MODEL ZOO: A GROWING "BRAIN" THAT LEARNS CONTINUALLY

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ABSTRACT

This paper argues that continual learning methods can benefit by splitting the capacity of the learner across multiple models. We use statistical learning theory and experimental analysis to show how multiple tasks can interact with each other in a non-trivial fashion when a single model is trained on them. The generalization error on a particular task can improve when it is trained with synergistic tasks, but can also deteriorate when trained with competing tasks. This theory motivates our method named Model Zoo which, inspired from the boosting literature, grows an ensemble of small models, each of which is trained during one episode of continual learning. We demonstrate that Model Zoo obtains large gains in accuracy on a wide variety of continual learning benchmark problems.

1 INTRODUCTION

A continual learner seeks to leverage data from past tasks to learn new tasks shown to it in the future, and in turn, leverage data from these new tasks to improve its accuracy on past tasks. It stands to reason that the performance of such a learner would depend upon the relatedness of these tasks. If the two sets of tasks are dissimilar, learning on past tasks is unlikely to benefit future tasks—it may even be detrimental. And similarly, new tasks may cause the learner to "forget" and result in deterioration of accuracy on past tasks. Our goal in this paper is to model the relatedness between tasks and develop new methods for continual learning that result in good forward-backward transfer by accounting for similarities and dissimilarities between tasks. Our contributions are as follows.

1. Theoretical analysis. We characterize when multiple tasks can be learned using a single model and, likewise, when doing so is detrimental to the accuracy of a particular task. The key technical idea here is to define a notion of relatedness between tasks. We first show how if the inputs of different tasks are "simple" transformations of each other (and likewise for the outputs), then one can learn a shared feature generator that generalizes better on every task, compared to training that task in isolation. Such tasks are strongly related to each other and therefore it is beneficial to fit a single model on all of them. We show that if tasks are not so strongly related, in particular if the optimal model for one task predicts poorly on another task, then fitting a single model on such tasks may be worse than training each task in isolation. Such tasks *compete* with each other for the fixed capacity in the single model. We also empirically study this competition using the CIFAR-100 dataset.

2. Algorithm development. The above analysis suggests that a continual learner could benefit from splitting its learning capacity across sets of synergistic tasks. We develop such a continual learner called Model Zoo. At each episode, a small multi-task model that is fitted to the current task and some of the past tasks is added to Model Zoo. This method is loosely inspired from AdaBoost in that it selects tasks that performed poorly in the past rounds and could therefore benefit the most from being trained with the current task. At inference time, given the task, we average predictions from all models in the ensemble that were trained on that task.

3. Empirical results. We comprehensively evaluate Model Zoo on existing task-incremental continual learning benchmark problems and show comparisons with existing methods. There is a wide variety in the problem settings used by existing methods, e.g., some replay data from past tasks (like Model Zoo is designed to do), some replay only a subset of data, some train only for one epoch in each episode, some use extremely small architectures, etc. We compare Model Zoo with existing methods in a number of these settings. **Model Zoo obtains better accuracy than existing methods on the evaluated benchmarks**. Improvement in average per-task accuracy is quite large in some cases, e.g., 30% for Split-miniImagenet. We also show that Model Zoo demonstrates strong forward and backward transfer.

4. A critical look at continual learning. We find that even an Isolated learner, i.e., one which trains a (small) model on tasks from each episode and does not perform any continual learning, significantly

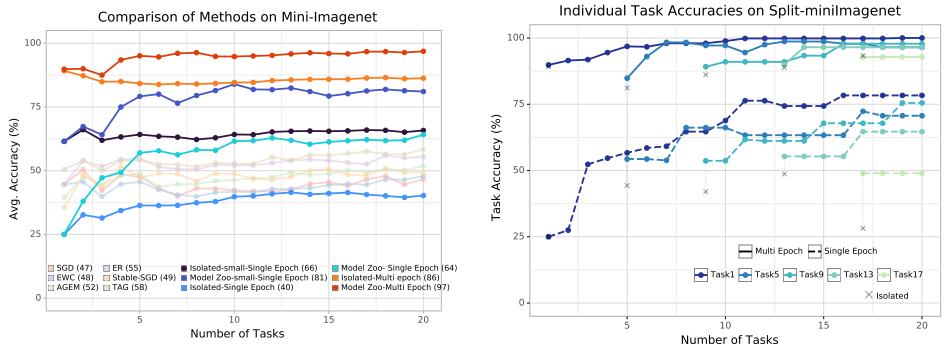

Figure 1: Left: How well do existing continual learning methods work? We track the average accuracy (over all tasks seen until the current episode) on the Split-miniImagenet dataset and compare our method Model Zoo and its variants (all in bold) to existing continual learning methods (faint lines, see Table 1 for references). All methods in this plot (unless specified otherwise) use the single epoch setting, i.e., each new task is allowed only 1 epoch of training. Isolated refers to a very simplistic realization of Model Zoo where a separate model is fitted at each episode without any continual learning, or data sharing between tasks; Isolated-small or Model Zoo-small refer to using a very small deep network with 0.12M weights. A number of surprising findings are seen here. (i) Isolated-small (black) outperforms existing methods by more than 10% margin, while having a faster training time, inference time, comparable model size and without performing any data replay. This indicates that **existing methods do not sufficiently leverage data from multiple tasks**. This also indicates the utility of simple methods like Isolated to perform a more prosaic, matter-of-fact, evaluation of continual learning. (ii) While the larger model with 3.6M weights per round, Isolated-Single Epoch (royal blue), performs poorly, its accuracy is better than all existing methods (Isolated-Multi Epoch) upon being trained for multiple epochs. This indicates that **existing methods may be severely under-trained in the single-epoch setting** and this may not be the appropriate setting to build continual learning methods; this was also noticed by Lopez-Paz and Ranzato (2017). (iii) Model Zoo and Model Zoo-small which replay all data from past tasks (A-GEM also replays 10% of the data), achieves around 10% improvement over its Isolated counterparts in both the single-epoch and multi-epoch setting; all these 4 **methods advocated in this paper are better than existing algorithms**. Even Model Zoo-single epoch which replays past data but trains on the new task only for 1 epoch outperforms existing methods significantly. This indicates that replaying data from past tasks is beneficial (Robins, 1995), even if replay may not conform to certain stylistic formulations of continual learning in the literature (Farquhar and Gal, 2019a; Kaushik et al., 2021). Not doing so significantly hurts forward and backward transfer, and average task accuracy.

