UNDERSTANDING THE COMPLEXITY GAINS OF CONTEXTUAL MULTI-TASK RL WITH CURRICULA

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ABSTRACT

Reinforcement learning (RL) problems can be challenging without well-shaped rewards. Prior work on provably efficient RL methods generally proposes to address this issue with dedicated exploration strategies, such as novelty-based bonuses. However, another way to tackle this challenge is to reformulate it as a multi-task RL problem, where the task space contains not only the challenging task of interest but also easier tasks that implicitly function as a curriculum. Such a reformulation opens up the possibility of running existing multi-task RL methods as a more efficient alternative to solving a single challenging task from scratch. In this work, we provide a theoretical framework that reformulates a single-task RL problem as a multi-task RL problem defined by a curriculum. Under mild regularity conditions on the curriculum, we show that sequentially solving each task in the multi-task RL problem is more computationally efficient than solving the original single-task problem, without any explicit exploration bonuses or other exploration strategies. We also show that our theoretical insights can be translated into an effective practical learning algorithm that can accelerate curriculum learning on robotic goal-reaching tasks.

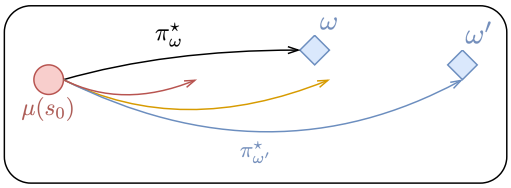
1 INTRODUCTION

Reinforcement learning (RL) provides an appealing and simple way to formulate control and decision-making problems in terms of reward functions that specify what an agent should do, and then automatically train policies to learn how to do it. However, in practice the specification of the reward function requires great care: if the reward function is well-shaped, then learning can be fast and effective, but if rewards are delayed, sparse, or can only be achieved after extensive explorations, RL problems can be exceptionally difficult (Kakade and Langford, 2002; Andrychowicz et al., 2017; Agarwal et al., 2019). This challenge is often overcome with either reward shaping (Ng et al., 1999; Andrychowicz et al., 2017; 2020) or dedicated exploration methods (Tang et al., 2017; Stadie et al., 2015; Bellemare et al., 2016; Burda et al., 2018), but reward shaping can bias the solution away from optimal behavior, while even the best exploration methods, in general, may require covering the entire state space before discovering high-reward regions.

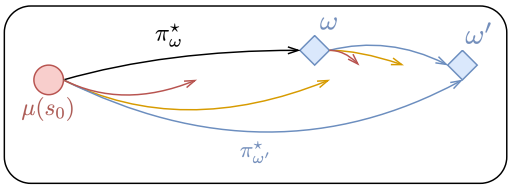
On the other hand, a number of recent works have proposed multi-task learning methods in RL that involve learning contextual policies that simultaneously represent solutions to an entire space of tasks, such as policies that reach any potential goal (Fu et al., 2018; Eysenbach et al., 2020b; Fujita et al., 2020; Zhai et al., 2022), policies conditioned on language commands (Nair et al., 2022), or even policies conditioned on the parameters of parametric reward functions (Kulkarni et al., 2016; Siriwardhana et al., 2019; Eysenbach et al., 2020a; Yu et al., 2020b). While such methods are often not motivated directly from the standpoint of handling challenging exploration scenarios, but rather directly aim to acquire policies that can perform all tasks in the task space, these multi-task formulations often present a more tractable learning problem than acquiring a solution to a single challenging task in the task space (e.g., the hardest goal, or the most complex language command).

We pose the following question: can we construct a multi-task RL problem with contextual policies that is easier than solving a single-task RL problem? In this work, we answer this question affirmatively by analyzing the sample complexity of a class of curriculum learning methods based on this principle.

To build the intuition for how reformulating a single-task problem into a multi-task problem enables efficient learning, consider the setting where the optimal state visitation distributions of two different contexts ω, ω' are "similar", and our goal is to learn the optimal policy $\pi^\star_{\omega'}$ w.r.t. ω'. Suppose we have learned the optimal policy $\pi^\star_\omega$, we can facilitate learning $\pi^\star_{\omega'}$ by: (1) using $\pi^\star_\omega$ as an initialization and (2) setting a new initial state distribution $\mu' = \beta d_\mu^{\pi^\star_\omega} + (1 - \beta_\mu)$ by mixing $d_\mu^{\pi^\star_\omega}$, the optimal state visitation distribution of $\pi^\star_\omega$, and μ, the initial distribution of interest. Using $\pi^\star_\omega$ as an initialization for learning $\pi^\star_{\omega'}$ facilitates the learning process as it guarantees the initialization is within some neighborhood of the optimality. Using the new initial distribution $\mu' = \beta d_\mu^{\pi^\star_\omega} + (1 - \beta)\mu$ also facilitates the learning of $\pi^\star_{\omega'}$ as it reduces the density mismatch ratio between $\left\| d_\mu^{\pi^\star_{\omega'}} / \mu' \right\|_\infty$, a key quantity that influences sample complexity. The new distribution $\mu' = \beta d_\mu^{\pi^\star_\omega} + (1 - \beta)\mu$ for learning $\pi^\star_{\omega'}$ could be understood as "rolling in $d_\mu^{\pi^\star_\omega}$ with probability β". we refer this approach, which consists of *rolling in a similar optimal state visitation distribution of another context*, as ROLLIN. We provide an illustration of ROLLINin Figure 1, which represents the basic idea of ROLLIN. Our paper analyzes this algorithm and proves that it can attain polynomial-time sample complexity, without any explicit exploration bonuses.



Learning $\pi^\star_{\omega'}$ from scratch



Learning $\pi^\star_{\omega'}$ with ROLLIN

Figure 1 Illustration of ROLLIN. The red circle represents the initial state distribution. **The dark curve** represents the optimal policy w.r.t. ω. **The blue diamonds** represent the optimal state distributions $d_\mu^{\pi^\star_\omega}, d_\mu^{\pi^\star_{\omega'}}$ of ω, ω' respectively. For learning $\pi^\star_{\omega'}$, when the optimal policy $\pi^\star_\omega$ of a similar context is known, ROLLIN rolls in the optimal policy of a near by context ω with probability β.

We formalize this intuition in the contextual MDP setting, where we assume each MDP $\mathcal{M}_\omega$ is uniquely defined by a context ω in the context space $\mathcal{W} \subset \mathbb{R}^n$, and we are given a curriculum $\{\omega_k\}_{k=0}^K$, with the last MDP $\mathcal{M}_{\omega_K}$ being the MDP of interest. To show our main results, we only require a Lipschitz continuity assumption on r_ω w.r.t. ω and some mild regularity conditions on the curriculum $\{\omega_k\}_{k=0}^K$. With the aforementioned assumptions, we show that learning $\pi^\star_K$ by recursively rolling in with a near optimal policy for ω_k to construct the initial distribution μ_{k+1} for the next context ω_{k+1}, is provably more efficient than learning $\pi^\star_{\omega_K}$ from scratch. In particular, we show that when an appropriate sequence of contexts is selected, we can reduce the iteration and sample complexity bounds of entropy regularized softmax policy gradient (with an inexact stochastic estimation of the gradient) from an original exponential dependency on the state space size, as suggested by Ding et al. (2021), to a polynomial dependency. We also prescribe a practical implementation of ROLLIN.

In summary, our contributions of this work can be stated as follows. We first provide a theoretical method (ROLLIN) that facilitates single-task policy learning by recasting it as a multi-task problem under entropy regularized softmax policy gradient (PG), which reduces the exponential complexity bound of the entropy regularized PG to a polynomial dependency on S. Lastly, we also provide a deep RL implementation of ROLLIN and demonstrate adding ROLLIN improves performance in simulated benchmark tasks in various settings, including goal relabeling (Andrychowicz et al., 2017) and Go-Explore (Ecoffet et al., 2019).

2 RELATED WORK

Convergence of policy gradient methods. Theoretical analysis of policy gradient methods has a long history (Williams, 1992; Sutton et al., 1999; Konda and Tsitsiklis, 1999; Kakade and Langford, 2002; Peters and Schaal, 2008). Motivated by the recent empirical success (Schulman et al., 2015; 2017) in policy gradient (PG) methods, the theory community has extensively studied the convergence of PG in various settings (Fazel et al., 2018; Agarwal et al., 2021; 2020; Bhandari and Russo, 2019; Mei et al., 2020; Zhang et al., 2020b; Agarwal et al., 2020; Zhang et al., 2020a; Li et al., 2021; Cen et al., 2021; Ding et al., 2021; Yuan et al., 2022; Moskovitz et al., 2022). Agarwal et al. (2021) established the asymptotic global convergence of policy gradient under different policy parameterizations. We extend the result of entropy regularized PG with stochastic gradient (Ding et al.,

2021) to the contextual MDP setting. In particular, our contextual MDP setting reduces the exponential state space dependency w.r.t. the iteration number and per iteration sample complexity suggested by Ding et al. (2021) to a polynomial dependency, when we have access to an appropriate curriculum. We shall also clarify that there are many existing convergence analysis on other variants of PG that produce an iteration number that do not suffer from an exponential state space dependency (Agarwal et al., 2021; Mei et al., 2020), but they assume access to the *exact* gradient during each update of PG, while we assume a stochastic estimation of the gradient, which is arguably more practical.

Contextual MDPs. Contextual MDPs (or MDPs with side information) have been studied extensively in the theoretical RL literature (Abbasi-Yadkori and Neu, 2014; Hallak et al., 2015; Dann et al., 2019; Jiang et al., 2017; Modi et al., 2018; Sun et al., 2019; Dann et al., 2019; Modi et al., 2020). We analyze the iteration complexity and sample complexity of (stochastic) policy gradient methods, which is distinct from these prior works that mainly focus on regret bounds (Abbasi-Yadkori and Neu, 2014; Hallak et al., 2015; Dann et al., 2019) and PAC bounds (Jiang et al., 2017; Modi et al., 2018; Sun et al., 2019; Dann et al., 2019; Modi et al., 2020). Several works assumed linear transition kernel and reward model (or generalized linear model (Abbasi-Yadkori and Neu, 2014)) with respect to the context (Abbasi-Yadkori and Neu, 2014; Modi et al., 2018; Dann et al., 2019; Modi et al., 2020; Belogolovsky et al., 2021). These assumptions share similarity to our assumptions — we have a weaker Lipschitz continuity assumption with respect to the context space (since linear implies Lipschitz) on the reward function and a stronger shared transition kernel assumption.

Exploration. A number of prior works have shown that one can reduce the complexity of learning an optimal policy with effective exploration methods (Azar et al., 2017; Jin et al., 2018; Du et al., 2019; Misra et al., 2020; Agarwal et al., 2020; Zhang et al., 2020d). The computational efficiency suggested by our work is different from some of the aforementioned prior methods that rely on adding exploration bonus for policy cover (Azar et al., 2017; Jin et al., 2018; Agarwal et al., 2020; Zhang et al., 2020d), as we assume access to a "good" curriculum which ensures the optimal policy defined by the next context is not too different from the optimal policy of the current context. The key intuition of our work is more related to the strategic exploration in the block MDP setting (Du et al., 2019; Misra et al., 2020). The block MDP setting assume the state space can be encoded into a feature space with a sequential block structure, and explores the next block in feature space after having a policy that covers the previous blocks. Such sequential exploration procedure is similar to ROLLIN, as our method also considers solving the multi-task contextual MDPs in a sequential perspective, but ROLLIN focuses on learning the optimal policy of the next context once we have a near-optimal policy of the current context.

Curriculum learning in reinforcement learning. Curriculum learning is a powerful idea that has been widely used in RL to address exploration challenges (Florensa et al., 2017; Kim and Choi, 2018; Omidshafiei et al., 2019; Ivanovic et al., 2019; Akkaya et al., 2019; Portelas et al., 2020; Fang et al., 2020; Klink et al., 2020; Dennis et al., 2020; Parker-Holder et al., 2022) (also see (Narvekar et al., 2020) for a detailed survey). Although curricula formed by well-designed reward functions (Vinyals et al., 2019; OpenAI, 2018; Berner et al., 2019; Ye et al., 2020; Zhai et al., 2022) are usually sufficient given enough domain knowledge, tackling problems with limited domain knowledge requires a more general approach where a suitable curriculum is automatically formed from a task space. In the goal-conditioned reinforcement learning literature, this corresponds to automatic goal proposal mechanisms (Florensa et al., 2018; Warde-Farley et al., 2018; Sukhbaatar et al., 2018; Ren et al., 2019; Ecoffet et al., 2019; Hartikainen et al., 2019; Pitis et al., 2020; Zhang et al., 2020c; OpenAI et al., 2021; Zhang et al., 2021).

Learning conditional policies in multi-task RL. Multi-task RL (Tanaka and Yamamura, 2003) approaches usually learn a task-conditioned policy that is shared across different tasks (Rusu et al., 2015; Rajeswaran et al., 2016; Andreas et al., 2017; D'Eramo et al., 2020; Yu et al., 2020a; Ghosh et al., 2021; Kalashnikov et al., 2021). Compared to learning each task independently, joint training enjoys the sample efficiency benefits from sharing the learned experience across different tasks as long as the policies generalize well across tasks. To encourage generalization, it is often desirable to condition policies on low dimensional feature representations that are shared across different tasks instead (e.g., using variational auto-encoders (Nair et al., 2018; Pong et al., 2019; Nair et al., 2020) or variational information bottleneck (Goyal et al., 2019; 2020; Mendonca et al., 2021)). The idea of learning contextual policies has also been discussed in classical adaptive control literature (Sastry et al., 1990; Tao, 2003; Landau et al., 2011; Åström and Wittenmark, 2013; Goodwin and Sin, 2014). Different from these prior works which have been mostly focusing on learning policies that can

generalize across different tasks, our work focuses on how the near-optimal policy from a learned task could be used to help the learning of a similar task.

