On the Effectiveness of Lipschitz-Driven Rehearsal in Continual Learning

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Abstract

Rehearsal approaches enjoy immense popularity with Continual Learning (CL) practitioners. These methods collect samples from previously encountered data distributions in a small memory buffer; subsequently, they repeatedly optimize on the latter to prevent catastrophic forgetting. This work draws attention to a hidden pitfall of this widespread practice: repeated optimization on a small pool of data inevitably leads to tight and unstable decision boundaries, which are a major hindrance to generalization. To address this issue, we propose Lipschitz-DrivEn Rehearsal (LiDER), a surrogate objective that induces smoothness in the backbone network by constraining its layer-wise Lipschitz constants w.r.t. replay examples. By means of extensive experiments, we show that applying LiDER delivers a stable performance gain to several state-of-the-art rehearsal CL methods across multiple datasets, both in the presence and absence of pre-training. Through additional ablative experiments, we highlight peculiar aspects of buffer overfitting in CL and better characterize the effect produced by LiDER.

1 Introduction

The last few years have seen a renewed interest in aiding Deep Neural Networks (DNNs) to acquire new knowledge and, at the same time, retain high performance on previously encountered data. In this regard, the mitigation of the longstanding problem of *catastrophic forgetting* [42] has driven the recent research towards novel incremental methods [33, 50, 72, 54, 11], often framed under the field of Continual Learning (CL) [72, 54].

Among other valid strategies, *rehearsal approaches* caught the attention of a large body of literature [50, 17, 11] thanks to their advantages. Simply, they maintain a small fixed-size buffer containing a fraction of examples from previous tasks; afterwards, these examples are mixed together with the ones of the current task, hence provided continuously as training data. In this respect, different approaches establish different regularization strategies on top of the retained examples [51, 40], as well as which kind of information to store (*e.g.* model responses [11, 9], explanations [21], etc.).

In spite of their widespread application, these approaches fall into a common and evident pitfall: as only a small fraction of past examples can be retained, overfitting of buffer datapoints is expected [60]. On the one hand, there are several approaches that mitigate this issue by extending the buffer through techniques of data-augmentation [5, 11], interpolation [10] and eventually the combination of the two [8]. On the other hand, other works [3, 4, 12] carefully selected which items have to be store for later replay, in an attempt to retain most of the knowledge lying past tasks.

In this work, we instead tackle buffer overfitting at its source: specifically, we argue that *catastrophic forgetting* can be linked to the heavy deterioration of the decision boundaries between classes observed as tasks progress. Indeed, we hold that the lack of access to the entire training set of past tasks causes

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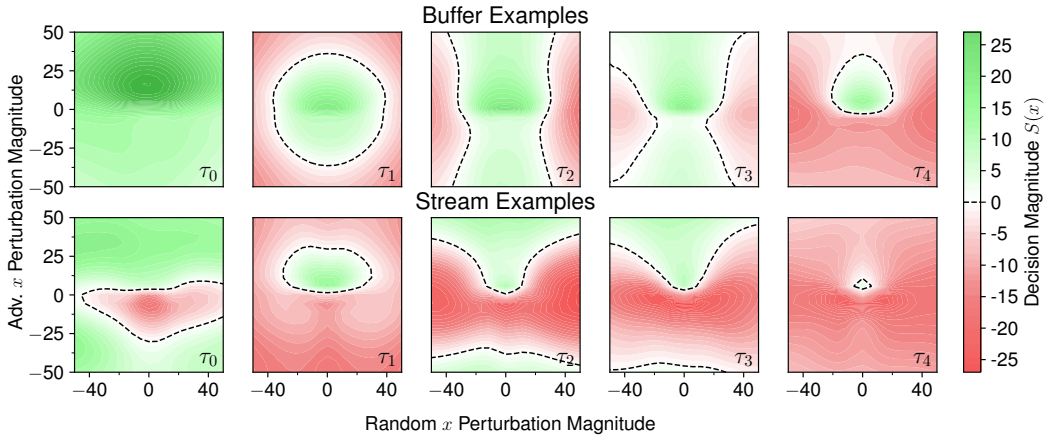
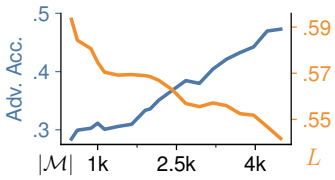


Figure 1: On Split CIFAR-10, depiction of the decision surface around examples of the first task while tasks progress from τ_0 (left) to τ_4 (right). The first row contains various snapshots around rehearsed examples, whereas the second one depicts the situation for non-rehearsed training examples. The diagrams derive from [69] and describe how perturbing the input influences model's prediction: in green areas, the correct response is preserved. As can be seen, the decision boundaries – indicated by dashed lines – are tighter around non-rehearsed examples. Refer to Sec. 5.5 for additional details.

erosion of the decision surface, which begins to *bubble* around buffer datapoints due to their repeated optimization, as highlighted in Fig.1. In other terms, we suspect that a loss of smoothness occurs on the decision surface, turning out to be fractured and jagged if compared to the optimal one attained at the end of the old task.



Recent study [67, 57, 36, 37, 65, 24] have highlighted the link between the Lipschitz continuity of a DNN and several properties, such as its generalization capabilities and robustness to adversarial attacks. On this latter point, controlling the Lipschitz constant of a DNN has shown to strengthen the robustness region around a point [18, 36, 37, 65, 74], thus preventing attacks of a certain magnitude from changing the output of the classifier. In this work, we exploit the Lipschitz constant as a proxy metric of how stable the prediction of the model is around buffer datapoints. As shown in the inset figure, when decreasing the capacity of the memory buffer, the corresponding Lipschitz constant of the model increases. This result suggests that repeated optimization on few examples leads to non-smooth decision surface, as also highlighted by the (low) accuracy obtained by the model in presence of adversarial attacks.

To the best of our knowledge, our work is the first attempt to assess how Lipschitz-constrained DNNs perform in a continual learning scenario. In this respect, we have conducted several experiments and ablative studies to support our idea. In particular, we have equipped several widely-know and state-of-the-art rehearsal approaches with our Lipschitz-guided optimization objective named **LiDER**, showing that it systematically leads to better results in several benchmarks.

2 Related works

2.1 Continual Learning

CL examines the capability of a deep model to learn from a sequence of non-i.i.d. classification tasks [19, 47] while preventing the onset of *catastrophic forgetting* [42]. To achieve this goal, models are trained according to specifically designed strategies, meant to influence their evolution and maximize the retention of previously acquired knowledge.