Right: Does the single-epoch setting show forward-backward transfer? The evolution of individual task accuracy of Model Zoo (the multi-epoch setting in bold and single-epoch setting in dotted), on the Split-miniImagenet dataset (only 5 tasks are plotted here, see Fig. A6 for the full version). The X markers denote the accuracy of Isolated. Accuracy of tasks improves with each episode which indicates backward transfer. Also, the X markers are often below the initial accuracy of the task during continual learning, which indicates forward transfer. While both single-epoch and multi-epoch Model Zoo show good forward-backward transfer, the accuracy of tasks for the former is about 25% worse than the latter; corresponding plots for other methods are in Appendix B.7. This indicates that we should also pay attention to under-training and per-task accuracy in continual learning.

outperforms *most* existing continual learning methods on the evaluated benchmark problems, e.g., by more than 8% in Fig. 1 and Tables 1 and A1. This strong performance is surprising because it is a very simple learner that has better training/inference time, no data replay, and a comparable number of weights as that of existing methods.

2 A THEORETICAL ANALYSIS OF HOW TO LEARN FROM MULTIPLE TASKS

In this section, we (i) formulate the problem of learning from multiple tasks, (ii) discuss a simple model that highlights when training one model on multiple tasks is beneficial, and (iii) show new results on how the fixed capacity of the model causes competition between tasks.

2.1 PROBLEM FORMULATION

A supervised learning task is defined as a joint probability distribution $P(x, y)$ of inputs $x \in X$ and labels $y \in Y$. The learner has access to m i.i.d samples $S = \{x_i, y_i\}_{i=1,\ldots,m}$ from the task. A hypothesis is a function $h : X \to Y$ with $h \in H$ being the hypothesis space. The learner may select a hypothesis that minimizes the empirical risk $\hat{e}_S(h) = \frac{1}{m}\sum_{i=1}^{m} \mathbf{1}_{\{h(x_i)\neq y_i\}}$ with the hope of achieving a small population risk $e_P(h) = \mathbb{P}(h(x) \neq y)$. Classical PAC-learning results (Vapnik, 1998) suggest that with probability at least $1-\delta$ over draws of the data S, uniformly for any $h \in H$, we have $e_P(h) \leq \hat{e}_S(h) + \epsilon$ if

$$m = \mathcal{O}\big((D - \log \delta)/\epsilon^2\big) \tag{1}$$

where $D = \text{VC}(H)$ is the VC-dimension of the hypothesis space H. We define the "excess risk" of a hypothesis as $\mathcal{E}_P(h) = e_P(h) - \inf_{h \in H} e_P(h)$. In the continual learning setting, a new task is shown to the learner at each episode (or round). Hence after n episodes, the learner is presented with n tasks $\bar{P} := (P_1, \ldots, P_n)$, with the corresponding training sets $\bar{S} := (S_1, \ldots, S_n)$, each with m samples, and the learner selects n hypotheses $\bar{h} = (h_1, \ldots, h_n) \in H^n$, each $h_i \in H$. If it seeks a small average population risk $e_{\bar{P}}(\bar{h}) = \frac{1}{n}\sum_{i=1}^n e_{P_i}(h_i)$, it may do so by minimizing the average empirical risk $\hat{e}_{\bar{S}}(\bar{h}) = \frac{1}{n}\sum_{i=1}^n \hat{e}_{S_i}(h_i)$. As Baxter (2000) shows, under very general conditions, if

$$m = \mathcal{O}\big(\epsilon^{-2}\,(d_H(n) - 1/n \log \delta)\big), \tag{2}$$

then we have $e_{\bar{P}}(\bar{h}) \leq e_{\bar{S}}(\bar{h}) + \epsilon$ for any $\bar{h} \in H^n$. The quantity $d_H(n)$ here is a generalized VC-dimension for the family of hypothesis spaces H^n, which depends on the joint distribution of tasks. Larger the number of tasks n, smaller the $d_H(n)$ (Ben-David and Borbely, 2008). Whether (2) is an improvement upon training the task in isolation as in (1) depends upon the hypothesis class H and the relatedness of tasks $P_1, \ldots, P_n$ through the quantity $d_H(n)$. The most important thing to note here is that according to these calculations, if one wishes to obtain a small *average* population risk across tasks, training multiple tasks together cannot be worse: $d_H(n) \leq \text{VC}(H)$.

2.2 Controlling the excess risk of a specific task for synergistic tasks

An important goal of continual learning is to have low risk on *all tasks*. This is a stronger requirement than for (2) which bounds the *average* population risk on all tasks.