3 PRELIMINARIES

We consider the contextual MDP setting, where a contextual MDP $\mathcal{M}_{\mathcal{W}} = (\mathcal{W}, \mathcal{S}, \mathcal{A}, \boldsymbol{P}, r_\omega, \gamma, \rho)$ consists of a context space $\mathcal{W}$, a state space $\mathcal{S}$, an action space $\mathcal{A}$, a transition dynamic function $\boldsymbol{P} : \mathcal{S} \times \mathcal{A} \to \mathcal{P}(\mathcal{S})$ (where $\mathcal{P}(X)$ denotes the set of all probability distributions over set X), a context-conditioned reward function $r : \mathcal{W} \times \mathcal{S} \times \mathcal{A} \to [0, 1]$, a discount factor $\gamma \in (0, 1]$, and an initial state distribution of interest ρ. For convenience we use $S = |\mathcal{S}|, A = |\mathcal{A}|$ to denote the number of states and actions. While some contextual MDP formulations (Hallak et al., 2015) have context-conditioned transition dynamics and reward functions, we consider the setting where only the reward function can change across contexts. We denote r_ω as the reward function conditioned on a fixed $\omega \in \mathcal{W}$ and $\mathcal{M}_\omega = (\mathcal{S}, \mathcal{A}, \boldsymbol{P}, r_\omega, \gamma, \rho)$ as the MDP induced by such fixed reward function. We use $\pi(a|s) : \mathcal{S} \to \mathcal{P}(\mathcal{A})$ to denote a policy and we adopt the softmax parameterization: $\pi_\theta(a|s) = \frac{\exp[\theta(s,a)]}{\sum_{a'} \exp[\theta(s,a')]}$, where $\theta : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}$. We use $d_\rho^\pi(s) := (1-\gamma) \sum_{t=0}^\infty \gamma^t \mathbb{P}^\pi(s_t = s | s_0 \sim \rho)$ to denote the discounted state visitation distribution and $V_\omega^\pi := \mathbb{E}\left[\sum_{t=0}^\infty \gamma^t r_\omega(s_t, a_t)\right] + \alpha \mathbb{H}(\rho, \pi)$ to denote the entropy regularized discounted return on $\mathcal{M}_\omega$, where $\mathbb{H}(\rho, \pi) := \mathbb{E}_{s_0 \sim \rho, a_h \sim \pi(\cdot|s_h)} \left[\sum_{h=0}^\infty -\gamma^h \log \pi(a_h|s_h)\right]$ is the discounted entropy term. We use $\pi_\omega^\star := \arg\max_\pi V_\omega^\pi$ to denote an optimal policy that maximizes the discounted return under $\mathcal{M}_\omega$. We assume all the contextual reward functions are bounded within $[0, 1]$: $r_\omega(s, a) \in [0, 1],\ \forall \omega \in \Omega, \forall (s, a) \in \mathcal{S} \times \mathcal{A}$. Similarly to previous analysis Agarwal et al. (2021); Mei et al. (2020); Ding et al. (2021), we assume the initial distribution ρ for PG or stochastic PG satisfies $\rho(s) > 0, \forall s \in \mathcal{S}$.

3.1 ASSUMPTIONS

Given a curriculum $\{\omega_k\}_{k=0}^K$, where the last context ω_K defines $\mathcal{M}_{\omega_K}$, the MDP of interest, our goal is to show that reformulating $\mathcal{M}_{\omega_K}$ into a multi-task problem $\{\mathcal{M}_{\omega_k}\}_{k=0}^K$, enjoys better computational complexity and sample complexity than solving the single-task problem $\mathcal{M}_{\omega_K}$ from scratch. As we will show in Section 4, if we have a "good" curriculum $\{\omega_k\}_{k=0}^K$ such that the optimal policies $\pi_{\omega_k}^\star, \pi_{\omega_{k+1}}^\star$ w.r.t. two consecutive contexts ω_k, ω_{k+1} are "close enough" to each other, then using an ε-optimal policy of ω_k as an initialization allows us to directly start from the near optimal regime of ω_{k+1}, hence only requiring polynomial complexity to learn $\pi_{\omega_{k+1}}^\star$. Formally, we use the two following assumptions to characterize the good properties of our curriculum.

Assumption 3.1 (Lipschitz reward in the context space) *The reward function is Lipschitz continuous w.r.t. the context:* $\max_{s,a} |r_\omega(s, a) - r_{\omega'}(s, a)| \le L_r \|\omega - \omega'\|_2, \forall \omega, \omega' \in \mathcal{W}$.

Assumption 3.2 (Similarity between two Consecutive Contexts) *We assume the given curriculum* $\{\omega_k\}_{k=0}^K$ *satisfies* $\max_{0 \le k \le K-1} \|\omega_{k+1} - \omega_k\|_2 \le O\left(S^{-2}\right)$, *and* $\pi_{\omega_0}^\star$ *is also provided by the curriculum.*

Intuitively, Assumption 3.1 defines the similarity between two tasks via a Lipschitz continuity in the context space. Assumption 3.1 and 3.2 together quantify the maximum difference between two consecutive tasks $\mathcal{M}_{\omega_{k-1}}, \mathcal{M}_{\omega_k}$, in terms of the maximum difference between their reward functions.

3.2 PRIOR ANALYSIS ON PG WITH STOCHASTIC GRADIENT

Ding et al. (2021) proposed a two-phased PG convergence analysis framework with stochastic gradient. In particular, the author demonstrate that with high probability, stochastic PG with arbitrary initialization achieves an ε-optimal policy can be achieved with an iteration number of T_1, T_2 and per iteration sample complexity of B_1, B_2 in two separate phases where $T_1 = \widetilde{\Omega}(S^{2S^3}), T_2 = \widetilde{\Omega}(S^{3/2})$[1] and $B_1 = \widetilde{\Omega}(S^{2S^3}), B_2 = \widetilde{\Omega}(S^5)$, respectively, and PG enters phase 2 only when the updating policy becomes ε_0-optimal, where ε_0 is a term depending on S (formally defined by (19) in Appendix A.3). For completeness, we restate the main theorem of Ding et al. (2021) in Theorem A.2, provide the details of such dependencies on S are provided in Corollary A.3, and describes the two phase procedure in Algorithm 4. The main implication of this two phase results is that, if we want to apply PG to learn an optimal policy from an arbitrary initialization, we suffer from an exponential iteration number and sample complexity, unless the initialization is ε_0-optimal. In the next section, we will discuss how a curriculum satisfying Assumption 3.1 and 3.2 enables an ε_0-optimal initialization for every ω_k, and provides an overall complexity for learning the target context ω_K.

[1] $\widetilde{\Omega}(\cdot)$ suppresses the $\log S$ and terms that do not contain S.

4 THEORETICAL ANALYSIS

In this section, we introduce ROLLIN, a simple algorithm that accelerates policy learning under the contextual MDP setup by bootstrapping new context learning with a better initial distribution (Algorithm 1). We also provide the total complexity analysis of applying ROLLIN to stochastic PG for achieving an ε-optimal policy.

4.1 ROLLIN

The theoretical version of ROLLIN is provided in Algorithm 1. Our intuition behind ROLLIN is that when two consecutive contexts in the curriculum $\{\omega_k\}_{k=0}^K$ are close, their optimal parameters $\theta^\star_{\omega_{k-1}}, \theta^\star_{\omega_k}$ should be close to each other. Let $\theta_t^{(k)}$ denote the parameters at the tth iteration of stochastic PG for learning $\theta^\star_{\omega_k}$, if we initialize the parameter $\theta_1^{(k)}$ as the optimal parameter of the previous context $\theta^\star_{\omega_{k-1}}$ (line 4 in Algorithm 1), and set the initial distribution μ_k as a mixture between the optimal state visitation distribution of the previous context $d_{\mu_{k-1}}^{\pi^\star_{\omega_{k-1}}}$ and the original distribution of interest ρ with ratio β (line 5 in Algorithm 1), such that

$$\mu_k = \beta d_{\mu_{k-1}}^{\pi^\star_{\omega_{k-1}}} + (1-\beta)\rho, \tag{1}$$

then stochastic PG enjoys faster convergence rate. This happens because setting $\theta_1^{(k)} = \theta^\star_{k-1}$ ensures a near-optimal initialization for $\theta_1^{(k)}$, and setting μ_k by mixing $d_{\mu_{k-1}}^{\pi^\star_{\omega_{k-1}}}$ and ρ further improves the rate of convergence by decreasing the density mismatch ratio $\left\| d_{\mu_k}^{\pi^\star_{\omega_k}} / \mu_k \right\|_\infty$ (a term with an S dependency that influences the rate of convergence).

Algorithm 1 Provably Efficient Learning via ROLLIN

1: **Input:** ρ, $\{\omega_k\}_{k=0}^K$, $\mathcal{M}_{\mathcal{W}}$, $\beta \in (0,1)$, $\theta^\star_{\omega_0}$.
2: Initialize $\mu_0 = \rho$, set $\theta_1^{(1)} = \theta^\star_{\omega_0}$.
3: **for** $k = 1, \ldots, K$ **do**
4: Set $\theta_1^{(k)} = \theta^\star_{\omega_{k-1}}$. ▷ $\pi^\star_{\omega_{k-1}} = \pi_{\theta^\star_{\omega_{k-1}}}$ is the optimal policy of ω_{k-1}.
5: Set $\mu_k = \beta d_{\mu_{k-1}}^{\pi^\star_{\omega_{k-1}}} + (1-\beta)\rho$. ▷ μ_k is the initial distribution for learning $\theta^\star_{\omega_k}$.
6: Use stochastic PG (Algorithm 4) with initialization $\theta_1^{(k)}$, $\mu_k, \mathcal{M}_{\omega_k}$ and obtain $\theta^\star_{\omega_k}$.
7: **end for**
8: **Output:** $\theta^\star_{\omega_K}$

4.2 MAIN RESULTS

We now discuss how to use a sequence of contexts to learn the target context ω_K with provable efficiency given the optimal policy $\pi^\star_{\omega_0}$ of the initial context ω_0, without incurring an exponential dependency on S (as mentioned in Section 3.2). Our polynomial complexity comes as a result of enforcing an ε_0-optimal initialization (ε_0 is the same as Section 3.2 and (19)) for running stochastic PG (line 6 of Algorithm 1). Hence, stochastic PG directly enters phase 2, with a polynomial dependency in S.

Our main results consists of two parts. We first show that when two consecutive contexts ω_{k-1}, ω_k are close enough to each other, using ROLLIN for learning $\theta^\star_k$ with initialization $\theta_1^{(k)} = \theta^\star_{\omega_{k-1}}$ and applying an initial distribution $\mu_k = \beta d_{\mu_{k-1}}^{\pi^\star_{\omega_{k-1}}} + (1-\beta)\rho$ improves the convergence rate. Specifically, the iteration number and complexity for learning $\theta^\star_{\omega_k}$ from $\theta^\star_{\omega_{k-1}}$ is stated as follows:

Theorem 4.1 (Complexity of Learning the Next Context) *Consider the context based stochastic softmax policy gradient (line 6 of Algorithm 1), if $\|\omega_k - \omega_{k-1}\|_2 \le O\left(S^{-2}\right)$, then the iteration number of obtaining an ε-optimal policy for ω_k from $\theta^\star_{\omega_{k-1}}$ is $\widetilde{\Omega}\left(S^{3/2}\right)$ and the per iteration sample complexity is $\widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)} S^3\right)$.*

In other words, Theorem 4.1 shows that when ω_{k-1}, ω_k are close enough, ROLLIN reduces the iteration and sample complexity from an exponential dependency of $\widetilde{\Omega}(S^{2S^3})$ to an iteration number of $\widetilde{\Omega}(S^{3/2})$ and per iteration sample complexity of $\widetilde{\Omega}(S^3)$. It is worth noting that the theorem above

only address the iteration numbers and sample complexity for learning $\theta^\star_{\omega_k}$ from $\theta^\star_{\omega_{k-1}}$. The next theorem provides the total complexity for learning $\theta^\star_{\omega_K}$ from $\theta^\star_{\omega_0}$ via recursively apply the results in Theorem 4.1. Concretely, we have:

Theorem 4.2 (Main Results: Total Complexity of ROLLIN) *Suppose the assumptions in Theorem 4.1 hold. Given a curriculum $\{\omega_k\}_{k=0}^K$ such that $\max_{1\le k\le K}\|\omega_k-\omega_{k-1}\|_2 \le O(S^{-2})$, the total number of iteration of learning $\pi^\star_{\omega_K}$ using Algorithm 1 is $\Omega(KS^{3/2})$ and the per iteration is $\widetilde{\Omega}\left(S^3\right)$, with high probability.*

A direct implication of Theorem 4.2 is that, with a curriculum $\{\omega_k\}_{k=0}^K$ satisfying Assumption 3.1 and 3.2, one can reduce the daunting exponential dependency on S caused by poor initialization to a polynomial dependency on S. Admittedly the state space S itself is still large in practice, but reducing the state space S itself requires extra assumptions on $\mathcal{S}$ which is beyond the scope of this work. We will provide a sketch proof of Theorem 4.1 and 4.2 in the next subsection and leave all the details proof to Appendix A.4, A.5 respectively.

4.3 PROOF SKETCH

Sketch proof of Theorem 4.1 The key insight for proving Theorem 4.1 is to show that ROLLIN provides a good initialization is to show that in MDP $\mathcal{M}_{\omega_k}$, the value function w.r.t. $\pi^\star_{\omega_k}, \pi^\star_{\omega_{k-1}}$ can be bounded by the ℓ^2 norm between ω_k and ω_{k-1}. In particular, we prove such a relation in Lemma A.5:

$$V^{\pi^\star_{\omega_k}}_{\omega_k}(\rho) - V^{\pi^\star_{\omega_{k-1}}}_{\omega_k}(\rho) \le \frac{2L_r\|\omega_k-\omega_{k-1}\|_2}{(1-\gamma)^2}. \tag{2}$$

By setting $\theta_1^{(k)} = \theta^\star_{\omega_{k-1}}$, Equation (2) directly implies $V^{\pi^\star_{\omega_k}}_{\omega_k}(\rho) - V^{\theta_1^{(k)}}(\rho) \le \frac{2L_r\|\omega_k-\omega_{k-1}\|_2}{(1-\gamma)^2}$. As suggested by Ding et al. (2021) stochastic PG can directly start from stage 2 with polynomial complexity of $T_2 = \widetilde{\Omega}(S^{3/2}), B_2 = \widetilde{\Omega}(S^5)$, if $V^{\pi^\star_{\omega_k}}_{\omega_k}(\rho) - V^{\theta_1^{(k)}}(\rho) \le \varepsilon_0$, where ε_0 (formally defined in Equation (19) in Appendix A.3) is a constant satisfying $\varepsilon_0 = O(S^{-2})$. Hence, by enforcing two consecutive contexts are close enough $\|\omega_k-\omega_{k-1}\|_2 \le O(S^{-2})$, we can directly start from a near-optimal initialization with polynomial complexity w.r.t. S. This largely explains the intuition behind the initialization in line 4 of ROLLIN: $\theta_1^{(k)} = \theta^\star_{\omega_{k-1}}$. It is worth highlighting that the per iteration sample complexity B_2 shown by Ding et al. (2021) scales as $\widetilde{\Omega}(S^5)$, while our result in Theorem 4.1 only requires a smaller sample complexity of $\widetilde{\Omega}(S^3)$. Such an improvement in the sample complexity comes from line 5 of ROLLIN: $\mu_k = \beta d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}} + (1-\beta)\rho$. Intuitively, by setting μ_k as $\beta d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}} + (1-\beta)\rho$ allows us to provide an upper bound on the density mismatch ratio:

$$\left\|d^{\pi^\star_{\mu_k}}_{\mu_k}/\mu_k\right\|_\infty \le \widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}\Delta^k_\omega S\right), \tag{3}$$

where $\Delta^k_\omega = \max_{1\le i\le k}\|\omega_i-\omega_{i-1}\|_2$. Since the sample complexity B_2 (provided in Corollary A.3) contains one multiplied of $\left\|d^{\pi^\star_{\mu_k}}_{\mu_k}/\mu_k\right\|_\infty$, by assuming $\Delta^k_\omega = O(S^{-2})$ immediately reduces the complexity by an order of S^2. The proof of the upper bound of the density mismatch ratio (Equation 3) is provided in Lemma A.1.