Among them, *regularization methods* work by introducing additional constraints in the form of loss terms; they are designed to limit the amount of total change either in parameter space [33, 71, 15, 2] or functional space [38, 7]. Differently, *structural methods* purposefully organize the allocation

of model capacity to prevent interference and facilitate parameter sharing [1, 41, 52, 30]. Lastly, *rehearsal methods* store and reuse a subset of previously seen data-points to prevent overfitting on current data and avert forgetting [12, 40, 4, 3]. While *rehearsal* strategies are by far the most frequently adopted thanks to their effectiveness and flexibility [22, 4], it is not uncommon to adopt solutions combining multiple approaches [27, 1, 11, 48, 14]. In this paper, we similarly propose a strategy that leverages an existing replay memory buffer to compute an additional regularization term, aimed at conditioning the learning dynamics and avoiding overfitting.

CL evaluations are often carried out in the so-called Task-Incremental setting (TIL) [33, 71, 40, 19, 15] – that is, the model is provided a *task identifier* at test time to restrict its predictions and avoid interference across logits of distinct tasks. However, recent works put an increasing focus on the harder Class-Incremental setting (CIL) [4, 27, 11, 66], which entails the production of a unified prediction encompassing all seen classes. W.r.t. the latter, TIL has been criticized as a less challenging and realistic benchmark [22, 59, 4]; we therefore conduct our main experiments on state-of-the-art CL models in the CIL setting[1].

2.2 Lipschitz-based Regularization

Naturally trained DNNs typically suffer from their overparametrization [73, 46], leading to the tendency to overfitt the training data by producing jagged decision boundaries that closely fit the seen examples. On the contrary, a model's reliability depends on its capability for generalization, which is linked to the appearance of smooth decision boundaries [67, 6, 68, 24]. Starting from the first studies focusing on this simple dichotomy, the Lipschitz constant L of a DNN has been established as a commonplace measure of both smoothness and generalization [67, 57, 31] and still constitutes a key ingredient for current evaluations of model capacity [6, 25].

Most notably, *Szegedy et al.* [57] verify that constraining L reduces the model's vulnerability to adversarial perturbations. Many current approaches to Adversarial Learning similarly operate either by minimizing L at the *global* or *local* level [37, 58, 36] or by devising models characterized by a small L by design [18, 28]. In other areas, the smoothing effect of L-based regularization has been favorably applied to both GAN training [44] and neural fields [39].

3 Method

A CL problem usually involves learning a function f from a stream of data, which we formalize as a succession of separate datasets $T = \{\tau_0, \tau_1, \dots, \tau_{|T|}\}$, where $\tau_t = \{(\mathbf{x_i}, y_i)\}_{i=1}^{N_t}$ and $\tau_i \cap \tau_j = \varnothing$; the label set Y_t for each τ_t are non-overlapping. In this setting, the ideal objective consists in minimizing the overall loss over all tasks experienced, formally:

$$f^* = \underset{f}{\operatorname{argmin}}\, \mathbb{E}_{t=0}^{|T|} \Big[\mathbb{E}_{(\mathbf{x},y)\sim\tau_t} \Big[\mathcal{L}(f(\mathbf{x}), y) \Big] \Big], \tag{1}$$

where $\mathcal{L}$ is an appropriate loss for solving the task at hand. In a continual scenario, only data from the current task τ_t is freely available; therefore, CL methods need to maintain knowledge from the past $t-1$ tasks in order to solve the overall problem.

For the sake of simplicity, we consider a feed forward neural network $f(\cdot) = (H^K \circ \sigma^K \circ H^{K-1} \circ \sigma^{K-1} \circ \dots H^1)(\cdot)$, *i.e.*, a sequence of σ-activated linear functions $H^k(\mathbf{h}) = \mathbf{W}_k^T \mathbf{h}$ (biases are omitted). A final projection head $g(\cdot) = \mathrm{softmax}(\cdot)$ is applied to produce per-class output probabilities. As stated in other works [24], other common transformations that make up DNNs (*e.g.*, convolutions, max-pool) can also be seen in terms of matrix multiplications, thus making our approach applicable to more complex networks.

Lipschitz continuity. A function f is said to be *Lipschitz continuous* if there exists a value $L \in \mathbb{R}^+$ such that the following inequality holds:

$$||f(\mathbf{x}) - f(\mathbf{y})||_2 \leq L||\mathbf{x} - \mathbf{y}||_2, \quad \forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^n. \tag{2}$$

[1] However, we remark that TIL can also be useful, as it reveals *forgetting* disentangled from other incremental learning effects. This motivates us to adopt TIL in some of our additional experiments.

If such a value exists, the smallest L that satisfies the condition is usually referred to as the Lipschitz norm $||f||_L$. Therefore, for a single point $x \in \mathbb{R}^n$, we obtain:

$$||f||_L = \sup_{x \neq y; y \in \mathbb{R}^n} \frac{||f(\mathbf{x}) - f(\mathbf{y})||_2}{||\mathbf{x} - \mathbf{y}||_2}. \tag{3}$$

Unfortunately, computing the Lipschitz constant of even the most simple multi-layer perceptron is a NP-hard problem [62]. Therefore, several works relied on its estimation by computing reliable upper bounds. As stated in [68, 53], an effective way to bound the Lipschitz constant of $f(\cdot)$ is to compute the constants of each linear projections H^k and then aggregate them. In more details:

$$||H^k||_L = \sup_{x \neq y; y \in \mathbb{R}^n} \frac{||W^T\mathbf{x} - W^T\mathbf{y}||_2}{||\mathbf{x} - \mathbf{y}||_2} = \sup_{\xi \neq 0; \xi \in \mathbb{R}^n} \frac{||\mathbf{W}_k \xi||_2}{||\xi||_2} = \sigma_{\max}(\mathbf{W}_k), \tag{4}$$

where $\sigma_{\max}(\mathbf{W}_k)$ is the largest singular value of the weight matrix $\mathbf{W}_k$ (also know as its spectral norm $||\mathbf{W}_k||_{SN}$). To account for non-linear composite functions (*e.g.*, the residual building blocks of most convolutional architecture), we leverage the following inequality:

$$\begin{aligned} ||g(z(\mathbf{x})) - g(z(\mathbf{y}))||_2 &\leq ||g||_L ||z(\mathbf{x}) - z(\mathbf{y})||_2 \\ &\leq ||g||_L ||z||_L ||\mathbf{x} - \mathbf{y}||_2 \Rightarrow ||z \circ g||_L \leq ||g||_L ||z||_L, \end{aligned}$$

where $\mathbf{g}(\cdot)$ and $\mathbf{z}(\cdot)$ are two Lipschitz continuous functions characterized by the constants $||g||_L$ and $||z||_L$. In the case of ReLU-activated networks (but the following result can be extended to other common non-linear functions), the forward pass through σ^l $l = 1, 2, \ldots, L$ can be re-arranged as a matrix multiplication by a diagonal matrix $D^l \in \mathbb{R}^{d_l \times d_{l+1}}$ whose diagonal elements equal either zero or one. Therefore, their corresponding Lipschitz constant $||\sigma^l||_L \leq 1$. On top of that, we can compute an upper bound on the Lipschitz constant of the entire network:

$$||f||_L \leq ||H^K||_L \cdot ||\sigma^K||_L \cdot \ldots \cdot ||H^1||_L \leq \prod_{k=1}^{K} ||H^k||_L = \prod_{k=1}^{K} ||\mathbf{W}^k||_{SN}. \tag{5}$$

Computing the spectral norm of weights matrices. The computation of $||\mathbf{W}_k||_{SN}$ can be done [44, 24] naively through the Singular Value Decomposition (SVD), yielding, among the others, the largest singular value. Such approach has been applied in recent works [44, 24]; however, for complex structures (*e.g.*, convolutions or entire residual blocks) the SVD decomposition is inaccessible. Hence, we rely on the approximation introduced in [53] and compute the largest eigenvalue λ_1^k of the Transmitting Matrix $\mathbf{TM}^k$ (which represents a good proxy of $||\mathbf{W}^k||_{SN}$[2]):

$$\mathbf{TM}^k \triangleq \left[(\mathbf{F}^k)^T (\mathbf{F}^{k-1})\right]^T \left[(\mathbf{F}^k)^T (\mathbf{F}^{k-1})\right], \tag{6}$$

where $\mathbf{F}^k \in \mathbb{R}^{B \times d_k}$ is the L2-normalized feature map produced by the l-th layer from a batch of B samples. Finally, our approach exploits the power iteration method [45] to compute the largest eigenvalue of $\mathbf{TM}^k$, which is backpropagation-friendly.

3.1 Lipschitz-Driven Rehearsal

In a continual setting, a model is asked *i)* to be adaptable to incoming samples from the stream (plasticity), and *ii)* to be accurate on past tasks (stability). We seek to ensure a balance between these clashing objectives through the two following loss terms.

Controlling Lipschitz-continuity. To mitigate overfitting on buffer datapoints, we firstly impose that each layer behaves as a c-Lipschitz continuous function, for a given real positive target constant c_k:

$$\mathcal{L}_{\text{c-Lip}} = \frac{1}{K} \sum_{k=0}^{K} |\lambda_1^k - c_k|. \tag{7}$$

During the computation of each λ_1^k, we discard the activation maps incoming from the examples of the current task. Indeed, as we have access to the entire training set (and not a subset as holds for

[2] We refer the reader to [53] for additional justifications for this step.

old tasks), additional regularization is not needed: the decision boundaries tied to the current task are less prone to the risk of over-adapting to certain points. Regarding the target constants c_k, we could fix them as hyperparameters of our learning objective (as done in [39]) and exploit them as a sort of budget assigned to each layer; however, we empirically observed that it is beneficial, instead, learning these targets by means of gradient descent (see Sec. 5.4), especially in a CL scenario where there is no access to the full data distribution. Indeed, these can be interpreted as additional learnable parameters, which represent the appropriate level of strictness each layer should be subjected to.

Therefore, to avoid trivial solutions, we add another regularization term, which asks these constant to be as much as possible close to zero:

$$\mathcal{L}_{\text{0-Lip}} = \frac{1}{K} \sum_{k=0}^{K} |c_k|. \tag{8}$$

Intuitively, when $\lambda_1^k \to 0$, the outputs of the corresponding k-th layer has low sensitivity to changes in its input. In our intentions, this could relieve continuous rehearsal from eroding the decision surface in a way that fits well only certain examples (*i.e.*, those retained in the memory buffer).

Overall objective. The overall objective combines the two introduced loss terms; formally:

$$\mathcal{L}_{\text{LiDER}} = \alpha \mathcal{L}_{\text{c-Lip}} + \beta \mathcal{L}_{\text{0-Lip}}. \tag{9}$$

This objective can be plugged in almost any rehearsal approach. For such a reason, we keep it general and avoid reporting the common loss terms asking for accurate predictions, as their form depend on the specific choices made by each approach. Finally, we further remark that the introduced loss terms require minimal additional computation. Moreover, they do not need additional samples to be retained, besides those that are already present in the memory buffer.

4 Experiments

To assess the effectiveness of our proposal, we introduce a suite of experiments encompassing the prevalent applications of rehearsal in modern literature. In particular, we show that our method can be easily applied to state-of-the-art replay methods and enhance their performance in a wide variety of challenging settings and backbone architectures. Moreover, we show that our proposal remains rewarding and can improve the generalization capabilities of CL models even when a pre-trained model is employed. Such scenario is important for a twofold reason: i) as shown in [43], pre-training implicitly mitigates forgetting by widening the local minima found in function space, thus making the model more robust to input perturbations; additionally, ii) we accommodate for real-world scenarios where pre-training is usually involved as an initial step. Due to space constraints, for additional experimental details we kindly refer the reader to the supplementary material.

Benchmarked models. We conduct experiments on multiple state-of-the-art rehearsal-based models. If not specified, all methods use reservoir sampling [63] to update the memory buffer.

- **Experience Replay with Asymmetric Cross-Entropy (ER-ACE)** [13]: the authors devise an improvement over standard ER with a separate loss for data coming from the stream and from the buffer. Consequently, they observe a significant performance gain, which shows the importance of reducing the drift in incrementally learned representations.
- **Dark Experience Replay (DER)** [11]: the authors propose an improvement over ER to take advantage high-level knowledge previously learned by means of self-distillation [23]. In addition to storing the one-hot labels, they also include the pre-softmax activations (logits) of the model sampled along the optimization trajectory; subsequently, they enforce consistent responses through time by minimizing the L2 norm between current and past logits. In this work, we use DER++, a stronger baseline that combines standard replay and self-distillation.
- **Incremental Classifier and Representational Learning (iCaRL)** [50]: instead of training a classifier to solve the task directly, the authors propose to learn a representation suitable for a nearest-neighbor classification w.r.t. class prototypes stored in the buffer. Additionally, forgetting is prevented by distilling the responses of the model's snapshot at the previous task to both current and replay examples. Since samples in the buffer have a major role as anchors for classifications, the buffer is managed through the herding strategy.

