NEURAL IMPLICIT SHAPE EDITING USING BOUNDARY SENSITIVITY

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ABSTRACT

Neural fields are receiving increased attention as a geometric representation due to their ability to compactly store detailed and smooth shapes and easily undergo topological changes. Compared to classic geometry representations, however, neural representations do not allow the user to exert intuitive control over the shape. Motivated by this, we leverage *boundary sensitivity* to express how perturbations in parameters move the shape boundary. This allows to interpret the effect of each learnable parameter and study achievable deformations. With this, we perform *geometric editing*: finding a parameter update which best approximates a globally prescribed deformation. Prescribing the deformation only locally allows to deform the rest of the shape according to some prior, such as *semantics* or *deformation* rigidity. Different to previous efforts, our method is model-agnostic and can be applied to a pre-trained NN and update it in-place. Furthermore, we show how boundary sensitivity helps optimize and constrain objectives (such as surface area and volume), which are difficult to compute without first converting to another representation, such as a mesh.

1 INTRODUCTION

A *neural field* is a neural network (NN) mapping every point in a domain of interest, typically of 2 or 3 dimensions, to one or more outputs, such as a signed distance function (SDF), occupancy probability, opacity or color. This allows to represent smooth, detailed, and watertight shapes with topological flexibility, while being compact to store compared to classic implicit representations (Davies et al., 2020). When the NN is trained not on a single shape but instead an entire collection, each shape is encoded in a latent vector, which is an additional input to the NN (Park et al., 2019; Chen & Zhang, 2019; Mescheder et al., 2019). As a result, neural fields are receiving increased interest as a geometric representation in numerous applications, such as shape generation (Park et al., 2019), shape completion (Chibane et al., 2020), shape optimization (Remelli et al., 2020), scene representation (Sitzmann et al., 2020), and view synthesis (Mildenhall et al., 2020). Some pioneering works have also investigated geometry processing, like smoothing and deformation, on neural implicit shapes (Yang et al., 2021; Remelli et al., 2020; Mehta et al., 2022), but these can be computationally costly or resort to intermediate mesh representations. In part, this difficulty stems from the shape being available only *implicitly* as the sub-level set of the field.

While classic explicit or parametric representations (like meshes, splines, or subdivision schemes) often have intuitive (often synonymous with local) geometric control as a key design principle, it is not trivial to edit even classic implicit representations, especially ones with global functions (Bærentzen & Christensen, 2002). Previous works on neural implicit shape editing have focused on the shape *semantics*, i.e. changing part-level features based on the whole shape structure, but achieve this through tailored training procedures or architectures.

We instead propose a framework which unifies geometric and semantic editing and which is agnostic to the model and its training and modifies the given model in-place akin to classic representations. To treat the geometry, not the field, as the primary object we consider *boundary sensitivity* to relate changes in the function parameters and the implicit shape. This allows us to express and interpret a basis for the displacement space.

In this framework, the user supplies a target displacement on (a part) of the shape boundary and through boundary sensitivity we find the parameter update which best approximates it. In *geometric*

editing, we prescribe an exact geometric update on the entirety of the boundary. Akin to local control in classic representations, we especially study the case where the prescribed displacement is local and the rest of the boundary is fixed. In *semantic* editing only a part of the boundary is prescribed a target displacement. The remaining unconstrained displacement is determined by leveraging the generalization capability of the model as an additional prior, producing semantically consistent results on the totality of the shape. Another prior often used in shape editing is based on *deformation* energy, such as as-rigid-as-possible (Sorkine & Alexa, 2007) or as-Killing-as-possible (Solomon et al., 2011), to generate physically plausible deformations minimizing stretch and bending. This is not trivially applicable to implicit surfaces, as there is no natural notion of stretch due to ambiguity in tangent directions. We discuss a few options to resolve this ambiguity and demonstrate that boundary sensitivity can be leveraged to optimize directly in the space of expressible deformations. Lastly, we use level-set theory and boundary sensitivity to constrain a class of functionals, which are difficult to compute without first converting to another representation, such as a mesh. As a specific use-case, we consider fixing the volume of a shape to prevent shrinkage during smoothing.

2 Related Work

Implicit Shape Representations and Manipulation Implicit shape representations or *level-sets* have been widely used in fields such as computer simulation (Sethian & Smereka, 2003), shape optimization (van Dijk et al., 2013), imaging, vision and graphics (Vese et al., 2003; Tsai & Osher, 2003). Clasically, an implicit function is represented as a linear combination of either a few global basis functions, such as in *blobby models* (Muraki, 1991), or many local basis functions supported on a (potentially adaptive) volumetric grid (Whitaker, 2002). While few global bases use less memory, the task of expressing local displacements is generally ill-posed (Whitaker, 2002). Hence, methods for interactive editing of implicit shapes are formulated for grid-supported level-sets (Museth et al., 2002; Bærentzen & Christensen, 2002). These methods use a prescribed velocity field, for which the *level-set-equation* – a partial differential equation (PDE) modelling the evolution the surface – is solved using numerical schemes on the discrete spatiotemporal grid.

