BUTTERFLY EFFECTS OF SGD NOISE: ERROR AMPLIFICATION IN BEHAVIOR CLONING AND AUTOREGRESSION

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ABSTRACT

This work studies training instabilities of behavior cloning with deep neural networks. We observe that minibatch SGD updates to the policy network during training result in sharp oscillations in long-horizon rewards, despite negligibly affecting the behavior cloning loss. We empirically disentangle the statistical and computational causes of these oscillations, and find them to stem from the chaotic propagation of minibatch SGD noise through unstable closed-loop dynamics. While SGD noise is benign in the single-step action prediction objective, it results in catastrophic error accumulation over long horizons, an effect we term *gradient variance amplification* (GVA). We show that many standard mitigation techniques do not alleviate GVA, but find an exponential moving average (EMA) of iterates to be surprisingly effective at doing so. We illustrate the generality of this phenomenon by showing the existence of GVA and its amelioration by EMA in both continuous control and autoregressive language generation. Finally, we provide theoretical vignettes that highlight the benefits of EMA in alleviating GVA and shed light on the extent to which classical convex models can help in understanding the benefits of iterate averaging in deep learning.

1 INTRODUCTION

Deep neural networks are increasingly used in machine learning tasks that contain *feedback loops* as a defining characteristic: outputs of language models depend on previously predicted tokens (Vaswani et al., 2017), recommendation systems influence the users to whom they give suggestions (Krauth et al., 2020; Dean & Morgenstern, 2022), and robotic policies take actions in reactive control environments (Ross & Bagnell, 2010; Laskey et al., 2017). Because these tasks are so complex, it is standard practice to optimize surrogate objectives, such as next-token prediction, that typically ignore feedback loops altogether (Pomerleau, 1988; Vaswani et al., 2017; Florence et al., 2022).

When training deep models by gradient updates on the surrogate objective, surrogate performance often improves more or less monotonically as training progresses. At the same time, successive iterates can exhibit wild variations in their performance on the task of interest. Because it is often impractical to evaluate the desired performance metric at multiple checkpoints, these oscillations imply that we have high risk of selecting and deploying a poor policy. Thus, in order to determine best practices, we must first understand whether *better training* or *better data* will fix these instabilities. This leads us to ask:

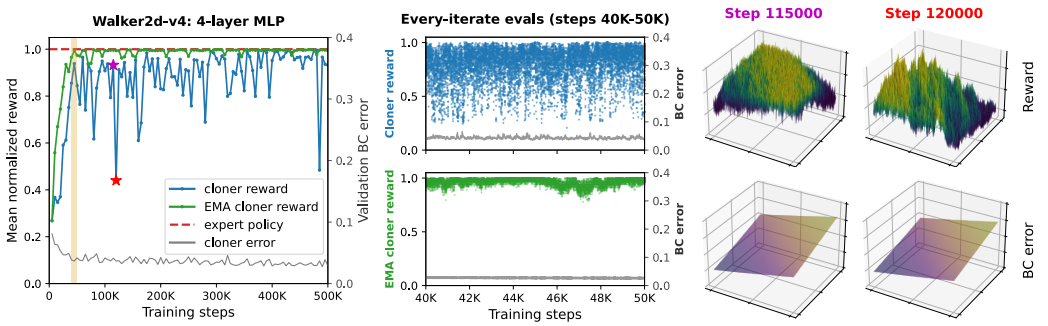


Figure 1: Typical reward instabilities over long-horizon ($H = 1000$) rollouts of neural behavior cloners for the `Walker2d-v4` MuJoCo locomotion task. *Left:* Rollout rewards (blue training curves) oscillate dramatically over the course of training (evaluated every 5000 iterations), while BC loss is stable. *Center:* Zoomed-in view of the highlighted region in (left). Large reward fluctuations are evident even between consecutive gradient iterates. *Right:* Exhaustive evaluation of small neighborhoods (in stochastic gradient directions) around iterates 115K and 120K, revealing a fractal reward landscape $\boldsymbol{\theta} \mapsto J_H(\pi_{\boldsymbol{\theta}})$; this jaggedness is invisible in the 1-step behavior cloning objective $\ell_{\mathrm{BC}}(\pi_{\boldsymbol{\theta}})$. Iterate averaging (EMA) drastically mitigates these effects (green training curves). Details are provided in Appendix B.1.1.

> *What causes instabilities in learning systems with feedback loops? To what extent can they be mitigated by algorithmic interventions alone, without resorting to collecting additional data?*

We explore this question in the context of behavior cloning (BC), a technique for training a policy to optimize a multi-step objective in a purely offline manner. This is achieved by introducing a surrogate loss function ℓ_{BC} (*behavior cloning loss*) that measures the distance between actions generated by some *expert policy* $\pi_{\boldsymbol{\theta}^\star}$ and those taken by the learner's policy, and then minimizing ℓ_{BC} over an offline dataset of expert trajectories. BC is sufficiently broad as to capture important tasks ranging from robotics and autonomous driving (Pomerleau, 1988; Codevilla et al., 2018; Chi et al., 2023) to autoregressive language generation (Chang et al., 2023a), and is popular in practice due to its simplicity and purely offline nature.

Our starting point is to observe that behavior cloning with deep neural networks exhibits **training instabilities** in which the multi-step objective (J_H), or *rollout reward*, of nearby checkpoints oscillates wildly during training, despite a well-behaved validation loss for ℓ_{BC}, even at the single iterate frequency; Figure 1 exhibits this phenomenon for a sample training curve of a behavior cloning policy in the `Walker2d-v4` MuJoCo locomotion task (Towers et al., 2023; Todorov et al., 2012). This oscillatory behavior is clearly undesirable; we cannot differentiate between low- and high-quality iterates based on (validation loss for) ℓ_{BC}, and thus cannot reliably select a high-quality policy.

