FBGA: Fast and Flexible Velocity Planning for Time-Optimal Maneuvers

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***Abstract*— Time-optimal velocity planning under generic acceleration constraints is critical for autonomous racing. Existing methods face a trade-off: optimal control handles complex constraints accurately but is computationally expensive, while fast semi-analytical methods are limited to conservative box constraints. We propose FBGA, a Forward-Backward algorithm with Generic Acceleration constraints, discretizing the path and performing forward-backward passes to maximize velocity profiles. Tested on five racetracks with racing cars and motorcycles, FBGA matches optimal control accuracy (within 0.11%-0.36%) while achieving 2-3 orders of magnitude speed-up, making it suitable for real-time multi-query trajectory planning. Open-source C++ implementation: `https://github.com/DRIVEWISE/FBGA`.**

I. INTRODUCTION

Time-optimal velocity planning under acceleration constraints is a key problem in robotics, with applications in autonomous racing (Fig. 1) [2]–[4], drone flight [5], manipulators [6], and mobile robot navigation [7], [8]. These applications use point-mass models to compute time-optimal velocity profiles along fixed paths, subject to acceleration limits. Fast online generation of such profiles is essential in dynamic environments, where trajectory planners must evaluate many candidate maneuvers, often with graph- or sampling-based frameworks [2], [5], [9], [10].

However, the existing methods for minimum-time velocity planning face a trade-off: Optimal Control Problems (OCPs) [11] and Quasi-Steady-State (QSS) [12], [13] offer high accuracy, but their high computational cost does not allow online multi-query planning. Conversely, semi-analytical Forward-Backward (FB) and sequential methods [3], [8] are faster but limited to box-shaped acceleration bounds, which are overly conservative for racing vehicles. Accurate performance modeling of racing vehicles requires complex g-g-v diagrams (Fig. 2a) [12], [14]–[16], describing the speed-dependent coupling of the lateral and longitudinal acceleration limits.

This paper introduces a real-time Forward-Backward method with Generic Acceleration constraints (FBGA), matching the accuracy of OCPs while being up to three orders of magnitude faster, making it a suitable building block for online sampling-based trajectory planners.

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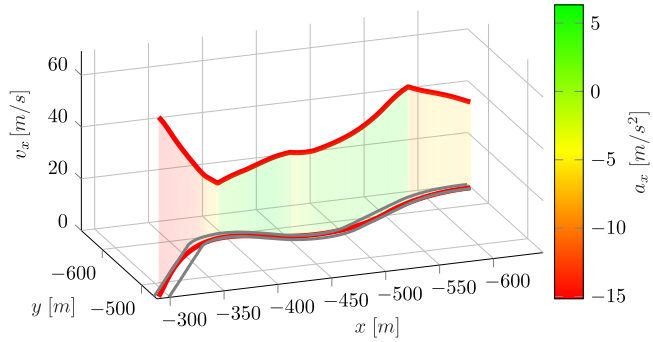
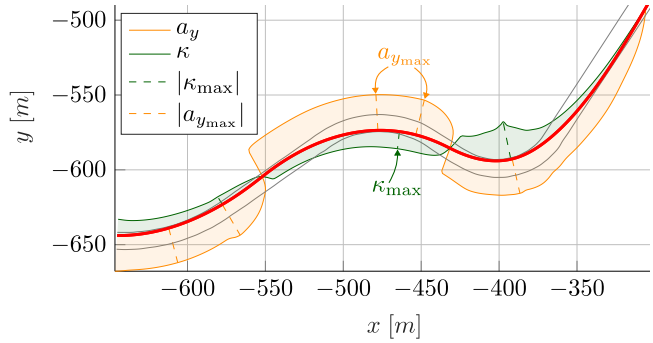
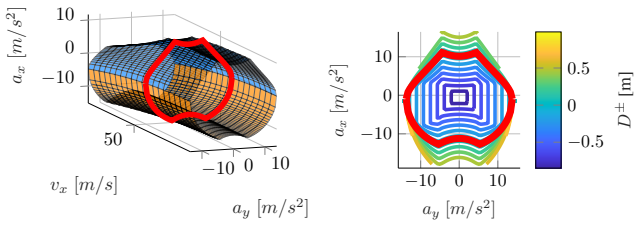


(a) Path, velocity (v_x) and longitudinal acceleration (a_x).