Sketch proof of Theorem 4.2 We obtain Theorem 4.2 by recursively applying Theorem 4.1. More precisely, we use an induction to show that, if we initialize the parameters of the policy as $\theta_1^{(k)} = \theta^\star_{\omega_{k-1}}$, when $t = \widetilde{\Omega}(S^{3/2})$, $\forall k \in [K]$, we have $V^{\pi^\star_{\omega_k}}_{\omega_k}(\rho) - V^{\pi_{\theta_t^{(k-1)}}}_{\omega_k}(\rho) < \varepsilon_0$. This implies that for any context $\omega_k, k\in[K]$, initializing $\theta_1^{(k)} = \theta_t^{(k-1)}$ from learning the previous context $\pi^\star_{\omega_{k-1}}$ using stochastic PG after $t = \Omega(S^{3/2})$ iteration, $\theta_1^{(k)}$ will directly start from the efficient phase 2 with a polynomial complexity. Hence, the total iteration number for learning the $\theta^\star_K$ is $\Omega(KS^{3/2})$, and the per iteration sample complexity $\widetilde{\Omega}\left(S^3\right)$ remains the same as Theorem 4.1.

5 PRACTICAL IMPLEMENTATION OF ROLLIN

In this section, we describe the practical implementation of ROLLIN using Soft-Actor-Critic (Haarnoja et al., 2018) algorithm when an oracle curriculum is available. SAC can be seen as a variant of entropy-regularized stochastic PG with the addition of the critics to reduce the gradient variance.

Recall that in the theoretical analysis, we learn a separate policy for each context that can start from the near-optimal state distribution of the previous context to achieve good return under the current context. However, in practice, we usually would want to have a policy that can directly start from the initial distribution ρ to obtain good return for the final context ω_K. In order to learn such policy, we propose to have two context-conditioned RL agents training in parallel, where the first agent π_{main} is the main agent that eventually will learn to achieve a good return from ρ, and the second agent π_{exp} is an exploration agent that learns to achieve a good return under the current context from the near-optimal state density of the previous context. Another purpose of the exploration agent (as the name suggests) is to provide better exploration experience for the main agent to learn the current context better. This is made convenient by using off-policy RL agent where the main agent can easily learn from data that is not on-policy.

Specifically, for each episode, there is a probability of β where we run the main agent conditioned on the previous context for a random number of steps until we switch to the exploration agent to collect experience for the current context until the episode ends. Otherwise, we directly run the main agent for the entire episode. Both agents are trained to maximize the return under the current context. Whenever the average return of the last 10 episodes exceed a performance threshold R, we immediately switch to the next context and re-initialize the exploration agent and its replay buffer. A high-level description is available in Algorithm 2 (a more detailed version in Algorithm 8).

Algorithm 2 Practical Implementation of ROLLIN

Input: $\{\omega_k\}_{k=0}^K$: input curriculum, R: near-optimal threshold, β: roll-in ratio, H: horizon, γ: discount factor.
Initialize $\mathcal{D} \leftarrow \emptyset, \mathcal{D}_{\text{exp}} \leftarrow \emptyset, k \leftarrow 0$, and two SAC agents π_{main} and π_{exp}.
for each episode **do**
  if average return of the last 10 episodes under context ω_k is greater than R **then**
    $k \leftarrow k+1$, $\mathcal{D}_{\text{exp}} \leftarrow \emptyset$, and re-initialize the exploration agent π_{exp}
  end if
  if $k > 0$ and with probability of β **then**
    $h \sim \text{Geom}(1-\gamma)$ (truncated at H)
    run $\pi_{\text{main}}(a|s, \omega_{k-1})$ from the initial state for h steps and switch to $\pi_{\text{exp}}(a|s, \omega_k)$ until the episode ends to obtain trajectory $\tau_{0:H} = \{s_0, a_0, r_0, s_1, a_1, \cdots, s_H\}$.
    record $\tau_{0:H}$ in $\mathcal{D}$, and $\tau_{h:H}$ in $\mathcal{D}_{\text{exp}}$.
  else
    run $\pi_{\text{main}}(a|s, \omega_k)$ to obtain trajectory $\tau_{0:H}$ and record $\tau_{0:H}$ in $\mathcal{D}$.
  end if
  at each environment step during the episode, update π_{main} using $\mathcal{D}$ and π_{exp} using $\mathcal{D}_{\text{exp}}$.
end for
Output: π_{main}

6 EXPERIMENTAL RESULTS

While the focus of our work is on developing a provably efficient approach to curriculum learning, we also conduct an experimental evaluation of our practical implementation of ROLLIN with soft actor-critic (SAC) (Haarnoja et al., 2018) as the RL algorithm on the `antmaze-umaze` environment (Fu et al., 2020). We use a hand-crafted path of contexts, where each context specifies the location that the ant needs to reach (as shown in Figure 2).[2] We consider a path of contexts $\omega(\kappa)$ parameterized by $\kappa \in [0, 1]$ where $\omega(0) = \omega_0$ and $\omega(1) = \omega_K$, and propose contexts along the path with a fixed step size Δ. See Appendix E for more implementation details.

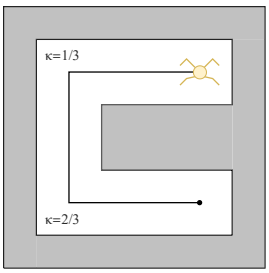


Figure 2 Oracle curriculum of desired goals on `antmaze-umaze`.

We can combine ROLLIN with a variety of prior methods, and we evaluate the following conditions: (1) standard goal reaching; (2) goal reaching with goal relabeling (Andrychowicz et al., 2017); (3) goal reaching with

[2]Note that in general, ROLLIN is *not* specific to goal-conditioned RL and could in principle utilize any type of context-parameterized reward function that satisfies its assumptions.

Go-Explore (Ecoffet et al., 2019). For goal relabeling, we adopt a similar relabeling technique as Pitis et al. (2020), where each minibatch contains $1/3$ original transitions, $1/3$ transitions with future state relabeling, and $1/3$ transitions with next state relabeling. We implemented the Go-Explore method by adding an additional standard Gaussian exploration noise (multiplied by a constant factor) to the agent for learning the next goal $\omega(k+1)$, once it reaches the current goal $\omega(k)$. We empirically observed that sampling the replay buffer from a geometric distribution with $p = 10^{-5}$ (more recent transitions are sampled more frequently) improves the overall performance. Hence, in all future experiments, we compare performance of ROLLIN with classic uniform sampling and the new geometric sampling. We demonstrate how adjusting the ROLLIN parameter ($\beta = 0.1, 0.2, 0.5, 0.75, 0.9$) impacts the learning speed on three different step sizes $\Delta = \frac{1}{24}, \frac{1}{18}, \frac{1}{12}$. We expect coarser curricula with larger steps to be more difficult.

Main comparison. We first provide an overview experiments that compares ROLLIN with a fixed $\beta = 0.1$ on different step sizes Δ in different settings. In each case we compare the prior method (vanilla, relabeled, or Go-Explore) with and without the addition of ROLLIN. Note that for the case without ROLLIN, we still provide the curriculum to the agent for training. As shown in Table 1, κ progresses further with ROLLIN in most presented settings (except Go-Explore with $\Delta = 1/12$). This result suggests that ROLLIN facilitates goal-conditioned RL with a curriculum, as the we only update the learning progress κ to $\kappa + \Delta$ when the return of the current policy reaches certain threshold R (See detailed update of κ in Algorithm 2). Since geometric sampling technique drastically improvement in the vanilla and the Go-Explore settings and slightly decreases the performance in the relabeling setting, we also want to point out that ROLLIN improves the learning progress of baselines at different levels (w/ and w/o geometric samplings). Note that $\beta = 0.1$ does not always produce the best result, we will provide more results comparing different βs in different settings later in this section, and we leave all the learning curves to Appendix F.

		w/o Geometric Sampling		w/ Geometric Sampling	
		$\Delta = 1/24$	$\Delta = 1/12$	$\Delta = 1/24$	$\Delta = 1/12$
Vanilla	Baseline	0.40 ± 0.02	0.36 ± 0.00	0.82 ± 0.08	0.38 ± 0.03
	ROLLIN	0.49 ± 0.04	0.44 ± 0.01	0.92 ± 0.02	0.55 ± 0.04
Relabeling	Baseline	0.89 ± 0.03	0.66 ± 0.04	0.76 ± 0.02	0.72 ± 0.03
	ROLLIN	0.91 ± 0.03	0.74 ± 0.01	0.78 ± 0.01	0.73 ± 0.00
Go-Explore Noise = 0.1	Baseline	0.37 ± 0.02	0.38 ± 0.01	0.82 ± 0.07	0.42 ± 0.03
	ROLLIN	0.52 ± 0.07	0.38 ± 0.01	0.95 ± 0.02	0.43 ± 0.02

Table 1 Learning progress κ at 3 million environment steps with varying curriculum step size Δ of different settings of goal reaching in `antmaze-umaze`. We pick $\beta = 0.1$ for all experiments using ROLLIN, the results of using other βs, Δs, and exploration noise can be found in Table 2, 3, 4. The standard error is computed over 8 random seeds.

Goal reaching in different settings. For the vanilla goal reaching case, we present the final κ values of 5 βs after 3 million environment steps in Table 2. Across all step sizes, ROLLIN is able to progress further in κ consistently compared to the baseline, demonstrating the effectiveness of ROLLIN in this curriculum learning setup. In particular, the first three rows of Table 2 show that ROLLIN achieves much better results than the baseline method on all tested βs, without geometric sampling. Note that the bottom three rows of Table 2 also suggests that ROLLIN largely improves the results on a better the baseline with geometric sampling (except for some extreme βs). We also observe similar phenomenon in the relabeling and Go-Explore settings (see details in Table 3, 4).

Geo	Δ	Baseline	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.9$
✗	$1/24$	0.40 ± 0.02	0.49 ± 0.04	0.51 ± 0.05	$\mathbf{0.57 \pm 0.04}$	0.55 ± 0.02	0.49 ± 0.02
✗	$1/18$	0.36 ± 0.01	0.39 ± 0.01	0.46 ± 0.01	$\mathbf{0.54 \pm 0.03}$	0.46 ± 0.03	0.50 ± 0.02
✗	$1/12$	0.36 ± 0.00	0.44 ± 0.01	0.41 ± 0.02	0.47 ± 0.02	$\mathbf{0.56 \pm 0.05}$	$\mathbf{0.56 \pm 0.02}$
✓	$1/24$	0.82 ± 0.08	0.92 ± 0.02	$\mathbf{0.95 \pm 0.02}$	0.88 ± 0.01	0.81 ± 0.01	0.70 ± 0.02
✓	$1/18$	0.68 ± 0.07	0.74 ± 0.07	0.76 ± 0.06	$\mathbf{0.78 \pm 0.03}$	0.75 ± 0.02	0.72 ± 0.02
✓	$1/12$	0.38 ± 0.03	0.55 ± 0.04	0.55 ± 0.04	0.64 ± 0.06	$\mathbf{0.69 \pm 0.06}$	0.67 ± 0.03

Table 2 Vanilla Goal reaching. Learning progress κ at 3 million environment steps with varying β and curriculum step size Δ of vanilla goal reaching task. `Geo` indicates the usage of geometric sampling. Baseline corresponds to $\beta = 0$, where no ROLLIN is used. The standard error is computed over 8 random seeds. We highlight the values that are larger than the baseline ($\beta = 0$) in purple, and the largest value in **bold font**.

Geo	Δ	$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.9$
✗	1/24	0.89 ± 0.03	$\mathbf{0.91 \pm 0.03}$	0.85 ± 0.04	0.86 ± 0.02	0.86 ± 0.02	0.80 ± 0.02
✗	1/18	0.76 ± 0.03	0.81 ± 0.01	$\mathbf{0.85 \pm 0.04}$	0.79 ± 0.01	0.84 ± 0.03	0.68 ± 0.04
✗	1/12	0.66 ± 0.04	$\mathbf{0.74 \pm 0.01}$	0.73 ± 0.03	0.66 ± 0.06	0.67 ± 0.05	0.69 ± 0.03
✓	1/24	0.76 ± 0.02	0.78 ± 0.01	0.78 ± 0.03	0.82 ± 0.02	$\mathbf{0.87 \pm 0.02}$	0.83 ± 0.02
✓	1/18	0.72 ± 0.02	0.76 ± 0.01	0.76 ± 0.02	0.78 ± 0.04	$\mathbf{0.85 \pm 0.03}$	0.81 ± 0.01
✓	1/12	0.72 ± 0.03	$\mathbf{0.73 \pm 0.00}$	$\mathbf{0.73 \pm 0.00}$	$\mathbf{0.73 \pm 0.03}$	0.69 ± 0.04	0.66 ± 0.04

Table 3 Goal relabeling. All other settings are the same as Table 2.

EN	Geo	Δ	$\beta = 0$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.9$
0.1	✗	1/24	0.37 ± 0.02	0.52 ± 0.07	$\mathbf{0.54 \pm 0.06}$	0.51 ± 0.06	0.39 ± 0.02	0.37 ± 0.01
0.1	✗	1/18	0.38 ± 0.01	0.37 ± 0.01	0.36 ± 0.01	0.38 ± 0.01	0.38 ± 0.01	0.35 ± 0.00
0.1	✗	1/12	0.38 ± 0.01	0.38 ± 0.01	0.36 ± 0.00	$\mathbf{0.39 \pm 0.01}$	0.36 ± 0.00	0.36 ± 0.00
0.1	✓	1/24	0.82 ± 0.07	$\mathbf{0.95 \pm 0.02}$	0.91 ± 0.02	0.92 ± 0.02	0.71 ± 0.02	0.45 ± 0.01
0.1	✓	1/18	0.57 ± 0.09	$\mathbf{0.71 \pm 0.08}$	0.65 ± 0.07	0.63 ± 0.07	0.62 ± 0.02	0.43 ± 0.01
0.1	✓	1/12	0.42 ± 0.03	0.43 ± 0.02	0.38 ± 0.04	$\mathbf{0.49 \pm 0.04}$	0.45 ± 0.02	0.39 ± 0.01
0.25	✗	1/24	0.38 ± 0.02	0.49 ± 0.06	0.48 ± 0.05	$\mathbf{0.55 \pm 0.07}$	0.43 ± 0.04	0.40 ± 0.02
0.25	✗	1/18	0.35 ± 0.00	$\mathbf{0.39 \pm 0.03}$	$\mathbf{0.39 \pm 0.02}$	0.36 ± 0.01	0.36 ± 0.01	0.35 ± 0.00
0.25	✗	1/12	0.36 ± 0.00	0.36 ± 0.00	0.36 ± 0.00	0.36 ± 0.00	0.36 ± 0.00	0.36 ± 0.00
0.25	✓	1/24	0.82 ± 0.10	0.97 ± 0.02	$\mathbf{1.00 \pm 0.00}$	0.94 ± 0.02	0.77 ± 0.02	0.49 ± 0.02
0.25	✓	1/18	0.64 ± 0.10	0.70 ± 0.07	$\mathbf{0.79 \pm 0.07}$	0.64 ± 0.06	0.63 ± 0.03	0.44 ± 0.01
0.25	✓	1/12	0.39 ± 0.01	$\mathbf{0.47 \pm 0.03}$	0.45 ± 0.02	$\mathbf{0.47 \pm 0.03}$	0.36 ± 0.04	0.40 ± 0.02
0.5	✗	1/24	0.49 ± 0.06	0.60 ± 0.08	$\mathbf{0.66 \pm 0.08}$	0.61 ± 0.08	0.65 ± 0.06	0.46 ± 0.04
0.5	✗	1/18	0.36 ± 0.01	$\mathbf{0.41 \pm 0.02}$	0.39 ± 0.01	0.38 ± 0.01	0.37 ± 0.01	0.35 ± 0.00
0.5	✗	1/12	0.36 ± 0.00	$\mathbf{0.38 \pm 0.01}$	$\mathbf{0.38 \pm 0.01}$	0.36 ± 0.00	$\mathbf{0.38 \pm 0.01}$	0.36 ± 0.00
0.5	✓	1/24	0.92 ± 0.08	$\mathbf{1.00 \pm 0.00}$	$\mathbf{1.00 \pm 0.00}$	$\mathbf{1.00 \pm 0.00}$	0.87 ± 0.03	0.55 ± 0.03
0.5	✓	1/18	0.66 ± 0.09	0.71 ± 0.08	$\mathbf{0.80 \pm 0.08}$	0.68 ± 0.08	0.67 ± 0.04	0.44 ± 0.02
0.5	✓	1/12	0.41 ± 0.02	0.44 ± 0.04	0.43 ± 0.03	$\mathbf{0.50 \pm 0.04}$	0.47 ± 0.03	0.39 ± 0.04

Table 4 Go-Explore with different exploration noise. EN represents the multiplier for the Gaussian exploration noise. All other settings are the same as Table 2.

