A SIMULATION-BASED FRAMEWORK FOR ROBUST FEDERATED LEARNING TO TRAINING-TIME ATTACKS

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ABSTRACT

Well-known robust aggregation schemes in federated learning (FL) are shown to be vulnerable to an informed adversary who can tailor training-time attacks (Fang et al., 2020; Xie et al., 2020). We frame robust distributed learning problem as a game between a server and an adversary that is able to optimize strong training-time attacks. We introduce RobustTailor, a simulation-based framework that *prevents the adversary from being omniscient*. The simulated game we propose enjoys theoretical guarantees through a regret analysis. RobustTailor improves robustness to training-time attacks significantly while preserving almost the same privacy guarantees as standard robust aggregation schemes in FL. Empirical results under challenging attacks show that RobustTailor performs similar to an *upper bound* with perfect knowledge of honest clients.

1 INTRODUCTION

In *federated learning* (FL), a global/personalized model is learnt from data distributed on multiple clients without sharing data (McMahan et al., 2017; Kairouz et al., 2021). Clients compute their (stochastic) gradients using their own local data and send them to a central server for aggregating and updating a model. While FL offers improvements in terms of privacy, it creates additional challenges in terms of robustness. Clients are often prone to the bias in the *stochastic gradient* updates, which comes not only from poor sampling or data noise but also from *malicious attacks of Byzantine clients* who may send arbitrary messages to the server instead of correct gradients (Guerraoui et al., 2018). Therefore, in FL, it is essential to guarantee some level of robustness to Byzantine clients that might be compromised by an adversary.

Compromised clients are vulnerable to data/model poisoning and tailored attacks (Fang et al., 2020). Byzantine-resilience is typically achieved by robust gradient aggregation schemes *e.g.,* Krum (Blanchard et al., 2017), Comed (Yin et al., 2018), and trimmedmean (Yin et al., 2018). These aggregators are resilient against attacks that are designed in advance. However, such robustness is insufficient in practice since a powerful adversary could learn the aggregation rule and tailor its training-time attack. It has been shown that well-known Byzantine-resilient gradient aggregation schemes are susceptible to an informed adversary that can tailor the attacks (Fang et al., 2020). Specifically, Fang et al. (2020) and Xie et al. (2020) proposed efficient and nearly optimal training-time attacks that circumvent Krum, Comed, and trimmedmean. A tailored attack is designed with a prior knowledge of the robust aggregation rule used by the server, such that the attacker has a provable way to corrupt the training process. Given the information leverage of the adversary, it is a significant challenge to establish successful defense mechanisms against such tailored attacks.

In this paper, we formulate robust distributed learning problem against training-time attacks as a game between a server and an adversary. To prevent the adversary from being omniscient, we propose to follow a *mixed strategy* using the existing robust aggregation rules. In real-world settings, both server and adversary have a number of aggregation rules and attack programs. How to utilize these aggregators efficiently and guarantee robustness is a challenging task. We address scenarios where neither the specific attack method is known in advance by the aggregator nor the exact aggregation rule used in each iteration is known in advance by the adversary, while the adversary and the server know the set of server's aggregation rules and the set of attack programs, respectively.[1]

[1]While this assumption is essential to frame our game, we provide experimental results on challenging settings where the server does not know the set of attack programs.

Due to information asymmetry between the server and the adversary, we assume every client donates *a small amount of honest data* to the server as the price to achieve some level of security more proactively and efficiently. Providing such public dataset to achieve robustness is a common assumption in FL (Fang & Ye, 2022; Huang et al., 2022; Kairouz et al., 2021; Yoshida et al., 2020; Zhao et al., 2018; Fang et al., 2020; Xie et al., 2020; Cao & Lai, 2019; Chang et al., 2019; Cao et al., 2020). We propose RobustTailor, a scheme based on simulating aggregation rules under different attacks. With minimal privacy leakage, RobustTailor realizes high resilience to training-time attacks. RobustTailor maintains stable performance under a challenging mixed attack, a strategy we propose for the adversary to simulate and design a successful attack when a smart server uses a mixed strategy to make the problem of attack design computationally harder. We emphasize that any deterministic Byzantine-resilient algorithm can be added in server's aggregation pool. Similarly, any attack can be used in the set of adversary's attack programs.

1.1 Summary of contributions

- We frame robust distributed learning problem as *a game* between a server and an adversary that tailors training-time attacks.
- We propose a simulation-based framework RobustTailor to improve robustness by preventing the adversary from being omniscient.
- The simulated game we propose enjoys theoretical guarantees through a regret analysis.
- Empirical studies validate our theory and show that RobustTailor *preforms similar to an upper bound* with perfect knowledge of all honest clients over the course of training. Even under a challenging mixed attack strategy, RobustTailor outperforms the robust baselines in terms of robustness and accuracy.

1.2 Related work

In this section, we provide a summary of related work. See Appendix A for complete related work.

Training-time attacks in FL. Federated learning (FL) usually suffers from training-time attacks (Biggio et al., 2012; Bhagoji et al., 2019; Sun et al., 2019; Bagdasaryan et al., 2020) because the server trains the model across various unreliable clients with private datasets. A strong adversary can potentially participate in every training round and adapt its attacks to an updated model. In *model update poisoning*, a class of training-time attacks, an adversary controls some clients and directly manipulates their outputs aiming to bias the global model towards opposite direction of honest training (Kairouz et al., 2021). If Byzantine clients have access to the updates of honest clients, they can tailor their attacks and make them difficult to detect (Fang et al., 2020; Xie et al., 2020).

