In What Ways Are Deep Neural Networks Invariant and How Should We Measure This?

Anonymous Author(s)
Affiliation
Address
email

Abstract

It is often said that a deep learning model is "invariant" to some specific type of transformation. However, what is meant by this statement strongly depends on the context in which it is made. In this paper we explore the nature of invariance and equivariance of deep learning models with the goal of better understanding the ways in which they actually capture these concepts on a formal level. We introduce a family of invariance and equivariance metrics that allows us to quantify these properties in a way that disentangles them from other metrics such as loss or accuracy. We use our metrics to better understand the two most popular methods used to build invariance into networks: data augmentation and equivariant layers. We draw a range of conclusions about invariance and equivariance in deep learning models, ranging from whether initializing a model with pretrained weights has an effect on a trained model's invariance, to the extent to which invariance learned via training can generalize to out-of-distribution data.

1 Introduction

The notions of invariance and equivariance have been guiding concepts across a diverse range of scientific domains, from physics to psychology. In machine learning (ML) the concept of invariance is frequently invoked to describe models whose output does not change when its input is transformed in ways that are irrelevant to the task the model was designed for. For a dog or cat image classifier, for example, it is desirable for the model to be reflection invariant so that the same prediction is made whether or not the input image is reflected across its vertical axis. This type of invariance is useful in this model because although an image generally changes when reflected, whether it contains a dog or cat does not. Equivariance is used to describe models whose output changes in a manner that is aligned with the way that input is transformed. For example, as an image is rotated, the position of bounding boxes predicted by an object detector should also rotate (thus, the object detector should be rotation equivariant).

While mathematicians have developed rigorous theory that can describe invariance and equivariance, the amount of this that is actually used in the context of ML varies dramatically among different works. Within computer vision, research on relatively simple image transformations (i.e., rotations or translations) is often presented within a solid group-theoretic framework [4; 44; 5]. Other work dealing with more complex types of transformations such as changes in image background have (by necessity) needed to be more informal [43]. Furthermore, while model invariance and equivariance are frequently a central component in a broad range of ML research, limited effort has been put into trying to measure them directly. Rather, most works measure them indirectly through other ML metrics that align with the ultimate purpose of the model (e.g., loss or accuracy). While such a strategy makes sense when optimizing model performance is the primary objective, we miss an opportunity to better understand how and why deep neural networks work (or fail). The purpose of this paper is to propose a group-theoretic family of metrics, associated to an arbitrary group

Submitted to 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Do not distribute.

of symmetries G, which we call *G-empirical equivariance deviation* (G-EED), that quantify the G-equivariance (and G-invariance) of a model.

Informally, the G-EED of a model measures the extent to which it fails to be equivariant on a specific data distribution and with respect to a specific notion of distance in output space. This aligns with needs in ML where the user of a model may care only that their model is equivariant on data that the model will actually encounter in practice and not on any possible input. Further, since invariance is a special case of equivariance, G-EED also measures the extent to which a model fails to be invariant to the action of G. To show the breadth of the G-EED concept, we give a number of different ways it can be applied to measure different aspects of equivariance in a model. For example, we show how G-EED can be applied to a model's latent space representations to measure the extent to which a model extracts G-invariant features.

Finally, we use G-EED to answer a range of questions about invariance and equivariance in neural networks, with a focus on the two most popular ways of inducing invariance in these models: data augmentation and equivariant architectures. Some of the conclusions we draw from our experiments include the following. (1) Training with augmentation does not tend to induce invariance through learned equivariant layers; rather, invariance arises through some other mechanism. (2) Invariance learned through augmentation does generalize mildly to out-of-distribution data (e.g., common image corruptions), but apparent invariance seen for far out-of-distribution data (e.g., images from a completely different domain) is probably a result of model insensitivity. (3) Invariance should not be assumed to correlate with model performance: models with random weights can be more invariant than models with learned or hard-coded invariance. (4) Models initialized with pretrained weights tend to have different invariance or equivariance properties than models initialized with random weights. Although the models and pretrained weights that we tested extracted fewer invariant features on average, their final predictions exhibited similar levels of invariance when compared to models initialized with random weights.

In summary, the contributions of this paper include:

- The introduction of G-empirical equivariance deviation, the first family of metrics that can rigorously and directly measure invariance and equivariance in deep learning models.
- A demonstration of the flexibility of G-EED, showing that it can be applied easily to a range of different components of a deep learning model to measure different forms of invariance and equivariance.
- Use of G-EED to answer a range of questions, shedding light on the extent to which neural networks are or are not invariant and equivariant.

2 Related Work

The literature on invariance and equivariance in machine learning can roughly be partitioned into two groups: those works that focus on how to build invariance and equivariance into a model or its components [21; 5; 44; 38; 6; 39; 15; 8; 31; 36; 1] and those works that focus on the theory behind invariance or equivariance [2; 4; 3; 28; 35].

Three common approaches are used to build invariance into deep learning models: data augmentation, feature averaging, and equivariant architectures. In this paper we focus on the first and third of these. Outside of a few ubiquitous layer types (such as standard translation equivariant convolutional layers), data augmentation is by far the most commonly used method, being a standard component of many training routines, particularly in computer vision. Although the details differ between different approaches, the basic idea is to randomly transform training examples as they are fed to the model, thereby encouraging the model to give the same prediction for different transformations of the same input [32]. Since it has become a common procedure when training deep learning models, data augmentation research has expanded in many directions [8; 31; 36; 1].