Suppose there exists a family F of functions $f_i : X \to X$ that map the inputs of one task to those of another, i.e., any task can be written as

$$P_j(A) = f[P_i](A) = \mathbb{P}_i(\{(f(x), y) : (x, y) \in A\})$$

for some function $f \in F$ for any set A. We can assume without loss of generality that F acts as a group over the hypothesis space and H is closed under its action. In simple words, this entails that given $h \in H$ suitable for task P, we can obtain a new hypothesis $h \circ f$ that is suitable for another task $f[P]$. Instead of searching over the entire space H^n like in §2.1, we now only need to find a hypothesis $h \in H$ such that its orbit

$$[h]_F = \{h' : \exists f \in F \text{ with } h' = h \circ f\}$$

contains hypotheses that have low empirical risk on each of the n tasks. Conceptually, this step learns the inductive bias (Baxter, 2000; Thrun and Pratt, 2012). The sample complexity of doing so is exactly (2). From within this orbit, we can select a hypothesis that has low empirical risk for a chosen task P_1. The sample complexity of this second step is

$$|S_1| = \mathcal{O}\big(\epsilon^{-2}\,(d_{\max} - \log \delta)\big) \tag{3}$$

where $d_{\max} = \sup_{h \in H} \text{VC}([h]_F)$. By uniform convergence, as Ben-David and Schuller (2003) show, this two-step procedure assures low excess risk for *every* task $P_1, \ldots, P_n$. We have

$$\sup_{h \in H} \text{VC}([h]_F) = d_{\max} \leq d_H(n+1) \leq d_H(n) \leq D = \text{VC}(H). \tag{4}$$

The total sample complexity is favorable to that of learning the task in isolation if both $d_H(n)$ and $d_{\max}$ are small. For instance, if F is finite and $n/\log n \geq D$, we have $d_H(n) \leq 2 \log |F|$ which indicates that we get a statistical benefit of learning with multiple tasks if $D \gg \log |F|$.

Remark 1 (Data from other tasks may not improve accuracy even if they are synergistic). Let us make a few observations using the above analysis. (i) From (4), number of samples per task m decreases with n; this is the benefit of the strong relatedness among tasks and as we see next, this is *not* the case in general. (ii) The number of tasks scales essentially linearly with D, which indicates that one should use a small model if we have few tasks. (iii) But we cannot always use a small model. If tasks are diverse and related by complex transformations with a large $|F|$, we need a large hypothesis space to learn them together. If $|F|$ is large and H is not appropriately so, the VC-dimension $d_{\max}$ is as large as D itself; in this case there is *again no statistical benefit* of training with multiple tasks together, but there is no deterioration either.

2.3 Task competition occurs for hypothesis spaces with limited capacity

There could be settings under which fitting one model on multiple tasks may not suffice. To study this, we consider a weaker notion of relatedness. We say that two tasks P_i, P_j are ρ_{ij}-related if

$$c\,\mathcal{E}_{P_i}^{1/\rho_{ij}}(h) \geq \mathcal{E}_{P_j}(h, h_i^*), \text{ for all } h \in H. \tag{5}$$

Here $\mathcal{E}_P(h, h') := e_P(h) - e_P(h')$ and $h_i^* = \text{argmin}_{h \in H}\, e_{P_i}(h)$ is the best hypothesis for task P_i; we set $c \geq 1$ to be a coefficient independent of i, j. Smaller the ρ_{ij}, more useful the samples from P_i to learn P_j. The definition suggests that all hypotheses h which have low excess risk on P_i also have low excess risk on P_j up to an additive term $e_{P_j}(h^*)$ and this effect becomes stronger as $\rho_{ij} \to 1_+$. Note that the definition of relatedness is not symmetric. Hanneke and Kpotufe (2020) call

this the transfer exponent. To gain some intuition, we can connect this definition to a certain triangle inequality between the tasks developed by Crammer et al. (2008): in the realizable setting where $e_{P_i}(h_i^*) = 0$, for $c, \rho_{ij} = 1$, we can write (5) as

$$e_{P_i}(h) + e_{P_j}(h_i^*) \geq e_{P_j}(h)$$

which is akin to a triangle with vertices at h, h_i^* and h_j^* with terms like $e_{P_i}(h)$ representing the length of the side between h and h_i^*. This definition therefore models a set of tasks and hypothesis space that is not unduly pathological, $e_{P_j}(h)$ cannot be much worse than the sum of the other two sides. We can now show the following theorem bounds the excess risk $\mathcal{E}_{P_1}(h)$ for a hypothesis h trained using data from multiple tasks. See Appendix C for the proof.

Theorem 2 (Task competition). Say we wish to find a good hypothesis for task P_1 and have access to n tasks $P_1, \ldots, P_n$ where each pair P_i, P_j are ρ_{ij}-related. Arrange tasks in an increasing order of ρ_{i1}, i.e., their relatedness to P_1. Let this ordering be $P_{(1)}, P_{(2)}, \ldots, P_{(n)}$ with $\rho_{(1)} \leq \rho_{(2)} \leq \ldots \leq \rho_{(n)}$ and $P_{(1)} \equiv P_1$ and $\rho_{(1)} = 1$. Let $\hat{h}^k$ be the hypothesis that minimizes the average empirical risk of the first $k \leq n$ tasks. Then, with probability at least $1 - \delta$ over draws of the training data,

$$\mathcal{E}_{P_1}(\hat{h}^k) \leq \frac{1}{k}\sum_{i=1}^{k} \mathcal{E}_{P_1}(h_{(i)}^*) + \frac{c}{k}\left(e_{\bar{S}}(h) + c'\left(\frac{D - \log\delta}{km}\right)^{1/2}\right)^{1/\rho_{\max}} \tag{6}$$

where $\rho_{\max}(k) = \max\left\{\rho_{(1)}, \ldots, \rho_{(k)}\right\}$ and c, c' are constants.

Notice that the first term grows with the number of tasks k because we pick tasks with lower ρ_{i1} that are more and more dissimilar to P_1. The second term typically decreases with k. The empirical risk $e_{\bar{S}}(h)$ is typically small; in our experiments with deep networks we achieve essentially zero training error on all. Increasing the number of tasks k, increases the effective number of samples km, thereby reducing the second term in totality. At the same time, these new samples are increasingly more inefficient because $\rho_{\max}(k)$ increases with k.

