# A STATISTICAL APPROACH FOR CONTROLLED TRAINING DATA DETECTION

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## ABSTRACT

Detecting training data for large language models (LLMs) is receiving growing attention, especially in applications requiring high reliability. While numerous efforts have been made to address this issue, they typically focus on accuracy without ensuring controllable results. To fill this gap, we propose **K**nockoff Inference-based **T**raining data **D**etector (KTD), a novel method that achieves rigorous false discovery rate (FDR) control in training data detection. Specifically, KTD generates synthetic knockoff samples that seamlessly replace original data points without compromising contextual integrity. A novel knockoff statistic, which incorporates multiple knockoff draws, is then calculated to ensure FDR control while maintaining high power. Our theoretical analysis demonstrates KTD's asymptotic optimality in terms of FDR control and power. Empirical experiments on real-world datasets such as WikiMIA, XSum and Real Time BBC News further validate KTD's superior performance compared to existing methods.

## 1 INTRODUCTION

Large language models (LLMs) have demonstrated exceptional performance across a variety of natural language processing (NLP) tasks, including machine translation (Wong et al., 2023), code completion (Chen et al., 2021; Li et al., 2022), and question answering (Dong et al., 2024; Li et al., 2024a). The success of LLMs is largely attributed to the use of incredibly massive language corpora, often at the trillion-token level (Computer, 2023). Although such extensive datasets endow LLMs with a broad knowledge base and robust text generation capabilities, they sometimes include private information (Carlini et al., 2021) or copyrighted content (Chang et al., 2023) collected from the Internet, leading to unexpected negative consequences. For example, LLMs may inadvertently memorize and reproduce sensitive information when prompted with carefully crafted inputs, thereby posing substantial risks of privacy leakage or copyright infringement (Carlini et al., 2021).

To address this issue, recent studies have focused on detecting training data from LLMs (Shi et al., 2023; Golchin & Surdeanu, 2023). Unfortunately, these approaches treat the problem as a binary classification task, solely aiming for accurately classifying training and non-training samples. However, we argue that in certain cases of copyright violation detection, accuracy alone is insufficient. For instance, if a copyright holder intends to sue a technology company for unauthorized use of proprietary content (Grynbaum & Mac, 2023), it is crucial to ensure that the majority of identified training samples were indeed used by the company for training. False detection in this context can result in unnecessary legal consequences. Therefore, controlling the false discovery rate (FDR) (Benjamini & Hochberg, 1995), also known as the false positive rate in binary classification, is crucial for training data detection and should not be overlooked.

In this paper, we study the problem of detecting training samples from LLMs with controllable FDR. Specifically, given a set of text samples, we aim to determine whether the model has been trained on them while ensuring the proportion of detected samples that are non-training sample is bounded. Inspired by controllable variable selection, we propose a Knockoff Inference-based Training Data Detector (KTD), which treats the problem of detecting training samples as an instance of relevant variable selection, thereby enabling the use of knockoff inference's (KI) robust capacity in FDR controlling. KI generates knockoff variables corresponding to each original variable and computes knockoff statistics for each pair of original and knockoff variables, which are then used to identify relevant variables. Building on the KI paradigm, KTD operates in two stages. In the first

stage, KTD generates knockoffs that preserve the semantics of the original text samples, ensuring they can substitute for the originals without altering the context. In the second stage, the knockoff statistics capturing the difference between original text samples and their corresponding knockoffs are computed to serve as a measure for identifying training samples.

In KTD, a fundamental component for successful detection is the construction of the knockoff statistic. While the knockoff statistic from the vanilla KI method can be directly applied to KTD, it has a significant drawback: the vanilla KI method uses only a single draw of the knockoff variable to calculate the knockoff statistic $W_j$. This approach results in high variance in $W_j$ due to the inherent randomness of the knockoff process, making it difficult to distinguish between training and non-training samples. Such indistinguishability can lead to overly conservative detection results, excluding too many true training samples to achieve the desired FDR control. Consequently, this significantly impairs detection efficiency, as measured by power (the proportion of actual training samples that are correctly detected).

To address this issue, KTD adopts a novel approach for calculating the knockoff statistic which utilizes multiple draws of the knockoff variable. This design effectively reduces the variance of the knockoff statistic, making it more centralized and thereby enhancing the separability between training and non-training samples. Specifically, KTD draws $m$ realizations of the knockoff variables and computes the knockoff statistic $\tilde{W}_j$ by comparing the importance score of the original variable to the average of the scores of these realizations.

To theoretically justify KTD, we first demonstrate that the knockoff statistic of KTD, i.e., $W_j^{\mathrm{KTD}}$ retains the symmetric property, ensuring that it can control the FDR just like the vanilla KI. Furthermore, we distinguish KTD from the vanilla KI by proving its asymptotically optimal property: as $m$ approaches infinity, the power converges to 1 and the FDR converges to 0.

We empirically evaluate KTD using three popular large language models on three real-world datasets, including the established benchmark WikiMIA. The experimental results demonstrate that KTD not only achieves the desired FDR control without relying on a validation set, which existing methods depend on but also exhibits significantly higher power compared to vanilla KI when achieving similar FDR levels.

To summarize, our contributions are as follows:

1. We address the problem of detecting training data from LLMs with FDR control through the perspective of knockoff inference and propose a knockoff inference-based detecting method KTD.
2. We theoretically justify our proposed KTD from two aspects. Firstly, we prove that the knockoff statistic in KTD possesses the symmetric property, which is essential for effective FDR control. Secondly, we show that KTD exhibits asymptotically optimal properties, distinguishing it from the vanilla KI.
3. Our experimental analysis validates the effectiveness of KTD by demonstrating its ability to achieve the desired FDR level with competitive power.

## 2 Related Work

### 2.1 Training Data Leakage in LLMs

Related to the training data leakage, memorization in language models has been extensively studied. Works such as Kandpal et al. (2022); Carlini et al. (2021; 2022b); Zeng et al. (2024) analyze the memorization behaviors of language models, providing insights into their underlying mechanisms. However, these studies primarily focus on exploring the characteristics and contributing factors of memorization, without proposing practical methods for detecting training samples.

Focusing on LLMs, other studies (Brown et al., 2020; Wei et al., 2021; Du et al., 2022) consider the potential impacts of training data leakage on evaluation results. To ensure the reliability of the results, these studies exclude test samples that have n-gram overlaps with any data used during pre-training. This approach requires access to pre-training datasets, making it impossible to detect training samples without support from the model provider.