In summary, we empirically demonstrated that ROLLIN improves the performance of goal reaching tasks in different settings with a vast range of selection of βs. Although ROLLIN introduces an extra parameter β for finetuning goal reaching tasks with a curriculum, our extensive experiments suggest that one could expect improvement by choosing a constant $\beta = 0.1$ or 0.2 for other goal reaching tasks. Still, for achieving the best performance in general task, one shall still consider finetuning the ROLLIN parameter β.

7 DISCUSSION AND LIMITATIONS

We presented ROLLIN, a simple algorithm that accelerates curriculum learning under the contextual MDP setup by rolling in a near optimal policy to bootstrap the learning of new nearby contexts with provable learning efficiency benefits. Our focus is on the theoretical analysis of ROLLIN, which analyzes the method instantiated with entropy-regularized policy gradients and a softmax policy parameterization. We show that ROLLIN attains polynomial sample complexity by utilizing adjacent contexts to initialize each policy. Since the key theoretical insight of ROLLIN suggests that one can reduce the density mismatch ratio by constructing a new initial distribution, it would be interesting to see how ROLLIN can affect other variants of convergence analysis of PG (e.g., NPG (Kakade, 2001; Cen et al., 2021) or PG in a feature space (Agarwal et al., 2021; 2020)) and we would like to leave it to future.

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APPENDIX A GENERALIZATION BETWEEN DIFFERENT TASKS IN THE CONTEXT SPACE

A.1 SUMMARIES OF NOTATIONS AND ASSUMPTIONS

1. The maximum entropy RL (MaxEnt RL) objective with initial state distribution ρ in reinforcement aims at maximizing (Equation 15 & 16 of Mei et al. (2020))

$$V^{\pi}(\rho) := \sum_{h=0}^{\infty} \gamma^h \mathbb{E}_{s_0\sim\rho, a_h\sim\pi(a_h|s_h)} \left[r(s_h, a_h)\right] + \alpha \mathbb{H}(\rho, \pi) \tag{4}$$

and $\mathbb{H}(\pi(a_h|s_h))$ is the discounted entropy term

$$\mathbb{H}(\rho, \pi) := \mathop{\mathbb{E}}_{s_0\sim\rho, a_h\sim\pi(\cdot|s_h)} \left[\sum_{h=0}^{\infty} -\gamma^h \log \pi(a_h|s_h)\right], \tag{5}$$

and α is the penalty term. For simplicity, we denote the optimization objective function in (4) as α-MaxEnt RL. Similar to Equation 18 & 19 of Mei et al. (2020), we also define the advantage and Q-functions and for MaxEnt RL as

$$\begin{aligned} A^{\pi}(s, a) &:= Q^{\pi}(s, a) - \alpha \log \pi(s, a) - V^{\pi}(s), \\ Q^{\pi}(s, a) &:= r(s, a) + \gamma \sum_{s'} P(s'|s, a) V^{\pi}(s). \end{aligned} \tag{6}$$

2. We let

$$d^{\pi}_{s_0}(s) = (1-\gamma) \sum_{t=0}^{\infty} \gamma^t \mathbb{P}^{\pi}(s_t = s|s_0), \tag{7}$$

to denote the discounted state visitation of policy π starting at state s_0, and let

$$d^{\pi}_{\rho}(s) = \mathbb{E}_{s\sim\rho}[d^{\pi}_{s}(s)] \tag{8}$$

denote the initial state visitation distribution under **initial state distribution** ρ.

3. We assume the reward functions under all context are bounded within $[0, 1]$:

$$r_{\omega}(s, a) \in [0, 1],\ \forall \omega \in \Omega, \forall (s, a) \in \mathcal{S} \times \mathcal{A}. \tag{9}$$

4. Similar to previous analysis in Agarwal et al. (2021); Mei et al. (2020); Ding et al. (2021), we assume the initial distribution μ for PG/stochastic PG satisfies $\rho(s) > 0, \forall s \in \mathcal{S}$.

A.2 MAIN RESULTS: MISMATCH COEFFICIENT UPPER BOUND

Lemma A.1 (Density Mismatch Ratio via ROLLIN) *Using* (1) *from* ROLLIN $\mu_k = \beta d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}} + (1-\beta)\rho$*, the density mismatch ratio* $\left\| d^{\pi^\star_{\omega_k}}_{\mu_k} / \mu_k \right\|_\infty$ *satisfies*

$$\left\| \frac{d^{\pi^\star_{\omega_k}}_{\mu_k}}{\mu_k} \right\|_\infty \leq \widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)} \Delta^k_\omega S\right), \tag{10}$$

where $\Delta^k_\omega = \max_{1\leq i\leq k} \|\omega_i - \omega_{i-1}\|_2$.

Proof By (1) from ROLLIN, we have

$$\begin{aligned} \left\| \frac{d^{\pi^\star_{\omega_k}}_{\mu_k}}{\mu_k} \right\|_\infty &= \left\| \frac{d^{\pi^\star_{\omega_k}}_{\mu_k} - d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}} + d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}}}{\mu_k} \right\|_\infty \\ &\overset{(i)}{\leq} \frac{\left\| d^{\pi^\star_{\omega_k}}_{\mu_k} - d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}} \right\|_1}{\min \mu_k} + \left\| \frac{d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}}}{\beta d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}} + (1-\beta)\rho} \right\|_\infty \\ &\overset{(ii)}{\leq} \frac{\left\| d^{\pi^\star_{\omega_k}}_{\mu_k} - d^{\pi^\star_{\omega_{k-1}}}_{\mu_{k-1}} \right\|_1}{\min \mu_k} + \frac{1}{\beta} \end{aligned} \tag{11}$$

where inequality (i) holds because of (1), and inequality (ii) holds by setting $\rho = \mathbf{0}$. Now it remains to bound $\left\| d_{\mu_{k+1}}^{\pi^\star_{\omega_{k+1}}} - d_{\mu_k}^{\pi^\star_{\omega_k}} \right\|_1$ using the difference $\|\omega_{k+1} - \omega_k\|_2$. Let $\mathbb{P}_h^k = \mathbb{P}_h^{\pi^\star_{\omega_k}}(s'|s_0 \sim \mu_k)$ denote the state visitation distribution resulting from $\pi^\star_{\omega_k}$ probability starting at μ_k, then we have

$$
\begin{aligned}
&\mathbb{P}_h^k(s') - \mathbb{P}_h^{k-1}(s') = \sum_{s,a} \left(\mathbb{P}_{h-1}^k(s)\pi^\star_{\omega_k}(a|s) - \mathbb{P}_{h-1}^{k-1}(s)\pi^\star_{\omega_{k-1}}(a|s) \right) P(s'|s,a) \\
&= \sum_{s,a} \left(\mathbb{P}_{h-1}^k(s)\pi^\star_{\omega_k}(a|s) - \mathbb{P}_{h-1}^k(s)\pi^\star_{\omega_{k-1}}(a|s) + \mathbb{P}_{h-1}^{k-1}(s)\pi^\star_{\omega_{k-1}}(a|s) - \mathbb{P}_{h-1}^{k-1}(s)\pi^\star_{\omega_{k-1}}(a|s) \right) P(s'|s,a) \\
&= \sum_{s} \mathbb{P}_{h-1}^k(s) \left[\sum_a \left(\pi^\star_{\omega_k}(a|s) - \pi^\star_{\omega_{k-1}}(a|s) \right) P(s'|s,a) \right] \\
&\quad + \sum_s \left(\mathbb{P}_{h-1}^k(s) - \mathbb{P}_{h-1}^{k-1}(s) \right) \left[\sum_a \pi^\star_{\omega_{k-1}}(a|s) P(s'|s,a) \right].
\end{aligned} \tag{12}
$$

Taking absolute value on both side, yields

$$
\begin{aligned}
&\left\| \mathbb{P}_h^k - \mathbb{P}_h^{k-1} \right\|_1 = \sum_{s'} \left| \mathbb{P}_h^k(s') - \mathbb{P}_h^{k-1}(s') \right| \\
&\leq \sum_s \mathbb{P}_{h-1}^k(s) \sum_a \underbrace{\left| \pi^\star_{\omega_k}(a|s) - \pi^\star_{\omega_{k-1}}(a|s) \right|}_{\leq c_1 \|\omega_k - \omega_{k-1}\|_2} \sum_{s'} P(s'|s,a) \\
&\quad + \sum_s \left| \mathbb{P}_{h-1}^k(s) - \mathbb{P}_{h-1}^{k-1}(s) \right| \left[\sum_{s'} \sum_a \pi^\star_{\omega_{k-1}}(a|s) P(s'|s,a) \right] \\
&\overset{(i)}{\leq} c_1 \|\omega_k - \omega_{k-1}\|_2 + \left\| \mathbb{P}_{h-1}^k - \mathbb{P}_{h-1}^{k-1} \right\|_1 \leq \cdots \leq c_1 h \|\omega_k - \omega_{k-1}\|_2 + \left\| \mathbb{P}_0^k - \mathbb{P}_0^{k-1} \right\|_1 \\
&\overset{(ii)}{=} c_1 h \|\omega_k - \omega_{k-1}\|_2 + \|\mu_k - \mu_{k-1}\|_1 ,
\end{aligned} \tag{13}
$$

where inequality (i) holds by applying Lemma B.2 with $c_1 = L_r/\alpha(1-\gamma)$ from and equality (ii) holds because the initial distribution of $\mathbb{P}_h^k$ is μ_k. By the definition of d_μ^π, we have

$$
d_{\mu_k}^{\pi^\star_{\omega_k}}(s) - d_{\mu_{k-1}}^{\pi^\star_{\omega_{k-1}}}(s) \overset{(i)}{=} d_k(s) - d_{k-1}(s) = (1-\gamma) \sum_{h=0}^{\infty} \gamma^h \left(\mathbb{P}_h^k(s) - \mathbb{P}_h^{k-1}(s) \right), \ \forall s \in \mathcal{S}. \tag{14}
$$

where in equality (i), we use d_k to denote $d_{\mu_k}^{\pi^\star_{\omega_k}}$. Adding ℓ^1 norm on both sides of (14) and applying (13), yields

$$
\begin{aligned}
&\|d_k - d_{k-1}\|_1 \leq (1-\gamma) \sum_{h=0}^{\infty} \gamma^h \left(c_1 h \|\omega_k - \omega_{k-1}\|_2 + \|\mu_k - \mu_{k-1}\|_1 \right) \\
&\overset{(i)}{=} \frac{\gamma c_1}{1-\gamma} \|\omega_k - \omega_{k-1}\|_2 + \|\mu_k - \mu_{k-1}\|_1 \overset{(ii)}{=} \frac{\gamma c_1}{1-\gamma} \|\omega_k - \omega_{k-1}\|_2 + \beta \|d_{k-1} - d_{k-2}\|_1 ,
\end{aligned} \tag{15}
$$

where equality (i) holds because $\sum_{h=0}^{\infty} \gamma^h h = \gamma/(1-\gamma)^2$ and equality (ii) holds because of (1). Hence, we know that

$$
\begin{aligned}
\|d_k - d_{k-1}\|_1 &\leq \frac{\gamma c_1}{1-\gamma} \|\omega_k - \omega_{k-1}\|_2 + \beta \|d_{k-1} - d_{k-2}\|_1 \\
&\leq \frac{\gamma c_1}{1-\gamma} \sum_{i=0}^{k-1} \left[\|\omega_{i+1} - \omega_i\|_2 \beta^{k-i} \right] + \beta^{k-1} \|d_1 - d_0\|_1 \\
&\leq \frac{\gamma c_1}{1-\gamma} \cdot \frac{1}{1-\beta} \Delta_\omega^k + \beta^{k-1} \|d_1 - d_0\|_1 \approx \frac{\gamma c_1}{(1-\gamma)(1-\beta)} \Delta_\omega^k
\end{aligned} \tag{16}
$$

where $\Delta_\omega^k = \max_{1\le i\le k} \|\omega_i - \omega_{i-1}\|_2$ and the last $\approx$ holds when k is large. Therefore, applying (16) back to (11), we know that

$$\begin{aligned}\left\|\frac{d_{\mu_k}^{\pi^\star_{\omega_k}}}{\mu_k}\right\|_\infty &\le \frac{\left\|d_{\mu_k}^{\pi^\star_{\omega_k}} - d_{\mu_{k-1}}^{\pi^\star_{\omega_{k-1}}}\right\|_1}{\min \mu_k} + \frac{1}{\beta} \\ &\overset{(i)}{\le} \frac{1}{\min\mu_k}\frac{\gamma c_1}{(1-\gamma)(1-\beta)}\Delta_\omega^k + \frac{1}{\beta} = \widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}\Delta_\omega^k S\right),\end{aligned} \tag{17}$$

where inequality (i) holds since Lemma B.2 implies $c_1 = L_r/\alpha(1-\gamma)$, and we omit the $1/(1-\gamma)^6$ and $\log$ in the $\widetilde{\Omega}$, which completes the proof. Note that we can only achieve the final bound $\widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}\Delta_\omega^k S\right)$ by setting β as a constant. If we pick an arbitrarily small β, then the $1/\beta$ term will dominate the complexity and we will not have the final bound of $\widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}\Delta_\omega^k S\right)$. ■