Remark 3 (Picking the size of the hypothesis space). The first and second terms characterize synergies and competition between tasks and balancing them is the key to good performance on a given task. Increasing the size of the hypothesis space reduces the first term since it allows a single hypothesis to more easily agree on two distinct distributions P_i and P_j. However, this comes at the cost of increasing the second term which grows with the size of the hypothesis space.

Remark 4 (The set of synergistic tasks can be different for different tasks). The right hand side in (6) is minimized for a choice of k (where $1 \leq k \leq n$) that balances the first and second terms. The optimal k can vary with the task, e.g., for generic tasks most other tasks will be synergistic and similarly a small optimal k indicates task dissonance where the particular task, say P_1 should be trained on with a specific set of other tasks. Even for typical datasets like CIFAR-100, it is highly nontrivial to understand the ideal set of tasks to train with; Fig. 2 studies this experimentally.

Remark 5 (Continual learning is particularly challenging due to task competition). Theorem 2 indicates that not only is the learner shown tasks sequentially, but it also may have to work against the competition between the current task and the representation learned on a past task. It does not have access to synergistic tasks from the future while learning on the current task. And further, in settings where there is no data replay, the learner cannot benefit from past synergistic tasks explicitly, other than the representation that it has already learnt. This suggests that one must be even more careful about how the representation in continual learning should be updated.

3 Model Zoo: A continual learner that grows its learning capacity

Theorem 2 can be thought of as a "no free lunch theorem". It indicates that ones should not always expect improved excess risk by combining data from different tasks. This theorem also suggests a way to work around the problem via Remarks 3 and 4. If we learn small models on synergistic tasks, we can hope to have each task benefit from the synergies without deterioration of accuracy due to task competition with dissonant tasks. Model Zoo is a simple method that is designed for this purpose.

Let us assume that tasks $P_1, \ldots, P_n$ are shown sequentially to the continual learner. We assume that all tasks have the same input domain X but may have different output domains $Y_1, \ldots, Y_n$. At each "episode" k, Model Zoo is designed to train using the current task P_k and a subset of the past tasks. For example, at episode $k = 2$, we train a model with a feature generator h and task-specific classifiers to obtain models $g_1 \circ h : X \mapsto Y_1$ and $g_2 \circ h : X \mapsto Y_2$. This model can classify inputs from both tasks and gives out a probability vector $p_{g_i \circ h}(y \mid x), \forall y \in Y_i$ depending upon the task. We assume that the identity of the task is known at the test time.

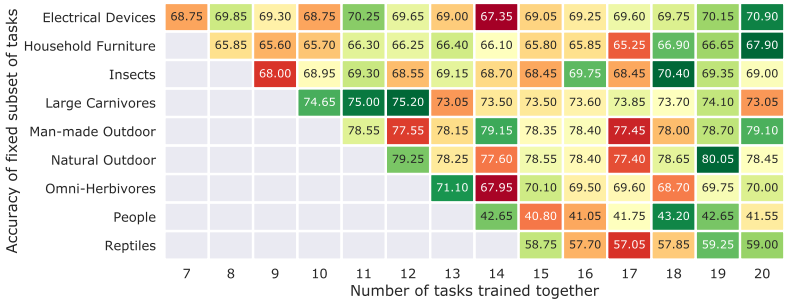

Figure 2: Competition between tasks in continual learning can be non-trivial. In order to demonstrate how some tasks help and some tasks hurt each other, we run a multi-task learner for a varying number of tasks (X-axis) and track the accuracy on a few tasks from CIFAR100 (each task is a superclass). Each cell represents a different experiment, i.e, there is no continual learning being performed here. Cells are colored warm if accuracy is worse than the median accuracy of that row. For instance, multi-task training with 11 tasks is beneficial for "Man-made Outdoor" but accuracy drops drastically upon introducing task #12, it improves upon introducing #14, while task #17 again leads to a drop. One may study the other rows to reach a similar conclusion: there is non-trivial competition between tasks, even in commonly used datasets. As we show, tackling this effectively is the key to obtaining good performance on multi-task learning problems. See Appendix B.1 for a more elaborate version.

Let the set of tasks considered at episode k be denoted by $\bar{P}_k = \{P_{\omega_k^1}, \ldots, P_{\omega_k^b}\}$ where $b \leq k$ is a hyper-parameter and $\omega_k^i \in \{1, \ldots, k\}$. Training on $\bar{P}_k$ will involve, like the example above, training one model with a feature generator h_k and task-specific classifiers g_{k,ω_k^i} for each task selected in that round. Such models, one trained in each round, together form the "Model Zoo". After k rounds, data from, say, P_i with $i \leq k$ can be predicted using the average of class probabilities output by all models that were fitted on that task, i.e.,

$$p_{k,i}(y \mid x) \propto \sum_{l=1}^{k} \mathbf{1}_{\{P_i \in \bar{P}_l\}} \, g_{l,i} \circ h_l(x). \qquad (7)$$

This expression is also used to predict at test time.

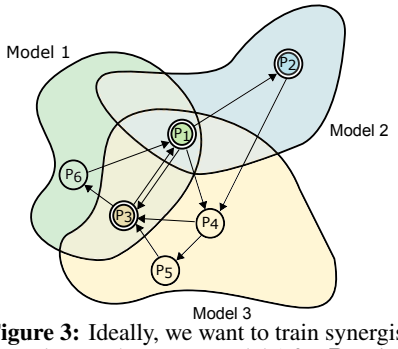

Figure 3: Ideally, we want to train synergistic tasks together, e.g., Model 1 for P_1 using P_3, P_6 and Model 3 for P_3 using P_1, P_4, P_5. At test time, all models (1, 2, 3) that were trained on a particular task, say P_1 would make predictions. Model Zoo is a simple, scalable instantiation of this idea. Discovering noncompeting tasks is difficult, so it selects tasks that have high training loss under the current ensemble.