Neural Fields In *neural fields*, a NN is used to represent an implicit function. Different from classic implicit representations, these are non-linear and circumvent the memory-expressivity dilemma Davies et al. (2020). In addition, automatic differentiation also provides easy access to differential surface properties, such as normals and curvatures, useful for smoothing and deformation (Yang et al., 2021; Mehta et al., 2022; Atzmon et al., 2021) or more exact shape fitting (Novello et al., 2022). Early works propose to use vanilla multilayer perceptrons (MLPs) to learn occupancy probability (Mescheder et al., 2019; Chen & Zhang, 2019) or signed distance functions (Park et al., 2019; Atzmon & Lipman, 2020), whose level-sets define the shape boundary. Conditioning the NN on a latent code as an additional input allows to decode a collection of shapes with a single NN (Chen & Zhang, 2019; Mescheder et al., 2019; Park et al., 2019). Later works build upon these constructions by introducing a spatial structure on the latent codes using (potentially adaptive) grids, affording more spatial control when generating novel shapes (Ibing et al., 2021) and allowing to reconstruct more complex shapes and scenes (Jiang et al., 2020; Peng et al., 2020; Chibane et al., 2020).

Neural Shape Manipulation Although there are many previous works on deformation of shapes with NNs, we focus only on methods that use neural implicit representations. These can roughly be sorted into two groups based on their guiding principle. Methods in the first group manipulate shapes based on a *semantic* principle. Hertz et al. (2022) create a generative framework with part-level control consisting of three NNs decomposing and mixing shapes in the latent space. Hao et al. (2020) use a shared latent space to express and manipulate fine and coarse shape features, the latter of which controls the semantics of the shape. Elsner et al. (2021) encourage the latent code to act as geometric control points, allowing to manipulate the geometry by moving the control points. Chen et al. (2021) demonstrate how to interpolate between shapes, while balancing their semantics by choosing which layer's features to track. Similar to our method, this is agnostic to the training and the model.

Methods in the second group are based on some well defined *energy*.Yang et al. (2021) demonstrate shape smoothing using a curvature-based loss. Similarly, shape deformation is performed by optimizing the deformation energy for which another invertible correspondence NN is needed. Niemeyer et al. (2019) train an additional NN as a spatio-temporally continuous velocity field, along which the

points of a shape are integrated. Atzmon et al. (2021) use approximate Killing vector fields (AKVFs) during training to encourage latent space interpolation to act as physically plausible deformation. Lastly, this can also be done by leveraging meshes as an intermediate representation to benefit from the well studied geometry processing operations on those. Remelli et al. (2020) propose differentiable mesh extraction to then propagate gradients from a differentiable operation on the mesh through to the implicit NN. Mehta et al. (2022) studies this link in more detail using classic theory of level-sets. They argue that the energy-minimization in several important mesh operations induces a geometric flow of the surface, which differs from that in a standard end-to-end loss minimization. The last three works use boundary sensitivity as part of their framework, which we more formally introduce together with editing in the next section.

3 Boundary Sensitivity

Let f be a differentiable function mapping a spatial coordinate $\mathbf{x} \in \mathcal{D} \subset \mathbb{R}^D$ to a scalar $f(\mathbf{x}; \mathbf{\Theta}) \in \mathbb{R}$ on a domain of interest $\mathcal{D}$. Let the vector $\mathbf{\Theta} \in \mathbb{R}^P$ gather the parameters of f. For a given $\mathbf{\Theta}$, the sign of f *implicitly* defines the shape $\Omega(\mathbf{\Theta}) = \{\mathbf{x} \in \mathcal{D} | f(\mathbf{x}; \mathbf{\Theta}) \leq 0\}$ and its boundary $\Gamma(\mathbf{\Theta}) = \{\mathbf{x} \in \mathcal{D} | f(\mathbf{x}; \mathbf{\Theta}) = 0\}$.

Consider the variation of $\Omega(\mathbf{\Theta})$ by a displacement field (also known as *velocity field*) $\delta\mathbf{x} : \mathcal{D} \mapsto \mathbb{R}^D$, denoted by $\Omega_{\delta\mathbf{x}}(\mathbf{\Theta}) = \{\mathbf{x} + \delta\mathbf{x}(\mathbf{x}) \in \mathcal{D} | f(\mathbf{x} + \delta\mathbf{x}(\mathbf{x}); \mathbf{\Theta}) \leq 0\}$. If $\delta\mathbf{x}$ is sufficiently small, the initial and the perturbed shapes are diffeomorphic Allaire et al. (2004), meaning there is a one-to-one correspondence between the points of both. Consider the displacement field $\delta\mathbf{x}$ induced by a sufficiently small parameter perturbation $\delta\mathbf{\Theta}$ and the perturbed shape $\Omega_{\delta\mathbf{x}}(\mathbf{\Theta}) = \Omega(\mathbf{\Theta} + \delta\mathbf{\Theta})$. To compute $\delta\mathbf{x}$, consider the total derivative of the boundary condition $f(\mathbf{x}; \mathbf{\Theta}) = 0$:

$$\mathrm{d}f(\mathbf{x}; \mathbf{\Theta}) = \nabla_{\mathbf{x}} f^\top \, \mathrm{d}\mathbf{x} + \nabla_{\mathbf{\Theta}} f^\top \, \mathrm{d}\mathbf{\Theta} = 0 \quad \forall \mathbf{x} \in \Gamma \, . \tag{1}$$