A.3 Complexity of Vanilla Stochastic PG

Theorem A.2 (Complexity of Stochastic PG (Theorem 5.1 of Ding et al. (2021))) *Consider an arbitrary tolerance level $\delta > 0$ and a small enough tolerance level $\varepsilon > 0$. For every initial point θ_1, if θ_{T+1} is generated by SPG (Algorithm 4) with*

$$\begin{aligned}&T_1 \ge \left(\frac{6D(\theta_0)}{\delta\varepsilon_0}\right)^{\frac{8L}{C_\delta^0 \ln 2}},\ T_2 \ge \left(\frac{\varepsilon_0}{6\delta\varepsilon} - 1\right)t_0,\ T = T_1 + T_2, \\ &B_1 \ge \max\left\{\frac{30\sigma^2}{C_\delta^0\varepsilon_0\delta}, \frac{6\sigma T_1 \log T_1}{\bar{\Delta} L}\right\},\ B_2 \ge \frac{\sigma^2 \ln(T_2 + t_0)}{6C_\zeta\delta\varepsilon}, \\ &\eta_t = \eta \le \min\left\{\frac{\log T_1}{T_1 L}, \frac{8}{C_\delta^0}, \frac{1}{2L}\right\}\ \forall 1 \le t \le T_1,\ \eta_t = \frac{1}{t - T_1 + t_0}\ \forall t > T_1,\end{aligned} \tag{18}$$

where

$$\begin{aligned}&D(\theta_t) = V^{\pi^\star}(\rho) - V^{\pi_{\theta_t}}(\rho),\ \varepsilon_0 = \min\left\{\left(\frac{\alpha \min_{s\in\mathcal{S}}\rho(s)}{6\ln 2}\right)^2\left[\zeta\exp\left(-\frac{1}{(1-\gamma)\alpha}\right)\right]^4, 1\right\}, \\ &t_0 \ge \sqrt{\frac{3\sigma^2}{2\delta\varepsilon_0}},\ C_\delta^0 = \frac{2\alpha}{S}\left\|\frac{d_\rho^{\pi^\star}}{\rho}\right\|_\infty^{-1}\min_{s\in\mathcal{S}}\rho(s)\min_{\theta\in\mathcal{G}_\delta^0}\min_{s,a}\pi_\theta(a|s)^2, \\ &C_\zeta = \frac{2\alpha}{S}\left\|\frac{d_\rho^{\pi^\star}}{\rho}\right\|_\infty^{-1}\min_{s\in\mathcal{S}}\rho(s)(1-\zeta)^2\min_{s,a}\pi^\star(a|s)^2, \\ &\mathcal{G}_\delta^0 := \left\{\theta\in\mathbb{R}^{S\times A} : \min_{\theta^\star\in\Theta^\star}\|\theta - \theta^\star\|_2 \le (1+1/\delta)\bar{\Delta}\right\},\ \bar{\Delta} = \left\|\log c_{\bar{\theta}_1,\eta} - \log\pi^\star\right\|_2, \\ &c_{\bar{\theta}_1,\eta} = \inf_{t\ge 1}\min_{s,a}\pi_{\theta_t}(a|s),\ \sigma^2 = \frac{8}{(1-\gamma)^2}\left(\frac{1+(\alpha\log A)^2}{(1-\gamma^{1/2})^2}\right),\ L = \frac{8+\alpha(4+8\log A)}{(1-\gamma)^3},\end{aligned} \tag{19}$$

then we have $\mathbb{P}(D(\theta_{T+1} \le \varepsilon)) \ge 1-\delta$.

Corollary A.3 (Iteration Complexity and Sample Complexity for ε-Optimal Policies) *Suppose we set the tolerance level $\varepsilon, \delta = O(S^{-1})$, the iteration complexity and sample complexity of obtaining an ε-optimal policy using stochastic softmax policy gradient (Algorithm 4) in phase 1 and phase 2 satisfies:*

- *Phase 1:* $T_1 = \widetilde{\Omega}\left(S^{2S^3}\right)$, $B_1 = \widetilde{\Omega}\left(S^{2S^3}\right)$,
- *Phase 2:* $T_2 = \widetilde{\Omega}\left(S^{3/2}\right)$, $B_2 = \widetilde{\Omega}\left(S^5\right)$,

with probability at least $1-\delta$.

Proof We first check the dependency of (19) on S. Notice that

- ε_0:

$$\frac{1}{\varepsilon_0} = \max\left\{\left(\frac{6\ln 2}{\alpha \min_{s\in\mathcal{S}}\rho(s)}\right)^2\left[\zeta\exp\left(-\frac{1}{(1-\gamma)\alpha}\right)\right]^{-4}, 1\right\} = \widetilde{\Omega}(S^2); \tag{20}$$

- t_0:

$$t_0 \geq \sqrt{\frac{3\sigma^2}{2\delta\varepsilon_0}} = \widetilde{\Omega}(S); \tag{21}$$

- C_δ^0:

$$\frac{1}{C_\delta^0} = \frac{S}{2\alpha}\left\|\frac{d_\rho^{\pi^\star}}{\rho}\right\|_\infty \max_{s\in\mathcal{S}}\rho(s)^{-1}\frac{1}{\min_{\theta\in\mathcal{G}_\delta^0}\min_{s,a}\pi_\theta(a|s)^2} = \widetilde{\Omega}(S^3); \tag{22}$$

- C_ζ:

$$\frac{1}{C_\zeta} = \frac{S}{2\alpha}\left\|\frac{d_\rho^{\pi^\star}}{\rho}\right\|_\infty \max_{s\in\mathcal{S}}\rho(s)^{-1}(1-\zeta)^{-2}\max_{s,a}\pi^\star(a|s)^{-2} = \widetilde{\Omega}(S^3). \tag{23}$$

Hence, the complexities in phase 1 scales at

$$T_1 \geq \left(\frac{6D(\theta_0)}{\delta\varepsilon_0}\right)^{\frac{8L}{C_\delta^0\ln 2}} = \widetilde{\Omega}\left(S^{2S^3}\right),\ B_1 \geq \max\left\{\frac{30\sigma^2}{C_\delta^0\varepsilon_0\delta}, \frac{6\sigma T_1\log T_1}{\bar{\Delta}L}\right\} = \widetilde{\Omega}\left(S^{2S^3}\right). \tag{24}$$

To enforce a positive T_2, the tolerance level ε,δ should satisfy $\frac{\varepsilon_0}{6\delta\varepsilon}\geq 1$, which implies $\frac{1}{\delta\varepsilon}=\Omega(S^2)$. Hence, if assuming $\frac{\varepsilon_0}{\delta\varepsilon}=o(S)$ the tolerance level $\varepsilon,\delta=O(S^{-1})$, the complexities in phase 2 scales at

$$T_2 \geq \left(\frac{\varepsilon_0}{6\delta\varepsilon}-1\right)t_0 = \widetilde{\Omega}\left(S^{3/2}\right),\ B_2 \geq \frac{\sigma^2\ln(T_2+t_0)}{6C_\zeta\delta\varepsilon} = \widetilde{\Omega}\left(S^5\right). \tag{25}$$

■

A.4 Complexity of Learning the Next Context

Theorem A.4 (Theorem 4.1: Complexity of Learning the Next Context) *Consider the context based stochastic softmax policy gradient (line 6 of Algorithm 1), if* $\|\omega_k-\omega_{k-1}\|_2\leq O\left(S^{-2}\right)$, *and the tolerance level* $\varepsilon,\delta=O(S^{-1})$, *then the iteration complexity of obtaining an* ε*-optimal policy for* ω_k *from an* $\theta^\star_{\omega_k-1}$ *is* $\Omega\left(S^{3/2}\right)$ *and the per iteration sample complexity is* $\widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}S^3\right)$.

We first introduce the following lemma to aid the proof of Theorem A.4.

Lemma A.5 (Bounded Optimal Values Between two Adjacent Contexts) *Under the same conditions as Theorem A.4, we have*

$$V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi^\star_{\omega_{k-1}}}(\rho) \leq \frac{2L_r\|\omega_k-\omega_{k-1}\|_2}{(1-\gamma)^2}. \tag{26}$$

Proof Let V_ω^π denote the value function of policy π with reward function r_ω. From (66) of Lemma B.3, we know that for any initial distribution ρ, we have

$$V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi^\star_{\omega_{k-1}}}(\rho) = \frac{1}{1-\gamma}\sum_s\left[d_\rho^{\pi^\star_{\omega_{k-1}}}(s)\cdot\alpha\cdot D_{\mathrm{KL}}\left(\pi^\star_{\omega_{k-1}}(\cdot|s)\|\pi^\star_{\omega_k}(\cdot|s)\right)\right]. \tag{27}$$

From (48) of Lemma B.1, we know that

$$\begin{aligned}\pi^\star_{\omega_{k-1}}(a|s) &= \left[\text{softmax}(Q^{\pi^\star_{\omega_{k-1}}}(\cdot,s)/\alpha)\right]_a := \frac{\exp\left[Q^{\pi^\star_{\omega_{k-1}}}(s,a)/\alpha\right]}{\sum_{a'}\exp\left[Q^{\pi^\star_{\omega_{k-1}}}(s,a')/\alpha\right]} \\ \pi^\star_{\omega_k}(a|s) &= \left[\text{softmax}(Q^{\pi^\star_{\omega}}(\cdot,s)/\alpha)\right]_a := \frac{\exp\left[Q^{\pi^\star_{\omega_k}}(s,a)/\alpha\right]}{\sum_{a'}\exp\left[Q^{\pi^\star_{\omega_k}}(s,a')/\alpha\right]},\end{aligned} \tag{28}$$

hence, we have

$$\begin{aligned}&D_{\text{KL}}\left(\pi^\star_{\omega_{k-1}}(\cdot|s)||\pi^\star_{\omega_k}(\cdot|s)\right) \\ &= \sum_a \pi^\star_{\omega_{k-1}}(a|s)\left\{\log\left(\left[\text{softmax}(Q^{\pi^\star_{\omega_{k-1}}}(a,s)/\alpha)\right]_a\right) - \log\left(\left[\text{softmax}(Q^{\pi^\star_{\omega_k}}(a,s)/\alpha)\right]_a\right)\right\}.\end{aligned} \tag{29}$$

Let $f(\boldsymbol{x})$ denote the log soft max function for an input vector $\boldsymbol{x} = [x_1, x_2, \ldots, x_A]^\top$ such that $x_i \geq 0$, then for a small perturbation $\boldsymbol{\Delta} \in \mathbb{R}^A$, the intermediate value theorem implies

$$|[f(\boldsymbol{x}+\boldsymbol{\Delta})]_i - [f(\boldsymbol{x})]_i| = \left|\boldsymbol{\Delta}^\top \nabla_{\boldsymbol{z}}\left[f(\boldsymbol{z})\right]_i\right|, \tag{30}$$

for some vector $\boldsymbol{z}$ on the segment $[\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{\Delta}]$. Now consider the Jacobian of the log softmax function $\partial[\nabla_{\boldsymbol{z}} f(\boldsymbol{z})]_i/\partial z_j$:

$$\frac{\partial[\nabla_{\boldsymbol{z}} f(\boldsymbol{z})]_i}{\partial z_j} = \begin{cases} 1 - p_i(\boldsymbol{z}) \in (0,1) & \text{if } i = j, \\ -p_j(\boldsymbol{z}) \in (-1,0) & \text{otherwise,}\end{cases} \tag{31}$$

where $p_i(\boldsymbol{z}) = \exp(z_i)/\sum_{k=1}^A \exp(z_k)$. hence, we know that

$$\begin{aligned}|[f(\boldsymbol{x}+\boldsymbol{\Delta})]_i - [f(\boldsymbol{x})]_i| &= \left|\boldsymbol{\Delta}^\top \nabla_{\boldsymbol{z}}\left[f(\boldsymbol{z})\right]_i\right| \leq \|\boldsymbol{\Delta}\|_\infty \sum_{k=1}^A \left|\frac{\partial[f(\boldsymbol{z})]_i}{\partial z_k}\right| \\ &= \|\boldsymbol{\Delta}\|_\infty \left(1 - p_i(\boldsymbol{z}) + \sum_{j\neq i} p_j(\boldsymbol{z})\right) \leq 2\|\boldsymbol{\Delta}\|_\infty.\end{aligned} \tag{32}$$

Now let

$$\begin{aligned}\boldsymbol{x} &= \frac{1}{\alpha}[Q^{\pi^\star_{\omega_{k-1}}}(s,a_1), Q^{\pi^\star_{\omega_{k-1}}}(s,a_2), \ldots, Q^{\pi^\star_{\omega_{k-1}}}(s,a_A)], \\ \boldsymbol{x}+\boldsymbol{\Delta} &= \frac{1}{\alpha}[Q^{\pi^\star_{\omega_k}}(s,a_1), Q^{\pi^\star_{\omega_k}}(s,a_2), \ldots, Q^{\pi^\star_{\omega_k}}(s,a_A)],\end{aligned} \tag{33}$$

(58) from Lemma B.2 implies that

$$\frac{1}{\alpha}\left\|Q^{\pi^\star_{\omega_k}} - Q^{\pi^\star_{\omega_{k-1}}}\right\|_\infty \leq \frac{L_r\|\omega_k - \omega_{k-1}\|_2}{\alpha(1-\gamma)}, \tag{34}$$

substituting (34) and (32) into (29), yields

$$D_{\text{KL}}\left(\pi^\star_{\omega_{k-1}}(\cdot|s)||\pi^\star_{\omega_k}(\cdot|s)\right) \leq \sum_a 2\pi^\star_{\omega_{k-1}}(a|s)\|\boldsymbol{\Delta}\|_\infty \leq 2\|\boldsymbol{\Delta}\|_\infty \leq \frac{2L_r\|\omega_k - \omega_{k-1}\|_2}{\alpha(1-\gamma)}. \tag{35}$$

Combine (35) with (27), we have

$$\begin{aligned}V^{\pi^\star_{\omega_k}}_{\omega_k}(\rho) - V^{\pi^\star_{\omega_{k-1}}}_{\omega_k}(\rho) &= \frac{1}{1-\gamma}\sum_s\left[d^{\pi^\star_{\omega_{k-1}}}_\rho(s)\cdot\alpha\cdot D_{\text{KL}}\left(\pi^\star_{\omega_{k-1}}(\cdot|s)||\pi^\star_{\omega_k}(\cdot|s)\right)\right] \\ &\leq \frac{2L_r\|\omega_k - \omega_{k-1}\|_2}{(1-\gamma)^2},\end{aligned} \tag{36}$$

which completes the proof. ■

Now we are ready to proceed to the proof of Theorem A.4.