Selecting tasks to train with for each round using boosting. In principle, we could use the transfer exponents ρ_{ij} to select synergistic tasks, but computing the transfer exponents is essentially as difficult as training on all tasks, a continual learning does not have access to all tasks *a priori*. We therefore develop an automatic way to select tasks in each round. We draw inspiration from boosting (Schapire and Freund, 2013) for this purpose. Recall the AdaBoost algorithm which builds an ensemble of weak learners (they can be any learner in principle Mason et al. (1999)), each of which is fitted upon iteratively re-weighted training data (Breiman, 1998). We think of the models learned at each episode of continual learning in Model Zoo as the "weak learners" and each round of boosting as the equivalent of each episode of continual learning. Let $\bar{w}_k \in \mathbb{R}^n$ be a normalized vector of task-specific weights. After episode k

$$\bar{w}_{k,i} \propto \exp\Big(-1/m \sum_{(x,y)\in S_i} \log p_{k,i}(y \mid x)\Big). \qquad (8)$$

for each task P_i with $i \leq k$; for $i > k$, $\bar{w}_{k,i} = 0$. Tasks for the next round $\bar{P}_{k+1}$ are drawn from a multinomial distribution with weights $\bar{w}_k$. Therefore, tasks with a low empirical risk under the current Model Zoo get a low weight for the next boosting round. Just like AdaBoost drives down the training error on *all* samples to zero exponentially (Schapire and Freund, 2013) by iteratively focusing upon difficult-to-classify samples, Model Zoo achieves a low empirical risk on *all* tasks as more models are added.

The key feature of Model Zoo is that it *automatically* splits the capacity across sets of tasks. Even if competing tasks are chosen in one round, which may result in high excess risk on some task, it will be chosen again in future rounds if it has a large error under the ensemble. Colloquially speaking, the ensemble in Model Zoo represents a "brain" that grows its learning capacity continually as more tasks are shown to it.

Remark 6 (Assumptions in the formulation of Model Zoo). We assume that, both at training time and test time, the identity of the task is known to the continual learner. Data from past tasks is also

stored with the task identity. This is known as the task-incremental setting in the literature (Van de Ven and Tolias, 2019). Recent work in continual learning also studies settings where such task identity is not known, e.g., (Kaushik et al., 2021), Model Zoo is not designed to handle such settings.

4 Empirical Validation

4.1 Setup

Datasets. * We evaluate on Rotated-MNIST (Lopez-Paz and Ranzato, 2017), Split-MNIST (Zenke et al., 2017), Permuted-MNIST (Kirkpatrick et al., 2017), Split-CIFAR10 (Zenke et al., 2017), Split-CIFAR100 (Zenke et al., 2017), Coarse-CIFAR100 (Rosenbaum et al., 2017; Yoon et al., 2019; Shanahan et al., 2021) and Split-miniImagenet (Vinyals et al., 2016; Chaudhry et al., 2019b). Split-MNIST, Split-CIFAR10, Split-CIFAR100 and Split-miniImagenet use consecutive groups of labels (2, 2, 5 and 10, respectively) to form tasks. Coarse-CIFAR100 is a variant of CIFAR100 where each super-class is considered a different task (Yoon et al., 2019; 2021; Shanahan et al., 2021). Our study in Fig. 2 has found that Coarse-CIFAR100 is a difficult dataset for continual learning, perhaps because of the semantic differences among the different super-classes.

Neural architectures and training methodology. We use a small wide-residual network of Zagoruyko and Komodakis (2016) (WRN-16-4 with 3.6M weights) with task-specific classifiers (one fully-connected layer). We also use an even smaller network (0.12M weights) with 3 convolution layers (kernel size 3 and 80 filters) interleaved with max-pooling, ReLU, batch-norm layers, with task-specific classifier layers. Stochastic gradient descent (SGD) with Nesterov's momentum and cosine-annealed learning rate is used to train all models in mixed precision. Ray Tune (Liaw et al., 2018) was used for hyper-parameter tuning using a multi-task learning model on all tasks from Coarse CIFAR-100. When we do full replay, Model Zoo samples $b = \min(k, 5)$ tasks at the kth episode; for problems with $n = 5$ tasks, we set $b = 2$; note that $b = 1$ indicates no data replay. **All hyper-parameters are kept fixed for all datasets and all experiments (see §4.2).**

See Appendix A for more details.

4.2 Evaluating continual learning methods

There is a wide variety of problem formulations in the continual learning literature (Farquhar and Gal, 2019a; Prabhu et al., 2020; Vogelstein et al., 2020; Lopez-Paz and Ranzato, 2017; Van de Ven and Tolias, 2019). Formulations vary with respect to whether they allow replaying data from past tasks, the number of epochs the learner is allowed to train each task for, and the capacity of the model being fitted. We next explain these different formulations, the rationale behind them, and how we execute Model Zoo to conform to each of these settings.

(i) The **strict formulation**, e.g., Kirkpatrick et al. (2017); Kaushik et al. (2021), does not allow any replay of data. For the strict formulation of Model Zoo, we simply set $\bar{w}_{k,i} = 0$ for all $i \neq k$ in (8). At each episode, a single model is trained on the current task and added to the zoo—we call this rather simplistic learner **Isolated**. From a practical standpoint, such a formulation imposes a constraint on the amount of computational resources (compute and/or memory) available during training.

(ii) One can **replay data to various degrees**, e.g., all of it (Nguyen et al., 2017; Guo et al., 2020b), or a subset of it (Chaudhry et al., 2019a). Just like AdaBoost, Model Zoo is fundamentally designed to allow full replay of past tasks. However, we can easily execute it with limited replay by only using a subset of the data to compute gradient updates and also the accuracy on past tasks in the kth episode. We use the nomenclature **Model Zoo (10% replay)** to indicate that only 10% of the data from past tasks is used; algorithms like A-GEM (Chaudhry et al., 2019a) also use 10% of past data on CIFAR100 datasets. See Appendix A.4 for implementation details. Note that Model Zoo without any data replay is simply Isolated. Let us emphasize that across all these problem settings, Model Zoo remains a legitimate continual learner because it gets access to each task sequentially and has a fixed computational budget (b tasks) at each episode. For a multi-task learner, the computational complexity scales with the number of tasks.