Robust aggregation and Byzantine resilience. To improve robustness under general Byzantine clients, a number of robust aggregation schemes have been proposed, which are mainly inspired by robust statistics such as median-based aggregators (Yin et al., 2018; Chen et al., 2017), Krum (Blanchard et al., 2017), trimmed mean (Yin et al., 2018). Moreover, Fang et al. (2020); Xie et al. (2020); Cao & Lai (2019); Cao et al. (2020) propose server-side verification methods using auxiliary data. Karimireddy et al. (2021) and Alistarh et al. (2018) propose history-aided aggregators. Ramezani-Kebrya et al. (2022) propose a framework based on randomization of multiple aggregation rules. However, none of them selects a proper aggregation rule proactively during training as our framework RobustTailor, and all of them can be used in RobustTailor while we mainly focus on statistical-based aggregators in this paper. Although past work has shown that these aggregators can defend successfully under specific conditions (Blanchard et al., 2017; Chen et al., 2017; Su & Vaidya, 2016), Fang et al. (2020) and Xie et al. (2020) argue that Byzantine-resilient aggregators can fail when an informed adversary tailor a careful attack and Gouissem et al. (2022) proves that such aggregation rules are vulnerable. Therefore, developing a robust and efficient algorithm under such strong tailored attacks is essential to improve security of FL, which is the goal of this paper.

Game theory in FL. Online convex optimization (OCO) framework (Zinkevich, 2003) is widely influential in the learning community (Hazan et al., 2016; Shalev-Shwartz et al., 2012), and bandit convex optimization (BCO) as an extension of OCO was proposed by Awerbuch & Kleinberg (2008) for decision making with limited feedback. Bandit paradigms paralleling FL framework are proposed by Shi & Shen (2021) and its extension under Byzantine attacks is proposed by Demirel et al. (2022). However, they account for uncertainties from both arm and client sampling rather than robust

aggregation in this paper. In this paper, we frame robust distributed learning problem as a game and consider the *bandit feedback model*.

2 PROBLEM SETTING

Under a synchronous setting in FL, clients compute their updates on their own local data and then aggregate from all peers to update model parameters. Consider a general distributed system consisting of a parameter server and n clients (Chen et al., 2017; Abadi et al., 2016; Li et al., 2014). Suppose that f Byzantine clients are controlled by an adversary and behave arbitrarily. Let $\mathbf{x} \in \mathbb{R}^d$ denote a machine learning model, *e.g.,* it represents all weights and biases of a neural network. We consider minimizing an overall empirical risk of multiple clients, which can be formulated as finite-sum problem:

$$\min_{\mathbf{x} \in \mathbb{R}^d} F(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} F_i(\mathbf{x}) \tag{FL}$$

where $F_i : \mathbb{R}^d \to \mathbb{R}$ denotes the training error (empirical risk) of $\mathbf{x}$ on the local data of client i.

At iteration t, *honest* clients compute and send *honest* stochastic gradients $\mathbf{g}_i(\mathbf{x}_t) = \nabla F_i(\mathbf{x}_t)$ for $i \in [n-f]$ while *Byzantine* clients, controlled by an informed adversary, output *attacks* $\mathbf{b}_j \in \mathbb{R}^d$ for $j \in [f]$. The server receives all n updates and aggregates them following a particular *robust* aggregation rule, which outputs an aggregated and updated model $\mathbf{x}_{t+1} \in \mathbb{R}^d$. Finally, the server broadcasts $\mathbf{x}_{t+1}$ to all clients.

2.1 GAME CONSTRUCTION

We frame this distributed learning problem under training-time attack as a game played by the adversary and the server. The informed adversary and training-time attacks are described in Section 2.1.1. The details of aggregators for the server are provided in Section 2.1.2. Though our formulation seems natural and intuitive, to the best of our knowledge, our work is the *first work* that frames robust learning problem under training-time tailored attacks as a game. The adversary aims at corrupting training while the server aims at learning an effective model, which achieves a satisfactory overall empirical risk over honest clients.

2.1.1 INFORMED ADVERSARY WITH ATTACKS

The adversary controls f out of n clients where these Byzantine clients collude aiming at disturbing the entire training process by sending training-time attacks (Biggio et al., 2012; Bhagoji et al., 2019; Sun et al., 2019; Bagdasaryan et al., 2020). We assume $n \geq 2f + 1$ which is a common assumption in the literature (Guerraoui et al., 2018; Blanchard et al., 2017; Alistarh et al., 2018; Rajput et al., 2019; Karimireddy et al., 2022); otherwise the adversary will be able to provably control the optimization trajectory and set the global model arbitrarily (Lamport et al., 1982).

An *informed adversary* controls the outputs of those compromised clients, *e.g.,* their gradients throughout the course of training. Moreover, the informed adversary has *full knowledge* of the outputs of $n - f$ honest clients across the course of training. Having access to the gradients of honest nodes, the adversary can compute the global aggregated gradient of an omniscient aggregation rule, which is the empirical mean of *all honest updates without an attack*:

$$\mathbf{g}^* = \frac{1}{n-f} \sum_{i=1}^{n-f} \mathbf{g}_i. \tag{1}$$

When an adversary knows a particular server's aggregation rule, it is able to design tailored attacks using $n - f$ honest gradients (Fang et al., 2020).

Definition 1 (Attack algorithm)**.** *Let $\{\mathbf{g}_1, \ldots, \mathbf{g}_{n-f}\}$ denote the set of honest updates computed by $n - f$ honest clients. The adversary designs f Byzantine updates using an* AT *algorithm:*

$$\{\mathbf{b}_{n-f+1}, \ldots, \mathbf{b}_n\} := \mathrm{AT}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-f}, \mathcal{A}). \tag{2}$$

where $\mathcal{A}$ denotes the set of aggregators formally defined in Section 2.1.2.

It is shown that several tailored attacks can be designed efficiently and provably fail well-known aggregation rules with a specific structure *e.g.,* Krum, Comed, and Krum + resampling (Fang et al., 2020; Xie et al., 2020; Ramezani-Kebrya et al., 2022). Suppose that the adversary has a set of S computationally tractable programs to design tailored attacks:

$$\mathcal{F} = \{\mathrm{AT}_1, \mathrm{AT}_2, \ldots, \mathrm{AT}_S\}. \tag{3}$$

2.1.2 Server with aggregators

The server aims at learning an effective model, which achieves a satisfactory overall empirical risk over honest clients comparable to that *under no attack*. To update the global model, the server aggregates all gradients sent by clients at each iteration.

