Exploring Non-Monotonic Latent Alignments for Non-Autoregressive Machine Translation

Abstract

Non-autoregressive translation (NAT) models are typically trained with the cross-entropy loss, which forces the model outputs to be aligned verbatim with the target sentence and will highly penalize small shifts in word positions. Latent alignment models relax the explicit alignment by marginalizing out all monotonic latent alignments with the CTC loss. However, they cannot handle non-monotonic alignments, which is non-negligible as there is typically global word reordering in machine translation. In this work, we explore non-monotonic latent alignments for NAT. We extend the alignment space to non-monotonic alignments to allow for the global word reordering and further consider all alignments that overlap with the target sentence. We non-monotonically match the alignments to the target sentence and train the latent alignment model to maximize the F1-score of non-monotonic matching. Extensive experiments on major WMT benchmarks show that our method substantially improves the translation performance and achieves comparable performance to the autoregressive Transformer with only one-iteration parallel decoding.

1 Introduction

Non-autoregressive translation (NAT) models achieve significant decoding speedup in neural machine translation [NMT, 1, 43] by generating target words simultaneously [17]. This advantage usually comes at the cost of translation quality due to the mismatch of training objectives. NAT models are typically trained with the cross-entropy loss, which forces the model outputs to be aligned verbatim with the target sentence and will highly penalize small shifts in word positions. The explicit alignment required by the cross-entropy loss cannot be guaranteed due to the multi-modality problem that there exist many possible translations for the same sentence [17], making the cross-entropy loss intrinsically mismatched with NAT.

As the cross-entropy loss can not evaluate NAT outputs properly, many efforts have been devoted to designing better training objectives for NAT [28, 38, 39, 13, 35, 9]. Among them, latent alignment models [28, 35] relax the alignment restriction by marginalizing out all monotonic latent alignments with the connectionist temporal classification loss [CTC, 15], which receive much attention for the ability to generate variable length translation.

Latent alignment models generally make a strong monotonic assumption on the mapping between model output and the target sentence. As illustrated in Figure 1a, the monotonic assumption holds in classic application scenarios of CTC like automatic speech recognition (ASR) as there is a natural monotonic mapping between the speech input and ASR target. However, non-monotonic alignments are non-negligible in machine translation as there is typically global word reordering, which is a common source of the multi-modality problem. As Figure 1b shows, when the target sentence is "I ate pizza this afternoon" but the model produces a translation with a different but correct word

(a) CTC for ASR   (b) CTC for NAT

Figure 1: Illustration of the monotonic alignment assumption of CTC: (a) CTC for ASR where there is a natural monotonic mapping between the speech input and ASR target, (b) CTC for NAT where there exists global word reordering that induces non-monotonic alignment. ϵ is the blank token that means 'output nothing'.

ordering "this afternoon I ate pizza", the CTC loss cannot handle this non-monotonic alignment between output and target and wrongly penalizes the model.

In this paper, we propose to model non-monotonic latent alignments for non-autoregressive machine translation. We first extend the alignment space from monotonic alignments to non-monotonic alignments to allow for the global word reordering in machine translation. Without the monotonic structure, we have to optimize the best alignment found by the Hungarian algorithm [42, 27, 7, 9] since it becomes difficult to marginalize out all non-monotonic alignments with dynamic programming. This difficulty can be overcome by not requiring an exact match between alignments and the target sentence. In practice, it is not necessary to have the translation include exact words as contained in the target sentence, but it would be favorable to have a large overlap between them. Therefore, we further extend the alignment space by considering all alignments that overlap with the target sentence. Specifically, we are interested in the overlap of n-grams, which is the core of some evaluation metrics (e.g., BLEU). We accumulate n-grams from all alignments regardless of their positions and non-monotonically match them to target n-grams. The latent alignment model is trained to maximize the F1-score of n-gram matching, which reflects the translation quality to a certain extent [31].

We conduct experiments on major WMT benchmarks for NAT (WMT14 En↔De, WMT16 En↔Ro), which shows that our method substantially improves the translation performance and achieves comparable performance to autoregressive Transformer with only one-iteration parallel decoding.

2 Background

2.1 Non-Autoregressive Machine Translation

[17] proposes non-autoregressive neural machine translation, which achieves significant decoding speedup by generating target words simultaneously. NAT breaks the dependency among target tokens and factorizes the joint probability of target words in the following form:

$$p(y|x,\theta) = \prod_{t=1}^{T} p_t(y_t|x,\theta), \tag{1}$$

where x is the source sentence belonging to the input space $\mathcal{X}$, $y = \{y_1, ..., y_T\}$ is the target sentence belonging to the output space $\mathcal{Y}$, and $p_t(y_t|x,\theta)$ indicates the translation probability in position t.

The vanilla-NAT has a length predictor that takes encoder states as input to predict the target length. During the training, the target length is set to the golden length, and the vanilla-NAT is trained with the cross-entropy loss, which explicitly aligns model outputs to target words:

$$\mathcal{L}_{CE}(\theta) = -\sum_{t=1}^{T} \log(p_t(y_t|x,\theta)). \tag{2}$$

2.2 NAT with Latent Alignments

The vanilla-NAT suffers from two major limitations. The first limitation is the explicit alignment required by the cross-entropy loss, which cannot be guaranteed due to the multi-modality problem

$$a = (\varepsilon, A, A, \varepsilon, A, B, B, C, \varepsilon, D)$$

$$(A, A, B, C, D) \quad (A, A, A, B, B, C, D)$$

CTC: $y = \beta^{-1}(a)$ SCTC: $y = \beta_s^{-1}(a)$

Figure 2: An example of target sentences obtained from collapsing functions of CTC and SCTC.

and therefore leads to the inaccuracy of the loss. The second limitation is the requirement of target length prediction. The predicted length may not be optimal and cannot be changed dynamically, so it is often required to use multiple length candidates and re-score them to produce the final translation.