The idea that G-invariance can be hard coded into a model by combining multiple G-equivariant layers with data reduction layers (e.g., pooling) has a long history in deep learning. The most famous example of this idea is the conventional convolutional neural network (CNN) [26]. Since then a multitude of other group equivariant layers have been designed including two-dimensional rotation equivariant layers [40; 42; 30; 33; 34], three-dimensional rotational equivariant layers [39; 7; 12],

layers equivariant to the Euclidean group and its subgroups [38] (which we test against in this paper), and layers that are equivariant with respect to the symmetric group [27].

Our work is not the first to analyze various aspects of invariance in neural network models. Lyle et al. [28] analyzed invariance with respect to the benefits and limitations of data augmentation and feature averaging, presenting both theoretical and empirical arguments for using feature averaging over data augmentation. More recently, Chen et al. [4] presented a useful group-theoretic framework with which to understand data augmentation. Relevant to the present work, Chen et al. [3] introduced a notion of *approximate invariance*. Unlike that work, however, which focused on theoretical results related to data augmentation, this paper aims to introduce metrics that can be applied to modern deep learning architectures and answers questions about invariance from an empirical perspective.

Finally, a range of recent works have shown that even beyond the standard evaluation statistics (e.g., accuracy), invariance is an important concept to consider when studying deep learning models. For example, Kaur et al. [20] showed that lack of invariance can be used to identify out-of-distribution inputs. A further series of works investigated whether excessive invariance can reduce adversarial robustness [18; 19; 35]. All of this work reinforces one of the primary messages of this paper, that it is important to be able to measure invariance and equivariance directly in a model.

3 Quantifying Invariance and Equivariance

We begin this section by recalling the mathematical definitions of equivariance and invariance. We present these definitions in terms of the mathematical concept of a group, which formally captures the notion of symmetry [11].

Assume that G is a group. We say that G *acts* on sets X and Y if there are maps $\phi_X : G \times X \to X$ and $\phi_Y : G \times Y \to Y$ that respect the composition operation of G. That is, for $g_1, g_2 \in G$ and $x \in X$,

$$\phi_X(g_2, \phi_X(g_1, x)) = \phi_X(g_2 g_1, x),$$

with an analogous condition for ϕ_Y. Whenever the meaning is clear, we simplify notation by writing $\phi_X(g, x) = gx$ (with an analogous convention for ϕ_Y). A map $f : X \to Y$ is said to be *G-equivariant* if for all $x \in X$ and $g \in G$,

$$f(gx) = gf(x). \tag{1}$$

In the case where the map ϕ_Y is trivial so that $gy = y$ for all $g \in G$ and $y \in Y$, we say that f is *G-invariant*. Thus, invariance is a special case of equivariance.

Assume that $f : X \to Y$ is a neural network where X is the ambient space of input data and Y is the target space. In many cases there is a natural way to factorize f into a composition $f = f_2 \circ f_1$ where $f_1 : X \to Z$ is known as the feature extractor, Z is the latent space of f, and $f_2 : Z \to Y$ is the classifier. For example, if f is a ResNet50 CNN [16] then f_1 may consist of all residual blocks while f_2 would consist of the final affine classification and softmax layers. We say that machine learning model f *extracts G-equivariant features* if f_1 is a G-equivariant function. This is an especially meaningful distinction in the context of transfer learning where invariance or equivariance can be transferred to a new task via the invariance or equivariance of f_1. Note that the definition of G-equivariant feature extraction requires a well-defined action of G on Z, which may not be obvious in many cases. Because the trivial action is defined for any G and Z, we can always ask whether f *extracts G-invariant features*.

The following proposition provides some insight into how the invariance (or lack of invariance) of f_1 relates to the invariance (or lack of invariance) of f.

Proposition 3.1. Let $f : X \to Y$ be a function that decomposes into $f = f_2 \circ f_1$ where $f_1 : X \to Z$ and $f_2 : Z \to Y$. Suppose that G acts on X, Z, and Y.

1. f can be G-invariant even if f_1 and f_2 are not.
2. If f_1 is G-invariant, than f is G-invariant.

A proof of Proposition 3.1 can be found in Section A.3. Note that Proposition 3.1.2 implies that if earlier layers of a network achieve invariance with respect to some group, then this invariance will persist into later layers. This may be seen as part of the justification of the common practice within

the equivariant architectures community of building invariance through successive combinations of G-equivariant layers and pooling layers. Of course, the statement holds only when exact invariance is achieved. We see below that this is not generally the case.

3.1 Measuring equivariance

In this section we assume that both X and Y are vector spaces and the action of G on both X and Y is linear. In the case that f is also a linear map, the equivariance of f can be checked directly by checking equivariance on a basis for X. By extension, the equivariance of many common types of neural network layers can be checked when these can be framed as linear maps (e.g., convolutional layers). However, there is no systematic procedure for checking the equivariance of a nonlinear function f. If f is the composition of a sequence of functions $f_1, f_2, \ldots, f_n$ and we can check that each is equivariant, then we know that f is equivariant, but we cannot prove that a function is not equivariant just by proving that each layer is not equivariant (this follows from Proposition 3.1, 1). We are thus motivated to introduce a family of metrics that can be used to empirically measure the extent to which a function deviates from being equivariant on a data distribution $\mathcal{D}$ on X.

