PRIOR PREFERENCE LEARNING FROM EXPERTS: DESIGNING A REWARD WITH ACTIVE INFERENCE

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ABSTRACT

Active inference may be defined as Bayesian modeling of a brain with a biologically plausible model of the agent. Its primary idea relies on the free energy principle and the prior preference of the agent. An agent will choose an action that leads to its prior preference for a future observation. In this paper, we claim that active inference can be interpreted using reinforcement learning (RL) algorithms and find a theoretical connection between them. We extend the concept of expected free energy (EFE), which is a core quantity in active inference, and claim that EFE can be treated as a negative value function. Motivated by the concept of prior preference and a theoretical connection, we propose a simple but novel method for learning a prior preference from experts. This illustrates that the problem with RL can be approached with a new perspective of active inference. Experimental results of prior preference learning show the possibility of active inference with EFE-based rewards and its application to an inverse RL problem.

1 INTRODUCTION

Active inference (Friston et al., 2009) is a theory emerging from cognitive science using a Bayesian modeling of the brain function (Friston et al., 2006; Friston, 2010; Friston et al., 2015; 2013), predictive coding (Friston et al., 2011; Lopez-Persem et al., 2016), and the free energy principle (Friston, 2012; Parr & Friston, 2019; Friston, 2019). It states that the agents choose actions to minimize an expected future surprise (Friston et al., 2012; 2017a;b), which is a measurement of the difference between an agent's prior preference and expected future. Minimization of an expected future surprise can be achieved by minimizing the *expected free energy* (EFE), which is a core quantity of active inference. Although active inference and EFE have been inspired and derived from cognitive science using a biologically plausible brain function model, its usage in RL tasks is still limited owing to its computational issues and prior-preference design. (Millidge, 2020; Fountas et al., 2020)

First, EFE requires heavy computational cost. A precise computation of an EFE theoretically averages all possible policies, which is clearly intractable as an action space $\mathcal{A}$ and a time horizon T increase in size. Several attempts have been made to calculate the EFE in a tractable manner, such as limiting the future time horizon from t to $t + H$ (Tschantz et al., 2019), and applying Monte-Carlo based sampling methods (Fountas et al., 2020; Çatal et al., 2020) for the search policies.

Second, it is unclear how the prior preferences should be set. This is the same question as how to design the rewards in the RL algorithm. In recent studies (Fountas et al., 2020; Çatal et al., 2020; Ueltzhöffer, 2018) the agent's prior preference is simply set as the final goal of a given environment for every time step. There are some environments in which the prior preference can be set as time independent. However, most prior preferences in RL problems are neither simple nor easy to design because prior preferences of short and long-sighted futures should generally be treated in different ways.

In this paper, we first claim that there is a theoretical connection between active inference and RL algorithms. We then propose prior preference learning (PPL), a simple and novel method for learning a prior-preference of an active inference from an expert simulation. In Section 2, we briefly introduce the concept of an active inference. From the previous definition of the EFE of a deterministic policy, in Section 3, we extend the previous concepts of active inference and theoretically demonstrate that it can be analyzed in view of the RL algorithm. We extend this quantity to a stochastic policy network and define an action-conditioned EFE for a given action and a given policy network.

Following Millidge (2020) using a bootstrapping argument, we show that the optimal distribution over the first-step action induced from active inference can be interpreted using Q-Learning. Consequently, we show that EFE can be treated as a negative value function from an RL perspective. From this connection, in Section 4, we propose a novel method for designing EFE-based rewards, by learning a prior preference from expert demonstrations. Through such expert demonstrations, an agent learns its prior preference given the observation to achieve a final goal, which can effectively handle the difference between local and global preferences. Our experiments in Section 5 show the applicability of active inference based rewards using EFE to an inverse RL problem.

2 ACTIVE INFERENCE

The active inference environment rests on the partially observed Markov decision process settings with an observation that comes from sensory input o_t and a hidden state s_t which is encoded in the agent's latent space. We will discuss a continuous observation/hidden state space, a discrete time step, and a discrete action space $\mathcal{A}$: At a current time $t < T$ with a given time horizon T, the agent receives an observation o_t.The agent encodes this observation to a hidden state s_t in its internal generative model, (i.e., a generative model for the given environment in an agent) and then searches for the action sequence that minimizes the expected future surprise based on the agent's prior preference $\tilde{p}(o_\tau)$ of a future observation o_τ with $\tau > t$.

