BOFORMER: LEARNING TO SOLVE MULTI-OBJECTIVE BAYESIAN OPTIMIZATION VIA NON-MARKOVIAN RL

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ABSTRACT

Bayesian optimization (BO) offers an efficient pipeline for optimizing black-box functions with the help of a Gaussian process prior and an acquisition function (AF). Recently, in the context of single-objective BO, learning-based AFs witnessed promising empirical results given its favorable non-myopic nature. Despite this, the direct extension of these approaches to multi-objective Bayesian optimization (MOBO) suffer from the *hypervolume identifiability issue*, which results from the non-Markovian nature of MOBO problems. To tackle this, inspired by the non-Markovian RL literature and the success of Transformers in language modeling, we present a generalized deep Q-learning framework and propose *BOFormer*, which substantiates this framework for MOBO via sequence modeling. Through extensive evaluation, we demonstrate that BOFormer constantly achieves better performance than the benchmark rule-based and learning-based algorithms in various synthetic MOBO and real-world multi-objective hyperparameter optimization problems.

1 INTRODUCTION

Bayesian optimization (BO) offers a sample-efficient pipeline for optimizing black-box functions in various practical applications, such as hyperparameter optimization (Lindauer et al., 2022; Snoek et al., 2012; Klein et al., 2017), analog circuit design (Lyu et al., 2018; Zhou et al., 2020), and automated scientific discovery (Ueno et al., 2016; Gómez-Bombarelli et al., 2018). Notably, these real-world engineering tasks usually involve multiple objective functions, which are potentially conflicting. To search for the set of candidate solutions under a sampling budget, *multi-objective BO* (MOBO) integrates the following two components: (i) MOBO utilizes Gaussian processes (GP) as a surrogate function prior for capturing the underlying structure of each objective function and thereby offering posterior predictive distributions in a compact manner; (ii) MOBO then iteratively determines the samples through an acquisition function (AF), which induces an index-type strategy based on the posterior distributions. The existing AFs are built on various design principles, such as maximizing one-step expected improvement (Emmerich & Klinkenberg, 2008; Yang et al., 2019) and maximizing one-step information gain (Hernández-Lobato et al., 2016; Belakaria et al., 2019; Tu et al., 2022). However, the existing AFs for MOBO are mostly handcrafted and *myopic*, i.e., greedily optimize a one-step surrogate and lack long-term planning capability. With that said, one important and yet under-explored challenge of MOBO lies in the design of *non-myopic* AFs.

In single-objective BO (SOBO), one promising non-myopic approach is to recast BO as a reinforcement learning (RL) problem, and several RL-based algorithms (Volpp et al., 2020; Hsieh et al., 2021; Shmakov et al., 2023) have recently witnessed competitive empirical results. Specifically, AFs could be parameterized by neural networks and learned by either actor-critic (Volpp et al., 2020) or valued-based RL (Hsieh et al., 2021), where the state-action representation consists of the posterior mean and variance of a candidate point as well as the best function value observed so far, and the reward is defined as a function of negative simple regret, as shown in Figure 1. However, a direct extension of these single-objective RL-based AFs to MOBO could suffer from the fundamental *hypervolume identifiability issue*. To better illustrate this, we provide a motivating example in Figure 1, which shows that one can construct a pair of scenarios that cannot be distinguished based solely on the current posterior distributions and the current best values. This example also highlights that the identifiability issue actually results from the inherent non-Markovianty in MOBO as the improvement in hypervolume is *history-dependent*. Note that this identifiability issue is much milder in the SOBO

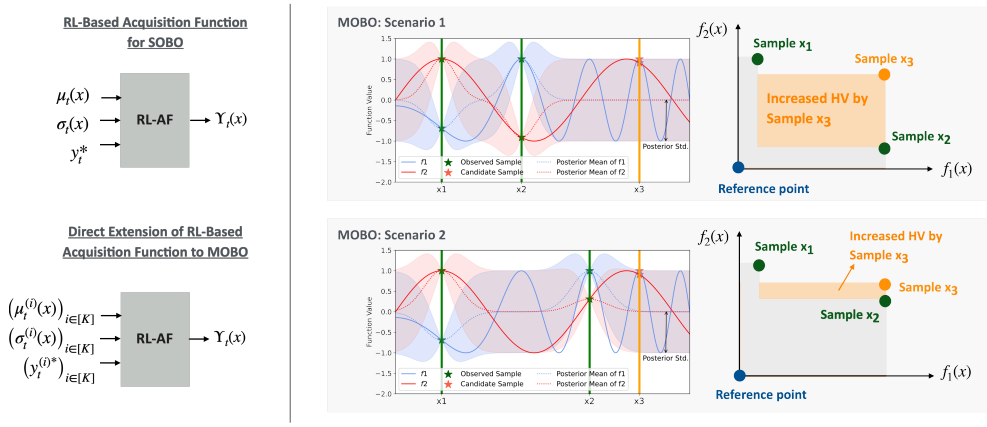


Figure 1: *Left*: In SOBO, an RL-based AF (e.g., FSAF (Hsieh et al., 2021)) takes the posterior mean and standard deviation $(\mu_t(x), \sigma_t(x))$ and the best function value observed so far y_t^* as input and then outputs the AF value $\Upsilon_t(x)$. An direct extension to MOBO simply takes into account the same set of information about all the K objective functions. *Right*: The hypervolume identifiability issue can be illustrated by comparing the hypervolume improvement incurred by the sample x_3 in the two different scenarios above. Clearly, despite that the AF inputs at x_3 are the same in both scenarios, the increases in hypervolume upon sampling x_3 are rather different. Hence, the increase in hypervolume is not identifiable solely based on the AF input $(\mu_t^{(i)}(x), \sigma_t^{(i)}(x)), y_t^{(i)*})_{i\in[K]}$ of the existing RL-based AFs.

setting since a good candidate point (i.e., with a high function value) remains good regardless of the history. As a result, there remains one important open challenge in MOBO:

How to learn a non-myopic AF for MOBO without suffering from the above identifiability issue?

To tackle the above challenge, we propose to rethink MOBO from the perspective of non-Markovian RL via sequence modeling. Specifically, motivated by the optimality equations in the general non-Markovian RL (Dong et al., 2022), we tackle the hypervolume identifiability issue by presenting the non-Markovian version of deep Q-network termed *Generalized DQN*, which extends the standard Markovian DQN (Mnih et al., 2013) by learning the generalized optimal Q-function defined on the history of observations and actions. To implement Generalized DQN, inspired by the significant success of the Transformers in language modeling, we propose BOFormer, which leverages the sequence modeling capability of the Transformer architecture and thereby minimizes the generalized temporal difference loss. As a general-purpose multi-objective optimization solver, the proposed BOFormer is trained solely on synthetic GP functions and can be deployed to optimize unseen testing functions. Moreover, to facilitate the training process, we present several useful and practical enhancements for BOFormer: (i) *Q-augmented observation representation*: Regarding the representation of per-step observation, we propose to use the posterior of the candidate point augmented with its Q-value, which serves as an informative indicator of the prospective improvement in hypervolume. Under this design, the representation is completely domain-agnostic and memory-efficient in the sense that it does not increase with the domain size. (ii) *Prioritized trajectory replay buffer and off-policy learning*: To improve convergence and data efficiency during training, we utilize a prioritized trajectory replay buffer, which can be viewed as a generalization of the typical replay buffer of vanilla DQN. Through this buffer, BOFormer naturally supports off-policy learning and more flexible reuse of training data. (iii) *Demo-policy-guided exploration*: While randomized exploration (e.g., epsilon-greedy) remains popular in many RL algorithms, this exploration scheme can be very inefficient as the sampling budget in BO is usually much smaller than the domain size (i.e., the number of actions). To achieve efficient exploration, we propose to collect part of the training trajectories through a helper demo policy. In practice, one can resort to a policy induced by any off-the-shelf rule-based AFs, such as Expected Hypervolume Improvement (Emmerich & Klinkenberg, 2008).