The two limitations can be addressed by using CTC-based latent alignment models [15, 28, 35], which extend the output space $\mathcal{Y}$ with a blank token ϵ that means 'output nothing'. We define the extended output space as $\mathcal{Y}^*$. Following prior works [14, 35], we refer to the elements $a \in \mathcal{Y}^*$ as alignments, since the location of the blank tokens determines an alignment between the extended output and target sentence. Assume the generated alignments have length T_a, we define a function $\beta(y)$ which returns a subset of $\mathcal{Y}^*$ including all possible alignments of length T_a for y, where $\beta^{-1} : \mathcal{Y}^* \mapsto \mathcal{Y}$ is the collapsing function that first collapses all consecutive repeated words in a and then removes all blank tokens to obtain the target sentence. As illustrated in Figure 2, the alignment $a = (\epsilon, A, A, \epsilon, A, B, B, C, \epsilon, D)$ is collapsed to the target sentence $y = (A, A, B, C, D)$ with a monotonic mapping from target positions to alignment positions. During the training, latent alignment models marginalize out all alignments with the CTC loss [15]:

$$\log p(y|x, \theta) = \log \sum_{a \in \beta(y)} p(a|x, \theta), \tag{3}$$

where the alignment probability $p(a|x, \theta)$ is modeled by a non-autoregressive Transformer:

$$p(a|x, \theta) = \prod_{t=1}^{T_a} p_t(a_t|x, \theta). \tag{4}$$

NAT models with latent alignments achieve superior performance by overcoming the two major limitations of NAT [28, 35, 16, 24, 32, 45]. The weakness in handling non-monotonic latent alignments is noticed in previous work [35, 16] but remains unsolved.

3 Approach

In this section, we explore non-monotonic latent alignments for NAT. We introduce a simplified CTC loss in section 3.1, which has a simpler structure that helps further analysis and derivation under the regular CTC loss. We explore non-monotonic alignments under the simplified CTC loss in section 3.2, and utilize the results to derive non-monotonic alignments under the regular CTC loss in section 3.3. Finally, we introduce the training strategy to combine monotonic and non-monotonic latent alignments in section 3.4.

3.1 Simplified Connectionist Temporal Classification

In CTC, the collapsing function β^{-1} maps an alignment to a target sentence in two steps: (1) collapses all consecutive repeated words, (2) removes all blank tokens. The two operations make the alignment space $\beta(y)$ complex. Therefore, we first introduce the simplified connectionist temporal classification loss (SCTC), which has a simpler structure that helps the derivation of non-monotonic alignments, and the results on SCTC are helpful for further analysis under the regular CTC loss.

To simplify the alignment structure, we consider only using one operation in the collapsing function. One option is only collapsing all consecutive repeated words in the alignment. The concern is that it will limit the expressive power of the model. For example, the target sentence $y = (A, A, B, C, D)$ has probability 0 since it has repeated words that cannot be mapped from any alignment. Therefore, we favor another option which only removes all blank tokens. In this way, the expressive power stays

104 the same and the alignment space becomes much simpler, where alignments of y simply contain T
105 target words and $T_a - T$ blank tokens.

106 We denote this simplified loss as SCTC, and illustrate the difference between CTC and SCTC in
107 Figure 2. The alignment space for the target sentence y is defined as $\beta_s(y)$, and the collapsing
108 function is defined as β_s^{-1}. We can still obtain the s translation probability $p(y|x, \theta)$ with dynamic
109 programming. We define the forward variable $\alpha_t(s)$ to be the total probability of $y_{1:s}$ considering all
110 possible alignments $a_{1:t}$, which can be calculated recursively from $\alpha_{t-1}(s)$ and $\alpha_{t-1}(s-1)$:

$$\alpha_t(s) = \alpha_{t-1}(s-1)p_t(y_s|x, \theta) + \alpha_{t-1}(s)p_t(\epsilon|x, \theta). \tag{5}$$

111 Finally, the total translation probability $p(y|x, \theta)$ is given by the forward variable $\alpha_{T_a}(T)$.

3.2 Non-Monotonic Alignments under SCTC

3.2.1 Bipartite Matching

114 In this section, we explore non-monotonic alignments under the SCTC loss, which are helpful for
115 further analysis under the regular CTC loss. We first extend the alignment space from monotonic
116 alignments $\beta_s(y)$ to non-monotonic alignments $\gamma_s(y)$ to allow for the global word reordering in
117 machine translation, where $\gamma_s(y)$ is defined as:

$$\gamma_s(y) = \underset{y' \in P(y)}{\cup} \beta_s(y'). \tag{6}$$

118 In the above definition, $P(y)$ represents all permutations of y. For example, $P((A, B)) =$
119 $\{(A, B), (B, A)\}$. By enumerating $P(y)$, we consider all possible word reorderings of the tar-
120 get sentence. Ideally, we want to traverse all alignments in $\gamma_s(y)$ to calculate the log-likelihood loss:
121

$$\mathcal{L}_{sum}(\theta) = -\log \sum_{a \in \gamma_s(y)} p(a|x, \theta). \tag{7}$$

122 However, without the monotonic structure, it becomes difficult to marginalize out all latent alignments
123 with dynamic programming. Alternatively, we can minimize the loss of the best alignment, which is
124 an upperbound of Equation 7:

$$\mathcal{L}_{max}(\theta) = -\log \max_{a \in \gamma_s(y)} p(a|x, \theta). \tag{8}$$

125 Following prior work [42, 7, 9], we formulize finding the best alignment as a maximum bipartite
126 matching problem and solve it with the Hungarian algorithm [27]. Specifically, we observe that the
127 alignment space $\gamma_s(y)$ is simply permutations of T target words and $T_a - T$ blank tokens. Therefore,
128 finding the best alignment is equivalent to finding the best bipartite matching between the T_a model
129 predictions and the T target words plus $T_a - T$ blank tokens, where the two sets of nodes are connected
130 by edges with the prediction log-probability as weights.