To evaluate training data leakage without access to the training dataset, several methods have been proposed for detecting training samples. For instance, Golchin & Surdeanu (2023) prompts a model

Table 1: Algorithmic properties

| Property | FX-Knockoff | MX-Knockoff | Contamination Test | Mink% | Time Traveling | KTD (Ours) |
|---|---|---|---|---|---|---|
| Metadata-free | ✓ | ✓ | ✓ | ✓ | ✗ | ✓ |
| Threshold-free | ✓ | ✓ | ✓ | ✗ | ✓ | ✓ |
| FDR Control | ✓ | ✓ | ✗ | ✗ | ✗ | ✓ |
| FDR analysis | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |
| Power analysis | ✗ | ✗ | ✗ | ✗ | ✗ | ✓ |

to generate completions using two types of instructions and identifies training samples by comparing the texts generated from these instructions. Shi et al. (2023) assumes that non-training texts are more likely to contain outlier tokens that cause significantly high loss and proposes detecting training samples using top-k token log probabilities. Oren et al. (2023) leverages the exchangeability of datasets, which implies that non-training samples are permutation invariant to the model, and identifies dataset-level contamination through hypothesis testing. Despite the empirical effectiveness of these methods, they do not provide theoretically guaranteed control of the FDR.

Related to training data detection, membership inference attack also has attracted much attention. Similar to our approach, Mattern et al. (2023); Fu et al. (2024) generate neighboring samples resembling the original ones and compare each sample with its generated neighbors to detect training data. However, these methods do not explicitly focus on controlling the FDR and are primarily driven by empirical observations rather than grounded in rigorous theoretical foundations. Some works consider FDR in their method, such as Carlini et al. (2022a); Mireshghallah et al. (2022). However, these works are not inherently designed for FDR control and often require either training a large number of shadow models or access to the distribution of data that was not used to train the target model. Such requirements render them unsuitable for LLM scenarios.

### 2.2 Knockoffs

The knockoff framework was first proposed in Barber & Candès (2015) as a data-driven method to control the FDR in variable selection for sparse regression problems. This framework was later extended to high-dimensional regression in Candes et al. (2018). Some works extend the application of knockoff inference to various tasks including multi-task regression (Dai & Barber, 2016), outlier detection (Xu et al., 2016), and sample selection (Wang et al., 2024). To the best of our knowledge, we are the first to apply the knockoff inference in the context of LLM training data detection.

To better illustrate the novelty of our proposed method, KTD, we compare it with works from the knockoff literature such as FX-Knockoff (Barber & Candès, 2015) and MX-Knockoff (Candes et al., 2018), as well as works from the training dataset detection literature such as Contamination Test (Oren et al., 2023), Mink% (Shi et al., 2023), and Time Traveling (Golchin & Surdeanu, 2023) in Table 1. In the table, "metadata-free" indicates that the method does not require meta information such as dataset name and partition name, and "threshold-free" means the method does not rely on heuristically determined thresholds. This table conclude the comprehensiveness of our proposed method, KTD, regarding both training data detection and knockoff properties analysis.

## 3 Background

**Notation** We use bold letters to represent vectors of random variables, e.g., $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}$. Furthermore, let $\mathbf{X}_{-j}$ denote the vector resulting from the exclusion of the $j$-th variable $X_j$, i.e., $\mathbf{X} \backslash \{X_j\}$. The independence between two random variables $X$ and $Y$ is symbolized as $X \perp Y$. Let $[n]$ represent the set $\{1, 2, \ldots, n\}$; for any given set $\mathcal{A}$, $|\mathcal{A}|$ denotes the cardinality of $\mathcal{A}$. For two number $a$ and $b$, let $a \vee b$ represent $\max(a, b)$.

**Problem Definition** Suppose we have $n$ potential training samples $X_1, X_2, \ldots, X_n$ to be tested. For the sake of clarity afterward, we defined $\mathcal{D}_{\text{total}}$ as the index set of these samples, i.e., $\mathcal{D}_{\text{total}} = [n]$. Depending on whether a sample has been used for training the model, $\mathcal{D}_{\text{total}}$ can be divided into two disjoint subsets: $\mathcal{D}_{\text{train}}$ and $\mathcal{D}_{\text{non-train}}$, which represent the index of training samples and non-training samples respectively. Given the dataset $\mathcal{D}_{\text{total}}$ and the language model $f_{\boldsymbol{\theta}}$, our objective is to identify an estimate of $\mathcal{D}_{\text{train}}$, denoted as $\hat{\mathcal{S}} \subset \mathcal{D}_{\text{total}}$, with an FDR bounded by a

predefined threshold $q$, while maintaining as high power as possible. Here, the power and FDR are defined as:

$$\text{Power} := \mathbb{E}\left[\frac{|\hat{\mathcal{S}} \cap \mathcal{D}_{\text{train}}|}{|\mathcal{D}_{\text{train}}| \vee 1}\right] \qquad \text{and} \qquad \text{FDR} := \mathbb{E}\left[\frac{|\hat{\mathcal{S}} \cap \mathcal{D}_{\text{non-train}}|}{|\hat{\mathcal{S}}| \vee 1}\right]. \tag{1}$$

**Auto-regressive LLMs** The goal of auto-regressive large language models is to capture the underlying language distribution $P_{\boldsymbol{\theta}}(X)$. They achieve this by predicting the next token in a sequence based on the preceding tokens. Specifically, given a sequence $X = (x_1, x_2, \cdots, x_T)$, these models represent the probability of $X$ using the chain rule:

$$P_{\boldsymbol{\theta}}(X) = \prod_{t=1}^{T} P_{\boldsymbol{\theta}}(x_t \mid x_1, x_2, \ldots, x_{t-1})$$

where $\boldsymbol{\theta}$ denotes the parameters of the language model. The parameters $\boldsymbol{\theta}$ are trained to maximize the log-likelihood of sequences in the training dataset.

# 4 METHODOLOGY

In this section, we introduce our knockoff inference-based training data detector, KTD. We begin by illustrating two critical procedures of KTD, which include synthetic knockoff generation and knockoff statistic calculation. Then, the asymptotic analysis for KTD is provided.

## 4.1 KTD: A NOVEL KNOCKOFF-BASED FRAMEWORK

**Motivation** Knockoff inference is a method originally designed for selecting variables relevant to certain output of interest while controlling FDR. In our settings, we can reformulate our problem as a variable selection problem by treating the training samples $\{X_j\}_{j \in \mathcal{D}_{\text{train}}}$ as relevant variables and model parameter $\boldsymbol{\theta}$ as the output of training algorithm $\mathcal{A}lg$ which takes training samples as input, i.e., $\boldsymbol{\theta} = \mathcal{A}lg(\{X_j\}_{j \in \mathcal{D}_{\text{train-total}}})$. Here, $\mathcal{D}_{\text{train-total}}$ represent the the model's entire training dataset and $\mathcal{D}_{\text{train}}$ is a subset of it. Through this reformulation, the robust FDR control ability of KI can be utilized in our context. Since in our approach, we treat samples as random variables, we will use these two terms interchangeably in the following text.

Intuitively, the fundamental idea behind the knockoff inference-based method is to identify relevant variables by comparing them with their noisy counterparts, known as knockoffs. As a result, knockoff inference-based method usually involves two critical procedures: first, generating knockoffs for the text samples to be tested; second, calculating the knockoff statistic for these text samples by comparing the scores assigned to them with the scores assigned to their knockoff counterparts. Next, we illustrate how these two stages work in vanilla KI and how they are instantiated in KTD.

