Simplified Models of Remaining Useful Life Based on Stochastic Orderings

Luciano Sánchez, Nahuel Costa
Departamento de Informática, Universidad de Oviedo
luciano, costanahuel@uniovi.es
Inés Couso
Departamento de Estadística, Universidad de Oviedo
couso@uniovi.es

Abstract

We introduce a method for designing *simplified* models of Remaining Useful Life (RUL) that is especially suited for deployment in resource-constrained environments. Instead of accurately predicting the RUL via complex nonlinear functions, our approach jointly learns (i) a probabilistic health state model and (ii) a stochastic ordering function — represented by a monotonic neural network — so that the resulting health indicator is *comonotonic* with the true RUL. Notably, this work is the first study where the learning task simultaneously optimizes both the predictive model and the criterion for comparing model quality. By co-optimizing these elements, our method selects the simplest representation that preserves the essential ordering of degradation, as measured by a smooth approximation of Kendall's τ statistic. Experiments on the CMAPSS benchmark and real-world datasets (including turbofan engines and road tunnel fans) demonstrate that our approach achieves competitive prediction accuracy with a drastic reduction in the number of parameters.

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1 Introduction

Modern techniques for estimating Remaining Useful Life (RUL)—such as convolutional neural networks, recurrent networks, and variational autoencoders (see, e.g., [1, 4])—often achieve high accuracy but require very large models with tens to hundreds of thousands of parameters. Figure 1 shows the trade-off between the number of parameters and prediction accuracy (measured by RMSE) for various RUL models. This model complexity is a significant challenge for embedded systems that must run real-time diagnostics on limited computing power.

Our approach tackles this issue by shifting the focus from accurately predicting exact RUL values to preserving the correct order of degradation in the estimated health signal. In simple terms, as a system's condition worsens, our health indicator decreases accordingly. To achieve this, we jointly learn a straightforward, interpretable probabilistic model and a stochastic ordering function. Unlike approaches that fix the comparison criterion in advance, our co-learning strategy enlarges the search space by optimizing both elements simultaneously. This extra flexibility lets us incorporate a regularization term in the loss function that steers the optimization toward a simpler model—even though, with a fixed criterion, the optimal model might have been more complex.

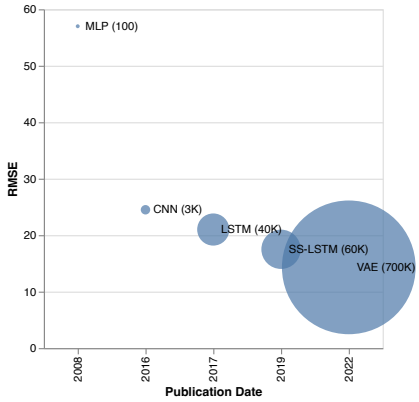

Figure 1: Historical evolution of the number of parameters (area of circles) versus RMSE for various RUL models. The proposed simplified ageing model attains competitive accuracy with dramatically lower complexity.

2 Proposed Method

Our approach consists of two components.

(i) Health State Model: We model the system's health at time t as a random variable following a Gamma distribution, i.e.,

$$\mathrm{SoH}_t \sim \Gamma(k, \theta_t), \quad E(\mathrm{SoH}_t) = k\theta_t.$$

Degradation is incorporated by updating the scale parameter as

$$\theta_{t+1} = \theta_t \cdot D_t, \quad \text{with} \quad \log D_t = -\| \operatorname{relu}(\alpha \cdot \dot{\mathbf{x}}_t - \beta)\|,$$

where $\dot{\mathbf{x}}_t$ is the time derivative of the measured signals, and α, β are learnable parameters.

(ii) Stochastic Ordering Function: To ensure that the estimated SoH is comonotonic with the true RUL, we introduce an ordering function $f(x, y)$ that is monotonically increasing in its first argument and decreasing in its second. This function is parameterized by a monotonic neural network (adapted from Sill's architecture [3]) and is used to compare pairs of systems. By maximizing a smooth approximation of Kendall's τ statistic [2] over these comparisons, the learning algorithm jointly optimizes the parameters of both the SoH model and the ordering function to preserve the correct ranking.

Figure 2 shows a block diagram of the method. During training, only the order of the available RUL data is used, so that the final health signal needs only to be monotonic with respect to the true RUL. A subsequent calibration step (via monotonic regression) converts this health signal into numerical RUL estimates.

3 Experimental Results

In addition to the real-world fan monitoring application that motivated this study, we evaluated our method on the widely used CMAPSS benchmark and on two real datasets. Table 1 summarizes the RMSE results for selected methods on the CMAPSS benchmark. Notably, our *Simple Ageing Model* achieves RMSE values comparable to those obtained by deep learning approaches (e.g., Deep LSTM, VAE+RNN), despite using orders of magnitude fewer parameters.

Experiments on real datasets further confirm the effectiveness of our approach. In turbofan engine diagnostics, our model successfully captures degradation trends in both the high-pressure compressor (HPC) and high-pressure turbine (HPT), yielding lower mean absolute errors than baseline cycle-counting and other data-driven approaches. Similarly, in road tunnel fan monitoring, our simplified model was implemented on a 30 kW axial fan for tunnel ventilation; even under strict computational constraints, the method produced a coherent health signal that closely correlates with observed deterioration, enabling timely fault alarms.

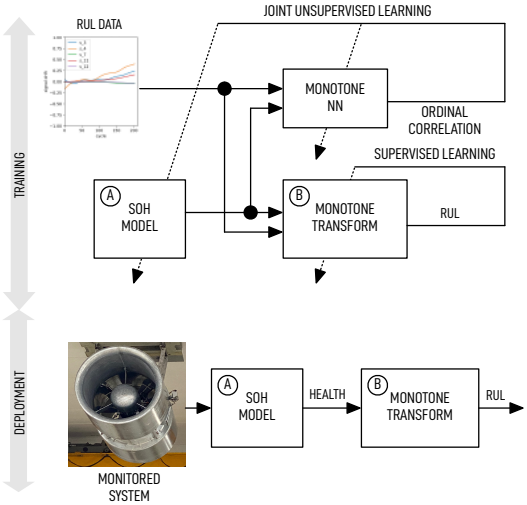

Figure 2: Block diagram of the proposed method. The learning process jointly estimates a probabilistic health state model and a stochastic ordering function, ensuring that the derived health indicator is comonotonic with the RUL.

Method	FD001	FD002	FD003	FD004
MLP [4]	37.56	80.03	37.39	77.37
CNN [4]	18.45	30.29	19.82	29.16
Deep LSTM [4]	16.14	24.49	16.18	28.17
VAE+RNN [1]	15.81	24.12	14.88	26.54
Simple Ageing Model	16.79	19.51	17.48	22.82

Table 1: RMSE comparison on the CMAPSS benchmark. The proposed method achieves competitive performance with greatly reduced complexity.

4 Conclusions and Future Work

We have presented a co-learning framework that simultaneously estimates a probabilistic health model and a stochastic ordering function, ensuring that the derived health indicator is comonotonic with the actual RUL. This strategy allows for a substantial reduction in model complexity without significant loss of diagnostic accuracy, making the approach ideal for embedded and low-power applications. Future research will explore alternative probability models and extensions to scenarios with missing or censored data.

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