(b) Lateral acceleration (a_y) and curvature (κ), with their local maxima.

Fig. 1. (a) Time-optimal velocity and longitudinal acceleration profiles computed by our FBGA along the first two corners of the Catalunya circuit. (b) Lateral acceleration and curvature profiles. Unlike QSS [17], FBGA does not assume that peak lateral accelerations ($a_{y_{\max}}$) occur at curvature peaks ($\kappa_{\max}$): in the second corner, $\kappa_{\max}$ happens between two $a_{y_{\max}}$ peaks.



(a) (b)

Fig. 2. (a) g-g-v diagram of the racing motorcycle model [18] used in Sec. III. (b) Output of the $D^{\pm}$ signed distance function (Algorithm **??**) for the red slice in (a). $D^{\pm}$ ranges from -1 (inside the g-g-v envelope) to $+\infty$ (outside). The red line denotes the envelope boundary, where $D^{\pm} = 0$.

A. Related Work and Main Limitation

Minimum-time velocity planning falls into two categories: (1) OCPs and QSS methods [11], [12], [17], which handle generic g-g-v constraints but are computationally expensive for online planning; (2) semi-analytical FB

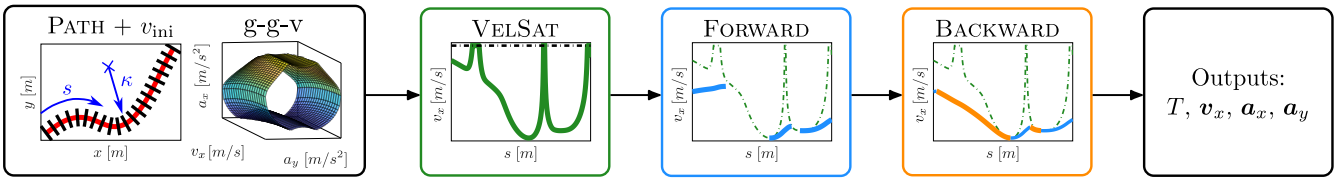


Fig. 3. Main phases of FBGA: path input, speed saturation, forward and backward passes to maximize velocity while satisfying g-g-v constraints.

methods [3], [8], [19], which are fast but limited to box-shaped acceleration constraints. The critical gap is that fast methods only support conservative rectangular g-g diagrams, while accurate methods cannot handle the complex, speed-dependent constraints of real racing vehicles at real-time speeds.

B. Contributions

We propose FBGA: (1) a new forward-backward algorithm for time-optimal velocity planning with generic acceleration constraints; (2) validation on five racetracks with racing cars and motorcycles; (3) three orders of magnitude computational speed-up over optimal control baselines with comparable accuracy.

II. METHODOLOGY

A. Time-Optimal Velocity Planning Problem

We consider the following velocity planning problem along a given path:

$$\min_{a_x} \quad T \tag{1a}$$

$$\text{s.t.} \quad \dot{s} = v_x, \quad \dot{v}_x = a_x, \tag{1b}$$

$$\Gamma_y^-(v_x) \leq a_y = \kappa(s)\, v_x^2 \leq \Gamma_y^+(v_x), \tag{1c}$$

$$\Gamma_x^-(a_y, v_x) \leq a_x \leq \Gamma_x^+(a_y, v_x), \tag{1d}$$

$$v_x(0) = v_{\text{ini}},\ s(0) = 0,\ s(T) = L. \tag{1e}$$

Where T is the maneuver time to be minimized. The system dynamics (1b) is a double integrator, where the states $\{s, v_x\}$ are the curvilinear abscissa along the path and the longitudinal velocity, while the control input a_x is the longitudinal acceleration. The control a_x is bounded by the acceleration constraints (1c)-(1d), where $\kappa(s)$ is the curvature of the path as a function of s, $a_y = \kappa(s)\, v_x^2$ is the lateral acceleration, and $\Gamma_x^{\pm}(\cdot)$ and $\Gamma_y^{\pm}(\cdot)$ are functions providing the lower and upper bounds of the lateral and longitudinal accelerations. Finally, the boundary conditions (1e) specify the initial velocity v_{ini} and the final travelled distance L. In (1), all the variables are time-dependent, and the notation $\dot{\mathrm{x}}$ denotes the time derivative of the variable x.

Assuming the lateral acceleration $a_y = \kappa(s)\, v_x^2$ places the model in the quasi-steady-state class, along a given path. Our formulation (1) condenses all dynamic nonlinearities in the acceleration constraints (1c)-(1d), which, for vehicles, define the g-g-v diagram. This diagram captures key physical effects, including tire saturation, actuation limits, aerodynamic drag, load transfer, and front/rear wheel lift in motorcycles, and expresses the maximum performance envelope in terms of center-of-mass accelerations.

Variants of the problem (1) were solved with optimal control for vehicle motion planning [11], [20], but their high computational cost hinders online planning of multiple trajectories in dynamic scenarios. This paper introduces FBGA, a novel algorithm that solves (1) with high accuracy and significantly lower computation time.

B. FBGA Algorithm Overview

The proposed FBGA method discretizes the path into short segments and performs three main phases:

1) calculation of maximum saturated speeds given lateral acceleration bounds;
2) forward pass to compute maximum feasible longitudinal accelerations;
3) backward pass to handle braking maneuvers.

For each segment, the algorithm enforces the g-g-v acceleration constraints using a custom signed-distance function. The total maneuver time is obtained as the sum of individual segment durations.

FBGA uses a piecewise-constant acceleration assumption on short path segments, enabling fast computation via analytical formulas. This semi-analytical approach matches optimal control accuracy while achieving three orders of magnitude speed improvement.

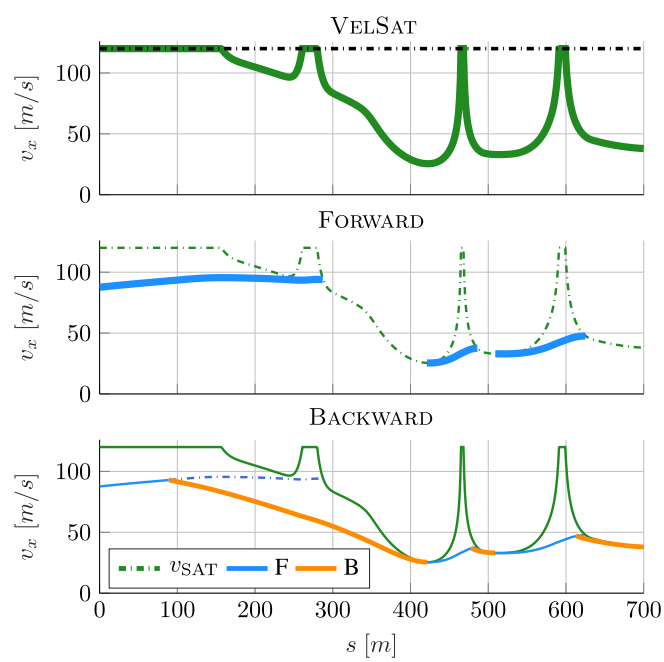


Fig. 4. Illustration of FBGA algorithm phases: velocity saturation, forward pass acceleration maximization, and backward pass for braking maneuvers.

1) Analytical Solution for Single Segment: The key challenge in velocity optimization is that the maneuver time T is unknown *a priori*, making direct time-based computation intractable. To overcome this, we employ a change of variables from time t to curvilinear abscissa s along the path. This transformation enables fast computation of velocity profiles for each segment while maintaining analytical tractability.

