Reliable Evaluation Protocol for Low-Precision Retrieval

Anonymous ACL submission

Abstract

Lowering the numerical precision of model parameters and computations is widely adopted to improve the efficiency of retrieval systems. However, when computing relevance scores between the query and documents in low-precision, we observe *spurious ties* due to the reduced granularity. This introduces high variability in the results based on tie resolution, making the evaluation less reliable. To address this, we propose a more robust retrieval evaluation protocol designed to reduce score variation. It consists of: (1) High-Precision Scoring (HPS), which upcasts the final scoring step to higher precision to resolve tied candidates with minimal computational cost; and (2) Tie-aware Retrieval Metrics (TRM), which report expected scores, range, and bias to quantify order uncertainty of tied candidates. Our experiments test multiple models with three scoring functions on twelve retrieval datasets to demonstrate that HPS dramatically reduces tie-induced instability, and TRM accurately recovers expected metric values. This combination enables a more consistent and reliable evaluation system for lower-precision retrieval.[1]

1 Introduction

Recent studies on low-precision techniques have been widely explored (e.g., quantization and compression) to enhance the efficiency and scalability of neural networks while reducing computational cost (Nagel et al.; Kurtic et al., 2024; Zhu et al., 2024; Hao et al., 2025). Without sacrificing performance, these methods aim to lower the numerical precision of model weights, gradients, and activations in training and inference, along with the retrieval stage (Choi et al., 2024; Lee et al., 2025) of retrieval-augmented generation (RAG) (Wang et al., 2024; Zhang et al., 2024, 2025). To generate informative responses, retrieving accurate candidates is crucial; otherwise, the following stages may be negatively affected and result in incoherent outputs (Chen et al., 2024b; Yadav et al., 2024; Sharma, 2025).

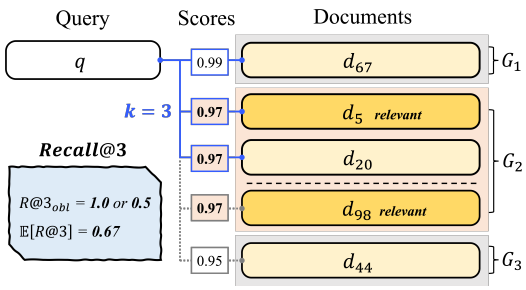

Figure 1: Example of tie-induced instability in evaluation metric. Three documents share the same score (G_2); two of them are relevant to the query. A tie-oblivious evaluation arbitrarily breaks the tie, so the reported R@3 depends on a random internal ordering. Instead, the tie-aware formulation deterministically reports the expectation over all permutations within the tie.

In neural retrieval systems, however, lowering numerical precision (e.g., `FP32` to `FP16`) inevitably reduces the granularity of representable floating point numbers (Shen et al., 2024; Hu et al., 2025) (see Appendix A); this coarser grid produces *spurious ties* among candidates by forcing many distinct relevance scores to quantize to the same value. Though resolving this issue can significantly affect evaluation scores (Figure 1), current mainstream retrieval evaluation systems (e.g., MTEB[2] (Muennighoff et al., 2023)) do not provide any principled mechanism for handling ties. Instead, they truncate the ranked list based on an arbitrary order (e.g., document IDs), which increases variances in results.

Thus, we propose a reliable evaluation protocol for low-precision retrieval. It is composed of (1) *High-Precision Scoring* (HPS) and (2) *Tie-aware Retrieval Metrics* (TRM). HPS upcasts the last scoring function into higher precision, to collapse spuri-

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[1]The source code will be available after the review period.

[2]`https://github.com/embeddings-benchmark/mteb`

ous ties (§ 2.2). TRM is an expectation-based evaluation augmented with extrema (i.e., maximum and minimum achievable scores) to quantify the order uncertainty of tied candidates (§ 2.3).

Our experiments demonstrate that evaluating low-precision models using conventional tie-oblivious metrics leads to misleading outcomes as shown in Figure 1. Adopting HPS significantly reduces score range variability, reducing MRR@10 range by 36.82%p. Meanwhile, TRM exposes biases inherent in tie-oblivious metrics, highlighting systematic overestimation by up to +9.08%p in BF16 evaluations. By contrast, our combined approach recovers near-FP32 stability and ordering, offering a consistent and discriminative framework for evaluating retrieval models in low-precision settings.

2 Reliable Evaluation Protocol

We first formalize the vulnerability of the current tie-oblivious evaluation, and then present *High-Precision Scoring* and *Tie-aware Retrieval Metric*. (See Appendix A for preliminaries.)

2.1 Spurious Ties in Low-Precision Evaluation

Let z denote the output of the linear layer after the last hidden state h. If the scoring function ϕ is softmax or sigmoid, then the cross-encoder takes the concatenated query and i-document pair $(q; d_i)$ as input and produces the logits z_i: two scalar values for softmax, or a single scalar value for sigmoid. If ϕ is a pairwise product, z_i denotes the pair of embeddings (h_q, h_{d_i}) obtained by encoding the query q and the document d_i independently with a bi-encoder. We denote the query-document relevance score $\tilde{s}_i$ as:

$$\tilde{s}_i = \phi^{(B)}(z_i) \tag{1}$$

where $\phi^{(B)}$ indicates that ϕ is operated entirely in a B-bit mantissa format.

Applied with low-precision inference (e.g., `BF16` (Burgess et al., 2019), `FP16`, etc), this maps theoretically continuous values onto a discrete set of representable scores; distinct true scores may collide: $\tilde{s}_i = \tilde{s}_j$ even with $z_i \neq z_j$, creating a *tie*. After sorting by $\tilde{s}$, we obtain ordered tie groups G_n consisting of scores s_i equivalent to v_n:

$$G_n = \{i \mid \tilde{s}_i = v_n\}. \tag{2}$$

If the relevant document at cutoff rank k falls inside a tie group G_n where $|G_n| \geq 2$, Any evaluation that disregards ties (*tie-oblivious*) may become stochastic and yield unpredictable results, as shown in Figure 1.

2.2 High-Precision Scoring (HPS)

Scoring functions such as softmax, sigmoid, and pairwise product compress logits into a narrow range. This effect is exacerbated under lower-precision formats due to fewer representable values resulting in coarser bucketization in $(0, 1)$ range (see examples in Appendix B).

For lower-precision models, HPS upcasts only the final scoring operation to `FP32`, leaving other layers unchanged, as defined in Appendix C. Concretely we replace the low-precision scoring function (Equation 1) with a higher-precision scoring function:

$$\hat{s}_i = \phi(\text{upcast}(z_i)), \tag{3}$$

and retain a more fine-grained score $\hat{s}_i$ for document candidate sorting. This significantly reduces the probability of tie collisions while preserving latency, since only a small logits tensor is upcast, requiring no re-training.

Advantages. HPS (i) leaves the forward pass intact and upcasts logits right before scoring, (ii) adds negligible memory and time overhead as described in Appendix E, (iii) collapses large tie groups, and (iv) restores alignment with deterministic and high-precision production sorting.

2.3 Tie-aware Retrieval Metric (TRM)

Existing tie-oblivious evaluation methods truncate the sorted list after a predefined cutoff k. If multiple candidates receive the same score, they are ordered arbitrarily before truncation, affecting which items are included in the top-k set. As a result, the evaluation results may vary depending on how ties are resolved as illustrated in Figure 1. To mitigate this problem, TRM supplies exact *expectations*, *range*, and a *bias*.