### 4.1.1 KNOCKOFF GENERATION

In vanilla KI, knockoffs are typically generated based on specific assumptions about the distribution of variables $\{X_j\}_{j=1}^n$, such as Gaussian (Candes et al., 2018), Markov model Sesia et al. (2018) and hidden Markov model (Sesia et al., 2018). However, due to the complexity of natural language, it is challenging to use common distributions to model the relationships between text samples, rendering existing methods unsuitable for knockoff generation in this context. Consequently, we directly adhere to the fundamental definition of knockoffs:

**Definition 1.** *Model-X Knockoffs, (Candes et al., 2018) Model-X knockoffs for a family of random variables* $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}$ *are a new family of random variables* $\tilde{\mathbf{X}} = \{\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n\}$ *satisfying:*

1. $\tilde{\mathbf{X}} \perp \boldsymbol{\theta} \mid \mathbf{X}$.
2. *For any* $s \subset [n]$, $(\mathbf{X}, \tilde{\mathbf{X}})_{\text{swap}(s)} \overset{d}{=} (\mathbf{X}, \tilde{\mathbf{X}})$.

*Here,* $(\mathbf{X}, \tilde{\mathbf{X}}) = (X_1, X_2, \ldots, X_n, \tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n)$*, and* $(\mathbf{X}, \tilde{\mathbf{X}})_{\text{swap}(s)}$ *is obtained by swapping* $X_j$ *with its corresponding knockoff* $\tilde{X}_j$ *for all* $j \in s$*. For example, when* $n = 3$*,* $(\mathbf{X}, \tilde{\mathbf{X}})_{\text{swap}(\{1,3\})} = (\tilde{X}_1, X_2, \tilde{X}_3, X_1, \tilde{X}_2, X_3)$*.*

This definition provides guidance for generating knockoff texts. Property (1) implies that the knockoff text should not be generated by the model being tested, and property (2) requires that the generated knockoff texts should be able to replace the original text samples without altering the overall joint distribution. In other words, the knockoff text should convey the same meaning as the original text but in a different manner. To meet these requirements, we generate knockoffs in the KTD framework using a natural language paraphraser, which restates or rephrases the text while preserving its original meaning.

#### 4.1.2 Knockoff Statistic Calculation

**Knockoff Statistic in Vanilla KI** After constructing the knockoffs, a test statistic known as the knockoff statistic is calculated for each text sample by comparing the importance of the original text sample with that of its knockoff counterparts. This statistic can be viewed as a relevance measure for each text sample and will serve as the basis for training sample selection, which is defined as

**Definition 2.** *Knockoff Statistic, (Candes et al., 2018) A knockoff statistic* $\mathbf{W} = \{W_1, W_2, \ldots, W_n\}$ *is a measure of variable importance that satisfies the following conditions:*

1. $\mathbf{W}$ *depends only on* $\mathbf{X}$*,* $\tilde{\mathbf{X}}$*, and* $\boldsymbol{\theta}$*:*

$$\mathbf{W} = f(\mathbf{X}, \tilde{\mathbf{X}}, \boldsymbol{\theta}). \tag{2}$$

2. *Swapping the original variable* $X_j$ *with its corresponding knockoff* $\tilde{X}_j$ *switches the sign of* $W_j$*:*

$$W_j([\mathbf{X}, \tilde{\mathbf{X}}]_{\text{swap}(s)}, \boldsymbol{\theta}) = \begin{cases} W_j([\mathbf{X}, \tilde{\mathbf{X}}], \boldsymbol{\theta}), & \text{if } j \notin s \\ -W_j([\mathbf{X}, \tilde{\mathbf{X}}], \boldsymbol{\theta}), & \text{if } j \in s. \end{cases} \tag{3}$$

Typically, the calculation of the knockoff statistic of each variable can be decomposed into two steps. First, assign importance scores $Z_j$ and $\tilde{Z}_j$ to each variable $X_j$ and its knockoff $\tilde{X}_j$ respectively, where the importance scores is calculated by a pre-defined scoring function $T$, i.e.,

$$Z_j = T_j([\mathbf{X}, \tilde{\mathbf{X}}], Y) \qquad \text{and} \qquad \tilde{Z}_j = T_{j+n}([\mathbf{X}, \tilde{\mathbf{X}}], Y). \tag{4}$$

Next, calculate the knockoff statistic of $j$-th sample by

$$W_j = Z_j - \tilde{Z}_j. \tag{5}$$

Intuitively, the scores $Z_j$ and $\tilde{Z}_j$ represent the importance of the original sample $X_j$ and its knockoff $\tilde{X}_j$, respectively. A positive $W_j$ ($W_j > 0$) indicates that the $j$-th sample is more relevant to the model parameter $\boldsymbol{\theta}$ than its knockoff, implying its membership in the training data. Conversely, a negative $W_j$ ($W_j < 0$) suggests that the $j$-th sample is more likely to be irrelevant to $\boldsymbol{\theta}$.

During this procedure, the key is to select an appropriate scoring function $t$ that can effectively measure the importance of each sample and its knockoff. In our scenario, we aim to ensure that samples seen by the model are assigned higher scores. Inspired by works using gradient information for OOD detection (Huang et al., 2021; Liang et al., 2018), we use the $L_2$ norm of the model's gradient as the score in KTD, which is defined as:

$$Z_j = -\left\| \frac{\partial \log P_{\boldsymbol{\theta}}(X_j)}{\partial \boldsymbol{\theta}} \right\|_2 \qquad \text{and} \qquad \tilde{Z}_j = -\left\| \frac{\partial \log P_{\boldsymbol{\theta}}(\tilde{X}_j)}{\partial \boldsymbol{\theta}} \right\|_2 \tag{6}$$

where $P_{\boldsymbol{\theta}}(\cdot)$ represents the probability distribution modeled by the model.

Finally, a threshold determined to thresholding knockoff statistic for training sample detection with FDR control guarantee. This procedure is illustrated as follows:

**Proposition 1.** *By choosing the threshold* $\tau$ *according to*

$$\tau = \arg\min_{t>0} \left\{ \frac{1 + |\{j \in [n] : W_j \leq -t\}|}{|\{j \in [n] : W_j \geq t\}| \vee 1} \leq q \right\}. \tag{7}$$

*and setting* $\hat{\mathcal{S}} = \{j : W_j \geq \tau\}$*, the procedure can control the FDR at* $\leq q$*.*

**Our Calculation of Knockoff Statistic** Despite the effectiveness of vanilla KI in controlling FDR, its knockoff statistic $W_j$ is prone to high variance due to the inherent randomness in the knockoff generation process. This drawback complicates the differentiation between training and non-training samples. Consequently, a conservative threshold $\tau$ is more likely to be selected, which excludes many true training samples to ensure FDR control, thereby reducing the proportion of training samples that are successfully detected, i.e., power.