With regard to the final performance of BC, it is well understood that scarce or low-quality data can lead to statistical challenges and consequent performance degradation of imitator policies; unsurprisingly, better data often improves the quality of a learned policy. Unfortunately, existing approaches to obtaining better data require either interactive access to the demonstrating expert (Ross & Bagnell, 2010; Laskey et al., 2017) or additional side information (Pfrommer et al., 2022; Block et al., 2023a); these interventions may be costly or impossible in many applications. Thus, in this work we treat the data generating process as fixed and aim to investigate whether we can mitigate oscillations and improve the performance of BC solely through the application of better algorithmic choices.

1.1 Contributions

In this paper, we aim to diagnose and ameliorate instabilities in behavior cloning that arise from training on the surrogate cost alone in the purely offline setting. Our findings are as follows.

Diagnosis of rollout oscillations: gradient variance amplification. In Section 3, we conduct an extensive empirical study (278 distinct interventions) of BC in continuous control tasks and inves-

tigate the effects that architecture, regularization, and optimization interventions have on training instability. We identify the presence of training oscillations and attribute them to *gradient variance amplification* (GVA): the propagation of minibatch SGD noise through closed-loop dynamics, leading to catastrophic error amplification resembling **butterfly effects** in chaotic systems. We ablate away much of the statistical difficulty, so that the presence of oscillations suggests that GVA is an *algorithmic* rather than *statistical* pathology.

Mitigating GVA: stabilizers for unstable optimizers. In Section 4, we investigate mitigations for GVA. Because GVA is caused by variance in the stochastic gradients, it can be ameliorated with variance reduction. Indeed, we observe (Section 3.2) that i) aggressively decaying the learning rate, and ii) greatly increasing the batch size through gradient accumulation, both have positive effects on the stability of training. Unfortunately, both of these interventions come at a great increase in compute cost. As such, our most significant finding (Section 4.1) is that *iterate averaging* by taking an Exponential Moving Average (EMA) of the optimization trajectory (Polyak & Juditsky, 1992; Ruppert, 1988), stabilizes training and mitigates GVA across a wide range of architectures and tasks, with essentially no downsides. While iterate averaging is popular in many deep learning research communities, this paper exposes iterate averaging as an *essential* design consideration when training any deep model in the presence of feedback loops.

A preliminary study of GVA in language generation. In Section 4.2, we broaden our focus by considering autoregressive sequence models. Our findings suggest that **unstable optimizers, when stabilized with iterate averaging to mitigate GVA, do not need full learning rate decay**, entailing potential computational and statistical benefits for training language models. For this reason, we suggest that EMA and related filters be designated as *stabilizers* in their own right and incorporated into deep learning pipelines in the same vein as modern optimizers and schedulers.

The applicability of convex theory. In Section 4.3, we complement our empirical results with theoretical vignettes. While the benefits of large learning rates cannot be explained in a convex setting, we demonstrate that—conditional on using theoretically suboptimal learning rates—stochastic convex optimization provides useful intuition for the causes and mitigations of GVA in deep learning. With our empirical results, these findings add to a line of work on surprising near-convex behavior in deep learning (Sandler et al., 2023; Frankle et al., 2020; Fang et al., 2022; Schaul et al., 2013).

1.2 Related work

Understanding and mitigating the effects of error amplification in behavior cloning has been the subject of much empirical work (Ross & Bagnell, 2010; Laskey et al., 2017), but most approaches use potentially impractical online query access to the expert policy; instead, we focus on a purely offline setting.

Complicated value function landscapes and their effect on training have been investigated in the context of planning in RL, with Dong et al. (2020) investigating natural examples of fractal reward functions, Wang et al. (2021) examining the instabilities arising from poor representations, and Emmons et al. (2021); Chang et al. (2023a) observing the fact that ℓ_{BC} is a poor proxy for J_H. To the best of our knowledge, there has not been a systematic study of training instability in the sense of rollout reward oscillation of nearby checkpoints.

In the context of stochastic optimization and optimization for deep learning, many previous works have attempted to reduce variance in theory (Polyak & Juditsky, 1992; Ruppert, 1988) and practice (Izmailov et al., 2018; Busbridge et al., 2023; Kaddour, 2022; Kaddour et al., 2023). Of particular note is Sandler et al. (2023), which demonstrates (empirically and in a toy theoretical setting) a form of equivalence between learning rate decay and iterate averaging. Our focus is not on variance reduction *per se*, but rather on the propagation of ,variance through unstable feedback loops. We expand on the relationship between our work and Sandler et al. (2023) and discuss other related work in Appendix A.

2 Preliminaries

MDP formalism. We let $\mathcal{M} = (\mathcal{S}, \mathcal{A}, P, r, H, \nu)$ denote a finite-horizon Markov decision process (MDP), where $\mathcal{S}$ is an abstract state space, $\mathcal{A}$ is an abstract action space, $P : \mathcal{S} \times \mathcal{A} \to \mathbf{\Delta}(\mathcal{S})$ is

a Markov transition operator. We denote by $r : \mathcal{S} \times \mathcal{A} \to [0, 1]$ a reward function and $H \in \mathbb{N}$ is the length of the horizon. Because we focus on continuous control tasks, we follow the notational conventions of control theory, denoting states by $\mathbf{x}$ and actions by $\mathbf{u}$. We let $\nu \in \mathbf{\Delta}(\mathcal{S})$ denote the initial distribution such that a trajectory from $\mathcal{M}$ consists of $\mathbf{x}_1 \sim \nu$ and $\mathbf{x}_{h+1} \sim P(\cdot \mid \mathbf{x}_h, \mathbf{u}_h)$ for all h.