(iii) To impose a strict constraint on the computational complexity of each episode some works, e.g., Chaudhry et al. (2019a), train each task for a single epoch. We therefore show results using both **Model Zoo (single epoch)** (where we replay past data for 1 epoch) and **Isolated (single epoch)** (no replay). Even if the rationale behind using each datum only once is well-taken, one single epoch is quite insufficient to train modern deep networks; if one thinks of biological considerations,

* Some works (Rebuffi et al., 2017; Lopez-Paz and Ranzato, 2017; Chaudhry et al., 2019a; Mirzadeh et al., 2020b) evaluate on a split of the CIFAR100 dataset where each task is random subset of 5 classes. We do not evaluate on this variant because it is difficult to exactly reproduce the composition of tasks; as Fig. 2 suggests different compositions can have vastly different task accuracy. This is also highlighted by large differences in the accuracy on Split-CIFAR100 and Coarse-CIFAR100 in our work.

Method	Replay	Single-epoch	Rotated-MNIST	Permuted-MNIST	Split-MNIST	Split-CIFAR10	Split-CIFAR100	Coarse-CIFAR100	Split-miniImagenet
EWC (Kirkpatrick et al., 2017)	✗	✗	•84	•96.9	-	-	•42.40	-	46.69
GEM (Lopez-Paz and Ranzato, 2017)	✓	✓	86.07	82.60	-	-	*67.8	-	51.86
A-GEM (Chaudhry et al., 2019a)	✓	✓	-	89.1	-	-	*62.3	-	61.13
Stable-SGD (Mirzadeh et al., 2020b)	✗	✓	70.8	80.1	-	-	*59.9	-	57.79
ER-Reservoir (Chaudhry et al., 2019b)	✓	✓	-	79.8	-	-	*68.5	-	64.03
MEGA-II (Guo et al., 2020a)	✓	✓	-	91.20	-	-	66.12	-	-
RMN (Kaushik et al., 2021)	✗	✗	-	97.73	99.5	-	80.01	-	-
APD (Yoon et al., 2019)	✗	✗	-	-	-	-	-	56.81	-
Our methods									
Isolated-small	✗	✗	-	-	-	96.88	90.18	69.07	82.48
Model Zoo-small	✓	✗	-	-	-	96.85	92.06	73.72	94.27
Model Zoo-small (10% replay)	✓	✗	-	-	-	96.58	89.76	77.18	84.6
Isolated-Resnet18-S	✗	✗	-	-	-	-	88.95	-	-
Model Zoo-Resnet18-S	✓	✗	-	-	-	-	93.15	-	-
Isolated	✗	✗	99.64	98.03	99.98	97.46	91.90	80.72	86.28
Model Zoo	✓	✗	99.66	97.71	99.97	98.68	94.99	84.27	96.84
Multi-Head (multi-task)			99.66	98.16	99.98	98.11	95.38	83.19	90.83

Table 1: Average per-task accuracy (%) at the end of all episodes. MNIST, Permuted-MNIST and Rotated-MNIST are not informative benchmarks for judging forward and backward transfer because even Isolated achieves 99%+ accuracy. Model Zoo outperforms, by significant margins, all existing continual learning methods on all datasets. Accuracy of existing methods is worse than Isolated which suggests little to no forward or backward transfer. Model Zoo-small and Isolated-small have comparable number of weights as that of existing methods, and in some cases, much fewer. Model Zoo-Resnet18-S and Isolated-Resnet18-S, make use of the Resnet18-S architecture from Lopez-Paz and Ranzato (2017). Both Model Zoo/Isolated have similar accuracies on Split-CIFAR100 with 3 different architectures and all of them are better than existing methods. This indicates that the improvement in accuracy is not a result of the specific choice of architecture. For single-epoch numbers refer to Fig. 1 and Table 2. **Note:** * indicates that the evaluation was on Split-CIFAR100 with each task containing randomly sampled labels and is hence it is not directly comparable to other methods. All numbers without a marker are from the paper cited in the first column. • denotes that the accuracy is not from the original paper but from one of (Nguyen et al., 2017; Serra et al., 2018; Chaudhry et al., 2019a). Numbers for other methods on Split-MiniImagenet were computed by us using open-source implementations of the original authors.

local-descent algorithms like stochastic gradient descent (SGD) are quite different from recurrent circuits in the biological brain (Kietzmann et al., 2019). We also run single epoch methods using a very small model (0.12M weights); these are **Model Zoo/Isolated-small (single epoch)**.

(iv) **Multi-Head** trains one single model on all tasks to minimize the average empirical risk with task-specific classifiers; mini-batches contain samples from different tasks. Since Multi-Head is trained on all tasks together, it is not a continual learner, but its accuracy is expected to be an upper bound on the accuracy of continual learning methods.

Evaluation criteria. We compare algorithms in terms of the validation accuracy averaged across all tasks at the end of all episodes, average per-task forward transfer (accuracy on a new task when it is first seen, larger this number more the forward transfer), average per-task forgetting (gap in the maximal accuracy of a task during continual learning and its accuracy at the end, larger this number more the forgetting and worse the backward transfer), training and inference time, and memory. Let us note that forward transfer is also sometimes called "learning accuracy" (Riemer et al., 2018), and another measure of backward transfer is the gap between the accuracy at the end of training and the initial accuracy of the task.

4.3 Results

Table 1 shows the validation accuracy of different continual learning methods on standard benchmark problems. There are many striking observations here.