We can replace the infinitesimal increments $\mathrm{d}\mathbf{x}$ and $\mathrm{d}\mathbf{\Theta}$ with sufficiently small perturbations $\nabla_{\mathbf{x}} f^\top \delta\mathbf{x} + \nabla_{\mathbf{\Theta}} f^\top \delta\mathbf{\Theta} = 0$. Assuming bounded gradients $\nabla_{\mathbf{x}} f = \nabla_{\mathbf{x}} f(\mathbf{x}; \mathbf{\Theta}) \in \mathbb{R}^D$ and $\nabla_{\mathbf{\Theta}} f = \nabla_{\mathbf{\Theta}} f(\mathbf{x}; \mathbf{\Theta}) \in \mathbb{R}^P$ allows to express $\delta\mathbf{x} = \delta\mathbf{x}(\mathbf{x}; \mathbf{\Theta}, \delta\mathbf{\Theta})$ as

$$\delta\mathbf{x} = \frac{-\nabla_{\mathbf{x}} f \nabla_{\mathbf{\Theta}} f^\top \delta\mathbf{\Theta}}{\|\nabla_{\mathbf{x}} f\|^2} + \delta\mathbf{x}_t \, . \tag{2}$$

This is what we refer to as *boundary sensitivity* – the movement of the boundary $\delta\mathbf{x}$ caused by the parameter perturbations $\delta\mathbf{\Theta}$. Equation 2 consists of normal and tangent components $\delta\mathbf{x} = \delta\mathbf{x}_n + \delta\mathbf{x}_t = \mathbf{n}\delta x_n + \mathbf{t}\delta x_t$. With $\mathbf{n} = \nabla_{\mathbf{x}} f / \|\nabla_{\mathbf{x}} f\|$ the outward facing normal, we rewrite the normal part as the weighted sum of the *basis* $\mathbf{b} = \mathbf{b}(\mathbf{x}; \mathbf{\Theta}) \in \mathbb{R}^P$

$$\delta x_n = \mathbf{b}^\top \delta\mathbf{\Theta} \quad , \quad \mathbf{b} = \frac{-\nabla_{\mathbf{\Theta}} f}{\|\nabla_{\mathbf{x}} f\|} \, . \tag{3}$$

For each positive parameter perturbation $\delta\Theta_p$, $p \in \{1..P\}$, a positive gradient $\partial f / \partial \Theta_p$ implies that the boundary moves inward along the normal, since the value of f at $\mathbf{x}$ increases. The total movement caused by all parameter perturbations is their superposition. We show some basis functions in Figures 2 and 3.

The tangential component $\delta\mathbf{x}_t$ is ambiguous since per definition $\nabla_{\mathbf{x}} f^\top \delta\mathbf{x}_t = 0$. This is transport of the surface along itself which has no effect on the implicit representation. However, this ambiguity can be solved to recover $\delta\mathbf{x}_t$ based on an additional assumption. In the context of classic implicit representations two such assumptions considered are that normals of points do not change during the deformation (Jos & Schmidt, 2011) and that are nearly-isometric (Tao et al., 2016).

Editing Let $\delta\bar{\mathbf{x}} : \bar{\Gamma} \mapsto \mathbb{R}^D$ be a prescribed deformation on (a part of) the boundary $\bar{\Gamma} \subseteq \Gamma$ and $E_C = \int_{\bar{\Gamma}} ||\delta\bar{\mathbf{x}} - \delta\mathbf{x}||^2 \, \mathrm{d}s$ the energy associated with the deviation from the prescribed displacement. The task in editing is to find a parameter update $\delta\mathbf{\Theta}$ inducing the displacement $\delta\mathbf{x}$ which minimizes this energy. Since a parameter update can cause movement only in the normal direction, it can only approximate the normal component of the target.

$$\begin{aligned}\delta\mathbf{\Theta} &= \underset{\delta\tilde{\mathbf{\Theta}}}{\operatorname{argmin}}\, E_C = \underset{\delta\tilde{\mathbf{\Theta}}}{\operatorname{argmin}} \int_{\bar{\Gamma}} \|\delta\bar{x}_n - \delta x_n + \delta\bar{x}_t - \delta x_t\|^2 \,\mathrm{d}s \\ &= \underset{\delta\tilde{\mathbf{\Theta}}}{\operatorname{argmin}} \int_{\bar{\Gamma}} \|\delta\bar{x}_n - \mathbf{b}^\top \delta\tilde{\mathbf{\Theta}}\|^2 \,\mathrm{d}s + \int_{\bar{\Gamma}} \|\delta\bar{x}_t - \delta x_t\|^2 \,\mathrm{d}s\end{aligned} \tag{4}$$