Notably, as a general-purpose multi-objective optimization solver, BOFormer enjoys the following salient features: (i) *No hypervolume identifiability issue*: Built on the proposed Generalized DQN framework, BOFormer systematically addresses the identifiability issue such that it can approximately

recover the Pareto front under a small sampling budget. (ii) *Zero-shot transfer*: BOFormer is trained solely on synthetic GP functions and can achieve zero-shot transfer to other unseen testing functions. With that said, it does not require any fine-tuning or any metadata during inference at deployment. (iii) *Cross-domain transfer capability*: BOFormer nicely supports cross-domain transfer in the sense that the domain size and dimensionality of the black-box functions for training can be different from those of the testing functions. (iv) *No Monte-Carlo estimation needed during inference*: As a learning-based AF, BOFormer completely obviates the need for the computationally heavy Monte-Carlo estimation required by many rule-based AFs and thereby enjoys efficient inference during deployment. The main contributions could be summarized as follows:

- We identify the critical hypervolume identifiability issue due to the inherent non-Markovianity in learning the AFs for MOBO. To resolve this, inspired by the literature of general RL, we present the Generalized DQN framework for non-Markovian environments.
- To substantiate the Generalized DQN framework, we propose BOFormer, which leverages the Transformer architecture and reinterprets MOBO as a sequence modeling problem. To the best of our knowledge, BOFormer serves as the first RL-based AF for MOBO. Moreover, several practical enhancements are proposed to facilitate the training of BOFormer.
- We construct a hypervolume optimization dataset called HPO-3DGS, consisting of 68000 different parameters on 3D Gaussian Splatting Kerbl et al. (2023) for 3D object reconstruction problems with 5 different scenes. We evaluate the proposed BOFormer on a variety of black-box functions, including both synthetic optimization functions and real-world hyperparameter optimization on HPO-3DGS. We demonstrate that the proposed AF significantly outperforms both existing rule-based AFs and other Transformer-based RL benchmark methods.

2 Related Work

2.1 Multi-Objective Bayesian Optimization

Random Scalarization: To leverage the plethora of AFs for SOBO in the MOBO setting, random scalarization addresses MOBO via iteratively solving single-objective BO subproblems under a scalarization function, such as a direct weighted sum or the Tchebycheff scalarization function (Miettinen, 1999). Notably, random scalarization was originally developed for recovering the Pareto front under evolutionary methods, such as the celebrated ParEGO (Knowles, 2006), MOEA/D (Zhang & Li, 2007), and RVEA (Cheng et al., 2016), and has been subsequently adapted to solving MOBO (Zhang et al., 2009; Paria et al., 2020). Despite its simplicity, as random scalarization enforces exploration mainly by the random sampling of the scalarization parameters, this approach is known to be sensitive to the scale of the different objective functions and could suffer under high-dimensional search spaces (Daulton et al., 2022).

Improvement Maximization: Another popular class of AFs is built on the maximization of improvement-based metrics, such as the expected one-step improvement in hypervolume (EHVI) in (Emmerich & Klinkenberg, 2008; Emmerich et al., 2011; Hupkens et al., 2015; Yang et al., 2019) (also known as the $\mathcal{S}$-metric in (Beume et al., 2007; Ponweiser et al., 2008)), sequential uncertainty reduction (Picheny, 2015), and the hypervolume knowledge gradient (Daulton et al., 2023). However, evaluating the one-step expected improvement typically involves a multi-dimensional integral, which is difficult to derive directly and hence needs to be approximated by the costly Monte Carlo estimation. Accordingly, to tackle the above computational complexity issue, differentiable methods have recently been developed to enable fast parallel evaluations of these AFs, such as qEHVI (Wada & Hino, 2019; Daulton et al., 2020) and qNEHVI (Daulton et al., 2021) in the BoTorch framework (Balandat et al., 2020).

Information-Theoretic Search Methods: Various information-theoretic criteria have been utilized in the context of MOBO. For example, Hernández-Lobato et al. (2016) proposes Predictive Entropy Search for MOBO (PESMO), which selects the candidate point with maximal reduction in the entropy of the posterior distribution over the Pareto-optimal input set, and is subsequently extended to the constrained setting (Garrido-Merchán & Hernández-Lobato, 2019). Subsequently, Belakaria et al. (2019) propose MESMO, which extends the Max-value Entropy Search approach (Wang & Jegelka, 2017) to the principle of output space entropy search for MOBO, i.e., utilizes the information gain about the Pareto-optimal output set as a more computationally tractable sampling criterion (Belakaria et al., 2021). Suzuki et al. (2020) propose Pareto-Frontier Entropy Search, which utilizes information

gain of the Pareto front in the AF design. Moreover, Joint Entropy Search (JES) further takes into account the joint information gain of the Pareto-optimal set of inputs and outputs (Tu et al., 2022; Hvarfner et al., 2022). On the other hand, USeMO (Belakaria et al., 2020) uses the volume of the uncertainty hyper-rectangle as an alternative uncertainty measure for sampling.

Despite the plethora of AFs developed for MOBO, most of them are built on optimizing one-step information-theoretic metrics and do not explore the possibility of multi-step look-ahead policies. By contrast, the proposed BOFormer takes the overall long-term effect of each sample into account through non-Markovian RL and sequence modeling.

2.2 Single-Objective Black-Box Optimization via Learning

Several recent attempts have tackled SOBO problems from the perspective of RL-based AFs. Volpp et al. (2020) propose MetaBO, which leverages actor-critic RL to learn AFs from GP functions for transfer learning. Subsequently, Hsieh et al. (2021) proposes a meta-RL framework termed Few-Shot Acquisition Function (FSAF), which learns a Bayesian deep Q-network as a differentiable AF and adapts the Bayesian model-agnostic meta-learning (Yoon et al., 2018) in order to enable few-shot fast adaptation to various black-box functions based on metadata. (Shmakov et al., 2023) proposes to solve SOBO through a combination of transformer-based deep kernels and RL-based acquisition functions. Despite the above, the existing solutions all focus on SOBO under the standard RL formulation and therefore cannot be directly applied to the non-Markovian problem of MOBO. Moreover, as optimization of single-objective black-box functions is essentially a sequential decision making problem, several recent attempts manage to learn sequence models in an end-to-end manner. For example, Chen et al. (2022) propose OptFormer, which focuses on hyperparameter optimization (HPO) and leverages Transformers through fine-tuning on an offline dataset to enable adaptation to the HPO tasks. More recently, Maraval et al. (2023) propose Neural Acquisition Process (NAP), which is a multi-task variant of Neural Process (NP) simultaneously learning an acquisition function and the predictive distributions, without using the surrogate GP model. To the best of our knowledge, our paper offers the first learning-based solution to MOBO.

Remarks on Application Scope and Objectives: Notably, there are two salient differences between BOFormer and the above two works: (i) *Application scope*: BOFormer is positioned as a general-purpose multi-objective black-box optimization solver with superior cross-domain capability (i.e., the size and the dimensionality of the input domains can be different between the training and deployment phases). In contrast, OptFormer is designed specifically for HPO, and NAP is built on the idea of transfer learning in BO and has limited cross-domain transferability. (ii) *Multiple objectives*: Both OptFormer and NAP focus on single-objective problems and are not readily applicable to recovering the Pareto front in the multi-objective setting. By contrast, BOFormer directly tackles multi-objective optimization and addresses the inherent identifiability issue. Therefore, we consider the above OptFormer and NAP as orthogonal directions to ours. Moreover, the proposed BOFormer can also benefit from the learned neural process in NAP and other variants of NPs (Garnelo et al., 2018; Kim et al., 2018) as surrogate models beyond GPs.

Due to the page limit, we defer the related works on sequence modeling for RL to Appendix B.1.

3 Preliminaries

In this section, we present the formulation of MOBO and the background of general RL. Throughout this paper, we let $\Delta(\mathcal{Z})$ denote the set of all probability distributions over a set $\mathcal{Z}$ and use $[K]$ as a shorthand for $\{1, \cdots, K\}$.

3.1 Multi-Objective Bayesian Optimization

The goal of MOBO is to design an algorithm that sequentially takes samples from the input domain $\mathbb{X} \subset \mathbb{R}^d$ to jointly optimize a black-box vector-valued function $\boldsymbol{f} : \mathbb{X} \to \mathbb{R}^K$, under a sampling budget $T \in \mathbb{N}$. For ease of exposition, we also write $\boldsymbol{f}(x) := (f_1(x), \cdots, f_K(x))$ as the tuple of the K scalar objective functions, for each $x \in \mathbb{X}$. At each step t, the algorithm selects a sample point $x_t \in \mathbb{X}$ and observes the corresponding function values $\boldsymbol{y}_t := (y_t^{(1)}, \cdots, y_t^{(K)})$, where $y_t^{(i)} = f_i(x_t) + \varepsilon_{t,i}$ is the noisy observation of the i-th entry of the function output and $\varepsilon_{t,i}$'s are i.i.d. zero-mean Gaussian noises. For notational convenience, we use $\mathcal{F}_t := \{(x_i, \boldsymbol{y}_i)\}_{i \in [t]}$ to denote the observations up to t.

