Knowledge-Aware Bayesian Deep Topic Model

Abstract

We propose a Bayesian generative model for incorporating prior domain knowledge into hierarchical topic modeling. Although embedded topic models (ETMs) and its variants have gained promising performance in text analysis, they mainly focus on mining word co-occurrence patterns, ignoring potentially easy-to-obtain prior topic hierarchies that could help enhance topic coherence. While several knowledge-based topic models have recently been proposed, they are either only applicable to shallow hierarchies or sensitive to the quality of the provided prior knowledge. To this end, we develop a novel deep ETM that jointly models the documents and the given prior knowledge by embedding the words and topics into the same space. Guided by the provided knowledge, the proposed model tends to discover topic hierarchies that are organized into interpretable taxonomies. Besides, with a technique for adapting a given graph, our extended version allows the provided prior topic structure to be finetuned to match the target corpus. Extensive experiments show that our proposed model efficiently integrates the prior knowledge and improves both hierarchical topic discovery and document representation.

1 Introduction

Probabilistic topic models (TMs) such as latent Dirichlet allocation (LDA) have enjoyed success in text mining and analysis in an unsupervised manner [Blei et al., 2003] . Typically, the goal of TMs is to infer per-document topic proportions, as well as a set of latent topics from the target corpus using word co-occurrences within each document. The extracted topics are widely used in various machine learning tasks [Kataria et al., 2011, Jin et al., 2018, Xu et al., 2018]. However, the objective function of most TMs is to maximize the likelihood of the observed data, which causes an over-concentration on high-frequency words. Moreover, the infrequent words might be assigned to irrelevant topics due to the lack of side information. Those two drawbacks could lead to human-unfriendly topics that fail to make sense to end users in practice. This issue is further exacerbated in hierarchical cases where a large number of topics and their relevance need to be modeled [Meng et al., 2020, Duan et al., 2021a].

In many cases, users with prior domain knowledge are concerned with a specific topic structure, as shown in Fig. 1. Such a topic hierarchy provides semantic common sense among topics and words, which can guide high-quality topic discovery. Therefore, several researches have attempted to exploit various prior knowledge into topic modeling to improve topic representation. For example, word correlation based topic models [Andrzejewski et al., 2009, Newman et al., 2011] use must-links and cannot-links to improve the interpretability of learned topics. Word semantic methods [Jagarlamudi et al., 2012, Doshi-Velez et al., 2015] allow users to predefine a set of seed words that could appear together in a certain topic to improve topic modeling. Furthermore, knowledge graph based topic models [Hu et al., 2016, Yao et al., 2017] combine LDA with entity vectors to inject multi-relational knowledge, achieving better semantic coherence. Although the above mentioned methods improve topic coherence in different ways, they only work with shallow topic models and ignore the relationships between topics.

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Figure 1: Motivation of the proposed model. We use the pre-specified topic hierarchy as the knowledge graph (left), where topics (colored nodes) and words (gray nodes) are organized into a tree-like taxonomy, and each topic is associated with a brief definition. Given the topic tree and the target corpus, our proposed model aims to *1)* retrieve a set of coherent words for each topic node (right); *2)* explore new topic hierarchies that are missed in the provided topic tree, e.g., the dashed boxes (right).

More recently, embedded topic models (ETMs) and their hierarchical variants [Dieng et al., 2020, Duan et al., 2021a] employ distributed embedding vectors to encode semantic relationship, which have gained growing research interest due to their effectiveness and better flexibility. Viewing words and topics as embeddings in a shared latent space makes it possible for ETMs to integrate prior knowledge in the form of pretrained word embeddings or semantic regularities [Dieng et al., 2020, Meng et al., 2020, Duan et al., 2021b]. The main idea behind those models is to constrain the topic embeddings under the predefined topic structure. Those models however are built on a strong assumption that the given topic hierarchy is well defined and matched to the target corpus, which is often unavailable in practice. Such mismatched structure might hinder the learning process and lead to sub-optimal results.

To address the above shortcomings, in this paper, we first propose TopicKG, a knowledge graph guided deep ETM that views topics and words as nodes in the predefined topic tree. As a Bayesian probabilistic model, TopicKG aims to model the document Bag-of-Word (BoW) vector and the given topic tree jointly by sharing the embedding vectors. Specifically, we first adopt a deep document generation model to discover multi-layer document representations and topics. To incorporate the human knowledge in the topic tree and guide the learning of topic hierarchy, a graph generative model is employed, which refines the node embeddings by exploring the belonging relations hidden in topic tree, resulting in more interpretable topics. Besides, to address the mismatch between the given topic tree and target corpus, we further extend TopicKG to TopicKGA, which adopts the graph **A**daptive technique to explore the missing links in topic tree from the target corpora, achieving better document representation. The final revised topic tree is obtained by combining the prior structure of the given topic tree with the added structures learned from the current corpus. Thus the topic tree in our TopicKGA has the ability to reinforces itself constantly according to the given corpora. Finally, both the proposed models are based on variational autoencoders (VAEs) [Kingma and Welling, 2014] and can be trained by maximizing the evidence lower bound (ELBO), enjoying promising flexibility and scalability.

