PRUDEX-Compass: Towards Systematic Evaluation of Reinforcement Learning in Financial Markets

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Abstract

The financial markets, which involve more than $90 trillion market capitals, attract the attention of innumerable investors around the world. Recently, reinforcement learning in financial markets (FinRL) has emerged as a promising direction to train agents for making profitable investment decisions. However, the evaluation of most FinRL methods only focuses on profit-related measures and ignores many critical axes, which are far from satisfactory for financial practitioners to deploy these methods into real-world financial markets. Therefore, we introduce **PRUDEX-Compass**, which has 6 axes, i.e., Profitability, Risk-control, Universality, Diversity, rEliability, and eXplainability, with a total of 17 measures for a systematic evaluation. Specifically, i) we propose AlphaMix+ as a strong FinRL baseline, which leverages mixture-of-experts (MoE) and risk-sensitive approaches to make diversified risk-aware investment decisions, ii) we evaluate 8 FinRL methods in 4 long-term real-world datasets of influential financial markets to demonstrate the usage of our PRUDEX-Compass, iii) PRUDEX-Compass[1] together with 4 real-world datasets, standard implementation of 8 FinRL methods and a portfolio management environment is released as public resources to facilitate the design and comparison of new FinRL methods. We hope that PRUDEX-Compass can not only shed light on future FinRL research to prevent untrustworthy results from stagnating FinRL into successful industry deployment but also provide a new challenging algorithm evaluation scenario for the reinforcement learning (RL) community.

1 Introduction

Quantitative trading (QT) is a type of market strategy that relies on mathematical and statistical models to automatically identify investment opportunities (Chan, 2021). With the advent of the AI age, it becomes more popular and accounts for more than 70% and 40% trading volumes, in developed markets (e.g., US) and developing markets (e.g., China), respectively (Karpoff, 1987). How to make profitable investment decisions against the various uncertainties in QT becomes one of the main challenges for financial practitioners (An et al., 2022). Among the various machine learning methods, such as deep learning (Xu & Cohen, 2018; Sawhney et al., 2021) and boosting decision trees (Ke et al., 2017), deep reinforcement learning (DRL) is attracting increasing attention from both academia and financial industries (Sun et al., 2021a) due to its stellar performance on solving complex sequential problems such as Go (Silver et al., 2017), StarCraft II (Vinyals et al., 2019), nuclear fusion (Degrave et al., 2022) and matrix mulplication (Fawzi et al., 2022).

DRL has achieved significant success in various QT tasks. Specifically, FDDR (Deng et al., 2016) and iRDPG (Liu et al., 2020b) are designed to learn financial trading signals and micmic behaviors of professional traders for algorithmic trading, respectively. For portfolio management, DRL methods are proposed to account for the impact of market risk (Wang et al., 2021b) and the commission fee (Wang et al., 2021a). A PPO-based framework (Lin & Beling, 2020) is proposed for order execution and a policy distillation mechanism is added to bridge the gap between imperfect market states and optimal execution actions (Fang et al., 2021). For market making, DRL methods are introduced from both game-theoretic (Spooner et al., 2018) and adversarial learning (Spooner & Savani, 2020) perspectives as an adaptation of traditional mathematical models.

[1] https://anonymous.4open.science/r/PRUDEX-Compass-68D5

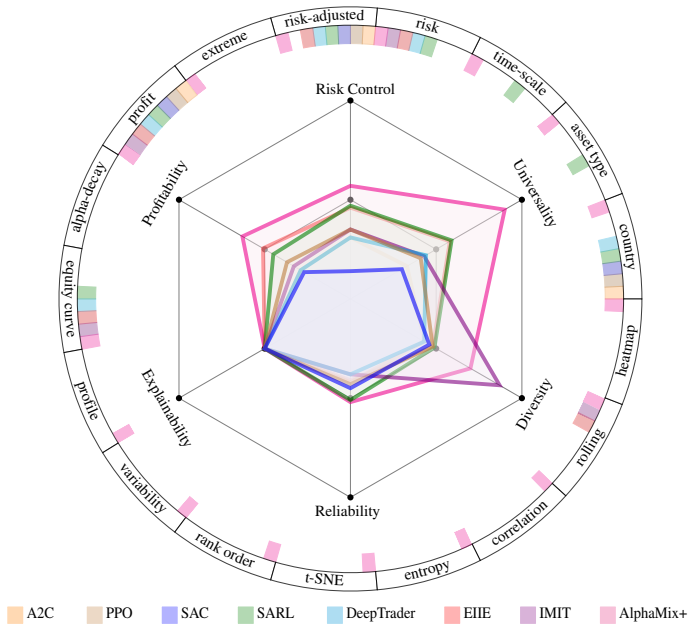


Figure 1: An illustration of our FinRL methods evaluation benchmark PRUDEX-Compass. The inner star plot provides a visual indication of the strength of different FinRL methods in terms of six axes. A mark on the star plot's inner circle suggests the market average. The outer level of the PRUDEX-Compass specifies which particular measures have been evaluated in practice to show the status of evaluation. We fill the compass with AlphaMix+ and 7 widely-used FinRL methods to provide an intuitive example. In general, AlphaMix+ gets the best performance (largest area in the inner level) with a comprehensive evaluation (much more markers in the outer level).

However, the evaluation of existing FinRL methods (Sun et al., 2022; Wang et al., 2021a; Fang et al., 2021) only focuses on profit-related measures, which ignores several critical axes, such as risk-control and reliability. In addition to profitability, financial practitioners care about many other aspects of FinRL methods, i.e., how much risk I need to take for per unit of profit; how FinRL algorithms behave when the market status changes. In practice, due to the low signal-to-noise and distribution shift nature of financial markets (Malkiel, 2003), FinRL methods with only high profit on backtesting are very likely to overfit on historical data and fail in real-world deployment (De Prado, 2018). As David Shaw (founder of a world-class hedge fund) said [2], he will never trade with a method that does not prove itself through a systematic evaluation. Therefore, a benchmark for the systematic evaluation of FinRL methods is urgently needed.

In this paper, we first propose AlphaMix+, a DRL method composed of diversified mixture-of-experts and risk-aware Bellman backup, as a strong FinRL baseline to mimic the bottom-up hierarchical trading strategy design workflow in real-world companies (Khorana et al., 2007). Then, we introduce PRUDEX-Compass, which has 6 axes with a total of 17 measures for systematic evaluation of FinRL methods. In addition, we evaluate 8 widely used FinRL methods together with AlphaMix+ on 4 long-term real-world datasets spanning over 15 years on portfolio management to demonstrate the usage of PRUDEX-Compass. Accompanied with an open-source library[1] of datasets, baseline implementation, RL environment and evaluation toolkits, we call for a change in how we evaluate FinRL methods to facilitate the industry deployment of FinRL methods. Moreover, PRUDEX-Compass also provide the RL community new algorithm evaluation scenarios to test the effectiveness of novel RL algorithms in the ever-changing financial markets.

2 Preliminaries

2.1 Quantitative Trading Preliminaries

Quantitative trading involves various trading tasks with different investment goals and risk tolerance (Sun et al., 2021a). We focus on portfolio management (PM), one of the most foundational and well-studied QT tasks (Wang et al., 2021a; Ye et al., 2020), in this work as a usage example. As a unified benchmark, evaluation metrics proposed in PRUDEX-Compass can be easily adopted to all QT tasks, For portfolio management, investors hold a number of financial assets, e.g., stocks, bonds, as well as cash, and reallocate the proportion of each asset periodically to maximize future profit.

[2] https://en.wikipedia.org/wiki/David_E._Shaw

OHLCV is a type of bar chart directly obtained from the financial market. OHLCV vector at time t is denoted as $\mathbf{x}_t = (p_t^o, p_t^h, p_t^l, p_t^c, v_t)$, where p_t^o is the open price, p_t^h is the high price, p_t^l is the low price, p_t^c is the close price and v_t is the volume.

Technical Indicator indicate features calculated by a formulaic combination of the original OHLCV to uncover the underlying pattern of the financial market. We denote the technical indicator vector at time t: $\mathbf{y}_t = \bigcup_k y_t^k$, where $y_t^k = f_k(\mathbf{x}_{t-h}, ..., \mathbf{x}_t, \theta^k)$, θ^k is the parameter of technical indicator k.

Portfolio is the proportion of capitals allocated to each asset that can be represented as a vector:

$$\mathbf{w_t} = [w_t^0, w_t^1, ..., w_t^M] \in R^{M+1} \quad and \quad \sum_{i=0}^{M} w_t^i = 1 \tag{1}$$

where $M+1$ is the number of portfolio's constituents, including cash and M financial assets. w_t^i represents the ratio of the total portfolio value invested at time t on asset i and w_t^0 represents cash.

Asset Price refers to the vector of close price for each financial asset defined as $\mathbf{p_t} = [p_t^0, p_t^1, ..., p_t^M]$, where p_t^i is the close price of asset i at time t. Note that the price of cash p_t^0 is a constant.

Portfolio Value v_{t+1} at time $t+1$ is defined based on the asset price change and portfolio weight as:

$$v_{t+1} = v_t \sum_{i=0}^{M} \frac{w_t^i \ p_{t+1}^i}{p_t^i} \tag{2}$$

The objective of portfolio management is to maximize the final portfolio value given a long time horizon by dynamically tuning the portfolio weight at each time step.

2.2 FinRL Preliminaries

RL Basics. We consider a standard RL scenario in which an agent (investor) interacts with an environment (the financial market) in discrete time. Formally, we introduce MDP, which is defined by the tuple: MDP $= (\mathcal{S}, \mathcal{A}, T, R, \gamma, T)$. Specifically, $\mathcal{S}$ is a finite set of states. $\mathcal{A}$ is a finite set of actions. $T : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \longrightarrow [0, 1]$ is a state transaction function, which consists of a set of conditional transition probabilities between states. $R : \mathcal{S} \times \mathcal{A} \longrightarrow \mathcal{R}$ is the reward function, where $\mathcal{R}$ is a continuous set of possible rewards and R indicates the immediate reward of taking an action in a state. $\gamma \in [0, 1)$ is the discount factor and T is a time horizon. A (stationary) policy $\pi_\theta : \mathcal{S} \times \mathcal{A} \longrightarrow [0, 1]$, parameterized by θ, assigns each state $s \in \mathcal{S}$ a distribution over actions, where $a \in \mathcal{A}$ has probability $\pi(a|s)$. A Q-value function gives expected accumulated reward when executing action a_t in state s_t and following policy π in the future, which is $Q^\pi(s_t, a_t) = \mathbb{E}_{(s_{t+1},...,\pi)} \left[\sum_{i=t}^{T} \gamma^i r(s_i, a_i)\right]$. The objective of the agent is to learn an optimal policy: $\pi_{\theta^*} = argmax_{\pi_\theta} \mathbb{E}_{\pi_\theta} \left[\sum_{i=t}^{T} \gamma^i r_{t+i} \mid s_t = s\right]$. For portfolio management at time t, the state $s_t \in \mathcal{S}$ is historical sequence of technical indicator vectors $(\mathbf{y_{t-h}}, ..., \mathbf{y_t})$ with length $h+1$. Action space is the portfolio weight $\mathbf{w}_t$. Reward is the change of portfolio value: $r_t = v_{t+1} - v_t$.

