Towards Optimal Communication Complexity in Distributed Non-Convex Optimization

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Abstract

We study the problem of distributed stochastic non-convex optimization in the intermittent communication setting, where M machines work in parallel over R communication rounds. In this setting, we propose and analyze a new algorithm, which improves upon existing methods by requiring fewer and lighter variance reduction operations. We also present a lower bound, showing that our algorithm simultaneously attains optimal communication and oracle complexity in practically relevant regimes. Numerical experiments demonstrate the superior performance of our algorithm.

1 Introduction

We consider the following distributed optimization problem with M machines:

$$\min_{x \in \mathbb{R}^d} F(x) := \frac{1}{M} \sum_{m=1}^{M} F_m(x), \tag{1.1}$$

where F_m, which denotes the objective on machine m, is a non-convex function for all m, as is the average objective F. We want to solve this problem in the *intermittent communication* (IC) setting [1, 2] where the machines work in parallel and are allowed to make K oracle calls between two communication rounds for R consecutive rounds. The IC setting has been widely studied [3, 4, 5, 6, 7, 8, 9, 10, 11, 2, 12] over the past decade, and many recent studies have focused on the problem with non-convex and heterogeneous objectives [13, 14, 15] which are common in cross-device federated learning (FL) [16, 17]. Towards this end, several algorithms [18, 19, 20, 21, 22, 23], all involving *local updates* (à la local-SGD [3, 16]), have been proposed and analyzed under assumptions bounding the heterogeneity of machines' objectives. Although these algorithms demonstrate promising empirical performances, it still remains elusive whether these algorithms provably dominate the embarrassingly parallelizable alternatives, i.e., *mini-batch* variants of the optimal sequential algorithms [24, 25, 26] (a.k.a. centralized algorithms).

Until very recently, the situation was similar even in the simpler *homogeneous* setting when F_m's are all identical and convex, where Woodworth et al. [2] showed that the optimal algorithm often does not require local updates at all. Even when F_m's are not identical, for high levels of heterogeneity, accelerated mini-batch SGD [27] is optimal [28]. Should we expect something similar in the non-convex setting? Or, can we prove that in some regime *local-update* algorithms do improve over the naive centralized baselines?

In this paper, we start by noting that in the absence of any heterogeneity assumption (c.f., Section 2), the centralized algorithms attain both optimal communication and oracle complexity at the same time. This in fact mimics the convex heterogeneous setting. Thus, *local-update* algorithms can only potentially improve when heterogeneity is low. This was indeed the motivation behind some of the

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recent works [18, 21, 22]. However, in the absence of any lower bound which explicitly depends on the heterogeneity parameter (such as in [15, 29]), it is not possible to definitively claim such an improvement. To alleviate this, **(C.1)** we provide a novel communication complexity lower bound, under the assumption that F_m's have bounded first-order or second-order heterogeneity (see Section 2). We show that centralized algorithms [24, 25, 26] can never achieve this optimal communication complexity, and most of the existing *local-update* algorithms cannot attain it either.

Table 1: Comparison of convergence rate for different algorithms in the intermittent communication setting. ζ and τ corresponds to the first and second order heterogeneity (see Section 2) of the problem. Note that $\tau \leq 2L$ and can be much smaller than L. The results suppress only and all numerical constants. * Although BVR-L-SGD and Algorithm 1 have the same fast convergence rate, BVR-L-SGD requires each client to compute large batch gradients for many rounds of communications and is thus less communication efficient in practice (see Section3 for a detailed comparison).

Method	Convergence Rate, i.e. $\mathbb{E}\left\|\nabla F(\widehat{x})\right\|^2 \leq$	Oracle
Full Participation Setting		
SCAFFOLD/Mini-batch SGD [18]	$\frac{\Delta L}{R} + \left(\frac{\sigma^2 \Delta L}{MKR}\right)^{1/2}$	Stochastic
Mini-batch STORM ([26], Theorem D.1)	$\frac{\Delta L}{R} + \left(\frac{\sigma \Delta L}{MKR}\right)^{2/3}$	Stochastic
Lower Bound (Centralized) Theorem 3.2	$\frac{\Delta L}{R} + \left(\frac{\sigma \Delta L}{MKR}\right)^{2/3} + \frac{\sigma^2}{MKR}$	-
STEM [20]	$(\Delta L + \sigma^2 + \zeta^2)\left(\frac{1}{R} + \frac{1}{(MKR)^{2/3}}\right)$	Stochastic
BVR-LSGD [22]* CE-LSGD, Theorem 3.1	$\frac{\Delta \tau}{R} + \frac{\Delta L}{\sqrt{K}R} + \left(\frac{\sigma \Delta L}{MKR}\right)^{2/3}$	Stochastic
CE-LSGD Theorem 3.1	$\frac{\Delta \tau}{R} + \frac{\Delta L}{KR}$	Exact
Lower Bound Theorem 3.2	$\min\left\{\frac{\Delta \tau}{R}, \frac{\zeta^2}{R}\right\} + \frac{\Delta L}{KR} + \frac{\sigma^2}{MKR} + \left(\frac{\sigma \Delta L}{MKR}\right)^{2/3}$	-
Partial Participation Setting		
FEDAVG [13]	$\frac{\Delta L}{R} + \left(\frac{\sigma^2 \Delta L}{mKR}\right)^{1/2} + \left(\frac{(\zeta\sqrt{K}+\sigma)\Delta L}{\sqrt{K}R}\right)^{2/3}$	Stochastic
Mini-batch STORM [18]	$\frac{\Delta L}{R} + \left(\frac{(\zeta\sqrt{K}+\sigma)\Delta L}{m\sqrt{K}R}\right)^{2/3}$	Stochastic
MIMEMVR [21]	$\frac{\Delta \tau}{R} + \frac{\Delta L}{KR} + \frac{\zeta^2}{mR} + \left(\frac{\zeta \Delta \tau}{\sqrt{m}R}\right)^{2/3}$	Exact
MIMELITEMVR [21]	$\frac{\Delta \tau}{R} + \frac{\Delta L}{KR} + \frac{\zeta^2+\sigma^2}{R} + \left(\frac{(\zeta+\sigma)\Delta \tau}{R}\right)^{2/3}$	Stochastic + Exact
CE-LSGD Theorem 3.3	$\frac{\Delta \tau}{R} + \frac{\Delta L}{\sqrt{K}R} + \left(\frac{\sigma \Delta L}{mKR}\right)^{2/3} + \left(\frac{(\tau\sigma + L\zeta)\Delta}{m\sqrt{K}R}\right)^{2/3} + \left(\frac{\zeta \Delta \tau}{mR}\right)^{2/3}$	Stochastic
CE-LSGD Theorem 3.3	$\frac{\Delta \tau}{R} + \frac{\Delta L}{KR} + \left(\frac{\zeta \Delta \tau}{mR}\right)^{2/3}$	Exact
Lower Bound Theorem 3.4	$\min\left\{\frac{\Delta \tau}{R}, \frac{\zeta^2}{R}\right\} + \frac{\Delta L}{KR} + \left(\frac{(\sigma+\zeta)\Delta L}{mKR}\right)^{2/3}$	-