Since we assume in this section that the action of G on Y is linear, we can define the kernel of this action, $\ker(\phi_Y)$, which is a subgroup of G. $\ker(\phi_Y)$ consists of all those $g \in G$ such that g acts as the identity on Y. For $x \in X$, we define $\hat{f}(x)$ to be the expected value of $f(gx)$ over $\ker(\phi_Y)$,

$$\hat{f}(x) := \mathbb{E}_{g \in \ker(\phi_Y)}(f(gx)) = \int_{g \in \ker(\phi_Y)} f(gx) d\mu \tag{2}$$

where μ is the usual normalized Haar measure on subgroup $\ker(G)$. Note that when f is G-equivariant, then for each $g \in \ker(\phi_Y)$, $f(gx) = gf(x) = f(x)$ and hence $\hat{f} = f$.

Let $(\mathcal{D}, \nu)$ be a probability distribution on X and let $m : Y \times Y \to \mathbb{R}_{\geq 0}$ be a distance function on Y. We can use m to measure the extent to which f deviates from being G-equivariant by computing

$$\int_{\mathcal{D}} \int_{g \in G} m(f(gx), g\hat{f}(x)) d\mu dx \tag{3}$$

where this time μ is the normalized Haar measure on G. Note that the argument $m(f(gx), g\hat{f}(x))$ measures the extent to which (1) fails to hold across distribution $\mathcal{D}$, except that $gf(x)$ is replaced by $g\hat{f}(x)$. We use $\hat{f}(x)$ because it averages over all values in the orbit of x (under G) that should map to $f(x)$, were f genuinely G-equivariant. This choice is supported by the fact that it naturally interpolates between the two extreme cases: G acts trivially on Y (invariance) and G acts faithfully on Y. In the former case $\hat{f}(x)$ is the average value of $f(gx)$ over all of $g \in G$ and in the latter case $\hat{f}(x) = f(x)$.

Proposition 3.2. Let $f : X \to Y$ be a continuous function, G a group that acts linearly on both X and Y, and $m : Y \times Y \to \mathbb{R}_{\geq 0}$ a metric. Let $\mathcal{D}$ be a distribution on X. Then (3) is zero if and only if f is G-equivariant almost surely, i.e., on a set of measure 1.

We provide a proof of Proposition 3.2 in Appendix A.4.

To approximate (3) for real models and data where we always work with finite groups and finite samples of $\mathcal{D}$, we define the G-EED of f with respect to m to be

$$\mathcal{E}(f, G) := \frac{1}{|D||G|} \sum_{x \in D} \sum_{g \in G} m(f(gx), g\hat{f}(x)). \tag{4}$$

Note that since G is a finite group (and hence discrete), the Haar measure turns into the usual counting measure.

In the remainder of this section we describe some specific types of G-EED that may be relevant to computer vision tasks. For channelwise G-EED, large values indicate more equivariance; for latent space G-EED and softmax G-EED, larger values indicate less invariance. The diagram in Figure 1 provides a summary of all of these.

The channelwise G-equivariance of convolutional layer activations: Throughout most layers of a CNN an individual image is represented as a 3-tensor. Let $f_\ell : X \to \mathbb{R}^{C_\ell \times H_\ell \times W_\ell}$ be the composition

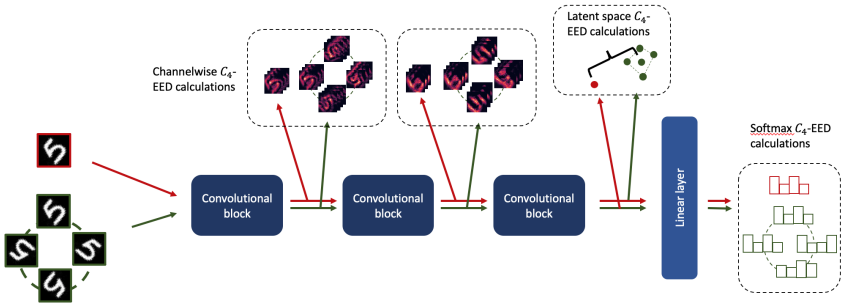

Figure 1: A diagram illustrating the different types of empirical equivariance deviation (EED) that we investigate in this paper.

of the first ℓ layers of a CNN such that for input image $x \in X$, $f_\ell(x)$ is a $C_\ell \times H_\ell \times W_\ell$ tensor where the first dimension corresponds to the channels of the representation and the second and third dimensions correspond to the two spatial dimensions.