Soft Actor-Critic. We introduce SAC (Haarnoja et al., 2018), which is the base model of many popular FinRL methods (Yuan et al., 2020; Wen et al., 2021). SAC is an off-policy actor-critic method based on the maximum entropy RL framework (Ziebart, 2010), which maximizes a weight objective of the reward and the policy entropy, to encourage robustness to noise and exploration. For parameter updating, SAC alternates between a soft policy evaluation and a soft policy improvement. At the soft policy evaluation step, a soft Q-function, which is modeled as a neural network with parameters θ, is updated by minimizing the following soft Bellman residual:

$$\mathcal{L}_{critic}^{\text{SAC}}(\phi) = \mathbb{E}_{\tau_t \sim \mathcal{D}}[(Q_\theta(s_t, a_t) - r_t - \gamma \bar{V}(s_{t+1}))^2] \tag{3}$$

$$where \quad \bar{V}(s_t) = \mathbb{E}_{a_t \sim \pi_\phi}[Q_{\bar{\theta}}(s_t, a_t) - \alpha \log \pi_\phi(a_t \mid s_t)] \tag{4}$$

where $\tau_t = (s_t, a_t, r_t, s_{t+1})$ is a transition, $\mathcal{D}$ is a replay buffer, $\bar{\theta}$ are the delayed parameters, and α is a temperature parameter. At the soft policy improvement step, the policy π with its parameter θ is updated by minimizing the following objective:

$$\mathcal{L}_{actor}^{\text{SAC}}(\theta) = \mathbb{E}_{s_t \sim \mathcal{D}}[\mathbb{E}_{a_t \sim \pi_\phi}[\alpha \log \pi_\phi(a_t \mid s_t) - Q_\theta(s_t, a_t)]] \tag{5}$$

To handle continuous action spaces, the policy is modeled as a Gaussian with mean and covariance given by neural networks. In addition to SAC, A2C (Mnih et al., 2016), a popular actor-critic RL method, shows stellar performance in algorithmic trading (Zhang et al., 2020). The simple and efficient policy gradient method PPO (Schulman et al., 2017) performs well in capturing trading opportunities for order execution (Lin & Beling, 2020). EIIE (Jiang et al., 2017) and Investor-Imitator (IMIT) (Ding et al., 2018) are two pioneering works that apply deep RL for quantitative trading. Furthermore, SARL (Ye et al., 2020) and Deeptrader (Wang et al., 2021b) are proposed with augmented market embedding to take market risk into account for portfolio management.

3 AlphaMix+: A Strong Baseline

In this section, we propose a novel RL algorithm based on ensemble learning that achieves good performance in terms of different evaluation axes (see Section 5 for details). Considering the limitation of existing FinRL methods, the major one is that investment decisions are made by a single agent with high potential risk. The success of real-world trading firms relies on an efficient bottom-up hierarchical workflow with risk management as illustrated in Figure 2). First, multiple experts conduct data analysis and build models independently based on personal trading style and risk tolerance. Later on, a senior portfolio manager summarizes their results, manage risk and makes final investment decisions.

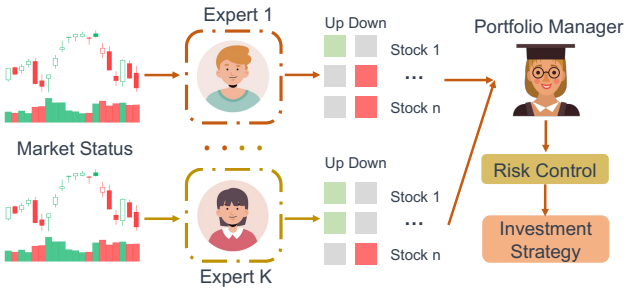


Figure 2: Workflow of real-world trading firms.

Inspired by it, we propose AlphaMix+, a universal DRL framework with diversified risk-aware mixture-of-experts (MoE) to mimic this efficient workflow. In principle, AlphaMix+ can be used in conjunction with most modern off-policy RL algorithms for any QT task. As SAC (Haarnoja et al., 2018) is a sample-efficiency algorithm in QT (Yuan et al., 2020), we pick it as the base model of AlphaMix+ here for exposition. An overview of AlphaMix+ is shown in Figure 3.

Risk-aware Bellman backup. We consider a trading firm with N trading experts, i.e., $\{Q_{\theta_i}, \pi_{\phi_i}\}_{i=1}^N$, where θ_i and ϕ_i denote the parameters of the i-th soft Q-function and policy. For conventional Q-learning based on the Bellman backup in Eq. (3), one major issue is the negative impact of error propagation that induces the "noise" to the learning "signal" (true Q-value) of the current Q-function (Kumar et al., 2020). This issue is more severe in the financial markets with a low signal-to-noise ratio and can cause unstable convergence (Kumar et al., 2020). To mitigate this issue, for each agent i, we consider a risk-aware Bellman backup (Lee et al., 2021b) as follows:

$$\mathcal{L}_{WQ}(\tau_t, \theta_i) = w(s_{t+1}, a_{t+1})(Q_{\theta_i}(s_t, a_t) - r_t - \gamma \bar{V}(s_{t+1}))^2 \tag{6}$$

where $\tau_t = (s_t, a_t, r_t, s_{t+1})$ is a transition, $a_{t+1} \sim \pi_{\phi_i}(a \mid s_t)$, and $w(s, a)$ is a confidence weight based on the ensemble of target Q-functions:

$$w(s, a) = \sigma(-\bar{Q}_{std}(s, a) * T) + k \tag{7}$$

where $\bar{Q}_{std}(s, a)$ is the empirical standard deviation of all target Q-functions $\{Q_{\bar{\theta}_i}\}_{i=1}^N$ and $T > 0$ is a temperature parameter (Hinton et al., 2015) to adapt the scale of $\bar{Q}_{std}(s, a)$. σ is the sigmoid function and $k > 0$ is used to control the value range of confidence weight. Note that the confidence weight is bounded in $[0.5, 0.5 + k]$ since $\bar{Q}_{std}(s, a)$ is always positive. The objective $\mathcal{L}_{WQ}$ down weights the sample transitions with inconsistent trading suggestions from different experts (high variance across target Q-functions), resulting in a loss function for the Q-updates with better risk management.

Diversified Experts. Bootstrap with random initialization (Osband et al., 2016) is applied to encourage the diversity between trading experts through two ideas: First, we initialize the model parameters of all trading experts with random parameter values for inducing an initial diversity in the models following (Lakshminarayanan et al., 2017; Wenzel et al., 2020). Second, we apply different

samples to train each agent based on similar idea in BatchEnsemble (Wen et al., 2019). Specifically, for each SAC agent i in each time step t, we draw the binary masks $m_{t,i}$ from the Bernoulli distribution with parameter $\beta \in (0, 1]$, and store them in the replay buffer. Then, when updating the model parameters of agents, we multiply the bootstrap mask to each objective function, such as: $m_{t,i}\mathcal{L}_\pi(s_t, \phi_i)$ and $m_{t,i}\mathcal{L}_{WQ}(\tau_t, \theta_i)$, respectively. This encourages each expert to think individually with diversified strategies. We find it sufficient for AlphaMix+ to have desired diversity (experiments in Section 5.7) with the two simple ideas. Other tricks such as a KL divergence (Yu et al., 2013) loss term is not further incorporated to keep simplicity. For inference, we propose a simple approximation scheme, which first generates N candidate action set from ensemble policies $\{\pi_{\phi_i}{}_{i=1}^{N}\}$, and then chooses the actions that maximize the UCB (Chen et al., 2017). We approximate the maximum posterior action by averaging the mean of Gaussian distributions modeled by each ensemble policy.

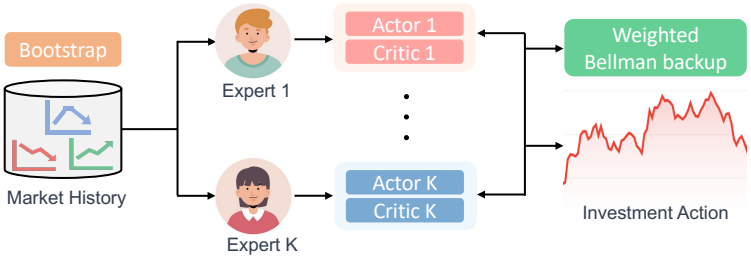


Figure 3: An illustration of AlphaMix+, a universal DRL framework with diversified risk-aware mixture-of-experts (MoE).

4 PRUDEX-Compass: Systematic Evaluation of FinRL

To provide a clear exposition of FinRL evaluation, we introduce PRUDEX-Compass to provide an intuitive visual means to give readers a sense of comparability and positioning of FinRL methods. PRUDEX-Compass is composed of two central elements: i) the axis-level (inner), which specifies the different axes considered for FinRL evaluation and ii) measure-level (outer), which specifies the measures used for benchmarking FinRL methods. Figuratively speaking, the axis-level maps out the relative strength of FinRL methods in terms of each axis, whereas the measure-level provides a compact way to visually assess which set-up and evaluation measures are practically reported to point out how comprehensive the evaluation are for FinRL algorithms. To provide a practical basis, we have directly filled the exemplary compass visualization in Figure 1.

4.1 Axes of PRUDEX-Compass

The axis-level of PRUDEX-Compass indicates the relative strength of FinRL methods in terms of 6 critical evaluation perspectives. We choose the design of the axis-level compass as a star diagram enhanced by contours with a mark in the inner circle to indicate the market average performance. The advantages of this design are three-fold: i) it provides an ideal visual representation when comparing plot elements (Fuchs et al., 2014), ii) it allows human perceivers to quickly learn more facts by fast visual search (Elder & Zucker, 1993), and iii) the geometric region boundaries in the star plot have high priority in perception (Elder & Zucker, 1998). To calculate the values of each axis, we generally normalize the numeric values of original experiment results to an integer score $t \in [0, 100]$, where $t = 50$ represents the market average performance (see Appendix A.2 for details). We introduce the 6 axis-level elements of PRUDEX-Compass as follows.

Profitability. Aligned with the key objective of QT, profitability focuses on the evaluation of FinRL methods' ability to gain market capital. Besides pure return, it also measures how stable (Franco & Leah, 1997) and persistent (Gârleanu & Pedersen, 2013) FinRL methods are to achieve high profit.

Risk-Control. Due to the well-known tradeoff between profit and risk in finance (Brink & McCarl, 1978), financial practitioners take great efforts on the assessment and control of both systematic risk and idiosyncratic risk (Goyal & Santa, 2003), which is also of vital importance in FinRL evaluation.

Universality. The financial markets is a complex ecosystem that involves innumerable assets, countries, time-scale and trading styles. Universality tries to evaluate FinRL's ability to achieve decent performance in various QT scenarios. Designing FinRL methods with better universality (Fang et al., 2021) is in line with popular ML topics such as transfer learning (Pan & Yang, 2009).

Diversity. In finance, diversification refers to the process of allocating capitals in a way that reduces the exposure to any one particular asset or risk. As Markowitz (Nobel Laureate in Economics) said (Tu & Zhou, 2011), diversity is the only free lunch in investing that plays an indispensable role on enhancing profitability and risk-control. In RL community, diversity is widely used to encourage exploration (Parker et al., 2020). This axis of PRUDEX-Compass tends to address the lack of diversity evaluation of FinRL methods.

Reliability. RL methods tend to be highly variable in performance and considerably sensitive to a range of different factors such as random seeds (Henderson et al., 2018). This variability hinders a reliable method and can be costly or even dangerous for high-stake applications such as QT. This axis introduces techniques on RL reliability evaluation (Agarwal et al., 2021) with a focus on QT.

Explainability. Psychologically speaking, *if the users do not trust a model, they will not use it* (Ribeiro et al., 2016). Explainability generally refers to any technique that helps users or developers of models understand why models behave the way they do. In FinRL, it can come in the form that tells traders which model is effective under what market conditions or why one trading action is mistaken and how to fix it. Rigorous regulatory requirements in financial markets further enhance its importance for model debugging, monitoring and audit (Bhatt et al., 2020).