In practice, we estimate this integral with discrete samples at locations $\{\mathbf{x}_i\}_{i=1..I} \subset \bar{\Gamma}$:

$$\delta\mathbf{\Theta} = \underset{\delta\tilde{\mathbf{\Theta}}}{\operatorname{argmin}} \sum_i \|\delta\bar{x}_n(\mathbf{x}_i) - \mathbf{b}(\mathbf{x}_i)^\top \delta\tilde{\mathbf{\Theta}}\|^2 \,. \tag{5}$$

This is a linear-least-squares problem $\mathbf{B}\delta\mathbf{\Theta} = \delta\bar{\mathbf{y}}$ with $\mathbf{B} = [\mathbf{b}(x_1)|\cdots|\mathbf{b}(x_I)]^\top \in \mathbb{R}^{I,P}$ and $\delta\bar{\mathbf{y}} = [\delta x_n(\mathbf{x}_1), \cdots, \delta x_n(\mathbf{x}_I)] \in \mathbb{R}^I$.

To improve the numerical stability one often uses Tikhonov regularization which penalizes the norm of the solution $\min_{\delta\mathbf{\Theta}} E_C + \lambda\|\delta\mathbf{\Theta}\|^2$ for some small regularization constant λ. In our setting, Tikhonov regularization serves an additional purpose: small $\delta\mathbf{\Theta}$ are necessary for the valididity of the linear expansion in Equation 1. Furthermore, especially in semantic editing, we might sample less points than parameters $I < N$, which would lead to an underdetermined system if regularization is not used. Lastly, some works also propose to make small updates in latent parameters to encourage a more stable latent space exploration (Remelli et al., 2020).

4 Applications

Having established the framework for editing implicit shapes through boundary sensitivity, we consider its applications. We emphasize geometric and semantic editing since our framework unifies both, while being agnostic to the architecture and training of the model. We also address deformation-rigidity-based editing since it classically is a widely used approach to shape editing. Lastly, we consider an efficient approach to fixing a certain class of shape functionals, and demonstrate this by fixing the volume during smoothing.

We first discuss some practical aspects. Despite the boundary sensitivity in Equation 1 being valid only for small displacements, large displacements can be achieved through several iterations. We sample points $\mathbf{x}_i$ on the boundary $\bar{\Gamma}$ via iterative rejection sampling on the domain of interest or near the farthest point samples of the existing points. When performing large deformations over several iterations, we seed the points from the previous iteration by moving them by $\delta\mathbf{x}$ of the previous iteration. This is a sufficiently efficient method for our needs, although more advanced methods exist (Hart et al., 2002). Target deformations are then prescribed on the sampled points. If these are not given as the magnitude along the normal deformation $\delta\bar{x}_n$, but as a vector $\delta\bar{\mathbf{x}}$, we project them $\delta\bar{x}_n = \mathbf{n}^\top\delta\bar{\mathbf{x}}$. With partially prescribed targets, one must be careful about the target being unintentionally restrictive if the normals at the sampled points are nearly orthogonal to the target. This can be remedied by additionally filtering the points based on their normals. In all cases, we include Tikhonov regularization with $\lambda = 0.1$ (see Appendix A), but note that the regularization behaviour depends on the number of points I and the magnitude of the prescribed displacement and in turn influences the number of iterations required and the approximation residual.

4.1 Geometric Editing

In *geometric* editing, the displacement is prescribed on the entirety of the boundary. However, we focus on studying the case where the prescribed displacement is local and the rest of the boundary is fixed akin to local control in classic representations. Deformation in Section 4.3 and smoothing in Section 4.4 are indirect examples of the target displacement being global.

The considered models are trained to fit the SDF and surface normals a shape and share the same architecture: sin activation functions and 3 hidden layers of 32 neurons each. In total, there are $P = 2273$ learnable parameters. We demonstrate several use-cases of geometric editing in Figure 1 with displacing parts of both man-made and organic neural implicit shapes. When the displacements are, loosely speaking, *natural* to the shape, the approximations recover the target well. However,

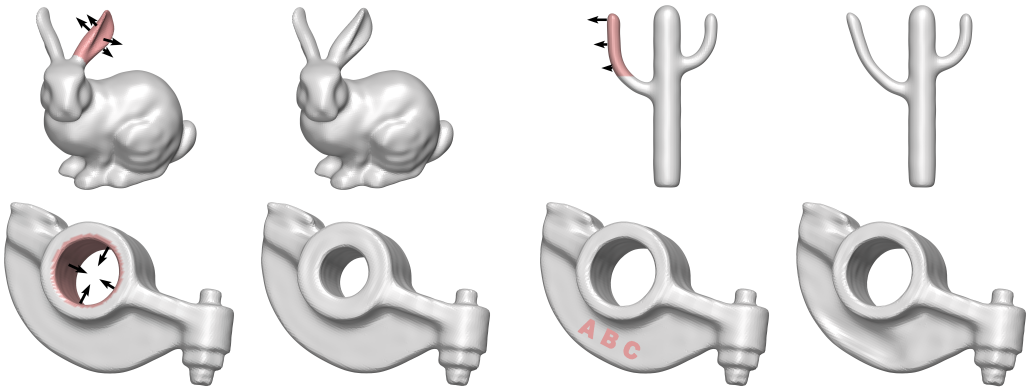

Figure 1: Geometric editing. Non-zero target displacement is prescribed on the highlighted regions and the rest is fixed with $\delta\bar{x}_n = 0$. Displacements that are natural are well approximated. However, the bottom-right figure serves to illustrate a counter-example, where the prescribed displacements are too complex and outside the limits of expressible deformations.