Definition 2 (Aggregation rule)**.** *Let $\mathbf{g}'_j \in \mathbb{R}^d$ denote an update received from client j, which can be either an honest or compromised client for $j \in [n]$. The server aggregates all updates from n clients and outputs a global update $\mathbf{g} \in \mathbb{R}^d$ using an aggregation rule* AG*:*

$$\mathbf{g} = \mathrm{AG}(\mathbf{g}'_1, \ldots, \mathbf{g}'_n, \mathcal{F}). \tag{4}$$

where $\mathcal{F}$ denotes the set of attacks defined in Section 2.1.1.

We assume that the server knows the number of compromised clients f or an upper bound on f, which is a common assumption in robust learning (Guerraoui et al., 2018; Blanchard et al., 2017; Alistarh et al., 2018; Rajput et al., 2019; Karimireddy et al., 2022).

However, the server does not know the specific Byzantine clients among n clients in this distributed system such that the server cannot compute $\mathbf{g}^*$ in Eq. (1) directly. To learn and establish some level of robustness against training-time attacks, several Byzantine-resilient aggregation rules have been proposed *e.g.,* Krum (Blanchard et al., 2017) and Comed (Yin et al., 2018). These methods inspired by robust statistics provide rigorous convergence guarantees under *specific settings* and have been shown to be vulnerable to tailored attacks (Fang et al., 2020; Xie et al., 2020). The set of M aggregators used by the server is denoted by

$$\mathcal{A} = \{\mathrm{AG}_1, \mathrm{AG}_2, \ldots, \mathrm{AG}_M\}. \tag{5}$$

Note that the pool of aggregators $\mathcal{A}$ and the set of attacks $\mathcal{F}$ are known by both the server and the adversary, but the specific AT^t are AG^t chosen at iteration t are unknown. Moreover, such powerful but *not omniscient* adversary does not have access to the random seed generator at the server. This is a mild and common assumption in cryptography, which requires a secure source of entropy to generate random numbers at the server (Ramezani-Kebrya et al., 2022).

To avoid trivial solutions, we assume each aggregation rule is robust (formal definition of robustness is provided in Appendix B) against a *subset of attack algorithms* in $\mathcal{F}$ while no aggregation rule is immune to all attack algorithms. Similarly, each attack program can *provably fail* one or more aggregation rules in $\mathcal{A}$ while no attack program can provably fail *all aggregation rules.*

2.2 Problem formulation

To evaluate the performance of an updated global model, *i.e.,* the output of AG in Eq. (4), we define a loss function, which measures the discrepancy of the output of AG and an omniscient model update *under no attack*.

Definition 3 (Loss function)**.** *The loss function associated with using aggregation rule* AG *under attack* AT *is defined as:*

$$\begin{aligned}\ell(\mathrm{AG}, \mathrm{AT}, \{\mathbf{g}'_i\}_{i=1}^n) &= ||\mathrm{AG}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-f}, \mathrm{AT}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-f}, \mathcal{F}), \mathcal{A}) - \mathbf{g}^*|| \\ &= ||\mathrm{AG}(\mathbf{g}_1, \ldots, \mathbf{g}_{n-f}, \mathbf{b}_{n-f+1}, \ldots, \mathbf{b}_n, \mathcal{A}) - \mathbf{g}^*||.\end{aligned} \tag{6}$$

where $\mathbf{g}^*$ is the ideal model under no attack which is computed in Eq. (1)*.*

To train the global model, the server takes multiple rounds of stochastic gradient descent by aggregating the stochastic gradients from clients. However, some gradients might be corrupt at each round, which are sent by compromised clients controlled by the adversary. We frame this robust distributed

learning scenario as a *game* between the adversary and the server. The server wants to minimize the loss defined in Definition 3, while the adversary aims to maximize it. This game as a *minimax optimization problem* is formulated as:

$$\min_{\mathrm{AG}\in\mathcal{A}} \max_{\mathrm{AT}\in\mathcal{F}} \ell(\mathrm{AG}, \mathrm{AT}, \{\mathbf{g}'_i\}_{i=1}^n). \tag{MinMax}$$

The entire process of model aggregation with T rounds is shown in Algorithm 1. Note that $\mathbb{E}_{\mathcal{G}}$ denotes the expectation with respect to randomness due to stochastic gradients. Ideally, the game in MinMax can reach a Nash equilibrium (NE) (Nash, 1950). However, the loss is not computable for the server, since the server cannot distinguish honest gradients such that $\mathbf{g}^*$ is unknown for it. Therefore, we will *simulate* the game in the following Section 3.

Algorithm 1 Update model from the perspective of server

Input: Initial weight vector $\mathbf{x}_0$, learning rate η_t, iteration rounds T, number of clients n, set of aggregation rules $\mathcal{A}$, set of attack algorithms $\mathcal{F}$, and public dataset.

for $t = 1$ ***to*** T **do**
  Server sends $\mathbf{x}_t$ to all clients.
  for $i = 1$ ***to*** $n - f$ **do**
    Honest client i computes local gradient $\mathbf{g}_i(\mathbf{x}_t) = \nabla F_i(\mathbf{x}_t)$.
  Compromised clients send attacks $\mathrm{AT}^t(\{\mathbf{g}_i\}_{i=1}^{n-f}, \mathcal{A})$.
  Sever receives gradients from all clients $\{\mathbf{g}'_i\}_{i=1}^n$.
  Server chooses AG^t by solving

$$\min_{\mathrm{AG}^t\in\mathcal{A}} \max_{\mathrm{AT}^t\in\mathcal{F}} \mathbb{E}_{\mathcal{G}}[\ell(\mathrm{AG}^t, \mathrm{AT}^t, \{\mathbf{g}'_i\}_{i=1}^n)].$$

  Server updates the model $\mathbf{x}_{t+1} = \mathbf{x}_t - \eta_t \mathrm{AG}^t(\{\mathbf{g}'_i\}_{i=1}^n, \mathcal{F})$.

3 Robust aggregation

Because MinMax cannot be solved during the process of updating the model, we propose to *simulate* it instead and obtain an optimized aggregator for model updates. As mentioned in Section 2, the informed adversary has *an advantage* over the server since it can perfectly estimate $\mathbf{g}^*$ in Eq. (1) while the server does not have such knowledge and cannot identify honest clients a priori. We assume that each client donates a *small amount* of data as a *public dataset* to the server to achieve some level of security by controlling the *information* gap between the server and adversary. Let $\widetilde{\mathbf{g}}$ denote the update computed at the server using the public dataset, which is a rough estimate of $\mathbf{g}^*$.