- **Greedy Sampler and Dumb Learner (GDumb)** [49]: the authors challenge the improvements made on CL literature, in which the authors propose to simply train the model at the task boundary solely on data from the buffer.

4.1 Datasets & measures

We focus our evaluation on the CIL setting and rely on commonly used and challenging image classification tasks. In each experiment, classes from the main dataset are split into separate and disjoint sets, which are then used sequentially to train the evaluated models.

Split CIFAR-100. An initial evaluation is carried by splitting the 32×32 images from the 100 classes of CIFAR-100 [34] into 10 tasks. In detail, we run two tests: *i)* using a randomly initialized ResNet18 [26] – which constitutes a common scenario in literature [71, 50, 17, 12]; *ii)* adopting the same backbone but with its parameters pre-trained on Tiny ImageNet [56].

Split *mini*ImageNet. This setting is designed to evaluate the robustness of models on longer sequences of tasks, by splitting the 100 classes of *mini*ImageNet [61] – a subset of the ImageNet dataset, where each image is resized to 84×84 – into 20 consecutive tasks. For this test, we opt for an EfficientGCN-B2 [55] backbone with no pre-train.

Split CUB-200. A final, more challenging benchmark involves classifying large-scale 224×224 images from the Caltech-UCSD Birds-200-2011 [64] dataset, organized in a stream of 10 20-fold classification tasks. This evaluation is designed to highlight the importance of protecting pre-trained weights from forgetting. Indeed, due to the limited size of the training set, competitive performance can only be achieved if each task can benefit from the initialization [16, 70]. As backbone network, we opt for the commonly available ResNet50 architecture pre-trained on the ImageNet dataset [20].

For each benchmark we measure performance in terms of the classification accuracy averaged across all seen tasks (Final Average Accuracy - FAA) and of Final Forgetting (FF) [15], formally defined as:

$$\text{FF} \triangleq \frac{1}{|T|-1} \sum_{i=0}^{|T|-2} \max_{t \in \{0,\dots,|T|-2\}} \{a_i^t - a_i^{|T|-1}\}, \tag{10}$$

where a_i^t indicates the accuracy on task τ_i after training on the tth task.

4.2 Results

Results of our evaluation on Split CIFAR-100 can be found in Tab. 1, while results on Split CIFAR-100, and Split *mini*ImageNet are in Tab. 2. For further reference, each table includes the results of a model jointly trained on all classes – which represents an ideal non-CL upper bound – and of a model sequentially trained on each task, with no countermeasure to forgetting.

Across the board, we find LiDER to be capable of improving the performance of all base methods in all evaluated scenarios, both in terms of FAA and FF metrics. Most notably, we find it especially beneficial for methods that feature the most compelling results – usually DER++ and iCaRL –, suggesting that their higher generalization capability can still benefit from increased smoothness in latent space. Differently, scenarios where a method fail to prevent forgetting *i.e.* GDumb with a reduced buffer size usually feature a lower degree of benefit from LiDER but still a notable improvement. However, as it takes advantage both from increased buffers and pre-trained initialization, we observe a considerable performance gap with our proposal. Notably, on Split CIFAR-100, GDumb with a buffer size of 500 moves from a performance gain of $+0.98\%$ to $+6.46\%$ when increasing the buffer to 2000 samples and $+2.99\%$ with pre-train.

Finally, we observe an average performance gain of 2.32%, 2.08%, and 4.36% on Split CIFAR-100, Split *mini*ImageNet, and Split CUB-200 respectively.

5 Ablation studies

In the following sections we provide further insights on the effectiveness of our proposal in controlling and improving the generalization capabilities of the base models.

Table 1: Final Average Accuracy (FAA) [↑] and Final Forgetting (FF) [↓] on Split CIFAR-100.

CIL FAA (FF)	**Split CIFAR-100**			
Method	*w/o pre-training*		*pre-tr. Tiny ImageNet*	
Joint (UB)	73.29 (–)		75.20 (–)	
Finetune	09.29 (86.62)		09.52 (92.31)	
Buffer Size	500	2000	500	2000
iCaRL [50]	44.04 (21.70)	50.23 (17.92)	56.00 (19.27)	58.10 (16.89)
+ **LiDER**	47.02 (21.89)	51.21 (17.13)	57.24 (19.16)	60.97 (15.49)
DER++ [11]	37.13 (49.80)	52.08 (31.10)	43.65 (48.72)	58.05 (29.65)
+ **LiDER**	39.25 (45.50)	53.27 (27.51)	45.37 (48.16)	60.88 (25.16)
GDumb [49]	09.28 (–)	19.69 (–)	23.09 (–)	36.05 (–)
+ **LiDER**	10.22 (–)	26.15 (–)	26.09 (–)	41.98 (–)
ER-ACE [13]	36.48 (38.21)	48.41 (27.90)	48.19 (31.84)	57.34 (25.48)
+ **LiDER**	38.43 (36.00)	50.32 (28.30)	48.97 (28.58)	57.39 (25.37)

Table 2: Final Avg. Accuracy (FAA) [↑] and Final Forgetting (FF) [↓] on Split *mini*ImageNet and Split CUB-200.