Suppose that G is a finite group that acts on images and other 2-tensors (e.g., rotations, translations, and reflections). To understand the extent to which the first ℓ layers of the network are G-equivariant, we can measure the G-EED of f_ℓ. Although there are numerous choices of m for (3) that could be used to measure the channelwise difference between $f_\ell(gx)$ and $gf_\ell(x)$, we choose the following: let $S : \mathbb{R}^{H_\ell \times W_\ell} \times \mathbb{R}^{H_\ell \times W_\ell} \to [0, 1]$ be the cosine similarity on individual channels treated as vectors in $\mathbb{R}^{H_\ell W_\ell}$. Write $[f_\ell(x)]_i$ for the ith channel of $f_\ell(x)$. Then we set

$$m(f_\ell(gx), g\hat{f}_\ell(x)) = \frac{1}{C}\sum_{i=1}^{C} S([f_\ell(gx)]_i, g[\hat{f}_\ell(x)]_i).$$

That is, let m be the average cosine similarity between individual channels of $f_\ell(gx)$ and $g\hat{f}_\ell(x)$. Note that since rotation, translation, and reflection groups all act faithfully on $\mathbb{R}^{H_\ell \times W_\ell}$, for these specific G, $\hat{f} = f$. We call this version of G-EED *channelwise G-EED*.

Invariance of latent space representations: Consider a model f consisting of feature extractor $f_1 : X \to \mathbb{R}^k$ and classifier $f_2 : \mathbb{R}^k \to \mathbb{R}^n$. If G is some group that acts on X, we can ask whether f_1 extracts G-invariant features. Define M to be the average ℓ_2-distance between distinct points in the set $f_1(D)$. By setting m to be ℓ_2-distance, we calculate the *latent space G-EED* as

$$\mathcal{E}_{\text{latent}}(f, G) := \frac{1}{M}\frac{1}{|D||G|}\sum_{x\in D}\sum_{g\in G} ||f_1(gx) - \hat{f}_1(x)||_2. \tag{5}$$

Note that because in this setting G acts trivially on the latent space $\mathbb{R}^k$ (because of the invariance assumption), $\hat{f}_1(x) = \frac{1}{|G|}\sum_{g\in G} f_1(gx)$.

We describe why the normalization term M is necessary in Section A.5 of the Appendix. See also Figure 7.

Invariance at the softmax layer of classifiers:

While the two examples above focused on calculating invariance and equivariance at different intermediate layers of computer vision models, invariance of classification models is most commonly considered with respect to the final softmax output. In this paper we choose to use KL-divergence for m because it is a distance function designed to handle probability distributions, which softmax outputs approximate. In particular,

$$\mathcal{E}_{\text{softmax}}(f, G) := \frac{1}{|D||G|}\sum_{x\in D}\sum_{g\in G} D_{KL}(f(gx), \hat{f}(x)).$$

We call this metric the *softmax G-EED*.

4 Understanding invariance and equivariance in deep learning

In this section we describe a range of experiments designed to probe the invariance and equivariance of computer vision models. To prototype G-EED we experiment with the finite cyclic group C_8 that acts on images by $45°$ rotations. This symmetry is relevant to datasets where imagery does not have a preferred orientation. The datasets we use include *Rotated MNIST* which consists of randomly rotated MNIST digits from classes 0-8 and *xView Maritime*, a chipped version of the xView object detector overhead dataset. We describe these and other datasets we use in detail in Section A.2.

To reduce the computational burden, when calculating (4) we only used 50 randomly chosen examples from the corresponding dataset. We found that evaluating at more points did not substantially change the results. When not otherwise stated all invariance and equivariance measurements were calculated using points from the test set, not the training set. The normalization constant for latent space G-EED is calculated over 200 randomly chosen points.

4.1 Do networks trained with augmentation learn equivariant layers?

In this section we apply our metrics to standard CNNs and C_8-rotation equivariant, steerable CNNs (C_8-CNN) [38] to compare the invariance or equivariance of a model trained with augmentation with the invariance or equivariance of a model that is explicitly designed to learn invariance through progressive equivariant layers and pooling. We train 10 copies of each model type for 2,000 iterations on rotated MNIST. We apply random rotation augmentation to the models during training and all achieve an accuracy of over 98% on the test set. By design, for any $1 \leq k \leq 6$, the first k-blocks of one of the C_8-CNN models will be approximately rotation equivariant. The standard wisdom is that, with pooling, this will lead to an invariant representation in the latent space.

Despite the fact that both families of models achieve similar performance, we note that their invariance and equivariance properties differ significantly. The channelwise C_8-EED of the composition of the first k blocks (where $1 \leq k \leq 6$) of the C_8-CNN is significantly higher than that of the CNN, indicating that (as claimed) the C_8-CNN is more equivariant with respect to rotation of input images of $45°$ (see Figure 2 for examples of output at blocks 2 (left) and 5 (right)). Beyond that we see that equivariance decays for both models as input travels through additional blocks. This is particularly true for the C_8-CNN. We conjecture that this is likely caused by the accumulation of interpolation artifacts associated with rotation. Perhaps even more interesting is the fact that for no layer of either model do the channelwise C_8-EED values increase beyond a small jump at the beginning of training. Indeed, for most of training, these values decay. This suggests that layerwise equivariance may not always be aligned with the learning objective and that learned invariance does not arise through learned equivariance.