Pareto Front and Hypervolume: To construct a (partial) ordering over the points of the input domain, we say that $\boldsymbol{f}(x)$ *dominates* $\boldsymbol{f}(x')$ if $f_i(x) \geq f_i(x')$ for all $i \in [K]$ and $f_j(x) > f_j(x')$ for at least one element j. For simplicity, we write $x \succ x'$ if $\boldsymbol{f}(x)$ dominates $\boldsymbol{f}(x')$. Based on this, the *Pareto front* (denoted by $\mathcal{X}^*$) is defined as the subset of $\mathbb{X}$ that cannot be dominated by any other point in $\mathbb{X}$, i.e., $\mathcal{X}^* := \{x \in \mathbb{X} | x' \not\succ x, \forall x' \in \mathbb{X}\}$. An alternative description of the goal of MOBO is to discover the Pareto front. Accordingly, MOBO algorithms are typically evaluated from the perspective of *hypervolume*, which offers a natural performance metric for capturing the inherent trade-off among different objective functions. Specifically, given a reference point $\boldsymbol{u} \in \mathbb{R}^K$ and any subset $\mathcal{X} \subseteq \mathbb{X}$, the hypervolume of $\mathcal{X}$ is defined as (Zitzler & Thiele, 1999):

$$\mathrm{HV}(\mathcal{X}; \boldsymbol{u}) := \lambda\Bigg(\bigcup_{\boldsymbol{y} \in \mathbb{R}^K} \Big\{ x' \big| \boldsymbol{f}(x) \succ \boldsymbol{y} \succ \boldsymbol{u}, x \in \mathcal{X} \Big\} \Bigg),$$

where $\lambda(\cdot)$ is the K-dimensional Lebesgue measureand x' are those elements satisfy $\boldsymbol{f}(x) \succ \boldsymbol{y} \succ \boldsymbol{u}, x \in \mathcal{X}$. In practice, the reference point can be configured as $\boldsymbol{u} = (\min_{x \in \mathbb{X}} f_1(x), \cdots, \min_{x \in \mathbb{X}} f_k(x))$. To evaluate a policy, we consider the *simple regret* defined as $\mathcal{R}(t) := \mathrm{HV}(\mathbb{X}) - \mathrm{HV}(\mathcal{X}_t)$, which measures the overall performance of the samples up to time step t. For brevity, we simply use $\mathrm{HV}(\mathcal{X})$ as a shorthand for $\mathrm{HV}(\mathcal{X}; \boldsymbol{u})$ in the sequel.

Gaussian Process as a Surrogate Model: To maximize hypervolume in a sample-efficient manner, MOBO imposes a function prior through GP, which serves as a surrogate probabilistic model for capturing the underlying structure of the objective functions. Specifically, as a Bayesian approach, the GP assumes that for each objective function $f_i(\cdot)$, the function values at any set of input points form a multivariate Gaussian distribution, which can be fully characterized by a mean function and a covariance kernel. Therefore, under a GP prior, given the observations $\mathcal{F}_t$ up to time t, the posterior predictive distribution of each $f_i(x)$ ($x \in \mathbb{X}$) remains Gaussian and can be written as $\mathcal{N}(\mu_t^{(i)}(x), \sigma_t^{(i)}(x)^2)$, where $\mu_t^{(i)}(x) := \mathbb{E}[f_i(x) | \mathcal{F}_t]$ and $\sigma_t^{(i)}(x) := \sqrt{\mathbb{V}[f_i(x) | \mathcal{F}_t]}$ can be derived in closed form through matrix operations (Williams & Rasmussen, 2006). For notational convenience, we let $\boldsymbol{\mu}_t(x) := (\mu_t^{(1)}(x), \cdots, \mu_t^{(K)}(x))$ and $\boldsymbol{\sigma}_t(x) := (\sigma_t^{(1)}(x), \cdots, \sigma_t^{(K)}(x))$.

Acquisition Functions: With the help of GPs, one natural way to address BO is through planning as in optimal control (e.g., via dynamic programming). However, finding the exact optimal policy for BO (either single- or multi-objective) is known to be computationally intractable in general due to the curse of dimensionality. To tackle this issue, BO resorts to *index-type* strategies induced by an acquisition function $\Upsilon(\boldsymbol{\mu}_t(x), \boldsymbol{\sigma}_t(x))$, which takes the posterior mean and variance as input and outputs an indicator for quantifying the usefulness of each potential candidate sample point $x \in \mathbb{X}$, typically based on some handcrafted design criteria. For example, the celebrated Expected Hypervolume Improvement (EHVI) method constructs an AF as $\Upsilon_{\text{EHVI}}(\boldsymbol{\mu}_t(x), \boldsymbol{\sigma}_t(x)) := \mathbb{E}[\mathrm{HV}(\mathcal{X}_t \cup \{x\}) - \mathrm{HV}(\mathcal{X}_t) | \mathcal{F}_t]$, which involves a multi-dimensional integral with respect to the posterior distribution characterized by $\boldsymbol{\mu}_t(x)$ and $\boldsymbol{\sigma}_t(x)$.

3.2 General RL in Non-Markovian Environments

To achieve RL without Markovianity, several generalizations of the standard Markov decision process (MDP) have been proposed, such as the classic partially-observable MDPs (Monahan, 1982; Jaakkola et al., 1994), the early works on general RL (Lattimore et al., 2013; Leike, 2016; Majeed, 2021), and the more recent attempts on RL for arbitrary environments (Dong et al., 2022; Lu et al., 2023; Bowling et al., 2023). In this paper, we consider the general RL formulation in (Dong et al., 2022; Lu et al., 2023) to address policy learning beyond Markovianity.

Environment: The general interaction protocol of the agent and the environment can be described as follows. Let $\mathcal{A}$ and $\mathcal{O}$ denote the set of actions and observations, respectively. At each time $t \in \mathbb{N}$, the agent first receives a new observation $O_t \in \mathcal{O}$ from the environment and takes an action $A_t \in \mathcal{A}$ based on the history $H_t := (A_0, O_1, A_1, \cdots, A_{t-1}, O_t)$ observed so far. For simplicity, we let the initial history H_0 be empty. We also define the set of all n-step histories as $\mathcal{H}^{(n)} := (\mathcal{A} \times \mathcal{O})^n$ and accordingly define the set of all finite histories as $\mathcal{H} := \bigcup_{n \geq 0} \mathcal{H}^{(n)}$. The transition dynamics of the environment is captured by the *transition function* $p : \mathcal{H} \times \mathcal{A} \to \Delta(\mathcal{O})$, which determines the transition probability $p(o|h, a) \equiv \mathbb{P}(O_{t+1} = o | H_t = h, A_t = a)$ of observing o upon applying action a under history h. Moreover, let $r : \mathcal{H} \times \mathcal{A} \times \mathcal{O} \to [-r_{\max}, r_{\max}]$ denote the reward function.

Notably, the reward function r in non-Markovian environments is allowed to be history-dependent and hence better suits the MOBO problems. Let $\gamma \in [0, 1)$ denote the discount factor for the rewards.

Policies and Value Functions: The agent specifies its strategy through a *policy* $\pi : \mathcal{H} \to \Delta(\mathcal{A})$, which maps each history to a probability distribution over the action set. Let Π denote the set of all policies. Similar to the MDP setting, we define value functions that reflect the long-term benefit of following a policy π. Given a τ-step history $h \in \mathcal{H}$,

$$V^\pi(h) := \mathbb{E}_\pi\Big[\sum_{t=\tau}^{\infty} \gamma^{t-\tau} r(H_t, A_t, O_{t+1})\Big| H_\tau = h\Big],$$

$$Q^\pi(h, a) := \mathbb{E}_\pi\Big[\sum_{t=\tau}^{\infty} \gamma^{t-\tau} r(H_t, A_t, O_{t+1})\Big| H_\tau = h, A_\tau = a\Big].$$

Moreover, we extend the definitions of the optimal value functions in MDPs to the non-Markovian setting as

$$V^*(h) := \sup_{\pi\in\Pi} V^\pi(h),\ Q^*(h, a) := \sup_{\pi\in\Pi} Q^\pi(h, a). \tag{1}$$

The proposition below offers a generalized version of the Bellman optimality equations and characterizes V^* and Q^*.