In detail, we can formally illustrate the active inference agent's process as follows: s_t and o_t are a hidden state and an observation at time t, respectively. In addition, $\pi = (a_1, a_2, ..., a_T)$ is a sequence of actions. Let $p(o_{1:T}, s_{1:T})$ be a generative model of the agent with its transition model $p(s_{t+1}|s_t, a_t)$, and $Q(o_{1:T}, s_{1:T}, \pi)$ be a variational density. A distribution over policies $Q(\pi)$ will be determined later. From here, we can simplify the parameterized densities as trainable neural networks with $p(o_t|s_t)$ as a decoder, $Q(s_t|o_t)$ as an encoder, and $p(s_{t+1}|s_t, a_t)$ as a transition network in our generative model.

First, we minimize the current surprise of the agent, which is defined as $-\log p(o_t)$. Its upper bound can be interpreted as the well-known negative ELBO term, which is frequently referred to as the variational free energy F_t at time t in studies on active inference.

$$-\log p(o_t) \leq \mathbb{E}_{Q(s_t|o_t)}[\log Q(s_t|o_t) - \log p(o_t, s_t)] = F_t$$

Minimizing F_t provides an upper bound on the current surprise, and makes our networks in the generative model well-fitted with our known observations of the environment and its encoded states. For the future action selection, the total EFE $\mathcal{G}(s_t)$ over all possible policies at the current state s_t at time t should be minimized.

$$\mathcal{G}(s_t) = \mathbb{E}_{Q(s_{t+1:T}, o_{t+1:T}, \pi)}[\log \frac{Q(s_{t+1:T}, \pi)}{\tilde{p}(s_{t+1:T}, o_{t+1:T})}]$$

Focusing on the distribution $Q(\pi)$, it is known that the total EFE $\mathcal{G}(s_t)$ is minimized when the distribution over policies $Q(\pi)$ follows $\sigma(-G_\pi(s_t))$, where $\sigma(\cdot)$ is a softmax over the policies and $G_\pi(s_t)$ is the EFE under a given state s_t for a fixed sequence of actions π at time t. (Millidge et al., 2020)

$$G_\pi(s_t) = \sum_{\tau > t} G_\pi(\tau, s_t) = \sum_{\tau > t} \mathbb{E}_{Q(s_\tau, o_\tau|\pi)}[\log \frac{Q(s_\tau|\pi)}{\tilde{p}(o_\tau) Q(s_\tau|o_\tau)}]$$

This means that a lower EFE is obtained for a particular action sequence π; a lower future surprise will be expected and a desired behavior $\tilde{p}(o)$ will be obtained. Several active inference studies introduce a temperature parameter $\gamma > 0$ such that $Q_\gamma(\pi) = \sigma(-\gamma G_\pi(s_t))$ to control the agent's behavior between exploration and exploitation. Because we know that the optimal distribution over the policies is $Q(\pi) = \sigma(-G_\pi(s_t))$, the action selection problem boils down to a calculation of the expected free energy $G_\pi(s_t)$ of a given action sequence π.

The learning process of active inference contains two parts: (1) learning an agent's generative model with its trainable neural networks $p(o_t|s_t)$, $Q(s_t|o_t)$, and $p(s_{t+1}|s_t, a_t)$ that explains the current observations and (2) learning to select an action that minimizes a future expected surprise of the agent by calculating the EFE of a given action sequence.

3 EFE AS A NEGATIVE VALUE: BETWEEN RL AND ACTIVE INFERENCE

In this section, we first extend the definition of an EFE to a stochastic policy. We then, propose an action-conditioned EFE that has a similar role as a negative action-value function in RL. Based on these extensions, we will repeat the arguments in the active inference and claim that the RL algorithm with EFE as a negative value is equivalent to controlling the policy network toward an ideal distribution in the active inference. Calculating the expected free energy $G_\pi(s_t)$ for all possible deterministic policies π is intractable even in a toy-example task because the number of policies rapidly increases as the time horizon T and the number of actions $|\mathcal{A}|$ increases. Instead of searching for a number of deterministic policies, we extend the concept of EFE to a stochastic policy based on a policy network $\phi = \phi(a_t|s_t)$. In this case, we also extend $Q(s_\tau|\pi)$ to $Q(s_\tau, a_{\tau-1}|\phi) := p(s_\tau|s_{\tau-1}, a_{\tau-1})\phi(a_{\tau-1}|s_{\tau-1})$, which is a distribution over the states and actions, where each action $a_{\tau-1}$ is only dependent on state $s_{\tau-1}$ with $\phi(a_{\tau-1}|s_{\tau-1})$. Plugging in a deterministic policy $\phi = \pi$ yields $Q(s_\tau, a_{\tau-1}|\pi) = p(s_\tau|s_{\tau-1}, a')$ with $\pi(a_{\tau-1}) = a'$, which is the same equation used in previous studies on active inference. Suppose we choose an action a_t based on the current state s_t and a given action network ϕ, its corresponding expected free energy term can then be written as follows. This can be interpreted as an EFE of a sequence of policies $(\phi, \phi, ..., \phi)$.