The main contributions of this paper can be summarized as follows: *1)*, A novel knowledge-aware deep ETM named TopicKG is proposed to incorporate prior domain knowledge into hierarchical topic modeling by accounting for the document and the given topic tree in a Bayesian generative framework. *2)*, To overcome the drawback of the mismatch issue between the prior structure of the provided topic tree and the current corpus, TopicKG is further extended to TopicKGA, which allows to revise the prior structure to better represent the current document. *3)*, Besides achieving promising performance on hierarchical topic discovery and document classification task, the proposed models give good flexibility and efficiency, providing a strong baseline for knowledge-based TMs.

2 Related work

The recent related work has focused on two different knowledge forms, as described below.

Pre-trained linguistic knowledge Pre-trained language models such as BERT [Devlin et al., 2019] and GPT [Brown et al., 2020] have been successful in various natural language processing tasks. Pre-trained on huge text, such models can serve as a powerful encoder that outputs contextual semantic embeddings. Behind this idea, combining a TM with BERT via knowledge distillation, Hoyle et al. [2020] treat BERT as the teacher model to improve the representation of topic models. CombinedTM [Bianchi et al., 2021] concatenates the Bag-of-Word (BoW) vector with the BERT contextual embedding as the final input of a TM, providing a consistent increase in topic coherence. Despite their improvement, those BERT-based TMs focus on the document latent representation, they ignore the relationship between words and topics. Moreover they require a large amount of memory to load the pre-trained BERT.

Pre-defined structural knowledge Concept hierarchies are key knowledge sources, and can be obtained easily from knowledge graph (*e.g.*, WordNet [Miller, 1995]). Few generative models have been developed to hierarchical topic mining task. For example, Meng et al. [2020] use the category tree (usually with simple and shallow structure) as the supervised information and preserve such relative hierarchical structure in the spherical embedding space to explore user interested topic structure. Duan et al. [2021b] view words and topics as the Gaussian distributions and employ the asymmetrical Kullback–Leibler (KL) divergence to measure the directional similarity in the given topic hierarchy. However, computing such KL divergence for each pair is time-consuming that limits its application to large-scale knowledge. Moreover, those knowledge-based models ignore the mismatch problem between the given topic tree and the current corpus, leading to sub-optimal results in practice.

3 The proposed model

To incorporate human knowledge into deep TMs, we first propose TopicKG that generates the target corpus and the given topic tree in the Bayesian manner. We further extend TopicKG to its adaptive version TopicKGA that allows the topic structure to be refined according to the current corpus, resulting in a better document representation. Below we introduce the details of our proposed model.

3.1 Problem formulation

Consider a corpus containing J documents with a vocabulary of V unique tokens. Each document is represented by the BoW vector $\boldsymbol{x} \in R_+^V$, where x_v denotes the number of occurrences of the vth word. Unlike other TMs that are purely data-driven, TopicKG intends to inject the knowledge graph to improve topic quality. Specifically, we introduce a topic tree $\mathcal{T}$ as our prior knowledge, where each node $e_i \in \mathcal{T}$ denotes a word or a topic. For each topic node, there is a corresponding definition as shown in Fig. 1 . Suppose there are $L + 1$ layers in the topic tree with the bottom word layer and L topic layers, and there are K_l nodes $\mathcal{T}_l : \{e_1^{(l)}, ..., e_{K_l}^{(l)}\}$ at the l-th layer, where $K_0 = V$. Generally, for the k-th topic node at the l-th layer $e_k^{(l)}$, we use $\mathcal{S}(e_k^{(l)})$ to denote the set of its child nodes, and use $\mathcal{C}(e_k^{(l)})$ to denote its key words extracted from the definition sentence. Mathematically, we adopt the binary matrix $\boldsymbol{S}^{(l)} = \{0,1\}^{K_{l-1} \times K_l}$ and $\boldsymbol{C}^{(l)} = \{0,1\}^{V \times K_l}, l = 1, ..., L$, to denote the belonging relations between two adjacent layers and the links between the topics and their concept words, respectively. Guided by this pre-specified topic tree, TopicKG expects to retrieve hierarchical topics from the target corpus that provide a clear taxonomy for the end users.

