DropCov: A Simple yet Effective Method for Improving Deep Architectures

Abstract

Previous works show global covariance pooling (GCP) has great potential to improve deep architectures especially on visual recognition tasks, where post-normalization of GCP plays a very important role in final performance. Although several post-normalization strategies have been studied, these methods pay more close attention to effect of normalization on covariance representations rather than the whole GCP networks, and their effectiveness requires further understanding. Meanwhile, existing effective post-normalization strategies (e.g., matrix power normalization) usually suffer from high computational complexity (e.g., $O(d^3)$ for d-dimensional inputs). To handle above issues, this work first analyzes the effect of post-normalization from the perspective of training GCP networks. Particularly, we for the first time show that *effective post-normalization can make a good trade-off between representation decorrelation and information preservation for GCP, which are crucial to alleviate over-fitting and increase representation ability of deep GCP networks, respectively*. Based on this finding, we can improve existing post-normalization methods with some small modifications, providing further support to our observation. Furthermore, this finding encourages us to propose a novel pre-normalization method for GCP (namely DropCov), which develops an adaptive channel dropout on features right before GCP, aiming to reach trade-off between representation decorrelation and information preservation in a more efficient way. Our DropCov only has a linear complexity of $O(d)$, while being free for inference. Extensive experiments on various benchmarks (i.e., ImageNet-1K, ImageNet-C, ImageNet-A, Stylized-ImageNet, and iNat2017) show our DropCov is superior to the counterparts in terms of efficiency and effectiveness, and provides a simple yet effective method to improve performance of deep architectures involving both deep convolutional neural networks (CNNs) and vision transformers (ViT).

1 Introduction

Global covariance pooling (GCP) has shown remarkable potential to improve performance of deep architectures in a variety of tasks, especially visual recognition [29, 21, 27, 28, 8, 43, 34, 42, 45]. One of core differences among various deep GCP methods is post-normalization for covariance representations, which plays a crucial role in final performance. The existing post-normalization approaches can be roughly categorized into element-wise post-normalization methods [29, 23, 2] and structure-wise ones [21, 27, 26, 23, 34, 42], which concentrate on normalizing entries and eigenvalues of covariance matrices, respectively. Some researches (e.g., [28]) perform two kinds of normalizations simultaneously. Although several post-normalization strategies have been studied, behavior of different approaches seem vary significantly and there is a lack of an intuitive and unified interpretation. Therefore, the true mechanism lying behind these post-normalization methods (especially under deep architectures) is still an open problem.

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Existing works understand the effect of post-normalization methods of GCP from different aspects. Specifically, [29, 23] claim that their element-wise normalization strategies can suppress the burstiness of covariance representations as in the classical bag-of-words [35] or actually perform a kind of whitening on representations. DeepO$_2$P [21] and MPN-COV [27] develop post-normalization methods to exploit geometry of covariances by considering the fact that space of covariance forms a Riemannian manifold [1, 32]. Besides, MPN-COV shows that matrix square root (sqrt-root) normalization accounts for a robust covariance estimation [41], while [28] claims spectral normalization compensates for burstiness of eigenvalues and shows effectiveness of matrix sqrt-root normalization via empirical comparisons. Although many efforts are made, the aforementioned researches try to explain why GCP works by studying how post-normalizations affect covariance representations, which lack a full view on the whole GCP networks. Therefore, this work analyzes effect of post-normalization approaches on optimizing deep GCP networks. Particularly, by taking matrix power normalization [27] as an example, we find that post-normalization tries to handle a paradox between representation decorrelation and information preservation for GCP, which are crucial to alleviate over-fitting and increase representation ability of deep GCP networks, respectively. Particularly, effective post-normalization approaches can achieve a good trade-off, and so lead to a better performance. Based on this finding, we introduce some modified versions of existing post-normalization methods (e.g., adaptive power normalization for MPN-COV, I-LogM for DeepO$_2$P, and linear transformation for B-CNN [29]), which achieve further improvement, providing support to our observation.

Another issue of existing post-normalization methods is high computational complexity. Particularly, effective structure-wise post-normalization strategies usually involve singular value decomposition (SVD) of covariance matrices or sequential matrix multiplications, resulting in the computational complexity of $O(d^3)$ for d-dimensional inputs. Inspired by finding above, this paper proposes an efficient pre-normalization method for GCP (namely DropCov) based on dropout [24], as it has the natural ability to perform feature decorrelation [4] and reduce over-fitting of training neural networks [24]. Specifically, our DropCov develops an adaptive channel dropout on features right before computation of covariance representations, where probability of dropout is adaptively decided by seeking a good trade-off between representation decorrelation and information preservation. Compared to existing post-normalization methods, our DropCov achieves superior or competitive performance with a linear computational complexity of $O(d)$, while it is only used for training.