Proposition 3.1 (Dong et al. (2022))**.** *The pair of* (V^*, Q^*) *is the unique solution to the following system of equations:*

$$V(h) = \max_{a'\in\mathcal{A}} Q(h, a') \tag{2}$$

$$Q(h, a) = \mathbb{E}_{o\sim p(\cdot|h,a), h'\equiv(h,a,o)}\big[r(h, a, o) + \gamma V(h')\big], \tag{3}$$

where $V : \mathcal{H} \to \mathbb{R}$ *and* $Q : \mathcal{H} \times \mathcal{A} \to \mathbb{R}$ *are bounded real-valued functions.*

4 Methodology

4.1 Generalized DQN for Non-Markovian Problems

Motivated by the optimality equations in (2)-(3), we convert these fundamental properties into a learning algorithm.

Loss Function: To learn Q^*, we adapt the loss function of the standard DQN to the generalized non-Markovian version by minimizing the residual of the optimality equation. Let $Q_\theta(h, a)$ denote the parameterized Q-function. Then, the loss function of the generalized DQN is designed as

$$\mathbb{E}_{(h,a,o)\sim\mathcal{D}}\left[\Big(r(h, a, o) + \gamma \max_{a'\in\mathcal{A}} Q_{\bar\theta}(h', a') - Q_\theta(h, a)\Big)^2\right], \tag{4}$$

where $\mathcal{D}$ is the underlying distribution of the observed histories during training, $h' = (h, a, o)$ is the history for the next Q-value, and $Q_{\bar\theta}$ is a copy of Q_θ with parameters frozen.

Remark 4.1. The above loss function bears some resemblance to that of the POMDP variant of DQN, such as Deep Recurrent Q-Networks (DRQN) in (Hausknecht & Stone, 2015). Despite this, one fundamental difference is: The POMDP formulation presumes that there exists a hidden true state, which determines the transitions and the reward function, and the hidden state is to be learned and deciphered by recurrent neural networks in DRQN. By contrast, Generalized DQN does not make this presumption and involves only the history of observations and actions.

Direct Implementation of Generalized DQN: To implement (4), one natural design is to leverage sequence modeling (e.g., Transformers) and directly use the full observations as the input of the sequence models. This design principle is widely adopted in Transformer-based RL (Chen et al., 2021; Janner et al., 2021; Chebotar et al., 2023) for various popular RL benchmark tasks (e.g., locomotion and robot arm manipulation in MuJoCo Todorov et al. (2012)). In the context of learning AFs for MOBO, one can apply this design principle and extend the representation design of AF for SOBO (cf. Figure 1) to the MOBO setting, and this amounts to taking the posterior distributions of all K objective functions at all the domain points along with $\{y_t^{(i)*} := \mathrm{argmax}_{j\le t-1}\, y_j^{(i)}\}_{i=i}^K$ the best function values

observed so far as the per-step observation, i.e., $o \equiv \{\mu^{(i)}(x), \sigma^{(i)}(x), y^{(i)*}\}_{x\in\mathcal{X}, i\in[K]}$. While being a natural variant of Transformer-based RL, this implementation of the Generalized DQN framework can be problematic in MOBO for two reasons: (i) *Limited cross-domain transferability*: As the observation representation is domain-dependent under this design, the learned model is tied closely to the training domain and has very limited transferability. As a result, retraining or customization is needed for every task at deployment. (ii) *Scalability issue in sequence length and memory requirement:* Under this design, the sequence length would grow linearly with the number of domain points and pose a stringent requirement on the hardware memory for training. Indeed, the domain size is at least on the order of thousands in practical BO problems (e.g., circuit design (Lyu et al., 2018) and hyperparameter optimization (Lindauer et al., 2022)).

To tackle the above issues, we propose an alternative design that better substantiates the Generalized DQN framework for MOBO with domain-agnostic representations and several practical enhancements, as detailed in Section 4.2.

4.2 BOFORMER: AN ENHANCED IMPLEMENTATION OF GENERALIZED DQN

To avoid the issues of the direct implementation, we propose BOFormer, which is built on the following enhancements. The pseudo code is in Algorithm 1 in the Appendix.

Q-Augmented Representation: Define

$$y_t^{(i)*} := \max_{1\le j\le t} y_i^{(i)}, \forall i \in [1, \cdots, K]$$

as the best observed function value of i-th objective at time t. Moreover, for each domain point $x \in \mathbb{X}$, let $o_t(x)$ denote the observation for x as

$$o_t(x) \equiv \left\{\mu_t^{(i)}(x), \sigma_t^{(i)}(x), y_t^{(i)*}, \frac{t}{T}\right\}_{i\in[K]}.$$

Moreover, in BOFormer, we use the normalized hypervolume improvement as the reward, i.e.,

$$r_t := \frac{\mathrm{HV}(\mathcal{X}_t) - \mathrm{HV}(\mathcal{X}_{t-1})}{\mathrm{HV}(\mathcal{X}^*) - \mathrm{HV}(\mathcal{X}_t)}.$$

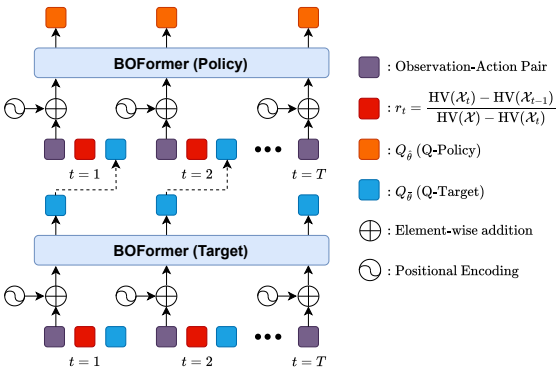


Figure 2: BOFormer comprises two distinct networks as shown above: The upper network functions as the policy network, utilizing the historical data and the Q-value predicted by the target network to estimate the Q-values for action selection. The lower network serves as the target network, responsible for constructing Q-values for past observation-action pairs.

Then, h_t, the history up to time t, is the concatenation of past observation-action pair representation defined as follows:

$$h_t = \left\{\mu_j^{(i)}(x_j), \sigma_j^{(i)}(x_j), y_{j-1}^{(i)*}, j/t, r_i, Q_{\bar{\theta}}\right\}_{i\in[k], j\in[t-1]}. \tag{5}$$

Notably, under this design, the representation is domain-agnostic and memory-efficient in the sense that its dimension does not increase with the domain size.

BOFormer as an Acquisition Function for MOBO: The model structure of BOFormer is provided in Figure 2. Denote $Q_\theta(\cdot) : \mathcal{H}^{(t-1)} \times \mathcal{O} \to \mathbb{R}$ to be the function of BOFormer parameterized by θ and let $\widehat{\theta}$ represent the parameters of BOFormer. The selected point x_t satisfies that $x_t := \mathrm{argmax}_{x\in\mathbb{X}} Q_{\widehat{\theta}}(h_t, o_t(x))$.

Then, $Q_{\bar{\theta}}$ considered in h_t can be implemented by a target network, as is commonly done in Deep Q-learning. In non-Markovian version, $Q_{\bar{\theta}}$ can be defined recursively, where

$$Q_{\bar{\theta}}(h_t, o_t(x_t)) := Q_{\bar{\theta}}\left(\{o_j(x_j), r_j, Q_{\bar{\theta}}(h_{j-1}, o_{j-1}(x_{j-1}))\}_{j=1}^{t-1}, o_t(x_t)\right).$$

Off-Policy Learning and Prioritized Trajectory Replay Buffer: We extend the concept of Prioritized Experience Replay (PER) Schaul et al. (2015) and introduce the Prioritized Trajectory Replay

Buffer (PTRB). The detailed modifications are as follows: (i) Elements pushed into this buffer are entire trajectories $\tau = \{o_i(x_i), r_i\}_{i=1}^{T}$. (ii) The TD-error considered in PER is replaced by $\delta(Q_{\theta_t}, \tau)$, which is the summation of the TD-error of the policy network for all transitions in this trajectory, i.e.,

$$\delta(Q, \tau) := \sum_{i=1}^{T-1} \left(Q\left(h_i, o_i(x_i)\right) - \left(r_i + \gamma \max_{x \in \mathbb{X}} Q_{\bar{\theta}}(h_{i+1}, o_{i+1}(x)) \right) \right)^2. \tag{6}$$

Let B denote the batch sampled from PTRB. The loss function of BOFormer is defined as $L(\theta) := \sum_{\tau \in B} \delta(Q_\theta, \tau)$.