3.2 Knowledge-aware Bayesian Deep Topic Model

Given the corpus $\mathcal{D}$ and topic tree $\mathcal{T}$, we aim to discover hierarchical document representations and topic hierarchy by jointly modeling the BoW vector $\boldsymbol{x}$, $\boldsymbol{S}^{(l)}$ and $\boldsymbol{C}^{(l)}, l = 1, ...L$ in a Bayesian framework. Specifically, we employ the gamma belief network (GBN) of Zhou et al. [2015a] to model the count vector, and use the semantic similarity between two node embeddings to generate

the topic tree $\mathcal{T}$. The whole generative model with L layers can be expressed as:

$$\boldsymbol{\theta}_j^{(L)} \sim \text{Gam}(\boldsymbol{\gamma}, 1/c_j^{(L+1)}), \quad \left\{\boldsymbol{\theta}_j^{(l)} \sim \text{Gam}(\boldsymbol{\Phi}^{(l+1)}\boldsymbol{\theta}_j^{(l+1)}, 1/c_j^{(l+1)})\right\}_{l=1}^{L-1},$$

$$\boldsymbol{x}_j \sim \text{Pois}(\boldsymbol{\Phi}^{(1)}\boldsymbol{\theta}_j^{(1)}) \quad \left\{\boldsymbol{\Phi}_k^{(l)} = \text{Softmax}(\boldsymbol{\Psi}_k^{(l)})\right\}_{l=1}^{L}, \quad \left\{\boldsymbol{\Psi}_{k_1 k_2}^{(l)} = e_{k_1}^{(l-1)^T} e_{k_2}^{(l)},\right\}_{k_1=1,k_2=1,l=1}^{K_{l-1},K_l,L},$$

$$\left\{S_{k_1,k_2}^{(l)} \sim \text{Bern}(\sigma(e_{k_1}^{(l-1)^T}\boldsymbol{W}e_{k_2}^{(l)})),\right\}_{k_1=1,k_2=1,l=1}^{K_{l-1},K_l,L}, \left\{C_{vk}^{(l)} \sim \text{Bern}(\sigma(e_v^{(0)^T}e_k^{(l)})),\right\}_{v=1,k=1,l=1}^{V,K_l,L},$$

$$\tag{1}$$

where $\boldsymbol{x}_j$ is generated as in Poisson factor analysis [Zhou et al., 2012], in which $\boldsymbol{x}_j$ is factorized as the product of the factor loading matrix (topics) $\boldsymbol{\Phi}^{(1)} \in \mathbb{R}_+^{K_0 \times K_1}$ and the gamma distributed factor scores (topic proportions) $\boldsymbol{\theta}_j^{(1)} \in \mathbb{R}_+^{K_1}$ under the Poisson likelihood; Then the GBN [Zhou et al., 2015b] is applied to explore multi-layer document representations by stacking hierarchical prior in the topic proportions $\boldsymbol{\theta}_j$. The topics at each layer $\boldsymbol{\Phi}^{(l)}$, $l = 1, ..., L$ is calculated by the semantic similarity (*e.g.*, the inner product) between corresponding topic embeddings, followed by a Softmax layer to satisfy the probability simplex: $\sum_{v=0}^{K_{l-1}} \boldsymbol{\Phi}_{vk}^{(l)} = 1$. $e_k^{(l)} \in R^d$ is the embedding vector of k-th topic at l-th layer (k-th node at l-th layer e_k^l in $\mathcal{T}$), d is the embedding dimension. $\sigma(\cdot)$ is the sigmoid function and $\text{Bern}(\cdot)$ denotes the Bernoulli distribution that is employed to model the two edge types $\boldsymbol{S}^{(l)}$ and $\boldsymbol{C}^{(l)}$. As suggested in Meng et al. [2020] and Duan et al. [2021b], we use asymmetry to capture the directed relations between two adjacent layers in $\boldsymbol{S}^{(l)}$, and use symmetry to capture the semantic similarity between topics and its concept words. $\boldsymbol{W}$ is a learnable parameter to guarantee the directed structure. Under the Bernoulli likelihood, our TopicKG has the ability to incorporate the topic hierarchical relations in $\mathcal{T}$ into topic modeling by sharing word and topic embeddings with $\boldsymbol{\Phi}$. In particular, for a node pair in $\mathcal{T}$, the embedding semantics of the source and destination nodes need to be similar enough to determine whether there is an edge, which will guide the learning of $\boldsymbol{\Phi}$, resulting in more coherent topics and some appealing model properties as described below.

The flexibility of human knowledge incorporation: As mentioned above, guided by the pre-defined topic tree, TopicKG models $\boldsymbol{x}$, $\boldsymbol{S}^{(l)}$ and $\boldsymbol{C}^{(l)}$, $l = 1, ...L$ jointly, making it possible for the learned hierarchical topics that not only satisfy the interpretable taxonomy structure but also prevent incoherent issues with the help of side information. Besides, the incorporation of $\boldsymbol{S}$ is simple and flexible. First of all, as shown in Eq. 1 and Sec. 4, the plug-and-play module of $\boldsymbol{S}$ can be applied to most of ETMs without changing their model structures, which provides a convenient alternative for introducing side information to TMs. Secondly, the pre-defined matrix $\boldsymbol{S}$ can be constructed in various ways so as to encode our beliefs about the graph structure, *e.g.*, the knowledge graph, taxonomy, and hierarchical label tree [Yao et al., 2017, Hu et al., 2016, Meng et al., 2020].