The learner has access to a class of policies $\pi : \mathcal{S} \times \Theta \to \mathbf{\Delta}(\mathcal{A})$, where Θ is the *parameter* space and $\pi_{\boldsymbol{\theta}} : \mathcal{S} \to \Delta(\mathcal{A})$ is the policy induced by parameter $\boldsymbol{\theta} \in \Theta$. Given a policy $\pi_{\boldsymbol{\theta}}$, we denote its expected cumulative reward by $J_H(\pi_{\boldsymbol{\theta}}) = \mathbb{E}[\sum_{h=1}^{H} r(\mathbf{x}_h, \mathbf{u}_h)]$ where $\mathbf{u}_h \sim \pi_{\boldsymbol{\theta}}(\cdot|\mathbf{x}_h)$ and the expectation is with respect to both the transition dynamics of $\mathcal{M}$ and the possible stochasticity of the policy. Our experiments focus on MDPs whose transition operators P are deterministic, i.e., there exists a function $f : \mathcal{S} \times \mathcal{A} \to \mathcal{S}$ such that $\mathbf{x}_{h+1} = f(\mathbf{x}_h, \mathbf{u}_h)$ for all h. In this case the only stochasticity of the system comes from the sampling of the initial state $\mathbf{x}_1 \sim \nu$ (and possibly the policy).

Imitation learning and behavior cloning. In imitation learning, we are given an offline data set of N trajectories $\mathcal{D}_{\text{off}} = \{(\mathbf{x}_h^{(i)}, \mathbf{u}_h^{(i)})_{1 \le h \le H} \mid 1 \le i \le N\}$ generated by an expert policy $\pi_{\boldsymbol{\theta}^\star}$ interacting with the MDP $\mathcal{M}$. In this work, we always consider *deterministic policies*, i.e., where for all $\mathbf{x}$, $\pi_{\boldsymbol{\theta}}(\mathbf{x})$ has support on a single action; in particular this holds for the expert $\pi_{\boldsymbol{\theta}^\star}$. The goal of the learner is to produce a policy $\pi_{\widehat{\boldsymbol{\theta}}}$ that maximizes the expected cumulative reward $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$ over an episode. We focus on the popular *behavior cloning* (BC) framework, where we fix a loss function $\ell_{\text{BC}} : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$ that measures the distance from the actions produced by $\pi_{\boldsymbol{\theta}^\star}$, and learn $\pi_{\widehat{\boldsymbol{\theta}}}$ by attempting to minimize the empirical risk of ℓ_{BC} over $\mathcal{D}_{\text{off}}$; we abuse notation by denoting $\ell_{\text{BC}}(\pi_{\boldsymbol{\theta}}) := \mathbb{E}_{\mathcal{D}_{\text{off}}}[\ell_{\text{BC}}(\pi_{\boldsymbol{\theta}}(\mathbf{x}), \mathbf{u})]$ The basic premise behind behavior cloning is that ℓ_{BC} should be chosen such that if $\ell_{\text{BC}}(\pi_{\widehat{\boldsymbol{\theta}}}) \ll 1$ then $J_H(\pi_{\widehat{\boldsymbol{\theta}}}) \approx J_H(\pi_{\boldsymbol{\theta}^\star})$; that is, imitation of the expert is a surrogate for large cumulative reward. In line with common practice in BC (Janner et al., 2021; Shafiullah et al., 2022; Chi et al., 2023), the imitator policies in our experiments augment the state with the previous action, i.e., $\pi_{\boldsymbol{\theta}} : \mathcal{S} \times \mathcal{A} \to \mathcal{A}$, which can be integrated into the previous formalism by expanding the state space. For the special case of the first state $\mathbf{x}_1$, we always let $\mathbf{u}_0 = \mathbf{0}$.

Notation. Throughout the paper, we denote vectors by bold lower case letters and matrices by bold upper case letters.[1] We reserve $\boldsymbol{\theta}$ for a parameter of our policy and J_H for the cumulative reward over a trajectory, omitting H when it is clear from context. For conciseness, we often refer to J_H as the *reward*; the per-step reward function r makes no appearance in the rest of the paper. Given a set $\mathcal{U}$, we let $\mathbf{\Delta}(\mathcal{U})$ denote the class of probability distributions on $\mathcal{U}$.

3 Diagnosis of rollout oscillations: gradient variance amplification

3.1 Instabilities in behavior cloning of MuJoCo tasks

Experimental setup. We investigate instabilities in behavior cloning for the {`Walker2d`, `Hopper`, `HalfCheetah`, `Humanoid`, `Ant`}`-v4` environments from the OpenAI Gymnasium (Towers et al., 2023), all rendered in MuJoCo (Todorov et al., 2012). We focus on `Walker2d-v4` for the discussion that follows, and defer detailed discussion of further environments (which exhibit similar behavior) to Appendix B. Our expert is a multilayer perceptron (MLP) trained with Soft Actor Critic (SAC) (Haarnoja et al., 2018) for 3M steps with `stable-baselines3` (Raffin et al., 2021), with out-of-the-box hyperparameters.[2] The *default* imitator is a 4 layer MLP; details are in Appendix B. We examine several widths and depths, as well as Transformer (Vaswani et al., 2017) imitators.