Proof From (19) we know that

$$\varepsilon_0 = \min\left\{\left(\frac{\alpha \min_{s\in\mathcal{S}} \rho(s)}{6\ln 2}\right)^2 \left[\zeta \exp\left(-\frac{1}{(1-\gamma)\alpha}\right)\right]^4, 1\right\} = O\left(\frac{1}{S^2}\right). \tag{37}$$

And from Section 6.2 of Ding et al. (2021), we can directly enter phase 2 of the stochastic PG when

$$V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi^\star_{\omega_{k-1}}}(\rho) \leq \varepsilon_0. \tag{38}$$

Hence, when $\Delta_\omega^k = \max_{1\leq i\leq k} \|\omega_i - \omega_{i-1}\|_2 = O(1/S^2)$, we have

$$V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi^\star_{\omega_{k-1}}}(\rho) \leq \frac{2L_r\Delta_\omega}{(1-\gamma)^2} \leq \frac{\varepsilon_0}{2}, \tag{39}$$

which implies we can directly enter phase 2 and enjoys the faster iteration complexity of $T_2 = \Omega\left(S^{3/2}\right)$ (by choosing $\delta = O(S^{-1})$) and the smaller batch size of

$$B_2 \geq \frac{\sigma^2 \ln(T_2+t_0)}{6C_\zeta \delta\varepsilon} \overset{(i)}{=} \widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}\Delta_\omega^k S^5\right) \overset{(ii)}{=} \widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}S^3\right), \tag{40}$$

where equation (i) holds by applying Lemma A.1 to (23):

$$\frac{\sigma^2 \ln(T_2+t_0)}{6C_\zeta \delta\varepsilon} = \widetilde{\Omega}\left(S^4 \cdot \left\|d_{\mu_k}^{\pi^\star_{\omega_k}}/\mu_k\right\|_\infty\right) = \widetilde{\Omega}\left(\frac{L_r}{\alpha(1-\beta)}\Delta_\omega^k S^5\right),$$

and equality (ii) holds by the assumption that $\Delta_\omega^k = O(S^{-2})$ and we omit the $\log$ term and components not related to S in $\widetilde{\Omega}$. ■

A.5 Total Complexity of Rollin

Theorem A.6 (Theorem 4.2: Total Complexity of Learning the Target Context) *Suppose the assumptions in Theorem 4.1 holds. Given a curriculum $\{\omega_k\}_{k=0}^K$ such that $\max_{1\leq k\leq K} \|\omega_k - \omega_{k-1}\|_2 \leq O(S^{-2})$, the total number of iteration of learning $\pi^\star_{\omega_K}$ using Algorithm 1 is $\Omega(KS^{3/2})$ and the per iteration is $\widetilde{\Omega}\left(S^3\right)$, with high probability.*

Proof From lemma A.5, we know that

$$V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi^\star_{\omega_{k-1}}}(\rho) \leq \frac{2L_r \|\omega_k - \omega_{k-1}\|_2}{(1-\gamma)^2}. \tag{41}$$

Suppose for each context ω_k, we initialize the parameters of the policy as $\theta_1^{(k)} = \theta^\star_{\omega_{k-1}}$, and let $\theta_t^{(k)}$ denote the parameters at the tth iteration of SPG. We will use induction to show that when $t = \widetilde{\Omega}(S^{3/2})$, $\forall k \in [K]$, we have

$$V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi_{\theta_t^{(k-1)}}}(\rho) < \varepsilon_0, \tag{42}$$

this implies that for any context $\omega_k, k \in [K]$, we can always find a good initialization by setting $\theta_1^{(k)} = \theta_t^{(k-1)}$ from learning $\pi^\star_{\omega_{k-1}}$ using SPG after $t = \Omega(S^{3/2})$ iteration. This result guarantees that every initialization $\theta_1^{(k)}$ for learning the optimal contextual policy $\pi^\star_{\omega_k}$ will directly start from the efficient phase 2.

Induction: $k = 1$**.** When $k = 1$, from (39) of Theorem A.4, we know that at $k = 1$, we have

$$V_{\omega_1}^{\pi^\star_{\omega_1}}(\rho) - V_{\omega_1}^{\pi^\star_{\omega_0}}(\rho) \leq \frac{\varepsilon_0}{2}. \tag{43}$$

Hence, if we initialize $\theta_1^{(1)}$ such that $\theta_1^{(1)} = \theta^\star_{\omega_0}$, then we have:

$$V_{\omega_1}^{\pi^\star_{\omega_1}}(\rho) - V_{\omega_1}^{\pi_{\theta_1^{(1)}}}(\rho) \leq \frac{\varepsilon_0}{2} < \varepsilon_0. \tag{44}$$

Induction: from $k-1$ to k. Suppose the result in (42) holds for $k-1$, then we know that

$$V_{\omega_{k-1}}^{\pi^\star_{\omega_{k-1}}}(\rho) - V_{\omega_{k-1}}^{\pi_{\theta_1^{(k-1)}}}(\rho) = V_{\omega_{k-1}}^{\pi^\star_{\omega_{k-1}}}(\rho) - V_{\omega_{k-1}}^{\pi_{\theta_t^{(k-2)}}}(\rho) < \varepsilon_0. \tag{45}$$

Select ε such that $\varepsilon \leq \varepsilon_0/2$. Theorem A.4 suggests that when $t' = \tilde{\Omega}(S^{3/2})$, with high probability, we have

$$V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi_{\theta_{t'}^{(k-1)}}}(\rho) < \varepsilon \leq \frac{\varepsilon_0}{2}. \tag{46}$$

Hence, if we initialize $\theta_1^{(k)} = \theta_t^{(k-1)}$, with high probability when $t' = \widetilde{\Omega}(S^{3/2})$, we have

$$\begin{aligned} &V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi_{\theta_{t'}^{(k-1)}}}(\rho) = V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi^\star_{\omega_{k-1}}}(\rho) + V_{\omega_k}^{\pi^\star_{\omega_{k-1}}}(\rho) - V_{\omega_k}^{\pi_{\theta_{t'}^{(k-1)}}}(\rho) \\ &\overset{(i)}{\leq} \frac{\varepsilon_0}{2} + V_{\omega_k}^{\pi^\star_{\omega_k}}(\rho) - V_{\omega_k}^{\pi_{\theta_{t'}^{(k-1)}}}(\rho) \overset{(ii)}{<} \varepsilon_0, \end{aligned} \tag{47}$$

where inequality (i) holds by equation (39) in Theorem A.4, inequality (ii) holds because of the induction assumption in (46). Therefore, we have shown (42) holds for $t = \widetilde{\Omega}(S^{3/2})$, $\forall k \in [K]$. Since we have K contexts in total, we know that Algorithm 1 can enforce a good initialization $\theta_1^{(k)}$ that directly starts from phase 2 for learning all $\pi^\star_{\omega_k}$, and for each $k \in [K]$, the iteration complexity is $\widetilde{\Omega}(S^{3/2})$. Hence the total iteration complexity of obtaining an ε-optimal policy for the final context ω_K is $\widetilde{\Omega}\left(KS^{3/2}\right)$, with per iteration sample complexity of $\widetilde{\Omega}\left(S^3\right)$. ■

Appendix B Key Lemmas

B.1 Optimal Policy of Maximum Entropy RL Nachum et al. (2017)

Lemma B.1 *The optimal policy $\pi^\star$ that maximizes the α-MaxEnt RL objective (4) with penalty term α satisfies:*

$$\pi^\star(a|s) = \exp\left[\left(Q^{\pi^\star}(s,a) - V^{\pi^\star}(s)\right)/\alpha\right] = \frac{\exp\left(Q^{\pi^\star}(s,a)/\alpha\right)}{\sum_a \exp\left(Q^{\pi^\star}(s,a)/\alpha\right)} \tag{48}$$

for all $h \in \mathbb{N}$, where

$$\begin{aligned} Q^{\pi^\star}(s,a) &:= r(s,a) + \gamma \mathbb{E}_{s' \sim P(s'|s,a)} V(s') \\ V^{\pi^\star}(s) &:= \alpha \log\left(\sum_a \exp\left(Q^{\pi^\star}(s,a)/\alpha\right)\right). \end{aligned} \tag{49}$$

Proof Similar proof appears in (Nachum et al., 2017), we provide the proof for completeness. At the optimal policy $\pi_\theta = \pi^\star$, take the gradient of (4) w.r.t. $p \in \Delta(\mathcal{A})$ and set it to 0, we have

$$\frac{\partial}{\partial p(a)}\left[\sum_{a\in\mathcal{A}} p(a)\left(Q^{\pi^\star}(s,a) - \alpha \ln p(a)\right)\right] = Q^{\pi^\star}(s,a) - \alpha \ln p(a) - \alpha = 0, \tag{50}$$

which implies

$$p(a) = \exp\left(\frac{Q^{\pi^\star}(s,a)}{\alpha} - 1\right) \propto \exp\left(\frac{Q^{\pi^\star}(s,a)}{\alpha}\right). \tag{51}$$

Hence, we conclude that $\pi^\star(a|s) \propto \exp(Q^\star(s,a)/\alpha)$. ■

B.2 Bounding the Difference between Optimal Policies

Lemma B.2 *Suppose Assumption 3.1 holds, let $\pi^\star_\omega(a|s), \pi^\star_{\omega'}(a|s)$ denote the optimal policy for α-MaxEnt RL (48), then $\forall (s,a) \in \mathcal{S} \times \mathcal{A}$, the optimal policies of α-MaxEnt RL under context ω, ω' satisfy:*

$$|\pi^\star_\omega(a|s) - \pi^\star_{\omega'}(a|s)| \leq \frac{L_r \|\omega - \omega'\|_2}{\alpha(1-\gamma)}. \tag{52}$$

Proof From Lemma C.1, we know that the soft value iteration

$$\mathcal{T}Q(s,a) = r(s,a) + \gamma\alpha\mathbb{E}_{s'}\left[\log\sum_{a'}\exp Q(s',a')/\alpha\right] \tag{53}$$

is a contraction. Let $Q^t_\omega, Q^t_{\omega'}$ denote the Q functions at the tth value iteration under context ω, ω' respectively, we know $Q^\infty_\omega = Q^{\pi^\star}_\omega$ and $Q^\infty_{\omega'} = Q^{\pi^\star}_{\omega'}$. Let $\varepsilon_t = \|Q^t_\omega - Q^t_{\omega'}\|_\infty$, then we have

$$\begin{aligned}
&\varepsilon_{t+1} = \left\|Q^{t+1}_\omega - Q^{t+1}_{\omega'}\right\|_\infty \\
&= \left\| r_\omega(s,a) - r_{\omega'}(s,a) + \gamma\alpha\mathbb{E}_{s'}\left[\log\sum_{a'}\exp\frac{Q^t_{\omega'}(s',a')}{\alpha}\right] - \gamma\alpha\mathbb{E}_{s'}\left[\log\sum_{a'}\exp\frac{Q^t_{\omega'}(s',a')}{\alpha}\right]\right\|_\infty \\
&\le \|r_\omega - r_{\omega'}\|_\infty + \gamma\alpha\left\|\mathbb{E}_{s'}\log\sum_{s'}\exp Q^t_\omega(s',a')/\alpha - \mathbb{E}_{s'}\log\sum_{s'}\exp Q^t_{\omega'}(s',a')/\alpha\right\|_\infty \\
&\le \|r_\omega - r_{\omega'}\|_\infty + \gamma\left\|Q^t_\omega - Q^t_{\omega'}\right\|_\infty = \|r_\omega - r_{\omega'}\|_\infty + \gamma\varepsilon_t,
\end{aligned} \tag{54}$$

where the last inequality holds because $f(\boldsymbol{x}) = \log\sum_{i=1}^n \exp(x_i)$ is a contraction. From (54), we have

$$\varepsilon_{t+1} \le \|r_\omega - r_{\omega'}\|_\infty + \gamma\varepsilon_t \le (1+\gamma)\|r_\omega - r_{\omega'}\|_\infty + \gamma^2\varepsilon_{t-1} \le \cdots \le \|r_\omega - r_{\omega'}\|_\infty \sum_{i=0}^t \gamma^i + \gamma^t\varepsilon_1, \tag{55}$$

which implies

$$\left\|Q^{\pi^\star}_\omega - Q^{\pi^\star}_{\omega'}\right\|_\infty = \varepsilon_\infty \le \frac{\|r_\omega - r_{\omega'}\|_\infty}{1-\gamma} \le \frac{L_r\|\omega-\omega'\|_2}{1-\gamma}, \tag{56}$$

where the last inequality holds by Assumption 3.1. Hence, we have

$$\frac{1}{\alpha}\left|Q^{\pi^\star_\omega}(s,a) - Q^{\pi^\star_{\omega'}}(s,a)\right| \le \frac{L_r\|\omega-\omega'\|_2}{\alpha(1-\gamma)},\ \forall s,a \in \mathcal{S}\times\mathcal{A} \tag{57}$$

which implies

$$\frac{1}{\alpha}\left\|Q^{\pi^\star_\omega} - Q^{\pi^\star_{\omega'}}\right\|_\infty \le \frac{L_r\|\omega-\omega'\|_2}{\alpha(1-\gamma)}. \tag{58}$$

Next, let $\pi^\star_\omega, \pi^\star_{\omega'}$ denote the maximum entropy policy RL under context ω, ω' respectively. Then for a fixed state action pair $(s,a) \in \mathcal{S}\times\mathcal{A}$, we have

$$\begin{aligned}
\pi^\star_\omega(a|s) &= \left[\text{softmax}(Q^{\pi^\star_\omega}(\cdot,s)/\alpha)\right]_a := \frac{\exp\left[Q^{\pi^\star_\omega}(s,a)/\alpha\right]}{\sum_{a'}\exp\left[Q^{\pi^\star_\omega}(s,a')/\alpha\right]}, \\
\pi^\star_{\omega'}(a|s) &= \left[\text{softmax}(Q^{\pi^\star_{\omega'}}(\cdot,s)/\alpha)\right]_a := \frac{\exp\left[Q^{\pi^\star_{\omega'}}(s,a)/\alpha\right]}{\sum_{a'}\exp\left[Q^{\pi^\star_{\omega'}}(s,a')/\alpha\right]},
\end{aligned} \tag{59}$$

where $Q^{\pi^\star_\omega}(\cdot,s), Q^{\pi^\star_{\omega'}}(\cdot,s) \in \mathbb{R}^A$, and we want to bound $|\pi^\star_\omega(a|s) - \pi^\star_{\omega'}(a|s)|$. Next we will use (58) to bound $|\pi^\star_\omega(a|s) - \pi^\star_{\omega'}(a|s)|$, where the last inequality holds by (57). Let $f(\boldsymbol{x})$ denote the softmax function for an input vector $\boldsymbol{x} = [x_1, x_2, \ldots, x_A]^\top$ such that $x_i \ge 0$, then for a small perturbation $\boldsymbol{\Delta} \in \mathbb{R}^A$, the intermediate value theorem implies

$$|[f(\boldsymbol{x}+\boldsymbol{\Delta})]_i - [f(\boldsymbol{x})]_i| = \left|\boldsymbol{\Delta}^\top\nabla_{\boldsymbol{x}}[f(\boldsymbol{z})]_i\right|, \tag{60}$$

for some vector $\boldsymbol{z}$ on the segment $[\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{\Delta}]$. Hence

$$\begin{aligned}
&|[f(\boldsymbol{x}+\boldsymbol{\Delta})]_i - [f(\boldsymbol{x})]_i| = \left|\boldsymbol{\Delta}^\top[\nabla_{\boldsymbol{x}}f(\boldsymbol{z})]_i\right| \le \|\boldsymbol{\Delta}\|_\infty\sum_{k=1}^A\left|\frac{\partial[f(\boldsymbol{z})]_i}{\partial z_k}\right| \\
&\le \|\boldsymbol{\Delta}\|_\infty\left(p_i(\boldsymbol{z})(1-p_i(\boldsymbol{z})) + \sum_{j\ne i}p_i(\boldsymbol{z})p_j(\boldsymbol{z})\right) < \|\boldsymbol{\Delta}\|_\infty\left(p_i(\boldsymbol{z}) + \sum_{j\ne i}p_j(\boldsymbol{z})\right) = \|\boldsymbol{\Delta}\|_\infty,
\end{aligned} \tag{61}$$

where the Jacobian of the softmax function $\partial\left[\nabla_{\boldsymbol{x}} f(\boldsymbol{z})\right]_i / \partial z_j$ satisfies:

$$\frac{\partial\left[\nabla_{\boldsymbol{x}} f(\boldsymbol{z})\right]_i}{\partial z_j}=\begin{cases} p_i(\boldsymbol{z})(1-p_i(\boldsymbol{z})) & \text{if } i=j, \\ p_i(\boldsymbol{z}) p_j(\boldsymbol{z}) & \text{otherwise,} \end{cases} \tag{62}$$

and $p_i(\boldsymbol{z})=\exp(z_i) / \sum_{k=1}^A \exp(z_k)$. Now let

$$\begin{aligned} \boldsymbol{x} &= \frac{1}{\alpha}[Q^{\pi_\omega^\star}(s, a_1), Q^{\pi_\omega^\star}(s, a_2), \ldots, Q^{\pi_\omega^\star}(s, a_A)], \\ \boldsymbol{x}+\boldsymbol{\Delta} &= \frac{1}{\alpha}[Q^{\pi_{\omega'}^\star}(s, a_1), Q^{\pi_{\omega'}^\star}(s, a_2), \ldots, Q^{\pi_{\omega'}^\star}(s, a_A)]. \end{aligned} \tag{63}$$