Remark 1. *The server may obtain such public dataset from other sources not necessarily the current clients. It is sufficient as long as the collected public dataset represents the clients' data distribution by some extent. To guarantee convergence, we only require that the update from the public dataset is a* **rough estimate** *of the ideal* $\mathbf{g}^*$*. In particular, the quality of such estimate directly impacts the convergence of our proposed algorithm (see Section 4 for details). As the quality of such estimate improves, the convergence of the global model to an effective model improves. The existence of such public dataset is a valid and common assumption in FL (Fang & Ye, 2022; Huang et al., 2022; Kairouz et al., 2021; Yoshida et al., 2020; Chang et al., 2019; Zhao et al., 2018).*

For the simulation, the server generates the simulated gradients $\{\widetilde{\mathbf{g}}_i\}_{i=1}^{n-f}$ based on the public dataset. The loss function in the simulated game becomes

$$\widetilde{\ell}(\mathrm{AG}, \mathrm{AT}, \{\widetilde{\mathbf{g}}_i\}_{i=1}^{n-f}) = ||\mathrm{AG}(\widetilde{\mathbf{g}}_1, \ldots, \widetilde{\mathbf{g}}_{n-f}, \mathrm{AT}((\widetilde{\mathbf{g}}_1, \ldots, \widetilde{\mathbf{g}}_{n-f}, \mathcal{A}), \mathcal{F}) - \frac{1}{n-f}\sum_{i=1}^{n-f} \widetilde{\mathbf{g}}_i|| \tag{7}$$

Let $\mathcal{L} \in \mathbb{R}_+^{M\times S}$ denote the loss of M aggregators corresponding to S attacks, and $\mathcal{L}(\mathrm{AG}_i, \mathrm{AT}_j)$ represents the loss associated with aggregation rule i in $\mathcal{A}$ under attack j in $\mathcal{F}$ in the simulation. After the adversary has committed to a probability distribution $\mathbf{q}$ over S attack algorithms, the server chooses a probability distribution $\mathbf{p}$ over M aggregation rules. Then, the server incurs the loss $\widetilde{\ell}(\mathbf{p}, \mathbf{q}) = \mathbf{p}\mathbb{E}_{\mathcal{G}}[\mathcal{L}]\mathbf{q}^\top$. We will solve Sim-MinMax below instead of MinMax.

$$\min_{\mathbf{p}\in\Delta_M} \max_{\mathbf{q}\in\Delta_S} \mathbf{p}\mathbb{E}_{\mathcal{G}}[\mathcal{L}]\mathbf{q}^\top. \tag{Sim-MinMax}$$

where Δ_M and Δ_S denote the probability simplex in $[M]$ and $[S]$, respectively.

In practice, it is computationally expensive to compute $\mathcal{L}^{M\times S}$ and meanwhile there is noise due to the gradients. Therefore, we consider *bandit feedback model* with limited feedback, in which the server and adversary can only observe the loss through exploration. To solve Sim-MinMax in the bandit feedback model, one player could implement the well-known *Exponential-weight Algorithm for Exploration and Exploitation (Exp3)* (Seldin et al., 2013) whose detailed description is deferred to Appendix C.

Due to two players (the server and the adversary) in our model, we propose an algorithm which simultaneously executes Exp3. We term our proposed *robust aggregation scheme* as $\mathrm{RobustTailor}$, which outputs an optimized AG at each iteration. The specific algorithm from perspective of the server is shown in Algorithm 2. Using the public dataset, the server generates $n-f$ noisy stochastic gradients $\widetilde{\mathbf{g}}_i = \nabla \widetilde{F}_i(\mathbf{x}_t)$ for $i \in [n-f]$ at iteration t. After K rounds of simulation on $\{\widetilde{\mathbf{g}}_i\}_{i=1}^{n-f}$, the server obtains a final probability distribution $\mathbf{p}$ and selects an aggregation rule by sampling from $\mathbf{p}$. The steps for our robust training procedure are summarized in Algorithm 3. Note that Appendix H demonstrates both theoretical analysis and empirical results of $\mathrm{RobustTailor}$'s computation complexity.

Algorithm 2 $\mathrm{RobustTailor}$

Input: Updating rates λ_1, λ_2, $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, simulation rounds K, simulated gradients $\{\widetilde{\mathbf{g}}_i\}_{i=1}^{n-f}$, $\mathcal{A}$, $\mathcal{F}$.

Initialize weight vector $w^0(i) = 1$ for $i \in [M]$ and $v^0(j) = 1$ for $j \in [S]$.

for $k = 1$ ***to*** K **do**

- Set $\mathbf{p}^k(\widetilde{\mathrm{AG}}_i) = (1-\lambda_1)\frac{w^k(i)}{\sum_{i=1}^{M} w^k(i)} + \lambda_1 \frac{1}{M}$ for $i \in [M]$.
- Set $\mathbf{q}^k(\widetilde{\mathrm{AT}}_j) = (1-\lambda_2)\frac{v^k(j)}{\sum_{j=1}^{S} v^k(j)} + \lambda_2 \frac{1}{S}$ for $j \in [S]$.
- Sample $\mathrm{AG}^k \sim \mathbf{p}^k$ and $\mathrm{AT}^k \sim \mathbf{q}^k$ respectively.
- Estimate the loss $\ell^k = \widetilde{\ell}(\mathrm{AG}^k, \mathrm{AT}^k, \{\widetilde{\mathbf{g}}_i\}_{i=1}^{n-f})$.
- Set $\hat{\ell}_1^k(i) = \frac{\mathbb{I}\{\widetilde{\mathrm{AG}}_i = \mathrm{AG}^k\}}{\mathbf{p}^k(\widetilde{\mathrm{AG}}_i)}\ell^k, \quad w^{k+1}(i) = w^k(i)\exp(-\tilde{\lambda}_1 \hat{\ell}_1^k(i)/M)$ for $i \in [M]$.
- Set $\hat{\ell}_2^k(j) = \frac{\mathbb{I}\{\widetilde{\mathrm{AT}}_j = \mathrm{AT}^k\}}{\mathbf{q}^k(\widetilde{\mathrm{AT}}_j)}\ell^k, \quad v^{k+1}(j) = v^k(j)\exp(\tilde{\lambda}_2 \hat{\ell}_2^k(j)/S)$ for $j \in [S]$.