4.2 Measures of PRUDEX-Compass

As the inner star plot contextualizes macroscopic axes of FinRL evaluation, the outer measure-level places emphasis on detailed evaluation set-up and metrics. In essence, a mark on the measure-level indicates that a method practically reports corresponding measures in its empirical investigation, where more marks indicate a more comprehensive evaluation. We list the 17 measures on the outer level of the PRUDEX-Compass in Table 2 with brief descriptions.

FinRL Explainability. DSP (Landajuela et al., 2021) is proposed to discover symbolic policy with expert knowledge. Differentiable decision trees are incorporated into RL for better explainability (Silva et al., 2020). Another line of works tries to discover interpretable features with techniques such as self-supervised learning (Shi et al., 2020) and adversarial learning (Gupta et al., 2020). Due to the lack of FinRL methods with solid design of explainability, we leave this axis as one further direction.

Table 1: Dataset statistics

Dataset	Market	Freq	Number	Days	From	To	Source
US Stock	US	1d	29	2517	12/01/03	21/12/31	Yahoo
China Stock	China	1h	47	1036	16/06/01	20/09/01	Yahoo
Crypto	-	1d	9	2014	16/01/01	21/07/06	Kaggle
FX	-	1d	22	5015	00/01/03	19/12/31	Kaggle

5 Demonstrative Usages of PRUDEX-Compass

We conduct experiments on portfolio management, one fundamental QT task that dynamically allocates different proportions of capital in financial assets to achieve investment goals, on real-world datasets of 4 influential financial markets to demonstrate the usage of PRUDEX-Compass.

5.1 Experiment Setup

Datasets and Features. We collect real-world financial datasets spanning over 15 years of US stock, China stock, Cryptocurrency (Crpyto) and Foreign Exhange (FX) from Yahoo Finance and Kaggle. All raw data and related processing scripts are publicly available. We summarize statistics of the 4 datasets with further elaboration in Table 1. US Stock dataset contains 10-year historical prices of 29 influential

Table 3: Features to describe the financial markets

Features	Calculation Formula
$z_{open}, z_{high}, z_{low}$ z_{close}	$z_{open} = open_t/close_t - 1$ $z_{close} = close_t/close_{t-1} - 1$
$z_{d_5}, z_{d_10}, z_{d_15}$ $z_{d_20}, z_{d_25}, z_{d_30}$	$z_{d_5} = \frac{\sum_{i=0}^{4} close_{t-i}/5}{close_t} - 1$

Table 2: Brief summary of evaluation measures in outer level of PRUDEX-Compass: Profitability, Risk-control, Universality, Diversity, rEliability, and eXplainability (see Appendix A.1 for details).

Axes	Measures	Descriptions
P	Profit	A class of metrics to assess FinRL's ability to gain market capital.
	Alpha Decay	Loss in the investment decision making ability of FinRL methods over time due to distribution shift in financial markets (Pénasse, 2022).
	Equity Curve	A graphical representation of the value changes over time.
R	Risk	A class of metrics to assess the risk level of FinRL methods (Shiller, 1992).
	Risk-adjusted Profit	A class of metrics that calculate the normalized profit with regards to different kinds of risks, i.e., volatility and downside risk (Magdon & Atiya, 2004).
	Extreme Market	The relative performance of FinRL methods on extreme market condition during black swan events (Aven, 2013) such as war and covid-19.
U	Country	Financial market across both developed countries (e.g., US and Europe) and developing countries (e.g., China and India).
	Asset Type	Various financial asset types, i.e., stock, future, FX and Crypto
	Time-Scale	Both coarse-grained (e.g., day level) and fine-grained (e.g., second level) financial data to match different trading styles.
	Rolling Window	Using rolling time window to retrain or fine-tune FinRL methods and evaluate the performance on multiple test periods (De Prado, 2018).
D	t-SNE	A statistical visualization tool to map high-dimensional time-series data points into 2-D dimension (Vander & Hinton, 2008) to assess the data-level diversity.
	Entropy	Entropy-based metrics from information theory (Reza, 1994) to show the diversity of FinRL methods' trading behaviors.
	Correlation	Metrics that account the correlation (Kirchner & Zunckel, 2011) between financial assets to assess the diversity of FinRL methods.
	Diversity Heatmap	A visualization tool to demonstrate the diversity of investment decisions among different financial assets with heatmap (Harris et al., 2020)
E	Performance Profile	A visualization of FinRL methods' empirical score distribution (Dolan & Moré, 2002), which is easy to read with qualitative comparisons.
	Variability	The performance standard deviation across different random seeds and hyper-parameters (Henderson et al., 2018).
	Rank Comparison	A visualization toolkit to show the rank of FinRL methods across different metrics, which will not be dominated by extreme values (Agarwal et al., 2021).
X	-	We discuss current status and highlight promising further directions.

stocks with top unit price as a strong assessment of the market's overall health and tendencies. China Stock dataset contains 4-year historical prices of 47 influential stocks with top capitalization from the Shanghai exchange. Both US and China stock data is collected from Yahoo-Finance[3]. Crypto dataset contains 6-year historical prices of 9 influential virtual currency with top unit price and trading volume collected from Kaggle[4]. FX dataset contains 20-year historical prices of 22 most popular currency with top foreign exchange reserves for US dollars collected from Kaggle[5].

Furthermore, we generate 11 temporal features as shown in Table 3 to describe the stock markets following (Yoo et al., 2021). z_{open}, z_{high} and z_{low} represent the relative values of the open, high, low prices compared with

[3] https://github.com/yahoo-finance
[4] https://www.kaggle.com/datasets/sudalairajkumar/cryptocurrencypricehistory
[5] https://www.kaggle.com/datasets/brunotly/foreign-exchange-rates-per-dollar-20002019

the close price at current time step, respectively. z_{close} and z_{adj_close} represent the relative values of the closing and adjusted closing prices compared with time step $t-1$. z_{dk} represents a long-term moving average of the adjusted close prices during the last k time steps compared to the current close price.

Baselines and Training Setup. We conduct experiments with 7 representative FinRL methods including 3 classic RL methods: A2C (Mnih et al., 2016), PPO (Schulman et al., 2017), SAC (Haarnoja et al., 2018), 4 RL-based trading methods: EIIE (Jiang et al., 2017), Investor-Imitator (IMIT) (Ding et al., 2018), SARL (Ye et al., 2020) and DeepTrader (DT) (Wang et al., 2021b) and our AlphaMix+. Descriptions of baselines are available in Appendix C.1. Non-RL methods are not included as baselines for two reasons: i) In general, RL methods outperform different types of non-RL methods (Moskowitz et al., 2012; Ke et al., 2017; Xu & Cohen, 2018) in different trading tasks. ii) PRUDEX-Compass focuses on the evaluation of RL methods in financial markets. We perform all experiments on an RTX 3090 GPU with 5 fixed random seeds. We apply grid search for AlphaMix+ to find suitable hyperparameters (details in Appendix C.2), where it takes about 30 minutes to train each random seed. For other FinRL methods, we follow their default settings in public implementations (Liu et al., 2020a) to keep a fair comparison.

5.2 A General Impression with PRUDEX-Compass

As shown in Figure 1, we fill the PRUDEX-Compass based on the experimental results of the 8 FinRL methods. For axis-level, it directly illustrates the relative performance of each method in terms of 6 axes to provide a general impression. We normalized the score into 0 to 100 with 50 as the market average (details in Appendix A.2). For explainability, all methods are scored 50 as we leave it as future direction. AlphaMix+ performs best in all 5 remaining axes. Specifically, it outperforms other FinRL methods 53% and 43% in universality and diversity, respectively, which demonstrates the effectiveness of the weighted Bellman backup and diversified bootsrap initialization.

For measure-level, we give a mark if one measure is used in the evaluation of the FinRL methods, the goal here is to show how comprehensive the methods are evaluated. Together with all measures we proposed, AlphaMix+ clearly has a more rigorous evaluation, which makes the results more trustworthy. Arguably, PRUDEX-Compass provides a compact visualization to evaluate FinRL methods that is much better than only looking at a result table of different metrics especially when lots of FinRL methods are involved. In other words, the compass highlights the required subtleties, that may otherwise be challenging to extract from text descriptions, potentially be under-specified and prevent readers to misinterpret results based on result table display. With PRUDEX-Compass, users can flexibly pick suitable methods with regards to their personal interests. Conservative traders may prefer methods with a relative stable profit rate and low risk. Aggressive traders may pay more attention on profitability, as they are willing to take high risk to pursue extremely high profit. For international trading firms, they may have high expectation on universality and diversity.

5.3 Visualizing Financial Markets with t-SNE

Even though it is a wide consensus that different financial markets share different trading patterns (Campbell et al., 1998), there is a lack of visualization tool to demonstrate how different are these markets. To show data-level diversity of evaluation, we use t-SNE here to map all 4 datasets into a 2-D dimension with the 11 temporal features described in Table 3 as the input. To avoid overlapping in financial data, we pick a data point every 30 time step for each asset across 4 financial markets. In Figure 4, the US stock and FX datasets lie in the lower left and upper right corner, respectively, as a whole cluster, which is consistent with their status as relative mature and stable markets (Emenyonu & Gray, 1996). For China stock and the Crypto, data points are scattered with more outliers that demonstrate their essence as emerging and violate markets (De Santis et al., 1997). The t-SNE plot is useful to provide an intuitive expression on the data-level diversity of different markets while evaluating FinRL methods.

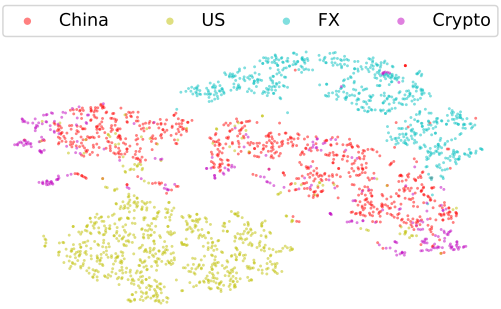


Figure 4: t-SNE market visualization.

5.4 PRIDE-Star for Evaluating Profitability, Risk-Control and Diversity

As the evaluation measures for Profitability, RIsk and DivErsity (PRIDE) are point-wise metrics with real number values, we use the PRIDE-star, which is a star plot to show the relative strength of 8 metrics including 1 profit metrics: total return (TR), 2 risk metrics: volatility (Vol) (Shiller, 1992) and maximum drawdown (MDD) (Magdon & Atiya, 2004), 3 risk-adjusted profit metrics: Sharpe ratio (SR) (Sharpe, 1998), Calmar ratio (SR) and Sortino ratio (SoR), and 2 diversity metrics: entropy (ENT) (Jost, 2006) and effect number of bets (ENB) (Kirchner & Zunckel, 2011). The mathematical definitions of these metrics are in Appendix A.1. We report the overall performance across the 4 financial markets of the 6 FinRL methods in Figure 5, where the inner circle represents market average. In general, AlphaMix+ performs best with the largest area in the PRIDE-Star plot. In addition, AlphaMix+ outperforms the second best by 25% in terms of ENT that shows the effectiveness of the boostrap with random initialization component in AlphaMix+.