$$G_\phi(s_t) = \mathbb{E}_{\prod_{\tau>t} Q(s_\tau, o_\tau, a_{\tau-1}|\phi)}[\sum_{\tau>t} \log \frac{Q(s_\tau, a_{\tau-1}|\phi)}{\tilde{p}(o_\tau)Q(s_\tau|o_\tau)}]$$

Note that the agent uses the same action network $\phi(a_\tau|s_\tau)$ for all τ, and we can therefore rewrite this equation in a recursive form.

$$\begin{aligned}
G_\phi(s_t) &= \mathbb{E}_{\prod_{\tau>t} Q(s_\tau, o_\tau, a_{\tau-1}|\phi)}[\sum_{\tau>t} \log \frac{Q(s_\tau, a_{\tau-1}|\phi)}{\tilde{p}(o_\tau)Q(s_\tau|o_\tau)}] \\
&= \mathbb{E}_{\prod_{\tau>t} Q(s_\tau, o_\tau, a_{\tau-1}|\phi)}[\log \frac{Q(s_{t+1}, a_t|\phi)}{\tilde{p}(o_{t+1})Q(s_{t+1}, a_t|o_{t+1})} + \sum_{\tau>t+1} \log \frac{Q(s_\tau, a_{\tau-1}|\phi)}{\tilde{p}(o_\tau)Q(s_\tau|o_\tau)}] \\
&= \mathbb{E}_{Q(s_{t+1}, o_{t+1}, a_t|\phi)}[\log \frac{Q(s_{t+1}, a_t|\phi)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})} + \mathbb{E}_{\prod_{\tau>t+1} Q(s_\tau, o_\tau, a_{\tau-1}|\phi)} \sum_{\tau>t+1} \log \frac{Q(s_\tau, a_{\tau-1}|\phi)}{\tilde{p}(o_\tau)Q(s_\tau|o_\tau)}] \\
&= \mathbb{E}_{Q(s_{t+1}, o_{t+1}, a_t|\phi)}[\log \frac{Q(s_{t+1}, a_t|\phi)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})} + G_\phi(s_{t+1})]
\end{aligned}$$

Replacing and fixing the first action $a_t = a$, an action-conditioned EFE $G_\phi(s_t|a)$ of a given action a and a policy network ϕ is then defined as follows:

$$\begin{aligned}
G_\phi(s_t|a) &:= \mathbb{E}_{Q(s_{t+1}, o_{t+1}, a_t|a_t=a)} \left[\log \frac{Q(s_{t+1}, a_t|a_t = a)}{\tilde{p}(o_{t+1}, s_{t+1})} + G_\phi(s_{t+1})\right] \\
&= \mathbb{E}_{p(s_{t+1}|s_t, a_t=a)p(o_{t+1}|s_{t+1})} \left[\log \frac{p(s_{t+1}|s_t, a_t = a)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})} + G_\phi(s_{t+1})\right]
\end{aligned}$$

Taking an expectation over $\phi(a|s_t)$, we obtain the relationship between $G_\phi(s_t)$ and $G_\phi(s_t|a_t)$.

$$\mathbb{E}_{\phi(a|s_t)}[G_\phi(s_t|a)] = G_\phi(s_t) + \mathcal{H}[\phi(a|s_t)]$$

We may consider separating the distribution over the first-step action from ϕ to find an alternative distribution that minimizes the EFE. Substituting the distribution over the first-step action as $Q(a_t)$ instead of $\phi(a_t)$, we obtain its one-step substituted EFE as indicated below. This can be interpreted as the EFE of a sequence of a policies $(Q(a_t), \phi, ..., \phi)$

$$\begin{aligned}
&\mathcal{G}^\phi_{1-step}(s_t) = \mathbb{E}_{Q(a_t)Q(s_{t+1}, o_{t+1}|a_t) \prod_{\tau>t+1} Q(s_\tau, o_\tau|\phi)}[\log \frac{Q(a_t)Q(s_{t+1}|a_t)Q(s_{t+2:T}, a_{t+1:T-1}|\phi)}{\tilde{p}(s_{t+1:T}, o_{t+1:T})}] \\
&= \mathbb{E}_{Q(a_t)Q(s_{t+1}, o_{t+1}|a_t) \prod_{\tau>t+1} Q(s_\tau, o_\tau|\phi)}[\log Q(a_t) + \log \frac{Q(s_{t+1}|a_t)}{\tilde{p}(o_{t+1}, s_{t+1})} + \log \prod_{\tau>t+1} \frac{Q(s_\tau, a_{\tau-1}|\phi)}{\tilde{p}(o_\tau)Q(s_\tau|o_\tau)}] \\
&= \mathbb{E}_{Q(a_t)} \left[\log Q(a_t) + G_\phi(s_t|a_t)\right]
\end{aligned}$$