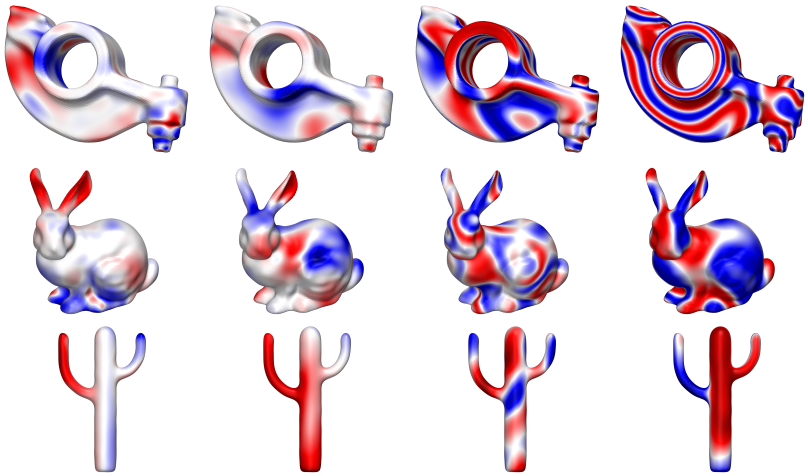

Figure 2: Basis functions of several neural implicit shapes. Each column depicts a weight from each layer increasing in depthg left-to-right. Red/blue denote positive/negative deformation w.r.t. the outward normal. These bases do not strongly encode semantic or geometric meaning, different to latent basis functions in decoders.

not arbitrarily complex displacements can be recovered as illustrated by the inscribed letters. The characteristic length of the detailed target is much finer than that of the shape, especially in that region. Certainly, the upper limit to how fine an arbitrary detail anywhere on the shape can be is given by A/P where A is the surface area of the shape. Furthermore, we hypothesize that the good memory-expressiveness trade-off of neural fields requires the NN to allocate geometric complexity where it is needed during training. To better illustrate this, some basis functions are shown in Figure 2. These do not encode obvious semantic or geometric meaning, but with increasing depth, the functions tend to become more complex. Nonetheless, the bases tend to have similar characteristic lengths as the shape itself.

4.2 Semantic Editing

In representation learning the aim is to explain observations with a small number of *latent variables* (Bengio et al., 2013). Generative models, such as those in (variational) auto-encoders or generative adversarial networks, attempt to map novel latent codes to novel outputs within the same distribution as the observations. When interpolating between two latent codes, the appearance of the generated

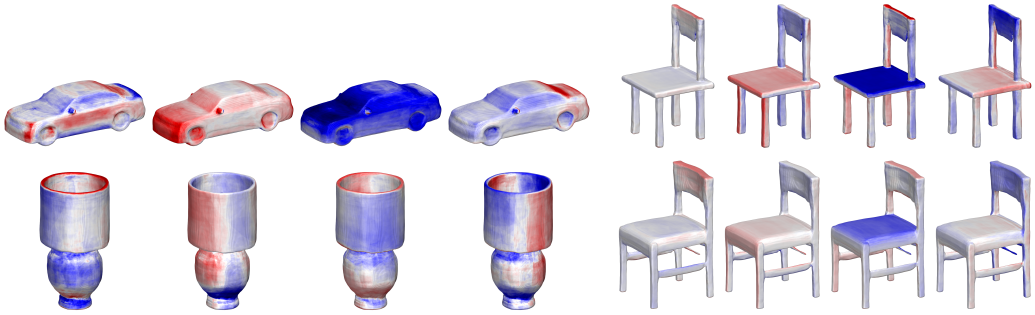

Figure 3: Basis functions of IM-Net. They partially encode semantics, such as rotation and reflection symmetries, show segmentation. Each column represents the same basis function. For similar shapes, such as the last two rows, the bases have a similar effect. Red/blue denote positive/negative deformation w.r.t. the normal.

output changes continuously (Shen et al., 2020a). Since this is valid for interpolation to many samples, we may also investigate the basis of the latent space as described by Equation 3. In Figure 3, we illustrate some of these basis functions for the decoder of the IM-Net architecture (Chen & Zhang, 2019). It is trained as part of the auto-encoder reconstructing the entire ShapeNet dataset (Chang et al., 2015). The decoder is an MLP with 8 layers of 1024 neurons each and leaky-ReLU activations, and, in addition to the three spatial coordinates, receives the $l \in \mathbb{R}^{256}$ latent code as an input. We use a pretrained model available at `https://github.com/czq142857/IM-NET-pytorch`.

These basis functions can be observed to encode semantics, such as rotation and reflection symmetries, and segment semantically meaningful parts, such as windscreen on the car, rim of the lamp and different surfaces on chairs. For similar parts, such as the two chairs, the bases show similar behaviour.