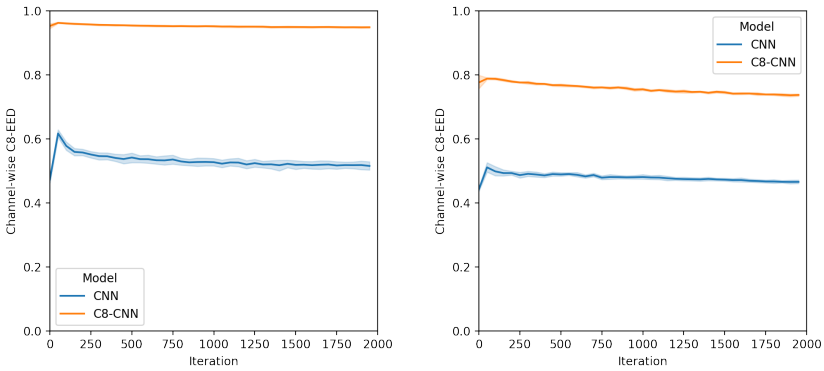

Figure 2: The channelwise C_8-EED for the first 2 blocks (left) and first 5 blocks (right) of a conventional and C_8-equivariant CNN (C_8-CNN) [38] with respect to the rotated MNIST dataset. Both plots include 95% confidence intervals. Larger values indicate more C_8-equivariance.

Both in terms of the latent space C_8-EED and softmax C_8-EED, the C_8-CNNs show more invariance than the conventional CNNs (see Figure 3) but the difference was less dramatic than it was for channelwise C_8-EED. This suggests that the CNNs likely caught up in terms of C_8-invariance using mechanisms distinct from layerwise C_8-equivariance. Notable also is the fact that unlike channelwise C_8-EED, which did not indicate increased C_8-equivariance over the course of training, both latent space and softmax C_8-EED decreased with training (suggesting learned invariance). So while C_8-equivariance is not learned by either model class, some C_8-invariance likely is.

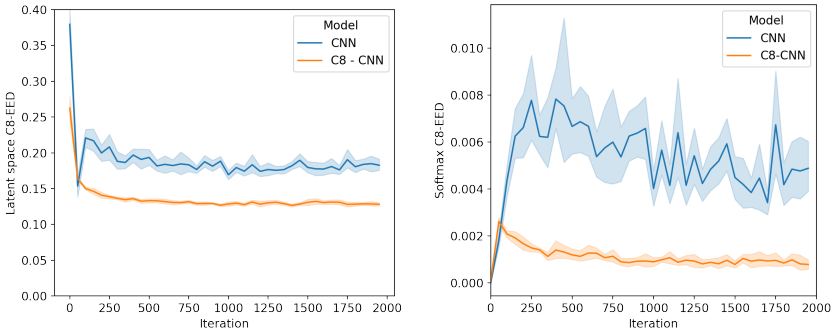

Figure 3: The latent space C_8-EED (left) and softmax C_8-EED (right) for conventional and C_8-equivariant CNNs (C_8-CNNs) [38] with respect to the rotated MNIST dataset. Both plots include 95% confidence intervals. Lower values for both plots indicate more C_8-invariance.

Summary: Equivariant architectures and nonequivariant architectures capture invariance in distinct ways. Nonequivariant networks do not learn layerwise equivariance as a path to invariance. Rather they achieve their learned invariance via some other mechanism. In our experiments, equivariant architectures were able to deliver models that were more invariant than their nonequivariant analog both in terms of the features that they extracted and their final softmax outputs.

4.2 Does learned invariance hold for out-of-distribution data?

It was noted in Lyle et al. [28] that while augmentation may be satisfactory for many tasks, there is a risk that learned invariance will not extend to out-of-distribution (OOD) data. We investigate this question using G-EED metrics, studying the conventional CNN and C_8-equivariant CNN from Section 4.1 and a conventional CNN with untrained weights that can be used for comparison. Note that because rotation invariance is at some level hard coded into the architecture of the C_8-CNN, our default assumption is that the C_8-CNN models will tend to be more invariant on OOD data than models with learned augmentation. Images of example OOD data as well as dataset descriptions can be found in Appendix A.2.

The left plot in Figure 4 shows the latent space C_8-EED for the three model families that we tested. We can see that random weights extract significantly less invariant latent space representations compared to both the augmentation-trained CNNs and C_8-CNNs. Although this difference is not surprising for the C_8-CNNs, for the standard CNNs it indicates that augmentation does learn some rotation-invariant features that generalize to OOD data. Furthermore, while C_8-CNNs do exhibit more latent space invariance than the augmentation-trained CNNs, the difference is insignificant compared to the difference between augmentation-trained CNNs and CNNs with random weights. Note that in these OOD experiments, the normalization term M in (5) is calculated using the OOD data (rather than the training or test set). This choice was made when considering situations in which a frozen network is used in combination with metric learning methods, as in Snell et al. [37], to solve a task with data drawn from a shifted distribution.