Expected Score. Let $G_1, \ldots, G_N$ be tie groups sorted in descending order, where $|G_n|$ is the group size and r_n is the number of relevant items. Following prior work (McSherry and Najork, 2008), we compute the expectation $\mathbb{E}[M]$ of an evaluation metric M in closed form. We then use it as a diagnostic reference to quantify ordering sensitivity via the *score range* and implementation-specific deviation due to tie breaking via the *score bias*. Explicit formulas are presented in Appendix D; the linear time complexity is analyzed in Appendix E.

Models	FP32	BF16				BF16 → FP32 (+HPS)			
	M	M_{obl}	$\mathbb{E}[M]$	Range(▼)	Bias(▼)	M_{obl}	$\mathbb{E}[M]$	Range(▼)	Bias(▼)
MIRACLReranking, $M = nDCG@10$									
Qwen3-Reranker-0.6B♣	73.53	75.04	68.38	25.59	6.66	73.59	73.35	**1.13**	**0.24**
bge-reranker-v2-m3◇	74.61	75.59	74.54	3.90	1.05	74.63	74.57	**0.16**	**0.06**
gte-multilingual-reranker-base◇	74.14	74.48	74.22	0.97	0.26	74.39	74.34	**0.14**	**0.05**
Qwen3-Embedding-0.6B♠	63.94	64.52	63.98	1.90	0.54	64.01	64.01	**0.00**	**0.00**
multilingual-e5-large-large♠	64.78	65.70	64.81	4.62	0.89	64.80	64.80	**0.00**	**0.00**
MIRACLReranking, $M = MRR@10$									
Qwen3-Reranker-0.6B♣	77.48	78.45	69.37	38.03	9.08	77.43	77.22	**1.21**	**0.21**
bge-reranker-v2-m3◇	79.58	80.68	79.17	6.72	1.51	79.66	79.56	**0.19**	**0.10**
gte-multilingual-reranker-base◇	79.39	79.75	79.47	0.85	0.28	79.59	79.52	**0.18**	**0.07**
Qwen3-Embedding-0.6B♠	68.97	69.54	68.91	2.23	0.63	69.02	69.02	**0.00**	**0.00**
multilingual-e5-large-large♠	71.37	71.84	71.28	4.61	0.56	71.18	71.18	**0.00**	**0.00**
AskUbuntuDupQuestions, $M = MAP@3$									
Qwen3-Reranker-0.6B♣	31.20	33.28	31.13	4.03	2.15	31.58	31.29	**0.57**	**0.29**
bge-reranker-v2-m3◇	31.91	32.26	31.83	0.83	0.43	31.89	31.84	**0.09**	**0.05**
gte-multilingual-reranker-base◇	30.83	31.23	30.75	0.93	0.48	30.69	30.67	**0.03**	**0.02**
Qwen3-Embedding-0.6B♠	29.54	30.10	29.65	0.87	0.45	29.69	29.69	**0.00**	**0.00**
multilingual-e5-large-large♠	29.13	31.31	29.47	3.54	1.84	29.70	29.70	**0.00**	**0.00**

Table 1: Results using metric M with its tie-oblivious version (M_{obl}), expectation ($\mathbb{E}[M]$), range ($M_{\max} - M_{\min}$), and bias ($M - \mathbb{E}[M]$) on MIRACLEReranking (nDCG@10 and MRR@10) and AskUbuntuDupQuestions (MAP@3) under three precision regimes, full FP32, BF16, and BF16→FP32 (with High-Precision Scoring). In full FP32 we empirically observe $M_{obl} = \mathbb{E}[M]$ with zero range and bias, so only M is shown. ♣, ◇, and ♠ indicate softmax, sigmoid, and pairwise product, respectively. Lower range and |bias| scores represent better stability.

Score Range. $M_{\max}$ places the query-relevant items in each partially included tie group as early as possible; $M_{\min}$ as late as possible. For each example i, we report the average range over I examples:

$$\text{Range}(M) = \frac{1}{I}\sum_{i=1}^{I}(M_{\max,i} - M_{\min,i}). \quad (4)$$

This metric quantifies uncertainty due solely to unresolved internal orderings. A smaller range indicates that results are more stable and reliable.

Score Bias. Let $M_{\text{obl},i}$ denote the tie-oblivious score for example i obtained using the original implementation's fixed (typically index-preserving) ordering. We define the score bias as

$$\text{Bias}(M) = \frac{1}{I}\sum_{i=1}^{I}(M_{\text{obl},i} - \mathbb{E}[M]_i). \quad (5)$$

A large positive bias implies that M_{obl} tends to overestimate the expected scores, while negative values indicate underestimation.

Reporting Protocol. For each cutoff k (or full ranking if required), we propose to report the expectation value and the range of score variance:

$$\big(\mathbb{E}[M],\ \text{Range}(M)\big), \quad (6)$$

optionally reporting the tie-oblivious value M_{obl}, discrepancy $\text{Bias}(M)$, the extrema $M_{\max}$ and $M_{\min}$. With expectation and range values, our proposed reporting protocol enables more reliable evaluation.

Models	ϕ	Size
Qwen3-Reranker-0.6B (Zhang et al., 2025)	Softmax♣	596M
bge-reranker-v2-m3 (Chen et al., 2024a)	Sigmoid◇	568M
gte-multilingual-reranker-base (Zhang et al., 2024)	Sigmoid◇	306M
Qwen3-Embedding-0.6B (Zhang et al., 2025)	Product♠	596M
multilingual-e5-large (Wang et al., 2024)	Product♠	560M

Table 2: Models used in our experiments and their corresponding scoring function and size.

3 Experiments

We evaluate to what degree our proposed evaluation protocol exposes and corrects reliability failures of existing tie-oblivious evaluation.

3.1 Experimental Setting

More detailed explanations of experimental settings and implementation are presented in Appendix F.

Models. We cover five models widely used in reranking and embedding with three prevalent scoring functions: Softmax♣, sigmoid◇, and pairwise product♠ as in Table 2.

Evaluation Metric. We evaluate the standard ranking metrics nDCG (Järvelin and Kekäläinen, 2002), MRR (VOORHEES, 2000), MAP (Salton, 1983), and Recall. Results for all metrics are deferred to Appendix G.

Datasets. We primarily utilize two publicly available datasets, **MIRACLReranking** (Zhang et al., 2023) and **AskUbuntuDupQuestions** (Lei et al.,

2016) that each supplies a fixed set of candidates per query. This enables us to assess the second-stage reranker or retriever independent of effects from the first-stage retriever. Further, we extend our experiments to the **MTEB-R benchmark** in Appendix H.

3.2 Results

Spurious Ties in Low-Precision Evaluation. When using full `BF16`, the results display significant uncertainty as shown in Table 1. Especially, `Qwen3-Reranker` model with softmax♣ shows the highest variation — 25.59%p in nDCG@10 and 38.03%p in MRR@10. These ranges exceed the margins typically used to distinguish model superiority.

Crucially, a striking decision error appears. Under the `BF16` and nDCG@10_{obl} evaluation, `Qwen3-Reranker` seems to beat `gte` (75.04 > 74.48). However, tie-aware metric $\mathbb{E}$[nDCG@10] flips the ranking (68.38 < 74.22), and our proposed protocol (HPS + TRM) confirms the reversal (73.35 < 74.34) within a narrow range, rendering the naive evaluation rankings unreliable.