Set $\mathbf{p}_i = \frac{\sum_{k=1}^{K} \mathbf{p}^k(\widetilde{\mathrm{AG}}_i)}{K}$ for $i \in [M]$.

Sample $\mathrm{AG} \sim \mathbf{p}$.

Output: AG.

Algorithm 3 Server's aggregation

Input: Learning rate η_t, n clients, f compromised clients, iteration rounds T, $\mathcal{A}$ and $\mathcal{F}$

Initialize model $\mathbf{x}_0$.

for $t = 1$ ***to*** T **do**

- Send $\mathbf{x}_t$ to all clients.
- Receive gradients from all clients $\{\mathbf{g}_i'\}_{i=1}^{n}$.
- Calculate simulated gradients $\{\widetilde{\mathbf{g}}_i\}_{i=1}^{n-f}$.
- Call Algorithm 2 to aggregate $\mathrm{AG}^t = \mathrm{RobustTailor}(\{\widetilde{\mathbf{g}}_i\}_{i=1}^{n-f}, \mathcal{A}, \mathcal{F})$.
- Update the global model by $\mathbf{x}_{t+1} = \mathbf{x}_t - \eta_t \mathrm{AG}^t(\{\mathbf{g}_i'\}_{i=1}^{n}, \mathcal{F})$.

The adversary can also perform simulation to *optimize* its attack at each iteration. The main differences for an adversarial simulation compared to $\mathrm{RobustTailor}$ include: 1) the adversary can use perfect honest stochastic gradients $\{\mathbf{g}_i\}_{i=1}^{n-f}$ instead of noisy estimates; 2) the probability output is $\mathbf{q}$ which is calculated by the weight vector of attacks $v(j)$ for $j \in [S]$. The details of the adversarial simulation are provided in Appendix D.

4 Theoretical Guarantees

To show convergence of the inner optimization in Algorithm 2, we first show how to turn two simultaneously played no-regret algorithms for a minimax problem into convergence to a Nash equilibrium (NE). To make the optimization problem shown in Algorithm 2 more general, we define

a new loss function $L : [M] \times [S] \to \mathbb{R}_+$. Consider simultaneously running two algorithms on the objective L, such that their respective expected regret is upper bounded by some quantities $\mathcal{R}_K^{\mathrm{i}}$ and $\mathcal{R}_K^{\mathrm{j}}$, i.e.,

$$\mathbb{E}\left[\sum_{k=1}^{K} L(i_k, j_k) - \sum_{k=1}^{K} L(i, j_k)\right] \leq \mathcal{R}_K^{\mathrm{i}}, \quad \mathbb{E}\left[\sum_{k=1}^{K} L(i_k, j) - \sum_{k=1}^{K} L(i_k, j_k)\right] \leq \mathcal{R}_K^{\mathrm{j}}, \tag{8}$$

for any $i \in [M]$ and $j \in [S]$ where the expectation is taken over the randomness of the algorithms.

Lemma 1. *Assume we run two algorithms simultaneously with regret as in (8) to obtain $\{(i_k, j_k)\}_{k=1}^K$. By playing $\bar{i}$ uniformly sampled from $\{i_k\}_{k=1}^K$, we can guarantee:*

$$\mathbb{E}_{\bar{i}}\left[L(\bar{i}, j)\right] \leq \mathbb{E}_{i^\star \sim \mathbf{p}^\star, j^\star \sim \mathbf{q}^\star}\left[L(i^\star, j^\star)\right] + \frac{1}{K}(\mathcal{R}_K^{\mathrm{i}} + \mathcal{R}_K^{\mathrm{j}}) \tag{9}$$

for any $j \in [S]$ where $(\mathbf{p}^\star, \mathbf{q}^\star)$ is a NE of the bilinear objective $\mathbb{E}_{i^\star \sim \mathbf{p}^\star, j^\star \sim \mathbf{q}^\star}\left[L(i^\star, j^\star)\right]$.

When the algorithms have sublinear regrets, we refer to them as *no-regret* algorithms. This condition ensures that the error term in (9) vanishes as $K \to \infty$. Exp3 (Auer et al., 2002), employed by both the attacker and aggregator in Algorithm 2, enjoys such a no-regret property.

Lemma 2 (Hazan et al. 2016, Lemma 6.3)**.** *Let K be the horizon, N be the number of actions, and $L_k : [N] \to \mathbb{R}_+$ be non-negative losses for all k. Then Exp3 with stepsize $\lambda = \sqrt{\frac{\log N}{KN}}$ enjoys the following regret bound:*

$$\mathbb{E}\left[\sum_{k=1}^{K} L_k\left(i_k\right) - \sum_{k=1}^{K} L_k(i)\right] \leq 2\sqrt{\mathrm{KN}\log N}, \tag{10}$$

for any $i \in [N]$, where the expectation is taken over the randomness of the algorithm.

Note that any two simultaneously played no-regret algorithms for a minimax problem can be turned into convergence to a NE following Lemmas 1 and 2. We obtain guarantees for the aggregation rule returned from Algorithm 2 as a direct consequence of Lemmas 1 and 2. Considering a specific situation in Algorithm 2, L is replaced by the simulation loss $\widetilde{\ell}$ shown in Eq. (7).