5 Experiments

In this section, we evaluate the proposed BOFormer against popular MOBO methods on both synthetic and hyperparameter optimization on 3D Gaussian Splatting (3DGS) (Kerbl et al., 2023). Unless stated otherwise, we report the average attained hypervolume at the final step over 100 evaluation episodes in the main text. Due to the space limit, all the statistics, including the hypervolume at each step and performance profiles (Agarwal et al., 2021) of hypervolume at final step are provided in the Appendix.

Benchmark Methods. We compare BOFormer with various classic and state-of-the-art benchmark methods, including: (i) *Rule-based methods*: NEHVI (Daulton et al., 2021), ParEGO (Knowles, 2006), NSGA-II (Deb et al., 2002), HVKG (Daulton et al., 2023), and JES (Tu et al., 2022; Hvarfner et al., 2022). Regarding NEHVI, ParEGO, and HVKG, we use the differentiable Monte-Carlo version, namely qNEHVI, qParEGO, and qHVKG, provided by BoTorch (Balandat et al., 2020). (ii) *Learning-based methods*: Given that BOFormer is the first learning-based MOBO method, we consider the direct multi-objective extension of FSAF (Hsieh et al., 2021), which achieves state-of-the-art results in SOBO. To showcase the design of BOFormer, we also adapt a popular RL Transformers, namely Decision Transformer (DT) (Chen et al., 2021), to the MOBO setting. Moreover, we also include Q-Transformer (QT) (Chebotar et al., 2023), a more recent Transformer design RL that uses a similar DQN loss (termed Autoregressive Discrete Q-Learning in their paper) and can be viewed as a variant of BOFormer without Q-augmented representation. To further demonstrate the competitiveness of BOFormer, we also compare it with OptFormer (Chen et al., 2022), a recent Transformer-based method designed specifically for hyperparameter optimization. All the learning-based methods are trained on GP functions under with the lengthscales drawn randomly from $[0.1, 0.4]$ for fairness. The detailed configuration is provided in Appendix A.

Q: Does BOFormer achieve sample-efficient MOBO on a variety of optimization problems?

Synthetic Functions: We answer this question by first evaluating BOFormer extensively on a diverse collection of synthetic black-box functions: (i) Combinations of functions with many local optima, including *Ackley-Rastrigin (ARa)*; (ii) Combination of smooth functions, including *Branin-Currin (BC) and Branin-Currin-Dixon*; (iii) Combination of non-smooth and smooth functions, including *Ackley-Rosenbrock (AR) and Dixon-Rastrigin (DRa)* Table 1 and Figure 5 shows the averaged hypervolume on synthetic problems. Based on Figure 5, we can observe that BOFormer constantly achieves the largest or among the largest hypervolume among all methods. In the cases of Branin and Currin, the performance of BOFormer is not as expected, which we attribute to the larger length scales of these functions compared to others. Then, QT, a variant of the generalized DQN but without Q-augmented representation, does not perform well. This indicates that sequence modeling itself does not necessarily guarantee an efficient search for the Pareto front, and Q-augmented representation are needed for solving MOBO, as showcased by the proposed BOFormer. Please see Appendix D for the performance profiles of hypervolume at final step.

Multi-Objective Hyperparameter Optimization on 3D Gaussian Splatting: We also evaluate our method on 3D object reconstruction problems. This task takes multiple images, which captures an object, as input, and the goal is to reconstruct a 3D representation for novel-view rendering. 3D Gaussian Splatting (3DGS) by Kerbl et al. (2023) is the recent state-of-the-art on this task. One practical issue of 3DGS is their sensitivity to the hyperparameters. To achieve the best quality, we typically require manual tuning for each capture, which is time consuming and requires expert knowledge. An automatically hyperparameters tuning pipeline thus become very useful. To evaluate on this task, we construct a dataset by consulting domain expert and dense-grid searching 1440

hyperparameters of 3DGS. We perform 30 samples for this task where we defer the details in the supplementary. The objectives are negative model size and novel-view rendering quality measured by PSNR. We use 4 different objects from Mildenhall et al. (2020) and 64 different chairs from Yu et al. (2023) to compare the performance of different hyperparameter tuning methods. Again, from Table 2 Figure 6, we can observe that BOFormer remains the best or among the best in all the tasks. This result demonstrates the wide applicability of the proposed BOFormer. The detailed hypervolume per step and performance profile of the final hypervolume for 3DGS dataset is provided in Appendix D.

Table 1: The average attained hypervolume at the 100-th sampling step under synthetic functions. Boldface and underlining denote performance within 1% of the best-performing method.

	AR	ARa	BC	DRa	RBF	Matern	BCD
Rule-based Methods							
qHVKG (Daulton et al. (2023))	0.5787	0.7008	**0.4751**	**0.9499**	0.8646	0.8506	0.3547
NSGA-II (Deb et al. (2002))	0.5557	0.7122	0.4271	**0.9573**	0.8603	0.8569	0.3400
qNEHVI (Daulton et al. (2021))	0.5428	0.6290	**0.4773**	0.9333	**0.8731**	**0.8696**	**0.3697**
JES (Tu et al. (2022))	0.4930	0.6207	0.4487	0.9392	0.8661	0.8594	0.3345
qParEGO (Knowles (2006))	0.5410	0.6111	0.4597	0.9347	**0.8730**	**0.8684**	0.3564
Learning-based Methods							
BOFormer (Ours)	**0.5900**	**0.7377**	0.4476	0.9461	**0.8751**	**0.8642**	0.3617
FSAF (Hsieh et al. (2021))	0.4424	0.4800	0.4175	0.9193	0.8381	0.8566	0.3578
OptFormer (Chen et al. (2022))	0.4448	0.4835	0.4143	0.8927	0.8558	0.8488	0.3437
DT (Chen et al. (2021))	0.4397	0.4855	0.4159	0.9141	0.8409	0.8359	0.2980
QT (Chebotar et al. (2023)	0.4630	0.4953	0.4086	0.8584	0.8685	0.8550	0.3320

Table 2: The average hypervolume at the 30-th step under 3DGS hyper-parameter optimization scenarios. Boldface and underlining denote performance within 1% of the best-performing method.

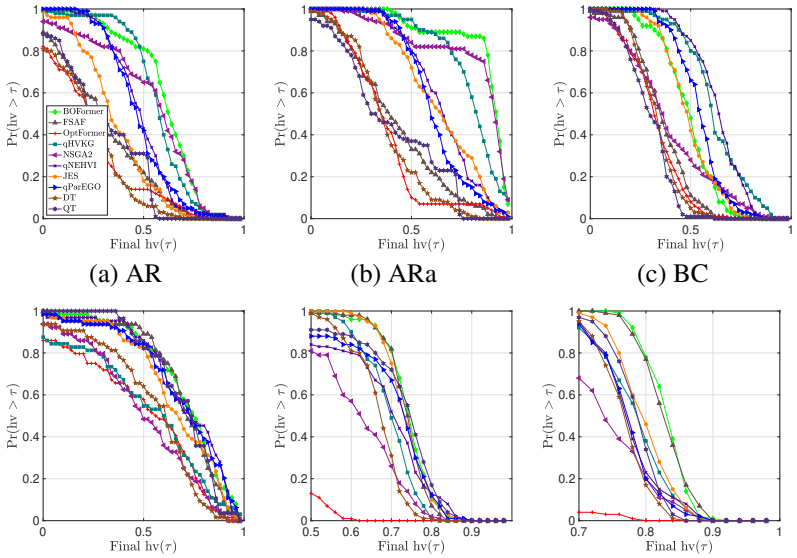
	Chairs	Lego	Materials	Mic	Ship
Rule-based Methods					
qHVKG (Daulton et al. (2023))	0.8508	0.9365	0.9088	0.8234	0.9630
NSGA-II (Deb et al. (2002))	0.8500	0.9192	0.8831	0.8051	0.9615
qNEHVI (Daulton et al. (2021))	**0.9159**	0.9344	0.9041	0.8353	**0.9661**
JES (Tu et al. (2022))	0.9003	**0.9409**	0.9217	0.8251	**0.9656**
qParEGO (Knowles (2006))	**0.9103**	0.9335	0.9098	0.8263	0.9635
Learning-based Methods					
BOFormer (Ours)	**0.9162**	**0.9508**	**0.9224**	**0.8816**	**0.9745**
FSAF (Hsieh et al. (2021))	**0.9120**	**0.9504**	**0.9213**	**0.8871**	**0.9737**
OptFormer (Chen et al. (2022))	0.8200	0.7875	0.7838	0.7865	0.9575
DT (Chen et al. (2021))	0.8602	0.9315	0.9005	**0.8786**	0.9409
QT (Chebotar et al. (2023))	0.9119	0.9368	0.9142	0.8731	0.9551

Q: The comparison between BOFormer and OptFormer. From Tables 1-2 and Figures 3-7, we observe that the performance of BOFormer surpasses that of OptFormer. We conjecture that the reasons are two-fold: (i) OptFormer takes a supervised learning perspective to learn the context that describes the HPO information while BOFormer leverages non-Markovian RL for better long-term planning. (ii) OptFormer reinterprets HPO as a language modeling problem in a text-to-text manner. This approach necessitates that the training dataset closely resembles the testing domain, and this requirement does not hold here (e.g., dimensionality of the training domain differs from that of the testing domain).