The relationship between $\{\boldsymbol{\Phi}^{(l)}\}_{l=1}^L$ and $\{\boldsymbol{S}^{(l)}\}_{l=1}^L$: The topic distribution matrix $\boldsymbol{\Phi}^{(l)}$ and the symmetry matrix $\boldsymbol{S}^{(l)}$ are related but play different roles. The sparse structure matrix $\boldsymbol{S}^{(l)}$ is constructed from the topic tree which will be used to guide the learning of the hierarchy of topics, while the learned topics $\boldsymbol{\Phi}^{(l)}$ contains more specific knowledge extracted from the text corpus, and can be viewed as the dense version of $\boldsymbol{S}^{(l)}$. In other words, the topics learned from TopicKG retains the semantic structure of $\boldsymbol{S}^{(l)}$ and enriches itself by the current corpora.

3.3 Encoder

The encoder of the proposed model is built around two main components: Weibull upward-downward variational encoder and the GCN-based topic aggregation module.

Weibull upward-downward variational encoder: To approximate the posterior of the topic proportions $\{\boldsymbol{\theta}_j^{(l)}\}$, like most of VAE-based methods, we define the variational distribution $q(\boldsymbol{\theta}_j|\boldsymbol{x}_j)$, which can be further factorized as [Vahdat and Kautz, 2020]:

$$q(\boldsymbol{\theta}_j|\boldsymbol{x}_j) = q(\boldsymbol{\theta}_j^{(L)}|\boldsymbol{x}_j) \prod_{l=1}^{L-1} q(\boldsymbol{\theta}_j^{(l)}|\boldsymbol{\theta}_j^{(l+1)}, \boldsymbol{x}_j).$$

In practice, it first obtains the latent feature by feeding the input $\boldsymbol{x}_j$ into a residual upward neural networks: $\boldsymbol{h}_j^{(l)} = \boldsymbol{h}_j^{(l-1)} + f_{\boldsymbol{W}_h}^{(l)}(\boldsymbol{h}_j^{(l-1)})$, where $l = 1, ..., L$, $\boldsymbol{h}_j^{(0)} = \boldsymbol{x}_j$, $f_{\boldsymbol{W}_h}^{(l)}(\cdot)$ is a two layer fully connected network parameterized by $\boldsymbol{W}_h^{(l)}$. To complete the variational distribution, we adopt the Weibull downward stochastic path [Zhang et al., 2018]:

$$q(\boldsymbol{\theta}_j^{(l)}|\boldsymbol{\Phi}^{(l+1)}, \boldsymbol{h}_j^{(l)}, \boldsymbol{\theta}_j^{(l+1)}) = \text{Weibull}(\boldsymbol{k}_j^{(l)}, \boldsymbol{\lambda}_j^{(l)}) \quad with$$

$$\boldsymbol{k}_j^{(l)} = \text{Softplus}(f_k^{(l)}(\boldsymbol{\Phi}^{(l+1)}\boldsymbol{\theta}_j^{(l+1)} \oplus \hat{\boldsymbol{k}}_j^{(l)})) \quad \boldsymbol{\lambda}_j^{(l)} = \text{Softplus}(f_\lambda^{(l)}(\boldsymbol{\Phi}^{(l+1)}\boldsymbol{\theta}_j^{(l+1)} \oplus \hat{\boldsymbol{\lambda}}_j^{(l)})) \quad (2)$$

$$\hat{\boldsymbol{k}}_j^{(l)} = \text{Relu}(\boldsymbol{W}_k^{(l)}\boldsymbol{h}_j + \boldsymbol{b}_k^{(l)}), \quad \hat{\boldsymbol{\lambda}}_j^{(l)} = \text{Relu}(\boldsymbol{W}_k^{(l)}\boldsymbol{h}_j + \boldsymbol{b}_k^{(l)}),$$

where $\oplus$ denotes the concatenation at topic dimension, and we use the Softplus to make sure the positive Weibull shape and scale parameters. f is a single layer fully connected network, correspondingly. The Weibull distribution is chosen mainly because it is reparameterizable and the KL divergence from the gamma to Weibull distributions has an analytic expression [Zhang et al., 2018, Fan et al., 2020].