To verify the effectiveness of our proposed DropCov, we conduct experiments using various deep architectures including CNNs [15] and vision transformers (ViT) [40, 30, 44] on ImageNet-1K [7], ImageNet-C [16], ImageNet-A [17], Stylized-ImageNet [12], and long-tailed iNat2017 [18]. The contributions of this work are summarized as follows: (1) To our best knowledge, this work for the first time analyzes the effect of post-normalization on GCP from the perspective of optimizing neural networks, and we show that effective post-normalization methods can make a good trade-off between representation decorrelation (*w.r.t* over-fitting reduction) and information preservation (*w.r.t* increase of representation ability) for GCP (*w.r.t* deep GCP networks); (2) Based on our finding, we make some small modifications to existing post-normalization methods (Sec. 2.2), e.g., adaptive power normalization and I-LogM & linear transformation. The comparisons in Sec. 5.2 show our modifications can bring further improvement, giving support on our observation; (3) Profiting from our finding, we propose a novel DropCov method by developing an adaptive channel dropout as pre-normalization of GCP, which has a linear complexity of $O(d)$ and is only used for training. Extensive experiments show our DropCov is superior or competitive to existing post-normalization approaches and provides a simple yet effective method to improve deep architectures.

2 Effect of Post-normalization on Deep GCP Networks

In this section, we first analyze the effect of post-normalization on optimizing deep GCP networks by taking matrix power normalization as an example. To further verify our finding, we spread it to existing post-normalization approaches, and make some modifications according to our observations.

2.1 How Does Matrix Power Normalization Impact GCP

Matrix Power Normalization   Both previous works [42, 34] and our experiments show structure-wise post-normalization methods are generally superior to element-wise ones, where matrix power normalization (MPN) usually performs the most competitively. Let $\mathbf{X} \in \mathbb{R}^{N \times d}$ be N d-dimensional

(a) Convergence curves of various α (b) Generalization *vs.* representation ability

Figure 1: Results of GCP networks with matrix power normalization (MPN) using the backbone of ResNet-18 ($d = 128$) on ImageNet-1K. (a) Convergence curves of various $\alpha \in (0, 1]$; (b) Comparison of generalization and representation abilities for various α. From them we can see that larger values of α result in higher training accuracies indicated by blue solid line in (b), *w.r.t* better representation or approximation ability, but suffer from heavier over-fitting (*w.r.t* worse generalization), aka larger differences of training accuracy over test one indicated by Δ. In contrast, smaller values of α have better generalization (aka smaller Δ), while obtaining lower classification accuracies (*w.r.t* weaker representation ability). Particularly, $\alpha = 0.5$ achieves the best trade-off and so leads the best results.

features with zero-mean those outputted by the last convolution layer of deep CNNs or the last transformer block of ViT (i.e., final word tokens), GCP computes $\mathbf{X}^T\mathbf{X}$ with post-normalization to generate a covariance representation $\mathbf{Z}$. Specifically, GCP with MPN is calculated as

$$\mathbf{Z} = (\mathbf{X}^T\mathbf{X})^{\alpha} = \mathbf{U}\mathbf{\Lambda}^{\alpha}\mathbf{U}^T, \tag{1}$$

where $\mathbf{Z} \in \mathbb{R}^{d \times d}$ is used for final classification. $\alpha > 0$ is a parameter of power. $\mathbf{U}$ and $\mathbf{\Lambda}$ are the matrix of eigenvectors and the diagonal matrix of eigenvalues of $\mathbf{X}^T\mathbf{X}$, respectively.

Effect of Power   For MPN, power α has a significant effect on the final results. Previous works [27, 28] conduct empirical comparison of α, and choose the best one (e.g., $\alpha = 0.5$). However, the effect brought by α is still not very clear. In this work, we first analyze the effect of $\alpha \in (0, 1]$ on optimizing deep GCP networks. It is clear from Eqn. (1) that α has the following effect on GCP from the mathematical perspective:

$$\begin{cases} \mathbf{Z} \longmapsto \mathbf{I}, & \text{if} \quad \alpha \longmapsto 0 \quad : \text{Representation Decorrelation \& Information Loss}, \\ \mathbf{Z} \longmapsto \mathbf{X}^T\mathbf{X}, & \text{if} \quad \alpha \longmapsto 1 \quad : \text{Information Preservation \& Strong Correlation}. \end{cases} \tag{2}$$

Specifically, when $\alpha \longmapsto 0$, the normalized representation $\mathbf{Z}$ will be decorrelated and tend to be an identity matrix $\mathbf{I}$. Such representation decorrelation can help optimization of neural networks [25, 20, 4] (in particular for combating over-fitting [5]), which, however, will lead to information loss for $\mathbf{X}^T\mathbf{X}$, hurting representation ability of covariances. Particularly, $\mathbf{Z}$ with MPN of $\alpha = 0$ becomes an identity matrix, and the information lying in $\mathbf{X}^T\mathbf{X}$ is completely lost. When $\alpha \longmapsto 1$, information of $\mathbf{X}^T\mathbf{X}$ will be gradually preserved, maintaining the correlations as characterized in the covariance matrices, which makes training of neural networks difficult (e.g., over-fitting). To further verify the above observation, we conduct experiments using ResNet-18 on ImageNet-1K. According to the results in Figs. 1(a) and 1(b), we draw the following conclusion:

- When $\alpha < 0.5$, post-normalization (i.e., MPN) tends to decorrelate representation. According to red line of Fig. 1(b), smaller values of α reduce over-fitting more significantly and have better generalization (aka smaller differences of training accuracy over test one [3, 5]). Meantime, indicated by blue lines of Fig. 1(b), smaller α leads lower accuracies (*w.r.t* weaker representation or approximation ability [33, 42]) due to information loss.