Q: A study on the effect of sequence length of BOFormer. We also conducted an ablation study on evaluating how the sequence length (denoted by w) would affect the hypervolume performance of BOFormer. Figure 4 shows that the hypervolume of non-Markovian BOFormer ($w > 1$) is superior to that of Markovian BOFormer ($w = 1$). The results also demonstrate that sequential modeling can successfully solve the hypervolume identifiability issue.

Q: A study on computational efficiency. We provide computation times per step in Table 3. BOFormer and qNEHVI are competitive in terms of final hypervolume (Tables 1-2), with the computation times of BOFormer being less sensitive to the number of objective functions and shorter than those of qNEHVI under the 3-objective function setting.

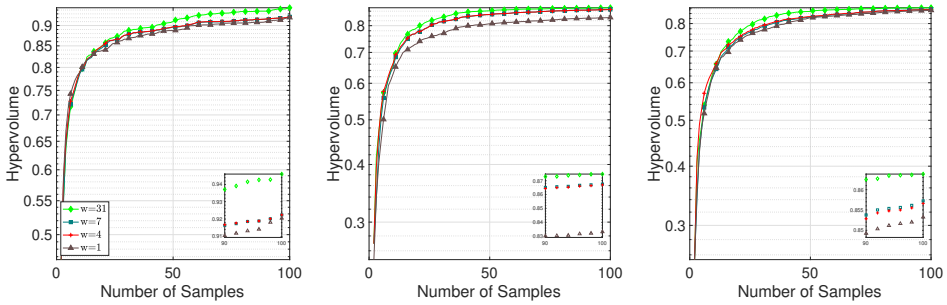
Q: How is the transfer ability of BOFormer under different numbers of objective functions? Although learning-based methods generally require training multiple models for different numbers of objective functions due to mismatched model structures, we conducted an experiment on BOFormer to determine whether it is possible to train only the state-action embedding layer while utilizing a pre-trained transformer model from a different number of objective functions. The results, shown in Figure 7, demonstrate that BOFormer exhibits excellent transfer ability across different numbers of objective functions. The detailed implementation of transferring on BOFormer is described in Appendix C.2.



(a) AR (b) ARa (c) BC

(d) Chairs (e) Materials (f) Lego

Figure 3: Performance profiles in terms of hypervolume at the final step under synthetic functions and HPO-3DGS.



(a) DRa (b) RBF (c) Matern52

Figure 4: Attained hypervolumes of BOFormer under various sequence lengths.

6 Conclusion

In this paper, we address MOBO problems from the perspective of RL-based AF by identifying and tackling the inherent hypervolume identifiability issue. We achieve this goal by first presenting a generalized DQN framework and implementing it through BOFormer, which leverages the sequence modeling capability of Transformers and incorporates multiple enhancements for MOBO. Our experimental results show that BOFormer is indeed a promising approach for general-purpose multi-objective black-box optimization.

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A Detailed Training Configuration

A.1 Environment

In each testing episode, the agent interacts with the environment for a total of $T = 100$ for synthetic functions and $T = 30$ time steps for HPO-3DGS. The observed function values are subject to noise $\epsilon \sim N(0, 0.1)$ for synthetic functions, while the function values for HPO-3DGS are observed without noise. For learning-based algorithms, there are K objective functions that are sample from GP with the kernel being randomly chosen from either the RBF or Matérn-5/2, and the length scale is sampled from Uniform$(0.1, 0.4)$. During testing and training, the objective functions of each episode are scaled by perturbation noise κ, which is sampled from Uniform$(0, 0.1)$ for synthetic functions and Uniform$(0, 0.01)$ for other functions.

For HPO-3DGS, there are about 68,000 data points related to 4 different objects: Lego, Materials, Mic, Ship, and Chairs. Additionally, there are 64 different scenes involving chairs. While testing on chairs, each episode is conducted with an individual scene.

A.2 Hyperparameter of Learning-Based Approaches

- **BOFormer:** hidden size: 128, learning rate: 10^{-5}, weight decay: 10^{-5}, $r_{\text{demo:}}$ 5, batch size: 8, # of attention layer: 8, # of head of attention layer: 4, window size $w = 31$, dropout: 0.1, buffer size: 64, training episode: 3000.
- **DT***: hidden size: 128, learning rate: 10^{-4}, weight decay: 10^{-4}, batch size: 16, # of attention layer: 3, # of head of attention layer: 2, embed dim: 500, dropout: 0.1, warmup steps: 10, max length: 100.
- **QT:** hidden size: 128, learning rate: 10^{-5}, weight decay: 10^{-5}, $r_{\text{demo:}}$ 5, batch size: 8, # of attention layer: 8, # of head of attention layer: 4, window size $w = 21$, dropout: 0.1, buffer size: 64, training episode: 2000.
- **FSAF:** alpha: 0.8, hidden size: 100, learning rate: 0.01, batch size: 128, few shot step: 5, # of particles: 5, total task: 3, size of meta data: 100, use demo: True, early terminate: False, select type: average, training episode: 300 for $K = 2$ and 500 for $K = 3$.
- **OptFormer:** string length: 128, learning rate: 10^{-2}, weight decay: 10^{-2}, batch size: 1, window size $w = 10$, training episode: 1000.
- **Common Hyperparameter:** optimizer: Adam Kingma & Ba (2014), ϵ-greedy rate: 0.1

A.3 Hyperparameter of Rule-Based Approaches

- **qNEHVI:** q(batch selection): 1
- **qHVKG:** q(batch selection): 1
- **qParEgo:** q(batch selection): 1, acquisition function: Expected Improvement
- **JES:** # of samples: 64, estimation type: LB
- **NSGA2:** population size: 10, # of generations: 10, sampling: random

B Additional Related Work

B.1 Transformers and Sequence Modeling for RL

Given the success of sequence-to-sequence models in language processing, RL has recently been addressed through the lens of sequence modeling, especially transformers. For example, Chen et al. (2021) propose Decision Transformer (DT), which addresses offline RL by mapping return-to-go to actions via transformers and thereby substantiating the concept of Upside Down RL (Schmidhuber, 2019). The design of DT has been subsequently extended to various settings, including online RL (Zheng et al., 2022), multi-game setting (Lee et al., 2022), and general information matching (Furuta et al., 2021). Concurrently, to tackle offline RL, Janner et al. (2021) propose Trajectory Transformer

*We reuse the open source implementation from https://github.com/jannerm/trajectory-transformer.

(TT), which serves as a predictive dynamics model and uses beam search as a trajectory optimizer. More recently, Chebotar et al. (2023) introduced Q-Transformer (QT), which is trained by offline temporal difference updates and achieves scalable representation for Q-functions by discretizing action dimensions as tokens on a Transformer, with a focus on robotic tasks. By contrast, the proposed BOFormer is built on the (online) generalized DQN framework and designed for addressing MOBO based on multiple enhancements.