On short path segments (typical discretization $\Delta s < 50$ m for racing applications), the longitudinal acceleration a_x can be treated as piecewise-constant, justifying the analytical solution that follows. This assumption enables closed-form expressions replacing costly numerical ODE solvers.

For each path segment with constant longitudinal acceleration a_x, the change of variables yields:

$$v_x(s) = \sqrt{2a_x s + v_{\text{ini}}^2} \tag{2}$$

Once the velocity profile is known as a function of position, the corresponding segment maneuver time can be computed as:

$$T = \frac{-v_{\text{ini}} + \sqrt{2a_x L + v_{\text{ini}}^2}}{a_x} \tag{3}$$

where L is the segment length. These closed-form expressions evaluate analytically, eliminating the expensive numerical integration required by optimal control solvers—the source of the 2-3 orders of magnitude speedup reported in Table I.

The acceleration a_x for each segment is determined by enforcing the g-g-v constraints at segment borders, where the lateral acceleration a_y is maximum. This analytical formulation underpins both the forward pass (maximizing acceleration for given entry speed) and backward pass (maximizing feasible speed by propagating constraints backward), enabling the FBGA algorithm to optimize the full velocity profile efficiently.

III. RESULTS

A. Setup and Benchmarking

We test FBGA on racing vehicles: a car (g-g-v from [11]) and a non-convex motorcycle envelope (from [18]), on five racetracks. Paths are generated from minimum-lap-time OCPs; then FBGA and the benchmark $\text{OCP}_{\text{bench}}$ (solved with PINS [21]) compute speed profiles on the same paths. Code is in C++ at `https://github.com/DRIVEWISE/FBGA`.

B. Maneuver Analysis

We now compare FBGA with $\text{OCP}_{\text{bench}}$ on five circuits with race car and motorcycle models. The two vehicles have distinct g-g-v envelopes (Fig. 2a, motorcycle; Fig. 7, car). The motorcycle envelope is non-convex due to wheel lift, limiting longitudinal accelerations.

Fig. 5-6 shows minimum-time maneuvers at turn 9 of Sepang circuit for both vehicles. FBGA's results closely match $\text{OCP}_{\text{bench}}$ with minor discrepancies in transition regions. Fig. 7a-7b shows the g-g-v envelope and FBGA

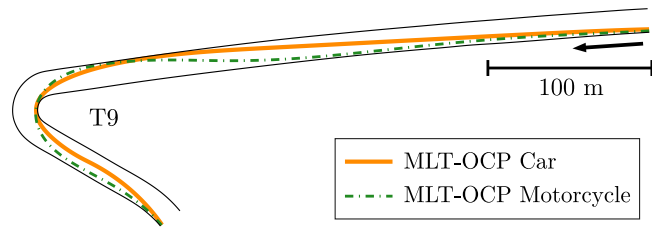
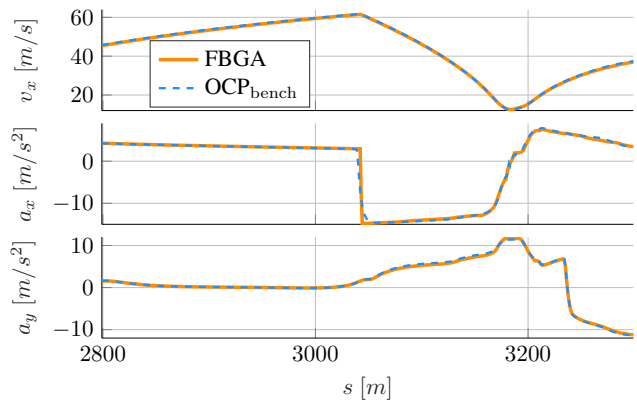
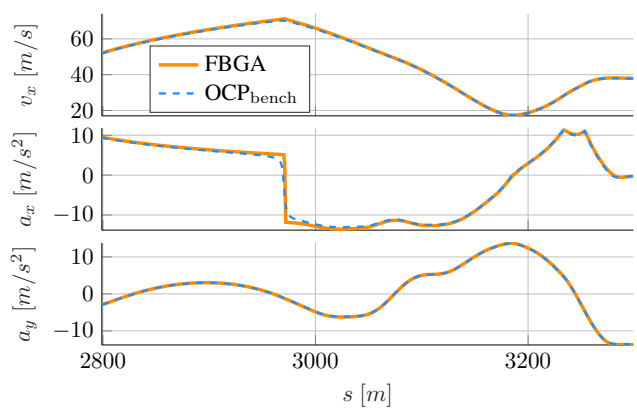