Under a given ϕ, the value above depends only on the distribution $Q(a_t)$. Thus, minimizing the quantity above will naturally introduce the distribution $Q^*(a_t) = \sigma_a(-G_\phi(s_t|a_t))$, which is known to be similar in terms of active inference to the $\gamma = 1$ case.

$$\mathcal{G}^{\phi}_{1-step}(s_t) = \mathbb{E}_{Q(a_t)}\Big[\log Q(a_t) + G_\phi(s_t|a_t)\Big] = KL(Q(a_t)||Q^*(a_t)) - \log \sum_{a_t \in \mathcal{A}} \exp(-G_\phi(s_t|a_t))$$

Therefore, $\mathcal{G}^{\phi}_{1-step}(s_t) \geq -\log \sum_{a_t \in \mathcal{A}} \exp(-G_\phi(s_t|a_t))$, and the equality holds if and only if $Q(a_t) = \sigma_a(-G_\phi(s_t|a_t))$. Let us further consider the temperature hyperparameter $\gamma > 0$ such that $Q^*_\gamma(a_t) = \sigma_a(-\gamma G(s_t|a_t))$. Similarly to the above, we obtain the following:

$$\begin{aligned}
&\mathcal{G}^{\phi}_{1-step}(s_t) = \frac{1}{\gamma}\mathbb{E}_{Q(a_t)}\Big[\gamma \log Q(a_t) + \gamma G_\phi(s_t|a_t)\Big] \\
&= \frac{1}{\gamma}\mathbb{E}_{Q(a_t)}\Big[(1-\gamma)(-\log Q(a_t)) + \log Q(a_t) - \log Q^*_\gamma(a_t)\Big] - \frac{1}{\gamma}\log \sum_{a_t \in \mathcal{A}} \exp(-\gamma G_\phi(s_t|a_t)) \\
&= (\frac{1}{\gamma} - 1)\mathcal{H}(Q(a_t)) + \frac{1}{\gamma} KL(Q(a_t)||Q^*_\gamma(a_t)) - \frac{1}{\gamma}\log \sum_{a_t \in \mathcal{A}} \exp(-\gamma G_\phi(s_t|a_t))
\end{aligned}$$

From the arguments above, we can conclude that (1) $\mathcal{G}^{\phi}_{1-step}(s_t)$ is minimized when $Q^*(a_t) = \sigma_a(-G(s_t|a_t))$, a 'natural case' in which $\gamma = 1$, which was heuristically set in the experiments described in Millidge (2020). (2) Plugging $Q^*_\gamma(a_t) = \sigma_a(-\gamma G(s_t|a_t))$ to $Q(a)$ in the equation above, we obtain

$$\begin{aligned}
\mathcal{G}^{\phi}_{1-step}(s_t) &= (1-\gamma)\mathbb{E}_{Q^*_\gamma(a)}[G_\phi(s_t|a_t)] - \log D_\gamma \\
&\approx (1-\gamma)\mathbb{E}_{Q^*_\gamma(a)}[G_\phi(s_t|a_t)] + \gamma \min_{a_t \in \mathcal{A}}\{G_\phi(s_t|a_t)\}
\end{aligned}$$

where $D_\gamma = \sum_{a_t \in \mathcal{A}} \exp(-\gamma G_\phi(s_t|a_t))$, and the last approximation comes from the known property of the log-sum-exp with

$$-\log D_\gamma = -\log \sum_{a_t \in \mathcal{A}} \exp(-\gamma G_\phi(s_t|a_t)) \approx -\max_{a_t \in \mathcal{A}}\{-\gamma G_\phi(s_t|a_t)\} = \gamma \min_{a_t \in \mathcal{A}} G_\phi(s_t|a_t)$$

When $\gamma = 1$, the quantity $\mathcal{G}^{\phi}_{1-step}(s_t)$ is minimized with $Q^*_1(a_t) = \sigma_a(-G(s_t|a_t))$, and its minimum value can be approximated as $-\log D_1 \approx \min_{a_t \in \mathcal{A}}\{G_\phi(s_t|a_t)\}$. When $\gamma \to \infty$, $Q^*_\gamma(a_t)$ can be considered as a deterministic policy seeking the smallest EFE, which leads to the following:

$$\mathcal{G}^{\phi}_{1-step}(s_t) = (\frac{1}{\gamma} - 1)\mathcal{H}(Q^*_\gamma(a_t)) - \frac{1}{\gamma}\log \sum_{a_t \in \mathcal{A}} \exp(-\gamma G_\phi(s_t|a_t)) \approx \min_{a_t \in \mathcal{A}} G_\phi(s_t|a_t)$$