To address this issue, we modify the calculation of the vanilla knockoff statistic $W_j$ by considering multiple draws of the knockoff variables $\tilde{\mathbf{X}}$. Specifically, we calculate the knockoff statistic in KTD as follows:

$$W_j^{\mathrm{KTD}} = Z_j - \frac{1}{m}\sum_{i=1}^{m} \tilde{Z}_j^{(i)} \tag{8}$$

where $\tilde{Z}_j^{(i)}$ is the score calculated based on the $\tilde{X}_j^{(i)}$, the $i$-th draw of $\tilde{X}_j$. Clearly, $W_j$ is a special case of $W_j^{\mathrm{KTD}}$ when $m = 1$. By taking multiple knockoff draws into consideration, we can reduce the variance of knockoff statistic $W_j^{\mathrm{KTD}}$, thereby enhancing the separability between training and non-training samples.

Next, we show that $W_j^{\mathrm{KTD}}$ can also select the appropriate threshold for FDR controlling as $W_j$ do in Proposition 1. We first give the independence assumption of $W_j^{\mathrm{KTD}}$ following Nguyen et al. (2020):

**Assumption 1.** *For any $j \in \mathcal{D}_{\mathrm{non-train}}$, the knockoff statistic $W_j^{\mathrm{KTD}}$ defined in Equation 8 are independent with each other.*

Next, we illustrate the symmetric property of $W_j^{\mathrm{KTD}}$:

**Lemma 1.** *$W_j^{\mathrm{KTD}}$ associated with irrelevant samples is symmetrically distributed around 0, i.e.,*

$$P(W_j^{\mathrm{KTD}} < -t) = P(W_j^{\mathrm{KTD}} > t) \quad \textit{for any } t > 0 \textit{ and } j \in \mathcal{D}_{\mathrm{non-train}}. \tag{9}$$

This Lemma is empirically validated by our experiments. For details please refer to the third part of our experimental results.

This Lemma, combined with the independence assumed in Assumption 1 implies that the number of non-training samples whose $W_j^{\mathrm{KTD}} > 0$ equals to the number of non-training samples whose $W_j^{\mathrm{KTD}} < 0$. This conclusion allows the use of the right-hand side of Equation 10 as an upper bound for FDR, thereby providing an FDR control guarantee.

Consequently, we can select training samples while controlling the FDR using $W_j^{\mathrm{KTD}}$ through a procedure similar to that described in Proposition 1, defined as follows:

**Proposition 2.** *Assume $\{W_j^{\mathrm{KTD}}\}_{j=1}^n$ are independent with each other, by choosing the threshold $\tau$ according to*

$$\tau = \min_{t>0}\left\{\frac{1 + |\{j \in [n] : W_j^{\mathrm{KTD}} \le -t\}|}{|\{j \in [n] : W_j^{\mathrm{KTD}} \ge t\}| \vee 1} \le q\right\} \tag{10}$$

*and setting $\hat{\mathcal{S}} = \{j : W_j^{\mathrm{KTD}} \ge \tau\}$, the procedure can control the FDR at $\le q$.*

### 4.2 The Asymptotic Optimal Property of knockoff statistic in KTD

Here, we provide analysis inspired by Zhao et al. (2022) to illustrates the asymptotic optimality of FDR and power in KTD. We begin by stating an assumption on which these theorems rely.

**Assumption 2.** *For any $j \in \mathcal{D}_{\mathrm{train}}$, we have $\mathbb{E}[W_j^{\mathrm{KTD}}] > 0$.*

**Remark 1.** *This assumption ensures that, on average, training samples will have higher importance scores than their knockoff counterparts. This is reasonable because a sample that has been seen by the model will induce fewer updates(thus higher $Z_j$ defined in Equation 6) compared to its knockoff. Like Lemma 1, we provide the empirical validation of this assumption in our experiment section.*

**Theorem 1.** *Assuming Assumption 2 holds, the variable selection procedure described in Proposition 2 satisfies*

$$\mathrm{Power} = \mathbb{E}\left[\frac{|\hat{\mathcal{S}} \cap \mathcal{D}_{\mathrm{train}}|}{|\mathcal{D}_{\mathrm{train}}|}\right] \to 1 \quad \textit{as} \quad m \to \infty. \tag{11}$$

**Theorem 2.** *Assuming Assumption 2 holds and the threshold $\tau$ found by Proposition 2 is not equal to 0, the variable selection procedure satisfies*

$$\mathrm{FDR} = \mathbb{E}\left[\frac{|\tilde{\mathcal{S}} \cap \mathcal{D}_{\mathrm{train}}|}{|\hat{\mathcal{S}}|}\right] \to 0 \quad as \quad m \to \infty. \tag{12}$$

**Remark 2.** *Intuitively, as $m$ approaches infinity, the values of $W_j^{\mathrm{KTD}}$ will become increasingly centralized around their expectations. Consequently, given Assumption 2, the $W_j^{\mathrm{KTD}}$ values corresponding to training and non-training samples will form distinct clusters. This separation allows the KTD method to identify a threshold that optimizes both FDR and power.*

## 5 Experiments

In this section, we conduct extensive experiments to validate the empirical efficacy of our proposed method. We begin by testing the effectiveness of our method in terms of FDR control. Following that, we empirically validate the symmetric property of our proposed knockoff statistic $W_j^{\mathrm{KTD}}$. Next, we investigate the influence of $m$, the number of knockoff draws, on FDR control performance and the trade-off between power and FDR. Finally, we conduct experiments using Pythia models with different numbers of parameters to examine how model size affects performance.

### 5.1 Setup

**Baselines** Since we are the first to realize FDR control in training data detection, there is no previous baseline we can compare. As a result, we first select several classic baselines from the membership inference attack (MIA) literature for comparison. Specifically, these baselines include **LOSS** (Yeom et al., 2018), which uses the auto-regressive loss of a sample to determine whether it has been seen during training; **MinK%** (Shi et al., 2023), which takes the average loss of the top k% tokens with the highest loss as the basis for detection; **Zlib** (Carlini et al., 2021), which compares the loss of text samples with their zlib compression entropy; and **Lowercase** (Carlini et al., 2021), which compares the loss of text samples with the loss of their lowercased versions. Then, we compare our method with **vanilla KI**, which is an instance of our method when setting $m = 1$.

**Models** We adopt three popular large language models to evaluate our detection algorithm: GPT-2 (137M parameters) (Radford et al., 2019), Pythia (1.4B parameters) (Biderman et al., 2023), and GPT-Neo (1.3B parameters) (Black et al., 2021). To generate reliable knockoffs for text samples, we use a paraphraser (Vladimir Vorobev, 2023) with the highest downloads on Hugging Face. This paraphraser is based on the T5-base model and fine-tuned with paraphrased texts generated by ChatGPT. Throughout our experiments, we use the model checkpoints provided by Hugging Face[1].