As a showcase of how a single model can display different types of invariance, the CNNs with random weights are far more invariant in terms of softmax C_8-EED than the trained C_8-CNN, which is more invariant than the trained CNN (right plot, Figure 4). We suspect that invariance of the CNNs with random weights arises from the fact that an untrained network tends to be insensitive to input. This observation is useful to keep in mind, especially as the concept of invariance is frequently treated as

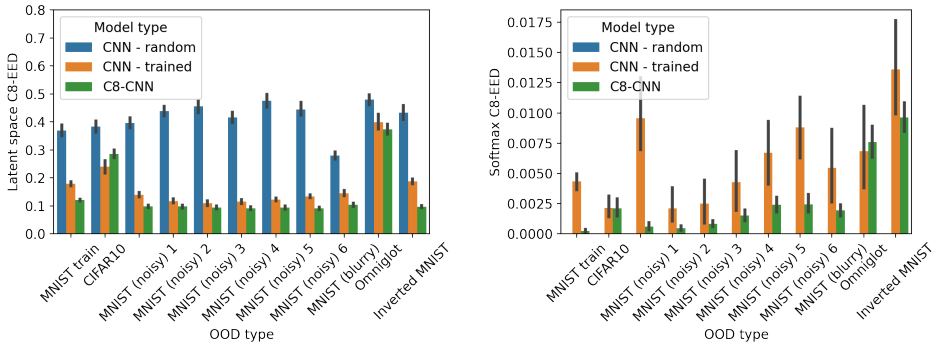

Figure 4: The latent space C_8-EED (left) and softmax C_8-EED (right) for conventional untrained CNNs, conventional CNNs trained on MNIST, and C_8-equivariant CNN (C_8-CNN) [38] trained on MNIST with respect to a range of in- and out-of-distribution datasets. Both plots include 95% confidence intervals. Lower values for both plots indicate more C_8-invariance.

a net benefit to any model. Invariance is useful when it is aligned with other metrics more directly related to the task the model was designed for.

Summary: Our experiments suggest that augmentation trained networks do learn invariance that mildly generalizes to OOD data. A single models can display differing kinds of invariance (an untrained network does not not extract invariant features but through insensitivity may make invariant predictions). In cases where invariance is critical and OOD data is expected, equivariant architectures may be a safer choice than training with augmentation.

5 How does the use of pretrained weights affect invariance and equivariance?

It has become standard practice in computer vision to initialize a model using weights trained on a large, diverse image dataset such as ImageNet [9]. It is reasonable to ask how such a strategy affects the invariance and equivariance of the resulting models. We choose four different image classifier architectures: ResNet50 [16], AlexNet [23], DenseNet121 [17], and LeViT192 [14]. For each of these architectures, we initialized five models with random weights and five models with weights from either the Torchvision package [29] (ResNet50, AlexNet, and DenseNet) or Timm [41] (LeViT192). We then trained all of these models on the xView Maritime training set. Next, we evaluated the latent space C_8-EED and softmax C_8-EED on the xView Maritime test set. It is important to note that pretrained weights, which were generated by training on ImageNet, might not be expected to have learned to extract fully rotationally invariant features. Indeed, the objects found in ImageNet mostly have a preferred orientation at which they generally are seen (there are very few instances of upside-down dogs or vehicles in ImageNet).

The results of our experiments are shown in Figure 5. Figure 5 shows the latent space C_8-EED for all eight model types every 200 iterations. We see that models trained from scratch extract more invariant features in their latent space. Moreover, while the invariance of models trained from scratch tends to decrease slightly over the course of training (latent space C_8-EED increases), the invariance of models trained using pretrained weights tends to increase slightly over the course of training. This may relate to ImageNet's lack of object classes that appear at a wide range of orientations. The models with pretrained weights may need to learn additional rotation invariance over the course of training. Contrast that with the weights trained from scratch that may learn rotation invariant features quickly which are later discarded in favor of less rotationally invariant features that better optimize the cross-entropy loss function used to train these networks. This, along with the fact that models using pretrained weights tend to have slightly higher accuracy, points to a potential tradeoff between invariance and task performance. Whether the robustness of the features captured by the pretrained networks is related to the fact that they are less invariant requires further exploration.

On the other hand, Figure 9 in the Appendix shows that the invariance properties of pretrained networks vs. networks trained from stratch is somewhat more ambiguous when considering softmax C_8-EED.

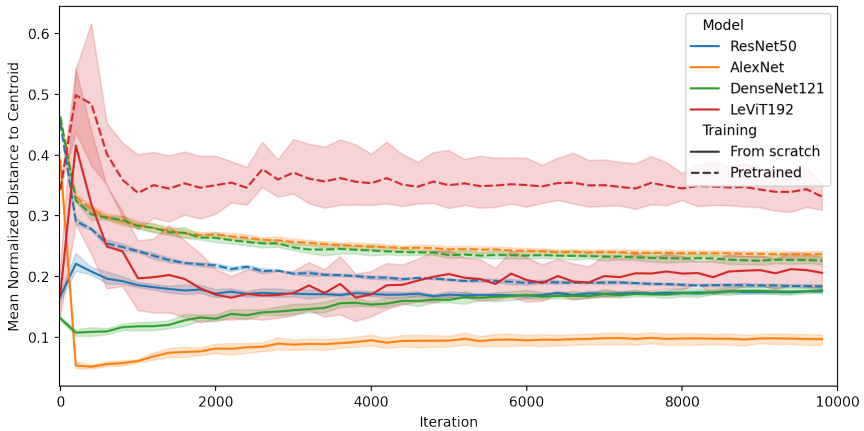

Figure 5: The latent space C_8-EED measured every 200 training iterations (for a total of 10,000 iterations) for a range of model architectures that were initialized with either random or pretrained weights generated using ImageNet.