CIL FAA (FF)	**Split *mini*ImageNet**		**Split CUB-200**	
Method	*w/o pre-training*		*pre-tr. ImageNet*	
Joint (UB)	53.55 (–)		78.54 (–)	
Finetune	04.51 (77.38)		08.56 (82.38)	
Buffer Size	2000	5000	400	1000
iCaRL [50]	22.58 (16.46)	22.78 (16.37)	56.52 (13.43)	60.09 (11.41)
+ **LiDER**	23.22 (11.21)	23.95 (11.18)	57.12 (14.31)	60.37 (10.89)
DER++ [11]	23.44 (46.69)	30.43 (37.11)	49.30 (36.05)	61.42 (19.95)
+ **LiDER**	28.33 (36.29)	35.04 (25.02)	57.90 (27.55)	67.97 (14.44)
GDumb [49]	15.22 (–)	27.79 (–)	09.36 (–)	18.98 (–)
+ **LiDER**	15.24 (–)	29.49 (–)	09.67 (–)	19.51 (–)
ER-ACE [13]	22.60 (23.74)	27.92 (19.72)	41.83 (26.42)	51.98 (18.79)
+ **LiDER**	24.13 (25.97)	30.00 (19.99)	50.89 (20.79)	60.92 (14.62)

5.1 Buffer Poisoning

To evaluate the performance of a ML model, practitioners usually resort to synthetic benchmarks that simulate the progressive arrival of novel knowledge from an unknown source. While this methodology is useful to compare the effectiveness of new proposals a key element is usually neglected: the labels given to each sample may – and in the real-world usually does – be incorrect. Notably, this aspect is even more relevant in a CL scenario and especially when a rehearsal strategy is employed. As previously shown, samples included in the buffer are the ones that are most likely to be overfitted, which would induce a severe loss of performance if the label is incorrect.

In the following, we propose to measure the negative effect of overfitting the wrong label for samples included in the memory buffer. Specifically, we simulate noise in the labels by randomly assigning with probability p a new label to the samples as these are added in the buffer, a technique which we name *label poisoning*. To better suit a CL scenario, poisoned labels are chosen from those of the current task.

Results reported in Tab. 3 show the evaluation performed on top of DER++ for the Split CIFAR-100 benchmark. Unexpectedly, performance degrades as p increases; however, we also observe that LiDER retains an higher level of accuracy against poisoning. This effect suggests that our proposal leads to a looser decision boundary around the items store in the buffer.

Table 3: FAA after poisoning the buffer for DER++ with and without LiDER.

p	DER++	+LiDER
.0%	37.14	39.25
.01%	36.13	38.08
.1%	31.35	35.53
.25%	28.74	30.78

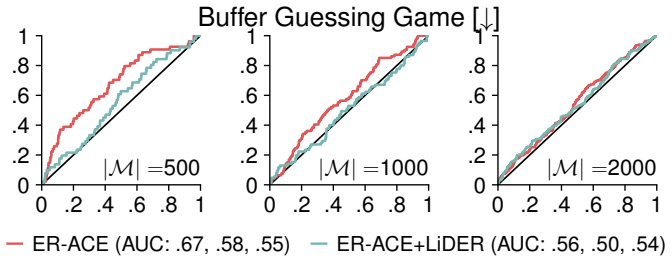


Figure 2: ROC curves for the Buffer Guessing Game, showing the likelihood of a given sample belonging in $\mathcal{M}$.

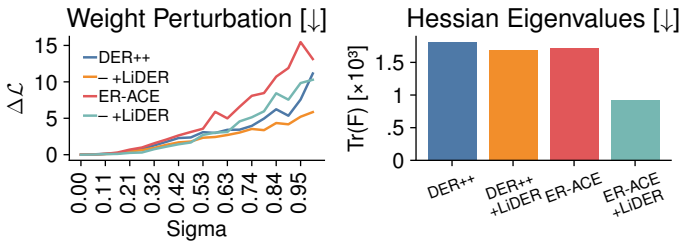


Figure 3: (Left) Robustness of models regularized with LiDER against weight perturbations (best seen in color). (Right) Flatness around the minima found during optimization, measured as the sum of the eigenvalues of the Hessian matrix.

Table 4: Performance comparison between LiDER and a regularization method obtained by fixing the Lipschitz targets in Eq. 7.

	Model	$\lvert\mathcal{M}\rvert$	Fixed tgt	*Eq. 9*
w/o pre-tr.	DER++	500	36.42	39.25
	ER-ACE	2000	34.99	38.43
	DER++	500	51.52	53.27
	ER-ACE	2000	46.70	48.97
pre-tr.	DER++	500	43.16	45.37
	ER-ACE	2000	45.21	48.97
	DER++	500	59.53	60.68
	ER-ACE	2000	54.82	57.39

5.2 Buffer Guessing Game

In this section, we set up a novel experiment aimed at further illustrating the peculiar overfitting behavior of rehearsal-based CL models. As mentioned in Sec. 1, we posit that a substantially different regime affects replay and stream examples: the former plays a much larger role in shaping the decision boundary w.r.t. the latter. To validate this intuition, we propose a simple *buffer guessing game*: given a rehearsal-based model f fully trained on a CL benchmark and the dataset τ_0 used in its first encountered task, we aim to find $\mathcal{M} \cap \tau_0$ (*i.e.* the subset of data-points that are included in the model's buffer).

We approach the game by associating each $x \in \tau_0$ with a score s_x computed as the mean height of the decision surface of f in a neighborhood of x[3]; finally, we evaluate the ROC curve obtained from these scores. In Fig. 2, we report the results obtained for ER-ACE and ER-ACE+LiDER on Split CIFAR-100 across different buffer sizes. We see that: *i)* ER-ACE makes it is easier to reconstruct the content of the buffer, as indicated by larger ROC-AUC scores w.r.t. ER-ACE+LiDER; *ii)* in line with our expectations, this effect is increased when employing smaller memory buffers, as this leads to the repeated optimization of a smaller pool of data.

5.3 Generalization Measures

Previous CL works proposed investigating the generalization capabilities of a CL model by evaluating the flatness of its attained minima [11, 9]. We likewise evaluate the effect of LiDER both in terms of its resilience to weight perturbations [46, 31] and the eigenvalues of the Hessian of the loss function [31, 29] in Fig. 3. We see that DER++ and ER-ACE combined with LiDER see an improvement of both metrics. This is expected, as the Lipschitz constant of a DNN has been commonly interpreted as a generalization measure [67, 57].

[3] We compute the decision surface by leveraging random perturbations as specified in [69] and compute it w.r.t. to the TIL prediction function to avoid the influence of inter-task biase on our results

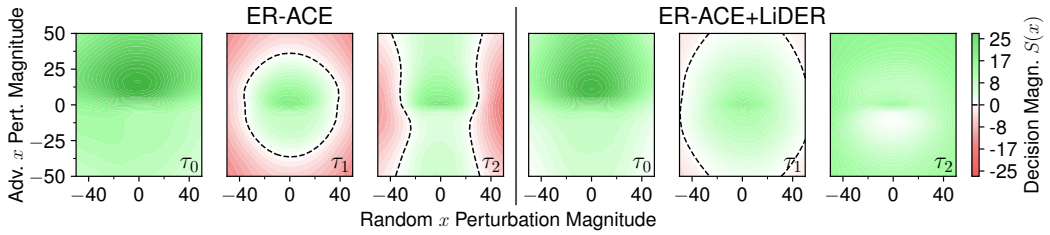


Figure 4: Effect of our proposal on the robustness of the decision boundary produced by ER-ACE across subsequent tasks. Higher values (in green) indicate areas of high confidence and span the possible directions where perturbations are not disruptive (best seen in color).