Our first suite of experiments aims to isolate instability from *statistical difficulties*. We set up the experiments to make the behavior cloning problem as easy as possible. First, we focus on the "large-data" regime $N = H = 1000$, in which overfitting with respect to the BC loss $\ell_{\text{BC}}(\pi_{\widehat{\boldsymbol{\theta}}})$ is not a problem (see Figure 1), and thus poor rollout performance for $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$ cannot be blamed on

[1] In particular, we denote states by $\mathbf{x}$ and actions by $\mathbf{u}$ in order to emphasize that, in our experiments, they are vectors in Euclidean space.

[2] By default, the Stable-Baselines3 SAC agent is stochastic, but we enforce determinism by selecting the mean action of the resulting policy. This results in negligible degradations to the rewards; see Figure 5.

insufficient data; this removes a typical source of statistical difficulty faced in applying behavior cloning to domains such as robotics (Chi et al., 2023; Pfrommer et al., 2022; Ross & Bagnell, 2010; Laskey et al., 2017). Beyond focusing on the large-data regime, (i) we consider only deterministic dynamics and deterministic experts, and (ii) we include within our default model the same class of MLPs that parameterize the expert policies, ensuring that expressivity is not an issue. As such, we have placed ourselves in perhaps the easiest possible setting for behavior cloning.

In Figure 1 (Left), we compare the evolution of the BC loss $\ell_{\mathrm{BC}}(\pi_{\widehat{\boldsymbol{\theta}}})$ (on a validation set) and reward $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$ for imitator policies in the `Walker2d-v4` MuJoCo locomotion task. In this figure, we observe extreme oscillatory behavior in $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$, juxtaposed with smoothly decaying $\ell_{\mathrm{BC}}(\pi_{\widehat{\boldsymbol{\theta}}})$. In Figure 1 (Middle), we zoom in on the training trajectory between iterates 40K and 50K and observe that the same instability persists even at the *every-iterate* level. Toward identifying what causes these instabilities, Figure 1 (Right) displays an experiment in which we independently sample two stochastic gradients of the training loss at a fixed checkpoint with good rollout reward. Policy weights are then perturbed by small steps in each of the two directions, and we evaluate the resulting reward $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$ over 20 rollouts, along with the BC loss $\ell_{\mathrm{BC}}(\pi_{\widehat{\boldsymbol{\theta}}})$ on a held-out validation set. We see that nearby models vary erratically in terms of rollout performance, but vary smoothly in validation BC loss. These findings are reproduced consistently across the other environments and architectures in Appendix B; thus, we conclude:

(R1) **The reward landscape is highly sensitive to small changes in policy parameters**: small perturbations in model weights induce *butterfly effects* in the reward $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$. In contrast, in the same regions, the *BC loss* landscape $\boldsymbol{\theta} \mapsto \ell_{\mathrm{BC}}(\pi_{\boldsymbol{\theta}})$ is well-behaved (nearly linear locally).

3.2 Instability is caused by gradient variance amplification

We now present compelling evidence that *variance in stochastic gradients* during training is responsible for training instability, because **gradient variance is amplified through the sensitivity of the rollout rewards to fluctuations in network parameters**. In Figure 2, we visualize evolution of both $\ell_{\mathrm{BC}}(\pi_{\widehat{\boldsymbol{\theta}}})$ and $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$ over training for a variety of potential algorithmic interventions. We find that neither changing the model architecture and scale (1st row) nor standard regularization techniques (2nd row) ameliorate the training instabilities observed. We do see, however, that aggressively decaying the learning rate and increasing the batch size (3rd row) significantly reduces oscillations (at least when measuring mean rewards), at the expense of substantially slowing down training. Thus, we conclude that fluctuations from stochasticity in the gradients are to blame for oscillations in rollout rewards, and term this phenomenon *gradient variance amplification* (GVA). To summarize:

(R2) **GVA arises from algorithmic suboptimality rather than an information-theoretic limit.** Even with "infinite" training data (i.e., fresh trajectories with i.i.d. initial conditions at each training step), rollout oscillations persist.

(R3) **Training oscillations are *not* mitigated by many standard approaches to regularization**, including architectural interventions and increased regularization. On the other hand, **oscillations are ameliorated by variance reduction techniques**, such as large batch sizes, learning rate decay, and iterate averaging.

Appendix B shows that (R2) and (R3) remain true across environments and model architectures. In addition, we find that training instability is not the result of inadequate network architecture; we observe oscillations across model scales, and for both MLP and Transformer architectures.

While Figure 2 shows that it is possible to quell GVA using small learning rates or large batch sizes, this may not always be practical, as both interventions can incur steep computational costs.[3] Even worse, the success of continuous optimization in deep learning depends on non-convex feature learning mechanisms (Chizat et al., 2019), and too small a learning rate or too large a batch size can have deleterious effects on generalization.[4] Thus, it is vital to seek interventions that are holistically compatible with existing deep learning pipelines. Among these, Figure 2 highlights that

[3] As another unsatisfactory compromise, we also find that shallower models are less susceptible to GVA.

[4] We refer to some theoretical and empirical accounts in Appendix A.