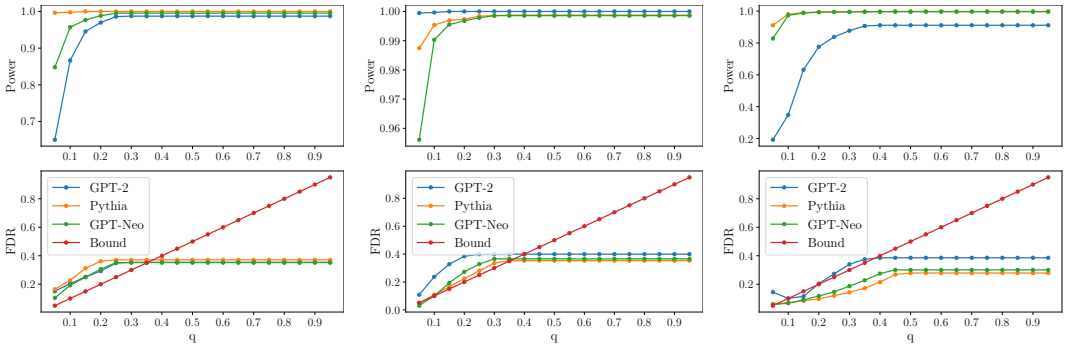
**Dataset** We conduct our experiments on three datasets: **WikiMIA** (Shi et al., 2023) includes texts collected from Wikipedia events. The dataset is separated into two disjoint parts: one corresponding to events happening before 2017 and the other to events happening after 2023. These two parts are used as training samples and non-training samples, respectively. **XSum** (Narayan et al., 2018) includes summaries of BBC news articles. We select the test set of this dataset and randomly separate it into two parts, corresponding to training and non-training samples. **BBC Real Time** (Li et al., 2024b) includes BBC articles from January 2017 to August 2024. Following the process in Shi et al. (2023), we use the articles published in 2017 as training samples and articles published in 2024 as non-training samples. To evaluate our method, we introduce the training samples in these datasets into the models' training data. Specifically, we fine-tune the models using the training samples while ensuring that the non-training samples remain unseen by the models.

**Computation and Hyperparameters** All the experiments are run with a single NVIDIA Tesla V100 32GB GPU and a 10-core Intel Xeon (Skylake IBRS) CPU. When the model is too large, we use 8-bit quantization to fit the model into GPU memory. Unless explicitly stated, we fix $m = 10$ for all experiments. All codes are implemented with Pytorch (Paszke et al., 2019).

[1] https://huggingface.co/

Table 2: Comparison between KTD and baselines. For each dataset and model, if any methods achieve FDR control, the one with the highest power among them will be bolded. Otherwise, the method with the best FDR will be bolded.

| | WikiMIA | | | | | | XSum | | | | | | BBC Real Time | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | GPT-2 | | Pythia | | GPT-Neo | | GPT-2 | | Pythia | | GPT-Neo | | GPT-2 | | Pythia | | GPT-Neo | |
| | FDR | Power | FDR | Power | FDR | Power | FDR | Power | FDR | Power | FDR | Power | FDR | Power | FDR | Power | FDR | Power |
| LOSS | 0.179 | 0.928 | **0.175** | **0.999** | 0.145 | 0.996 | **0.153** | **0.150** | 0.163 | 0.938 | 0.133 | 0.843 | 0.135 | 0.950 | 0.122 | 0.999 | 0.123 | 0.998 |
| MinK% | 0.193 | 0.232 | 0.193 | 0.987 | 0.120 | 0.749 | 0.313 | 0.024 | 0.165 | 0.060 | 0.203 | 0.293 | 0.245 | 0.279 | 0.103 | 0.958 | 0.130 | 0.964 |
| Zlib | **0.000** | **0.060** | 0.187 | 0.984 | **0.096** | **0.881** | 0.237 | 0.499 | 0.164 | 0.880 | 0.164 | 0.801 | 0.122 | 0.800 | 0.142 | 0.997 | 0.112 | 0.993 |
| Lowercase | 0.484 | 0.995 | 0.485 | 0.991 | 0.487 | 0.983 | 0.570 | 0.096 | 0.697 | 0.019 | 0.654 | 0.081 | 0.807 | 0.005 | 0.495 | 0.016 | 0.671 | 0.017 |
| Vanilla KI | 0.207 | 0.476 | 0.194 | 0.991 | 0.198 | 0.972 | 0.194 | 0.998 | 0.117 | 0.973 | 0.109 | 0.936 | 0.161 | 0.223 | 0.083 | 0.915 | 0.083 | 0.873 |
| KTD (Ours) | 0.197 | 0.869 | 0.230 | 0.998 | 0.193 | 0.958 | 0.238 | 1.000 | **0.109** | **0.995** | **0.101** | **0.990** | **0.089** | **0.412** | **0.071** | **0.980** | **0.067** | **0.973** |




(a) WikiMIA (b) XSum (c) BBC Real Time

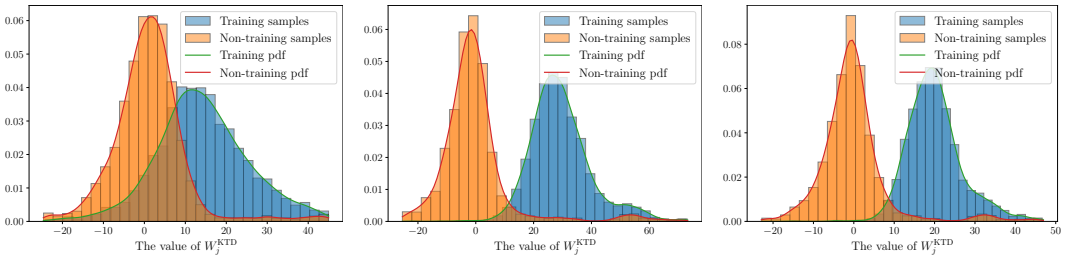
Figure 1: The FDR control results on three datasets. We vary the FDR bound $q$ from 0.05 to 0.95 and calculate its corresponding FDR and power. Each subplot represent results on a dataset and each line in the subplots represents the results of a model. To clearly visualize the bound, we also plot the red line ($y = x$) in each subplots. If a model's FDR is bounded, its corresponding line should be below the red line.

## 5.2 Results

**The Effectiveness of FDR Control with fixed $q$** We fix the FDR bound $q = 0.1$ and present the comparison results between our method and the baselines in Table 2. Since the MIA baselines (LOSS, MinK%, Zlib, and Lowercase) only output a confidence score for training data membership inference, they require a validation set to determine the threshold. For these methods, we sample a 100-sample validation set and select the thresholds that result in a bounded FDR with the highest power on the validation set. From the Table, we can observe that our method, KTD, successfully controls the FDR on the XSum and BBC Real-Time datasets while maintaining relatively stable performance across different datasets and models. In contrast, baselines such as Zlib, MinK%, and Lowercase sometimes exhibit exceptionally poor FDR or power, indicating that the success of these baselines heavily relies on the selection of validation set. Although some baselines achieve more favorable results on WikiMIA, we argue that this is not a completely fair comparison, as these baselines require access to a validation set with ground truth membership labels, which may not be available in practical settings.