5.4 Optimization with a fixed target

Instead of constraining the optimization of the Lipschitz targets of Eq. 7 by an additional contribution, one might argue that a similar effect could be obtained by fixing an initial value and force the model to reduce its capacity until it meets the desiderata [39]. In Tab. 4 we empirically show that such approach does not lead to satisfactory results in a CL setting, with our proposal consistently exceeding its performance on various settings and base methods.

5.5 Decision Landscape of LiDER

In Fig. 1, we presented a pictorial demonstration of a model's tolerance to input perturbations in the form of a decision surface plot [69]. Such a visualization is constructed by focusing on a set of perturbations $x_p \triangleq x + i \cdot \alpha + j \cdot \beta$ computed around a data-point x (α is a random divergence direction and β corresponds to the direction induced by the first step of a non-targeted FGSM attack [35]). The plot shows the respective values of the decision function $S(x_p)$, where $S(x) \triangleq f(x)_t - \max_{i \neq t} f(x)_i$ (with $f(\cdot)$ indicating pre-softmax responses) and highlights decision boundary of the model (*i.e.* the locus of $\{x_p | S(x_p) = 0\}$), in correspondence of which the model accuracy fails.

In Fig. 4, we adopt the same approach to compare the decision boundaries around rehearsed τ_0 examples for ER-ACE with and without LiDER. It can be observed that, while both models start with a similar robust decision landscape in τ_0, later tasks reveal a clear shrinking behavior in ER-ACE. On the contrary, introducing L-based regularization hardens the backbone against adversarial perturbations, leading to minimal decision boundary deterioration in later tasks.

6 Conclusions

We present LiDER, a novel regularization strategy to compensate for the phenomenon of buffer overfitting in rehearsal-based Continual Learning; with its carefully-designed loss term it enforces the decision boundary of replayed samples to be smoother by bounding the complexity of the model. We show that such approach can be readily applied to diverse state-of-the-art models and remains competitive across various settings and backbone architecture. Finally, we provide an extensive investigation to assess the validity of theoretical assumptions.

Limitations & Societal Impact

Our approach heavily relies on the estimation of the Lipschitz constant and, in particular, on its upper bound (as reported in the previous section, the exact computation is a NP-hard problem). We highlight that tighter bounds have been recently proposed, at the expense of higher complexity and hence slower computation speed. Moreover, the approximation exploited in this work cannot be naively applied to those architectures comprising of not-Lipschitz continuous layers (*e.g.*, cross-attention) [32].

Concerning societal impact, we do not feel that this work will have detrimental applications that might affect any public. We only remark that, as our approach is based on rehearsal techniques that store in-plain data, it cannot be used in those scenario where privacy constraints are crucial.

References

[1] Davide Abati, Jakub Tomczak, Tijmen Blankevoort, Simone Calderara, Rita Cucchiara, and Babak Ehteshami Bejnordi. Conditional channel gated networks for task-aware continual learning. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2020.

[2] Rahaf Aljundi, Francesca Babiloni, Mohamed Elhoseiny, Marcus Rohrbach, and Tinne Tuytelaars. Memory aware synapses: Learning what (not) to forget. In *Proceedings of the European Conference on Computer Vision*, 2018.

[3] Rahaf Aljundi, Eugene Belilovsky, Tinne Tuytelaars, Laurent Charlin, Massimo Caccia, Min Lin, and Lucas Page-Caccia. Online continual learning with maximal interfered retrieval. In *Advances in Neural Information Processing Systems*, 2019.

[4] Rahaf Aljundi, Min Lin, Baptiste Goujaud, and Yoshua Bengio. Gradient based sample selection for online continual learning. In *Advances in Neural Information Processing Systems*, 2019.

[5] Jihwan Bang, Heesu Kim, YoungJoon Yoo, Jung-Woo Ha, and Jonghyun Choi. Rainbow memory: Continual learning with a memory of diverse samples. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2021.

[6] Peter L Bartlett, Dylan J Foster, and Matus J Telgarsky. Spectrally-normalized margin bounds for neural networks. In *Advances in Neural Information Processing Systems*, 2017.

[7] Ari S Benjamin, David Rolnick, and Konrad Kording. Measuring and regularizing networks in function space. In *International Conference on Learning Representations Workshop*, 2019.

[8] David Berthelot, Nicholas Carlini, Ian Goodfellow, Nicolas Papernot, Avital Oliver, and Colin A Raffel. Mixmatch: A holistic approach to semi-supervised learning. *Advances in Neural Information Processing Systems*, 2019.

[9] Matteo Boschini, Lorenzo Bonicelli, Pietro Buzzega, Angelo Porrello, and Simone Calderara. Class-incremental continual learning into the extended der-verse. *arXiv preprint arXiv:2201.00766*, 2022.

[10] Matteo Boschini, Pietro Buzzega, Lorenzo Bonicelli, Angelo Porrello, and Simone Calderara. Continual semi-supervised learning through contrastive interpolation consistency. *arXiv preprint arXiv:2108.06552*, 2021.

[11] Pietro Buzzega, Matteo Boschini, Angelo Porrello, Davide Abati, and Simone Calderara. Dark Experience for General Continual Learning: a Strong, Simple Baseline. In *Advances in Neural Information Processing Systems*, 2020.

[12] Pietro Buzzega, Matteo Boschini, Angelo Porrello, and Simone Calderara. Rethinking Experience Replay: a Bag of Tricks for Continual Learning. In *International Conference on Pattern Recognition*, 2020.

[13] Lucas Caccia, Rahaf Aljundi, Nader Asadi, Tinne Tuytelaars, Joelle Pineau, and Eugene Belilovsky. New Insights on Reducing Abrupt Representation Change in Online Continual Learning. In *International Conference on Learning Representations Workshop*, 2022.

[14] Hyuntak Cha, Jaeho Lee, and Jinwoo Shin. Co2l: Contrastive continual learning. In *IEEE International Conference on Computer Vision*, 2021.