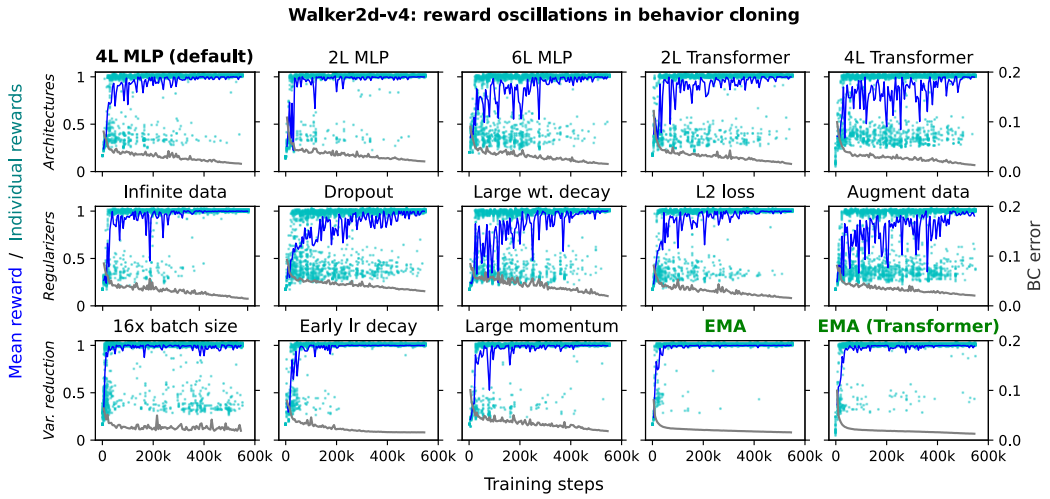


Figure 2: Highlights from a large suite of experiments, suggesting an algorithmic (rather than statistical) origin of reward oscillations. All plots use the 4-layer MLP architecture unless otherwise specified. Blue curves show mean rewards over 20 initial conditions, while teal dots show disaggregated per-episode rewards (such that each point represents the rollout reward of a fixed initial condition of the policy at the current iterate). These oscillations persist across dataset sizes, architectures, model scales, and choices of regularizers, and diminish toward the end of training as the learning rate decays to 0. They are most strongly mitigated by **variance reduction strategies**. Here, we opt for direct visualizations, providing a qualitative demonstration of GVA and its mitigations. We accompany these with quantitative comparisons in Appendix B.1.2.

a large momentum coefficient is mildly helpful, but taking an exponential moving average (EMA) of iterates (Polyak & Juditsky, 1992) is *extremely* effective. This motivates us to take a closer look at the latter in Section 4 through another suite of experiments.

3.3 Understanding GVA: mismatch between BC loss and rollout reward

The disparity between behavior cloning loss $\ell_{\mathrm{BC}}(\pi_{\widehat{\boldsymbol{\theta}}})$ and rollout reward $J_H(\pi_{\widehat{\boldsymbol{\theta}}})$ has long been appreciated in the imitation learning literature, and is understood to be caused by *error amplification*, the process by which mildly erroneous predictions, when fed repeatedly through feedback loops, result in highly suboptimal performance (Chen & Hazan, 2021; Wang et al., 2020a). More precisely, for given ℓ_{BC} and J_H as well as a policy $\pi_{\boldsymbol{\theta}^\star}$ and $\delta > 0$, we define the *error amplification constant* at scale δ to be the maximal value (with respect fo $\boldsymbol{\theta}$) for $J_H(\pi_{\boldsymbol{\theta}^\star}) - J_H(\pi_{\boldsymbol{\theta}})$ such that $\ell_{\mathrm{BC}}(\pi_{\boldsymbol{\theta}}) - \ell_{\mathrm{BC}}(\pi_{\boldsymbol{\theta}^\star}) < \delta$. The following proposition provides a simple theoretical illustration for how small fluctuations in BC loss can be drastically amplified by feedback between imperfectly-imitated policies and system dynamics.

Proposition 3.1 (Example of exponential error amplification)**.** *Let $\mathcal{B}_\delta$ denote the set of δ-Lipschitz functions $\Delta : \mathcal{S} \to \mathcal{A}$ with $\Delta(\mathbf{0}) = \mathbf{0}$. For any $\delta > 0$, there exists a deterministic MDP with horizon H and an expert policy $\pi_{\boldsymbol{\theta}^\star}$ such that the dynamics are Lipschitz in both state and action and $\pi_{\boldsymbol{\theta}^\star}$ is Lipschitz in the state, and such that*

$$\sup_{\Delta \in \mathcal{B}_\delta} \{J_H(\pi_{\boldsymbol{\theta}^\star}) - J_H(\pi_{\boldsymbol{\theta}^\star} + \Delta)\} \geq \Omega(H) \cdot \left(e^{\Omega(H\delta)} - 1\right),$$

yet $\sup_{\Delta \in \mathcal{B}_\delta} \ell_{\mathrm{BC}}(\pi_{\boldsymbol{\theta}^\star} + \Delta) \leq \mathcal{O}\left(H \cdot \delta^2\right)$, where ℓ_{BC} is the ℓ_2 loss. Thus, the error amplification constant is exponential *in the time horizon.*

Working model for GVA. Proposition 3.1 shows that even when ℓ_{BC} is uniformly small in a neighborhood around $\pi_{\boldsymbol{\theta}^\star}$, the rollout loss can be *exponentially large* in the same neighborhood. At the same time, there are good subsets of parameter space that do not experience this worst-case error amplification in our construction. We therefore hypothesize that, when stochastic optimization converges to a small neighborhood around zero-BC error models, it bounces between low-BC error