We know that $f(\boldsymbol{x})=\pi_\omega^\star(a|s)$ and $f(\boldsymbol{x}+\boldsymbol{\Delta})=\pi_{\omega'}^\star(a|s)$. Then (58) implies that

$$\|\boldsymbol{\Delta}\|_\infty \leq \frac{L_r\|\omega-\omega'\|_2}{\alpha(1-\gamma)}, \tag{64}$$

substituting this bound on $\|\boldsymbol{\Delta}\|_\infty$ into (61), we have

$$|\pi_\omega^\star(a|s)-\pi_{\omega'}^\star(a|s)|=|f(\boldsymbol{x})-f(\boldsymbol{x}+\boldsymbol{\Delta})| \leq\|\boldsymbol{\Delta}\|_\infty \leq \frac{L_r\|\omega-\omega'\|_2}{\alpha(1-\gamma)}, \tag{65}$$

which completes the proof. ■

B.3 Soft Sub-Optimality lemma (Lemma 25 & 26 of Mei et al. (2020))

Lemma B.3 *For any policy π and any initial distribution ρ, the value function $V^\pi(\rho)$ of the α-MaxEnt RL (49) satisfies:*

$$V^{\pi^\star}(\rho)-V^\pi(\rho)=\frac{1}{1-\gamma} \sum_s\left[d_\rho^\pi(s) \cdot \alpha \cdot D_{\mathrm{KL}}\left(\pi(\cdot|s)\|\pi^\star(\cdot|s)\right)\right], \tag{66}$$

where $\pi^\star$ is the optimal policy of the α-MaxEnt RL (4).

Proof Similar proof appears in Lemma 25 & 26 of Mei et al. (2020), we provide the proof here for completeness.

Soft performance difference. We first show a soft performance difference result for the MaxEnt value function (Lemma 25 of Mei et al. (2020)). By the definition of MaxEnt value function and Q-function (4), (6), $\forall \pi, \pi'$, we have

$$\begin{aligned} & V^{\pi'}(s)-V^\pi(s) \\ =& \sum_a \pi'(a|s) \cdot\left[Q^{\pi'}(s, a)-\alpha \log \pi'(a|s)\right]-\sum_a \pi(a|s) \cdot\left[Q^\pi(s, a)-\alpha \log \pi(a|s)\right] \\ =& \sum_a\left(\pi'(a|s)-\pi(a|s)\right) \cdot\left[Q^{\pi'}(a|s)-\alpha \log \pi'(a|s)\right] \\ & +\sum_a \pi(a|s) \cdot\left[Q^{\pi'}(s, a)-\alpha \log \pi'(a|s)-Q^\pi(s, a)+\alpha \log \pi(a|s)\right] \\ =& \sum_a\left(\pi'(a|s)-\pi(a|s)\right) \cdot\left[Q^{\pi'}(a|s)-\alpha \log \pi'(a|s)\right]+\alpha D_{\mathrm{KL}}\left(\pi(\cdot|s)\|\pi'(\cdot|s)\right) \\ & +\gamma \sum_a \pi(a|s) \sum_{s'} P(s'|s, a) \cdot\left[V^{\pi'}(s')-V^\pi(s')\right] \\ =& \frac{1}{1-\gamma} \sum_{s'} d_s^\pi(s')\left[\sum_{a'}\left(\pi'(a'|s')-\pi(a'|s')\right)\left[Q^{\pi'}(s', a')-\alpha \log \pi'(a'|s')\right]\right. \\ & \left.+\alpha D_{\mathrm{KL}}\left(\pi(\cdot|s')\|\pi'(\cdot|s')\right)\right], \end{aligned} \tag{67}$$

where the last equality holds because by the definition of state visitation distribution

$$d_{s_0}^{\pi}(s) = (1-\gamma)\sum_{t=0}^{\infty}\gamma^t \mathbb{P}^{\pi}(s_t = s|s_0), \tag{68}$$

taking expectation of s w.r.t. $s \sim \rho$, yields

$$\begin{aligned}
&V^{\pi'}(\rho) - V^{\pi}(\rho) \\
=&\frac{1}{1-\gamma}\sum_{s'} d_{\rho}^{\pi}(s')\Bigg[\sum_{a'}(\pi'(a'|s') - \pi(a'|s'))\cdot\Big[Q^{\pi'}(s',a') - \alpha\log\pi'(a'|s')\Big] \\
&+ \alpha D_{\mathrm{KL}}\left(\pi(\cdot|s')||\pi'(\cdot|s')\right)\Bigg],
\end{aligned} \tag{69}$$

and (69) is known as the soft performance difference lemma (Lemma 25 in Mei et al. (2020)).

Soft sub-optimality. Next we will show the soft sub-optimality result. By the definition of the optimal policy of α-MaxEnt RL (48), we have

$$\alpha\log\pi^{\star}(a|s) = Q^{\pi^{\star}}(s,a) - V^{\pi^{\star}}(s). \tag{70}$$

Substituting $\pi^{\star}$ into the performance difference lemma (69), we have

$$\begin{aligned}
&V^{\pi^{\star}}(s) - V^{\pi}(s) \\
=&\frac{1}{1-\gamma}\sum_{s'} d_{s}^{\pi}(s')\cdot\Bigg[\sum_{a'}(\pi^{\star}(a'|s') - \pi(a'|s'))\cdot\underbrace{\Big[Q^{\pi^{\star}}(s',a') - \alpha\log\pi^{\star}(a'|s')\Big]}_{=V^{\pi^{\star}}(s')} \\
&+ \alpha D_{\mathrm{KL}}\left(\pi(\cdot|s')||\pi^{\star}(\cdot|s')\right)\Bigg] \\
=&\frac{1}{1-\gamma}\sum_{s'} d_{s}^{\pi}(s')\cdot\Bigg[\underbrace{\sum_{a'}(\pi^{\star}(a'|s') - \pi(a'|s'))}_{=0}\cdot V^{\pi^{\star}}(s') + \alpha D_{\mathrm{KL}}\left(\pi(\cdot|s')||\pi^{\star}(\cdot|s')\right)\Bigg] \\
=&\frac{1}{1-\gamma}\sum_{s'}\left[d_{s}^{\pi}(s')\cdot\alpha D_{\mathrm{KL}}\left(\pi(\cdot|s')||\pi^{\star}(\cdot|s')\right)\right],
\end{aligned} \tag{71}$$

taking expectation $s \sim \rho$ yields

$$V^{\pi^{\star}}(\rho) - V^{\pi}(\rho) = \frac{1}{1-\gamma}\sum_{s}\left[d_{\rho}^{\pi}(s)\cdot\alpha\cdot D_{\mathrm{KL}}\left(\pi(\cdot|s)||\pi^{\star}(\cdot|s)\right)\right], \tag{72}$$

which completes the proof. ■

APPENDIX C SUPPORTING LEMMAS

C.1 BELLMAN CONSISTENCY EQUATION OF MAXENT RL

Lemma C.1 (Contraction of Soft Value Iteration) *From* (49) *and* (6)*, the soft value iteration operator $\mathcal{T}$ defined as*

$$\mathcal{T}Q(s,a) := r(s,a) + \gamma\alpha\mathbb{E}_{s'}\left[\log\sum_{a'}\exp\left(Q(s',a')/\alpha\right)\right] \tag{73}$$

is a contraction.

Proof A similar proof appears in Haarnoja (2018), we provide the proof for completeness. To see (73) is a contraction, for each $(s,a) \in \mathcal{S} \times \mathcal{A}$, we have

$$\begin{aligned}
&\mathcal{T}Q_1(s,a) = r(s,a) + \gamma\alpha \log \sum_{a'} \exp\left(\frac{Q_1(s,a)}{\alpha}\right) \\
\leq & r(s,a) + \gamma\alpha \log \sum_{a'} \exp\left(\frac{Q_2(s,a) + \|Q_1 - Q_2\|_\infty}{\alpha}\right) \\
\leq & r(s,a) + \gamma\alpha \log \left\{\exp\left(\frac{\|Q_1 - Q_2\|_\infty}{\alpha}\right) \sum_{a'} \exp\left(\frac{Q_2(s,a)}{\alpha}\right)\right\} \\
= & \gamma \|Q_1 - Q_2\|_\infty + r(s,a) + \gamma\alpha \log \sum_{a'} \exp\left(\frac{Q_2(s,a)}{\alpha}\right) = \gamma \|Q_1 - Q_2\|_\infty + \mathcal{T}Q_2(s,a),
\end{aligned} \tag{74}$$

which implies $\mathcal{T}Q_1(s,a) - \mathcal{T}Q_2(s,a) \leq \gamma \|Q_1 - Q_2\|_\infty$. Similarly, we also have $\mathcal{T}Q_2(s,a) - \mathcal{T}Q_1(s,a) \leq \gamma \|Q_1 - Q_2\|_\infty$, hence we conclude that

$$|Q_1(s,a) - Q_2(s,a)| \leq \gamma \|\mathcal{T}Q_1 - \mathcal{T}Q_2\|_\infty, \ \forall (s,a) \in \mathcal{S} \times \mathcal{A}, \tag{75}$$

which implies $\|Q_1 - Q_2\|_\infty \leq \gamma \|\mathcal{T}Q_1 - \mathcal{T}Q_2\|_\infty$. Hence $\mathcal{T}$ is a γ-contraction and the optimal policy $\pi^\star$ of it is unique. ■

C.2 Constant Minimum Policy Probability

Lemma C.2 (Lemma 16 of Mei et al. (2020)) *Using the policy gradient method (Algorithm 3) with an initial distribution ρ such that $\rho(s) > 0, \forall S$, we have*

$$c := \inf_{t \geq 1} \min_{s,a} \pi_{\theta_t}(a|s) > 0 \tag{76}$$

is a constant that does not dependent on t.

Remark C.3 (State Space Dependency of constant c) *Note that assuming c is independent of S is quite strong and does not generally hold as suggested by Li et al. (2021). Still, if one replace the constant c with other S dependent function $f(S)$, one still can apply a similar proof technique for Theorem 4.1 to show that* ROLLIN *reduces the iteration complexity, and the final iteration complexity bound in Theorem 4.1 will include an additional $f(S)$.*

Appendix D Supporting Algorithms

Algorithm 3 PG for α-MaxEnt RL (Algorithm 1 in Mei et al. (2020))

Input: $\rho, \theta_0, \eta > 0$.
for $t = 0, \ldots, T$ **do**
  $\theta_{t+1} \leftarrow \theta_t + \eta \cdot \frac{\partial V^{\pi_{\theta_t}}(\rho)}{\partial \theta_t}$
end for

Algorithm 4 Two-Phase SPG for α-MaxEnt RL (Algorithm 5.1 in Ding et al. (2021))

Input: $\rho, \theta_0, \alpha, B_1, B_2, T_1, T, \{\eta_t\}_{t=0}^T$
for $t = 0, 1, \ldots, T$ **do**
  if $t \leq T_1$ **then**
    $B = B_1$ ▷ Phase 1
  else
    $B = B_2$ ▷ Phase 2
  end if
  Run random horizon SPG with $\rho, \alpha, \theta_t, B, t, \eta_t$ ▷ Algorithm 5
end for

Algorithm 5 Random-horizon SPG for α-MaxEnt RL Update (Algorithm 3.2 in Ding et al. (2021))

1: **Input:** $\rho, \alpha, \theta_0, B, t, \eta_t$
2: **for** $i = 1, 2, ..., B$ **do**
3: $\quad s^i_{H_t}, a^i_{H_t} \leftarrow \text{SamSA}(\rho, \theta_t, \gamma)$ ▷ Algorithm 6
4: $\quad \hat{Q}^{\pi_{\theta_t}, i} \leftarrow \text{EstEntQ}(s^i_{H_t}, a^i_{H_t}, \theta_t, \gamma, \alpha)$ ▷ Algorithm 7
5: **end for**
6: $\theta_{t+1} \leftarrow \theta_t + \frac{\eta_t}{(1-\gamma)B} \sum_{i=1}^{B} \left[\nabla_\theta \log \pi_{\theta_t}(a^i_{H_t}|s^i_{H_t}) \left(\hat{Q}^{\pi_{\theta_t}, i} - \alpha \log \pi_{\theta_t} \right) (a^i_{H_t}|s^i_{H_t}) \right]$

Remark D.1 *Lemma 3.4 in Ding et al. (2021) implies that the estimator*

$$\frac{1}{(1-\gamma)} \left[\nabla_\theta \log \pi_{\theta_t}(a^i_{H_t}|s^i_{H_t}) \left(\hat{Q}^{\pi_{\theta_t}, i} - \alpha \log \pi_{\theta_t} \right) (a^i_{H_t}|s^i_{H_t}) \right] \tag{77}$$

in line 6 of Algorithm 6 is an unbiased estimator of the gradient $\nabla_\theta V^{\pi_\theta}(\rho)$.