Working with these limitations in mind, our main contribution is **(C.2)** a new algorithm **CE-LSGD** that attains this optimal communication complexity as well as optimal oracle complexity in regimes of practical interest. Our algorithm, like many other *local-update* algorithms, uses variance-reduction techniques [24, 26] but requires both fewer and lighter "heavy-batch" operations compared to the existing methods (see discussion in Section 3). In fact we show that when endowed with exact gradient oracles, our method is min-max optimal. **(C.3)** We also analyze CE-LSGD in the partial client participation setting, which is of particular interest in cross-device federated learning (FL)[17]

Table 2: Comparison of optimal communication and oracle complexity required by different algorithms to attain $\mathbb{E}\|\nabla F(\widehat{x})\|_2^2 \leq \epsilon$ in the green regime described in figure 1. ζ and τ are the heterogeneity (see Section 2) of the problem. $\tau \leq 2L$ and can be much smaller than L. The results suppress only numerical constants and assume that $M\epsilon^{1/2} \leq \sigma$ and $\sigma\epsilon^{1/2} \leq \Delta L$.

Methods	Communication Complexity (R)	Oracle Complexity (N)
Full Participation Setting		
SCAFFOLD/Mini-batch SGD [18]	$\frac{\Delta L}{\epsilon}$	$\frac{\sigma^2 \Delta L}{\epsilon^2}$
Mini-batch STORM ([26], Theorem D.1)	$\frac{\Delta L}{\epsilon}$	$\frac{\sigma \Delta L}{\epsilon^{3/2}}$
STEM [20]	$\frac{\Delta L + \sigma^2 + \zeta^2}{\epsilon}$	$\frac{(\Delta L)^{3/2} + \sigma^3 + \zeta^3}{\epsilon^{3/2}}$
BVR-L-SGD [22] CE-LSGD, Theorem 3.1	$\frac{\Delta \tau}{\epsilon}$	$\frac{\sigma \Delta L}{\epsilon^{3/2}}$
Lower Bound CE-LSGD, Theorem 3.2	$\min\left\{\frac{\Delta \tau}{\epsilon}, \frac{\zeta^2}{\epsilon}\right\}$	$\frac{\sigma \Delta L}{\epsilon^{3/2}}$
Partial Participation Setting		
FEDAVG [13]	$\frac{\Delta L}{\epsilon} + \frac{\zeta \Delta L}{\epsilon^{3/2}}$	$\frac{\sigma^2 \Delta L}{\epsilon^2}$
Mini-batch STORM [26]	$\frac{\Delta L}{\epsilon} + \frac{\zeta \Delta L}{m\epsilon^{3/2}}$	$\frac{\sigma^2 \Delta L}{m\epsilon^2}$
MIMEMVR [21]	$\frac{\Delta \tau \zeta}{m^{1/2}\epsilon^{3/2}} + \frac{\zeta^2}{m\epsilon} + \frac{\Delta \tau}{\epsilon}$	Uses Exact Oracle
MIMELITEMVR [21]	$\frac{\Delta(\zeta+\sigma)\tau}{\epsilon^{3/2}} + \frac{\zeta^2+\sigma^2}{m\epsilon} + \frac{\Delta \tau}{\epsilon}$	Uses Exact Oracle
CE-LSGD Theorem 3.3	$\frac{\zeta \Delta \tau}{m\epsilon^{3/2}} + \frac{\Delta \tau}{\epsilon}$	$\frac{\Delta^2 L^2 m}{\epsilon^2}$
Lower Bound Theorem 3.4, [30, 31]	$\min\left\{\frac{\Delta \tau}{\epsilon}, \frac{\zeta^2}{\epsilon}\right\}$	$\frac{(\sigma+\zeta)\Delta L}{\epsilon^{3/2}}$

when there is an extremely large number of clients. Not only does CE-LSGD improve over the best-known communication complexity, but it is in fact the only algorithm which doesn't require exact oracle queries for variance reduction in this setting. When endowed with an exact oracle, CE-LSGD is again min-max optimal when $\tau K < L$. We summarize our results and the comparison to some important baselines in Tables 1 and 2. **(C.4)** As an auxiliary contribution we provide a variant of our algorithm for the online setting which is of interest in memory-constrained cross-device FL. We also empirically compare our method against centralized and *local-update* algorithms, demonstrating faster convergence and better communication efficiency.