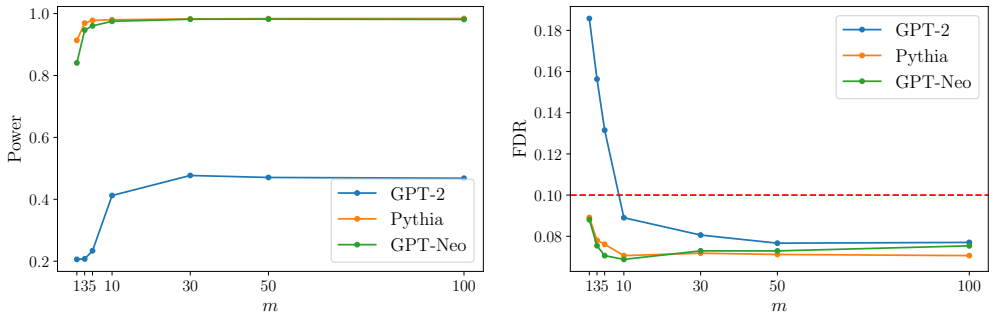
**The Effectiveness of FDR Control under varying $q$** We vary $q$ and plot the corresponding power and FDR against it in Figure 1. We find that our method effectively controls the FDR in most cases. Although the FDR of three models on the WikiMIA dataset is not strictly bounded by $q$, their corresponding lines exhibit a similar tendency to red line, indicating that they are bounded by $q$ plus some constant. Moreover, from this figure, we can also observe that the model size affects the effectiveness of FDR control. It is more challenging to control the FDR in smaller models like GPT-2, and models of similar size like Pythia and GPT-Neo exhibit similar FDR tendencies.

**The Symmetric Property of $W_j^{\text{KTD}}$** To empirically validate Lemma 1 and Assumption 2 which are associated with our knockoff statistic $W_j^{\text{KTD}}$, we plot the distribution of $W_j^{\text{KTD}}$ calculated on the BBC Real Time dataset in Figure 2. From the figure, we can make three observations. Firstly, the $W_j^{\text{KTD}}$ of non-training samples are symmetrically distributed around 0, which is aligned with




(a) GPT-2 (b) Pythia (c) GPT-Neo

Figure 2: The distribution of our knockoff statistic $W_j^{\mathrm{KTD}}$.




(a) Relationship between power and $m$ (b) Relationship between FDR and $m$

Figure 3: $m$'s influence on power and FDR. We fix the bound $q$ at 0.1 and vary $m$ in $\{1, 3, 5, 10, 30, 50, 100\}$ to calculate the power and FDR for each model. The red line in the figure represents the FDR bound given by $q$. Dots below the red line indicate successful bounding.

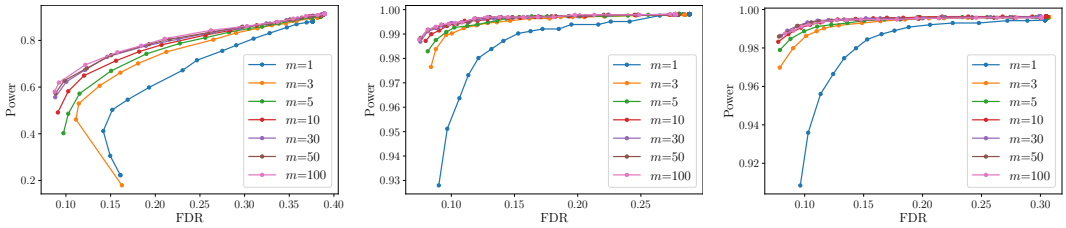
Lemma 1. Secondly, the expectation of non-training samples' $W_j^{\mathrm{KTD}}$ is greater than 0, illustrating the validity of Assumption 2, which Theorems 1 and 2 rely on. Lastly, larger models (Pythia and GPT-Neo) exhibit better separability between training and non-training samples. This is expected, as larger models have a higher probability of memorizing their training data (Carlini et al., 2022b), thereby showing different behavior for training versus non-training samples.

**The Influence of the Number of Knockoff Draws** In KTD, we use multiple knockoff draws to calculate KTD statistic $W_j^{\mathrm{KTD}}$. In this part, we justify our design by investigating two problems: (1) what is the influence of $m$ on the FDR control and power; (2) what is the influence of $m$ on the power-FDR trade-off? Please note that vanilla KI is an instance of KTD corresponding to $m = 1$.

For the first point, we plot the power and FDR against $m$ in Figure 3a and Figure 3b respectively. From these two figures, we can make several observations. Firstly, KTD provide better FDR control compared to vanilla KI. Vanilla KI fails to control the FDR for GPT-2 while KTD bound the FDR when $m$ is greater than or equal to 10. Secondly, the increase of $m$ not only benefits FDR but also power. As the $m$ increases, both power and FDR get improved consistently. Finally, for Pythia and GPT-Neo, the power and FDR approaches 1 and 0 as $m$ goes to infinity, which validate the asymptotic optimal property given in Theorem 1 and Theorem 2.

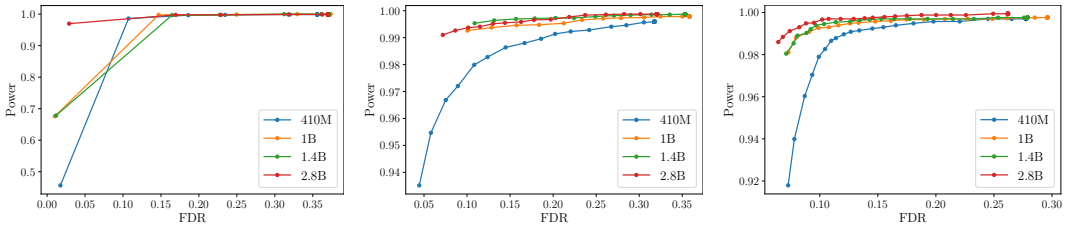
To validate the second point, we plot the power against FDR for different values of $m$ in Figure 4. From this figure, we observe that as $m$ increases, the curves move closer to the upper left corner, indicating that increasing $m$ benefits the power-FDR trade-off. Even relatively small values of $m$, such as $m = 3$, result in significant improvements. This observation illustrates that the introduction of multiple draws can significantly enhance the trade-off between power and FDR. Moreover, as depicted in the figure, GPT-2 exhibits an unusual tendency when the number of knockoff draws $m$ is small (1 or 3): occasionally, the FDR decreases as the target FDR level $q$ increases. We hypothesize that this tendency is due to the high instability associated with small $m$.

Since the computation time is primarily dominated by the calculation of gradient norms for the knockoffs. As a result, Figure 3 also shows the trade-off between computation time and performance




(a) GPT-2 (b) Pythia (c) GPT-Neo

Figure 4: Trade-off between FDR and power under different $m$. For each $m$, we vary FDR bound $q$ from 0.1 to 1 and calculate corresponding the trade-off between power and FDR. Each subplot represent a model and each line in the subplots represents the trade-off curve under certain $m$. The closer the curve is to the upper left corner, the better the trade-off between power and FDR.




(a) WikiMIA (b) XSum (c) BBC Real Time

Figure 5: Trade-off between FDR and power under different model size. For each model size, varying FDR bound $q$ are applied to compute the trade-off between power and FDR. Each subplot presents the results on a different dataset, with each line representing the trade-off curve of a model. Curves closer to the upper-left corner indicate a more favorable balance between power and FDR.

of power and FDR control. From the figure, we can find that $m = 10$ strikes an optimal balance, achieving good performance within a relatively short computation time.