3.2.2 N-Gram Matching

132 Without the monotonic structure desired for dynamic programming, calculating Equation 7 becomes
133 difficult. This difficulty is also caused by the strict requirement of exact match for alignments, which
134 makes it intractable to simplify the summation of probabilities. In practice, it is not necessary to force
135 the exact match between alignments and the target sentence since a large overlap is also favorable.
136 Therefore, we further extend the alignment space by considering all alignments that overlap with the
137 target sentence. Specifically, we are interested in the overlap of n-grams, which is the core of some
138 evaluation metrics (e.g., BLEU).

139 We propose a non-monotonic and non-exclusive n-gram matching objective based on SCTC to
140 encourage the overlap of n-grams. Following the underlying idea of probabilistic matching [37], we
141 introduce the probabilistic variant of the n-gram count to make the objective differentiable.

142 Specifically, we use $C_g(y)$ to denote the occurrence count of n-gram $g = (g_1, ..., g_n)$ in the target
143 sentence y, and use $C_g^s(\theta)$ to denote the probabilistic count of n-gram g for the model with input x
144 and parameter θ, where the superscript s indicates SCTC. For simplicity, we omit the source sentence

x in $C_g^s(\theta)$ and the following notations. The probabilistic n-gram count is obtained by accumulating n-grams from all possible target sentences:

$$C_g^s(\theta) = \sum_{y \in \mathcal{Y}} p(y|x,\theta) C_g(y) = \sum_{a \in \mathcal{Y}^*} p(a|x,\theta) C_g(\beta_s^{-1}(a)). \tag{9}$$

The calculation of $C_g^s(\theta)$ is the core of this method and will be described in detail later. We use $M_g^s(\theta)$ to denote the match count of n-gram g between the latent alignment model and the target sentence y (we omit y in $M_g^s(\theta)$ for simplicity), which is defined as follows:

$$M_g^s(\theta) = \min(C_g(y), C_g^s(\theta)). \tag{10}$$

Our objective is to maximize the n-gram overlap between the target sentence and model output, so we can train the model to maximize the precision or recall of n-gram matching:

$$P_n^s(\theta) = \frac{\sum_{g \in G_n} M_g^s(\theta)}{\sum_{g \in G_n} C_g^s(\theta)}, R_n^s(\theta) = \frac{\sum_{g \in G_n} M_g^s(\theta)}{\sum_{g \in G_n} C_g(y)}, \tag{11}$$

where we use G_n to denote the set of all n-grams. However, since the latent alignment model have the ability to control the translation length, both precision and recall cannot accurately reflect the translation quality. The precision will encourage short translations to reduce the denominator, while the recall prefers long translations. Therefore, we consider both of them and maximize the F1 score:

$$\mathcal{L}_n^s(\theta) = -\text{F1}_n^s(\theta) = -\frac{2 \cdot \sum_{g \in G_n} M_g^s(\theta)}{\sum_{g \in G_n} (C_g(y) + C_g^s(\theta))}, \tag{12}$$

where we use G_n to denote the set of all n-grams. In the numerator of Equation 12, $M_g^s(\theta)$ is non-zero only if $C_g(y)$ is non-zero, so we only need to calculate probabilistic n-gram counts $C_g^s(\theta)$ for n-grams g in the target sentence. In the denominator, $\sum_{g \in G_n} C_g(y)$ equals to the constant $T-n+1$. Therefore, there are only two tasks left: calculating $C_g^s(\theta)$ and the summation $\sum_{g \in G_n} C_g^s(\theta)$.

For the first task of calculating $C_g^s(\theta)$, we first consider the simple case $n=1$. The main difference between 1-gram matching and bipartite matching is that 1-gram matching is non-exclusive, which allows us to marginalize out all alignments using the property of non-autoregressive generation. Due to the space limit, we refer readers to Appendix.A for the derivation and directly give the probabilistic 1-gram count as follows:

$$C_g^s(\theta) = \sum_{t=1}^{T_a} p_t(g_1|x,\theta), g \in G_1, \tag{13}$$

where G_1 denotes the set of all 1-grams $g = (g_1)$. Both 1-gram matching and bipartite matching are completely non-monotonic, which aim at generating correct target words regardless of the word ordering. When the target sentence is "I ate pizza this afternoon", they will think that "pizza ate I this afternoon" is also a good translation. We consider 2-gram matching to overcome this limitation, which is also non-monotonic but takes target dependency into consideration. We first introduce a transition matrix A of size $T_a \times T_a$, where $A_{i,j}$ is the probability that all positions between i and j are blank tokens:

$$A_{i,j} = \begin{cases} 0 & j < i+1 \\ 1 & j = i+1 \\ \prod_{t=i+1}^{j-1} p_t(\epsilon|x,\theta) & j > i+1 \end{cases}. \tag{14}$$

Then we give the probabilistic 2-gram count for $g = (g_1, g_2)$ (see Appendix.A for derivation):

$$C_g^s(\theta) = \sum_{i=1}^{T_a} \sum_{j=1}^{T_a} p_i(g_1|x,\theta) A_{i,j} p_j(g_2|x,\theta), g \in G_2. \tag{15}$$

For a larger n, the probabilistic n-gram count is:

$$C_g^s(\theta) = \sum_{i_1=1}^{T_a} \sum_{i_2=1}^{T_a} \dots \sum_{i_n=1}^{T_a} p_{i_1}(g_1|x,\theta) \times \prod_{k=2}^{n} A_{i_{k-1},i_k} p_{i_k}(g_k|x,\theta), \ g \in G_n. \tag{16}$$

174 Equation 15 can be efficiently calculated in matrix form with $\mathcal{O}(T_a^2)$ time complexity. For Equation
175 16, we need $\mathcal{O}(nT_a^n)$ operations to calculate it by brute force, which is impractical when n is large.
176 We refer readers to Appendix.B for the $\mathcal{O}(nT_a^2)$ algorithm, which is similar to calculating the n-step
177 transition probability of a Markov chain.