Note that Q-learning with a negative EFE can be interpreted as Q^*_γ with the case of $\gamma = \inf$. When $\gamma \searrow 0$, $Q^*_\gamma(a_t)$ converges to a uniform distribution over the action space $\mathcal{A}$, and the temperature hyperparameter γ motivates the agent to explore other actions with a greater EFE. Its weighted sum also converges to an average of the action-conditioned EFE.

Considering the optimal policy ϕ^* that seeks an action with a minimum EFE, we obtain the following:

$$\begin{aligned}
G_{\phi^*}(s_t) &= \min_a G_{\phi^*}(s_t|a) \\
&= \min_a \mathbb{E}_{p(s_{t+1}|s_t, a_t = a)p(o_{t+1}|s_{t+1})}\left[\log \frac{p(s_{t+1}|s_t, a_t = a)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})} + G_{\phi^*}(s_{t+1})\right]
\end{aligned}$$

This equation is extremely similar to the Bellman optimality equation. Here, $G_\phi(s_t|a)$ and $G_\phi(s_{t+1})$ correspond to the action-value function and the state value function, respectively. Based on this similarity, we can consider the first term $\log \frac{p(s_{t+1}|s_t, a_t = a)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})}$ as a one-step negative reward (because active inference aims to minimize the expected free energy of the future) and EFE as a negative value function.

4 PPL: PRIOR PREFERENCE LEARNING FROM EXPERTS

From the previous section, we verified that using $\log \frac{p(s_{t+1}|s_t,a_t)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})}$ as a negative reward can handle EFE with traditional RL methods. Through a simple calculation, the given term can be decomposed as follows:

$$R_t := -\log \frac{p(s_{t+1}|s_t, a_t)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})} = \log \tilde{p}(o_{t+1}) + (-\log \frac{p(s_{t+1}|s_t, a_t)}{Q(s_{t+1}|o_{t+1})}) = R_{t,i} + R_{t,e}$$

The first term measures the similarity between a preferred future and a predicted future, which is an intuitive value. The second term is called the epistemic value, which encourages the exploration of an agent. (Friston et al., 2015) The epistemic value can be computed either with prior knowledge on the environment of an agent or with various algorithms to learn a generative model. (Igl et al., 2018; Kaiser et al., 2019)

The core key to calculating the one-step reward and learning EFE is the prior preference $\tilde{p}(o_t)$, which includes information about the agent's preferred observation and goal. Setting a prior preference for an agent is a challenging point for active inference. A simple method that has been used in recent studies (Ueltzhöffer, 2018; Çatal et al., 2020) is to set a prior preference as a Gaussian distribution of the mean as goal observation. However, it would be inefficient to use the same prior preference for all observations o. Local preferences may differ from a global preference. Taking a mountain-car environment as an example, to reach the goal position, the car must move away from the goal position in an early time step. It is also difficult to directly design the prior preference for some observation space, such as in an image.

To solve the problems arising from active inference, we introduce prior preference learning from experts (PPL). Suppose that an agent can access expert simulations $S = \{(o_{i,1}, ..., o_{i,T}\}_{i=1}^{N}$, where N is the number of simulations. Note that the agent does not need an access to the expert's action. (Edwards et al., 2019) From the expert simulations S, an agent can learn an expert's prior $p(o_{t+1}|o_t)$ based on the current observation o_t. Model $p(o_{t+1}|o_t)$ captures the expert's local preference, which is more effective than the global preference. We can state that this method can be applied only with expert simulations, regardless of the knowledge on the forward dynamics of the environment.

5 EXPERIMENTS

In this section, we discuss and compare the experimental results of the classical control environments. First, we aim to compare the conventional global preference method to the PPL. We expect the PPL to be effective in environments where local and global preferences are different. We also observe how an epistemic value in a one-step reward term influences the learning process and its performance.

In addition, we use expert simulations for batch sampling when learning EFE and a forward dynamic model. This allows an EFE network and a dynamic model to be trained even in states that do not reach the early stage of learning. Note that our dynamic model needs access to expert actions. We use the Q-learning algorithm to learn the EFE network. Commonly used techniques in RL, such as replay memory and target network (Mnih et al., 2015) are used.