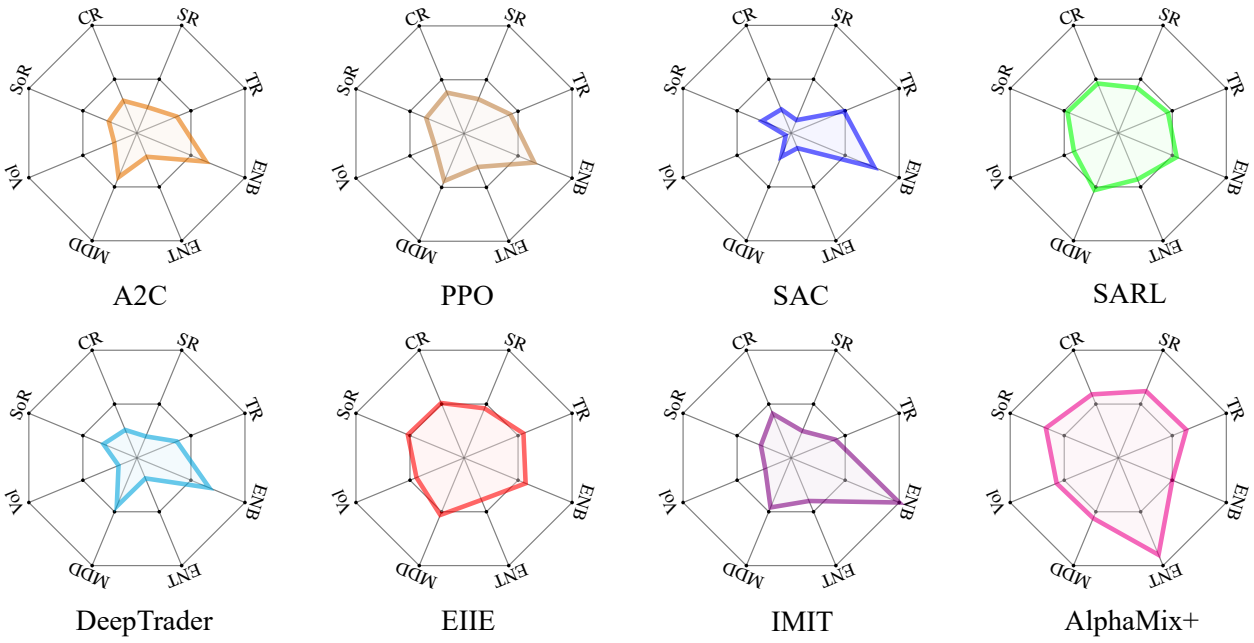
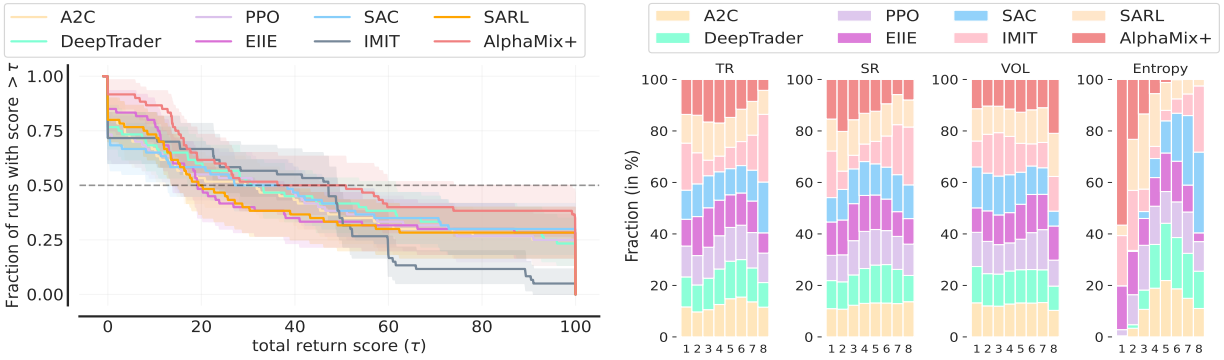


Figure 5: Overall performance across 4 financial markets on PRIDE-Star to evaluate profitability, risk-control and diversity, where AlphaMix+ achieves the best performance in 7 out of 8 metrics.



(a) (b)

Figure 6: (a) Performance profile of total return score distributions across 4 financial markets. Shaded regions show pointwise 95% confidence bands based on percentile bootstrap with stratified sampling. (b) Rank distribution in terms of TR, SR, Vol and ENT across 4 financial markets.

5.5 Performance Profile: An Unbiased Approach to Report Performance

The performance profile reports the score distribution of all runs across the 4 financial markets that are statistically unbiased, more robust to outliers and require fewer runs for lower uncertainty compared to conventional point estimates such as mean. Performance profiles proposed herein visualize the empirical tail distribution function of a random score (higher curve is better), with point-wise confidence bands based on stratified bootstrap (Efron, 1979). A score distribution shows the fraction of runs above a certain normalized score that is an unbiased estimator of the underlying performance distribution. As shown in Figure 6a, AlphaMix+ is generally a robust but conservative FinRL methods that shows the least bad runs, which makes it an attractive option for conservative investors that care more about risk. However, radical investors may pick SAC as it has the largest probability of achieving score 100, which indicates a return rate higher than twice the market average.

5.6 Rank Distribution to Demonstrate the Rank of FinRL methods

In Figure 6b, we plot the rank distribution of 6 FinRL methods in terms of TR, SR, Vol and ENT across 4 financial markets, where the i-th column in the rank distribution plot shows the probability that a given method is assigned rank i in the corresponding metrics. For TR and SR, AlphaMix+ slightly outperforms other methods with 41% and 47% probability to achieve top 2 performance. For Vol, SAC gets the overall best performance while AlphaMix+ goes through higher volatility. For ENT, AlphaMix+ significantly outperforms other FinRL methods with over 80% probability for rank 1, which demonstrates its ability to train mixture of diversified trading experts.

5.7 Visualizing Strategy Diversity with Heatmap

To demonstrate the overall investment diversity of FinRL methods, we show the average portfolio across test period as a heatmap in Figure 7. For IMIT, it puts all capital in one or two assets. The portfolio of SARL and EIIE is not that diversified with near 0 weight on many assets more (red). For A2C, DT, PPO and SAC, the portfolio is closed to uniform, which is not desirable due to poor profitability. Our AlphaMix+ achieves an ideal investment portfolio, which is generally diversified and allocate more weights on a few bullish stocks.

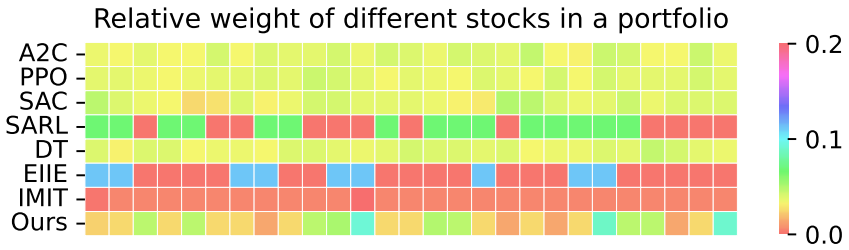
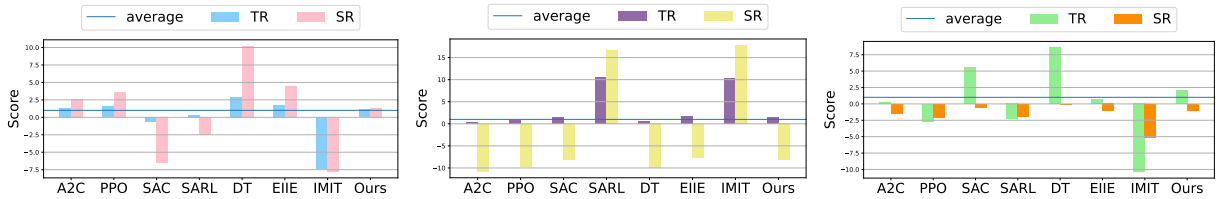


Figure 7: Heatmap of average portfolio on China stock market.

5.8 The Impact of Extreme Market Conditions

To further evaluate the risk-control and reliability, we pick three extreme market periods with black swan events. For China stock market, the period is from February 1st to March 31st 2021 when strict segregation policy is implemented in China to fight against the COVID-19 pandemic (Lee et al., 2021a). For US stock market, the period is from March 1st to April 30th 2020, when the global financial markets are violate due the global pandemic of COVID-19 (Mazur et al., 2021). For Crypto, the period is from April 1st to May 31st 2021 when many countries posed regulation against Crypto oligarch. In Figure 8, we plot the bar chart of TR and SR during the period of extreme market conditions. As a conservative method, AlphaMix+'s performance is unsatisfactory in extreme market conditions, which proves the general consensus that radical methods such as DeepTrader (DT) and SARL are more suitable for extreme markets (Marimoutou et al., 2009). Analyzing the performance on extreme market conditions can shed light on the design of FinRL methods, which is in line with economists' efforts on understanding the financial markets. For instance, incorporating volatility-aware auxiliary task (Sun et al., 2021b) and multi-objective RL (Hayes et al., 2022) in AlphaMix+ may further

make it be aware of extreme market conditions in advance and behave as a profit-seeking agents to achieve better performance during extreme market conditions.



(a) US (b) China (c) Crypto

Figure 8: Performance of FinRL methods during extreme market conditions.

6 Discussion

Complementary Related Efforts. Apart from the above aspects described in PRUDEX-Compass, there also exist several orthogonal perspectives for FinRL evaluation, which encompass a *check-list* (Pineau et al., 2021) for quantitative experiments, the construction of elaborate *dataset sheets* (Gebru et al., 2021), and the creation of *model cards* (Mitchell et al., 2019). We stress that these perspectives remain indispensable, as novel datasets and their variants are regularly suggested in FinRL and the PRUDEX-Compass does not disclose intended use with respect to human-centered application domains, ethical considerations, or their caveats. We believe that it is best to report both the prior works and PRUDEX-Compass together to further improve the evaluation quality.

Potential Impact. We hope that PRUDEX-Compass could encourage both researchers and financial practitioners to avoid fooling themselves by evaluating FinRL methods in a systematic way and facilitate the design of stronger FinRL methods. While accounting for all elements in PRUDEX-Compass is not a panacea, it provides a strong foundation for trustworthy results on which the community can build on and further increase the confidence for real-world industry deployment. For RL community, PRUDEX-Compass introduces a new challenging scenario with different evaluation axes for the test of novel RL algorithms in financial market. In addition, the usage of PRUDEX-Compass is not limited in RL settings, most elements of PRUDEX-Compass can be easily generalized to supervised learning settings with broader impact.

Auxiliary Experiments. Due to space limitations, we have included some auxiliary yet important experiments in Appendix D. Specifically, the result tables with mean and standard deviation of the 8 metrics in PRIDE-Star are reported in Appendix D.1. The equity curves with standard deviation shades on the 4 financial markets are included in Appendix D.2. We plot the PRIDE-Star, performance profile and rank comparison of each financial market individually in Appendix D.3, D.4, D.5, respectively.

Hosting, Maintenance, Licensing. The PRUDEX-Compass datasets is hosted on Google Drive. The source code are publicly available at https://anonymous.4open.science/r/PRUDEX-Compass-68D5. The authors will provide important bug fixes to the community as commits to the GitHub repository. There will be summary of changes to the code and the datasets in the README web page of the GitHub repository. In the unlikely case that the Google Drive link stops operating, we will migrate the dataset to another hosting and announce the new links in the GitHub repository. The provided source code and dataset are copyrighted by us and under the MIT license [6]. Users have the permission to reuse the codes for any purpose.

[6] https://opensource.org/licenses/MIT

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Appendix

A PRUDEX-Compass

A.1 PRUDEX-Compass Measure

We elaborate 16 measures in Table 2 with mathematical definitions and detailed descriptions.

Profit measure contains metrics to evaluate FinRL methods' ability to gain market capital. Total return (TR) is the percent change of net value over time horizon h. The formal definition is $TR = (n_{t+h} - n_t)/n_t$, where n_t is the corresponding value at time t.

Alpha Decay indicates the loss in the investment decision making ability of FinRL methods over time due to distribution shift in financial markets. In finance, information coefficient (IC) across time is widely-used to measure alpha decay (Pénasse, 2022).

Equity Curve is a graphical representation of the value changes of trading strategies over time. An equity curve with a consistently positive slope typically indicates that the trading strategies of the account are profitable. In RL settings, we usually plot equity curves with mean and standard deviation of multiple random seeds.