Notation. We use $\mathcal{B}$ to denote the index set and $|\mathcal{B}|$ to denote its cardinality. For $x \in \mathbb{R}^d$, we use $\|x\|$ to denote its ℓ_2-norm. For $A \in \mathbb{R}^{d\times d}$, $\|A\|$ denotes the operator norm. $[n]$ denotes the set $\{1, 2, \ldots, n\}$.

2 Our Setting and the Centralized Baselines

In this section, we introduce some definitions and assumptions that will be used in our analysis. Our goal is to find an ϵ-approximate stationary point of F, i.e., a point $x \in \mathbb{R}^d$ such that $\mathbb{E}[\|\nabla F(x)\|^2] \leq \epsilon$, where the expectation is w.r.t. any randomness in the choice of x. We consider client objectives in the class $\mathcal{F}(L, \Delta)$ of differentiable and L-smooth functions, which have bounded sub-optimality at 0, i.e., for all $m \in [M]$, $\Delta \geq 0$, $F_m(0) - \inf_{x\in\mathbb{R}^d} F(x) \leq \Delta$. We also make assumptions that relate

the functions of different clients to one another. These are typically known as assumptions on the *"heterogeneity"* of the problem, and we consider two classes of problems.

Definition 1. *$\{F_m \in \mathcal{F}(L,\Delta)\}_{m=1}^M$ are first-order ζ-heterogeneous, if $\sup_{x\in\mathbb{R}^d}\sum_{m=1}^M \|\nabla F_m(x) - \nabla F(x)\|^2/M \le \zeta^2$, and we say that $\{F_m\}_{m\in[M]} \in \boldsymbol{\mathcal{F}_M^1(L,\Delta,\zeta)}$.*

Definition 2. *Twice-differentiable $\{F_m \in \mathcal{F}(L,\Delta)\}_{m=1}^M$ are second-order τ-heterogeneous, if $\sup_{m\in[M],x\in\mathbb{R}^d}\|\nabla^2 F_m(x) - \nabla^2 F(x)\| \le \tau$. We say that $\{F_m\}_{m\in[M]} \in \boldsymbol{\mathcal{F}_M^2(L,\Delta,\tau)}$.*

We assume that each machine has access to the following multi-point oracle [31] [Section 5.3, 2].

Definition 3. *Given a function $G \in \mathcal{F}(L,\Delta)$, $\mathcal{O}_G^{n,L,\sigma} : (\mathbb{R}^d)^n \times \mathcal{Z} \to (\mathbb{R})^n \times (\mathbb{R}^d)^n$ is a multi-point stochastic first order oracle if for some distribution $\mathcal{D}$ on $\mathcal{Z}$, and for all $x_1,\ldots,x_n \in \mathbb{R}^d$, the oracle samples a random seed $z \sim \mathcal{D}$ and returns estimators $\mathcal{O}_G^{n,L,\sigma}(x_1,\ldots,x_n,z) = (\{f(x_i;z)\}_{i\in[n]},\{g(x_i;z)\}_{i\in[n]})$ such that $\forall i \in [n]$, $\mathbb{E}_{z\sim\mathcal{D}}(f(x_i;z),g(x_i;z)) = (G(x),\nabla G(x))$ and $\mathbb{E}_{z\sim\mathcal{D}}\|g(x_i;z) - \nabla G(x_i)\|^2 \le \sigma^2$. Furthermore, the unbiased gradients satisfy mean smoothness, i.e., $\sup_{x,y\in\mathbb{R}^d}\mathbb{E}_{z\sim\mathcal{D}}[\|g(x;z) - g(y;z)\|] \le L\|x-y\|$.*

Finally, we consider a generalization of zero respecting algorithms denoted by $\mathcal{A}_{ZR}$ (see appendix A) in the intermittent communication (IC) setting. This class captures a very wide variety of distributed optimization algorithms, including mini-batch SGD, accelerated mini-batch SGD, local SGD, as well as all the variance-reduction algorithms. Algorithms which are not distributed zero-respecting are those whose iterates have components in directions about which the algorithm has no information, meaning that in some sense, it is just "wild guessing". We also denote the class of centralized algorithms in $\mathcal{A}_{ZR}$ by $\mathcal{A}_{ZR}^{cent}$ (see appendix A). This includes algorithms such as mini-batch SARAH [24] and mini-batch STORM [26]. We first present a lower bound result applicable to centralized algorithms.

Theorem 2.1 (Centralized Lower Bound)**.** *For all $\tau,\Delta,\zeta,\sigma \ge 0$, and $L \ge \tau$, every algorithm $A \in \mathcal{A}_{ZR}^{cent}$ optimizing a problem in $\mathcal{F}_M^1(L,\Delta,\zeta) \cup \mathcal{F}_M^2(L,\Delta,\tau)$, with access to an oracle $\mathcal{O}_{F_m}^{2,L,\sigma}$ over $R > 1$ communication rounds must output x_R^A such that*

$$\mathbb{E}\left[\left\|\nabla F(x_R^A)\right\|^2\right] \ge c\cdot\left(\frac{\Delta L}{R} + \frac{\sigma^2}{MKR} + \left(\frac{\sigma\Delta L}{MKR}\right)^{2/3}\right),$$

where c is a numerical constant.