[15] Arslan Chaudhry, Puneet K Dokania, Thalaiyasingam Ajanthan, and Philip HS Torr. Riemannian walk for incremental learning: Understanding forgetting and intransigence. In *Proceedings of the European Conference on Computer Vision*, 2018.

[16] Arslan Chaudhry, Marc'Aurelio Ranzato, Marcus Rohrbach, and Mohamed Elhoseiny. Efficient Lifelong Learning with A-GEM. In *International Conference on Learning Representations Workshop*, 2019.

[17] Arslan Chaudhry, Marcus Rohrbach, Mohamed Elhoseiny, Thalaiyasingam Ajanthan, Puneet K Dokania, Philip HS Torr, and Marc'Aurelio Ranzato. On tiny episodic memories in continual learning. In *International Conference on Machine Learning Workshop*, 2019.

[18] Moustapha Cisse, Piotr Bojanowski, Edouard Grave, Yann Dauphin, and Nicolas Usunier. Parseval networks: Improving robustness to adversarial examples. In *International Conference on Machine Learning*, 2017.

[19] Matthias De Lange, Rahaf Aljundi, Marc Masana, Sarah Parisot, Xu Jia, Ales Leonardis, Greg Slabaugh, and Tinne Tuytelaars. A continual learning survey: Defying forgetting in classification tasks. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2021.

[20] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2009.

[21] Sayna Ebrahimi, Suzanne Petryk, Akash Gokul, William Gan, Joseph E Gonzalez, Marcus Rohrbach, and Trevor Darrell. Remembering for the right reasons: Explanations reduce catastrophic forgetting. *Applied AI Letters*, 2021.

[22] Sebastian Farquhar and Yarin Gal. Towards Robust Evaluations of Continual Learning. In *International Conference on Machine Learning Workshop*, 2018.

[23] Tommaso Furlanello, Zachary C Lipton, Michael Tschannen, Laurent Itti, and Anima Anandkumar. Born again neural networks. In *International Conference on Machine Learning*, 2018.

[24] Henry Gouk, Eibe Frank, Bernhard Pfahringer, and Michael J Cree. Regularisation of neural networks by enforcing lipschitz continuity. *Machine Learning*, 2021.

[25] Florian Graf, Sebastian Zeng, Marc Niethammer, and Roland Kwitt. On measuring excess capacity in neural networks. *arXiv preprint arXiv:2202.08070*, 2022.

[26] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2016.

[27] Saihui Hou, Xinyu Pan, Chen Change Loy, Zilei Wang, and Dahua Lin. Learning a unified classifier incrementally via rebalancing. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2019.

[28] Yujia Huang, Huan Zhang, Yuanyuan Shi, J Zico Kolter, and Anima Anandkumar. Training certifiably robust neural networks with efficient local lipschitz bounds. In *Advances in Neural Information Processing Systems*, 2021.

[29] Stanislaw Jastrzebski, Zachary Kenton, Devansh Arpit, Nicolas Ballas, Asja Fischer, Yoshua Bengio, and Amos Storkey. Three factors influencing minima in sgd. In *International Conference on Artificial Neural Networks*, 2018.

[30] Zixuan Ke, Bing Liu, and Xingchang Huang. Continual learning of a mixed sequence of similar and dissimilar tasks. In *Advances in Neural Information Processing Systems*, 2020.

[31] Nitish Shirish Keskar, Dheevatsa Mudigere, Jorge Nocedal, Mikhail Smelyanskiy, and Ping Tak Peter Tang. On large-batch training for deep learning: Generalization gap and sharp minima. In *International Conference on Learning Representations Workshop*, 2017.

[32] Hyunjik Kim, George Papamakarios, and Andriy Mnih. The lipschitz constant of self-attention. In *International Conference on Machine Learning*, 2021.

[33] James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, et al. Overcoming catastrophic forgetting in neural networks. *Proceedings of the National Academy of Sciences*, 2017.

[34] Alex Krizhevsky et al. Learning multiple layers of features from tiny images. Technical report, Citeseer, 2009.

[35] Alexey Kurakin, Ian Goodfellow, Samy Bengio, et al. Adversarial examples in the physical world. In *International Conference on Learning Representations Workshop*, 2016.

[36] Sungyoon Lee, Jaewook Lee, and Saerom Park. Lipschitz-certifiable training with a tight outer bound. In *Advances in Neural Information Processing Systems*, 2020.

[37] Klas Leino, Zifan Wang, and Matt Fredrikson. Globally-robust neural networks. In *International Conference on Machine Learning*, 2021.

[38] Zhizhong Li and Derek Hoiem. Learning without forgetting. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2017.

[39] Hsueh-Ti Derek Liu, Francis Williams, Alec Jacobson, Sanja Fidler, and Or Litany. Learning smooth neural functions via lipschitz regularization. *arXiv preprint arXiv:2202.08345*, 2022.

[40] David Lopez-Paz and Marc'Aurelio Ranzato. Gradient episodic memory for continual learning. In *Advances in Neural Information Processing Systems*, 2017.

[41] Arun Mallya and Svetlana Lazebnik. Packnet: Adding multiple tasks to a single network by iterative pruning. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2018.

[42] Michael McCloskey and Neal J Cohen. Catastrophic interference in connectionist networks: The sequential learning problem. *Psychology of learning and motivation*, 1989.

[43] Sanket Vaibhav Mehta, Darshan Patil, Sarath Chandar, and Emma Strubell. An empirical investigation of the role of pre-training in lifelong learning. In *International Conference on Machine Learning*, 2021.

[44] Takeru Miyato, Toshiki Kataoka, Masanori Koyama, and Yuichi Yoshida. Spectral normalization for generative adversarial networks. In *International Conference on Learning Representations Workshop*, 2018.

[45] Yuji Nakatsukasa and Nicholas J Higham. Stable and efficient spectral divide and conquer algorithms for the symmetric eigenvalue decomposition and the svd. *SIAM Journal on Scientific Computing*, 2013.

[46] Behnam Neyshabur, Srinadh Bhojanapalli, David Mcallester, and Nati Srebro. Exploring Generalization in Deep Learning. In *Advances in Neural Information Processing Systems*, 2017.

[47] German I Parisi, Ronald Kemker, Jose L Part, Christopher Kanan, and Stefan Wermter. Continual lifelong learning with neural networks: A review. *Neural Networks*, 2019.