Risk includes a class of metrics to assess the risk level of FinRL methods.

- **Volatility (Vol)** is the variance of the return vector $\mathbf{r}$. It is widely used to measure the uncertainty of return rate and reflects the risk level of strategies. The definition is $Vol = \sigma[\mathbf{r}]$
- **Maximum drawdown (MDD)** measures the largest decline from the peak in the whole trading period to show the worst case. The formal definition is $MDD = \max_{\tau \in (0,t)}[\max_{t \in (0,\tau)} \frac{n_t - n_\tau}{n_t}]$
- **Downside deviation (DD)** refers to the standard deviation of trade returns that are negative.

Risk-adjusted Profit calculates the potential normalized profit by taking one share of the risk. We define three metrics with different types of risk:

- **Sharpe ratio (SR)** is a risk-adjusted profit measure, which refers to the return per unit of deviation: $SR = \frac{\mathbb{E}[\mathbf{r}]}{\sigma[\mathbf{r}]}$
- **Sortino ratio (SoR)** is a variant of risk-adjusted profit measure, which applies DD as risk measure: $SoR = \frac{\mathbb{E}[\mathbf{r}]}{DD}$
- **Calmar ratio (CR)** is another variant of risk-adjusted profit measure, which applies MDD as risk measure: $CR = \frac{\mathbb{E}[\mathbf{r}]}{MDD}$

Extreme Market. It is necessary to evaluate on extreme market conditions with black swan events to show the reliability of FinRL methods .By analyzing the trading behaviors during extreme markets, we can understand their cons and pros and further design better FinRL methods. There are a few potential testbed such as COVID-19 pandemic, financial crisis, government regulation and war.

Countries. Financial markets in different countries have different trading patterns, where markets in developed countries is more "effcient" with high proportion of institutional investors and markets in developing countries is more noisy with high personal investors. It is necessary to evaluate FinRL methods on multiple mainstream financial markets in different countries, such as US, Europe and China, to evaluate universality.

Asset Type. A financial asset is a liquid asset that derives its value from any contractual claim. Different asset types have different liquidity, trading rules and value models. It is necessary to evaluate FinRL methods on various financial asset types to evaluate universality.

Time Scale. We can evaluate FinRL methods on multiple trading scenarios with financial data on different time-scale (both coarse-grained and fine-grained). For instance, second-level data can be used for high

frequency trading; minute-level data is suitable for intraday trading; day-level data can be applied for long-term trend trading.

Rolling Window. Due to the remarkable distribution shift in financial markets, researchers need to train and evaluate FinRL methods in a rolling time window, which means retrain or fine-tune RL models periodically to fit on current market status. Backtest with rolling window can evaluate the reliability of FinRL.

t-SNE is a statistical method for visualizing high-dimensional data by giving each datapoint a location in a two-dimensional map. First, t-SNE constructs a probability distribution over pairs of high-dimensional objects where similar objects are assigned a higher probability. Second, t-SNE defines a similar probability distribution over the points in the low-dimensional map, and it minimizes the KL divergence between the two distributions with respect to the locations of the points in the map. In FinRL, we use t-SNE to visualize financial datasets to show the relative position of them.

Entropy. (Shannon, 1948) is applied in finance to measure the amount of information give by observing the financial market. In a portfolio, it is defined as $H(\omega) = -exp\,(\sum_{i=1}^{n} \omega_i \, log\, \omega_i)$, where $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is a portfolio among N financial assets. This measure reach a minimum value 1 if a portfolio is fully concentrated in one single asset and a maximum equal to N that representation the equally weighted portfolio.

Correlation. As entropy ignore the correlation between different financial assets, effective number of bets (ENB) is proposed to remove the correlation while calculating entropy. We first define diversification distribution as $p_i(\omega) = \omega_{F_i}^2 \lambda_i^2 / \sum_{i=1}^{n} \omega_{F_i}^2 \lambda_i^2$. We formulate the covariance matrix of N assets Σ as $E'\Sigma E = \lambda$ where λ is a diagonal matrix,λ_i is the element of the diagonal matrix and $\omega_f = E^{-1}\omega$ where ω is the our original portfolios' weights. The effective number of bets is defined as: $ENB(\omega) = -exp\,(\sum_{i=1}^{n} p_i(\omega)\, log\, p_i(\omega))$, where $p_i(\omega)$ is the ith diversification distribution for the portfolios' weights ω.

Diversity Heatmap is a visualization tool to demonstrate the diversity of investment decisions among different financial assets with heatmap (Harris et al., 2020). The x-axis refers to the relative weight of each asset in the portfolio. The y-axis includes the results of different FinRL algorithms.

Performance Profile reports the score distribution of all runs across the 4 financial markets that are statistically unbiased, more robust to outliers and require fewer runs for lower uncertainty compared to point estimates such as mean. Performance profiles proposed herein visualize the empirical tail distribution function of a random score (higher curve is better), with point-wise confidence bands based on stratified bootstrap [15]. A score distribution shows the fraction of runs above a certain normalized score that is an unbiased estimator of the underlying performance distribution.

Variability refers the variance of performance across different random seeds in RL. As a high stake domain, it is important to test the variability, which is closely relevant to reliability.

Rank Comparison

In the rank distribution plot, the ith column shows the probability that a given method is assigned rank i in the corresponding metrics, which provides a indication on the overall rank of FinRL methods.

A.2 Creating a PRUDEX-Compass

To make the PRUDEX-Compass as accessible as possible and disseminate in a convenient way, we provide two options for practical use.

- We provide a **LaTeX template** for PRUDEX-Compass, making use of the TikZ library to draw the compass with LaTeX. We envision that such a template makes it easy for other authors to include a compass into their future research, where they can adapt the naming and values of the entries respectively.

- We further provide a **Python script** to generate the PRUDEX-Compass. In fact, because the use of drawing in LaTeX with TikZ may be unintuitive for some, we have written a Python script that automatically fills above LaTeX template, so that it can later simply be included into a LaTeX

document. The Python script takes a path to a JSON file that needs to be filled by the user. There is a default JSON file that is easy to adopt.

Next, we concentrate on explaining on how the values of the inner part is calculated based the original experiment results. Generally speaking, we normalize the performance of different measures on the 6 axes to a score from 0 to 100. We mark market average strategy (evenly invest on all assets) as 50. All scores are calculated with the average of 4 financial markets. We define the metrics value for FinRL methods and market average as m_{rl} and m_{ave}, respectively. For the 4 profit-related metrics (TR, SR, CR, SoR), we normalized them into a score S_{pro} with range $[0, 100]$: $S_{pro} = (m_{rl}/m_{ave} - 0.8) * 250$, where 20% higher profit than market average is scored 100. We clip values lower than 0 and higher than 100 as 0 and 100, respectively. For the 2 risk metrics (Vol, MDD), we normalized them into a score S_{risk} with range $[0, 100]$: $S_{risk} = (1.2 - m_{rl}/m_{ave}) * 250$, where 20% lower risk than market average is scored 100. We clip values lower than 0 and higher than 100 as 0 and 100, respectively.

For universality, we directly use the raw return rate and calcualte the 4 indicators of profitability, we then plot a rank graph and for each measures of profitability, we multiply the probability obtained from the rank matrix and the rankscore(if it's 1st, the score is 100 and if 6th, the score is 0) to get a score for that measure. Then we average the 4 measures together to get the university score for that algorithm. For the 2 diversity metrics (ENT, ENB), we normalized them into a score S_{div}:

$$S_{div} = \begin{cases} m_{rl}/m_{ave} \times 100, & \text{for ENT} \\ m_{rl}/m_{ave} \times 50, & \text{for ENB} \end{cases}$$

where the diversity of uniform policy is scored 100 for ENT and 50 for ENB. For explainability, we set the explainability score 50 for all FinRL methods. For reliability, we use the total return rate score we just normalized using average policy as a indicator and draw a performance profile graph, then we use the area under the curve for each algorithm to calculate the reliability.

A.3 Future Version of PRUDEX-Compass

We acknowledge that the present version of the PRUDEX-Compass is not perfect nevertheless contains room to grow. We plan to improve PRUDEX-Compass from the following perspectives: i) For axis-level, we plan to explore the evaluation of FinRL explainability with measures and plots; ii) For measure-level, we plan to include metrics to evaluate alpha decay and more metrics, e.g., optimality gaps, for profits and risks. iii) For visualization, we plan to further develop a GUI software version accompanied with a website to further lower the barrier for dissemination and use; iv) As most axes and measures in PRUDEX-Compass are key points for all trading scenarios, we plan to bring PRUDEX-Compass into more general machine learning settings for broader impacts.

B Related Works

FinRL Methods. Recent years have witnessed the successful marriage of reinforcement learning and quantitative trading. Neuneier (1996) made the first attempt to learn trading strategies using Q-learning. Moody & Saffell (1998) proposed a policy-based method, namely recurrent reinforcement learning (RRL), for quantitative trading. However, traditional RL approaches have difficulties in selecting market features and learning good policy in large scale scenarios. To tackle these issues, many DRL approaches have been proposed to learn market embedding through high dimensional data. Jiang et al. (2017) used DDPG to dynamically optimize cryptocurrency portfolios. Deng et al. (2016) applied fuzzy learning and deep learning to improve financial signal representation. Yu et al. (2019) proposed a model-based RL framework for daily frequency portfolio trading. Liu et al. (2020b) proposed an adaptive DDPG-based framework with imitation learning. A comprehensive survey of FinRL is at (Sun et al., 2021a).

Rigorous Evaluation in RL. There are many prior works focusing on rigorous evaluation in RL. Henderson et al. (2018) highlights various reproducibility issues in RL. Colas et al. (2018) studies the minimum number

of random seeds required to report results with statistical significance. Agarwal et al. (2021) recommend for reporting interval estimates of aggregate performance and propose performance profiles for reliable RL evaluation with a handful of runs. Chan et al. (2019) propose metrics to measure the reliability of RL algorithms in terms of their stability during training and their variability and risk in returns across multiple episodes. Jordan et al. (2020) propose a game-theoretic evaluation procedure for "complete" algorithms that do not require any hyperparameter tuning and recemmend evaluating between 1000 to 10000 runs per task to detect statstically significant results.

C Experiment Setup

C.1 Baseline

During the experiment, we use 7 baselines to compare with AlphaMix+ under the systematic evaluation. For all 4 datasets, the hyper-parameters are the same for each algorithms and here is the

- A2C (Mnih et al., 2016) is a classic actor-critic RL algorithms that introduce an advantage function to enhance policy gradient update by reducing variance.
- PPO (Schulman et al., 2017) is a proximal policy optimization that constrain the difference between current policy and updated policy. It also simplify the objective with a clipping term into the objective function.
- SAC (Haarnoja et al., 2018) is an off-policy actor-critic method based on the maximum entropy RL framework, which encourages the robustness to noise and exploration by maximizing a weight objective of the reward and the policy entropy.
- SARL (Ye et al., 2020) propose a State-Augmented RL (SARL) framework based on DPG. SARL learns the price movement prediction with financial news as additional states.
- DeepTrader (DT) (Wang et al., 2021b) is a PG-based DRL method, to tackle the risk-return balancing problem in PM. The model simultaneously uses negative maximum drawdown and price rising rate as reward functions to balance between profit and risk with an asset scoring unit.
- EIIE (Jiang et al., 2017) is a DPG-based RL framework, which contains: 1) the Ensemble of Identical Independent Evaluators (EIIE) topology; 2) a Portfolio Vector Memory (PVM); 3) an Online Stochastic Batch Learning (OSBL) scheme.
- Investor-Imitator (IMIT) (Ding et al., 2018) imitates behaviors of different investors (oracle investor, collaborator investor, public investor) by designing investor-specific reward functions with a set of logic descriptors.