**The influence of Model Size** We evaluate the FDR in terms of the power-FDR trade-off across three datasets using different-sized Pythia models (440M, 1B, 1.4B, 2.8B). Figure 5 shows a clear trend that the trade-off improves as model size increases. This aligns with our expectations, as larger models are more likely to memorize training data, making it easier to distinguish between training and non-training samples. Moreover, models with sizes larger than 1 billion parameters exhibit relatively high power even when a strict FDR bound is imposed. This observation suggests that 1 billion parameters may serve as a threshold, beyond which models can easily memorize samples from these three datasets, thereby making the distinction between training and non-training samples exceptionally clear.

## 6 Conclusion and Limitation

In this paper, we tackled the critical issue of detecting training data for LLMs with a focus on controlling FDR and introduced a novel knockoff-based method, KTD. KTD instantiates the KI framework in the context of training data detection and employs a novel calculation method that leverages multiple knockoff draws to address the high variance of the knockoff statistic in vanilla KI. To support KTD, we provided theoretical guarantees for KTD, demonstrating that it not only effectively controls the FDR but also possesses asymptotic optimal properties. Our empirical evaluations on three datasets further validated the efficacy of KTD, showcasing its superior performance in terms of FDR control and the power-FDR trade-off compared to existing methods. The limitations of our method primarily stem from two factors. First, the effectiveness of our approach depends on access to the gradients of LLMs, which may not be available for certain proprietary models where gradient information is inaccessible. This dependency also limits the applicability of our method to tasks such as paraphrased text detection. Second, our method assumes the availability of a high-quality paraphraser to generate knockoff samples. This reliance on paraphraser quality introduces a potential bottleneck in achieving optimal performance.

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# A Appendix

## A.1 The Proof of Lemma 1

**Lemma 1.** *$W_j^{\mathrm{KTD}}$ associated with irrelevant samples is symmetrically distributed around 0, i.e.,*

$$P(W_j^{\mathrm{KTD}} < -t) = P(W_j^{\mathrm{KTD}} > t) \quad \textit{for any } t > 0 \textit{ and } j \in \mathcal{D}_{\mathrm{non-train}}. \tag{13}$$

*Proof.* For simplicity, let $W_j^{(i)}$ denote $Z_j - Z_j^{(i)}$. Then,

$$\begin{aligned} P(W_j^{\mathrm{KTD}} < -t) &= P\left(\frac{1}{m}\sum_{i=1}^{m} W_j^{(i)} < -t\right) \\ &= \int_{-\infty}^{-t}\int_{D_v} P(W_j^{(1)} = a_1, W_j^{(2)} = a_2, \ldots, W_j^{(m)} = a_m)\, da_1\, da_2\, \ldots\, da_m\, dv, \end{aligned} \tag{14}$$

where $D_v = \{(a_1, a_2, \ldots, a_m) \mid \frac{1}{m}\sum_{i=1}^{m} a_i = v\}$.

According to Lemma 3.3 in Candes et al. (2018), the signs of $W_j^{(i)}$ are independent of their magnitudes. As a result, we can express the probability $P(W_j^{(1)} = a_1, W_j^{(2)} = a_2, \ldots, W_j^{(m)} = a_m)$ as the product of the following two terms:

1. $P(|W_j^{(1)}| = |a_1|, |W_j^{(2)}| = |a_2|, \ldots, |W_j^{(m)}| = |a_m|)$,
2. $\prod_{i=1}^{m} P(\mathrm{sign}(W_j^{(i)}) = \epsilon_i)$, where $\epsilon_i = \mathrm{sign}(a_i) \in \{-1, 1\}$.

Given the symmetric property of standard knockoff statistic $W_j$, we have $P(\mathrm{sign}(W_j^{(i)}) = -1) = P(\mathrm{sign}(W_j^{(i)}) = 1)$. Therefore, switching the sign of $W_j^{(i)}$ will not affect the probability above. Consequently, for any element $(a_1, a_2, \ldots, a_m)$ in $D_v$, there exists a corresponding element $(-a_1, -a_2, \ldots, -a_m)$ in $D'_v = \{(a'_1, a'_2, \ldots, a'_m) \mid \frac{1}{m}\sum_{i=1}^{m} a'_i = -v\}$ satisfying $P(W_j^{(1)} = a_1, W_j^{(2)} = a_2, \ldots, W_j^{(m)} = a_m) = P(W_j^{(1)} = a'_1, W_j^{(2)} = a'_2, \ldots, W_j^{(m)} = a'_m)$. As a result, Equation 14 equals:

$$\begin{aligned} &\int_{-\infty}^{-t}\int_{D_v} P(W_j^{(1)} = a_1, W_j^{(2)} = a_2, \ldots, W_j^{(m)} = a_m)\, da_1\, da_2\, \ldots\, da_m\, dv \\ &= \int_{-\infty}^{-t}\int_{D'_v} P(W_j^{(1)} = a'_1, W_j^{(2)} = a'_2, \ldots, W_j^{(m)} = a'_m)\, da_1\, da_2\, \ldots\, da_m\, dv \\ &= \int_{t}^{\infty}\int_{D_v} P(W_j^{(1)} = a_1, W_j^{(2)} = a_2, \ldots, W_j^{(m)} = a_m)\, da_1\, da_2\, \ldots\, da_m\, dv \\ &= P\left(\frac{1}{m}\sum_{i=1}^{m} W_j^{(i)} > t\right) \\ &= P(W_j^{\mathrm{KTD}} > t). \end{aligned} \tag{15}$$

□

### A.2 THE PROOF OF THEOREM 1

**Theorem 1.** *Assuming Assumption 2 holds, the variable selection procedure described in Proposition 2 satisfies*

$$\text{Power} = \mathbb{E}\left[\frac{|\hat{\mathcal{S}} \cap \mathcal{D}_{\text{train}}|}{|\mathcal{D}_{\text{train}}|}\right] \to 1 \quad \textit{as} \quad m \to \infty. \tag{16}$$

*Proof.* The main idea of the proof comes from (Zhao et al. (2022), Theorem 6). For the sake of clarity of the article, we provide the complete proof here. Let $\xi_j$ denote the expectation of $W_j^{\text{KTD}}$, i.e., $\xi_j = \mathbb{E}[W_j^{\text{KTD}}]$, and let $\xi$ be the minimum $\xi_j$ among the relevant variable set $\mathcal{D}_{\text{train}}$, i.e., $\xi = \min_{j \in \mathcal{D}_{\text{train}}} \xi_j$. Use $\sigma_j$ to represent the standard error of $W_j^{\text{KTD}}$.