[48] Quang Pham, Chenghao Liu, and Steven Hoi. Dualnet: Continual learning, fast and slow. In *Advances in Neural Information Processing Systems*, 2021.

[49] Ameya Prabhu, Philip HS Torr, and Puneet K Dokania. GDumb: A simple approach that questions our progress in continual learning. In *Proceedings of the European Conference on Computer Vision*, 2020.

[50] Sylvestre-Alvise Rebuffi, Alexander Kolesnikov, Georg Sperl, and Christoph H Lampert. icarl: Incremental classifier and representation learning. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2017.

[51] Matthew Riemer, Ignacio Cases, Robert Ajemian, Miao Liu, Irina Rish, Yuhai Tu, and Gerald Tesauro. Learning to Learn without Forgetting by Maximizing Transfer and Minimizing Interference. In *International Conference on Learning Representations Workshop*, 2019.

[52] Joan Serra, Didac Suris, Marius Miron, and Alexandros Karatzoglou. Overcoming Catastrophic Forgetting with Hard Attention to the Task. In *International Conference on Machine Learning*, 2018.

[53] Yuzhang Shang, Bin Duan, Ziliang Zong, Liqiang Nie, and Yan Yan. Lipschitz continuity guided knowledge distillation. In *IEEE International Conference on Computer Vision*, 2021.

[54] Konstantin Shmelkov, Cordelia Schmid, and Karteek Alahari. Incremental learning of object detectors without catastrophic forgetting. In *IEEE International Conference on Computer Vision*, 2017.

[55] Yi-Fan Song, Zhang Zhang, Caifeng Shan, and Liang Wang. Constructing Stronger and Faster Baselines for Skeleton-based Action Recognition. *arXiv preprint arXiv:2106.15125*, 2021.

[56] Stanford. Tiny ImageNet Challenge (CS231n), 2015. `https://www.kaggle.com/c/tiny-imagenet`.

[57] Christian Szegedy, Wojciech Zaremba, Ilya Sutskever, Joan Bruna, Dumitru Erhan, Ian Goodfellow, and Rob Fergus. Intriguing properties of neural networks. In *International Conference on Learning Representations Workshop*, 2014.

[58] Yusuke Tsuzuku, Issei Sato, and Masashi Sugiyama. Lipschitz-margin training: Scalable certification of perturbation invariance for deep neural networks. In *Advances in Neural Information Processing Systems*, 2018.

[59] Gido M van de Ven and Andreas S Tolias. Three continual learning scenarios. In *Neural Information Processing Systems Workshops*, 2018.

[60] Eli Verwimp, Matthias De Lange, and Tinne Tuytelaars. Rehearsal revealed: The limits and merits of revisiting samples in continual learning. In *IEEE International Conference on Computer Vision*, 2021.

[61] Oriol Vinyals, Charles Blundell, Timothy Lillicrap, Daan Wierstra, et al. Matching networks for one shot learning. In *Advances in Neural Information Processing Systems*, 2016.

[62] Aladin Virmaux and Kevin Scaman. Lipschitz regularity of deep neural networks: analysis and efficient estimation. In *Advances in Neural Information Processing Systems*, 2018.

[63] Jeffrey S Vitter. Random sampling with a reservoir. *ACM Transactions on Mathematical Software*, 1985.

[64] C. Wah, S. Branson, P. Welinder, P. Perona, and S. Belongie. The Caltech-UCSD Birds-200-2011 Dataset. Technical report, California Institute of Technology, 2011.

[65] Lily Weng, Huan Zhang, Hongge Chen, Zhao Song, Cho-Jui Hsieh, Luca Daniel, Duane Boning, and Inderjit Dhillon. Towards fast computation of certified robustness for relu networks. In *International Conference on Machine Learning*, 2018.

[66] Yue Wu, Yinpeng Chen, Lijuan Wang, Yuancheng Ye, Zicheng Liu, Yandong Guo, and Yun Fu. Large scale incremental learning. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2019.

[67] Huan Xu and Shie Mannor. Robustness and generalization. *Machine Learning*, 2012.

[68] Yuichi Yoshida and Takeru Miyato. Spectral norm regularization for improving the generalizability of deep learning. *arXiv preprint arXiv:1705.10941*, 2017.

[69] Fuxun Yu, Zhuwei Qin, Chenchen Liu, Liang Zhao, Yanzhi Wang, and Xiang Chen. Interpreting and evaluating neural network robustness. In *International Joint Conference on Artificial Intelligence*, 2019.

[70] Lu Yu, Bartlomiej Twardowski, Xialei Liu, Luis Herranz, Kai Wang, Yongmei Cheng, Shangling Jui, and Joost van de Weijer. Semantic drift compensation for class-incremental learning. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, 2020.

[71] Friedemann Zenke, Ben Poole, and Surya Ganguli. Continual learning through synaptic intelligence. In *International Conference on Machine Learning*, 2017.

[72] Mengyao Zhai, Lei Chen, Frederick Tung, Jiawei He, Megha Nawhal, and Greg Mori. Lifelong gan: Continual learning for conditional image generation. In *IEEE International Conference on Computer Vision*, 2019.

[73] Chiyuan Zhang, Samy Bengio, Moritz Hardt, Benjamin Recht, and Oriol Vinyals. Understanding deep learning requires rethinking generalization. In *International Conference on Learning Representations Workshop*, 2017.

[74] Hongyang Zhang, Yaodong Yu, Jiantao Jiao, Eric Xing, Laurent El Ghaoui, and Michael Jordan. Theoretically principled trade-off between robustness and accuracy. In *International Conference on Machine Learning*, 2019.

Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes]
   (c) Did you discuss any potential negative societal impacts of your work? [Yes]
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes]
   (b) Did you include complete proofs of all theoretical results? [Yes]
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] The code for reproducing our experiments can be found in the supplemental material and will be made publicly available at publication.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] We cover these aspects in the main paper (Sec. 4) and, more in depth, in the supplemental materials.
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] Due to space constraints, variance values for our experiments are included in the supplemental material.

(d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No] Experiments were conducted on an on-premise desktop machine, equipped with a RTX 2080 consumer GPU. However, we did not keep a detailed record of total computation time.

4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...

   (a) If your work uses existing assets, did you cite the creators? [Yes] In our work we use original code, with the only exceptions for the evaluation datasets whose original papers are cited.

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5. If you used crowdsourcing or conducted research with human subjects...

   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]

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