C.2 Training Setup

Table 4: Hyperparameters of AlphaMix+

Hyperparameter	**Value**	**Hyperparameter**	**Value**
Random crop	True	Observation	Number of assets $\times$ 11
Replay buffer size	10000	Initial step	10000
Layer(MLP)	(128,128)	Stacked frame	3
Evaluation episodes	10	Optimizer	Adam
Temperature	20	Uncertainty	0.5
Actor learning rate	0.0007	Critic learning rate	0.0007
Batch size	256	Action numbers	29(US) 47(China) 22(FX) 9(Crypto)
Discount γ	0.99	Ber_mean	0.5
Non-linear	Sigmoid		

Table 5: Hyperparameters of other 7 FinRL methods

Hyperparameter	**Value**	**Hyperparameter**	**Value**
Random Crop	True	Observation	Number of assets $\times$ 11
Replay buffer size	10000	Initial step	10000
Layer(MLP)	(256, 256)	Non-linear	Tanh
Evaluation episodes	10	Optimizer	Adam
Learning rate	0.0001	Discount γ	0.99
Batch size	200	Action numbers	29(US) 47(China) 22(FX) 9(Crypto)

D Experimental Results

D.1 Result Table

In this subsection, we report detailed results of 8 metrics in the four financial markets. Since we apply 1 year rolling window during training, each financial market has 3 tables for 3 consecutive years.

Table 6: US Stock 2021

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	12.4±6.3	13.1±4.4	20.0±10.3	17.3±2.0	17.2±7.4	16.9±1.2	4.0±10.1	20.7±1.8
SR	1.03±0.50	0.94±0.26	1.30±0.57	1.37±0.12	1.22±0.50	1.39±0.08	0.35±0.64	1.64±0.11
CR	0.81±0.34	0.83±0.13	0.91±0.19	1.06±0.03	0.84±0.23	1.06±0.02	0.47±0.61	1.09±0.02
SoR	1.52±0.70	1.38±0.44	1.89±0.87	2.02±0.21	1.81±0.75	2.03±0.12	0.52±0.91	2.36±0.17
MDD(%)	15.0±1.9	15.3±2.5	19.7±4.6	15.6±1.2	19.1±3.9	15.2±0.6	14.6±3.7	17.9±1.4
VOL(%)	0.78±0.06	0.87±0.03	0.91±0.02	0.76±0.02	0.86±0.03	0.73±0.03	1.0±0.12	0.74±0.02
ENT	2.26±0.26	1.82±0.02	1.67±0.02	2.79±0.11	1.82±0.01	2.90±0.26	1.89±0.1	3.25±0.12
ENB	1.34±0.10	1.49±0.01	1.61±0.02	1.14±0.06	1.51±0.02	1.19±0.10	1.73±0.10	1.11±0.03

Table 7: US Stock 2020

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	7.46±4.86	18.3±8.4	7.77±21.1	9.23±8.79	10.2±10.5	15.8±16.1	-20.6±9.5	12.7±1.7
SR	0.36±0.12	0.62±0.19	0.37±0.52	0.41±0.21	0.42±0.25	0.55±0.30	-0.28±0.21	0.52±0.04
CR	0.36±0.12	0.54±0.15	0.30±0.48	0.38±0.19	0.42±0.22	0.49±0.22	-0.28±0.22	0.47±0.03
SoR	0.44±0.14	0.76±0.23	0.50±0.63	0.50±0.26	0.56±0.33	0.68±0.38	-0.36±0.27	0.62±0.05
MDD(%)	36.6±2.4	42.2±2.4	42.8±5.3	37.8±3.19	34.9±3.91	38.9±6.69	43.5±6.42	38.2±0.96
VOL(%)	2.25±0.09	2.32±0.03	2.45±0.16	2.26±0.17	2.31±0.09	2.21±0.23	2.9± 0.39	2.16±0.04
ENT	1.96±0.20	1.82±0.02	1.61±0.03	2.07±0.67	1.82±0.02	2.41±0.67	1.85 ±0.21	3.22±0.03
ENB	1.05±0.01	1.05±0.004	1.1±0.003	1.05±0.03	1.05±0.002	1.05±0.05	1.13± 0.04	1.0± 0.006

Table 8: US Stock 2019

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	20.5±9.27	21.5±10.1	27.3±9.79	27.7±1.63	22.9±3.51	30.2±8.42	20.6±9.52	25.1±1.42
SR	1.47±0.58	1.53±0.62	1.72±0.54	2.00±0.15	1.60±0.24	2.12±0.27	1.28±0.21	1.98±0.06
CR	1.00±0.06	1.01±0.06	1.05±0.09	1.02±0.05	1.01±0.05	1.06 ±0.05	0.28±0.22	1.03±0.007
SoR	1.96±0.78	2.04±0.83	2.17±0.78	2.52±0.24	2.16±0.43	2.82±0.42	0.36±0.27	2.47±0.09
MDD(%)	18.8±5.95	19.4±6.26	23.4±4.70	24.7±2.63	21.2±2.41	25.2±4.83	43.6±6.42	22.3±0.99
VOL(%)	0.82±0.01	0.83±0.02	0.91±0.05	0.79±0.10	0.84±0.03	0.79±0.11	1.9±0.39	0.73±0.02
ENT	1.83±0.01	1.82±0.01	1.63±0.02	2.08±0.67	1.81±0.01	2.32±0.48	1.89±0.16	3.27±0.03
ENB	1.28±0.01	1.27±0.01	1.38±0.01	1.29±0.21	1.26±0.005	1.15±0.06	1.73±0.10	1.05± 0.01

Table 9: China Stock 2020

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	5.52±4.82	6.20±7.57	13.4±20.1	11.4±8.84	12.3±14.2	13.0±3.8	52.6±20.8	14.8±3.30
SR	0.35±0.21	0.40±0.37	0.59±0.68	0.63±0.41	0.62±0.60	0.73± 0.17	2.68±0.83	0.79±0.12
CR	0.25±0.15	0.29±0.27	0.41±0.48	0.42±0.27	0.36±0.39	0.51±0.08	1.18±0.24	0.53±0.06
SoR	0.40±0.24	0.45±0.42	0.71±0.81	0.71±0.46	0.71±0.69	0.80±0.18	4.04±1.2	0.89±0.15
MDD(%)	28.1±3.68	25.3±1.59	29.9±7.38	26.5±5.09	31.5±6.02	27.0±2.41	34.2±9.16	29.1±1.85
VOL(%)	0.74±0.04	0.68±0.004	0.81±0.06	0.68±0.02	0.76±0.01	0.67±0.02	0.66±0.09	0.69±0.01
ENT	1.85±0.42	2.82±0.01	1.10±0.02	2.60±0.17	1.53±0.001	2.39±0.10	1.30 ±0.85	3.12±0.02
ENB	1.12±0.05	1.04±0.01	1.27±0.01	1.04±0.01	1.16±0.004	1.06± 0.01	2.82± 0.85	1.02±0.01

Table 10: China Stock 2019

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	31.9±10.6	29.7±8.42	25.4±10.4	32.6±5.78	22.1±13.6	36.2± 7.09	-7.14±0.82	32.2±2.20
SR	1.59±0.46	1.65±0.39	1.13±0.38	1.80±0.24	1.19±0.59	1.77± 0.05	-0.34±0.1	1.79±0.10
CR	0.90±0.16	0.94±0.13	0.79±0.23	1.01±0.07	0.74±0.21	1.04± 0.05	-0.29±0.07	1.01±0.02
SoR	2.32±0.75	2.41±0.63	1.70±0.64	2.63±0.34	1.72±0.91	2.57± 0.15	-0.37± 0.08	2.62±0.14
MDD(%)	31.7±2.85	28.6±2.82	29.8±2.38	28.9±2.38	27.5±4.92	30.5±3.74	20.4± 0.63	28.9±0.90
VOL(%)	0.65±0.02	0.58±0.004	0.74±0.02	0.57±0.01	0.63±0.02	0.64±0.11	0.77± 0.06	0.57±0.01
ENT	1.54±0.01	2.85±0.005	1.02±0.02	2.47±0.17	1.53±0.009	1.97±0.90	1.30±0.85	3.15±0.07
ENB	1.18±0.009	1.05±0.002	1.32±0.009	1.05±0.01	1.18±0.004	1.21 ±0.18	1.19±0.85	1.04±0.01

Table 11: China Stock 2018

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	-6.86±11.30	-6.56±3.66	-4.05±10.77	-5.77±4.04	5.67±5.95	-2.27±1.99	-7.14± 0.82	-0.70±1.38
SR	-0.28±0.61	-0.31±0.22	-0.12±0.50	-0.25±0.21	0.37±0.30	-0.05± 0.12	-0.34±0.1	0.03±0.08
CR	-0.14±0.50	-0.25±0.16	-0.06±0.42	-0.20±0.15	0.40±0.32	-0.04±0.11	-0.29±0.07	0.04±0.10
SoR	-0.33±0.78	-0.41±0.26	-0.15±0.75	-0.35±0.29	0.54±0.45	-0.07± 0.17	-0.37±0.08	0.04±0.11
MDD(%)	22.3±5.52	20.5±1.55	25.9±4.95	20.2±3.49	19.1±3.39	17.5±1.52	20.4±0.63	16.1±1.45
VOL(%)	0.67±0.03	0.59±0.01	0.75±0.03	0.61±0.03	0.66±0.02	0.59± 0.03	0.77±0.06	0.57±0.02
ENT	1.54±0.01	2.85±0.007	1.01±0.03	2.41±0.40	1.53±0.007	2.55±0.15	1.30± 0.85	3.12±0.02
ENB	1.31±0.01	1.08±0.005	1.57±0.009	1.11±0.10	1.30±0.009	1.05± 0.01	1.19 ±0.08	1.03±0.003

Table 12: Crypto 2021

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	223±346	146±129	377±982	199±195	398±344	146±137	140± 155	291±71
SR	1.38±0.74	1.22±0.56	1.13±0.67	1.41±0.45	1.71±0.45	1.46± 0.42	1.24±0.34	1.87±0.09
CR	1.77±1.02	1.48±0.53	2.18±2.05	1.75±0.74	2.18±0.84	1.44±0.49	1.39±0.58	2.02±0.26
SoR	2.15±1.23	2.13±1.11	3.16±3.31	2.42±1.24	2.97±1.35	2.42± 1.22	2.03± 0.80	3.14±0.56
MDD(%)	77.1±15.7	74.9±10.4	80.1±15.6	79.4±10.0	85.1±8.42	72.5±10.2	71.0± 11.2	85.7±4.14
VOL(%)	7.30±1.91	6.97±0.94	10.94±8.13	7.26±2.31	7.85±2.05	5.22±1.06	5.56± 1.61	6.80±0.84
ENT	1.02±0.31	0.72±0.05	0.47±0.07	1.42±0.25	0.56±0.04	1.53±0.23	0.58±0.23	2.09±0.02
ENB	1.79±0.15	1.94±0.04	1.99±0.05	1.67±0.40	1.97±0.03	1.12±0.05	1.66±0.38	1.47±0.15

Table 13: Crypto 2020

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	61.4±58.1	59.1±58.4	16.0±38.4	31.4±12.8	37.9±27.1	30.7± 11.7	55.3± 17.6	33.8±6.16
SR	1.12±0.66	1.12±0.67	0.51±0.50	0.81±0.24	0.85±0.38	0.79±0.17	1.03± 0.08	0.86±0.08
CR	1.06±0.47	1.02±0.45	0.59±0.58	0.86±0.23	0.97±0.42	0.82±0.19	0.73± 0.07	0.87±0.09
SoR	1.58±1.20	1.62±1.22	0.59±0.63	0.89±0.29	1.04±0.54	0.92±0.18	1.49± 0.32	0.94±0.10
MDD(%)	52.9±7.58	52.2±8.03	55.7±2.12	46.6±8.08	50.3±8.04	44.6±3.22	56.6± 4.43	46.5±2.43
VOL(%)	3.86±0.22	3.75±0.17	4.46±0.35	3.66±0.76	4.05±0.32	3.37± 0.36	2.86± 0.53	3.46±0.22
ENT	0.59±0.04	0.59±0.04	0.45±0.02	1.32±0.45	0.60±0.01	1.24±0.52	0.58±0.52	2.17±0.07
ENB	1.05±0.01	1.06±0.03	1.07±0.03	1.01±0.005	1.05±0.01	1.01±0.01	1.01±0.01	1.00±0.001