(i) **Accuracy of existing methods** compared to in Table 1 (see Table A1 as well) **is poorer than Isolated**. This is surprising because Isolated can be thought of as the simplest possible continual learner—one that unfreezes new capacity at each episode and does not replay data. This indicates that existing methods may be failing to achieve forward or backward transfer compared to simply training the task in isolation; Table 2 investigates this further.

(ii) In comparison, **Model Zoo (all three variants**: small, small with 10% data replay and the standard method) **has better accuracy compared to both existing methods as well as Isolated**. This shows the utility of splitting the capacity of the learner across multiple tasks.

(iii) **Model Zoo matches the accuracy of the multi-task learner** in the last row of Table 1 which has access to all tasks beforehand. Surprisingly, **Model Zoo performs better than Multi-Head in spite of being trained in continual fashion**, especially on harder problems like Coarse-CIFAR100 and Split-miniImagenet. This is a direct demonstration of the effectiveness of Model Zoo in mitigating task competition: the capacity splitting mechanism not only avoids catastrophic forgetting, but it can also leverage data from other tasks even if they are shown sequentially.

Table 2 shows a comparison of the methods developed in this paper with existing methods on Split-CIFAR100 in terms of continual-learning specific metrics. We find:

(i) There are no significant differences in the forward transfer performance in the single epoch setting; larger variants of Isolated and Model Zoo do not work well here because a **single epoch is not sufficient to train modern deep networks**. But **Model Zoo and variants show less forgetting**, it is essentially zero. This indicates that although existing methods are designed to avoid forgetting (the single epoch setting aids this directly), say, A-GEM, or EWC, they do forget. Forgetting can be mitigated by the capacity splitting mechanism in Model Zoo. The per-task accuracy of existing methods is also rather low compared to Model Zoo variants.

(ii) If our methods are implemented in the **multi-epoch setting**, then the **forward transfer** is exceptionally good and **almost as good as the average accuracy** of the task. Surprisingly, this does not come at the cost of **forgetting, which is again essentially zero**.

(iii) Even if Model Zoo and its variants are implemented with **very small models** (0.12M weights/episode, which is 2.42M weights/20 episodes), the **accuracy is better** (Table 1). This suggests that Model Zoo is a performant and viable approach to continual learning. In fact, even the larger model used in Model Zoo is a WRN-16-4 with 3.6M weights and therefore we can train multiple models on the same GPU easily; this is why the training time of Model Zoo is about the same as that of Model Zoo-small.

(iv) The simplicity of Model Zoo and its variants results in much smaller training times and comparable inference times as compared to existing methods.

Method	Inference time (ms/sample)	Training time (min)	Storage Samples (%)	Storage #Weights (M)	Metrics (Multi Epoch) Accuracy (%)	Metrics (Multi Epoch) Forgetting (%)	Metrics (Multi Epoch) Forward (%)	Metrics (Single Epoch) Accuracy (%)	Metrics (Single Epoch) Forgetting (%)	Metrics (Single Epoch) Forward (%)
EWC	10.34	50	0	1.6	-	-	-	42.4	17.52	67.76
Prog-NN	-	82	0	23.7	-	-	-	59.2	0.0	59.2
GEM	10.34	1048	5–10	1.6	-	-	-	61.2	6.0	67.61
A-GEM	10.34	88	5–10	1.6	-	-	-	62.3	7.0	70.13
RMN	2712.4	-	0	11.5	80.01	-	-	-	-	-
Our methods										
Isolated-small	2.34	17.09	0	2.42	90.18	0.0	91.18	71.6	0.0	71.6
Model Zoo-small	11.70	31.71	100	2.42	92.28	0.17	90.0	73.67	0.20	71.91
Model Zoo-small (10% replay)	11.70	22.41	10	2.42	89.76	0.22	89.8	71.09	0.69	70.5
Isolated	2.34	20.76	0	54.8	91.9	0.0	91.0	50.43	0.0	50.43
Model Zoo	31.84	41.86	100	54.8	94.99	0.21	94.02	57.67	0.81	56.58

Table 2: A comparison of **continual learning evaluation metrics on Split-CIFAR100** for existing methods and the methods developed in this paper. Our methods demonstrate strong forward and backward transfer, high per-task accuracy, smaller training times and comparable inference times. Training times of other methods are from Chaudhry et al. (2019a) and it is the total training time in minutes for all tasks. The Inference time is the per sample prediction latency averaged over 50 mini-batches of size 16. See Appendix A.5 for more details.

Replay (%)	Split-CIFAR100	Split-miniImagenet
0	71.91	65.80
1	70.48	67.18
5	71.33	70.71
10	71.97	74.22
100	73.67	81.05

# Tasks (b) (100% replay)	Split-CIFAR100	Split-miniImagenet
1	71.91	65.02
2	72.26	67.33
5	73.67	81.05
7	73.97	88.76
9	74.13	84.9

Method	Model Zoo	Ensemble of Isolated (100×)
Split-CIFAR100	73.67	71.46
Split-miniImagenet	81.05	67.26

Figure 4: Ablation studies that show the average per-task accuracy as we vary the size of data replay for Model Zoo (left), the number of past tasks sampled at each episode (middle, $b = 1$ implies no replay), and compare Model Zoo with an ensemble of Isolated models (right). These results are for the single-epoch setting and are therefore directly comparable to those in Table 2 and Table 1 as far as comparison to other methods is concerned. Accuracy is roughly the same on Split-CIFAR100 across varying degrees of replay while it improves significantly on Split-miniImagenet; this suggests that Model Zoo also works with very small amounts of data replay. Accuracy on Split-CIFAR100 is consistent as the number of replay tasks is changed but increases on larger datasets like Split-miniImagenet where there are many more tasks. Finally, the performance of Model Zoo is not merely an artifact of ensembling. Even if Isolated is a strong model, a very large ensemble of Isolated compares poorly to Model Zoo with 100% replay; this indicates that Model Zoo can effectively leverage data from past tasks without forgetting. See the Appendix for more ablation studies.