- When $\alpha > 0.5$, post-normalization (i.e., MPN) tends to preserve information. The training accuracies (*w.r.t* representation or approximation ability) of deep GCP networks indicated by blue solid line of Fig. 1(b) increase along values of α enlarge, but heavier over-fitting (aka larger values of Δ) occur because effect of representation decorrelation decreases.

119 According to Eqn. (2) and above observation, we have

120 **Corollary 1.** *Effective post-normalization (e.g., matrix power normalization (MPN) with $0 < \alpha \leq 1$)*
121 *can achieve a good trade-off between representation decorrelation and information preservation*
122 *for GCP, which are crucial to alleviate over-fitting and increase representation ability of deep GCP*
123 *networks, respectively. Particularly, MPN with $\alpha = 0.5$ achieves the best trade-off for $\alpha \in (0, 1]$*
124 *(without considering other factors), which is proved to be the widely used choice of α [27, 28].*

125 The Corollary 1 illuminates the effect of MPN on deep GCP networks from the perspective of model
126 training, and gives an intuitive explanation about the choice of parameter α.

2.2 Extension of Corollary 1 to Existing Post-normalization Methods

128 To further verify Corollary 1, we extend it to analyze behavior of existing post-normalization.
129 Accompanied by these analyses, we can improve previous methods with some small modifications.

130 **Adaptive Power Normalization (APN)** Above all, Corollary 1 shows MPN with $\alpha = 0.5$ is
131 the best trade-off for $\alpha \in (0, 1]$ if no other factors are considered. To further verify the effect of
132 trade-off, we propose an adaptive power normalization (APN) by considering the effect of inputs
133 (i.e., eigenvalues Λ of covariances). Specifically, we formulate the trade-off between representation
134 decorrelation and information preservation as the following objective function:

$$\min_{\alpha} \log \left(\frac{\lambda_{max}^{\alpha}}{\lambda_{min}^{\alpha}} \right), \quad s.t. \ \max_{\alpha} \sum_i \frac{\lambda_i^{\alpha}}{\sum_i \lambda_i^{\alpha}} \log \left(\frac{\lambda_i^{\alpha}}{\sum_i \lambda_i^{\alpha}} \right), \quad (3)$$

135 where λ_{max} and λ_{min} are maximum and minimum of Λ, respectively. Particularly, we minimize
136 the ratio of λ_{max}^{α} to λ_{min}^{α} (in logarithmic coordinates) to enforce representation decorrelation. Here,
137 smaller values of the objective function indicate stronger decorrelation, and $\alpha = 0$ achieves minimum
138 (i.e., zero). On the other hand, we maximize negative entropy of normalized eigenvalues to keep
139 discrepancy among eigenvalues and so preserve information, which achieves maximum value (i.e.,
140 zero) for $\alpha \to +\infty$. Considering that previous works [27, 28] show relation between performance
141 and α is generally convex, we exploit a grid search strategy to obtain a suboptimal solution for
142 $\alpha \in (0, 1]$ with the interval of 0.01^1. However, λ_{min} is usually nearby zero, leading to computational
143 instability. Therefore, we optimize the objective function (3) as

$$\min_{\alpha} \Big[\underbrace{\alpha(\log(\lambda_{max}) - \log(\max(C, \lambda_{min})))}_{\text{representation decorrelation}} \underbrace{- \tau \sum_{i=1}^{K} (\lambda_i^{\alpha} / \sum_{i=1}^{K} \lambda_i^{\alpha}) \log(\lambda_i^{\alpha} / \sum_{i=1}^{K} \lambda_i^{\alpha})}_{\text{information preservation}} \Big], \quad (4)$$

144 where we set minimum of λ_{min} to a constant C (e.g., 1e-2), and compute the entropy using Top-K
145 eigenvalues. τ is a balance parameter, which is set to 1.2. Note that Eqn. (4) is computed efficiently
146 for small K ($K \leq 3$ in our work) using about a hundred trials, given the eigenvalues Λ.

147 **MPN with $\alpha > 1$** Theoretically, α of MPN could be larger
148 than one, which, however, violates the principle of representa-
149 tion decorrelation. Meanwhile, it disrupts equilibrium between
150 eigenvalues, hurting information inherent in covariances. As
151 illustrated in Fig. 2, MPN with $\alpha = 2$ suffers from heavy
152 over-fitting, and is clearly worse than one with $\alpha = 0.5$.