Table 14: Crypto 2019

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	82.8±51.5	85.8±57.1	73.9±33.9	61.0±19.3	39.1±54.5	50.8±13.2	110.6± 17.6	68.3±17.8
SR	1.37±0.50	1.39±0.55	1.27±0.35	1.26±0.26	0.83±0.59	1.15±0.20	2.06± 0.08	1.43±0.22
CR	1.14±0.30	1.14±0.33	1.06±0.24	1.06±0.11	0.73±0.39	0.98±0.10	1.45± 0.07	1.08±0.11
SoR	2.19±0.98	2.22±1.02	2.10±0.65	1.89±0.39	1.30±1.12	1.77±0.28	2.99± 0.32	2.16±0.35
MDD(%)	59.5±9.64	59.9±10.5	59.1±7.74	52.4±6.49	51.3±11.3	49.9±5.80	56.6± 4.43	55.5±3.58
VOL(%)	3.69±0.36	3.69±0.35	3.63±0.27	3.29±0.35	3.46±0.24	3.15±0.30	2.86± 0.53	3.09±0.18
ENT	0.60±0.02	0.62±0.03	0.43±0.03	1.21±0.40	0.59±0.02	1.24±0.52	0.58±0.52	2.15±0.10
ENB	1.15±0.006	1.14±0.01	1.15±0.008	1.06±0.02	1.14±0.01	1.01±0.01	1.01±0.01	1.01±0.01

Table 15: Foreign Exchange 2019

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	-0.94±3.14	-1.49±2.68	-1.02±3.11	0.96±0.36	-0.19±2.88	1.42±0.24	-5.53± 0.05	1.22±0.46
SR	-0.22±0.73	-0.34±0.64	-0.21±0.64	0.29±0.09	-0.03±0.66	0.39±0.05	-0.97± 0.01	0.41±0.16
CR	-0.07±0.42	-0.16±0.35	-0.02±0.43	0.22±0.08	0.04±0.46	0.32±0.07	-0.49± 0.01	0.36±0.17
SoR	-0.32±1.05	-0.51±1.00	-0.26±0.86	0.50±0.17	0.06±1.11	0.65±0.07	-1.5± 0.02	0.74±0.31
MDD(%)	7.90±1.85	6.28±1.37	6.94±2.07	4.60±0.47	6.84±1.38	4.65± 0.43	11.17± 0.05	3.77±0.56
VOL(%)	0.27±0.01	0.28±0.01	0.31±0.01	0.22±0.01	0.26±0.01	0.23±0.01	0.25 ±0.01	0.19±0.01
ENT	1.44±0.08	1.37±0.01	0.93±0.04	2.55± 0.10	1.35±0.02	2.23±0.08	3.08±0.01	2.97±0.04
ENB	1.65±0.06	1.73±0.01	2.02±0.04	1.18±0.10	1.71±0.03	1.16±0.06	1.08±0.01	1.18±0.06

Table 16: Foreign Exchange 2018

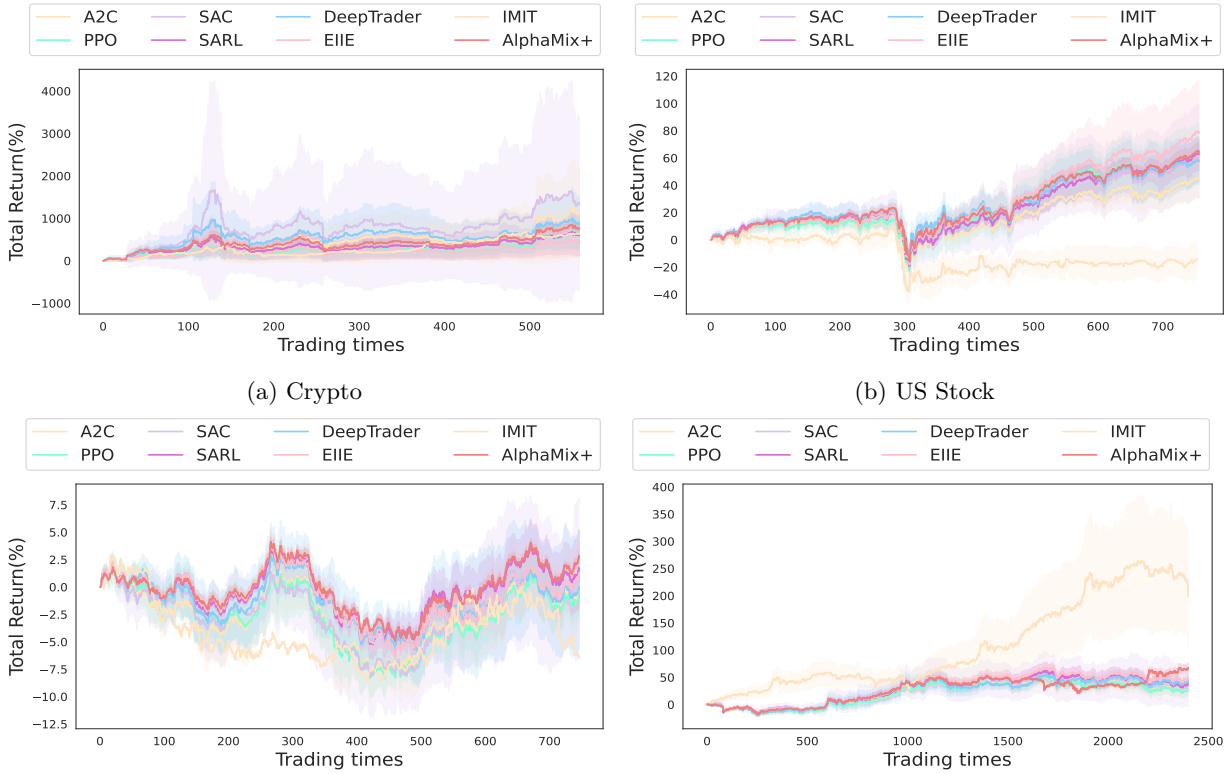
Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	-5.43±2.29	-5.25±2.45	-5.61±3.02	-4.52±1.91	-4.99±2.82	-6.15±0.28	-5.53± 0.05	-4.92±0.32
SR	-1.01±0.45	-0.97±0.49	-0.91±0.43	-0.93±0.41	-0.91±0.55	-1.15±0.14	-0.97± 0.01	-1.15±0.10
CR	-0.50±0.14	-0.48±0.16	-0.50±0.18	-0.48±0.19	-0.49±0.21	-0.59±0.04	-0.49± 0.01	-0.57±0.03
SoR	-1.43±0.64	-1.36±0.68	-1.23±0.44	-1.42±0.62	-1.29±0.74	-1.66±0.26	-1.5± 0.02	-1.77±0.16
MDD(%)	10.4±1.67	10.4±1.85	10.7±1.80	9.08±0.96	9.41±2.20	10.6±0.91	11.2± 0.05	8.66±0.14
VOL(%)	0.34±0.02	0.34±0.02	0.37±0.03	0.31±0.02	0.34±0.01	0.34±0.03	0.25± 0.0	0.27±0.01
ENT	1.38±0.03	1.34±0.03	0.94±0.03	2.23±0.37	1.37±0.01	1.55±0.72	3.08± 0.01	2.98±0.03
ENB	1.45±0.01	1.46±0.03	1.65±0.05	1.16±0.09	1.45±0.02	1.39±0.27	1.08±0.01	1.11±0.01

Table 17: Foreign Exchange 2017

Metrics	A2C	PPO	SAC	SARL	DeepTrader	EIIE	IMIT	AlphaMix+
TR(%)	6.93±1.71	6.85±1.44	8.25±3.62	5.81±2.26	5.94±2.50	6.71±2.72	6.42± 0.16	7.16±0.36
SR	1.34±0.29	1.32±0.24	1.41±0.48	1.34±0.57	1.13±0.51	1.26±0.47	0.81± 0.02	1.72±0.13
CR	0.82±0.12	0.81±0.12	0.88±0.17	0.79±0.27	0.77±0.19	0.88±0.11	0.73± 0.01	0.93±0.02
SoR	2.15±0.45	2.14±0.42	2.17±1.01	2.28±0.97	1.74±0.89	1.89±0.80	1.21± 0.03	3.04±0.27
MDD(%)	8.21±1.28	8.20±1.12	9.01±2.45	7.12±1.27	7.46±1.87	7.33±2.08	8.98± 0.12	7.51±0.43
VOL(%)	0.32±0.01	0.32±0.01	0.35±0.03	0.28±0.04	0.33±0.02	0.34±0.08	0.36± 0.01	0.25±0.01
ENT	1.34±0.02	1.35±0.02	0.90±0.03	2.17±0.45	1.37±0.01	1.41±0.57	3.08±0.01	2.98±0.10
ENB	1.58±0.01	1.59±0.04	1.82±0.02	1.29±0.15	1.57±0.02	1.55±0.18	1.05±0.04	1.10±0.04

D.2 Equity Curve

In subsection, we plot the equity curve of the 4 financial markets. Each line is the mean of 5 individual runs with the shaded area as the standard deviation.