178 For the second task of efficiently calculating the summation of probabilistic n-gram counts
179 $\sum_{g \in G_n} C_g^s(\theta)$, we also consider the simple case $n = 1$ first, where the summation can be cal-
180 culated by directly summing all probabilities:

$$\sum_g C_g^s(\theta) = \sum_{t=1}^{T_a} \sum_g p_t(g_1|x,\theta), g \in G_1. \tag{17}$$

181 For $n > 1$, the search space is too big for the direct summation. Fortunately, the definition of $C_g^s(\theta)$
182 in Equation 9 allows us to safely extend the result of $n = 1$ to a larger n since n-grams count in a
183 sentence is always $n - 1$ less than 1-grams count.

$$\sum_{g \in G_n} C_g^s(\theta) = \sum_{g \in G_1} C_g^s(\theta) - n + 1. \tag{18}$$

184 In summary, with the above results (Equation 13-18), we are able to efficiently calculate the loss
185 defined in Equation 12 to maximize the F1 score non-monotonic n-gram matching, which reflects the
186 translation quality to a certain extent.

3.3 Non-Monotonic Alignments under CTC

188 In this section, we explore non-monotonic latent alignments under the regular CTC loss. Under
189 SCTC, we can use both the bipartite matching and n-gram matching to train the latent alignment
190 model. However, bipartite matching is infeasible under the regular CTC loss. Recall that the search
191 space $\gamma_s(y)$ is simply permutations of target words and $T_a - T$ blank tokens, which allows us to
192 find the best alignment with the Hungarian algorithm. However, as consecutive repeated words will
193 be collapsed in the regular CTC loss, alignments in $\gamma(y)$ may contain different words, so the best
194 alignment cannot be found by the Hungarian algorithm.

195 Fortunately, for n-gram matching, we can utilize previous results of SCTC to help the analysis and
196 derivation under the regular CTC loss. As the output of the collapsing function $\beta_s^{-1}(a)$ simply
197 contains more repeated words than $\beta^{-1}(a)$, we can first identify how much influence these repeated
198 words have, and then remove this influence from the results of SCTC. We first formally define the
199 probabilistic n-gram count under the regular CTC:

$$C_g(\theta) = \sum_{a \in \mathcal{Y}^*} p(a|x,\theta)C_g(\beta^{-1}(a)). \tag{19}$$

200 We define $R_g(\theta)$ to be the gap bewteen $C_g(\theta)$ and $C_g^s(\theta)$ when $n = 1$, which represents the number
201 of repeated words g_1 removed by the collapsing function. $R_g(\theta)$ can be efficiently calculated as
202 follows (see Appendix.C for derivation):

$$R_g(\theta) = C_g^s(\theta) - C_g(\theta) = \sum_{t=1}^{T_a-1} p_t(g_1|x,\theta)p_{t+1}(g_1|x,\theta), g \in G_1. \tag{20}$$

203 The above equation enables us to efficiently calculate $C_g(\theta)$ and $\sum_g C_g(\theta)$ when $g \in G_1$. For
204 2-grams $g = (g_1, g_2)$, we divide it into two cases $g_1 = g_2$ and $g_1 \neq g_2$. The collapsing of consecutive
205 repeated words will only remove 2-grams of the first case, where each removed word corresponds
206 to a removed 2-gram. Taking Figure 2 as an example, the output of SCTC $\beta_s^{-1}(a)$ have two more
207 2-grams $\{(A, A), (B, B)\}$ of the $g_1 = g_2$ case than the output of CTC $\beta^{-1}(a)$, and they contain
208 exactly the same 2-grams of the $g_1 \neq g_2$ case. Therefore, we can directly reuse previous results of
209 SCTC and the number of repeated words $R_g(\theta)$ to calculate the probabilistic 2-gram count:

$$C_g(\theta) = \begin{cases} C_g^s(\theta) - R_g(\theta), & g_1 = g_2 \\ C_g^s(\theta), & g_1 \neq g_2 \end{cases}, g \in G_2. \tag{21}$$

210 Unfortunately, for $n > 2$, there is no such a clear relationship between $C_g(\theta)$ and $C_g^s(\theta)$, where the
211 CTC output can contain n-grams (for example, (A, B, C) in Figure 2) that the SCTC output does not

have. If we directly formulize $C_g(\theta)$ like in Equation 16, it will be more complex and the $\mathcal{O}(nT_a^2)$ algorithm is no longer applicable. Therefore, we do not use n-gram matching with $n > 2$ under the regular CTC loss. Finally, we define the match count $M_g(\theta)$ similarly as in Equation 10 and train the CTC-based latent alignment model to maximize the F1 score:

$$\mathcal{L}_n(\theta) = -\frac{2 \cdot \sum_{g \in G_n} M_g(\theta)}{\sum_{g \in G_n}(C_g(y) + C_g(\theta))}. \tag{22}$$

3.4 Training

In the above, we propose non-monotonic training objectives for the latent alignment model. They need to be combined with monotonic training objectives, otherwise, they may generate translations like "pizza ate I this afternoon". Therefore, we first pretrain the latent alignment model with monotonic training objectives like SCTC or CTC. The model pretrained by SCTC can be finetuned with the bipartite matching objective (Equation 8) or the n-gram matching objective (Equation 12). The model pretrained by CTC can be finetuned with the n-gram matching objective (Equation 22), and the choices of n are limited to $\{1, 2\}$.

4 Experiments

4.1 Experimental Setup

Datasets We evaluate our methods on the most widely used public benchmarks in previous NAT studies: WMT14 English↔German (En↔De, 4.5M sentence pairs) [5] and WMT16 English↔Romanian (En↔Ro, 0.6M sentence pairs) [6]. For WMT14 En↔De, the validation set is *newstest2013* and the test set is *newstest2014*. For WMT16 En↔Ro, the validation set is *newsdev-2016* and the test set is *newstest-2016*. Following prior works, we use tokenized BLEU [31] to evaluate the translation quality. Considering that BLEU might be biased as our method optimizes the n-gram overlap, we also report METEOR [2], which is not directly based on n-gram overlap. We learn a joint BPE model [36] with 32K operations to process the data and share the vocabulary for source and target languages.

Knowledge Distillation Following prior works [17], we apply sequence-level knowledge distillation [25] to reduce the complexity of training data. We use Transformer-base setting for the teacher model and train NAT on the distilled data.