Lemma 3. *Let $\widetilde{\ell}$ be the simulation loss in Eq. (7). Sample $\mathrm{AG} \sim \mathbf{p}$ as defined in Algorithm 2 with $\tilde{\lambda}_1 = \sqrt{\frac{\log M}{KM}}$ and $\tilde{\lambda}_2 = \sqrt{\frac{\log S}{KS}}$. Then the loss is bounded in expectation for any attack $\mathrm{ATK} \in \mathcal{F}$ as,*

$$\mathbb{E}_{\mathrm{AG},\mathcal{G}}\left[\widetilde{\ell}\left(\mathrm{AG}, \mathrm{AT}, \{\mathbf{g}_i'\}_{i=1}^n\right)\right] \leq \mathbf{p}^\star \mathbb{E}_{\mathcal{G}}[\mathcal{L}](\mathbf{q}^\star)^\top + 2\frac{\sqrt{M\log M} + \sqrt{S\log S}}{\sqrt{K}}, \tag{11}$$

where $(\mathbf{p}^\star, \mathbf{q}^\star) \in \Delta_M \times \Delta_S$ is a Nash equilibrium of the zero-sum game with stochastic payoff matrix $\mathbb{E}_{\mathcal{G}}[\mathcal{L}]$ as defined in Sim-MinMax.

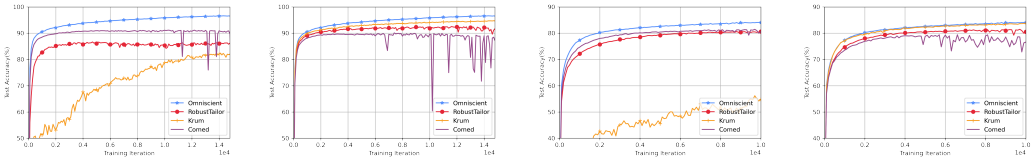
Lemma 3 implies that the simulated loss approaches the NE value even under the worst-case attack. The proofs of Lemma 1 and Lemma 3 are provided in Appendix E and Appendix F respectively. Importantly, a sufficient condition for almost sure convergence for the *outer* loop is provided in Appendix B.

5 Experimental evaluation

In this section, we evaluate the resilience of RobustTailor against tailored attacks. To provide intuitive results and show benefits of simulation in terms of robustness, we first construct a simple pool of aggregators including only Krum (Blanchard et al., 2017) and Comed (Yin et al., 2018). Concerning the adversary's tailored attacks, we consider two different types of attacks, which can successfully ruin Krum and Comed, respectively. As described in (Fang et al., 2020; Xie et al., 2020), an adversary with *ϵ-reverse attack* computes the average of honest update, scales the average with a parameter ϵ, and sends scaled updates to the server to induct the global model towards the inverse of the direction along the one without attacks. It is known that a small ϵ corrupts Krum, while a large one corrupts Comed (Fang et al., 2020; Xie et al., 2020).

We simulate training with a total of 12 clients, 2 of which are compromised by an informed adversary. We train a CNN model on MNIST (Lecun et al., 1998) under both iid and non-iid settings and meanwhile training on Fashion-MNIST (FMNIST) (Xiao et al., 2017) and CIFAR10 (Krizhevsky et al., 2009) is under iid setting. Note that the dataset is shuffled and equally partitioned among clients in the iid settings (Fig. 1). In addition, all honest clients donate 5% of their local training dataset to the server as a public dataset as specified in Remark 1, and the informed adversary has access to the gradients of honest clients. Note that the details of the model and training hyper-parameters are provided in Appendix G.1.

Single tailored attacks. RobustTailor successfully decreases the capability of the adversary to launch tailored attacks. RobustTailor maintains stability in Fig. 1a when Krum fails catastrophically under a small ϵ attack. Fig. 1b shows that RobustTailor has much less fluctuations in terms of test accuracy compared to Comed when facing a large ϵ attack. In addition, on average, RobustTailor has 70.68% probability of choosing Comed under $\epsilon = 0.5$ attack while 65.49% probability of choosing Krum under $\epsilon = 100$ attack, which proves that the server successfully learns how to defend. Training on FMNIST shows consistent results as seen in Fig. 1c and Fig. 1d and further results on CIFAR10 are in Appendix G.2. Our goal here is to show that RobustTailor is stable and performs close to the optimal method even under a simple set of rules with only Krum and Comed. Note that RobustTailor will outperform both Krum and Comed if there is a larger pool to select aggregators, which results are shown in Appendix G.2.



(a) *MNIST,* $\epsilon = 0.5$ **(b)** *MNIST,* $\epsilon = 100$ **(c)** *FMNIST,* $\epsilon = 0.1$ **(d)** *FMNIST,* $\epsilon = 100$

Figure 1: ***Test accuracy on MNIST and FMNIST under iid setting.*** *Tailored attacks* ($\epsilon = 0.1/0.5, 100$) *are applied.* RobustTailor *selects an aggregator from Krum and Comed based on the simulation at each iteration.*

Mixed attacks. We now consider additional and stronger attack strategies beyond vanilla ϵ-reverse attacks. We assume the adversary has a set of attacks including $\epsilon = 0.5$ and $\epsilon = 100$ attacks. The first attack strategy called **StochasticAttack** picks an attack from its set *uniformly at random* at each iteration. Fig. 2 shows the performance of different aggregators under StochasticAttack. RobustTailor still shows a similar level of robustness as in the case of a single tailored attack shown in Fig. 1. Another attack strategy **AttackTailor** optimizes an attack based on simulation at each iteration. Please see its detailed algorithm in Appendix D. Compared to all previous attacks, AttackTailor is much stronger since it can pick a proper attack at each iteration based on simulation under perfect knowledge of honest updates. The poison of AttackTailor shown in Fig. 3 is almost as effective as the most targeted attack tailored against a single deterministic aggregator, which shows that the adversary successfully learns how to attack. For example, Comed attacked by AttackTailor shows more and deeper fluctuations than StochasticAttack, which are similar to those under $\epsilon = 100$ attack in Fig. 1b. Importantly, RobustTailor shows impressive robustness against AttackTailor as shown in Fig. 3 since it keeps high and stable accuracy even when facing such a strong adversary.