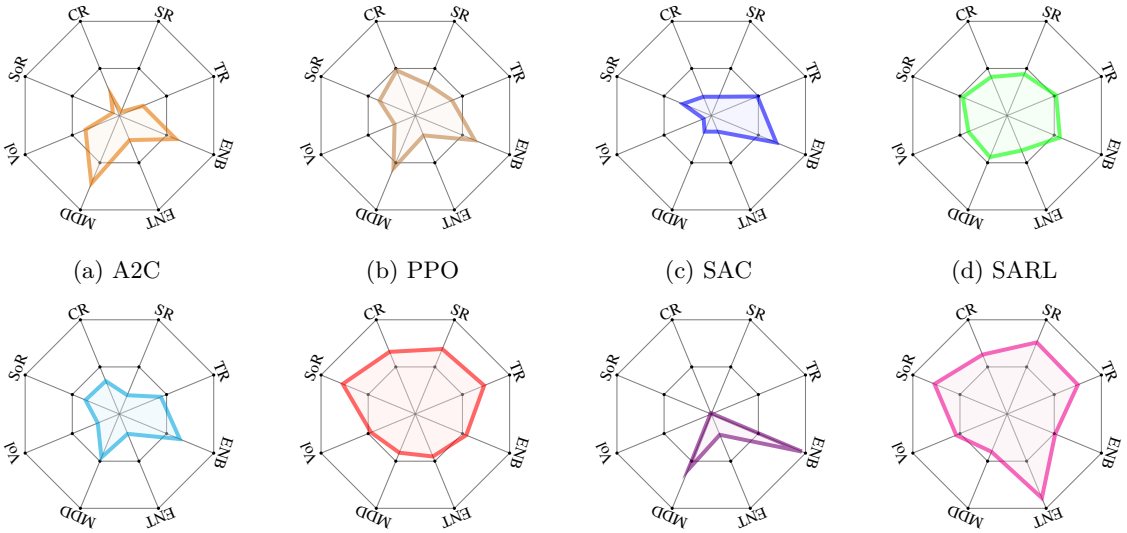


(a) Crypto (b) US Stock

(c) FX (d) China Stock

Figure 9: Equity curve on 4 influential financial markets

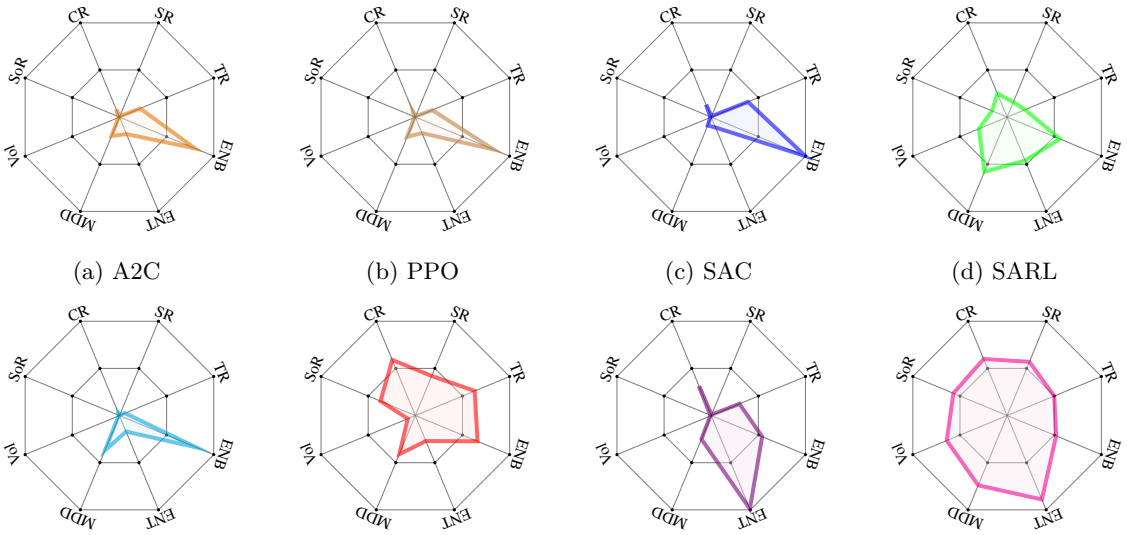
D.3 PRIDE-Star



(a) A2C (b) PPO (c) SAC (d) SARL

(e) DeepTrader (f) EIIE (g) IMIT (h) AlphaMix+

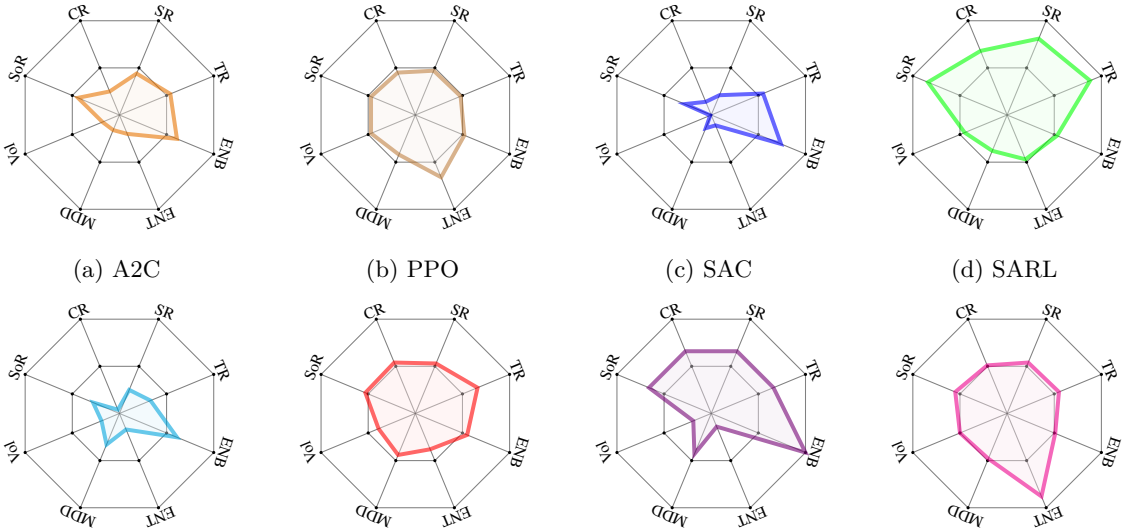
Figure 10: PRIDE-Star on US Stock



(a) A2C (b) PPO (c) SAC (d) SARL

(e) DeepTrader (f) EIIE (g) IMIT (h) AlphaMix+

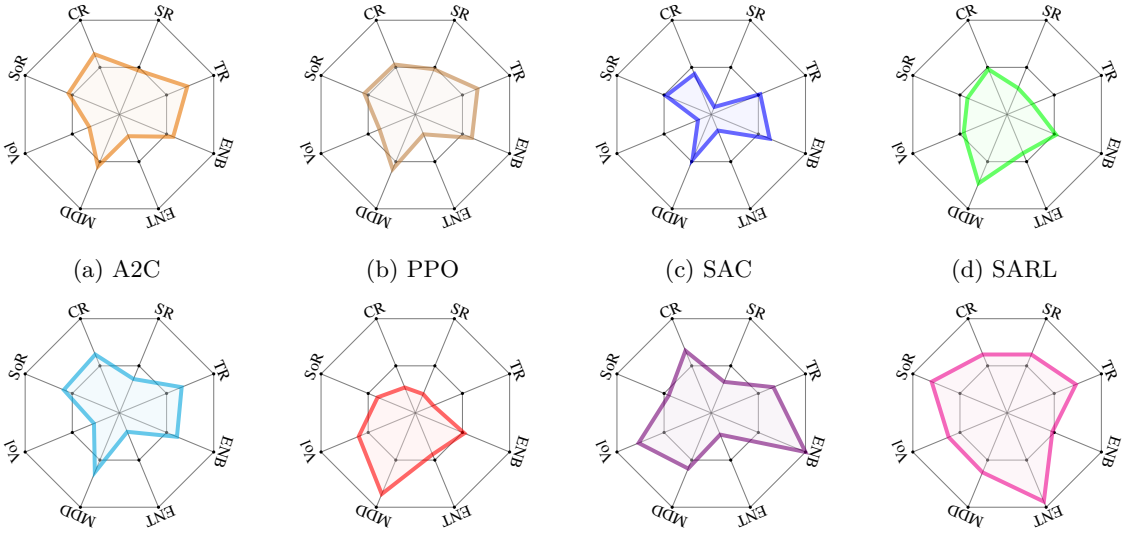
Figure 11: PRIDE-Star on FX



(a) A2C (b) PPO (c) SAC (d) SARL

(e) DeepTrader (f) EIIE (g) IMIT (h) AlphaMix+

Figure 12: PRIDE-Star on China Stock

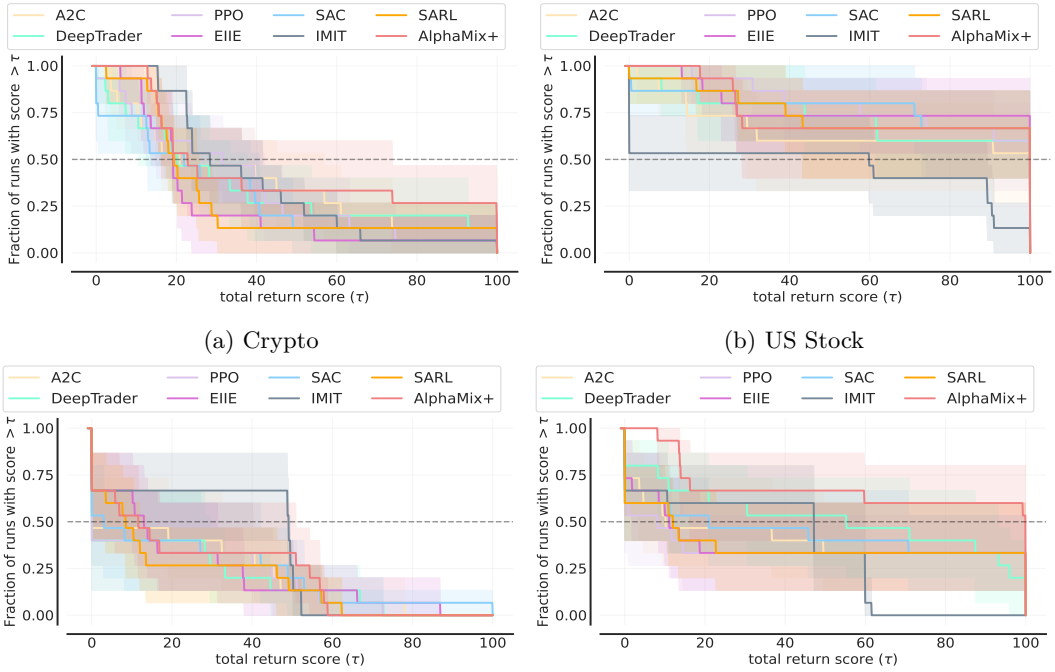


(a) A2C (b) PPO (c) SAC (d) SARL

(e) DeepTrader (f) EIIE (g) IMIT (h) AlphaMix+

Figure 13: PRIDE-Star on Crypto

D.4 Performance Profile

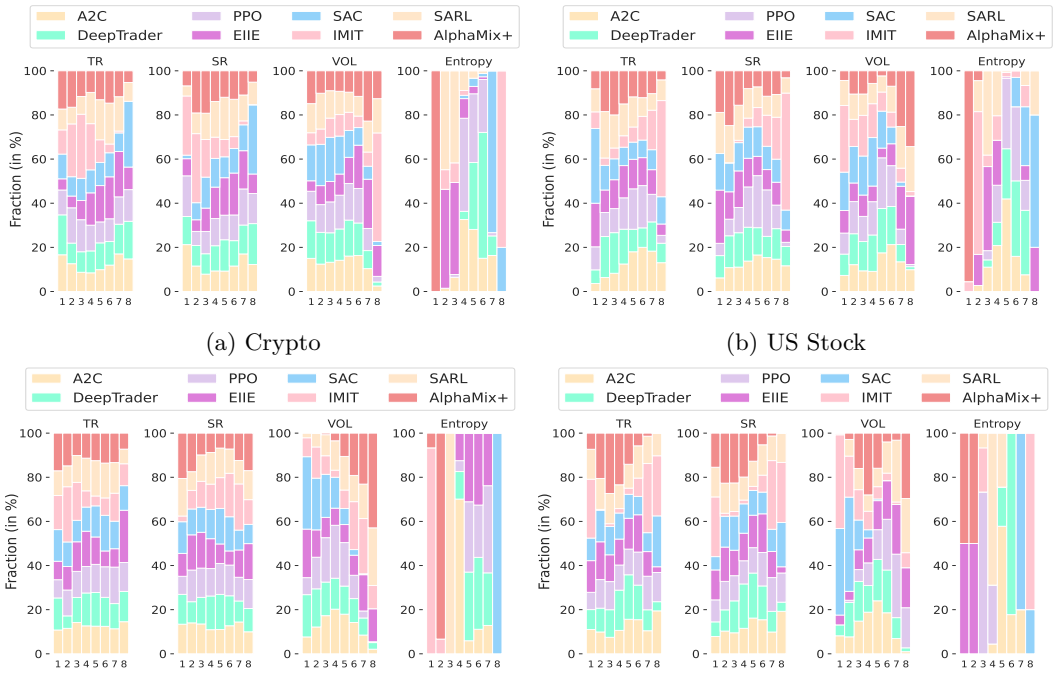


(a) Crypto (b) US Stock

(c) FX (d) China Stock

Figure 14: Performance profile on 4 influential financial markets

D.5 Rank Distribution



(a) Crypto (b) US Stock

(c) FX (d) China Stock

Figure 15: Rank distribution on 4 influential financial markets