Implementation Details We adopt Transformer-base [43] as our autoregressive baseline as well as the teacher model. We uniformly copy encoder outputs to construct decoder inputs. For CTC-based NAT models, the length for decoder inputs is $3\times$ as long as the source length. On WMT14 En↔De, we use a dropout rate of 0.2 to train NAT models and use a dropout rate of 0.1 for finetuning. On WMT16 En↔Ro, the dropout rate is 0.3 for both the pretraining and finetuning. We use the batch size and train NAT models for 300K steps on WMT14 En↔De and 150K steps on WMT16 En↔Ro. During the finetuning, we train NAT models for 2K steps with the batch size 256K. All models are optimized with Adam [26] with $\beta = (0.9, 0.98)$ and $\epsilon = 10^{-8}$. The learning rate warms up to $5 \cdot 10^{-4}$ within 10K steps in the pretraining and warms up to $2 \cdot 10^{-4}$ within 500 steps in the finetuning. We use the GeForce RTX 3090 GPU to train models and measure the translation latency. We implement our models based on the open-source framework of `fairseq` [MIT License, 30].

Decoding For autoregressive models, we use beam search with beam size 5 for the decoding. For Vanilla-NAT, we use argmax decoding to generate the translation. For CTC-based models, we can also use argmax decoding to generate the alignment and then collapse it to obtain the translation. Besides, following [28, 16], we can use beam search decoding combined with a 4-gram language model [20] to find the translation that maximizes:

$$\log p(y|x, \theta) + \alpha \log p_{LM}(y) + \beta \log(|y|), \tag{23}$$

where α and β are hyperparameters for language model scores and length bonus. We use a fixed beam size 20 with grid-search α, β based on the performance on the validation set. The beam search does not contain any neural network computations and can be implemented efficiently in C++[1].

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[1] https://github.com/parlance/ctcdecode

4.2 Main Results

We first compare the performance of different non-monotonic training objectives for latent alignment models. Baseline models include the Transformer, vanilla-NAT, SCTC, and CTC, and we finetune latent alignment models with bipartite matching and n-gram matching objectives. In Table 1, we report BLEU and METEOR scores along with the speedup on WMT14 En-De test set.

Table 1: Performance comparison between baselines and our methods on WMT14 En-De test set.

	Model	BLEU	METEOR	Speed
Base	Transformer	27.54	54.38	1.0×
	Vanilia-NAT	19.32	45.79	15.5×
SCTC	SCTC	25.26	52.06	
	+bipartite	26.13	53.17	
	+1-gram	26.22	53.20	
	+2-gram	**26.79**	53.54	14.7×
	+3-gram	26.66	53.48	
	+4-gram	26.62	53.43	
CTC	CTC	26.34	53.15	
	+1-gram	27.16	53.95	14.7×
	+2-gram	**27.57**	**54.50**	

Table 2: Performance comparison between our models and existing methods. The speedup is measured on WMT14 En-De test set with batch size 1. AT means autoregressive. **Iter.** denotes the number of iterations at inference time. '–' indicates that the result is not reported. '12-1' means the Transformer with 12 encoder layers and 1 decoder layer [23]. 'beam&lm' means beam search decoding combined with a 4-gram language model.

Models		Iter.	Speed	WMT14		WMT16	
				EN-DE	DE-EN	EN-RO	RO-EN
AT	Transformer (teacher)	N	1.0×	27.54	**31.57**	**34.26**	**33.87**
	Transformer (12-1)	N	2.6×	26.09	30.30	32.76	32.39
	+ KD	N	2.7×	**27.61**	31.48	33.43	33.50
Non-CTC NAT	NAT-FT [17]	1	15.6×	17.69	21.47	27.29	29.06
	AXE [13]	1	–	23.53	27.90	30.75	31.54
	GLAT [32]	1	15.3×	25.21	29.84	31.19	32.04
	AligNART [41]	1	13.4×	26.40	30.40	32.50	33.10
	CMLM [12]	10	–	27.03	30.53	33.08	33.31
	LevT [18]	2.05	4.0×	27.27	–	–	33.26
	JM-NAT [19]	10	–	27.69	**32.24**	33.52	33.72
	RewriteNAT [11]	2.70	–	**27.83**	31.52	**33.63**	**34.09**
CTC-based NAT	CTC [28]	1	–	16.56	18.64	19.54	24.67
	Imputer [35]	1	–	25.80	28.40	32.30	31.70
	REDER [45]	1	15.5×	26.70	30.68	33.10	33.23
	+ beam&reranking	1	5.5×	27.36	31.10	33.60	34.03
	Fully-NAT [16]	1	16.8×	27.20	31.39	33.71	**34.16**
	Imputer [35]	8	–	**28.20**	**31.80**	**34.40**	34.10
Our Implementation	Bag-of-ngrams [38]	1	15.5×	25.28	29.66	31.37	31.51
	+ rescore 5 candidates	1	9.5×	25.95	30.43	32.78	32.92
	OAXE [9]	1	15.3×	24.70	29.30	31.06	31.37
	+ rescore 5 candidates	1	14.0 ×	25.78	30.29	32.45	32.63
	GLAT+CTC [32]	1	14.7×	26.71	29.93	32.85	33.12
	+ beam&lm	1	5.0×	**27.58**	**31.25**	**33.59**	**33.98**
Our work	Vanilia-NAT	1	15.5×	19.32	24.23	29.03	29.46
	CTC	1	14.7×	26.34	29.58	33.45	33.32
	NMLA	1	14.7×	27.57	31.28	33.86	33.94
	NMLA + beam&lm	1	5.0×	**28.35**	**32.27**	**34.72**	**34.95**

From Table 1, we have the following observations: (1) Non-monotonic training objectives can effectively improve the performance of latent alignment models, which improves SCTC by 1.53 BLEU and 1.48 METEOR and improves CTC by 1.23 BLEU and 1.35 METEOR, illustrating the importance of non-monotonic alignments. (2) Despite having the same expressive power, SCTC underperforms CTC in the task of non-autoregressive translation. We speculate that it is because of the difference in the distribution of the alignment space. Most target sentences do not contain repeated words, in which case the alignment space of SCTC $\beta_s^{-1}(y)$ is a subset of $\beta^{-1}(y)$, making the ability of SCTC relatively weak. (3) SCTC achieves the best performance with 2-gram finetuning, and then the performance decreases with the increase of n due to the sparsity of higher rank n-grams. It suggests that we do not need a large n and relieves the concern on CTC that we cannot calculate n-gram matching for $n > 2$. (4) The correlation between the reported BLEU and METEOR scores is very strong, suggesting that BLEU is not biased when evaluating our methods. Therefore, we can safely use BLEU to evaluate the translation quality in the following experiments.