Fig. 5. Comparison of the MLT-OCP maneuvers for the racing car and motorcycle models of this paper, on the turn n.9 of the Sepang circuit.



(a) Results for the racing car model.



(b) Results for the racing motorcycle model.

Fig. 6. Velocity, longitudinal and lateral acceleration profiles of our FBGA and the benchmark $\text{OCP}_{\text{bench}}$, for a racing car (a) and motorcycle (b) at turn 9 of the Sepang circuit (Fig. 5).

solution on Catalunya, confirming adherence to constraints. To further validate FBGA, we compare it with a high-fidelity double-track model (MLT-DT) with Pacejka MF5.2 tire model on Catalunya circuit. Figure 8 shows FBGA matches both MLT-DT and $\text{OCP}_{\text{bench}}$, with lap time difference of only 73 ms.

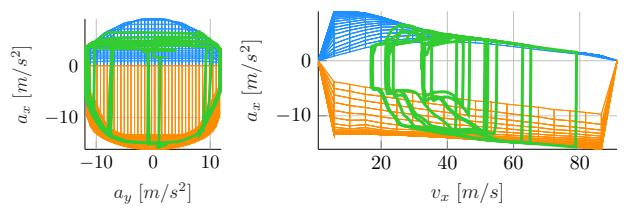
C. Results

Table I compares lap times and CPU times on five racetracks with two vehicle models. FBGA lap times match $\text{OCP}_{\text{bench}}$ within 0.094-0.449 s (0.11-0.36% difference), while CPU times are 2-3 orders of magnitude faster (2.39-11.44 ms). Table II shows 300 m short-horizon planning: FBGA achieves 550× speed-up with only 0.19% maneuver time difference.

TABLE I

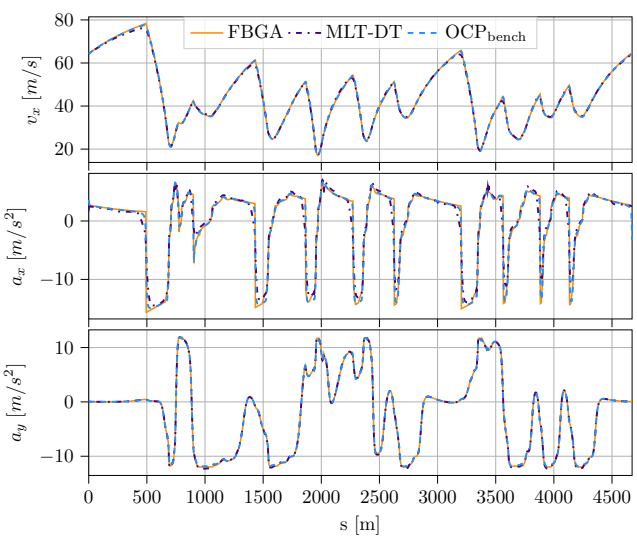
LAP TIMES AND CPU TIMES FOR FBGA VS. BENCHMARK $\text{OCP}_{\text{bench}}$ ON FIVE RACETRACKS WITH CAR AND MOTORCYCLE MODELS (M2 MAX CHIP).