However, it is known that not all directions in the latent space are (equally) viable (Chen et al., 2022; Vyas et al., 2021), hence, a similar argument can be made for the bases. Furthermore, while each basis might cause meaningful deformation, they are not necessarily disentangled, i.e. each basis does not explain a single generative factor, unless latent disentanglement is used in training (Bengio et al., 2013; Shen et al., 2020b; Tschannen et al., 2018), which would further increase their interpretability.

Despite this, our method allows to intuitively traverse the latent space by considering the link between geometric and latent code changes. We present several examples in Figure 4, where the highlighted areas of an initial shape are prescribed a local, high-level displacement, such as shortening a leg of a chair or squeezing the sides of the boat. We sample roughly 100 points on these areas, prescribe the deformations, and leave the remaining boundary unconstrained. After finding the best fit parameter update according to Equation 5, we repeat this process for a few (< 15) iterations. Despite only prescribing local geometric deformation, we observe global and semantically consistent changes. Not only are obvious symmetries preserved, but also the morphology of the shape can change significantly, such as with the boat.

Compared to geometric editing, semantic editing has the computational advantage of sampling points on just a small part of the boundary and the number of latent parameters being much smaller than number of NN parameters, leading to a very small least-squares system. Apart from that, the method only requires a single backward pass through the model, altogether easily allowing $< 0.1s$ and enabling interactive usage.

4.3 Rigid Editing

As a last approach to editing, we briefly address deformation-rigidity since it classically is a widely used approach to shape editing. One the on hand, deformation energy is one of the many alternative priors to semantic prior discussed previously. A simple approach is to consider pure bending energy, since it can be computed only from the implicit representation itself. On the other hand, typical applications, such as as-rigid-as-possible (Sorkine & Alexa, 2007) or as-Killing-as-possible (Solomon et al., 2011), also include the stretching energy. This is not trivially applicable to implicit surfaces, as there is no natural notion of stretch due to ambiguity in the tangent directions. As discussed

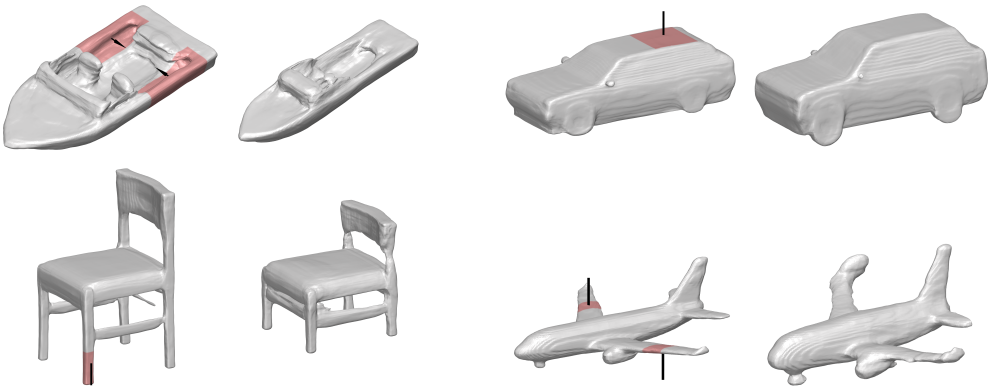

Figure 4: Semantic editing. Non-zero target displacement is prescribed on the highlighted regions $\bar{\Gamma}$ and the rest is *unconstrained*. The right image in each pair shows to result after < 15 iterations. In most cases, a semantically plausible result is produced as a result of locally prescribed plausible displacement. The bottom-right figure serves to illustrate a counter-example: prescribing each wing to move in opposite directions is not a semantically viable displacement and the method fails to struggles to approximate the displacements and produce meaningful results.

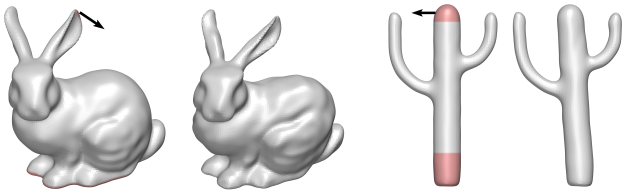

Figure 5: Rigid editing. Target displacement is prescribed in the highlighted regions $\bar{\Gamma}$ and the rest is unconstrained. In both cases, the bottom of the shapes is anchored.

in Section 3, there are several choices to recovering tangential displacements, but we consider perhaps the simplest one – using another independent function to provide the tangential component $\delta\mathbf{x}_t(\mathbf{x};\Xi) = \left(\mathbf{I} - \mathbf{n}(\mathbf{x})\mathbf{n}(\mathbf{x})^\top\right)\mathbf{f}_t(\mathbf{x};\Xi)$ as the tangential projection of a sufficiently smooth NN $\mathbf{f}_t(\mathbf{x};\Xi) : \mathbb{R}^3 \mapsto \mathbb{R}^3$ parameterized in Ξ.