We call the best method among non-monotonic training methods for latent alignment models NMLA, and report the performance of NMLA comparing with AT and existing NAT approaches in Table 2. Notably, NMLA achieves comparable performance to autoregressive Transformer on all benchmarks, demonstrating the effectiveness of our method. With the help of beam search and 4-gram language model, NMLA even outperforms other iterative NAT models and the autoregressive Transformer with only one-iteration parallel decoding. Compared to our implementations of prior works that explore non-monotonic matching for NAT [38, 9], we can see that they underperform the CTC baseline even when rescoring 5 candidates, and their speedup after rescoring also lays behind CTC. It demonstrates the importance of exploring non-monotonic matching on latent alignment models, which is more desirable for its flexibility of generating predictions with variable length.

5 Related Work

[17] first proposes non-autoregressive machine translation to accelerate the decoding process of NMT. The cross-entropy loss is applied to train the Vanilla-NAT, which requires the explicit alignment between the model output and target sentence. The explicit alignment cannot be guaranteed due to the multi-modality problem, and many efforts have been devoted to mitigating this issue.

One research direction is to enable explicit alignment by leaking part of the target-side information during the training. [17] uses fertilities as a latent variable to specify the number of output words corresponding to each input word. [22, 34, 4] apply vector quantization to train NAT with discrete latent variables, and [29, 40, 16] apply variational inference to train NAT with continuous latent variables. [33, 3, 41] also introduce latent variables that carry the reordering information. Besides using latent variables, [12] directly feeds the partially masked target sentence as the decoder input, and [32] applies a glancing sampling strategy to control the mask ratio.

Another research direction is to propose better training objectives for NAT. [44] introduces two auxiliary regularization terms to reduce errors of repeated translations and incomplete translations. [13] introduces the aligned cross-entropy loss, which trains the model with the best monotonic alignment found by dynamic programming. [21] trains NAT with deep supervision and feed additional layer-wise predictions. Our approach is most related to [28, 38, 35, 9]. [28, 35] introduces latent alignment models for NAT, which relax the alignment restriction of the cross-entropy loss and allow for generating predictions with variable length. [38] introduces the bag-of-ngrams loss and [9] introduces the order-agnostic cross-entropy loss for NAT. Both the two training objectives perform non-monotonic matching between the model output and the target sentence. The main difference of our work is that we explore non-monotonic matching on latent alignment models, which is a more complex scenario and more desirable due to its superior performance and the flexibility of generating predictions with variable length.

6 Conclusion

Latent alignment models cannot handle non-monotonic alignments during the training, which is nonnegligible in machine translation. In this paper, we explore non-monotonic latent alignments for NAT and propose two matching objectives named bipartite matching and n-gram matching. Experiments show that our method achieves strong improvements on latent alignment models.

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A Proof of Equation 13, Equation 15, and Equation 16

Equation 13 and and Equation 15 are the special cases of Equation 16 when n is 1 or 2, so we directly provide the proof of Equation 16:

$$C_g^s(\theta) = \sum_{i_1=1}^{T_a} \sum_{i_2=1}^{T_a} \cdots \sum_{i_n=1}^{T_a} p_{i_1}(g_1|x,\theta) \times \prod_{k=2}^{n} A_{i_{k-1},i_k} p_{i_k}(g_k|x,\theta), g \in G_n.$$

We begin with the definition of $C_g^s(\theta)$, which accumulates the n-gram g from all alignments:

$$C_g^s(\theta) = \sum_{a \in \mathcal{Y}^*} p(a|x,\theta) C_g(\beta_s^{-1}(a)),$$

where $C_g(\beta_s^{-1}(a))$ is the occurrence count of n-gram g in the collapsed sentence $\beta_s^{-1}(a)$. We use $i_{1:n} = \{i_1, ..., i_n\}$ to denote the possible position of $g = \{g_1, ..., g_n\} \in G_n$. There is an n-gram $g \in G_n$ in the position $i_{1:n}$ if and only if $a_{i_k} = g_k$ for all $k \in \{1, ..., n\}$ and all other words between them are blank tokens. By traversing all possible positions, we can obtain the n-gram count as follows:

$$C_g(\beta_s^{-1}(a)) = \sum_{i_1=1}^{T_a} \sum_{i_2=1}^{T_a} \cdots \sum_{i_n=1}^{T_a} 1(a_{i_1} = g_1) \times \prod_{k=2}^{n} 1(i_{k-1} < i_k, a_{i_k} = g_k, a_{i_{k-1}+1:i_k-1} = \epsilon), g \in G_n.$$

where $1(\cdot)$ is the indicator function that takes value 1 when the inside condition holds, otherwise it takes value 0. Recall that A_{ij} is the probability that all positions between i and j are blank tokens, which satisfies the following equation:

$$A_{i_{k-1},i_k} = p(i_{k-1} < i_k, a_{i_{k-1}+1:i_k-1} = \epsilon|x,\theta).$$

Using the above equations, we can rewrite $C_g^s(\theta)$ as:

$$
\begin{aligned}
C_g^s(\theta) &= \sum_a p(a|x,\theta) C_g(\beta_s^{-1}(a)) \\
&= \sum_{i_1=1}^{T_a} \sum_{i_2=1}^{T_a} \cdots \sum_{i_n=1}^{T_a} \sum_a p(a|x,\theta) 1(a_{i_1} = g_1) \times \prod_{k=2}^{n} 1(i_{k-1} < i_k, a_{i_k} = g_k, a_{i_{k-1}+1:i_k-1} = \epsilon) \\
&= \sum_{i_1=1}^{T_a} \sum_{i_2=1}^{T_a} \cdots \sum_{i_n=1}^{T_a} p(a_{i_1} = g_1|x,\theta) \times \prod_{k=2}^{n} p(a_{i_k} = g_k|x,\theta) p(i_{k-1} < i_k, a_{i_{k-1}+1:i_k-1} = \epsilon) \\
&= \sum_{i_1=1}^{T_a} \sum_{i_2=1}^{T_a} \cdots \sum_{i_n=1}^{T_a} p_{i_1}(g_1|x,\theta) \times \prod_{k=2}^{n} A_{i_{k-1},i_k} p_{i_k}(g_k|x,\theta), g \in G_n. \quad \square
\end{aligned}
$$

B The $\mathcal{O}(nT_a^2)$ algorithm for Equation 16

Here we give the $\mathcal{O}(nT_a^2)$ algorithm to calculate $C_g^s(\theta)$ according to Equation 16. Notice that Equation 16 has a similar form with the n-step transition probability of a Markov chain, so we can try a similar method as the calculating the transition probability for a Markov chain. Specifically, we introduce a set of state vectors s_k, where $k \in \{1, 2, ..., T_a\}$ and the dimension of each vector s_k is T_a. We define the state vectors as follows:

$$s_k(i_k) = \begin{cases} p_{i_k}(g_k|x,\theta), & k = 1 \\ \sum_{i_{k-1}=1}^{T_a} s_{k-1}(i_{k-1}) A_{i_{k-1},i_k} p_{i_k}(g_k|x,\theta), & k > 1 \end{cases}.$$

From the above equation, the time complexity to calculate s_k given s_{k-1} is $\mathcal{O}(T_a^2)$ and can be efficiently calculated in matrix form. Therefore, the time complexity to sequentially calculate $s_{1:n}$ is $\mathcal{O}(nT_a^2)$. Finally, we can easily obtain $C_g^s(\theta)$ by summing the last vector s_n:

$$C_g^s(\theta) = \sum_{i_1=1}^{T_a} \sum_{i_2=1}^{T_a} \cdots \sum_{i_n=1}^{T_a} p_{i_1}(g_1|x,\theta) \times \prod_{k=2}^{n} A_{i_{k-1},i_k} p_{i_k}(g_k|x,\theta)$$

$$= \sum_{i_n=1}^{T_a} \cdots \sum_{i_2=1}^{T_a} \sum_{i_1=1}^{T_a} s_1(i_1) \prod_{k=2}^{n} A_{i_{k-1},i_k} p_{i_k}(g_k|x,\theta)$$

$$= \sum_{i_n=1}^{T_a} \cdots \sum_{i_3=1}^{T_a} \sum_{i_2=1}^{T_a} s_2(i_2) \prod_{k=3}^{n} A_{i_{k-1},i_k} p_{i_k}(g_k|x,\theta)$$

$$= \ldots\ldots$$

$$= \sum_{i_n=1}^{T_a} \sum_{i_{n-1}=1}^{T_a} s_{n-1}(i_{n-1}) \times A_{i_{n-1},i_n} p_{i_n}(g_n|x,\theta)$$

$$= \sum_{i_n=1}^{T_a} s_n(i_n), g \in G_n. \quad \square$$

C Proof of Equation 20

Here we provide the proof of Equation 20:

$$R_g(\theta) = C_g^s(\theta) - C_g(\theta) = \sum_{t=1}^{T_a-1} p_t(g_1|x,\theta) p_{t+1}(g_1|x,\theta), g \in G_1,$$

which is equivalent to proving the following equation:

$$C_g(\theta) = p_1(g_1|x,\theta) + \sum_{t=2}^{T_a} p_t(g_1|x,\theta)(1 - p_{t-1}(g_1|x,\theta)), g \in G_1.$$

We begin with the definition of $C_g(\theta)$, which accumulates the n-gram g from all alignments:

$$C_g(\theta) = \sum_{a \in \mathcal{Y}^*} p(a|x,\theta) C_g(\beta^{-1}(a)).$$

$C_g(\beta^{-1}(a))$ is the occurrence count of n-gram g in the collapsed sentence $\beta^{-1}(a)$. Besides blank tokens, repeated words will also be removed by β^{-1}. Therefore, we call a word in the alignment a valid word when it does not repeat the word before it. We can calculate the 1-gram count by collecting all valid words, which will not be removed by the collapsing function:

$$C_g(\beta^{-1}(a)) = 1(a_1 = g_1) + \sum_{t=2}^{T_a} 1(a_t = g_1, a_{t-1} \neq g_1), g \in G_1.$$

Using the above equation, we can rewrite $C_g(\theta)$ as:

$$C_g(\theta) = \sum_a p(a|x,\theta) C_g(\beta^{-1}(a)) = p(a_1 = g_1|x,\theta) + \sum_{t=2}^{T_a} p(a_t = g_1, a_{t-1} \neq g_1|x,\theta)$$

$$= p_1(g_1|x,\theta) + \sum_{t=2}^{T_a} p_t(g_1|x,\theta)(1 - p_{t-1}(g_1|x,\theta)), g \in G_1. \quad \square$$

D Analysis

To better understand our method, we present a qualitative analysis to provide some insights on where modeling non-monotonic latent alignments improves over the CTC baseline. We analyze the generated outputs of the WMT14 En-De test set.