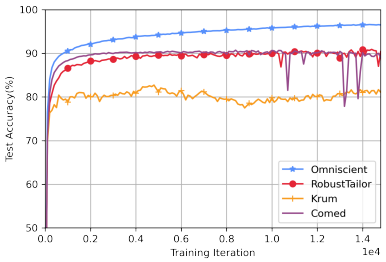


Figure 2: StochasticAttack. StochasticAttack *applies* $\epsilon = 0.5$ *and* $\epsilon = 100$ *uniform randomly.*

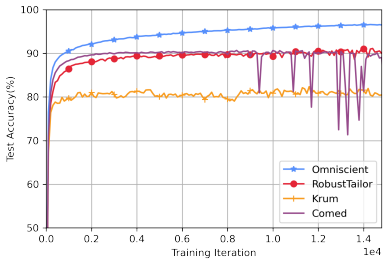
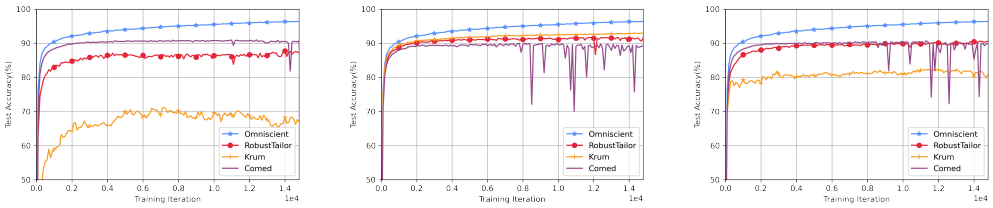


Figure 3: AttackTailor. AttackTailor *applies* $\epsilon = 0.5$ *and* $\epsilon = 100$ *based on the simulation.*

Non-iid settings. Considering non-identical but independent (non-iid) data distribution across clients, we use the *heterogeneous degree* $\mu \in [0, 1]$ to represent the level of disparity among clients' local data. To be specific, 100μ % of local data for each client is drawn in a non-iid manner from a particular class corresponding to the client index and $100(1-\mu)$ % of the local data is drawn iid from all classes. Fig. 4 shows the setting with $\mu = 0.9$ and we surprisingly observe that RobustTailor shows a satisfactory level of robustness even under heterogeneous data settings. Note that additional results and analysis with $\mu = 0.1, 0.5$ are illustrated in Appendix G.2.



(a) $\epsilon = 0.5$ **(b)** $\epsilon = 100$ **(c)** AttackTailor

Figure 4: ***Non-iid setting with*** $\mu = 0.9$.

Aggregators with auxiliary data. Public dataset can be used not only for simulation but also to assist with aggregation. Fang et al. (2020) have proposed two server-side verification methods: error rate based rejection (ERR) and loss function based rejection (LFR). In ERR and LFR, the server first uses error rates or loss values to reject potentially harmful gradients, and then applies an aggregation rule. In Table 1, we provide experiments with the setup as in Fig. 3. These results provide further evidence that RobustTailor delivers superior performance over existing techniques. By additional experiments, we observe that ERR/LFR helps aggregator achieve around 97% accuracy when facing $\epsilon = 0.5$ attack while it is totally ruined when against $\epsilon = 100$. In this more sensitive situation, AttackTailor is easily to break single aggregation rules but RobustTailor still performs well.

Table 1: Comparison of aggregators with auxiliary data.

Aggregator	Accuracy
Omniscient	96.13%
RobustTailor	90.28%
ERR/LFR + Krum/Comed	$\approx$ 10%

Additional experiments. To further validate the performance of RobustTailor, we set up additional experiments in Appendix G.2 including 1) iid setting on three datasets; 2) non-iid settings; 3) more Byzantines; 4) more aggregation rules added in RobustTailor; 5) unknown attacks for the server; 6) the impact of the proportion of public data.

6 Conclusions and Future Work

We formulate the robust distributed learning problem as a game between a server and an adversary. We propose RobustTailor, which achieves robustness by simulating the server's aggregation rules under different attacks optimized by an informed and powerful adversary. RobustTailor provides theoretical guarantees for the simulated game through a regret analysis. We empirically demonstrate the significant superiority of RobustTailor over baseline robust aggregation rules. Any Byzantine-resilient scheme with a given structure can be added to RobustTailor 's framework.

Yoshida et al. (2020) and Zhao et al. (2018) proposed mechanisms using public data to handle statistical heterogeneity problem in FL. Therefore, further improving RobustTailor's performance in the non-iid setting based on the public dataset can be a future work. In addition, although the increased computation complexity of RobustTailor is acceptable for the great robustness which is analyzed in Appendix H, it is also a future work to develop efficient and secure protocols to apply RobustTailor *e.g.,* using multi-party computation (Boneh et al., 2019).

REFERENCES

Martín Abadi, Paul Barham, Jianmin Chen, Zhifeng Chen, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Geoffrey Irving, Michael Isard, et al. {TensorFlow}: A System for {Large-Scale} Machine Learning. In *12th USENIX symposium on operating systems design and implementation (OSDI 16)*, 2016.

Scott Alfeld, Xiaojin Zhu, and Paul Barford. Data poisoning attacks against autoregressive models. In *AAAI Conference on Artificial Intelligence*, 2016.

Dan Alistarh, Zeyuan Allen-Zhu, and Jerry Li. Byzantine stochastic gradient descent. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2018.

Zeyuan Allen-Zhu, Faeze Ebrahimian, Jerry Li, and Dan Alistarh. Byzantine-resilient non-convex stochastic gradient descent. In *International Conference on Learning Representations (ICLR)*, 2021.

Peter Auer, Nicolo Cesa-Bianchi, Yoav Freund, and Robert E Schapire. The nonstochastic multiarmed bandit problem. *SIAM journal on computing*, 32(1):48–77, 2002.

Baruch Awerbuch and Robert Kleinberg. Online linear optimization and adaptive routing. *Journal of Computer and System Sciences*, 74(1):97–114, 2008.

Eugene Bagdasaryan, Omid Poursaeed, and Vitaly Shmatikov. Differential privacy has disparate impact on model accuracy. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2019.

Eugene Bagdasaryan, Andreas Veit, Yiqing Hua, Deborah Estrin, and Vitaly Shmatikov. How to backdoor federated learning. In *International Conference on Artificial Intelligence and Statistics (AISTATS)*, 2020.

Arjun Nitin Bhagoji, Supriyo Chakraborty, Prateek Mittal, and Seraphin Calo. Analyzing federated learning through an adversarial lens. In *International Conference on Machine Learning (ICML)*, 2019.