We evaluate our approach with a classic control environment implemented in Open AI Gym (Brockman et al., 2016). Expert simulations were obtained from Open AI RL baseline zoo (Raffin, 2018). Table 1 contains four experimental settings, where the setting 1 is our proposed method and the others are experimental groups to compare the effects of PPL, an expert batch, and the epistemic value.

5.1 CLASSIC CONTROL ENVIRONMENT

We tested three classical control environments: Acrobot, Cartpole, and MountainCar. We used 5000 pairs of (o_t, o_{t+1}) to train the prior preference $\tilde{p}(o_{t+1})$. We used the same neural network architecture for all environments, except for the number of input and output dimensions. During the training process, we clip the epistemic value to prevent a gradient explosion while using the epistemic value in settings 1, 2, and 3.

Algorithm 1: Inverse Q Learning with Prior Preference Learning

Learning prior preference from expert simulations;
Input: Expert simultaions $S = (o_{i,1}, ...o_{i,T})_{i=1}^{N}$
Initialize a prior preference network $\tilde{p}_\theta(o)$;
while *not converge* **do**
  Compute loss $L_{ppl}(\tilde{p}(o_t), o_{t+1})$;
  Update $\theta \leftarrow \theta - \alpha \nabla L_{ppl}$
end
Output: $\tilde{p}_\theta(o)$

Learning EFE and forward dynamic of an environment;
Input: Prior preference $\tilde{p}_\theta(o)$
Initialize the forward dynamic $p_\eta(o|s), T_\eta(s_{t+1}|s_t, a_t), Q_\eta(s|o)$, and EFE network $G_\xi(s_t, a_t)$.
while *not converge* **do**
  Reset environment;
  for $t \leftarrow 0$ **do**
    Select action $a_t = \arg\max_a G(s_t, a)$ with ϵ-greedy;
    Observe new observation o_{t+1};
    Compute $R_t = \log \frac{T(s_{t+1}|s_t,a_t)}{\tilde{p}(o_{t+1})Q(s_{t+1}|o_{t+1})}$;
    Compute the environment model loss $L_{model}\left((p \circ T \circ Q)(o_t), o_{t+1}\right)$;
    Compute the EFE network loss $L_{efe}(G_\xi(s_t, a_t), R_t + \max_a G_\xi(s_{t+1}, a))$;
    Update $\eta \leftarrow \eta - \alpha \nabla L_d$ and $\xi \leftarrow \xi - \alpha \nabla L_e fe$.
  end
end

5.1.1 ACROBOT

Acrobot is an environment with a two-link pendulum, where the second link is actuated. The goal of this environment is to swing the end effector up to the horizontal line. The state space has six dimensions: $(\cos\theta_1, \sin\theta_1, \cos\theta_2, \sin\theta_2, \dot{\theta}_1, \dot{\theta}_2)$, where θ_1 and θ_2 are the two rotational joint angles. The action space consists of three actions: accelerating +1, 0, or -1 torque on the joint between the two links. We did not run setting 2 in this study, because Acrobot is ambiguous in defining the global preference of the environment.

5.1.2 CARTPOLE

Cartpole is an environment with a cart and a pole on the cart. The goal is to balance the pole by controlling the cart. The state space is of 4 dimensions: $(x, \dot{x}, \theta, \dot{\theta})$, where x is the position of the cart and θ is the angle of the pole from the vertical line. The action space consists of two actions: accelerating the cart to the left or right. The global preference is given by a Gaussian distribution with mean (0, 0, 0, 0) with a fixed standard deviation 0.1 on each dimension.

5.1.3 MOUNTAINCAR

MountainCar is an environment with a car on the valley. The goal is to make the car reach the goal position, the right peak of the mountain. The state space is of two dimensions $(x, \dot{x})$ where x is the position of the car. The action space consists of three actions: accelerating the car to the left or right, or leaving it. The global preference is given by a Gaussian distribution with mean (0.5, 0) with a fixed standard deviation 0.1 on each dimension. Note that $(0.5, 0)$ is the state of the goal position with zero velocity.

5.2 RESULTS AND DISCUSSIONS

Figure 1 shows the experimental results on Acrobot, Cartpole, and MountainCar. Their performances are compared and benchmarked with the default reward of the environments.