Summary: In our experiments, models that used ImageNet pretrained weights tended to extract somewhat less invariant features, even if the invariance of their softmax outputs tended to be comparable. It may be worth considering developing large datasets for pretraining that contain the types of symmetries that are relevant to specific domains (e.g., a version of ImageNet designed for rotation invariant imagery such as overhead imagery and microscopy).

6 Limitations

Although the G-EED family of metrics that we introduce is flexible in many ways, it is not capable of capturing notions of invariance that do not have a known underlying symmetry group (e.g., changes in image background). An important next step would be to bring our metrics to a broader notion of invariance. Future research should explore how invariance to more complex sets of transformations can be measured without losing too much of the associated rigor in the metrics.

Despite our focus on classification models in computer vision in this paper, there is no reason that G-EED cannot be applied to other model types in other domains. Testing G-EED in other ML settings where the concepts of invariance or equivariance are important would bolster the evidence that G-EED is an important evaluation tool to use when studying deep neural networks.

7 Conclusion

In this paper we described a novel family of metrics meant to measure equivariance in deep learning models. We use these metrics to suggest answers to questions related to the extent to which neural networks are or are not invariant or equivariant to different types of transformations. Surprisingly, while invariance and equivariance are fundamental topics within machine learning, this is one of the first works (to our knowledge) that tries to measure these properties empirically in modern neural networks. We hope that this work will open a conversation on this important topic that will ultimately lead to a better understanding of how deep learning is able to achieve the results that it does.

References

[1] A. Antoniou, A. Storkey, and H. Edwards. Data augmentation generative adversarial networks. *arXiv preprint arXiv:1711.04340*, 2017.

[2] B. Bloem-Reddy and Y. W. Teh. Probabilistic symmetries and invariant neural networks. *J. Mach. Learn. Res.*, 21:90–1, 2020.

[3] S. Chen, E. Dobriban, and J. H. Lee. Invariance reduces variance: Understanding data augmentation in deep learning and beyond. 2019.

[4] S. Chen, E. Dobriban, and J. Lee. A group-theoretic framework for data augmentation. *Advances in Neural Information Processing Systems*, 33:21321–21333, 2020.

[5] T. Cohen and M. Welling. Group equivariant convolutional networks. In *International conference on machine learning*, pages 2990–2999. PMLR, 2016.

[6] T. S. Cohen and M. Welling. Steerable cnns. *arXiv preprint arXiv:1612.08498*, 2016.

[7] T. S. Cohen, M. Geiger, J. Köhler, and M. Welling. Spherical cnns. *arXiv preprint arXiv:1801.10130*, 2018.

[8] E. D. Cubuk, B. Zoph, D. Mane, V. Vasudevan, and Q. V. Le. Autoaugment: Learning augmentation policies from data. *arXiv preprint arXiv:1805.09501*, 2018.

[9] J. Deng, W. Dong, R. Socher, L.-J. Li, K. Li, and L. Fei-Fei. ImageNet: A Large-Scale Hierarchical Image Database. In *CVPR09*, 2009.

[10] L. Deng. The mnist database of handwritten digit images for machine learning research. *IEEE Signal Processing Magazine*, 29(6):141–142, 2012.

[11] D. S. Dummit and R. M. Foote. *Abstract algebra*, volume 3. Wiley Hoboken, 2004.

[12] C. Esteves, C. Allen-Blanchette, A. Makadia, and K. Daniilidis. Learning so (3) equivariant representations with spherical cnns. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 52–68, 2018.

[13] G. B. Folland. *Real analysis: modern techniques and their applications*, volume 40. John Wiley & Sons, 1999.

[14] B. Graham, A. El-Nouby, H. Touvron, P. Stock, A. Joulin, H. Jégou, and M. Douze. Levit: a vision transformer in convnet's clothing for faster inference. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 12259–12269, 2021.

[15] S. Hauberg, O. Freifeld, A. B. L. Larsen, J. Fisher, and L. Hansen. Dreaming more data: Class-dependent distributions over diffeomorphisms for learned data augmentation. In *Artificial Intelligence and Statistics*, pages 342–350. PMLR, 2016.

[16] K. He, X. Zhang, S. Ren, and J. Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 770–778, 2016.

[17] G. Huang, Z. Liu, L. Van Der Maaten, and K. Q. Weinberger. Densely connected convolutional networks. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 4700–4708, 2017.

[18] J.-H. Jacobsen, J. Behrmann, R. Zemel, and M. Bethge. Excessive invariance causes adversarial vulnerability. *arXiv preprint arXiv:1811.00401*, 2018.

[19] S. Kamath, A. Deshpande, and K. Subrahmanyam. Invariance vs robustness of neural networks. 2019.

[20] R. Kaur, S. Jha, A. Roy, S. Park, E. Dobriban, O. Sokolsky, and I. Lee. idecode: In-distribution equivariance for conformal out-of-distribution detection, 2022.

[21] R. Kondor and S. Trivedi. On the generalization of equivariance and convolution in neural networks to the action of compact groups. In *International Conference on Machine Learning*, pages 2747–2755. PMLR, 2018.

[22] A. Krizhevsky, G. Hinton, et al. Learning multiple layers of features from tiny images. 2009.

[23] A. Krizhevsky, I. Sutskever, and G. E. Hinton. Imagenet classification with deep convolutional neural networks. In *Proceedings of the 25th International Conference on Neural Information Processing Systems - Volume 1*, NIPS'12, page 1097–1105, Red Hook, NY, USA, 2012. Curran Associates Inc.