GCN-based topic aggregation: Attracted by the excellent ability of graph neural network (GCN) [Kipf and Welling, 2016a,b] in propagating graph structure information, and the predefined topic tree can be viewed as a directed graph. We thus construct a deterministic aggregating module for node embeddings with GCN:

$$\boldsymbol{E}^{(t)} = \text{GCN}(\tilde{\boldsymbol{A}}, \boldsymbol{E}^{(t-1)}), t = 1, ..., T \quad (3)$$

where T is the number of GCN layer, $\boldsymbol{E}^{(0)} \in R^{d \times N}$ is the node embedding matrix, and $N = \sum_{l=0}^{L} K_l$. $\tilde{\boldsymbol{A}} = \boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{A}\boldsymbol{D}^{-\frac{1}{2}}$ is the normalized adjacent matrix with degree matrix $\boldsymbol{D}$, $e.g.$, $\boldsymbol{D}_{ii} = \sum_{j=1}^{N} \boldsymbol{A}_{ij}$, $\boldsymbol{A} \in \{0,1\}^{N \times N}$ is the adjacent matrix of the topic tree $\mathcal{T}$. Unlike previous ETMs that view embeddings as independent learnable parameters, the GCN module in Top-

Figure 2: Overview of the proposed model. TopicKG contains a Weibull upward-downward variational encoder (right part), a GCN-based topic aggregation module(left-top part). It jointly models the topic tree and document via the shared node embeddings $\boldsymbol{E}$.

icKG updates the node embeddings by considering their child-level topics and related words, resulting in more meaningful topic embeddings. We calculate $\boldsymbol{\Phi}$ via the updated node embeddings:

$$\boldsymbol{\Phi}_k^{(l)} = \text{Softmax}(\boldsymbol{e}^{(T)^{(l-1)T}}\boldsymbol{e}_k^{(T)^{(l)}}) \quad (4)$$

3.4 TopicKG with adaptive structure

One of the main assumptions in the previous section is that the pre-defined topic tree is helpful for the current corpus and the corresponding edges are highly reliable. However, this is generally unrealistic in practical applications, as $\mathcal{T}$ may be (i) noisy, (ii) built on an ad hoc basis, (iii) not closely related to the topic discovering task [Elinas et al., 2020, Chen et al., 2020]. Consequently, we further propose TopicKGA that overcomes the above mismatching issue and revises the structure of the given topic tree based on the corpus at hand.

In detail, TopicKGA first randomly initializes a learnable node embedding dictionaries $\boldsymbol{E}_A \in R^{d \times N}$ for all nodes in $\mathcal{T}$. Then an adaptive graph $\boldsymbol{A}^{\text{ada}}$ is generated based on $\boldsymbol{E}_A$ using the certain kernel function $k : R^d \times R^d \longrightarrow R$:

$$\tilde{\boldsymbol{A}}^{\text{ada}} = \text{Softmax}(k(\boldsymbol{E}_A, \boldsymbol{E}_A)) \quad (5)$$

Here, we choose the consine similarity to define our kernel function: $k(\boldsymbol{E}_A, \boldsymbol{E}_A) = \frac{\boldsymbol{E}_A\boldsymbol{E}_A^T}{||\boldsymbol{E}_A||||\boldsymbol{E}_A||}$, and Softmax function is used to normalize the adaptive matrix. Note that, instead of generating $\boldsymbol{A}^{\text{ada}}$, we directly generate its normalized version to avoid unnecessary calculations [Bai et al., 2020]. During training, $\boldsymbol{E}_A$ will be updated automatically to learn the hidden dependencies which are ignored by $\boldsymbol{A}$.

Following Yu et al. [2020], the final revised adjacency matrix is formed as (here we still use $\boldsymbol{A}$ to denote the revised graph for convenience):

$$\tilde{\boldsymbol{A}} = \tilde{\boldsymbol{A}} + \tilde{\boldsymbol{A}}^{\mathrm{ada}} \tag{6}$$

which will enhance the GCN in Eq. 3 by replacing the adjacency matrix with the revised one.

4 Training

As a deep ETM, TopicKG intends to learn the latent document-topic distribution and the deterministic hierarchical topic embeddings. Like other ETMs, the posterior of the topic proportions and topic embeddings are intractable to compute. We thus derive an efficient algorithm for approximating the posterior with amortized variational inference [Blei et al., 2017], which makes the proposed model flexible for downstream task. The resulting algorithm can either use pre-trained word embeddings, or train them from scratch. The EBLO of the proposed model can be expressed as:

$$
\begin{aligned}
\mathcal{L} = \sum_{j=1}^{J} \mathrm{E}_{q(\boldsymbol{\theta}|\boldsymbol{x}_j)}[\log p(\boldsymbol{x}_j|\boldsymbol{\Phi}^{(1)}, \boldsymbol{\theta}_j^{(1)})] + \beta \sum_{l=1}^{L} \sum_{k_2=1}^{K_l} \sum_{k_1=1}^{K_{l-1}} \log p(\boldsymbol{S}_{k_1 k_2}^{(l)})|\boldsymbol{e}_{k_1}^{(l-1)}, \boldsymbol{e}_{k_2}^{(l)}) \\
+ \sum_{v=1}^{V} \log p(\boldsymbol{C}_{v k_2}^{(l)}|\boldsymbol{e}_v^{(0)}, \boldsymbol{e}_{k_2}^{(l)})) - \sum_{j=1}^{J} \sum_{l=1}^{L} \mathrm{KL}(q(\boldsymbol{\theta}_j^{(l)})||p(\boldsymbol{\theta}_j^{(l)}|\boldsymbol{\Phi}^{(l+1)}, \boldsymbol{\theta}_j^{(l+1)})))
\end{aligned} \tag{7}
$$