The proof of this theorem follows the known oracle complexity lower bounds [30, 31] and is included in appendix B. This theorem shows that when heterogeneity is large i.e., $\tau \approx L$, mini-batch SARAH/STORM already achieve the optimal communication and oracle complexity (see Table 1). Therefore, most existing methods including FEDAVG[16], SCAFFOLD [18] and FEDPAGE [19] can not outperform these centralized baselines as their analyses do not benefit from small heterogeneity. At the same time this highlights the limitation of the centralized baselines, which also do not improve with lower heterogeneity. Certain existing *local-update* algorithms such as MIMEMVR [21] and BVR-L-SGD [22] can indeed improve upon centralized algorithms in the low heterogeneity regime. In the next section we quantify this improvement, and show that our algorithm strictly dominates the centralized baselines and almost matches our lower bound for algorithms in $\mathcal{A}_{ZR}$.

3 Our Algorithm and Min-max Optimality

In this section we present our communication-efficient algorithm abbreviated CE-LSGD and illustrate it in Algorithm 1. Note that for $m \in [M]$, we use the notation $\nabla F_{m,\mathcal{B}^m}(x) := \sum_{l\in\mathcal{B}^m} g(x;z_l \sim \mathcal{D}_m)/|\mathcal{B}^m|$, to denote the stochastic mini-batch gradient obtained by querying $\mathcal{O}_{F_m}^{2,l,\sigma}$ for $|\mathcal{B}^m|$ many times, and $\nabla F_{m,z}(x)$ to denote a single stochastic gradient $g(x;z\sim\mathcal{D}_m)$.

At each iteration of Algorithm 1, we need **two rounds** of communication, i.e., two back and forth communications between the server and all clients. Our method uses the extra round of communication, i.e., line 4 to line 11, to update the variance reduced gradient v_r using the current and previous server models, x_r, x_{r-1} respectively. In the following discussion we will use the iteration number R and communication complexity of Algorithm 1 interchangeably.

Algorithm 1 Communication Efficient Local Stochastic Gradient Descent (CE-LSGD)

input Initialization x_0, communication round R, local steps K, step size η, momentum $\beta \in [0,1]$

1: Let $x_{-1} = x_0$
2: **for** $r = 0, 1, \ldots, R-1$ **do**
3: **if** $r = 0$ set $\beta = 1, K = 1, B = b_0$ **else** set β, K as the input values, $B = K$
4: **Communicate (send)** (x_r, x_{r-1}) to clients
5: **on client** $m \in [M]$ **do**
6: Compute $\nabla F_{m,\mathcal{B}_r^m}(x_r)$ and $\nabla F_{m,\mathcal{B}_r^m}(x_{r-1})$, where $|\mathcal{B}_r^m| = B$
7: **Communicate (rec)** $\left(\nabla F_{m,\mathcal{B}_r^m}(x_r), \nabla F_{m,\mathcal{B}_r^m}(x_{r-1})\right)$ to the server
8: **end on client**
9: $v_r = \frac{1}{M}\sum_{m=1}^{M} \nabla F_{m,\mathcal{B}_r^m}(x_r) + (1-\beta)\left(v_{r-1} - \frac{1}{M}\sum_{m=1}^{M} \nabla F_{m,\mathcal{B}_r^m}(x_{r-1})\right)$
10: **Communicate (send)** (x_r, v_r) to client $\widetilde{m}_r$, where $\widetilde{m}_r \sim Unif\left([M]\right)$
11: **on client** $\widetilde{m}$ **do**
12: $w_{r+1,1}^{\widetilde{m}_r} := w_{r+1,0}^{\widetilde{m}_r} := x_t, v_{r,0}^{\widetilde{m}_r} := v_r$
13: **for** $k = 1, \ldots, K$ **do**
14: $v_{r,k}^{\widetilde{m}_r} = \nabla F_{\widetilde{m}_r, z_{r,k}^{\widetilde{m}_r}}(w_{r+1,k}^{\widetilde{m}_r}) + v_{r,k-1}^{\widetilde{m}_r} - \nabla F_{\widetilde{m}_r, z_{r,k}^{\widetilde{m}_r}}(w_{r+1,k-1}^{\widetilde{m}_r})$
15: $w_{r+1,k+1}^{\widetilde{m}_r} = w_{r+1,k}^{\widetilde{m}_r} - \eta v_{r,k}^{\widetilde{m}_r}$
16: **end for**
17: **Communicate (rec)** $\left(w_{r+1,K+1}^{\widetilde{m}_r}\right)$ to the server
18: **end on client**
19: Let $x_{r+1} = w_{r+1,K+1}^{\widetilde{m}_r}$
20: **end for**

output Choose $\widetilde{x}$ uniformly from $\{w_{r,k}^{\widetilde{m}_r}\}_{r\in[R],k\in[K]}$

At the core of our proposed method is the variance reduction term v_r and the local gradient estimator $v_{r,k}^m$ (lines 10 and 17 in Algorithm 1). The construction of the local gradient estimator is motivated by the variance reduction technique of SARAH [24, 25]. Intuitively, the estimation error between the proposed local gradient estimator $v_{r,k}^m$ and the full gradient $\nabla F(w_{r+1,k}^m)$, i.e., $\mathbb{E}\|v_{r,k}^m - \nabla F(w_{r+1,k}^m)\|$, can be decomposed into two dominating terms: $\mathbb{E}\|v_r - \nabla F(x_r)\|^2$ and $\tau^2 K \sum_{k=1}^{K} \mathbb{E}\|w_{r+1,k}^m - w_{r+1,k-1}^m\|^2$. The first term is the estimation error between the variance reduction term v_r and the full gradient $\nabla F(x_r)$. Since v_r is updated based on the momentum-based variance reduction technique [26], this estimation error is dominated by $L^2\mathbb{E}\|x_r - x_{r-1}\|^2$, which approaches zero as the algorithm converges. Similarly, in the second term $\mathbb{E}\|w_{r+1,k}^m - w_{r+1,k-1}^m\|^2$ approaches zero as the algorithm converges and the τ factor controls the benefit we can obtain from small heterogeneity. Intuitively, for smaller values of τ we can make more local updates and the algorithm converges faster. We note that, our method reduces to mini-batch STORM if we choose the number of local updates K to be one (see Appendix D). Our formal convergence result is as follows:

Theorem 3.1 (Simplified Statement)**.** *Suppose* $\{F_m\}_{m\in[M]} \in \mathcal{F}_M^2(L, \Delta, \tau)$ *for* $L, \Delta, \tau \geq 0$ *then,*

(a) if each client $m \in [M]$ *has a stochastic oracle* $\mathcal{O}_{F_m}^{2,L,\sigma}$*, and assuming* $1 \leq (\Delta L/\sigma^2) \cdot MKR \leq R^3$*, then the output* $\widetilde{x}$ *of Algorithm 1 using* $\beta = \frac{(\Delta L)^{2/3}(MK)^{1/3}}{\sigma^{4/3}R^{2/3}} \leq 1$*,* $b_0 = \frac{\sigma^{8/3}(KR)^{1/3}}{(\Delta L)^{4/3}M^{2/3}}$*, and* $\eta = \min\left\{\frac{1}{L}, \frac{1}{K\tau}, \frac{1}{\sqrt{K}L}, \frac{M^{2/3}}{L(KR)^{1/3}}\right\}$ *satisfies,*

$$\mathbb{E}\|\nabla F(\widetilde{x})\|^2 \leq c_1 \cdot \left(\frac{\Delta\tau}{R} + \frac{\Delta L}{\sqrt{K}R} + \left(\frac{\sigma\Delta L}{MKR}\right)^{2/3}\right);$$

(b) if each client $m \in [M]$ *has a deterministic oracle* $\mathcal{O}_{F_m}^{2,L,0}$*, then the output* $\widetilde{x}$ *of Algorithm 1 using* $\beta = 1$ *and* $\eta = \min\left\{\frac{1}{L}, \frac{1}{K\tau}\right\}$ *satisfies,*

$$\mathbb{E}\|\nabla F(\widetilde{x})\|^2 \leq c_2 \cdot \left(\frac{\Delta\tau}{R} + \frac{\Delta L}{KR}\right);$$

where c_1, c_2 is a numerical constant.

Note that we don't actually need the assumption $1 \leq (\Delta L/\sigma^2) \cdot MKR \leq R^3$ which ensures that $\beta \leq 1$ in the stochastic setting. We use it here to present a simpler version of our convergence rate. In appendix C, we derive our general result by carefully tuning β, b_0 in different regimes as highlighted in Figure 1. We show that the convergence rate attained by our algorithm is *almost optimal* by proving the following lower bound result.

Theorem 3.2. *For all $L, \sigma, \tau, \Delta, \zeta \geq 0$ every algorithm $A \in \mathcal{A}_{zr}$ optimizing a problem in $\mathcal{F}_M^1(L, \Delta, \zeta) \cup \mathcal{F}_M^2(L, \Delta, \tau)$ with $K > 0$ intermittent accesses to two-point first-order oracles $\{\mathcal{O}_{F_m}^{2,L,\sigma}\}_{m \in [M]}$ on all the machines outputs x_R^A after $R \geq c_1$ rounds such that,*

$$\mathbb{E}\left[\left\|\nabla F(x_R^A)\right\|^2\right] \geq c_2 \cdot \left(\min\left\{\frac{\zeta^2}{R}, \frac{\Delta\tau}{R}\right\} + \frac{\Delta L}{KR} + \frac{\sigma^2}{MKR} + \left(\frac{\sigma \Delta L}{MKR}\right)^{2/3}\right),$$

where c_1, c_2 are numerical constants.

By comparing the upper and lower bounds for problems in $\mathcal{F}_M^2(L, \Delta, \tau)$, we can make two observations. First, in the deterministic setting ($\sigma = 0$), our upper bound matches the lower bound and hence CE-LSGD is min-max optimal. Thus, our result improves over all the existing results in this setting, including MIMEMVR [21]. Second, in the stochastic setting ($\sigma > 0$), when we restrict to the regime when $\sigma^4/(\Delta L)^2 \leq MKR$ (which is almost w.l.o.g.), our algorithm's upper bound is optimal except for the second term in Theorem 3.1, which has a $L/\sqrt{K}R$ factor versus the L/KR term in the lower bound. We conjecture that the lower bound is tight, and like the deterministic setting our algorithm is also min-max optimal in the stochastic setting.

Our construction for theorem 3.2 uses the non-convex hard instance proposed by Carmon et al.[30] and splits it across different machines to get a communication complexity lower bound. This idea has been used previously to give lower bounds in the heterogeneous setting [32, 15, 33]. We prove the result in Appendix B. From looking at Table 1, we can note that BVR-L-SGD [22] also attains a similar upper bound as our method. In fact in Appendix C, we show that with deterministic oracle BVR-L-SGD also attains the min-max optimal rate. This is not surprising, knowing that several variance-reduced algorithms [25, 26, 24] are simultaneously optimal even in the sequential setting, but our method requires fewer and lighter variance reduction operations, which lead to better scalability from the algorithmic design perspective. In the next section, we examine the difference between both these methods more carefully.