[24] B. M. Lake, R. Salakhutdinov, and J. B. Tenenbaum. Human-level concept learning through probabilistic program induction. *Science*, 350(6266):1332–1338, 2015.

[25] D. Lam, R. Kuzma, K. McGee, S. Dooley, M. Laielli, M. Klaric, Y. Bulatov, and B. McCord. xview: Objects in context in overhead imagery. *arXiv preprint arXiv:1802.07856*, 2018.

[26] Y. LeCun, P. Haffner, L. Bottou, and Y. Bengio. Object recognition with gradient-based learning. In *Shape, contour and grouping in computer vision*, pages 319–345. Springer, 1999.

[27] J. Lee, Y. Lee, J. Kim, A. Kosiorek, S. Choi, and Y. W. Teh. Set transformer: A framework for attention-based permutation-invariant neural networks. In K. Chaudhuri and R. Salakhutdinov, editors, *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pages 3744–3753. PMLR, 09–15 Jun 2019. URL https://proceedings.mlr.press/v97/lee19d.html.

[28] C. Lyle, M. van der Wilk, M. Kwiatkowska, Y. Gal, and B. Bloem-Reddy. On the benefits of invariance in neural networks. *arXiv preprint arXiv:2005.00178*, 2020.

[29] S. Marcel and Y. Rodriguez. Torchvision the machine-vision package of torch. In *Proceedings of the 18th ACM international conference on Multimedia*, pages 1485–1488, 2010.

[30] D. Marcos, M. Volpi, N. Komodakis, and D. Tuia. Rotation equivariant vector field networks. In *Proceedings of the IEEE International Conference on Computer Vision*, pages 5048–5057, 2017.

[31] A. J. Ratner, H. Ehrenberg, Z. Hussain, J. Dunnmon, and C. Ré. Learning to compose domain-specific transformations for data augmentation. *Advances in neural information processing systems*, 30, 2017.

[32] J. Schmidhuber. Deep learning in neural networks: An overview. *Neural networks*, 61:85–117, 2015.

[33] L. Sifre and S. Mallat. Combined scattering for rotation invariant texture analysis. In *ESANN*, volume 44, pages 68–81. Citeseer, 2012.

[34] L. Sifre and S. Mallat. Rotation, scaling and deformation invariant scattering for texture discrimination. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 1233–1240, 2013.

[35] V. Singla, S. Ge, B. Ronen, and D. Jacobs. Shift invariance can reduce adversarial robustness. *Advances in Neural Information Processing Systems*, 34, 2021.

[36] L. Sixt, B. Wild, and T. Landgraf. Rendergan: Generating realistic labeled data. *Frontiers in Robotics and AI*, 5:66, 2018.

[37] J. Snell, K. Swersky, and R. Zemel. Prototypical networks for few-shot learning. *Advances in neural information processing systems*, 30, 2017.

[38] M. Weiler and G. Cesa. General e (2)-equivariant steerable cnns. *Advances in Neural Information Processing Systems*, 32:14334–14345, 2019.

[39] M. Weiler, M. Geiger, M. Welling, W. Boomsma, and T. Cohen. 3d steerable cnns: Learning rotationally equivariant features in volumetric data.

[40] M. Weiler, F. A. Hamprecht, and M. Storath. Learning steerable filters for rotation equivariant cnns. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 849–858, 2018.

[41] R. Wightman. Pytorch image models. `https://github.com/rwightman/pytorch-image-models`, 2019.

[42] D. E. Worrall, S. J. Garbin, D. Turmukhambetov, and G. J. Brostow. Harmonic networks: Deep translation and rotation equivariance. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 5028–5037, 2017.

[43] K. Xiao, L. Engstrom, A. Ilyas, and A. Madry. Noise or signal: The role of image backgrounds in object recognition. *arXiv preprint arXiv:2006.09994*, 2020.

[44] R. Zhang. Making convolutional networks shift-invariant again. In *International conference on machine learning*, pages 7324–7334. PMLR, 2019.

Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] See Section 6
   (c) Did you discuss any potential negative societal impacts of your work? [No] As the work investigates how deep learning algorithms work in a non-application specific setting, we did not see this as relevant.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] In both Proposition 3.1 and Proposition 3.2 we give a rigorous and specific set of assumptions.
   (b) Did you include complete proofs of all theoretical results? [Yes] See Appendix A.3 and A.4
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] We include explicit definitions for all metrics in Section 3.1 and include details that would allow the experiments to be recreated in Section 4 and Appendix A.1
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [No] We specify all training and dataset details in Appendix A.1 and A.2 except for the training/test splits.
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] We include confidence intervals in all of our figures.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [Yes] See Appendix A.1
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes] We cited all datasets and pretrained models that we used in our experiments.
   (b) Did you mention the license of the assets? [Yes] We cite licenses for MNIST, xView, CIFAR10, and Omniglot.
   (c) Did you include any new assets either in the supplemental material or as a URL? [No] The only asset is our version of the public xView dataset, xView Maritime. We do describe how we created our version of this dataset.
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]

(e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]

5. If you used crowdsourcing or conducted research with human subjects...

(a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]

(b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]

(c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]