5 Related Work

Theoretical work on learning from multiple tasks. Works such as Baxter (2000); Maurer (2006), or recent ones like Du et al. (2020); Tripuraneni et al. (2020) study a shared feature generator with task-specific classifiers, and show that the sample complexity of learning a task improves if true task-specific classifiers are diverse enough. It is also appreciated that such a shared feature generator may not exist for dissimilar tasks. So a different perspective on the problem can found in Crammer et al. (2008); Ben-David et al. (2010); Ben-David and Borbely (2008) who show that learning diverse

tasks requires a larger feature generator and, thereby, more samples; we discuss this in §2.2. We build upon Hanneke and Kpotufe (2019; 2020) to construct the transfer exponent in §2; their work shows that even in very favorable settings, e.g., when all tasks have the same optimal classifier, having access to a large number of tasks may not help. Model Zoo is strongly influenced from these results and we think of it as essentially a way to circumvent them.

There are a number of algorithmic tools to estimate task relatedness, e.g., Evgeniou et al. (2005); Cavallanti et al. (2010); Kumar and Daume III (2012), and although such methods are popular in transfer learning (Pentina and Lampert, 2015; Jaakkola and Haussler, 1999), one cannot apply them in continual learning because we do not know the tasks beforehand. As §2 shows, task relatedness is critical to good learning. So, taking inspiration from AdaBoost (Schapire and Freund, 2013), Model Zoo uses a simple indicator of which past tasks can benefit from future ones, these are the ones with low accuracy under the current ensemble.

Catastrophic forgetting. has been the focus of a number of continual learning techniques, e.g., episodic memory-based ones (Lopez-Paz and Ranzato, 2017; Chaudhry et al., 2019a; Farajtabar et al., 2020; Guo et al., 2020a), data replay (Robins, 1995; Shin et al., 2017; Lee et al., 2017), new architectures (Serra et al., 2018), generative replay-based (Mocanu et al., 2016; Shin et al., 2017; Liu et al., 2020; Ven et al., 2020), ensemble-based (Aljundi et al., 2017; Wen et al., 2020) and methods that select locally-redundant directions in the weight space (Kirkpatrick et al., 2017; Aljundi et al., 2018; Mallya et al., 2018; Zenke et al., 2017; Chaudhry et al., 2018). Variational methods, e.g., (Nguyen et al., 2017; Farquhar and Gal, 2019b), sequentially update a posterior over the weights and have an elegant foundation in Bayesian methods but implementing them for large datasets remains a challenge. In spite of intense activity, an effective solution to forgetting remains largely unknown.

Model Zoo embraces the fact that forgetting is a fundamental phenomenon of learning multiple tasks and therefore splitting the capacity may be essential; our results indicate that this approach is effectively at tackling forgetting. This approach also significantly improves other key metrics, e.g., forward-backward transfer and computational complexity of training and inference that have received limited attention (Díaz-Rodríguez et al., 2018). Let us note that Model Zoo is designed for the task-incremental continual learning setting (Van de Ven and Tolias, 2019).

Parameter sharing/isolation. A single shared feature generator (i.e., hard parameter sharing) is a popular architecture (Kirkpatrick et al., 2017; Lopez-Paz and Ranzato, 2017; Rebuffi et al., 2017; Nguyen et al., 2017; Mirzadeh et al., 2020b; Chaudhry et al., 2019b). It has been recognized that this is not sufficient; this has given rise to methods for soft-parameter sharing that either design or learn specialized routing architectures (Rosenbaum et al., 2017; Sun et al., 2019; Fernando et al., 2017; Devin et al., 2017; Misra et al., 2016; Vandenhende et al., 2019). Model Zoo is a very simplistic instantiation of parameter isolation, or growing (Rusu et al., 2016; Mallya and Lazebnik, 2018; Xu and Zhu, 2018). Model Zoo trains on one episode and never updates the model again but its accuracy does play a role in determining whether a *new* model should be used for that past task, or not. To extend the analogy, just like soft-parameter sharing architectures use, say gradient conflict (Aljundi et al., 2018) or attention (Serra et al., 2018), to determine which synapses to share, Model Zoo uses the training loss of the ensemble to decide what task the new model should be trained upon. It is remarkable that such a simple instantiation performs so well; this has not been noticed before.

6 Discussion

Continual learning is an important problem as deep learning systems transition from the traditional paradigm of having a fixed model that makes inferences on user queries to settings where we would like to update the model to handle new types of queries. The key desiderata of such a system are clear: it must display high per-task accuracy and strong forward-backward transfer. This paper seeks to develop such a continual learner and investigates the problem using the lens of task relatedness. It argues that the learner must split its capacity across sets of tasks to mitigate competition between tasks and benefit from synergies among them. We develop Model Zoo, which is a continual learning algorithm inspired by AdaBoost, that grows an ensemble of models, each of which is trained on data from the current episode along with a subset of past tasks. **We show that across a wide variety of datasets, problem formulations, and evaluation criteria, Model Zoo and its variants outperform existing continual learning methods.** We also show that a simple baseline method, where a separate, small model is trained independently in each episode, outperforms a number of existing continual methods. Appendix D discusses these results further.

7 Ethics Statement

This work does not raise any ethical concerns.

8 Reproducibility Statement

To ensure the reproducability of our work, the full source code is available at `https://github.com/rahul13ramesh/modelzoo_continual`. The code contains scripts to download all datasets, implementations of different variants of our method discussed in the paper, scripts for hyper-parameter tuning, and data analysis. We note here that we use the same set of hyper-parameters for all our experiments; this will help other researchers reproduce our results.

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