153 **Matrix Logarithm Normalization (LogM)** LogM [21] is
154 one another popular structure-wise post-normalization. Math-
155 ematically, logarithm has similar behavior with power function
156 of $\alpha = 0.3$ when eigenvalues $\lambda > 1$, but reverses amplitudes of
157 eigenvalues $\lambda < 1$ (i.e., neither decorrelation nor information

Figure 2: Results of $\alpha > 1$ & LogM

158 preservation). However, small eigenvalues frequently occur in deep architectures [27], especially for
159 large feature dimension (d). Accordingly, as illustrated in Fig. 2, LogM achieves similar results with
160 MPN of $\alpha = 0.3$ for $d = 128$. For $d = 256$, GCP network with LogM fails to converge due to the

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[1]Note that it is unnecessary to obtain exactly the optimal solution, since performance gap is negligible when
difference of α is less than 0.01.

161 side effect brought by small eigenvalues ($\lambda < 1$). *Therefore, we modified LogM with a shift operation*
162 *(namely I-LogM) to make all inputs of LogM be larger than one, i.e., $\log(\lambda_i) \longrightarrow \epsilon \log((\lambda_i + \epsilon)/\epsilon)$.*
163 Our I-LogM brings promising gains when $d = 256$ (see Sec. 5.2 and Table 4).

Element-wise Normalization (EwN)   EwN could be roughly written in a unified formulation:
$\boldsymbol{\beta} \frac{f(\mathbf{z})-\mu}{\|f(\mathbf{z})-\mu\|_*} + \boldsymbol{\gamma}$, where $\mathbf{z}$ is vectorization of $\mathbf{X}^T\mathbf{X}$. f is a mapping function and $\|\mathbf{a}\|_*$ indicates a
certain norm of $\mathbf{a}$. $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ perform a linear transformation (LT). Particularly, for element-wise power
normalization (EPN) [29], f is a signed power function and $\mu = 0$. $\|\mathbf{a}\|_*$ is a ℓ_2 norm, and no LT is
performed. For LN [2], f is an identity mapping and μ is mean of $f(\mathbf{z})$. $\|\mathbf{a}\|_*$ is a ℓ_2 norm. According
to above formulation of EwN and Corollary 1, EwN performs representation decorrelation based on
zero-mean (i.e., $f(\mathbf{z}) - \mu$) and standardization (i.e., normalized by $\|f(\mathbf{z}) - \mu\|_*$), while LT operation
aims to recover information. Compared to MPN, element-wise normalization lacks an effective
mechanism to balance representation decorrelation and information preservation. Particularly, as
shown in Sec. 5.2, EPN does not work well, since only representation decorrelation is performed.
Therefore, we introduce a LT for EPN, and it achieves promising improvement (see Table 4).

3 DropCov: A Simple yet Effective Normalization

3.1 Normalizing GCP via Dropout

Although structure-wise normalization (e.g., MPN) achieves advanced performance, it suffers from
high computational complexity (i.e., $O(d^3)$). Corollary 1 shows the core effect of MPN lies in a
good trade-off between representation decorrelation and information preservation for optimizing
deep GCP networks. Meanwhile, previous works [38, 5, 4] show that dropout can reduce correlation
among features. Therefore, one question is naturally raised up: *Could we use efficient dropout to*
perform representation decorrelation? Given probability ρ of dropout, $\rho \to 1$ leading to sparser, less
correlated features and more information loss. On the contrary, $\rho \to 0$ preserves more information
while performing less decorrelation. Above observations show consistence with Corollary 1. There
exist two issues to apply dropout for normalizing GCP, i.e., (1) how to perform dropout and (2) how
to choose probability ρ of dropout. A naive method is dropout after covariance with a fixed ρ (as
compared in Table 2). However, it has several limitations: first, dropout for covariance representations
will break the structure of covariances (e.g., row i of covariance means correlations of i-th channel
and all ones); second, fixed ρ hardly achieves trade-off between representation decorrelation and
information preservation; third, such strategy has the same complexity of $O(d^2)$ with EwN.

3.2 Pre-Normalization via Adaptive Channel Dropout (ACD)

To handle above issues, this work proposes a pre-normalization method by performing channel
dropout with an adaptive probability ρ, which aims to reach a good trade-off between representation
decorrelation and information preservation. This method focuses on feature decorrelation and thus
can maintain structure of covariance representations. Meanwhile, it has a linear computational
complexity of $O(d)$, and so is more efficient. To adaptively decide probability ρ, we compute it by
considering two factors: (1) feature dimension (d) and (2) relationship between feature correlation
and feature importance. As such, probability ρ of dropout is defined as

$$\rho = 1 - \frac{D}{\log(d)} \left(\frac{\boldsymbol{\omega}^T \boldsymbol{\pi}}{\|\boldsymbol{\omega}\|\|\boldsymbol{\pi}\|} \right), \tag{5}$$
$$\boldsymbol{\omega} = \sigma(\mathrm{C1D}_3(\mathrm{GAP}(\mathbf{X}))), \quad \boldsymbol{\pi} = \mathrm{SUM}_{\mathrm{row}}(\mathbf{X}^T\mathbf{X}),$$

where $\boldsymbol{\omega}$ and $\boldsymbol{\pi}$ represent feature importance and feature correlation, respectively. D is a scaling
parameter about feature dimension ($D = 10$ throughout all experiments). For computing feature
importance, we modify a channel attention module [19], which involves global average pooling
(GAP), a 1D convolution with kernel size of 3 (C1D$_3$), and softmax function (σ). Then, we compute
feature correlation by summarizing the elements along row of $\mathbf{X}^T\mathbf{X}$ (i.e., $\mathrm{SUM}_{\mathrm{row}}(\mathbf{X}^T\mathbf{X})$).