Albeit bias can be positive or negative, all `BF16` biases are positive, implying that tie-oblivious M_{obl} is overestimated (up to +9.08%p). This positive trend is likely a result of errors in dataset construction, coupled with deterministic tie-breaking, as the positive items are more consistently placed earlier in the dataset to create preferential tie groups.

High-Precision with Low-Cost. High precision scoring (HPS) collapses the large tie groups while keeping the bulk of computation in `BF16`. Softmax♣ ranges shrink from 25.59 to 1.13%p at nDCG@10 and from 38.03 to 1.21%p at MRR@10; sigmoid♢ model ranges drop roughly an order of magnitude (e.g., 3.90 to 0.16%p in nDCG@10); pairwise product♠ models become perfectly deterministic (range = bias = 0). The remaining softmax residual range (∼ 1%p) lies within ordinary inter-model differences, making rank reversals highly unlikely.

Compared to full `FP32` inference (stable but computationally costlier), HPS recovers near-`FP32` stability and ordering with negligible time and space overhead, as described in Appendix E. Consequently, while pure low-precision scoring erodes evaluation reliability, adopting our protocol, HPS with reporting ($\mathbb{E}[M]$, Range), restores precise and discriminative comparisons. We further demonstrate the superiority of this protocol over alternative baselines in Appendix I.

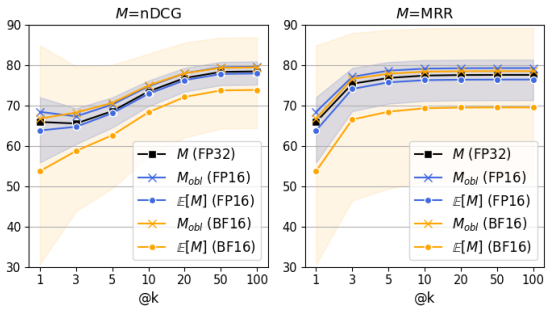


Figure 2: Tie-oblivious and expectation scores of nDCG and MRR at k of `Qwen3-Reranker-0.6B`♣ model when scored with each **dtype** on MIRACLReranking.

Impact of Precision across Cutoffs. Figure 2 shows nDCG and MRR metrics across various k-rank cutoffs, illustrating increased variance ranges and biases under lower-precision computations. Consistent with our observations in Appendix A, the `BF16` inference displays significant fluctuations and uncertainty (wide shaded areas), whereas `FP16` demonstrates intermediate stability, and `FP32` offers empirically stable results with negligible ties. This reflects the coarser bucketization induced by fewer mantissa bits in lower-precision formats (`BF16` ≪ `FP16` ≪ `FP32`).

Notably, under M_{obl}, the `BF16` curves surpass the `FP32` baseline at every cutoff. Such results would incorrectly indicate better performance, highlighting the unreliability of tie-oblivious evaluation due to reduced precision. Conversely, the tie-aware expectation $\mathbb{E}[M]$ consistently places `BF16` below `FP32`, accurately reflecting the true model performance, shown in Appendix G.

4 Conclusion

We demonstrate that current retrieval evaluations under low-precision settings overlook tied candidates, resulting in unstable outcomes. To address this, we proposed two concise yet effective remedies: High-Precision Scoring (HPS) and Tie-aware Retrieval Metrics (TRM). HPS upcasts the final scoring function to collapse spurious ties with negligible cost, and TRM reports the expectation value of scores with range and bias. Our proposed combination mitigates spurious ties across precision formats and provides a more reliable alternative to previous naive methods. Our method enables more stable document retrieval in tasks such as retrieval augmented generation (RAG), while preserving the efficiency and memory savings offered by low-precision models.

Limitations

Our remedy targets the inference stage and does not explore how low-precision training influences ranking stability, nor whether mixed-precision training combined with HPS inference yields further gains. Finally, TRM's outputs, expectations with ranges, are richer than single scalars, yet we have not conducted user-centered studies to assess their interpretability in practical evaluation pipelines.

References

Neil Burgess, Jelena Milanovic, Nigel Stephens, Konstantinos Monachopoulos, and David Mansell. 2019. Bfloat16 processing for neural networks. In *2019 IEEE 26th Symposium on Computer Arithmetic (ARITH)*, pages 88–91. IEEE.

Jianlv Chen, Shitao Xiao, Peitian Zhang, Kun Luo, Defu Lian, and Zheng Liu. 2024a. Bge m3-embedding: Multi-lingual, multi-functionality, multi-granularity text embeddings through self-knowledge distillation. *Preprint*, arXiv:2402.03216.

Weijie Chen, Ting Bai, Jinbo Su, Jian Luan, Wei Liu, and Chuan Shi. 2024b. Kg-retriever: Efficient knowledge indexing for retrieval-augmented large language models. *CoRR*.

Chanyeol Choi, Junseong Kim, Seolhwa Lee, Jihoon Kwon, Sangmo Gu, Yejin Kim, Minkyung Cho, and Jy-yong Sohn. 2024. Linq-embed-mistral technical report. *arXiv preprint arXiv:2412.03223*.

Zhiwei Hao, Jianyuan Guo, Li Shen, Yong Luo, Han Hu, Guoxia Wang, Dianhai Yu, Yonggang Wen, and Dacheng Tao. 2025. Low-precision training of large language models: Methods, challenges, and opportunities. *arXiv preprint arXiv:2505.01043*.

Weiming Hu, Haoyan Zhang, Cong Guo, Yu Feng, Renyang Guan, Zhendong Hua, Zihan Liu, Yue Guan, Minyi Guo, and Jingwen Leng. 2025. M-ant: Efficient low-bit group quantization for llms via mathematically adaptive numerical type. In *2025 IEEE International Symposium on High Performance Computer Architecture (HPCA)*, pages 1112–1126. IEEE.

Kalervo Järvelin and Jaana Kekäläinen. 2002. Cumulated gain-based evaluation of ir techniques. *ACM Transactions on Information Systems (TOIS)*, 20(4):422–446.

Eldar Kurtic, Alexandre Marques, Shubhra Pandit, Mark Kurtz, and Dan Alistarh. 2024. " give me bf16 or give me death"? accuracy-performance trade-offs in llm quantization. *arXiv preprint arXiv:2411.02355*.

Jinhyuk Lee, Feiyang Chen, Sahil Dua, Daniel Cer, Madhuri Shanbhogue, Iftekhar Naim, Gustavo Hernández Ábrego, Zhe Li, Kaifeng Chen, Henrique Schechter Vera, and 1 others. 2025. Gemini embedding: Generalizable embeddings from gemini. *arXiv preprint arXiv:2503.07891*.

Tao Lei, Hrishikesh Joshi, Regina Barzilay, Tommi Jaakkola, Kateryna Tymoshenko, Alessandro Moschitti, and Lluís Màrquez. 2016. Semi-supervised question retrieval with gated convolutions. In *Proceedings of the 2016 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pages 1279–1289.

Frank McSherry and Marc Najork. 2008. Computing information retrieval performance measures efficiently in the presence of tied scores. In *European conference on information retrieval*, pages 414–421. Springer.

Niklas Muennighoff, Nouamane Tazi, Loic Magne, and Nils Reimers. 2023. Mteb: Massive text embedding benchmark. In *Proceedings of the 17th Conference of the European Chapter of the Association for Computational Linguistics*, pages 2014–2037.