Algorithm 6 SamSA: Sample s, a for SPG (Algorithm 8.1 in Ding et al. (2021))

1: **Input:** ρ, θ, γ
2: Draw $H \sim \text{Geom}(1-\gamma)$ ▷ $\text{Geom}(1-\gamma)$ geometric distribution with parameter $1-\gamma$
3: Draw $s_0 \sim \rho, a_0 \sim \pi_\theta(\cdot|s_0)$
4: **for** $h = 1, 2, \ldots, H-1$ **do**
5: $\quad$ Draw $s_{h+1} \sim \mathbb{P}(\cdot|s_h, a_h), a_{h+1} \sim \pi_{\theta_t}(\cdot|s_{h+1})$
6: **end for**
7: **Output:** s_H, a_H

Algorithm 7 EstEntQ: Unbiased Estimation of MaxEnt Q (Algorithm 8.2 in Ding et al. (2021))

1: **Input:** $s, a, \theta, \gamma, \alpha$
2: Initialize $s_0 \leftarrow s, a_0 \leftarrow a, \hat{Q} \leftarrow r(s_0, a_0)$
3: Draw $H \sim \text{Geom}(1-\gamma)$
4: **for** $h = 0, 1, \ldots, H-1$ **do**
5: $\quad s_{h+1} \sim \mathbb{P}(\cdot|s_h, a_h), a_{h+1} \sim \pi_\theta(\cdot|s_{h+1})$
6: $\quad \hat{Q} \leftarrow \hat{Q} + \gamma^{h+1}/2 \left[r(s_{h+1}, a_{h+1}) - \alpha \log \pi_\theta(a_{h+1}|s_{h+1}) \right]$
7: **end for**
8: **Output:** $\hat{Q}$

APPENDIX E EXPERIMENTAL DETAILS

For our `antmaze-umaze` experiments, We use a sparse reward function where the reward is 0 when the distance D between the ant and the goal is greater than 0.5 and $r = \exp(-5D)$ when the distance is smaller than or equal to 0.5. The performance threshold is set to be $R = 200$. Exceeding such threshold means that the ant stays on top of the desired location for at least 200 out of 500 steps, where 500 is the maximum episode length of the `antmaze-umaze` environment. We use the average return of the last 10 episodes and compare it to the performance threshold R. For both of the SAC agents, we use the same set of hyperparameters shown in Table 5. We use the SAC implementation from `https://github.com/ikostrikov/jaxrl`. See Algorithm 8, for a more detailed pseudocode.

Initial Temperature		1.0
Target Update Rate	update rate of target networks	0.005
Learning Rate	learning rate for the Adam optimizer	0.0003
Discount Factor		0.99
Batch Size		256
Warmup Period	number of steps of initial random exploration (random actions)	10000
Network Size		$(256, 256)$

Table 5 Hyperparameters used for the SAC algorithm (Haarnoja et al., 2018)

Algorithm 8 Practical Implementation of ROLLIN

Input: $\{\omega_k\}_{k=0}^{K}$: input curriculum, ρ: initial state distribution, R: near-optimal threshold, β: roll-in ratio, discount factor γ.
Initialize $\mathcal{D} \leftarrow \emptyset, \mathcal{D}_{\text{exp}} \leftarrow \emptyset, k \leftarrow 0$, and two off-policy RL agents π_{main} and π_{exp}.
for each environment step **do**
  if episode terminating or beginning of training **then**
    if average return of the last 10 episodes under context ω_k is greater than R **then**
      $k \leftarrow k + 1, \mathcal{D}_{\text{exp}} \leftarrow \emptyset$
      Re-initialize the exploration agent π_{exp}
    end if
    Start a new episode under context ω_k with $s_0 \sim \rho$, $t \leftarrow 0$
    if $k > 0$ and with probability of β **then**
      enable **Rollin** for the current episode.
    else
      disable **Rollin** for the current episode.
    end if
  end if
  if Rollin is enabled for the current episode **then**
    if Rollin is stopped for the current episode **then**
      $a_t \sim \pi_{\text{exp}}(a_t|s_t, \omega_k)$
    else
      $a_t \sim \pi_{\text{main}}(a_t|s_t, \omega_{k-1})$
      with probability of $1 - \gamma$, stop **Rollin** for the current episode
    end if
  else
    $a_t \sim \pi_{\text{main}}(a_t|s_t, \omega_k)$
  end if
  take action a_t in the environment and receives s_{t+1} and $r_t = r_{\omega_k}(s_t, a_t)$
  add (s_t, a_t, s_{t+1}, r_t) in replay buffer $\mathcal{D}$
  if Rollin is disabled for the current episode **then**
    update π_{main} using $\mathcal{D}_{\text{main}}$.
  end if
  if π_{exp} was used to produce a_t **then**
    add (s_t, a_t, s_{t+1}, r_t) in replay buffer $\mathcal{D}_{\text{exp}}$
    update π_{exp} using $\mathcal{D}_{\text{exp}}$.
  end if
  $t \leftarrow t + 1$
end for
Output: π_{main}

APPENDIX F ADDITIONAL LEARNING CURVES

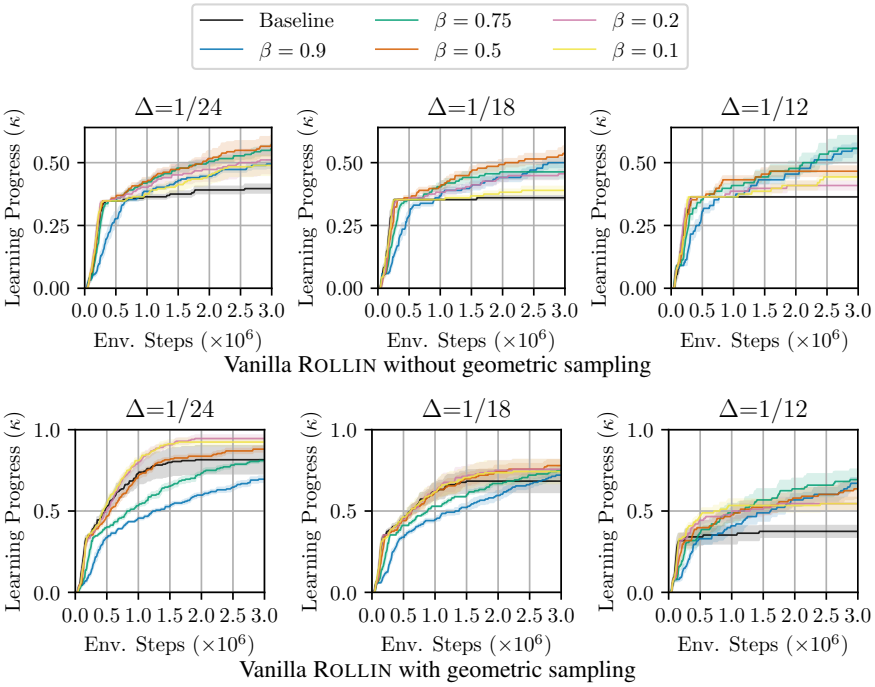

Figure 3 Vanilla Goal reaching. Accelerating learning on `antmaze-umaze` with ROLLIN on an oracle curriculum in Figure 2. The confidence interval represents the standard error computed over 8 random seeds.

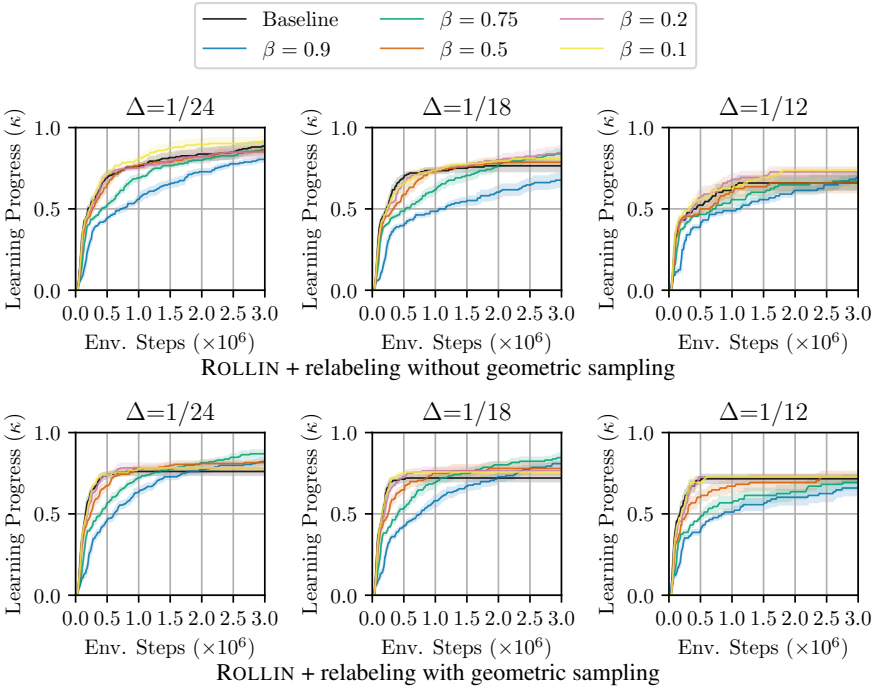

Figure 4 Goal relabeling. Accelerating learning on `antmaze-umaze` with ROLLIN on an oracle curriculum in Figure 2. The confidence interval represents the standard error computed over 8 random seeds.

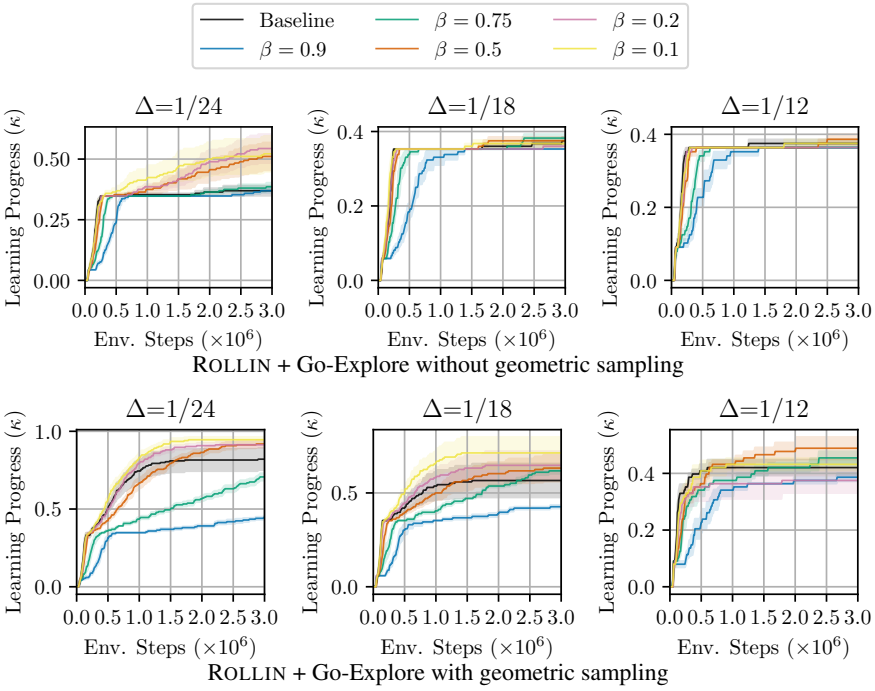

Figure 5 Go-Explore (exploration noise = 0.1). Accelerating learning on `antmaze-umaze` with ROLLIN on an oracle curriculum in Figure 2. The confidence interval represents the standard error computed over 8 random seeds.

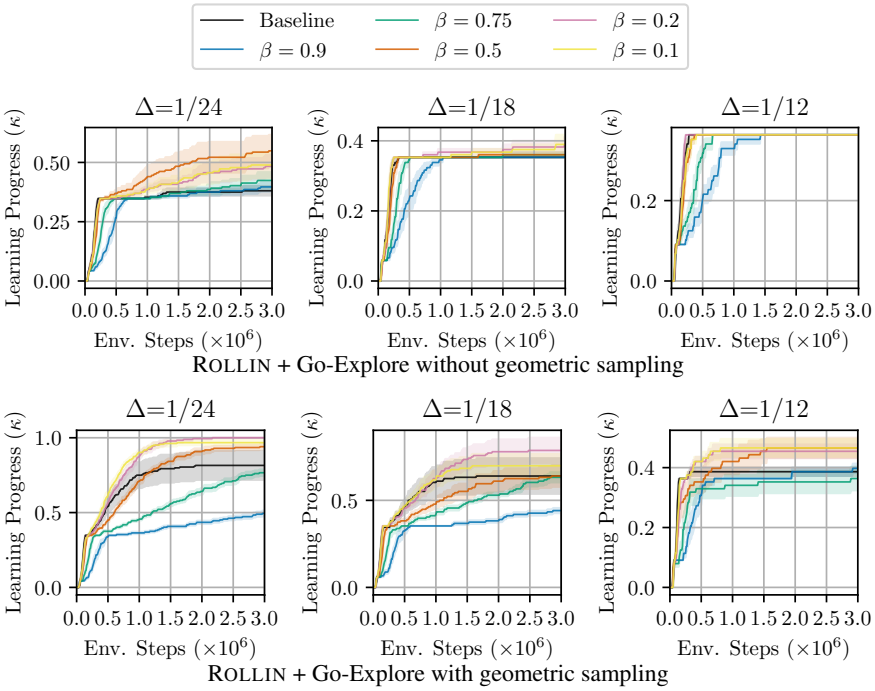

Figure 6 Go-Explore (exploration noise = 0.25). Accelerating learning on `antmaze-umaze` with ROLLIN on an oracle curriculum in Figure 2. The confidence interval represents the standard error computed over 8 random seeds.

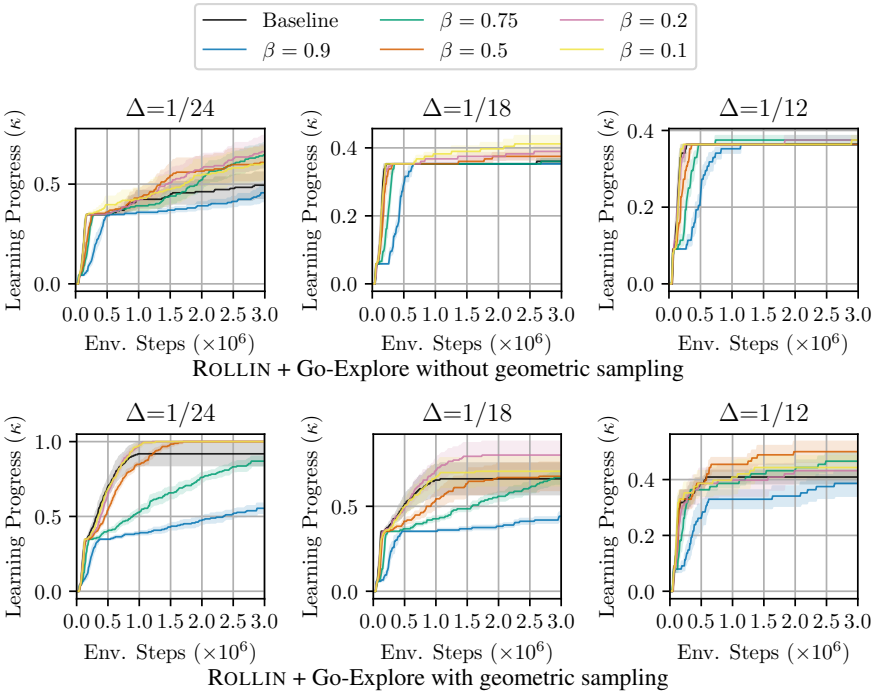

Figure 7 Go-Explore (exploration noise = 0.5). Accelerating learning on `antmaze-umaze` with ROLLIN on an oracle curriculum in Figure 2. The confidence interval represents the standard error computed over 8 random seeds.