As shown in Eqn. (5), bigger probability of dropout is performed on larger feature dimension. When
feature correlation is irrelevant to feature importance, we perform dropout more safely. Otherwise,
we need to carefully adopt dropout and set a small ρ. Based on Eqn. (5), our DropCov is achieved by

$$\mathbf{z} = \mathrm{V}(\delta_\rho(\mathbf{X})^T \delta_\rho(\mathbf{X})), \tag{6}$$

Table 1: Comparison of our DropCov with close related works in terms of formulation, computational complexity of post-normalization and usage mode. Please refer to Sec. 4 for the details of symbols.

	Method	Formulation	Complexity	Train	Infer.
Element-wise Post-Norm.	B-CNN [29]	$\ell_2(sqrt(V(\mathbf{X}^T\mathbf{X})))$	$O(d^2)$	✓	✓
	SigmE [23]	$2/(1 + e^{-\beta \cdot V(\mathbf{X}^T\mathbf{X})}) - 1$	$O(d^2)$	✓	✓
	LN [2]	$\boldsymbol{\beta} \odot (V(\mathbf{X}^T\mathbf{X}) - \mu)/\sigma) \oplus \boldsymbol{\gamma}$	$O(d^2)$	✓	✓
Structure-wise Post-Norm.	DeepO$_2$P [21]	$V(LogM(\mathbf{X}^T\mathbf{X}))$	$O(d^3)$	✓	✓
	MPN-COV [27]	$V((\mathbf{X}^T\mathbf{X})^\alpha)$	$O(d^3)$	✓	✓
	IB-CNN [28]	$\ell_2(sqrt(V(\mathbf{X}^T\mathbf{X})^{1/2}))$	$O(d^3)$	✓	✓
	iSQRT-COV [26]	$V(\approx (\mathbf{X}^T\mathbf{X})^{1/2})$	$O(d^3)$	✓	✓
	MaxExp [23]	$\mathbf{I} - (\mathbf{I} - \mathbf{X}^T\mathbf{X}/(\mathbf{X}^T\mathbf{X} + \varepsilon))^\alpha$	$O(d^3)$	✓	✓
Ours	DropCov	$V(\delta_\rho(\mathbf{X})^T \delta_\rho(\mathbf{X}))$	$O(d)$	✓	×

✱: Note β and ε are parameters. $\odot$ and $\oplus$ indicate element-wise multiplication and addition, respectively.

where δ_ρ is a channel dropout operation with the probability of ρ. V is a vectorization operation followed by triangulation, leading to a $d(d + 1)/2$-dimensional covariance representation $\mathbf{z}$. Note that our DropCov (6) is only adopted to train GCP networks, which can be easily implemented by standard back-propagation. During inference stage, we simply use $V(\mathbf{X}^T\mathbf{X})$ for final classification.

4 Related Works

In this section, we review some works closely related to our DropCov. Table 1 gives a detailed comparison of our DropCov with existing post-normalization methods in terms of formulation, computational complexity of post-normalization and usage mode. For element-wise post-normalization methods, B-CNN [29] performs element-wise signed sqrt-root ($sqrt$) followed by a ℓ_2 normalization on covariance representations. [23] proposes to use some surrogate functions (e.g., SigmE) to handle issue of negative evidence inherent in power function. Layer normalization (LN) [2] is proposed to normalize each feature with its mean (μ) and variance γ followed by a linear transformation. Since these methods perform normalization on each element of covariance representations, they have the computational complexity of $O(d^2)$ for d-dimensional features $\mathbf{X}$. For structure-wise post-normalization methods, DeepO$_2$P and MPN-COV [27] perform matrix logarithm (LogM) and matrix power for covariance representations, respectively. IB-CNN [28] combines matrix sqrt-root normalization with element-wise one, while iSQRT-COV [26] proposes an approximate matrix sqrt-root normalization via fast iterative matrix multiplications. Song et al. further analyze the effect of iSQRT-COV [36] and develop a faster variant [37]. [23] introduces several spectral power functions (e.g., MaxExp) for normalizing covariance matrices. These structure-wise approaches involve SVD of covariances or sequential matrix multiplications, having the computational complexity of $O(d^3)$. ReDro [34] proposes a relation dropout scheme to divide a large-sized covariance into a group of small-sized ones, aiming to mitigate the computational issue of matrix normalization [27, 28, 26, 10]. ReDro reduces computational complexity of structure-wise approaches from $O(d^3)$ to $O(d^3/G^2)$, where G is the number of groups (e.g., $G = 4$). Besides, all existing works perform post-normalization methods during both training and inference (Infer.) stages. Different from aforementioned works, our DropCov performs adaptive channel dropout δ on features $\mathbf{X}$ right before GCP, which has a linear complexity $O(d)$ and is free during inference stage. Experiments in Sec. 5.3 show our DropCov with much less computational complexity is superior or competitive to previous works.