It consists three main parts: the expected log-likelihood of $\boldsymbol{x}$ (first term), the concept structure log-likelihood (the two middle terms), and the KL divergence from prior $p(\boldsymbol{\theta}_j^{(l)})$ to $q(\boldsymbol{\theta}_j^{(l)})$. The graph weight β denotes the belief about the predefined topic tree. Notably, the two middle terms distinguish the proposed models from previous deep ETMs. On the one hand, it acts as a regularization that constrains the embedding vectors to conform to the provided prior structure, and on the other hand, it provides an alternative for ETM to introduce side information to improve its interpretability.

Annealed training for TopicKGA: In TopicKGA, the structure of the revised graph changes during the training. To incorporate the new edges that are inferred from the target corpus, we develop an annealed training algorithm for TopicKGA. Specifically, we update $\boldsymbol{S}^{(l)}$ and $\boldsymbol{C}^{(l)}$ in Eq. 7 every M iterations:

$$
\boldsymbol{S}_{k_1 k_2}^{(l)} = \begin{cases} 1, & \text{if} \quad \tilde{\boldsymbol{A}}_{k_1 k_2}^{(l)} > s; \\ 0, & else \end{cases} \quad \boldsymbol{C}_{v k_2}^{(l)} = \begin{cases} 1, & \text{if} \tilde{\boldsymbol{A}}_{v k_2}^{(l)} > s; \\ 0, & else \end{cases} \tag{8}
$$

where s is the threshold, and $\tilde{\boldsymbol{A}}^{(l)}$ is the corresponding sub-block in $\tilde{\boldsymbol{A}}$. As $\boldsymbol{E}_A$ becomes stable, the structure of the graph will converge to a good blueprint that balances the prior graph and the current corpus. We summarize all training algorithm in Appendix.

5 Experiment

In this section, we conduct extensive experiments on several benchmark text datasets to evaluate the performance of the proposed models against other knowledge-based TMs, in terms of topic interpretability and document representations.

5.1 Corpora

Our experiments are conducted on four widely-used benchmark text datasets, varying in scale. The datasets include 20 Newsgroups (20NG) [Lang, 1995], Reuters extracted from the Reuters-21578 dataset, R8, and Reuters Corpus Volume 2 (RCV2) [Lewis et al., 2004], where 20NG, R8 and RCV2 are corpus that associate with the document label. R8 is a subset of Reuters that collected from 8 different classes. For the multi-label RCV2 dataset, we follow previous works [Miyato et al., 2017] in which documents with a single label at the second level topics are left, resulting in 0.1M documents totally. Both R8 and RCV2 are already pre-processed. For other two datasets, we tokenize and clean text by excluding standard stop words and low-frequency words appearing less than 20 times [Yao et al., 2019]. The statistics of the preprocessed datasets are summarized in Appendix.

Figure 3: Topic coherence (TC, top row), topic diversity (TD, middle row), and word embedding coherence (WE, bottom row) results for various deep topic models on four datasets. In each subfigure, the horizontal axis indicates the layer index of the topics. For all metrics, higher is better.

5.2 Baselines

To demonstrate the effectiveness of incorporating human knowledge into deep TMs, we consider several baselines for a fair comparison, including representative ETMs and recent knowledge-based topic models, described as follows: *1)*, **ETM** [Dieng et al., 2020], the first neural embedded topic model that assumes topics and words live in the same embedding space. The topic distributions in ETM are calculated by the inner product of the word embedding matrix and the topic embedding. *2)*, **CombinedTM** [Bianchi et al., 2021], a BERT-based neural topic model that uses the pre-trained BERT as its contextual encoder. We use it as our BERT baseline. *3)*, **SawETM** [Duan et al., 2021a], a hierarchical ETM that employs the Poisson and gamma distributions to model the BoW vector and the latent representation, respectively. *4)*, **JoSH** [Meng et al., 2020], a knowledge-based hierarchical topic mining method that uses category hierarchy as the side information and employs the EM algorithm to learn the spherical tree and text embedding. *5)*, **TopicNet** [Duan et al., 2021b], a semantic graph guided topic model that views the words and topics as the Gaussian distributions and uses the KL divergence to regularize the structure of the pre-defined tree. For all baselines, we employ their official codes and default settings obtained from their release repositories.