Markus Nagel, Marios Fournarakis, Rana Ali Amjad, Yelysei Bondarenko, Mart van Baalen, and Tijmen Blankevoort. A white paper on neural network quantization. arxiv 2021. *arXiv preprint arXiv:2106.08295*, 4.

Gerard Salton. 1983. Modern information retrieval. *(No Title)*.

Chaitanya Sharma. 2025. Retrieval-augmented generation: A comprehensive survey of architectures, enhancements, and robustness frontiers. *arXiv preprint arXiv:2506.00054*.

Haihao Shen, Naveen Mellempudi, Xin He, Qun Gao, Chang Wang, and Mengni Wang. 2024. Efficient post-training quantization with fp8 formats. *Proceedings of Machine Learning and Systems*, 6:483–498.

EM VOORHEES. 2000. The trec-8 question answering track report. In *Proc. Eighth Text REtrieval Conference (TREC-8), NIST Special Publication 500-246*, pages 77–82.

Liang Wang, Nan Yang, Xiaolong Huang, Linjun Yang, Rangan Majumder, and Furu Wei. 2024. Multilingual e5 text embeddings: A technical report. *arXiv preprint arXiv:2402.05672*.

Wikipedia contributors. 2025. Bfloat16 floating-point format.

Neemesh Yadav, Sarah Masud, Vikram Goyal, Md Shad Akhtar, and Tanmoy Chakraborty. 2024. Tox-bart: Leveraging toxicity attributes for explanation generation of implicit hate speech. In *ACL (Findings)*.

Xin Zhang, Yanzhao Zhang, Dingkun Long, Wen Xie, Ziqi Dai, Jialong Tang, Huan Lin, Baosong Yang, Pengjun Xie, Fei Huang, and 1 others. 2024. mgte: Generalized long-context text representation and reranking models for multilingual text retrieval. In

Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing: Industry Track, pages 1393–1412.

Xinyu Zhang, Nandan Thakur, Odunayo Ogundepo, Ehsan Kamalloo, David Alfonso-Hermelo, Xiaoguang Li, Qun Liu, Mehdi Rezagholizadeh, and Jimmy Lin. 2023. Miracl: A multilingual retrieval dataset covering 18 diverse languages. *Transactions of the Association for Computational Linguistics*, 11:1114–1131.

Yanzhao Zhang, Mingxin Li, Dingkun Long, Xin Zhang, Huan Lin, Baosong Yang, Pengjun Xie, An Yang, Dayiheng Liu, Junyang Lin, Fei Huang, and Jingren Zhou. 2025. Qwen3 embedding: Advancing text embedding and reranking through foundation models. *arXiv preprint arXiv:2506.05176*.

Xunyu Zhu, Jian Li, Yong Liu, Can Ma, and Weiping Wang. 2024. A survey on model compression for large language models. *Transactions of the Association for Computational Linguistics*, 12:1556–1577.

A Preliminaries

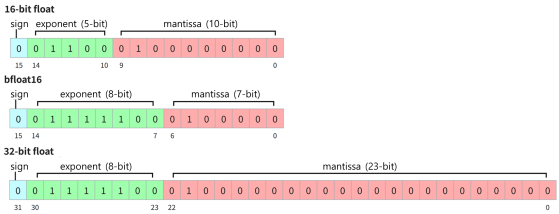


Figure 3: Bit layouts of FP16, BF16, and FP32 formats (Wikipedia contributors, 2025)

Floating-Point Value. A floating-point value is a way to represent numbers in computer systems, and typically encoded as three fields—*sign*, *exponent*, and *mantissa* (also called the fraction)—as illustrated in Figure 3. The exponent determines the dynamic range, the largest and smallest magnitudes that can be represented, whereas the mantissa governs the *precision* attainable within that range. Since a shorter mantissa implies coarser quantization, multiple real numbers inevitably collapse into the same representable bin, producing tied values.

After the common 1-bit sign, FP16 allocates 5 exponent bits and 10 mantissa bits, BF16 uses 8 and 7 bits respectively, and FP32 retains 8 exponent bits alongside a much longer 23-bit mantissa. By preserving the full 8-bit exponent of FP32, BF16 inherits the same dynamic range as single precision, which is widely credited with stabilizing training and thereby aiding generalization.

However, when outputs are confined to the range $(0, 1)$—as with the probabilities emitted by softmax or sigmoid scoring functions—the short 7-bit mantissa of BF16, and to a lesser extent the 10-bit mantissa of FP16, sharply reduces resolution. This loss of granularity, particularly severe in BF16, exacerbates the tied-score phenomenon and makes it difficult to distinguish among retrieval candidates that quantize to identical values.

B Examples of Relevance Scores

The example lists below show raw relevance scores for the first query of the MIRACLReranking test split produced by the `Qwen3-Reranker-0.6B` model where relevant values for the given are in **bold**. The first list (`scores_bf16`) is obtained with both the model and scoring function executed entirely in BF16, while the second (`scores_hps`) applies High Precision Scoring (HPS). Tie group sizes shrink considerably under HPS.

```
scores_bf16 = [
1.00000000,  1.00000000,  1.00000000,  1.00000000,  1.00000000,
1.00000000,  1.00000000,  1.00000000,  1.00000000,  1.00000000,
0.99609375,  0.99609375,  0.99609375,  0.99609375,  0.99609375,
0.99609375,  0.99609375,  0.99609375,  0.99609375,  0.99609375,
0.99609375,  0.99609375,  0.99609375,  0.99609375,  0.99609375,
0.99609375,  0.99609375,  0.99609375,  0.99609375,  0.99218750,
0.99218750,  0.99218750,  0.99218750,  0.99218750,  0.99218750,
0.99218750,  0.99218750,  0.99218750,  0.99218750,  0.99218750,
0.99218750,  0.99218750,  0.98828125,  0.98828125,  0.98828125,
0.98828125,  0.98828125,  0.98828125,  0.98828125,  0.98437500,
0.98437500,  0.98046875,  0.97656250,  0.97656250,  0.97265625,
0.96875000,  0.96875000,  0.96875000,  0.96875000,  0.96875000,
0.96875000,  0.96093750,  0.96093750,  0.96093750,  0.96093750,
0.95703125,  0.95703125,  0.95703125,  0.95703125,  0.95703125,
0.95312500,  0.95312500,  0.94921875,  0.94921875,  0.94531250,
0.94531250,  0.94531250,  0.94140625,  0.94140625,  0.93359375,
0.92578125,  0.92578125,  0.91796875,  0.91406250,  0.88671875,
0.88671875,  0.87890625,  0.87890625,  0.87500000,  0.86718750,
0.77734375,  0.60937500,  0.51562500,  0.46875000,  0.34960938,
0.30664062,  0.28125000,  0.17285156,  0.08496094,  0.02441406,
]
```

```
scores_hps = [
0.99948066,  0.99933332,  0.99929035,  0.99919587,  0.99914408,
0.99883050,  0.99883050,  0.99875510,  0.99858958,  0.99829930,
0.99767691,  0.99767691,  0.99752742,  0.99752742,  0.99736834,
0.99719906,  0.99719906,  0.99701905,  0.99682730,  0.99662340,
0.99662340,  0.99592990,  0.99566853,  0.99566853,  0.99509466,
0.99509466,  0.99477994,  0.99444515,  0.99444515,  0.99408901,
0.99408901,  0.99408901,  0.99330717,  0.99330717,  0.99330717,
0.99242276,  0.99142247,  0.99142247,  0.99142247,  0.99087441,
0.99087441,  0.99029154,  0.98967183,  0.98901308,  0.98901308,
0.98901308,  0.98831278,  0.98831278,  0.98756832,  0.98593640,
0.98409361,  0.98201376,  0.97838473,  0.97702265,  0.97404259,
0.97068775,  0.96885622,  0.96885622,  0.96885622,  0.96691406,
0.96691406,  0.96267307,  0.96036118,  0.96036118,  0.96036118,
0.95791227,  0.95791227,  0.95791227,  0.95791227,  0.95531917,
0.95257413,  0.95257413,  0.94966936,  0.94966936,  0.94659668,
0.94659668,  0.94659668,  0.93991333,  0.93991333,  0.93245327,
0.92414182,  0.92414182,  0.91964257,  0.91490096,  0.88720459,
0.88720459,  0.88079703,  0.88079703,  0.87407720,  0.86703575,
0.77729988,  0.60766321,  0.51561993,  0.46879065,  0.34864515,
0.30735803,  0.28140560,  0.17328820,  0.08509904,  0.02442309,
]
```