3.1 The Perspective of Reducing Communication

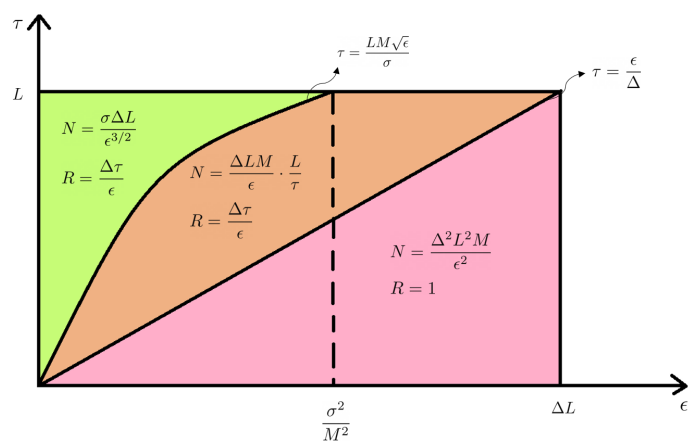

Figure 1: Illustration of the best communication complexity R and oracle complexity N that our method can obtain for different ϵ and τ. Green regime: Our method can obtain the optimal communication and oracle complexity. Orange regime: Our method can obtain the optimal communication using a larger oracle complexity. Red regime: Our method only needs one round of communication using a larger oracle complexity.

So far, we have looked at convergence rates in the intermittent communication model, where K, R are fixed. However, another perspective that is often useful is to reduce the communication complexity to the minimum possible with the minimum required oracle complexity. Both these complexities can be expressed in terms of ϵ using the convergence guarantees we showed, where we want to attain an

ϵ-approximate stationary point. This view is often more useful when communication rounds comprise the bulk of the required physical time. This scenario is common in FL, where devices become available intermittently which delays the synchronous updates. With this in mind, we summarize the communication and oracle complexities attained by both our method and BVR-L-SGD [22] in Figure 1 when optimizing with stochastic oracles. Notice that the figure has three different regimes based on the relative scaling of τ versus ϵ. We focus on the green regime characterized by $\epsilon^{1/2} \in (0, \tau\sigma/(LM)]$. This regime is of most practical interest in, say, deep learning, where modern over-parameterized models often drive ϵ to really small values. And when ϵ is small enough, this regime covers a wide range of values of τ.[1].

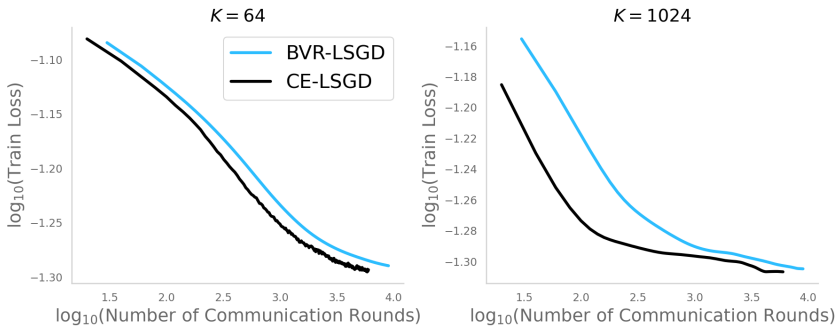

Figure 2: Training loss of CE-LSGD and BVR-L-SGD on CIFAR-10 data-set versus the number of communication rounds in the intermittent communication setting with different local-updates K. We use $M = 10$ machines, and synthetically generate heterogeneous data-sets (see Section 4) with $q = 0.1$. All oracle queries use a mini-batch of size $b = 16$, i.e., there are Kb oracle queries on each machine between two communication rounds. We note that in all the settings, our method has a faster convergence, which highlights its communication efficiency. Fixed step-sizes η for both the methods were tuned in $\{0.001, 0.005, 0.01, 0.05, 0.1, 0.5\}$ (to obtain best loss) following [22], our method set the momentum $\beta = 0.3$, $b_{max}^{our} = K$, while $b_{max}^{BVR} = 5000$ according to [22].

In the green regime, both CE-LSGD and BVR-L-SGD require $K = \sigma L/(\tau M \epsilon^{1/2})$ local steps to achieve the optimal communication and oracle complexities. However, BVR-L-SGD requires multiple heavy-batch stochastic gradient computations on each machine with batch size b_{max}. In particular, for BVR-L-SGD, we have $\rho_{\text{BVR}} = b_{max}/K = \sigma\tau/(L\epsilon^{1/2})$, which suggests that for $S = L\Delta/\sigma\sqrt{\epsilon}$ communication rounds, it requires each machine to compute ρ_{BVR} times heavier batch stochastic gradients compared to the other communication rounds. As for CE-LSGD, we have $b_0 = \sigma^3/(L\Delta M\epsilon^{1/2})$, which gives us $\rho_{\text{our}} = b_0/K = \sigma^2\tau/(L^2\Delta)$. This suggests that our method only requires each machine to compute ρ_{our} times larger batch stochastic gradient, and that too only once. Furthermore, $\rho_{\text{our}}/\rho_{\text{BVR}} = \sigma\epsilon^{1/2}/(L\Delta) \leq 1$. Thus, the size of our large batch gradient is also smaller than the one for BVR-L-SGD and **our method has fewer and lighter heavy-batch operations**.

If one implements both these methods in the intermittent communication model, i.e., by breaking the large batch computation across multiple rounds, with local budget $K = \sigma L/(\tau M \epsilon^{1/2})$, then the effective communication complexity of both methods is $\Delta\tau/\epsilon$, and this subtle difference gets washed away. However, in Figure 2, we show that this equivalence up to numerical constants doesn't hold in practice, where our method indeed converges faster than BVR-L-SGD. In Table 2, we summarize the communication and oracle complexities attained by different algorithms in the green regime.