For any $j \in \mathcal{D}_{\text{train}}$, we have

$$\begin{aligned} P(W_j^{\text{KTD}} > -\frac{\xi}{2}) &\geq P(W_j^{\text{KTD}} > \frac{\xi_j}{2}) \\ &\geq P(|W_j^{\text{KTD}} - \xi_j| < \frac{\xi_j}{2}) \\ &\geq 1 - \frac{4\sigma_j^2}{\xi_j^2 m} \\ &\geq 1 - \frac{4\sigma_j^2}{\xi^2 m}. \end{aligned} \tag{17}$$

Due to the symmetric property of elements in $\mathcal{D}_{\text{non-train}}$, we have $\xi_j = 0$ for all $j \in \mathcal{D}_{\text{non-train}}$. Hence, similar to the above equation, the following formula holds:

$$P(W_j^{\text{KTD}} > -\frac{\xi}{2}) \geq 1 - \frac{4\sigma_j^2}{\xi^2 m} \qquad \forall j \in \mathcal{D}_{\text{non-train}}. \tag{18}$$

Combining Equation 17 and Equation 18, we get:

$$\begin{aligned} P(\min_j W_j^{\text{KTD}} < -\frac{\xi}{2}) &= P(W_1^{\text{KTD}} < -\frac{\xi}{2} \vee W_2^{\text{KTD}} < -\frac{\xi}{2} \vee \cdots \vee W_n^{\text{KTD}} < -\frac{\xi}{2}) \\ &= \sum_{j=1}^{n} P(W_j^{\text{KTD}} < -\frac{\xi}{2}) \\ &\leq \frac{4}{\xi^2 m} \sum_{j=1}^{n} \sigma_j^2. \end{aligned} \tag{19}$$

If $\min_j W_j^{\text{KTD}} < -\frac{\xi}{2}$, according to the procedure described in Proposition 2 to determine the threshold, we have $\tau \leq \max\{0, -\min_j W_j^{\text{KTD}}\}$. Therefore, we can derive a lower bound for the power:

$$\text{Power} = \mathbb{E}\left[\frac{|\mathcal{D}_{\text{train}} \cap \hat{\mathcal{S}}|}{|\mathcal{D}_{\text{train}}|}\right] \geq \mathbb{E}\left[\frac{|\mathcal{D}_{\text{train}} \cap \hat{\mathcal{S}}|}{|\mathcal{D}_{\text{train}}|} \middle| \min_j W_j^{\text{KTD}} > -\frac{\xi}{2}\right] \cdot P(\min_j W_j^{\text{KTD}} > -\frac{\xi}{2}). \tag{20}$$

Note that $\tau < \frac{\xi}{2}$. Thus, the above formula is less than or equal to

$$\frac{1}{|\mathcal{D}_{\text{train}}|} \sum_{j \in \mathcal{D}_{\text{train}}} P(W_j^{\text{KTD}} > \frac{\xi}{2}) \cdot P(\min_j W_j^{\text{KTD}} > -\frac{\xi}{2}). \tag{21}$$

This lower bound approaches 1 as $m \to \infty$. □

### A.3 THE PROOF OF THEOREM 2

**Theorem 2.** *Assuming Assumption 2 holds and the threshold $\tau$ found by Proposition 2 is not equal to 0, the variable selection procedure satisfies*

$$\mathrm{FDR} = \mathbb{E}\left[\frac{|\tilde{\mathcal{S}} \cap \mathcal{D}_{\mathrm{train}}|}{|\hat{\mathcal{S}}|}\right] \to 0 \quad as \quad m \to \infty. \tag{22}$$

*Proof.* Similar to Equation 19, we have:

$$P(\min_{j \in \mathcal{D}_{\mathrm{train}}} W_j^{\mathrm{KTD}} < \frac{\xi}{2}) \leq \frac{4}{\xi^2 m} \sum_{j \in \mathcal{D}_{\mathrm{train}}} \sigma_j^2. \tag{23}$$

This probability approaches 0 as $m \to \infty$.

We then consider FDR:

$$\begin{aligned}
&\mathbb{E}\left[\frac{|\mathcal{D}_{\mathrm{non-train}} \cap \hat{\mathcal{S}}|}{|\hat{\mathcal{S}}|}\right] = \mathbb{E}\left[\frac{|\mathcal{D}_{\mathrm{non-train}} \cap \hat{\mathcal{S}}|}{|\hat{\mathcal{S}}|}\Big| \min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}\right] \cdot P(\min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}) \\
&+ \mathbb{E}\left[\frac{|\mathcal{D}_{\mathrm{non-train}} \cap \hat{\mathcal{S}}|}{|\hat{\mathcal{S}}|}\Big| \min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}, \min_{j \in \mathcal{D}_{\mathrm{train}}} W_j^{\mathrm{KTD}} < \frac{\xi}{2}\right] \cdot P(\min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}, \min_{j \in \mathcal{D}_{\mathrm{train}}} W_j^{\mathrm{KTD}} < \frac{\xi}{2}) \\
&+ \mathbb{E}\left[\frac{|\mathcal{D}_{\mathrm{non-train}} \cap \hat{\mathcal{S}}|}{|\hat{\mathcal{S}}|}\Big| \min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}, \min_{j \in \mathcal{D}_{\mathrm{train}}} W_j^{\mathrm{KTD}} > \frac{\xi}{2}\right] \cdot P(\min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}, \min_{j \in \mathcal{D}_{\mathrm{train}}} W_j^{\mathrm{KTD}} > \frac{\xi}{2}).
\end{aligned} \tag{24}$$

The probabilities in the first and second terms go to 0 as $m \to \infty$. Hence, we focus on the expectation in the third term. Under the condition $\min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}$, we have $\tau < \frac{\xi}{2}$. Additionally, considering $\min_{j \in \mathcal{D}_{\mathrm{train}}} W_j^{\mathrm{KTD}} > \frac{\xi}{2}$, it follows that $|\hat{\mathcal{S}}| > |\mathcal{D}_{\mathrm{train}}|$.

$$\begin{aligned}
&\mathbb{E}\left[\frac{|\mathcal{D}_{\mathrm{non-train}} \cap \hat{\mathcal{S}}|}{|\hat{\mathcal{S}}|}\Big| \min_j W_j^{\mathrm{KTD}} > -\frac{\xi}{2}, \min_{j \in \mathcal{D}_{\mathrm{train}}} W_j^{\mathrm{KTD}} > \frac{\xi}{2}\right] \\
&\leq \frac{1}{|\mathcal{D}_{\mathrm{train}}|} \sum_{j \in \mathcal{D}_{\mathrm{non-train}}} P(W_j^{\mathrm{KTD}} > \tau).
\end{aligned} \tag{25}$$

Since we assume $\tau > 0$, there exists a $\delta > 0$ such that $\tau > \delta > 0$. Therefore, the above formula is less than or equal to:

$$\frac{1}{|\mathcal{D}_{\mathrm{train}}|} \sum_{j \in \mathcal{D}_{\mathrm{non-train}}} \frac{\sigma_j}{m\delta}, \tag{26}$$

which goes to 0 as $m \to \infty$. □

### A.4 PARAMETER SETTINGS

For fine-tuning, we used the following settings:

- warmup_step = 100
- weight_decay = 0.01
- batch_size = 2
- num_epochs = 2

All other hyperparameters were set to the default values provided by the 'TrainingArguments' class in the Transformers library.

For paraphrasing, we applied the following configurations:

- Top-k sampling with 'topk' = 50
- Top-p sampling with 'topp' = 0.95
- Temperature scaling with 'temperature' = 1.9