C Upcast Operation

Definition. Upcast(·) is a function that converts the inputs to a higher-precision floating-point datatype while preserving their real-valued magnitude. For example, a single-scalar logit of -1.25 that is internally represented in FP16 as the bit pat-

tern `1 01111 0100000000` is upcast to FP32 as `1 01111111 01000000000000000000000`. Although the underlying bit representation changes, the value remains exactly -1.25, because every FP16 and BF16 values is exactly representable in FP32.

Ease of Implementation. Our proposed HPS method is designed for seamless integration into existing evaluation pipelines. As demonstrated in Figure 4, adopting HPS requires minimal modification to the codebase, often replacing a single line of code within the standard evaluation script. This drop-in compatibility ensures that researchers can reproduce our high-precision results without architectural overhaul.

```
# Forward pass
outputs = model(**inputs)
logits = outputs.logits
+ logits = logits.to(dtype=torch.float32)

# Calculate probabilities
probs = F.sigmoid(logits, dim=-1)
```

Figure 4: Implementation of HPS. We enforce FP32 precision to ensure reproducibility.

D Closed-form Expectations

Let the tie groups be $G_1, \dots, G_N$ in descending score order. Each group G_n has size $|G_n|$ and r_n relevant items ($0 \le r_n \le |G_n|$). Define the per-group relevance probability

$$p_n = \frac{r_n}{|G_n|}, \tag{7}$$

and the cumulative size

$$c_n = \sum_{m \le n} |G_m|, \qquad c_0 = 0. \tag{8}$$

For a cutoff rank k, the number of items from group G_n that appear within the top-k list is

$$t_n = \max\{0,\ \min(|G_n|,\ k - c_{n-1})\}. \tag{9}$$

Count-based Metrics.

With $N_+ = \sum_m r_m$,

$$\mathbb{E}[\text{Hits@}k] = \sum_{n:t_n>0} p_n t_n, \tag{10}$$

$$\mathbb{E}[\text{Recall@}k] = \frac{\sum_n p_n t_n}{N_+}, \tag{11}$$

$$\mathbb{E}[\text{Precision@}k] = \frac{\sum_n p_n t_n}{k}, \tag{12}$$

$$\mathbb{E}[\text{F1@}k] = \frac{2\sum_n p_n t_n}{k + N_+}. \tag{13}$$

nDCG.

With binary gains and weights $w_r = \frac{1}{\log_2(r+1)}$, define

$$W(a,b) = \sum_{r=a}^{b} w_r. \tag{14}$$

Then

$$\mathbb{E}[\text{DCG@}k] = \sum_{n:t_n>0} p_n\, W(c_{n-1}+1,\ c_{n-1}+t_n), \tag{15}$$

$$\text{IDCG@}k = \sum_{r=1}^{\min(N_+,k)} w_r, \tag{16}$$

$$\mathbb{E}[\text{nDCG@}k] = \frac{\mathbb{E}[\text{DCG@}k]}{\text{IDCG@}k}. \tag{17}$$

Reciprocal Rank.

Let $n^* = \min\{n \mid r_n > 0\}$ be the first group containing a relevant item and $\binom{x_a}{x_b}$ be the binomial coefficient. If $k \le c_{n^*-1}$ then $\mathbb{E}[\text{RR@}k] = 0$; otherwise

$$u = \min(|G_{n^*}| - 1,\ k - c_{n^*-1} - 1) \tag{18}$$

$$r_t = c_{n^*-1} + t + 1 \tag{19}$$

$$\pi_t = \frac{\binom{|G_{n^*}| - r_{n^*}}{t}}{\binom{|G_{n^*}|}{t}} \tag{20}$$

$$\lambda_t = \frac{r_{n^*}}{|G_{n^*}| - t} \tag{21}$$

$$\mathbb{E}[\text{RR@}k] = \sum_{t=0}^{u} \frac{1}{r_t}\, \pi_t \lambda_t. \tag{22}$$

Average Precision.

For rank $r = c_{n-1} + t + 1$ with $0 \le t < t_n$ in group G_n,

$$A_{n,t} = R_{n-1} + 1 + t\frac{r_n - 1}{|G_n| - 1}, \tag{23}$$

$$D_{n,t} = c_{n-1} + t + 1, \tag{24}$$

where $R_{n-1} = \sum_{m<n} r_m$. The expected AP@k is

$$\mathbb{E}[\text{AP@}k] = \frac{1}{N_+} \sum_{n:r_n>0} \sum_{t=0}^{t_n-1} p_n \frac{A_{n,t}}{D_{n,t}}. \tag{25}$$

E Time and Space Complexity

Let the ranked list for one query contain L candidate documents and let the evaluation cutoff be k. The list is partitioned into N tie groups $G_1, \ldots, G_N$ of sizes $|G_1|, \ldots, |G_N|$ with $\sum_{n=1}^{N} |G_n| = L$. All complexities below are per query.

High-Precision Scoring (HPS). Only the final logits are upcast to `FP32` and passed once through a scoring function ϕ, so the time cost is $O(L)$ with negligible extra memory. In our implementation, converting 1,000,000 (batch size) × 1,024 (hidden size) FP16 elements to FP32 takes about 5 ms end-to-end on a single NVIDIA H200. From a memory standpoint, the impact is transient and bounded. If a temporary FP32 buffer is materialized for the top-k block, the peak extra footprint is k × d × (4–2) bytes (e.g., 2 MB at k=1,024, d=1,024), and no FP32 state is persisted after scoring.

Tie-aware Metrics (TRM). All computations occur after sorting, so no additional log L factor is introduced. (i) A single left-to-right scan gathers the pairs $(|G_n|, r_n)$ for every tie group in $O(L)$ where r_n refers to the number of relevant items in the n-th tie group G_n. (ii) Closed-form expressions let nDCG, MAP, Recall, Precision, and F1 be evaluated in $O(\min\{k, N\})$ time. (iii) MRR examines only the first tie group containing a relevant document, costing $O(|G_{j^*}|) \leq O(k)$ where j^* is the index of the tie group that includes the first relevant item. (iv) Max, min, and range scores need only the tie group that straddles rank k, again $O(k)$.

In total TRM adds at most $O(k + N) \subseteq O(L)$ lightweight arithmetic per query, far below the cost of the forward pass or initial sort, while providing tie-robust evaluation.