3.2 The Partial Participation Setting

In settings such as cross-device federated learning [17], there are often tens of thousands of clients (think of android mobile users) and it is not feasible to consider training on all of the clients synchronously. It is more natural to consider a partial sampling of clients for each communication

[1]We talk about the other regimes while giving the full statement of Theorem 3.1 in C

round. More formally, we can re-state our distributed optimization problem as follows:

$$\min_{x\in\mathbb{R}^d} F(x) := \mathbb{E}_{n\sim\mathcal{P}}\left[F_n(x)\right], \tag{3.1}$$

where $\mathcal{P}$ is a probability distribution on the clients, and we assume at each communication round, we can sample m clients independently from $\mathcal{P}$. This is w.l.o.g. because in settings like cross-device FL, there is an extremely large number of devices, and most devices participate only once. We also need to modify the IC setting: during each communication round, $S_r \sim \mathcal{P}^m$ clients participate, and each queries their oracle K times. This setting has been considered in several works [18, 21], and we can also adapt Algorithm 1 to this setting by communicating with only m clients at each round and using m_0 clients for the first round to initialize the variance reduction term (see Appendix F). Specially, we can guarantee the following convergence rate where $\mathcal{F}^1_{\mathcal{P}}(L,\Delta,\zeta)$ and $\mathcal{F}^2_{\mathcal{P}}(L,\Delta,\tau)$ are natural generalizations of $\mathcal{F}^1_M(L,\Delta,\zeta)$ and $\mathcal{F}^2_M(L,\Delta,\tau)$ to the partial participation setting, formally defined in Appendix F.

Theorem 3.3. *Suppose for all n in support of $\mathcal{P}$, $F_n \in \mathcal{F}^1_{\mathcal{P}}(L,\Delta,\zeta)\cap\mathcal{F}^2_{\mathcal{P}}(L,\Delta,\tau)$ then,*

(a) if each client n has a stochastic oracle $\mathcal{O}^{2,L,\sigma}_{F_n}$, and assuming that $\Delta\sqrt{m}(\sigma\tau + \zeta L) \le R\zeta^2\sigma$ the output $\widetilde{x}$ of Algorithm 1 using $b_0 = K$, $m_0 = m/(\beta^2 R)$, $\beta = \left(\frac{\Delta\sqrt{m}}{R}\left(\tau + \frac{L}{\sqrt{K}}\right)\cdot\min\left\{\frac{K}{\sigma^2},\frac{1}{\zeta^2}\right\}\right)^{2/3}$, and $\eta = \min\left\{\frac{1}{K\tau},\frac{1}{\sqrt{K}L},\frac{\sqrt{\beta m}}{\sqrt{K}L},\frac{\sqrt{\beta m}}{\tau K}\right\}$ satisfies

$$\mathbb{E}\|\nabla F(\widetilde{x})\|^2 \le c_1\cdot\left(\frac{\Delta\tau}{R} + \frac{\Delta L}{\sqrt{K}R} + \left(\frac{\sigma\Delta L}{mKR}\right)^{2/3} + \left(\frac{(\sigma\tau+\zeta L)\Delta}{m\sqrt{K}R}\right)^{2/3} + \left(\frac{\zeta\Delta\tau}{mR}\right)^{2/3}\right);$$

(b) if each client n has a deterministic oracle $\mathcal{O}^{2,L,0}_{F_n}$, and assuming that $\Delta\tau\sqrt{m} \le \zeta^2 R$, then the output $\widetilde{x}$ of Algorithm 1 using $m_0 = (mR)^{1/3}\cdot\left(\frac{\zeta^2}{\Delta\tau}\right)^{4/3}$, $\beta = \left(\frac{\Delta\tau\sqrt{m}}{\zeta^2 R}\right)^{2/3}$, and $\eta = \min\left\{\frac{1}{L},\frac{1}{K\tau},\frac{m^{2/3}}{R^{1/3}K\tau}\cdot\left(\frac{\Delta\tau}{\zeta^2}\right)^{1/3}\right\}$ satisfies

$$\mathbb{E}\|\nabla F(\widetilde{x})\|^2 \le c_2\cdot\left(\frac{\Delta\tau}{R} + \frac{\Delta L}{KR} + \left(\frac{\zeta\Delta\tau}{mR}\right)^{2/3}\right);$$

where c_1, c_2 are numerical constants.

It follows from Theorem 3.3 that our method can achieve $R = \zeta\Delta\tau/(m\epsilon^{3/2})$ communication complexity with $N = \Delta^2L^2m/\epsilon^2$ stochastic oracle calls when we choose K appropriately. This communication complexity improves over the result of $R = L\Delta\zeta/(m\epsilon^{3/2})$ using mini-batch STORM when τ is small. As far as we know, our algorithm is the only algortihm in the partial participation setting which uses just stochastic oracles and achieves such an improvement. Theorem 3.3 also suggests that with a deterministic oracle, our method attains a strictly faster convergence rate than the best-known result MIMEMVR [21], which also uses a deterministic oracle. More specifically, the term $(\zeta\Delta\tau)/(mR)^{2/3}$ in our rate improves the term $(\zeta\Delta\tau)/(\sqrt{m}R)^{2/3}$ in their rate (see Table 1 and Table 2 for detailed comparisons). Furthermore, we have the following lower bound result to show that the convergence rate of our algorithm is *almost optimal*.