5 Experiments

In this section, we conduct experiments to evaluate effectiveness of our DropCov in improving deep architectures on image classification tasks. Specifically, we first describe implementation details, and then make ablation study on ImageNet-1K (IN-1K) [7] using backbone of ResNet-18. Additionally, we apply our DropCov to both ResNets [15] and ViT models [40, 30, 44], while comparing them on several benchmarks (i.e., IN-1K, ImageNet-C (IN-C) [16], ImageNet-A (IN-A) [17] and Stylized-ImageNet (Sty.-IN) [12]) to assess generalization and robustness of our DropCov. Finally, we assess generalization of DropCov models by transferring them to long-tailed species classification [18].

Table 2: Comparisons (% in Top-1 accuracy) of adaptive normalization methods with the fixed ones using ResNet-18 on ImageNet-1K.

Method	$d = 64$	$d = 128$	$d = 256$
DropElement ($\rho = 0.5$)	73.4	74.6	74.0
DropChannel ($\rho = 0.5$)	70.1	73.1	75.1
ACD (Ours)	**73.5**	**75.0**	**75.2**
MPN ($\alpha = 0.5$)	73.1	74.4	74.9
APN (Ours)	73.3	74.5	75.0

Table 3: Comparisons (% in Top-1 accuracy) of various dropout strategies using ResNet-18 on ImageNet-1K.

Method	$d = 64$		$d = 256$	
	Element	Channel	Element	Channel
Large ($\rho = 0.5$)	72.2	71.3	71.4	70.9
Small ($\rho = 0.5$)	68.4	71.2	66.4	71.5
Uniform ($\rho = 0.5$)	72.8	72.1	74.0	74.4
ACD (Ours)	**73.5**		**75.2**	

5.1 Implementation Details

In this work, we apply the proposed DropCov to several ResNets [15] and ViT models [40, 30, 44]. Specifically, we construct DropCov models based on ResNet following exactly the same schemes in [27, 42]. Given a CNN backbone, we first introduce a 1×1 convolution after the last convolution layer for reducing dimension of features to d, and then replace the original GAP by our DropCov followed by a fully-connected (FC) layer and softmax for classification. All models are optimized using the same hyper-parameter settings and data augmentations as suggested in [15, 27], where stochastic gradient descent (SGD) with initial learning rate (lr) of 0.1 is used to train the networks within 100 epochs and lr is decayed by 10 every 30 epochs. We combine DropCov with ViT models as in [37], where one FC layer is employed to reduce dimension of final word tokens, and our DropCov replaces GAP and classification token for Swin Transformers [30] and DeiT/T2T-ViT [40, 44], respectively. These models are trained by using the same hyper-parameter settings in [40, 30, 44]. All programs are implemented by using PyTorch [31], and run a sever equipped with eight Nvidia RTX 3090 GPUs and 128G RAM. The source code and models will be released.

5.2 Ablation Study

In this subsection, we conduct experiments on ImageNet-1K using lightweight ResNet-18 to assess effect of adaptive probability ρ of dropout and dropout strategies on our method as well as effect of our modifications on existing post-normalization approaches.

Fixed ρ vs. ACD. Our DropCov performs pre-normalization for GCP by using an adaptive channel dropout (ACD) method, which automatically determines probability ρ of dropout by considering trade-off between representation decorrelation and information preservation. We compare with two kinds of baseline dropout methods using fixed $\rho = 0.5$, i.e., element-wise dropout for covariance representations (DropElement) and channel dropout for input features (DropChannel). As shown in the upper part of Table 2, our ACD achieves the best results for various feature dimensions (d) and clearly outperforms the naive DropElement in Sec. 3.1, verifying ACD can achieve a better trade-off.

Effect of Various Dropout Strategies. We compare with several specific dropout strategies. First, we perform element-wise (Element) dropout on some specific entries of covariance with $\rho = 0.5$. Specifically, we rank all entries of covariance in a descending order, and then perform dropout on top/bottom/uniform half of the ordered entries (namely Large/Small/Uniform). We adopt similar strategies to feature channels (Channel), where channel indicators are decided by attention module of our ACD. Note that 'Element' and 'Channel' dropout methods can be regarded as special cases of ACD, where only feature correlation or feature importance are considered, respectively. As compared in Table 3, uniform dropout clearly outperforms those only focus on largest and smallest entries, indicating equilibrium of feature correlation or importance is crucial to final performance. Furthermore, our ACD is superior to uniform dropout by a clear margin, verifying effectiveness of our ACD in adaptively deciding ρ while randomness inherent in ACD further improves generalization.