We leverage Killing vector field (KVF) which generate isometric deformations. $\delta\mathbf{x}$ is a KVF if it has an anti-symmetric Jacobian $\mathbf{J}(\delta\mathbf{x}) = \mathbf{J}(\delta\mathbf{x})(\mathbf{x}) \in \mathbb{R}^{3,3}$ everywhere on the boundary $\mathbf{J}(\delta\mathbf{x}) + \mathbf{J}(\delta\mathbf{x})^\top = 0 \; \forall \mathbf{x} \in \Gamma$ Solomon et al. (2011). A vector fields' deviation from being Killing can be measured with its *Killing energy*

$$E_K(\delta\mathbf{x}, \Gamma) = \int_\Gamma \|\mathbf{J}(\delta\mathbf{x}) + \mathbf{J}(\delta\mathbf{x})^\top\|^2 \, \mathrm{d}x \; . \tag{6}$$

Classic works compute the Jacobians using discrete operators on spatial discretizations while Atzmon et al. (2021) formulates AKVF on a neural implicit directly. We follow this approach and seek an AKVF respecting the prescribed boundary deformations $\delta\bar{\mathbf{x}}(\mathbf{x}_i)$.

$$\underset{\delta\mathbf{x}}{\operatorname{argmin}} \, E_K + \alpha E_C \tag{7}$$

Boundary sensitivity allows to search for the energy minimizing deformation directly in the space of expressible deformations and directly optimize the normal component over $\delta\mathbf{\Theta}$ using Equation 2. The E_C in Equation 4 is expressed trivially but can now also accomodate the tangential component. To express E_K, we can leverage $\mathbf{J}$ being a linear operator $\mathbf{J}(\delta\mathbf{x}) = \mathbf{J}(\delta\mathbf{x}_n) + \mathbf{J}(\delta\mathbf{x}_t)$. Here $\mathbf{J}(\delta\mathbf{x}_n) = \mathbf{J}(\mathbf{b}^\top\delta\mathbf{\Theta}) = \mathbf{J}(\mathbf{b}^\top)\delta\mathbf{\Theta}$ with $\mathbf{J}(\mathbf{b}^\top) \in \mathbb{R}^{3,3,P}$ and $\mathbf{J}(\delta\mathbf{x}_t) = \mathbf{J}(\mathbf{f}_t)$ straight-forward. Figure 5 illustrates the results of AKVF deformation.

4.4 CONSTRAINTS

We demonstrate how boundary sensitivity can be used to fix a value of a functional during parameter updates. If the functional is expressed as a surface or volume integral, this can be done without computing the integrals themselves. An example of this is volume preservation, which can simulate incompressible material behaviour (Desbrun & Gascuel, 1995) and is desirable in smoothing to prevent shrinkage (Taubin, 1995). Fixing the volume of an implicit shape is difficult without going to an intermediate representation, such as a mesh (Remelli et al., 2020; Mehta et al., 2022), since there is no trivial way to differentiably compute the volume of an implicit shape.

Shape Derivatives To this end, we loosely introduce shape derivatives and refer to (Allaire et al., 2021) for more detail. $H'(\Omega)(\delta\mathbf{x})$ denotes a *shape derivative* of the functional $H(\Omega) \in \mathbb{R}$ if the expansion $H(\Omega_{\delta\mathbf{x}}) = H(\Omega) + H'(\Omega)(\delta\mathbf{x})$ holds for small $\delta\mathbf{x}$. We rewrite this using perturbations as $\delta H(\Omega)(\delta\mathbf{x}) = H(\Omega_{\delta\mathbf{x}}) - H(\Omega)$. For several functionals the shape derivatives are known analytically. For the functional $H(\Omega) = \int_\Omega h \,\mathrm{d}x$ defined as a volume integral of $h = h(\mathbf{x}) \in \mathbb{R}$ the shape derivative is $\delta H(\Omega)(\delta\mathbf{x}) = \int_\Omega \mathrm{div}\,(h\delta\mathbf{x}) \;\mathrm{d}x$. To compute the *volume* $H = V$, we integrate over the multiplicative identity $h = 1$. Inserting the boundary sensitivity from Equation 2 we arrive at

$$\delta V = \delta\mathbf{\Theta}^\top \int_\Omega \mathbf{b} \,\mathrm{d}x =: \mathbf{b}_V^\top \delta\mathbf{\Theta} \tag{8}$$

where $\mathbf{b}_V = \mathbf{b}_V(\mathbf{x}) \in \mathbb{R}^P$ again acts as a basis, but for the volume. $\delta V = 0$ can now be enforced both as soft constraint or as a hard constraint by projecting any parameter update $\delta\mathbf{\Theta}$ onto the $\mathbb{R}^{P-1}$ linear subspace of constant volume $\delta\mathbf{\Theta}_{\delta V=0} = (\mathbf{I} - \mathbf{b}_V\mathbf{b}_V^\top)\delta\mathbf{\Theta}$.