F Experimental Settings

F.1 Implementation Details

We use a maximum input length of 4,096 tokens[3] and a batch size of 16[4]. All models are run under three data types: `BF16`, `FP16`, and `FP32`. HPS is implemented by upcasting the final scoring operation to `FP32`. Baseline tie-oblivious scores rely on the framework's predefined index order inside ties. In contrast, tie-aware expectations and extrema are computed with TRM (Section 2.3).

[3]Only `multilingual-e5-large` is truncated to 512 tokens due to its length constraints.

[4]Batch size affects the representations produced by low-precision inference, even with identical inputs.

F.2 Datasets

MIRACLReranking. We adopt the English subset of the **MIRACLReranking** test split (Zhang et al., 2023), derived from an open-domain Wikipedia. After discarding queries without a relevant passage, 717 of the original 799 queries remain; each with exactly 100 candidate passages (≈ 2.9 relevant passages on average).

AskUbuntuDupQuestions. For evaluation, each query is a concise **AskUbuntuDupQuestions** (Lei et al., 2016) question with at least one manually annotated duplicate. The test split contains 375 queries, each accompanied by 20 candidate questions (≈ 6 true duplicates on average).

G Detailed Experimental Results

We present the full experimental results for the both datasets in Figure 5 and 6. We attach results for `Qwen3-Reranker-0.6B`, which is known as the state-of-the-art in general text retrieval tasks. Panels (a)-(d) report nDCG, MRR, MAP, and Recall. Each marker shows the tie-oblivious score M_{obl} (×) and the tie-aware expectation $\mathbb{E}[M]$ (●). The legend entry indicates the data types of the model and scoring function, respectively. For example, `BF16_FP32` denotes that the model operates in `BF16` precision, while the scoring function is upcast to `FP32`, corresponding to the HPS setting.

H Extending to MTEB Retrieval

To demonstrate the generalizability of our method across retrieval evaluation frameworks, we extended our experiments to the **MTEB Retrieval (MTEB-R)** task. This evaluation covers ten diverse English datasets: ArguAna, ClimateFEVERHardNegatives, CQADupstackGamingRetrieval, CQADupstackUnixRetrieval, FEVERHardNegatives, FiQA2018, HotpotQAHardNegatives, SCIDOCS, Touche2020Retrieval.v3, and TREC-COVID.

Table 3 presents the quantitative stability results. We observe that standard `BF16` inference introduces high score range compared to the full precision (`FP32`) baseline. This significant metric instability is particularly pronounced in `Qwen3-Reranker-0.6B`, which exhibits the highest volatility with a range of 10.09 in nDCG@10 and 17.14 in MRR@10. In contrast, proposed HPS (`BF16`→`FP32`) effectively mitigates the instability, drastically reducing the range and absolute bias.

Figure 7 further visualizes this phenomenon for `Qwen3-Reranker-0.6B`. While the standard `BF16` regime (yellow) suffers from a wide variance in scores, our method (green) aligns closely with the ground truth `FP32` trajectory, ensuring reliable and consistent ranking evaluations.

I Comparison with Alternative Baselines

In this section, we discuss why alternative tie-breaking strategies or numerical formulations are less ideal compared to the proposed HPS and TRM frameworks. We categorize these alternatives into stochastic, deterministic, numerical, and precision-based approaches.

I.1 Stochastic Tie-breaking

A straightforward baseline to handle ties is to randomly permute the tied documents. However, this introduces non-determinism and sampling variance. The introduced *expected score* in TRM represents the analytic expectation of such a random permutation process. Mathematically, if we denote the score of a random permutation baseline as M_{rand}, our expected score is equivalent to $\mathbb{E}[M_{\text{rand}}]$. While the empirical average of repeated random trials would converge to the expectation by the *Law of Large Numbers*, it requires significant computational overhead to reduce variance. In contrast, TRM provides a closed-form, deterministic, and variance-free evaluation metric at negligible additional cost.

I.2 Deterministic Heuristics

Deterministic tie-breaking policies often rely on the inherent storage order or external metadata (e.g., document IDs, timestamps). The *tie-oblivious* baseline (M_{obl}) discussed in Section 3 effectively corresponds to the former index-preserving policy.

A critical flaw in these heuristics is their susceptibility to spurious correlations. As shown in Figure 6, in datasets like AskUbuntuDupQuestions, positive passages are systematically indexed earlier than negative ones due to the data collection pipeline. Consequently, an index-preserving policy in this case acts identically to an optimistic $M_{\max}$ policy, leading to artificially inflated metrics. Relying on external metadata (e.g., chronological ordering) suffers from similar biases.

In contrast, HPS, being strictly content-dependent and metadata-independent, ensures fair and reproducible scoring regardless of the storage implementation or data curation history.

I.3 Alternative Numerical Formulations

Raw Logits vs. HPS (Sigmoid). One might suggest using raw logits z_i directly for ranking instead of upcasting followed by a sigmoid function, $\phi(\text{upcast}(z_i))$, arguing that logits offer a wider numerical range. However, we prove that distinct FP16 logits map to distinct FP32 sigmoid outputs, making the ranking identical.

Let $\sigma(x)$ be the sigmoid function. Its derivative is bounded by $0 < \sigma'(x) \leq 1/4$. By the *Mean Value Theorem*, for any two logits z_i, z_j, there exists c such that:

$$|\sigma(z_i) - \sigma(z_j)| = \sigma'(c)|z_i - z_j| \leq \frac{1}{4}|z_i - z_j|. \quad (26)$$

For two FP32 sigmoid outputs to collapse to the same value, their difference must be smaller than one FP32 Unit in the Last Place (ULP), which is approximately 10^{-7} in the $(0, 1)$ interval. Combining this with the inequality:

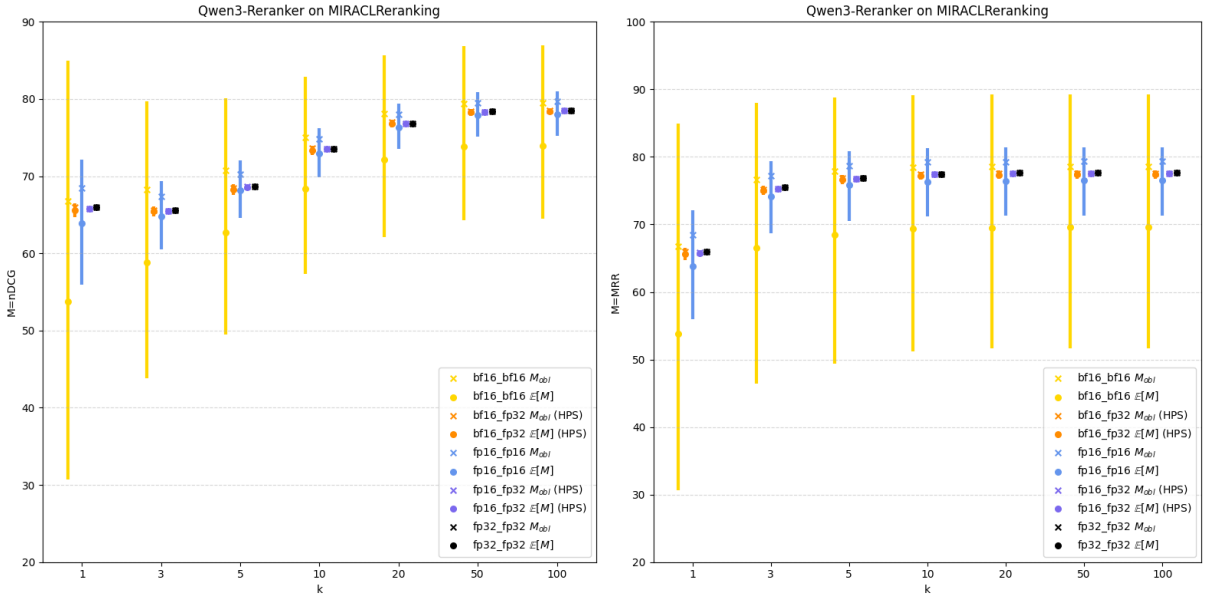
$$|z_i - z_j| \geq 4|\sigma(z_i) - \sigma(z_j)| \approx 4 \times 10^{-7}. \quad (27)$$