Effect of Modifications. According to Corollary 1, we make several modifications to existing post-normalization approaches. Beyond MPN, we propose an APN method by considering effect of eigenvalues Λ (see Eqns. (3) and(4)). As compared in the bottom part of Table 2, our APN slightly improves MPN with $\alpha = 0.5$ for various feature dimensions (d). Note that average values

Table 4: Comparison of DropCov with several post-normalization in terms of Top-1 accuracy (Acc.) and running speed using ResNet-18 on ImageNet-1K. *: According to Sec.2.2 and Corollary 1, we modify B-CNN and IB-CNN with a linear transform (LT), while modifying DeepO$_2$P with I-LogM.

Method	$d = 64$			$d = 256$		
	Top-1 Acc. (%)	Train (µs)	Test (µs)	Top-1 Acc. (%)	Train (µs)	Test (µs)
Plain GCP	71.1	3.45	1.04	70.0	35.08	11.89
B-CNN [29]	38.3	4.31	1.23	41.1	44.97	14.38
B-CNN + LT*	68.3	4.45	1.28	73.2	47.37	15.24
LN [2]	71.7	4.10	1.13	70.2	40.79	13.59
DeepO$_2$P [21]	70.1	922.50	910.95	*Not Converge*	9731.71	9638.71
I-LogM*	71.2	922.53	910.98	72.0	9732.46	9639.65
MPN-COV [27]	73.1	925.16	913.87	74.9	9735.11	9642.43
iSQRT-COV [26]	73.4	12.35	4.94	75.2	193.65	77.46
IB-CNN [28]	*Not Converge*	13.93	5.16	36.1	202.38	80.55
IB-CNN + LT*	70.0	14.16	5.21	72.8	207.48	82.98
DropCov (Ours)	**73.5**	**3.54**	**1.04**	**75.2**	**36.20**	**11.89**

$\Diamond$: Note we compute running speed (µs) of single GCP module with post-normalization on a 2080Ti GPU.
$\sharp$: The original ResNet-18 with GAP achieves 70.2% in Top-1 accuracy.

of α achieved by APN are 0.53, 0.49 and 0.45 for $d = 256$, $d = 128$ and $d = 64$, respectively. These comparisons above demonstrate that more appropriate trade-off leads to better results, though MPN has achieved strong performance. Besides, as discussed in Sec.2.2, we introduce an I-LogM and a linear transform (LT) based on BN [20] for improving LogM normalization (DeepO$_2$P) and element-wise post-normalization (i.e., B-CNN and IB-CNN), respectively. As shown in Table 4, DeepO$_2$P performs poorly troubled by small eigenvalues (especially for $d = 256$), while our I-LogM obtains promising gains for both low- and high-dimensional features. Furthermore, B-CNN and IB-CNN achieve very unsatisfied results; in contrast, our simple modification brings significant improvement for B-CNN and IB-CNN. Above results clearly verify our finding in Corollary 1.

5.3 Comparisons with Counterparts

Here, we conduct experiments on ImageNet-1K using ResNet-18 for comparing with existing post-normalization methods in terms of Top-1 accuracy and running speed, including B-CNN [29], LN [2], DeepO$_2$P [21], MPN-COV [27] with $\alpha = 0.5$, iSQRT-COV [26] and IB-CNN [28]. The results of all compared methods are listed in Table 4, where we can see that our DropCov outperforms plain GCP (i.e., GCP without normalization) by 2.4% (5.2%) for $d = 64$ ($d = 256$) with similar running time. Compared with element-wise post-normalization (i.e., LN, B-CNN and IB-CNN), our DropCov achieves significant performance gains while requiring less running time. For structure-wise post-normalization, our DropCov is remarkably superior to matrix logarithm normalization (DeepO$_2$P) in terms of both accuracy and running time. Meanwhile, DropCov is superior or comparable to MPN-COV and iSQRT-COV using much less running time, especially during inference stage. Above results show our DropCov performs better or on par with the counterparts in terms of efficiency and effectiveness, providing an very promising normalization method for deep GCP networks.

5.4 Comparisons with Various Deep Architectures

CNNs. Here we apply our DropCov with $d = 128$ to various CNNs, including ResNet-34, ResNet-50 and ResNet-101. Meanwhile, we compare DropCov with the original CNNs on IN-1K, IN-C, IN-A and Sty.-IN. As shown in the upper part of Table 5, DropCov respectively brings about 2.6%, 2.2% and 1.8% gains over ResNet-34, ResNet-50 and ResNet-101 on IN-1K, while showing stronger robustness on IN-C, IN-A and Sty.-IN. Particularly, DropCov with ResNet-50 and ResNet-101 are respectively superior to the original ResNet-101 and ResNet-152, but having less model complexity.

ViT. For ViT models, we apply our DropCov with $d = 128$ to DeiT-S [40], Swin-T [30] and T2T-ViT-14 [44], and compare the original models as well as ConViT [6]. As shown in the bottom part of Table 5, DropCov brings about 2.7%, 1.2% and 1.2% gains over DeiT-S, Swin-T and T2T-ViT-14 on

Table 5: Comparison of our DropCov with various ResNet and ViT models on four datasets, where results in terms of mCE and Top-1 accuracy (%) are reported on IN-C and reminding ones, receptively.