Circuit (length)	**N. mesh points**	**Vehicle**	**Lap time [s]**			**CPU time [ms]**	
			$\text{OCP}_{\text{bench}}$	**FBGA** (ours)	Δ (FBGA $-$ $\text{OCP}_{\text{bench}}$)	$\text{OCP}_{\text{bench}}$	**FBGA** (ours)
Catalunya (4.66 km)	4660	Car	112.461	112.204	−0.257	8017.15	9.86
		Motorcycle	105.381	104.999	−0.382	693.91	3.31
Valencia (4.00 km)	4000	Car	104.742	104.520	−0.222	7411.99	8.85
		Motorcycle	96.752	96.434	−0.318	830.82	3.40
Misano (4.16 km)	4160	Car	107.740	107.451	−0.289	6805.87	9.66
		Motorcycle	98.814	98.497	−0.317	602.19	3.37
Sepang (5.52 km)	5520	Car	135.480	135.086	−0.394	8239.47	11.44
		Motorcycle	125.483	125.034	−0.449	1282.58	4.30
Palm Beach (3.17 km)	3170	Car	79.963	79.869	−0.094	4819.43	6.38
		Motorcycle	74.790	74.537	−0.253	706.43	2.39



(a) Top view (b) Lateral view

Fig. 7. g-g-v envelope (blue-orange lines) for the race car model, and solution computed by our FBGA (green line) on the Catalunya circuit.



FBGA	MLT-DT	$\text{OCP}_{\text{bench}}$
113.96 s	114.05 s	114.16 s

Fig. 8. Comparing our FBGA, the $\text{OCP}_{\text{bench}}$ and the MLT-DT OCP, which is solved with a double-track (DT) vehicle model, on the Catalunya circuit.

TABLE II

SHORT-HORIZON PLANNING (300 M) ON CATALUNYA CIRCUIT: MANEUVER TIME AND CPU TIMES FOR DIFFERENT MESH DENSITIES.

$\text{OCP}_{\text{bench}}$		**FBGA** (ours)		
T [s]	**CPU [ms]**	**N. points**	**T [s]**	**CPU [ms]**
8.7936	97.533	300	8.7765	0.177
		200	8.7830	0.131
		100	8.8029	0.062

D. Sensitivity Analysis

Fig. 9 shows FBGA's robustness to path discretization resolution. CPU time scales linearly with segment count, while lap time decreases hyperbolically and changes by only 0.07 s (0.06%) when increasing segments by 10×. Minor oscillations at coarse resolutions stem from segment-level curvature variations affecting root-finding iterations. These results confirm suitability for coarse-to-fine mesh strategies: coarse meshes enable rapid planning with negligible accuracy loss, while fine meshes maximize solution quality and fully exploit acceleration limits.

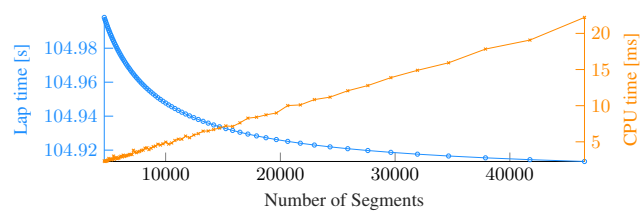


Fig. 9. Lap times and CPU times vs. mesh segments (racing motorcycle, Catalunya circuit).

IV. CONCLUSIONS

We introduced FBGA, a forward-backward method for real-time time-optimal velocity planning with generic acceleration constraints. Tested on five racetracks with racing vehicles, FBGA matches optimal control accuracy (0.11-0.36% lap time deviation) while achieving 2-3 orders of magnitude speed-up. Results hold across tracks and vehicle types without convexity assumptions on constraints. The method's effectiveness extends to short-horizon planning, where FBGA maintains three orders of magnitude speed-up with maneuver times within 0.19% of benchmark solutions.

FBGA is suitable for online multi-query trajectory planning, warm-starting complex OCPs, and short-horizon planning for autonomous racing, unlocking new capabilities for sampling-based trajectory planners in dynamic environments. Future work will extend the method to handle jerk limits and provide formal optimality guarantees.

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