This implies that for ranking collisions to occur in HPS where they do not occur in raw logits, the input logits must differ by less than $\approx 4 \times 10^{-7}$. Since the precision grid of BF16/FP16 is orders of magnitude coarser than this threshold (except in a negligible neighborhood around zero), **distinct 16-bit logits map to distinct FP32 sigmoid scores**. Therefore, using raw logits provides no ranking benefit over HPS. Moreover, HPS preserves the $(0, 1)$ probability scale, maintaining interpretability and consistency across different models and datasets.

Temperature Scaling. Applying temperature scaling to logits can mitigate saturation but does not solve the fundamental issue of the limited bucket count in 16-bit arithmetic. Furthermore, temperature is a hyperparameter that requires dataset-specific tuning. In contrast, HPS is a zero-parameter solution that universally mitigates spurious ties.

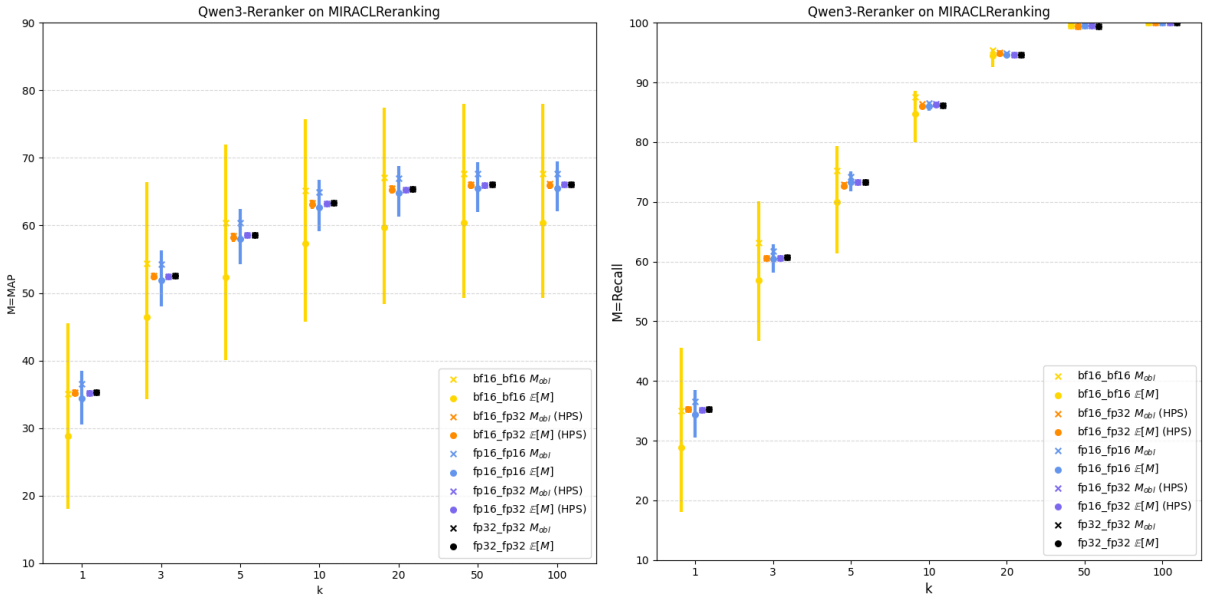
I.4 Full Precision Computation

Performing all computations in FP32 from the start is the ideal solution for numerical accuracy. However, this negates the efficiency benefits of modern low-precision accelerators. As illustrated in Figure 7, proposed HPS yields performance trajectories that closely match the full FP32 baseline, but with significantly lower memory bandwidth and computational costs as discussed in Section E. Thus, HPS offers the practically optimal trade-off between precision and efficiency.