NMLA Handles Long Sequences Better We first investigate the performance of AT and NAT models for different sequence lengths. We split the test set into different buckets based on target sequence length and calculate BLEU for each bucket. We report the results in Table 3. The first observation is that the performance of Vanilla-NAT drops drastically as the sequence length increases. It is a well known property of NAT and can be explained as the increasing probability of observing misalignment between model output and target as the target sequence becomes longer. The misalignment in long sequences causes the inaccuracy of loss and therefore harms the translation quality. We have a similar observation when comparing CTC with NMLA. The performance of CTC is close to NMLA on short sequences, but the gap becomes larger as the sequence length increases. Our explanation is that the longer the sequence, the more likely the monotonic assumption fails due to the global word reordering, which causes the inaccuracy of loss and therefore harms the translation quality. NMLA models non-monotonic alignments and alleviates the inaccuracy of loss on long sequences, so it achieves strong improvements on long sequences and even outperforms AT by more than 2 BLEU, which suffers from the error accumulation on long sequences.

Table 3: The performance of AT and NAT models with respect to the target sequence length. 'Vanilla' means Vanilla-NAT. 'N' denotes the target sequence length. 'AT' means autoregressive Transformer.

Length	Vanilla	CTC	AT	NMLA
$1 \leq N < 20$	21.27	24.83	25.68	25.55
$20 \leq N < 40$	19.49	26.81	28.05	27.82
$40 \leq N < 60$	16.21	26.54	26.71	27.74
$60 \leq N$	10.69	26.69	26.44	28.89
All	19.32	26.34	27.54	27.57

NMLA Improves Model Confidence NAT suffers from the multi-modality problem, which makes the model consider many possible translations at the same time and become less confident in generating outputs. We investigate whether our method alleviates the multi-modality problem by measuring how confident the model is during the decoding. We use the information entropy to measure the confidence, which is calculated as $H(X) = -\sum_{x \in \mathcal{X}} p(x) \log p(x)$, and lower entropy indicates higher confidence. We calculate the entropy in every position when decoding the WMT14 En-De test set and use the average entropy to represent the model confidence. We do not report the entropy of AT since it varies with the decoding algorithm. The average entropy scores of NAT models are reported in Table 4. We can see that the Vanilla-NAT has low prediction confidence, which is substantially improved by CTC. NMLA further reduces the average entropy to 0.061, showing that the model confidence can be improved by modeling non-monotonic latent alignments.

Table 4: The average prediction entropy of NAT models. 'Vanilla' means Vanilla-NAT.

	Vanilla	CTC	NMLA
Entropy	2.098	0.260	0.061

NMLA Improves Generation Fluency Since NAT generates target words independently, generation fluency is a main weakness of NAT. The multi-modality problem makes NAT consider many possible translations at the same time and generate an inconsistent output, which further reduces the generation fluency. We investigate whether our method improves the generation fluency with an n-gram language model [20] trained on the target-side of WMT14 En-De training data. We use the language model to calculate the perplexity scores of model outputs and report the results in Table 5. We can see that Vanilla-NAT and CTC have high perplexity that indicates low fluency. NMLA substantially reduces the perplexity score, which is only 42.6 higher than the gold reference. For AT, its perplexity score is even lower than the gold reference, which is consistent with prior works that autoregressive NMT generally improves fluency at the cost of adequacy [8, 10].

Table 5: The perplexity scores of AT and NAT models on WMT14 En-De test set.

	Vanilla	CTC	NMLA	AT	Gold
PPL	597.0	315.5	258.8	197.0	216.2

E   Training Cost

Our method requires to pretrain the CTC model first and then finetune it with non-monotonic training objectives, so more training time is required. We may also use a larger batch size for the training or finetuning to improve model performance at the cost of more training time. In Table 6, we use different batch sizes and report the BLEU scores along with the training time on WMT14 En-De. We can see that NMLA improves the CTC baseline by 1.23 BLEU with only $1.28\times$ training cost, which cannot be achieved by simply increasing the batch size for the training or finetuning.

Table 6: BLEU scores of different models on WMT14 En-De test set along with the training time measured on 8 GeForce RTX 3090 GPUs.

Model	BLEU	Time
64K GLAT+CTC [32]	26.39	–
128K Fully-NAT [16]	27.20	–
128K CTC	26.59	52.8h
64K CTC	26.34	26.4h
+512K CTC-Tune	26.56	27.8h
+64K Bigram-Tune	27.32	28.3h
+256K Bigram-Tune	27.57	33.9h

F   Finetuning Steps

To illustrate the impact of the finetuning steps, we use the bigram matching objective to finetune SCTC and CTC baselines, and draw their BLEU curves on WMT14 En-De validation set in Figure 3. For both SCTC and CTC, their BLEU scores rise rapidly in the first 500 steps, and then the curves become more steady. After 2000 steps, continuing finetuning can barely bring a performance gain, so it is cost-effective to finetune the baseline for only 2000 steps.

Figure 3: BLEU scores of CTC and SCTC under different finetuning steps of bigram matching.

G   Combination of N-grams

In Table 1, we only finetune the baseline with different n-gram matching objectives respectively. Here we combine diffent n-gram matching objectives to see whether the combination can improve the model performance. First, we linearly combine 1-gram and 2-gram matching with a hyperparameter α:

$$\mathcal{L}(\theta) = \alpha \cdot \mathcal{L}_1(\theta) + (1 - \alpha) \cdot \mathcal{L}_2(\theta), \quad \mathcal{L}^s(\theta) = \alpha \cdot \mathcal{L}_1^s(\theta) + (1 - \alpha) \cdot \mathcal{L}_2^s(\theta).$$

We train CTC and SCTC baselines with different α and report the results in Table 7, which shows that the combination of 1-gram and 2-gram underperforms simply using 2-gram matching.

As $n > 2$ is allowed in SCTC, we further try to combine higher rank n-grams. We consider $n \in \{1, 2, 3, 4\}$ and combine them with arithmetic average and geometric average respectively. The

Table 7: BLEU scores of SCTC and CTC under different α on WMT14 En-De validation set.

α	0	0.25	0.5	0.75	1
SCTC	25.09	24.98	24.85	24.70	24.54
CTC	25.90	25.74	25.66	25.51	25.42

BLEU scores of arithmetic average and geometric average are respectively 24.94 BLEU and 24.88 BLEU on WMT14 En-De validation set, which also underperforms simply using 2-gram matching. Therefore, we conclude that simply combining different n-gram matching objectives cannot improve the model performance.