learning algorithms do not perform long term mapping and or path planning. We provide evidence that the algorithms are learning to perform

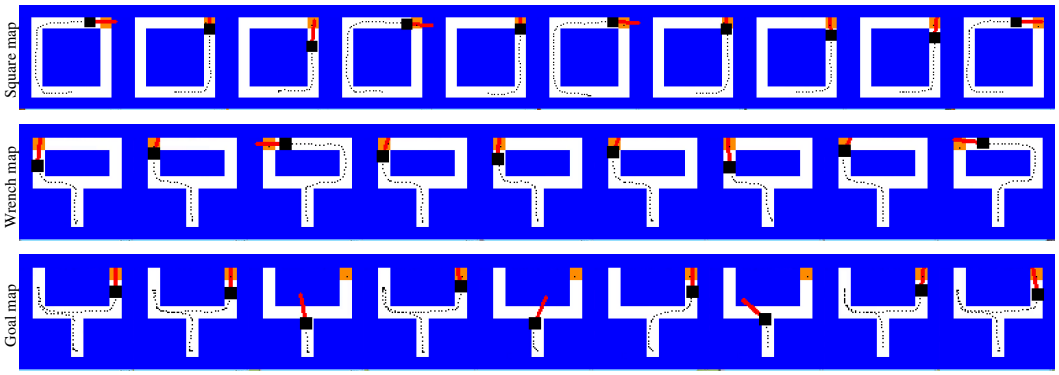

Figure 6: Snapshots of path taken by the agent to reach the goal in a single episode when model trained on 1000 maps is evaluated Square, Wrench and Goal map. The top row shows an evaluation example on Square map, the agent takes the shortest path 6/10 times but when averaged over 100 episodes, the percentage of shortest path taken is not better than random 50.4% ($\pm 12.8\%$). Although for the example of Wrench map the agent takes the shortest path 8/10 times but when averaged over 100 episodes, the percentage of shortest path taken is reduced to 32.9% ($\pm 25.1\%$). For the Goal map, the example chosen here shows that the shortest path is only taken 1/6 times, on an average over 100 episodes, the shortest path is taken 42.6% ($\pm 35.1\%$) times.

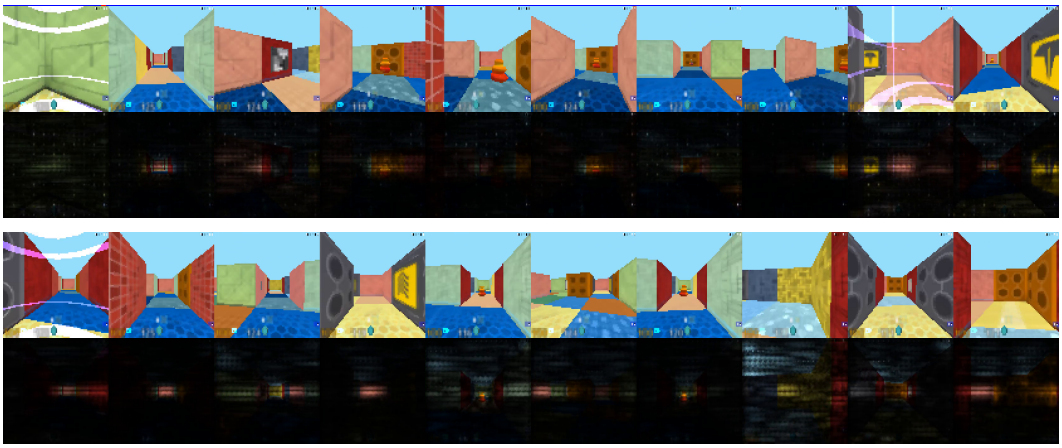

Figure 7: Visualizing attention for two sequences. The first two rows show the sequance when the model is trained on and evaluated on the same map. The last two rows shows the sequence for a model trained on 1000 maps and evaluated on one of the maps. We observe that the attention is uniformly distributed on the image when the agent spawns. The attention narrows down few pixels in the center when the agent is navigating through the corridor. It spreads to the entire image around turns and junctions. The attention also pays close attention to important objects like goal, apples and unique decals.

bug exploration algorithms when the position of goal is random. When the position of goal and structure of map is known at training time, deep reinforcement learning algorithms are able to learn near optimal policy and able to repeat it at test time.

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