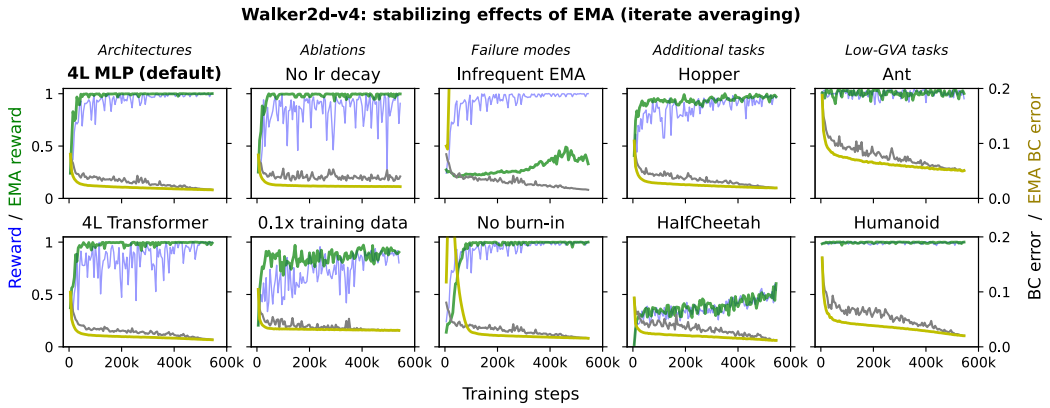


Figure 3: Iterate averaging significantly mitigates GVA-induced reward oscillations, **without needing to change the learning rate schedule or batch size**. These improvements hold across architectures, dataset sizes, and *some* tasks. *Column 2, bottom:* Algorithmic instabilities are more pronounced at smaller sample sizes; thus, **stabilization can lead to improved sample efficiency**. *Column 3:* We recommend updating the EMA at every iterate, with an initial burn-in phase, and with a tuned $\gamma^{(t)} = t^{-\alpha}$ decay, to avoid divergence or slower progress. *Columns 4-5*: We verify that the benefits of EMA are not exclusive to the `Walker2d-v4` task; for some other tasks (including the higher-dimensional `Humanoid-v4`), oscillations are more benign.

models that experience large error amplification, and those that do not. To recapitulate: *GVA is the phenomenon in which gradient stochasticity leads to optimization trajectories repeatedly visiting regions of parameter space with catastrophic error amplification.* Because our MuJoCo environments involve nonlinear contact dynamics (while the example in Proposition 3.1 is linear), oscillations in Figure 1 are even more chaotic than this example may suggest. We elaborate on this point further by studying the advantages of EMA on a discontinuous "cliff loss" problem in Section 4.3.

4 Mitigating GVA: Stabilizers for Unstable Optimizers

In Section 3.2, we isolated GVA as the primary cause of observed instabilities in BC (cf. Fig. 1) and identified iterate averaging with EMA (Polyak & Juditsky, 1992) as a promising remedy. In this section, we conduct an in-depth investigation of EMA as a mitigation. We start in continuous control (Section 4.1), and find EMA works almost unreasonably well at reducing GVA in the experimental testbed described in the prequel. Next, moving beyond continuous control (Section 4.2), we observe analogous effects in autoregressive language generation. In both settings, we find iterate averaging works so well as to **eliminate the need for full learning rate decay**; this leads us to recommend a conceptual reframing of EMA as a *stabilizer* for training neural networks, akin to (and interacting with) conventional optimizers and schedulers. We conclude (Section 4.3) by exploring the extent to which intuition on benefits of iterate averaging from the theory of stochastic convex optimization applies in our empirical settings.

4.1 The Outsized Benefit of Iterate Averaging

We recall the definition of the EMA method for iterate averaging (Polyak & Juditsky, 1992). Given an optimization trajectory $(\boldsymbol{\theta}^{(t)})_{0\le t} \subset \mathbb{R}^d$ and a sequence $(\gamma_t)_{1\le t} \subset [0,1]$, the EMA iterates $(\widetilde{\boldsymbol{\theta}}_\gamma^{(t)})$ are[5]

$$\widetilde{\boldsymbol{\theta}}_\gamma^{(0)} = \boldsymbol{\theta}^{(0)}, \quad \text{and } \widetilde{\boldsymbol{\theta}}_\gamma^{(t+1)} = (1-\gamma_t)\cdot \widetilde{\boldsymbol{\theta}}_\gamma^{(t)} + \gamma_t \cdot \boldsymbol{\theta}^{(t+1)}. \tag{4.1}$$

Many prior works have detailed the benefits of iterate averaging in stochastic convex optimization and beyond (see Appendix A). Here, we investigate its effect on GVA. We begin by considering the

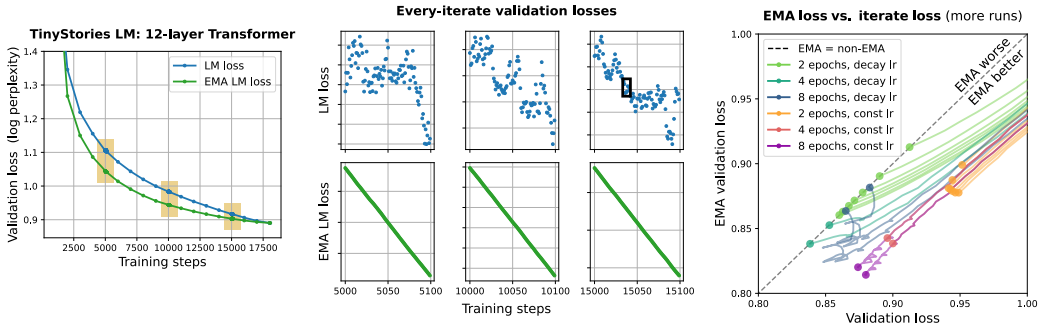
[5]Common heuristics include updating the EMA only after an initial "burn-in", and annealing γ with a polynomial decay: $\gamma^{(t)} = \max(t^{-\alpha}, \gamma_{\min})$. It is also customary to use $\beta^{(t)}$ to denote $1-\gamma^{(t)}$.

same MuJoCo framework as in Section 3. In Figure 3, we produce similar plots to those in Section 3, but this time juxtapose the vanilla trained models with an EMA of their iterates (further results and details are deferred to Appendix B). We observe the following:

(R4) **EMA iterate averaging strongly mitigates rollout oscillations**. *In every setup we consider,* across a variety of architectures and environments, EMA significantly reduces the oscillations in rollout reward; in no instance does it hurt performance.