5.3 External knowledge

Since the learning of TopicKG involves a pre-defined topic tree, some generic knowledge graphs such as WordNet [Miller, 1995] provide a convenient way to bring in external knowledge. WordNet is a large lexical database that groups semantically similar words into synonym sets, these sets are further linked by the hyponymy relation. For example, in WordNet the category furniture includes bed, which in turn includes bunkbed. With these relations, the topic structure can be easily defined according to certain heuristic rules. Specifically, for each dataset we first get the word intersection of the vocabulary and WordNet, the chosen words are then considered as leaf nodes of the topic hierarchy. Afterwards, each leaf node can continuously find its ancestor nodes based on hyponymy relations and finally all leaf nodes converge to the same root node, resulting in an adaptive topic structure.

5.4 Settings

For all experiments, we set the embedding dimension as $d = 50$, the knowledge confidence hyper-parameter as $\beta = 50.0$, the threshold as $s = 0.4$. Empirically, we find that the above settings work

well for all datasets, we refer readers to the appendix for more detailed analysis. We initialize the node embedding from the Gaussian distribution $\mathcal{N}(0, 0.02)$. We set the batch size as 200 and use the AdamW [Loshchilov and Hutter, 2019] optimizer with learning rate 0.01. We generate a 7-layer topic hierarchy for each dataset with the method described in Sec. 5.3. From the top to bottom, the number of topics at each layer are: 20NG: [1,2,11,66,185,277,270]; R8: [1,2,11,77,287,501,547]; RCV2: [1,2,11,68,263,469,562]; Reuters: [1,2,11,78,300,526,584]. For the single-layer methods (ETM and CombinedTM), we set the number of topics as the same of the first layer of deep models. For all methods, we run the algorithms in comparison five times by modifying only the random seeds and calculate the mean and standard deviation. All experiments are performed on an Nvidia RTX 3090-Ti GPU and our proposed models are implemented with PyTorch.

Table 1: Micro F1 and Macro F1 score of different models on three datasets. The digits in brackets indicate the number of layers. Micro F1 /Macro F1.

Model	20NG	R8	RCV2
ETM	50.25 $\pm$0.42 / 47.44 $\pm$0.21	88.10 $\pm$0.45 / 59.67 $\pm$0.24	68.63 $\pm$0.15 / 24.40 $\pm$0.11
CombinedTM	56.43 $\pm$0.14 / 54.95 $\pm$0.11	93.69 $\pm$0.09 /84.14 $\pm$0.10	84.85 $\pm$0.11 / 51.47 $\pm$0.21
Sawtooth(3)	52.41 $\pm$0.08 / 51.53 $\pm$0.10	90.04 $\pm$0.15 / 78.84 $\pm$0.21	82.54 $\pm$0.11 / 49.25 $\pm$0.10
TopicNet(3)	55.16 $\pm$0.22 / 54.78 $\pm$0.34	89.95 $\pm$0.17 / 64.15 $\pm$0.16	84.15 $\pm$0.25 / 50.37 $\pm$0.22
TopicKG(3)	55.73 $\pm$0.15 / 54.48 $\pm$0.08	93.6 $\pm$0.05 / 83.32 $\pm$0.07	84.75 $\pm$0.16 / 50.51 $\pm$0.41
TopicKGA(3)	**58.63** $\pm$0.15 / **57.90** $\pm$0.10	**93.70** $\pm$0.52 / **84.50** $\pm$0.11	**85.34** $\pm$0.14 / **52.35** $\pm$1.10
ETM	47.79 $\pm$0.12 / 44.19 $\pm$1.01	86.54 $\pm$0.84 / 59.88 $\pm$1.11	63.77 $\pm$0.14 / 21.44 $\pm$1.04
CombinedTM	58.16 $\pm$0.15 / 58.10 $\pm$0.10	93.50 $\pm$0.13 /84.84 pm0.11	82.91 $\pm$0.11 / 48.17 $\pm$0.05
Sawtooth(7)	53.71 $\pm$0.11 / 53.02 $\pm$0.47	92.86 $\pm$0.07 / 82.54 $\pm$0.41	82.46 $\pm$0.15 / 49.34 $\pm$0.34
TopicNet(7)	56.13 $\pm$0.19 / 55.41 $\pm$0.39	90.65 $\pm$0.00 / 66.57 $\pm$0.00	82.81 $\pm$0.00 / 49.44 $\pm$0.00
TopicKG(7)	56.32 $\pm$0.12 / 57.35 $\pm$0.04	94.04 $\pm$0.12 / 85.04 $\pm$0.11	82.48 $\pm$0.11 / 48.24 $\pm$0.09
TopicKGA(7)	**60.04** $\pm$0.34 / **59.12** $\pm$0.13	**94.10** $\pm$0.08 / **85.50** $\pm$0.10	**83.08** $\pm$0.23 / **50.50** $\pm$0.08