Table 1: Experment setting for classic control environments

	Preference	Batch sampling	Reward
Setting 1	PPL	Replay memory + Experts	$R_{t,i} + R_{t,e}$
Setting 2	PPL	Replay memory	$R_{t,i} + R_{t,e}$
Setting 3	PPL	Replay memory + Experts	$R_{t,i}$
Setting 4	Global preference	Replay memory + Experts	$R_{t,i} + R_{t,e}$

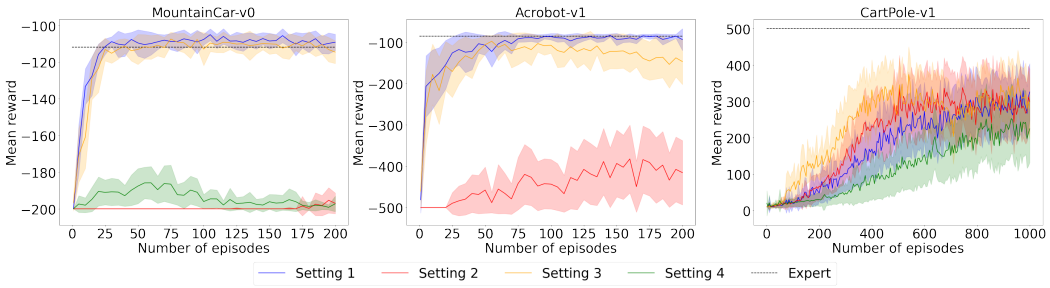

Figure 1: Experiment results on three classical control environments: MountainCar-v0, Acrobot-v1, and CartPole-v1. The curves in the figure were averaged over 50 runs, and the standard deviation of 50 runs is given as a shaded area. Each policy was averaged out of 5 trials. All rewards of the environment follow the default settings of Open AI Gym.

- **PPL and global preference**: Comparing setting 1 (blue line) and setting 2 (red line), it can be seen that PPL is more efficient than the conventional global preference as expected. In particular, in MountainCar, we can see that little learning is achieved. This seems to be because the difference between global and expert preferences is greater in the MountainCar environment. In the Cartpole, setting 2 learned more slowly than setting 1.
- **Expert Batch**: Comparing setting 1 (blue line) and setting 3 (orange line), it can be seen that using the expert batch is helpful for certain tasks. With Acrobot and MountainCar, the use of an expert batch performs better than the case without an expert batch. However, the results without expert batch are marginally better than those of setting 1 for Cartpole. This is because an agent of Cartpole only moves near the initial position, and thus there is no need for an expert batch to discover the dynamics of the generative model.
- **Epistemic Value**: We found that the epistemic value in the EFE term does not significantly impact the training process. Comparing setting 1 (blue line) and setting 4 (green line), the results were similar regardless of whether the epistemic value was used. In Acrobot and MountainCar, standard deviations were marginally smaller, but there were no significant differences between them. In the result of CartPole-v1, we found that setting 4 with no epistemic value term learned faster than our proposed setting 1 at the beginning of the training process. We deduce that this initial performance drop is due to the instability of the epistemic term. At the beginning of the training process, the generative model is not learned, and thus the related epistemic term becomes unstable. We leave this issue to a future study.

6 Conclusion

In this paper, we introduced the use of active inference from the perspective of RL. Although active inference emerged from the Bayesian model of cognitive process, we show that the concepts of active inference, especially for EFE, are highly related to RL using the bootstrapping method. The only difference is that, the value function of RL is based on a reward, while active inference is based on the prior preference. We also show that active inference can provide insights to solve the inverse RL problems. Using expert simulations, an agent can learn a local prior preference, which is more effective than the global preference. Furthermore, our proposed active inference based reward with

a prior preference and a generative model makes the previous IRL problems free from an ill-posed state. Our work on active inference is complementary to RL because it can be applied to model-based RL for the design of reward and model-free RL for learning of generative models.

REFERENCES

O. Çatal, T. Verbelen, J. Nauta, C. D. Boom, and B. Dhoedt. Learning perception and planning with deep active inference. In *ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 3952–3956, 2020.

Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. Openai gym, 2016.

Ashley Edwards, Himanshu Sahni, Yannick Schroecker, and Charles Isbell. Imitating latent policies from observation. In *International Conference on Machine Learning*, pp. 1755–1763, 2019.

Zafeirios Fountas, Noor Sajid, Pedro A. M. Mediano, and Karl J. Friston. Deep active inference agents using monte-carlo methods. *CoRR*, abs/2006.04176, 2020. URL https://arxiv.org/abs/2006.04176.

K. Friston, J. Kilner, and L. Harrison. A free energy principle for the brain. *Journal of Physiology-Paris*, 100:70–87, 2006.

Karl Friston. The free-energy principle: a unified brain theory? *Nature reviews. Neuroscience*, 11: 127–38, 02 2010. doi: 10.1038/nrn2787.

Karl Friston, Spyridon Samothrakis, and Read Montague. Active inference and agency: Optimal control without cost functions. *Biological cybernetics*, 106:523–41, 08 2012. doi: 10.1007/s00422-012-0512-8.