Theorem 3.4. *For all $L,\sigma,\tau,\Delta,\zeta \ge 0$ every algorithm $A \in \mathcal{A}_{zr}$ optimizing a problem in $\mathcal{F}^1_{\mathcal{P}}(L,\Delta,\zeta)\cup\mathcal{F}^2_{\mathcal{P}}(L,\Delta,\tau)$ with $K > 0$ intermittent accesses to two-point first-order oracles $\{\mathcal{O}^{2,L,\sigma}_{F_m}\}_{m\in[M]}$ on all the machines outputs x^A_R after $R \ge c_1$ rounds such that,*

$$\mathbb{E}\left[\left\|\nabla F(x^A_R)\right\|^2\right] \ge c_2\cdot\left(\min\left\{\frac{\zeta^2}{R},\frac{\Delta\tau}{R}\right\} + \frac{\Delta L}{KR} + \frac{\sigma^2}{mKR} + \left(\frac{(\sigma+\zeta)\Delta L}{mKR}\right)^{2/3}\right),$$

where c_1, c_2 are numerical constants.

Note that in the deterministic setting (i.e., $\sigma = 0$) the gap between the lower bound and our upper bound, is due to the terms $(\zeta\Delta\tau/mR)^{2/3}$ v/s $((\sigma+\zeta)\Delta L/mKR)^{2/3}$. When $\tau \le L/K$ then our method is min-max optimal. We conjecture that our method is always min-max optimal, and the correct term in the lower bound should be $(\zeta\Delta\tau/mR)^{2/3}$. We defer proving this to future work.

4 Numerical Experiments

We evaluate the performance of our method by optimizing a two-layer fully connected network for multi-class classification on the CIFAR-10 [34] data-set. Since we are in the heterogeneous setting, we need to artificially generate a data-set. We follow the same data processing procedure as in [22]. We first make sure that all the ten classes in CIFAR-10 have the same number of samples (roughly around 5000), and assign $q \times 100\%$ of class m's samples to client $m \in [10]$ where q is chosen from $\{0.1, 0.35, 0.6, 0.85\}$. For each class m, we evenly split the remaining $(1 - q) \times 100\%$ samples to the other 9 clients except client m. Thus, q controls the heterogeneity of our data-set, with small q corresponding to small heterogeneity.

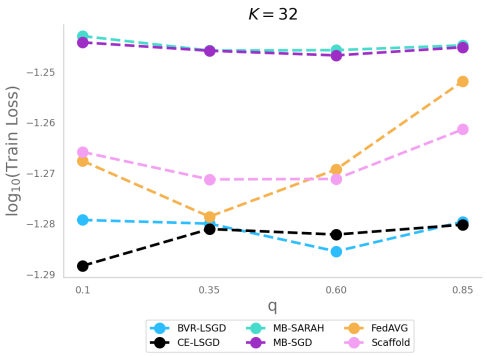

Figure 3: Comparing CE-LSGD to centralized and local-update methods, for fixed $K = 32$ and varying heterogeneity controlled by q on CIFAR-10 [34] data-set. Like figure 2 we use mini-batch size $b = 16$ for each oracle query. Thus each method makes Kb oracle queries every round per machine. All the methods for different q are tuned separately, following a similar hyper-parameter search as in figure 2.

We perform two different experiments. In the first experiment, we directly compare our method, i.e., CE-LSGD, with BVR-L-SGD in the intermittent communication setting (see Figure 2). We observe that while both the methods converge to a similar quality of solution eventually, our method is more communication efficient. In the second experiment, we compare our method with BVR-L-SGD [22] as well as FedAvg [16], SCAFFOLD [18], MB-SARAH [24] and MB-SGD [5] for the same number of updates/iterations. The last two methods are centralized baselines, and we use the local computation to compute a mini-batch stochastic gradient. We again observe that CE-LSGD and BVR-L-SGD have comparable performance which is better than all the other methods.

5 Discussion and Open Problems

In this paper, we provide a new communication-efficient local update algorithm CE-LSGD and analyze it in the full and partial client participation setting with intermittent communication. In the full-participation setting with access to exact oracles, our algorithm is the first and only known min-max optimal algorithm. And in the partial participation setting with exact oracles, it significantly improves over the existing results [21] and is almost optimal. In both types of participation, we either match or improve upon the existing results when equipped with stochastic oracles. Our lower bound results provide a much-needed baseline to measure algorithmic developments in non-convex distributed optimization.

In Appendix D, we provide an extension of CE-LSGD which uses a stochastic Hessian vector product oracle [12, 35] instead of a multi-point oracle, and recovers similar optimal communication complexity. This is relevant for memory-constrained online settings where it might not be feasible to preserve several copies of a model on the client device for simultaneous queries in variance reduction algorithms.

Our work also leaves some open questions. In the full-participation setting, do CE-LSGD and/or BVR-LSGD attain the min-max optimal rate? As mentioned in section 3, the second terms in their upper bound and lower bound have a $1/\sqrt{K}$ gap, which vanishes in the deterministic setting. We don't know if this gap is "real" or a result of our analysis. Can we use a simple single machine baseline in the red regime in Figure 1? We are not able to show this with only assumptions on τ. However, in Appendix C, we discuss this regime when ζ is also small and show how a single machine baseline can be used here. In general, it is unclear if the set of problems in $\mathcal{F}_M^1(L, \Delta, \zeta)$ is indeed simpler than $\mathcal{F}_M^2(L, \Delta, \tau)$. Finally, our lower bound for partial participation can most probably be improved, replacing L with τK. We hope to tackle these questions in our future work.

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   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes]
   (c) Did you discuss any potential negative societal impacts of your work? [N/A]
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes]
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   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No]
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