B.2 A More Detailed Comparison to Q-Transformer

Q-Transformer (QT) Chebotar et al. (2023) similarly incorporates the idea of utilizing a Transformer in learning the Q-function. Despite this high-level design resemblance, the proposed BOFormer is different from QT in multiple aspects:

- **Problem Setting:** QT is designed mainly to address offline RL, where the performance is highly correlated to the quality of prior data, as a robotic learning approach and required to address high-dimensional action spaces. By contrast, our work introduces BOFormer, which is trained to solve MOBO by learning a generalized Q function based on the online interactions with synthetic GP functions, and hence this can be viewed as an instance of online RL.
- **Network Architecture:** QT uses state-action sequences as the input of the Transformer. By contrast, to resolve the hypervolume identifiability issue and achieve cross-domain transferability simultaneously, the proposed transformer of BOFormer implements a generalized DQN and uses the Q-augmented observation representation as the input of the transformer. This approach can better address the non-Markovian property in MOBO.
- **Training Algorithm:** Compared to QT, the proposed BOFormer incorporates multiple practical enhancements, including Q-augmented representation, reward normalization, and demo policy and prioritized trajectory replay buffer for off-policy learning.

C Pseudo Code and Additional Implementation Details of BOFormer

C.1 Pseudo Code

The detailed pseudo code of the training processes for BOFormer under off-policy learning and on-policy learning setting are provided in Algorithms 1 and 2, respectively.

C.2 Additional Implementation Details of BOFormer

Reward Signal With Normalization: In the MOBO setting, one natural reward design is the one-step improvement in hypervolume, i.e., $\hat{r}_t := \mathrm{HV}(\mathcal{X}_t) - \mathrm{HV}(\mathcal{X}_{t-1})$. However, as the achieved hypervolume increases, the reward signal $\hat{r}_t$ can get weaker in the later stage of an episode, making it difficult to recover the whole Pareto front. To address this, we construct r_t, which is $\hat{r}_t$ but scaled by the difference between the current hypervolume and optimal hypervolume, as the reward signal for RL in MOBO, i.e.,

$$r_t := \frac{\mathrm{HV}(\mathcal{X}_t) - \mathrm{HV}(\mathcal{X}_{t-1})}{\mathrm{HV}(\mathcal{X}^*) - \mathrm{HV}(\mathcal{X}_t)}. \tag{7}$$

Remark C.1. The information about $\mathrm{HV}(\mathcal{X}^*)$ in (7) is used only during training and can be easily pre-computed or approximated given the knowledge about the domain and the black-box functions.

Demo-Policy-Guided Exploration: To facilitate the off-policy learning of BOFormer, one natural approach is to adopt a behavior policy with randomized exploration (e.g., epsilon-greedy) for collecting trajectories from the environment. However, such a randomized exploration scheme can be very inefficient as the sampling budget in MOBO is usually much smaller than the domain size (i.e., the number of actions). To better guide the exploration, we propose to use a demo policy, which is possibly sub-optimal but of sufficient strength in exploring regions near the Pareto front. In practice, we set r_{demo} to be the probability of using a demo policy induced by an off-the-shelf AF for MOBO

(e.g., EHVI) in this training episode. The training processes of BOFormer for off-policy learning and on-policy learning are provided in Algorithms 1 and 2, respectively.

Transfer Learning Across Different Numbers of Objective Functions As discussed in the main text, the input dimension of BOFormer varies depending on the number of objective functions. For instance, a BOFormer model trained on 2 objective functions cannot be directly applied to 3 objective functions due to the mismatch in input dimensions, which requires reshaping of the embedding layer. To address this issue and reduce the computational burden of training BOFormer, we developed an efficient transfer learning strategy that enables model transfering across different numbers of objective functions.

Our approach first loads the pre-trained weights for the transformer component from another trained BOFormer, then initializes a linear embedding layer to match the dimension of the state-action pair. Only the linear embedding layer is trained. This strategy significantly reduces training episodes and computational requirements by allowing us to leverage pre-trained models for different numbers of objective functions with minimal adjustments.

The efficiency of this transfer learning method is evident in the reduced number of training episodes. While the original model requires approximately 3000 episodes to converge, the transfer model achieves comparable performance after only 400 episodes of fine-tuning. This substantial reduction in training episodes demonstrates the effectiveness of our approach in rapidly adapting pre-trained models to new objective function configurations.

D Detailed Experimental Results

Due to the space limit, in this section, we provide all the detailed experimental results as follows:

D.1 Detailed Hypervolume Statistics and Inference time

Due to the space limit, in this section, we further provide all the detailed statistics of the final hypervolume and the inference time average over 1 episode:

Table 3: Average inference time for one trial.

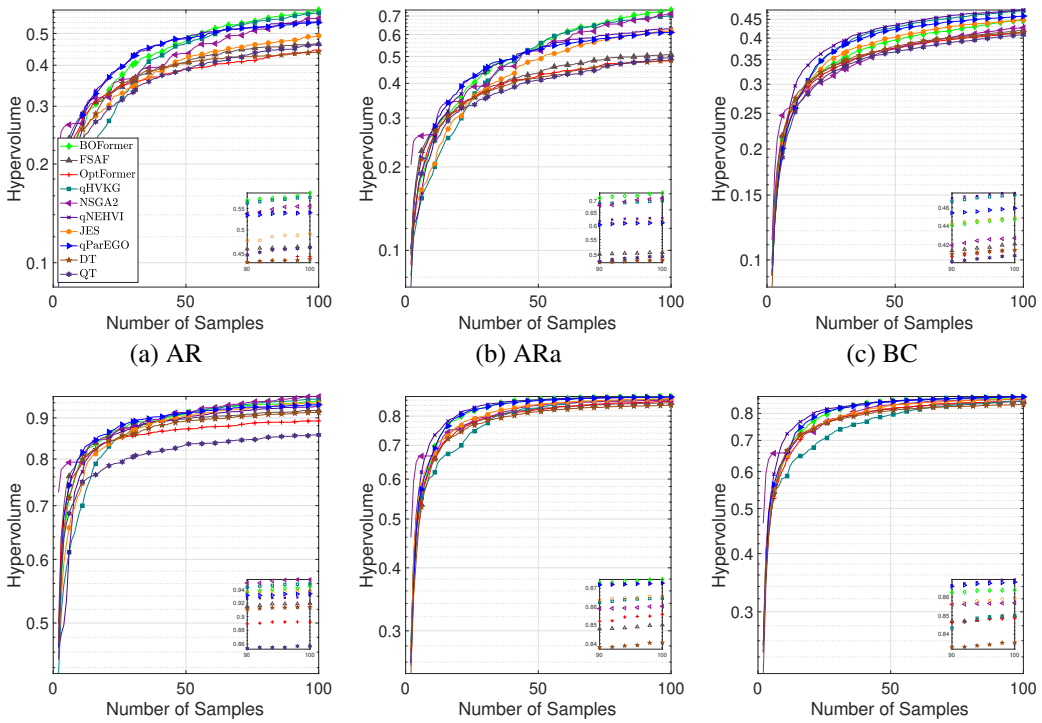
	Time(s)	
Algorithms	2 Obj.	3 Obj.
BOFormer	0.3495	0.3529
qNEHVI	0.2296	2.9048
JES	0.6124	0.7367
FSAF	0.0172	0.0173
DT	0.0054	0.0081
NSGA2	0.0002	0.0003
qParEGO	0.2159	0.2300
OptFormer	2.6013	3.8589
qHVKG	4.5816	6.5375
QT	0.1451	0.1427