5.5 Topic interpretability

Generally, topic models are evaluated based on perplexity. However, perplexity on the held-out test is not an appropriate measure of the topic quality and sometimes can even be contrary to human judgements [Chang et al., 2009, Yao et al., 2017]. To this end, we instead adopt three common metrics, including Topic coherence (**TC**), Topic diversity (**TD**) and word embedding topic coherence (**WE**), to evaluate the learned topics from various aspects [Dieng et al., 2020, Bianchi et al., 2021]. TC measures the average Normalized Pointwise Mutual Information (NPMI) over the top 10 words of each topic, and a higher score indicates more interpretable topics. TD denotes the percentage of unique words in the top 25 words of the selected topics. WE, as its name implies, provides an embedded measure of how similar the words in a topic are. WE is calculated by the average pairwise cosine similarity of the word embeddings of the top-10 words in a topic. We report the topic quality results at the first five layer in Fig. 3 [1]. We here focus on topic hierarchy and ignore the single layer methods. We find that: *1)*, Knowledge-based methods including JoSH, CombinedTM, TopicNet and our proposed models are generally better than other likelihood-based ETMs, which illustrates the benefit of incorporating side information; *2)*, Our proposed TopicKG and TopicKGA achieve higher performance than other knowledge-based models in most cases, especially on TC and WE tasks, which means our proposed model prefer to mine more coherent topics while achieving comparable TD. We attribute this to the joint topic tree likelihood that provides an efficient alternative to integrate human knowledge into ETMs. *3)*, Compared to TopicKG, its adaptive version TopicKGA discovers better topics at higher layers. It is not surprise that TopicKGA gives greater robustness and flexibility by refining the topic structure according to the current corpus.

5.6 Document classification

Besides the topic quality evaluation, we also use document classification to compare the extrinsic predictive performance. In detail, we collect the inferred topic proportions, $e.g.$, $\boldsymbol{\theta}^{(1)}$, and than apply logistic regression to predict the document label. We report the Micro F1 and Macro F1 score on 20NG, R8 and RCV2 datasets in Table. 1. Overall, the proposed models outperform the baselines on all corpora, which confirms the effectiveness of our innovation of combining human knowledge

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[1]We can't report the results of JoSH on RCV2 as JoSH requires the sequential text, but only the BoW form is available for RCV2.

Figure 4: The learned hierarchical topic structure from TopicKG (a) and TopicKGA(b) on 20NG. Each topic box contains its concept name and the corresponding keywords. Topics in dashed box (free topics) are newly added by TopicKGA from the target corpus. The thickness of the arrow represents the relation weight and different colors denote different layers.

and ETMs in improving document latent representations. Moreover, with the revised topic structure fine-tuned to the current corpus, TopicKGA surpasses other knowledge graph fixation methods with significant gaps. This conclusion is consistent with one of our motivations that the predefined concept tree may not match the target corpus, leading to suboptimal document representations.

5.7 Qualitative analysis

We visualize the learned topic hierarchies of TopicKG and TopicKGA on 20NG in Fig. 4(a-b), respectively. Each topic box consists of the pre-specified concept name on the top bar and its keywords listed in the bottom content. We can observe that *1)*, The mined keywords are highly relevant to the corresponding topics, providing a clean description of their concepts. *2)*, Guided by human knowledge, the connections between topics at two adjacent layer are highly interpretable, resulting in human-friendly topic taxonomies. *3)*, More interestingly, to further verify the adaptive ability of our proposed TopicKGA, we manually added several free topics (dashed boxes in Fig. 4(b)) to each layer of the given topic tree, which can be viewed as the missing topics in the predefined knowledge and need to be learned from the target corpus. We find that our TopicKGA indeed has the ability to capture the missing concepts, and revise the prior graph to match the current corpus.

Figure 5: Negative log-likelihood curves of various methods on 20NG.

5.8 Time efficiency

To demonstrate the time efficiency of incorporating human knowledge into TMs, we run TopicKG, TopicKGA and TopicNet (as they are implemented in PyTorch, JoSH is in C) on 20NG dataset with a 7-layer topic tree. Fig. 5 shows the Negative log-likelihood of documents x, which shows that the proposed models not only have faster learning speed than TopicNet, but also achieve better reconstruction performance. This result illustrates the efficiency of the introduced graph likelihood which is important in real-time applications.

6 Conclusion

We develop an efficient Bayesian probabilistic model that integrates pre-specified domain knowledge into hierarchical ETMs. The core idea is the joint document and topic tree modeling with the shared word and topic embeddings. Besides, with the graph adaptive technique, TopicKGA has the ability to revise the given prior structure according to the target corpus, enjoying robustness and flexibility in practice. Extensive experiments show that the proposed models outperform competitive methods in term of both topic quality and document classification task. Moreover, thanks to the efficient knowledge incorporation algorithm, our proposed models achieve faster learning speed than other knowledge-base models.

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