Karl Friston, Philipp Schwartenbeck, Thomas Fitzgerald, Michael Moutoussis, Tim Behrens, and Raymond Dolan. The anatomy of choice: active inference and agency. *Frontiers in Human Neuroscience*, 7:598, 2013. ISSN 1662-5161. doi: 10.3389/fnhum.2013.00598. URL https://www.frontiersin.org/article/10.3389/fnhum.2013.00598.

Karl Friston, Francesco Rigoli, Dimitri Ognibene, Christoph Mathys, Thomas Fitzgerald, and Giovanni Pezzulo. Active inference and epistemic value. *Cognitive Neuroscience*, 6(4):187–214, 2015. doi: 10.1080/17588928.2015.1020053.

Karl Friston, Thomas FitzGerald, Francesco Rigoli, Philipp Schwartenbeck, and Giovanni Pezzulo. Active inference: A process theory. *Neural Computation*, 29(1):1–49, 2017a. doi: 10.1162/NECO_a_00912. URL https://doi.org/10.1162/NECO_a_00912. PMID: 27870614.

Karl J. Friston. A free energy principle for biological systems. *Entropy*, 14(11):2100–2121, 2012. doi: 10.3390/e14112100. URL https://doi.org/10.3390/e14112100.

Karl J. Friston. A free energy principle for a particular physics, 2019. URL https://arxiv.org/abs/1906.10184.

Karl J Friston, Jean Daunizeau, and Stefan J Kiebel. Reinforcement learning or active inference? *PloS one*, 4(7):e6421, 2009.

Karl J. Friston, Jérémie Mattout, and James Kilner. Action understanding and active inference. *Biol. Cybern.*, 104(1-2):137–160, 2011. doi: 10.1007/s00422-011-0424-z. URL https://doi.org/10.1007/s00422-011-0424-z.

Karl J. Friston, Marco Lin, Christopher D. Frith, Giovanni Pezzulo, J. Allan Hobson, and Sasha Ondobaka. Active inference, curiosity and insight. *Neural Computation*, 29(10):2633–2683, 2017b. doi: 10.1162/neco_a_00999. URL https://doi.org/10.1162/neco_a_00999.

Maximilian Igl, Luisa Zintgraf, Tuan Anh Le, Frank Wood, and Shimon Whiteson. Deep variational reinforcement learning for pomdps. *arXiv preprint arXiv:1806.02426*, 2018.

Lukasz Kaiser, Mohammad Babaeizadeh, Piotr Milos, Blazej Osinski, Roy H Campbell, Konrad Czechowski, Dumitru Erhan, Chelsea Finn, Piotr Kozakowski, Sergey Levine, et al. Model-based reinforcement learning for atari. *arXiv preprint arXiv:1903.00374*, 2019.

Alizée Lopez-Persem, Philippe Domenech, and Mathias Pessiglione. How prior preferences determine decision-making frames and biases in the human brain. *Neuroscience*, 5:e20317, 2016. ISSN 2050-084X. doi: 10.7554/eLife.20317. URL https://doi.org/10.7554/eLife.20317.

Beren Millidge. Deep active inference as variational policy gradients. *Journal of Mathematical Psychology*, 96:102348, 2020. ISSN 0022-2496. doi: https://doi.org/10.1016/j.jmp.2020.102348. URL http://www.sciencedirect.com/science/article/pii/S0022249620300298.

Beren Millidge, Alexander Tschantz, and Christopher L. Buckley. Whence the expected free energy? *CoRR*, abs/2004.08128, 2020. URL https://arxiv.org/abs/2004.08128.

Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin A. Riedmiller, Andreas Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadik, Ioannis Antonoglou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg, and Demis Hassabis. Human-level control through deep reinforcement learning. *Nat.*, 518(7540):529–533, 2015. doi: 10.1038/nature14236. URL https://doi.org/10.1038/nature14236.

Thomas Parr and Karl J. Friston. Generalised free energy and active inference. *Biological Cybernetics*, 113(5-6):495–513, 2019. doi: 10.1007/s00422-019-00805-w. URL https://doi.org/10.1007/s00422-019-00805-w.

Antonin Raffin. Rl baselines zoo. https://github.com/araffin/rl-baselines-zoo, 2018.

Alexander Tschantz, Manuel Baltieri, Anil K. Seth, and Christopher L. Buckley. Scaling active inference. *CoRR*, abs/1911.10601, 2019. URL http://arxiv.org/abs/1911.10601.

Kai Ueltzhöffer. Deep active inference. *Biol. Cybern.*, 112(6):547–573, December 2018. ISSN 0340-1200. doi: 10.1007/s00422-018-0785-7. URL https://doi.org/10.1007/s00422-018-0785-7.