Algorithm 1 Off-Policy BOFormer

Input: $\boldsymbol{\theta}_1$, Training Episodes E, Time Horizon T, Demo Rate r_{demo}, Target Rate r_{target}, Target Network $Q_{\bar{\theta}}$, Batch Size B, Replay buffer $\mathcal{B}$, environment $\mathcal{E}$.
for $e = 1, 2, \cdots, E$ **do**
  $\mathcal{E}$.reset()
  $s_1^e = \mathcal{E}$.state
  demo $=$ random.binomial(r_{demo})
  $\pi_t^e =$ ExpectedHypervolumeImprovement
  for $t = 1, 2, \cdots, T$ **do**
    if demo **then**
      $x_t^e = \pi_t^e(s_t^e)$
    else
      for $i = 1, 2, \cdots, t-1$ **do**
        Compute $Q_{\bar{\theta}}(h_i^e, o_i^e(x_i^e)) = Q_{\bar{\theta}}\left(\left\{o_j^e(x_j^e), r_j^e, Q_{\bar{\theta}}(h_{j-1}^e, o_{j-1}^e(x_{j-1}^e))\right\}_{j=1}^{i-1}, o_i^e(x_i^e)\right)$.
      end for
      Select $x_t^e := \operatorname{argmax}_{x \in \mathbb{X}} Q_{\widehat{\theta}}\left(h_t^e, o_t^e(x)\right)$
    end if
    $s_{t+1}^e, r_t^e = \mathcal{E}$.step$(x_t^e)$
  end for
  if e mod $r_{\text{target}} == 0$ **then**
    $Q_{\bar{\theta}} = Q_{\boldsymbol{\theta}_e}$
  end if
  $\mathcal{B}$.append$(\tau_e = \{o_i^e(x_i^e), r_i^e\}_{i=1}^T)$
  Sample B batches from $\mathcal{B}$.
  for $b = 1, 2, \cdots, B$ **do**
    Compute $Q_{\bar{\theta}}^b(h_1^b, o_1^b(x_1^b)) = Q_{\bar{\theta}}\left(o_1^b(x_1^b)\right)$
    for $i = 1, 2, \cdots, T-1$ **do**
      Compute $Q_{\bar{\theta}}^b(h_i^b, o_i^b(x_i^b)) = Q_{\widehat{\theta}}\left(\left\{o_i^b(x_j^b), r_j^b, Q_{\bar{\theta}}(h_{j-1}^b, o_{j-1}^b(x_{j-1}^b))\right\}_{j=1}^{i-1}, o_i^b(x_i^b)\right)$.
      Compute $Q_{\bar{\theta}}^b(h_i^b, o_i^b(x_i^b)) = Q_{\bar{\theta}}\left(\left\{o_j^b(x_j^b), r_j^b, Q_{\bar{\theta}}(h_{j-1}^b, o_{j-1}^b(x_{j-1}^b))\right\}_{j=1}^{i-1}, o_i^b(x_i^b)\right)$.
    end for
  end for
  Loss$(\theta) = \sum_{b=1}^{B} \sum_{t=1}^{T-1} \left(Q_{\widehat{\theta}}^b(h_i^b, o_i^b(x_i^b)) - (r_t^b + \max_{x \in \mathbb{X}} Q_{\bar{\theta}}^b(h_i^b, o_i^b(x)))\right)^2$
  $\boldsymbol{\theta}_{e+1} = \operatorname{argmin}_{\boldsymbol{\theta}} \text{Loss}(\theta)$
end for

Algorithm 2 On-Policy BOFormer

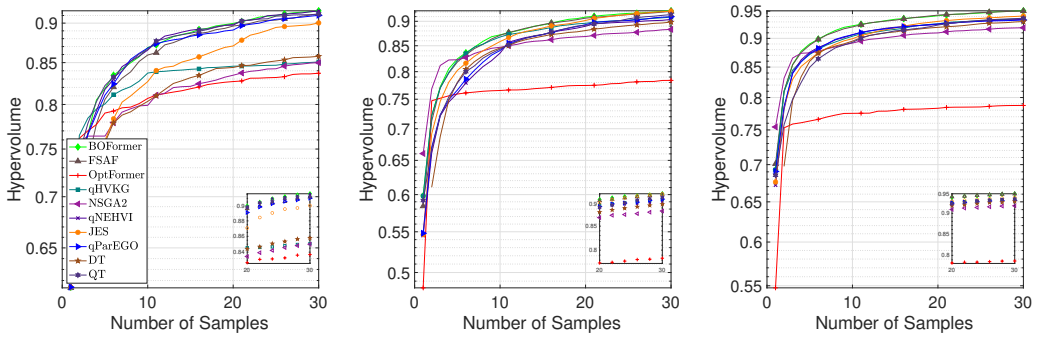
Input: $\boldsymbol{\theta}_1$, Training Episodes E, Time Horizon T, environment $\mathcal{E}$,
for $e = 1, 2, \cdots$,E **do**
  $\mathcal{E}$.reset()
  $s_1 = \mathcal{E}$.state
  for $t = 1, 2, \cdots, T$ **do**
    Select $x_t := \operatorname{argmax}_{x \in \mathbb{X}} Q_{\theta_t}\left(h_t, o_t(x)\right)$
    $o_{t+1}(x), r_t = \mathcal{E}$.step$(x_t)$
    $\theta_{t+1} = \operatorname{argmin}_{\boldsymbol{\theta}} \left(Q_\theta\left(h_t, o_t(x_t)\right) - (r_t + \gamma \max_{x \in \mathbb{X}} Q_{\boldsymbol{\theta}_t}(h_{t+1}, o_{t+1}(x)))\right)^2$
  end for
end for



(a) AR (b) ARa (c) BC

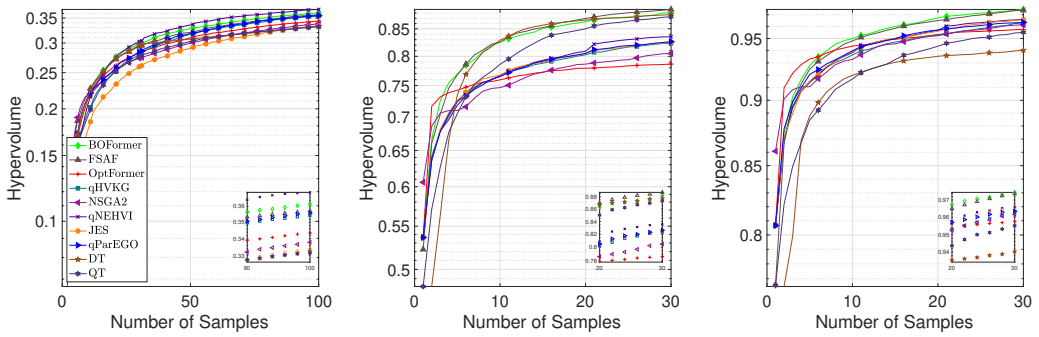
(d) DRa (e) RBF (f) Matern52

Figure 5: Averaged attained hypervolume under synthetic functions with two objectives.



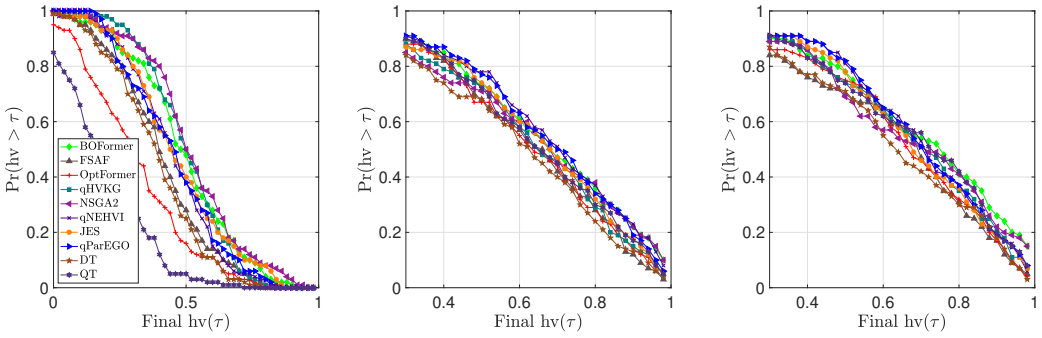
(a) Chairs (b) Materials (c) Lego

Figure 6: Averaged attained hypervolume under HPO-3DGS with two objectives.

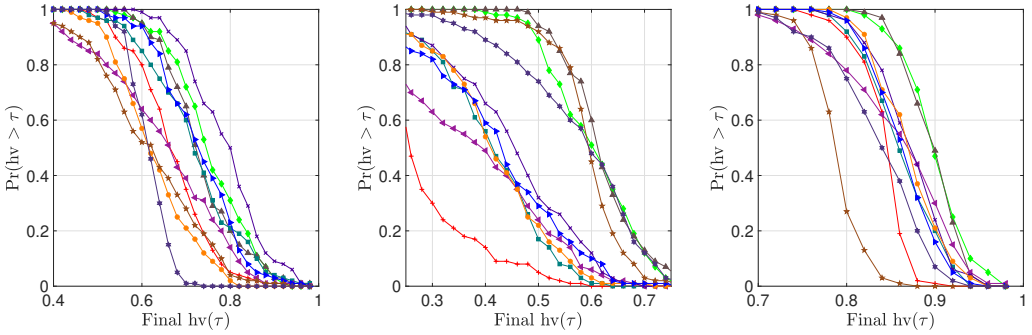


(a) BCD (b) Mic (c) Ship

Figure 7: Averaged attained hypervolume under synthetic and HPO-3DGS with three objectives.



(d) DR (e) Matern52 (f) RBF



(a) BCD (b) Mic (c) Ship

Figure 8: Performance profiles in terms of hypervolume at the final step under synthetic functions and HPO-3DGS.