(a) nDCG

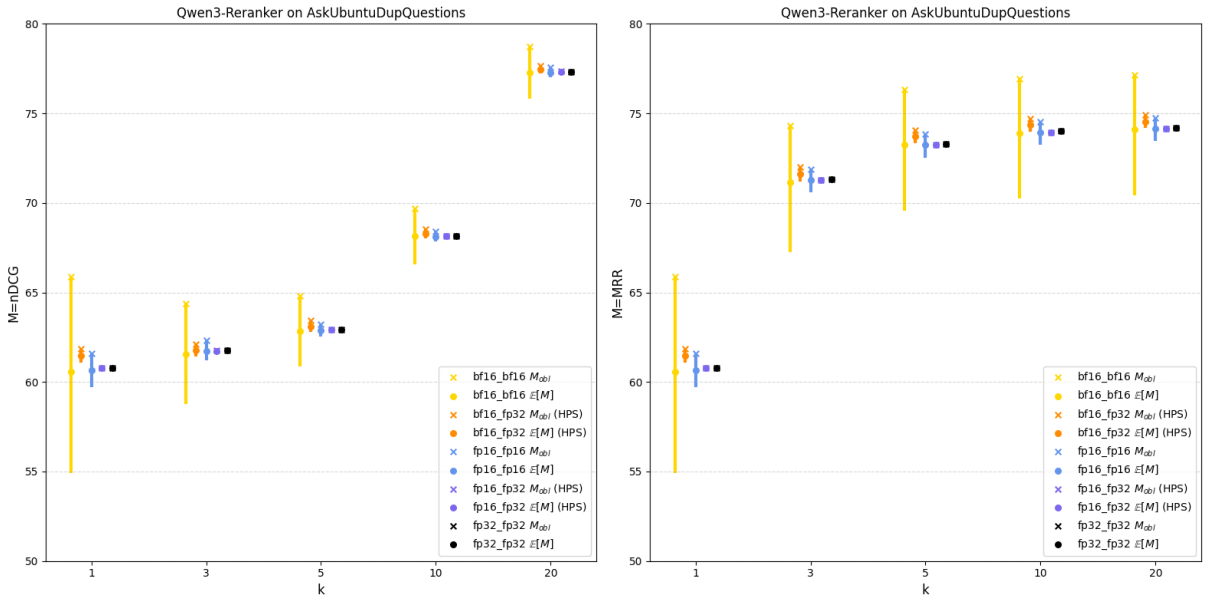
(b) MRR



(c) MAP

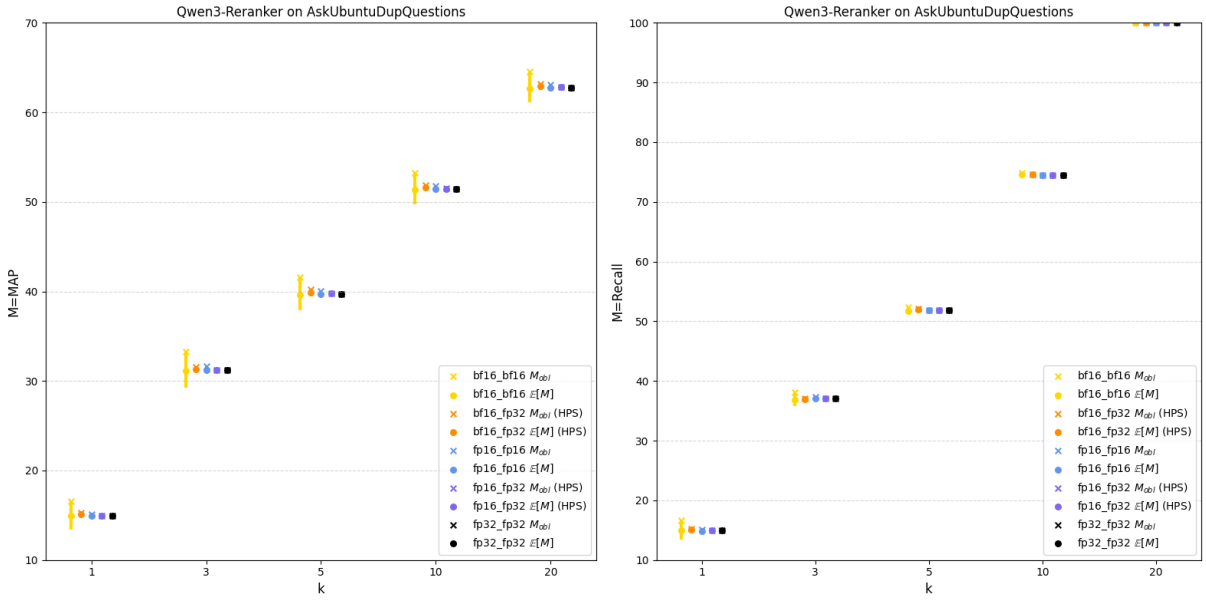
(d) Recall

Figure 5: Metric scores for cutoff k of `Qwen3-Reranker-0.6B` on **MIRACLReranking** dataset.



(a) nDCG

(b) MRR



(c) MAP

(d) Recall

Figure 6: Metric scores for cutoff k of `Qwen3-Reranker-0.6B` on **AskUbuntuDupQuestions** dataset. In this dataset, all tie-oblivious metrics attain their maximum possible value (being overestimated) because, during candidate construction, every relevant item is concatenated ahead of all non-relevant ones.[5]

[6] https://github.com/embeddings-benchmark/mteb/blob/1.38.38/mteb/evaluation/evaluators/RerankingEvaluator.py#L175

Models	FP32	BF16				BF16 → FP32 (+HPS)			
	M	M_{obl}	$\mathbb{E}[M]$	Range(▼)	|Bias|(▼)	M_{obl}	$\mathbb{E}[M]$	Range(▼)	|Bias|(▼)
				$M = nDCG@10$					
Qwen3-Reranker-0.6B♣	47.51	46.92	46.24	10.09	0.68	47.56	47.55	**0.65**	**0.01**
bge-reranker-v2-m3◇	43.94	43.70	43.81	2.13	0.11	43.96	43.92	**0.18**	**0.03**
gte-multilingual-reranker-base◇	46.72	46.71	46.62	0.93	0.09	46.70	46.69	**0.12**	**0.01**
Qwen3-Embedding-0.6B♠	45.59	45.55	45.50	1.19	0.06	45.50	45.50	**0.00**	**0.00**
multilingual-e5-large-large♠	43.20	43.66	43.13	4.46	0.53	43.35	43.35	**0.00**	**0.00**
				$M = MRR@10$					
Qwen3-Reranker-0.6B♣	49.47	48.14	47.44	17.14	0.70	49.48	49.42	**0.79**	**0.05**
bge-reranker-v2-m3◇	45.38	44.92	45.34	4.27	0.42	45.36	45.35	**0.17**	**0.00**
gte-multilingual-reranker-base◇	49.42	49.11	49.14	1.41	0.03	49.37	49.36	**0.14**	**0.01**
Qwen3-Embedding-0.6B♠	47.23	47.36	47.22	1.47	0.14	47.31	47.31	**0.00**	**0.00**
multilingual-e5-large-large♠	44.57	45.65	44.83	5.60	0.82	44.95	44.95	**0.00**	**0.00**

Table 3: Evaluation stability results on the MTEB-R benchmark. All reported scores are averaged across ten diverse datasets, re-ranking top-1000 passages retrieved by BM25. We compare the tie-oblivious metric (M_{obl}), expectation ($\mathbb{E}[M]$), range (▼), and absolute bias (▼) under standard BF16 and our BF16→FP32 (+HPS). Symbols ♣, ◇, and ♠ denote softmax, sigmoid, and pairwise product scoring, respectively. Bold values indicate the lowest range and absolute bias.

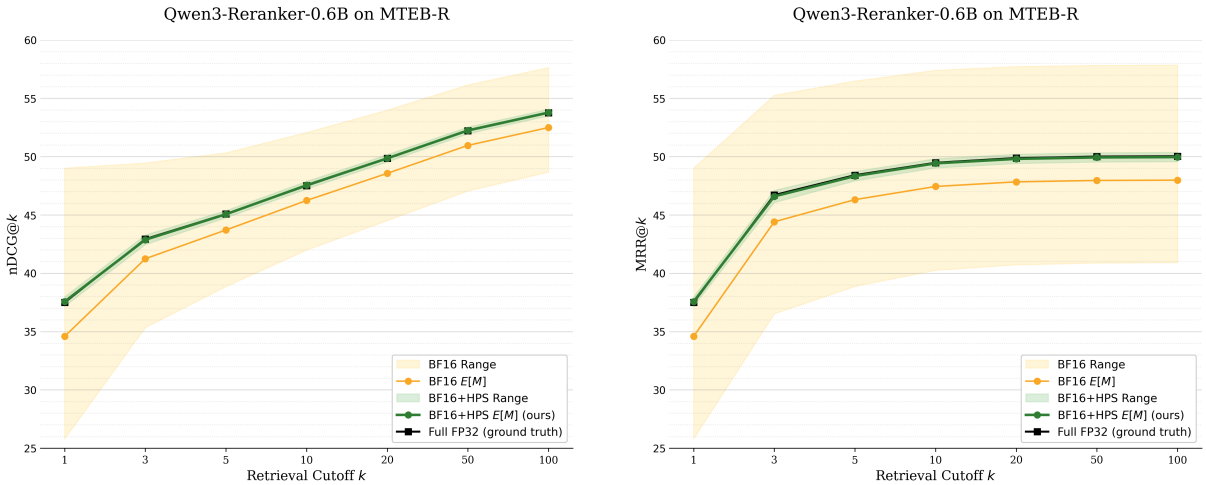


Figure 7: Visualization of **nDCG@k (left)** and **MRR@k (right)** fluctuations for Qwen3-Reranker-0.6B under the BF16 precision regime on MTEB-R. The high variance in scores (yellow shaded area) demonstrates the inherent risk of reaching inconsistent ranking conclusions when using low-precision inference without a reliable protocol. In contrast, our proposed HPS yields performance trajectories that closely match the full FP32 baseline.