We provide quantitative comparisons for a wide range of interventions in Figures 8 to 11.

4.2 AUTOREGRESSIVE SEQUENCE MODELS AND LANGUAGE GENERATION



[...] *One day, Tim and Spot were playing near the pit when they saw a big ball. They both wanted to play with...*

15027-15035: ...the ball, but they didn't want to share. Tim said, "I want the ball!" Spot barked, "No, I want the ball!" They started to fight over the ball. Tim's mom saw them fighting and said, "Stop fighting! You can both play with the ball if you share." Tim and Spot stopped fighting and [...]

15036-15037: ...it. Tim said, "Let's share the ball, Spot!" Spot barked happily, and they both played with the ball together. As they played, a little girl named Sue came by. She saw Tim and Spot sharing the ball and wanted to join them. Tim said, "Yes, you can play with us!" So, Tim, [...]

15038-15039: ...it. Tim said, "I want the ball!" Spot barked and wagged his tail. They both ran to the ball and started to play. As they played, the ball went into the pit. Tim and Spot looked at each other, not knowing what to do. Then, a friendly frog jumped out of the pit with the ball [...]

15040-15049: ...the ball, but they didn't want to share. Tim said, "I want the ball!" Spot barked, "No, I want the ball!" They started to fight over the ball. As they fought, the ball rolled into the pit. Tim and Spot stopped fighting and looked at the pit. They were sad because they couldn't [...]

Figure 4: **GVA in natural language generation**, with 270M-parameter Transformer models trained on TinyStories. *(Top row) Left:* Validation loss curves with and without EMA. *Center:* Zooming in on *(left)*, evaluations at every update demonstrate small per-iterate loss fluctuations, which are even smaller if EMA is applied; note that the green "lines" are also scatter plots. *Right:* Training paths in (model loss, EMA loss) space. EMA enables training without learning rate decay; this mitigates overfitting, resulting in the lowest-perplexity model. *(Bottom)* Examples of autoregressively generated text (with argmax decoding), where nearby training iterates can bifurcate. See Appendix B.2 for full results, including quantitative evaluations of these *"butterfly effects"* in generation.

We posit that GVA is a generic phenomenon that can manifest in disparate settings: whenever a model's predictions are applied within a (marginally stable or unstable) feedback loop, the closed-loop dynamics can amplify small fluctuations in a deleterious manner. A natural and timely setting with this structure—which complements continuous control–is autoregressive language modeling. Here, a network's parameters $\boldsymbol{\theta}$ are optimized on a 1-step prediction loss, which takes the role of $\ell_{\mathrm{BC}}(\pi_{\boldsymbol{\theta}})$. The network $\pi_{\boldsymbol{\theta}}$ is then used to generate a sequence of symbols $w_{1:H}$ by iteratively rolling out $\pi_{\boldsymbol{\theta}} : w_{1:h} \mapsto w_{h+1}$. Such models have been paradigm-shattering in NLP, code synthesis, and beyond. Motivated by the similarity of this pipeline to behavior cloning,[6] we perform a smaller set of analogous experiments on language generation. Our findings here parallel our findings for continuous control, and show (i) the presence of GVA, and (ii) substantial benefits of iterate averaging. In more detail, we train 270M-parameter 12-layer Transformer models on the TinyStories dataset (Eldan & Li, 2023), which serves as an inexpensive surrogate for a full-scale pretraining pipeline. Highlights are shown in Fig. 4, while Appendix B.2 provides full documentation, including larger-scale training runs with a non-synthetic corpus (Wikipedia). We summarize our findings below:

[6]Many works have investigated GPT-style pretraining through the lens of offline IL (Chang et al., 2023a). There are many degrees of freedom in evaluating performance; thus, we do not commit to a canonical notion of reward and measure GVA-induced oscillations via disagreements in long-horizon rollouts.

(R5) Autoregressive LMs exhibit significant rollout oscillations throughout training. **EMA stabilizes the trajectory, accelerates training, and improves generalization**, complementing (and potentially obviating) standard practices in learning rate annealing.