Smoothing without Shrinkage We demonstrate fixing the volume with smoothing. Smoothing on neural fields has been previously formulated using gradient descent on an objective penalizing the deviation from a specified mean-curvature (Yang et al., 2021) and as mean-curvature flow computed on an intermediate mesh Mehta et al. (2022). We also use mean-curvature flow without resorting to the intermediate mesh, but the proposed volume-constraint is agnostic to the choice of the smoothing method. For that, we first compute the mean curvature $\kappa(\mathbf{x}) \in \mathbb{R}$ by expanding $\kappa(\mathbf{x}) = \mathrm{div}(\mathbf{n}(\mathbf{x}))$. From there, we can reuse the geometric editing framework with the globally prescribed target displacement $\delta x_n(\mathbf{x}) = -\kappa(\mathbf{x}) \;\forall\; \mathbf{x} \in \bar{\Gamma} = \Gamma$. Note, that this approach induces the correct geometric flow as discussed by Mehta et al. (2022). Figure 6 compares smoothing with and without this constraint. The volume after 86 iterations decreases by 1.6% with the constraint and 80% without. Note, that without a volume constraint the shape ultimately converges to a singular point.

Similarly, constant area can simulate the behaviour of unstretchable, but bendable materials, such as rope in 2D or textile in 3D. For this, one would leverage the shape derivative of the functional $G(\Omega) = \int_\Gamma g \,\mathrm{d}s$ defined as a surface integral of $g = g(\mathbf{x}) \in \mathbb{R}$ $\delta G(\Omega)(\delta\mathbf{x}) = \int_\Gamma (\partial g/\partial\mathbf{n} + \kappa g)\, \delta\mathbf{x}^\top \mathbf{n} \,\mathrm{d}s$, where $\partial g/\partial\mathbf{n}$ is the directional derivative of g along $\mathbf{n}$. More generally, a similar constraint can be formulated for any functional of the presented forms.

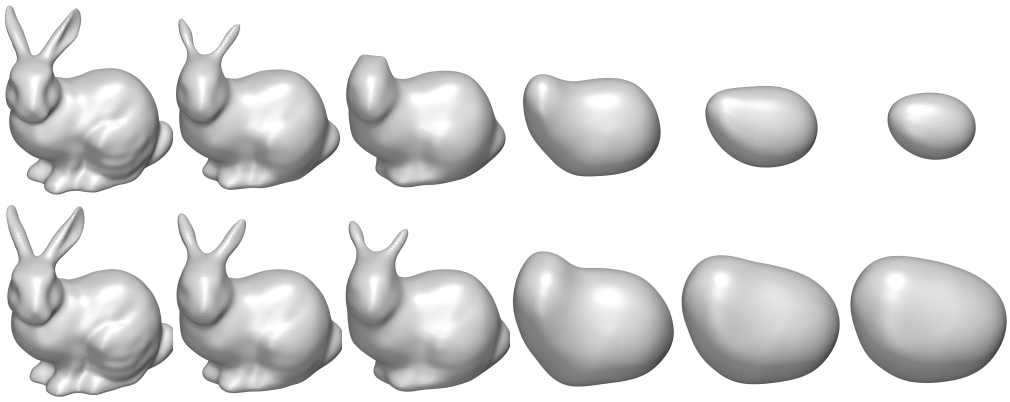

Figure 6: Select iterations of smoothing via mean-curvature flow without and with a volume constraint formulated through boundary sensitivity by projecting the parameter update onto the locally linear isovolumetric subspace.

5 Conclusions

With implicit neural shapes becoming a widespread representation, we have demonstrated a unifying approach to perform geometric and semantic editing without the need for tailored training or architectures, while being simple to implement, and, especially in the case of semantic editing, fast and fit for interactive use. While we touched upon optimizing directly in the deformation space with a rigidity-prior, using other priors for unconstrained and tangential deformations remains an interesting problem. We used signed-distance fields and occupation probabilities, but the editing framework can be also extend to other neural fields, where NeRFs especially provide tantalizing options. We hope that formulating a basis for the deformation space allows future work to further study and build models with desirable properties, such as interpretability, tailored degrees-of-freedom, linear-independence, compactness or use them for segmentation or symmetry detection.

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A Effect of Tikhonov regularization on semantic editing

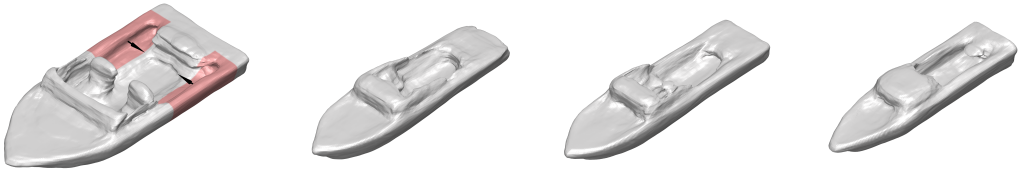

Figure 7: Effect of Tikhonov regularization on semantic editing. Left: source shape is prescribed an inward displacement on the highlighted regions while the rest is unconstrained. The three figures on the right have decreasing amount of Tikhonov regularization $\lambda = 10^1, 10^{-1}, 10^{-3}$. Stronger regularization better preserves similarity to the source shape, which is especially noticeable on the unconstrained front of the boat remaining wider. Stronger regularization also requires much more iterations to converge to a similar result (860 for $\lambda = 10^1$ compared to < 10 for the other two) since the the constraint violation E_C is weighted less heavily than regularization.