Method	Params.	FLOPs	IN-1K (↑)	IN-C (↓)	IN-A (↑)	Sty.-IN (↑)
ResNet-34 [15]	21.8 M	3.66 G	74.19	77.9	1.63	7.59
ResNet-50 [15]	25.6 M	3.86 G	76.02	76.7	2.47	7.15
ResNet-101 [15]	44.6 M	7.57 G	77.67	70.3	4.15	9.51
ResNet-152 [15]	60.2 M	11.28 G	78.13	69.3	5.98	10.09
ResNet-34+DropCov (Ours)	29.6 M	5.56 G	$76.81_{(2.62)}$	$71.1_{(6.8)}$	$3.45_{(1.82)}$	$11.16_{(3.57)}$
ResNet-50+DropCov (Ours)	32.0 M	6.19 G	$78.19_{(2.17)}$	$69.8_{(6.9)}$	$5.08_{(2.61)}$	$9.90_{(2.75)}$
ResNet-101+DropCov (Ours)	51.0 M	9.90 G	$\mathbf{79.51}_{(1.84)}$	$\mathbf{65.8}_{(4.5)}$	$7.54_{(3.39)}$	$\mathbf{11.41}_{(1.90)}$
DeiT-S [40]	22.1 M	4.6 G	79.8	54.6	18.9	14.91
Swin-T [30]	28.3 M	4.5 G	81.2	62.0	21.6	13.40
T2T-ViT-14 [44]	21.5 M	5.2 G	81.5	53.2	23.9	15.80
DeiT-B [40]	86.6 M	17.6 G	82.0	48.5	27.4	17.94
ConViT-B [6]	86.5 M	17.7 G	82.4	$\mathbf{46.9}$	29.0	$\mathbf{19.67}$
DeiT-S+DropCov (Ours)	25.6 M	5.5 G	$82.5_{(2.7)}$	$52.6_{(2.0)}$	$31.2_{(12.3)}$	$17.10_{(2.19)}$
Swin-T+DropCov (Ours)	31.6 M	6.0 G	$82.4_{(1.2)}$	$54.8_{(7.2)}$	$33.1_{(11.5)}$	$14.13_{(0.73)}$
T2T-ViT-14+DropCov (Ours)	24.9 M	5.4 G	$\mathbf{82.7}_{(1.2)}$	$52.1_{(1.1)}$	$\mathbf{31.7}_{(7.8)}$	$18.81_{(3.01)}$

IN-1K, respectively. Meanwhile, it shows stronger robustness on IN-C, IN-A and Sty.-IN. Particularly, DropCov with DeiT-S is superior to DeiT-B and ConViT-B, but having less model complexity. Above results show our DropCov provides a simple yet effective method to improve deep architectures.

5.5 Comparisons on Long-tailed Benchmark

Finally, we transfer our DropCov models to iNat2017 [18], and compare with several deep architectures, i.e., ResNet-101, ResNet-152, Inception V3 (IncV3) [39] with SE [19] as well as the recently proposed ViT [9] and TransFG [14]. Specifically, we perform fine-tuning of our DropCov models with backbones of ResNet-101 and DeiT-S on iNat2017, namely DropCov (ResNet-101) and DropCov (DeiT-S). As shown in Table 6, our DropCov (ResNet-101) obtains 5.5% gains over the original ResNet-101, while outperforming ResNet-152 and IncV3 SE. Besides, our DropCov (DeiT-S) respectively outperforms ViT-B and TransFG by 4.3% and 1.3%, which are pre-trained on ImageNet-21K. The results further verify generalization of DropCov.

Table 6: Comparisons (%) on iNat2017.

Method	Top-1	Top-5
ResNet-101	62.4	84.1
ResNet-152	64.2	85.5
IncV3 SE	66.3	86.7
ViT-B_16 (IN-21K) [9]	68.7	N/A
TransFG (IN-21K) [14]	71.7	N/A
DropCov (ResNet-101)	67.9	87.3
DropCov (DeiT-S)	$\mathbf{73.0}$	$\mathbf{88.5}$

6 Conclusion

In this paper, we first analyze the effect of post-normalization on GCP from the perspective of optimizing deep GCP networks. Particularly, we find that effective post-normalization methods have a good ability to balance representation decorrelation and information preservation, which can reduce over-fitting and increase representation ability of deep GCP networks, respectively. According to our finding, we introduce some strategies to improve existing normalization methods, further verifying our finding. More importantly, we propose a novel pre-normalization for GCP (namely DropCov) based on adaptive channel dropout, which achieves very competitive performance with a linear complexity and training-only mode. Extensive experiments identify that our DropCov provides a simple yet effective solution to improve existing deep architectures on image classification tasks.

Although GCP normally produces a high-dimensional covariance representation that brings a certain amount of computational cost, our work makes use of high-dimensional GCP more flexible. Meanwhile, the results in Table 4 show our DropCov can achieve clear gains for low-dimensional GCP ($d = 64$), where little extra computational cost is introduced. Additionally, our DropCov is potentially applicable to compact GCP methods [11, 22] and encourages other exploration of structure-level dropout strategies (e.g., DropBlock [13]), which will be investigated in our future work.

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