4.3 To what extent does convex theory explain the benefits of EMA?

We close by providing mathematical intuition as to why iterate averaging with EMA can reduce the oscillations caused by GVA. As discussed in Section 3.3, oscillations can occur when there is a disparity between the BC loss $\ell_{\mathrm{BC}}(\pi_{\widehat{\boldsymbol{\theta}}})$ on which we train and the rollout reward function $J(\pi_{\widehat{\boldsymbol{\theta}}})$ on which we evaluate. To study this phenomenon, we a consider simple, horizon-one behavior cloning problem with a single action determined by the model parameter $\boldsymbol{\theta}$. We take the *training loss* to be a quadratic $\ell_{\mathrm{BC}}(\boldsymbol{\theta}) = \frac{1}{2} \cdot \|\boldsymbol{\theta} - \boldsymbol{\mu}\|^2$, and the *rollout reward* $J(\cdot)$ to be

$$J(\theta) = \begin{cases} -\|\boldsymbol{\theta} - \boldsymbol{\mu}\|^2, & \|\boldsymbol{\theta} - \boldsymbol{\mu}\| \leq \epsilon \\ -C, & \text{otherwise} \end{cases}, \tag{4.2}$$

where $C \gg \epsilon^2 > 0$ are constants. Here the training loss is convex, but rollout reward is not; the latter exhibits a "cliff," dropping sharply from $-\epsilon^2$ to $-C$ once $\|\boldsymbol{\theta} - \boldsymbol{\mu}\| > \epsilon$. The pair (ℓ_{BC}, J) may be thought of as a discontinuous, horizon-one analogue of the example in Proposition 3.1, illustrating the contrast between extreme sensitivity of reward and insensitivity of the loss to the parameter of interest. The reward function encapsulates discontinuities arising in control tasks from, e.g., contract forces. In the MuJoCo walker, "cliff"-type behavior may come from an expert policy close to overbalancing the agent, with the learner's policy falling down if the parameter is "over the cliff."

We analyze SGD iterates $\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t+1)} - \eta(\boldsymbol{\theta}^{(t)} - \mu + \mathbf{w})$, where $\eta > 0$ is a constant step size and $\mathbf{w} \sim \mathcal{N}(0, \mathbf{I})$. This corresponds to SGD on a noisy version of the BC loss given by $\tilde{\ell}_{\mathrm{BC}}(\boldsymbol{\theta}) := \mathbb{E}[\|\boldsymbol{\theta}_t - \mathbf{u} + \mathbf{w}\|^2]$, which satisfies $\mathbb{E}_{\mathbf{w}}[\tilde{\ell}_{\mathrm{BC}}(\boldsymbol{\theta})] = \ell_{\mathrm{BC}}(\boldsymbol{\theta}) + \text{constant}$. We show that applying EMA to the resulting iterates achieves substantially higher rollout reward than vanilla SGD.

Proposition 4.1 (Informal version of Proposition C.6)**.** *Consider the setting in Eq. (4.2) for parameters $C \gg \epsilon^2 > 0$ in dimension one, and let $\boldsymbol{\theta}^{(T)}$ denote the SGD iterate with learning rate $\eta > 0$ as described above. Let $\tilde{\boldsymbol{\theta}}_\gamma^{(T)}$ denote the EMA iterate (4.1) with fixed parameter $\gamma_t \equiv \gamma \leq \eta$ satisfying $\gamma \gg 1/T$. Then, $\mathbb{E}[\ell_{\mathrm{BC}}(\boldsymbol{\theta}^{(T)})]$ scales as $\Theta(\eta)$, while $\mathbb{E}[\ell_{\mathrm{BC}}(\tilde{\boldsymbol{\theta}}_\gamma^{(T)})]$ scales as $\Theta(\gamma) \leq \eta$. In particular, when $\eta > c_1\epsilon$, and $\gamma \log(C/\gamma) \leq c_2\epsilon$, for absolute constants $c_1, c_2 > 0$, we find that*

$$\mathbb{E}[J(\boldsymbol{\theta}^\star) - J(\boldsymbol{\theta}^{(T)})] \geq \tfrac{C}{2}, \quad \textit{but} \quad \mathbb{E}[J(\boldsymbol{\theta}^\star) - J(\tilde{\boldsymbol{\theta}}_\gamma^{(T)})] \leq \mathcal{O}(\gamma).$$

This proposition holds, which shows that the rollout performance for EMA can be arbitrarily small relative to that of SGD, holds even in the regime where SGD is initialized at $\boldsymbol{\theta}^{(0)} = \boldsymbol{\mu}$ (so that both $\boldsymbol{\theta}^{(T)}$ and $\boldsymbol{\theta}_\gamma^{(T)}$ are unbiased estimates of $\boldsymbol{\mu}$), and thus highlights that EMA can reduce the variance that arises from accumulation of SGD noise.[7] Notice that Proposition 4.1 requires $\eta \geq \gamma \gg 1/T$, which is above the optimal step size of $\eta_t = 1/t$.[8] Indeed, in Appendix C we show that EMA, with the parameters we find empirically successful, only benefits optimization *above* these aggressively-decayed theoretically optimal learning rate schedules. Thus, we conclude that **convex theory reveals the variance-reducing benefit of *either* learning rate decay or EMA, but does not suggest which one is better.** We defer further theoretical results to Appendix C, and present an empirical study of a system motivated by the cliff loss in Appendix B.5; in particular, our analysis provides a simple example where GVA provably occurs, both theoretically and empirically.

The above example reveals the difference between the **statistical and algorithmic difficulties** of BC: with enough data, the empirical risk minimizer (sample mean) $\widehat{\boldsymbol{\theta}}$ of BC loss exhibits $\ell_{\mathrm{BC}}(\boldsymbol{\theta}) \sim 1/T \ll \epsilon$, which ensures J_H is small; on the other hand, with minibatch SGD and too large a learning rate, there is a noise floor on how close the non-EMA'd iterate $\widehat{\boldsymbol{\theta}}$ will be to $\boldsymbol{\theta}^\star$, ensuring that J_H is large.

[7] We compare to similar findings (Sandler et al., 2023) in Appendix A.

[8] Note that the $\eta_t = \frac{1}{t}$ step size schedule gives the sample mean, which is the maximum likelihood estimator for our objective.

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