Battista Biggio, Blaine Nelson, and Pavel Laskov. Poisoning attacks against support vector machines. *arXiv preprint arXiv:1206.6389*, 2012.

Peva Blanchard, El Mahdi El Mhamdi, Rachid Guerraoui, and Julien Stainer. Machine learning with adversaries: Byzantine tolerant gradient descent. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2017.

Keith Bonawitz, Vladimir Ivanov, Ben Kreuter, Antonio Marcedone, H Brendan McMahan, Sarvar Patel, Daniel Ramage, Aaron Segal, and Karn Seth. Practical secure aggregation for privacy-preserving machine learning. In *proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*, 2017.

Dan Boneh, Elette Boyle, Henry Corrigan-Gibbs, Niv Gilboa, and Yuval Ishai. Zero-knowledge proofs on secret-shared data via fully linear PCPs. In *Annual International Cryptology Conference*, 2019.

Léon Bottou. Online learning and stochastic approximations. *On-line learning in neural networks*, 1998.

Xiaoyu Cao, Minghong Fang, Jia Liu, and Neil Zhenqiang Gong. FLTrust: Byzantine-robust federated learning via trust bootstrapping. *arXiv preprint arXiv:2012.13995*, 2020.

Xinyang Cao and Lifeng Lai. Distributed gradient descent algorithm robust to an arbitrary number of Byzantine attackers. *IEEE Transactions on Signal Processing*, 67(22):5850–5864, 2019.

Nicholas Carlini and Andreas Terzis. Poisoning and backdooring contrastive learning. *arXiv preprint arXiv:2106.09667*, 2021.

Nicholas Carlini and David Wagner. Towards evaluating the robustness of neural networks. In *2017 ieee symposium on security and privacy (sp)*, 2017.

Hongyan Chang, Virat Shejwalkar, Reza Shokri, and Amir Houmansadr. Cronus: Robust and heterogeneous collaborative learning with black-box knowledge transfer. *arXiv preprint arXiv:1912.11279*, 2019.

Yudong Chen, Lili Su, and Jiaming Xu. Distributed statistical machine learning in adversarial settings: Byzantine gradient descent. *Proceedings of the ACM on Measurement and Analysis of Computing Systems*, 1(2):1–25, 2017.

Deepesh Data and Suhas Diggavi. Byzantine-resilient high-dimensional SGD with local iterations on heterogeneous data. In *International Conference on Machine Learning (ICML)*, 2021.

Ilker Demirel, Yigit Yildirim, and Cem Tekin. Federated multi-armed bandits under byzantine attacks. *arXiv preprint arXiv:2205.04134*, 2022.

El Mahdi El Mhamdi, Rachid Guerraoui, and Sébastien Louis Alexandre Rouault. Distributed momentum for Byzantine-resilient stochastic gradient descent. In *International Conference on Learning Representations (ICLR)*, 2021.

Minghong Fang, Xiaoyu Cao, Jinyuan Jia, and Neil Gong. Local model poisoning attacks to Byzantine-robust federated learning. In *29th USENIX Security Symposium (USENIX Security 20)*, 2020.

Xiuwen Fang and Mang Ye. Robust federated learning with noisy and heterogeneous clients. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 2022.

Donald L. Fisk. Quasi-martingales. *Transactions of the American Mathematical Society*, 120(3): 369–389, 1965.

Ian J Goodfellow, Jonathon Shlens, and Christian Szegedy. Explaining and harnessing adversarial examples. *arXiv preprint arXiv:1412.6572*, 2014.

Ala Gouissem, Khalid Abualsaud, Elias Yaacoub, Tamer Khattab, and Mohsen Guizani. Federated learning stability under Byzantine attacks. In *2022 IEEE Wireless Communications and Networking Conference (WCNC)*, 2022.

Tianyu Gu, Kang Liu, Brendan Dolan-Gavitt, and Siddharth Garg. Badnets: Evaluating backdooring attacks on deep neural networks. *IEEE Access*, 7:47230–47244, 2019.

Rachid Guerraoui, Sébastien Rouault, et al. The hidden vulnerability of distributed learning in Byzantium. In *International Conference on Machine Learning (ICML)*, 2018.

Elad Hazan et al. Introduction to online convex optimization. *Foundations and Trends® in Optimization*, 2(3-4):157–325, 2016.

Ling Huang, Anthony D Joseph, Blaine Nelson, Benjamin IP Rubinstein, and J Doug Tygar. Adversarial machine learning. In *Proceedings of the 4th ACM workshop on Security and artificial intelligence*, 2011.

Wenke Huang, Mang Ye, and Bo Du. Learn from others and be yourself in heterogeneous federated learning. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 2022.

Matthew Jagielski, Alina Oprea, Battista Biggio, Chang Liu, Cristina Nita-Rotaru, and Bo Li. Manipulating machine learning: Poisoning attacks and countermeasures for regression learning. In *2018 IEEE Symposium on Security and Privacy (SP)*, 2018.

P. Kairouz, H. B. McMahan, B. Avent, A. Bellet, M. Bennis, A. N. Bhagoji, K. Bonawitz, Z. Charles, G. Cormode, R. Cummings, R. G. L. D'Oliveira, S. E. Rouayheb, D. Evans, J. Gardner, Z. Garrett, A. Gascón, B. Ghazi, P. B. Gibbons, M. Gruteser, Z. Harchaoui, C. He, L. He, Z. Huo, B. Hutchinson, J. Hsu, M. Jaggi, T. Javidi, G. Joshi, M. Khodak, J. Konečný, A. Korolova, F. Koushanfar, S. Koyejo, T. Lepoint, Y. Liu, P., M. Mohri, R. Nock, A. Özgür, R. Pagh, M. Raykova, H. Qi, D. Ramage, R. Raskar, D. Song, W. Song, S. U. Stich, Z. Sun, A. T. Suresh, F. Tramèr, P. Vepakomma, J. Wang, L. Xiong, Z. Xu, Q. Yang, F. X. Yu, H. Yu, and S. Zhao. Advances and open problems in federated learning. *Foundations and Trends® in Machine Learning*, 14(1–2):1–210, 2021.

Sai Praneeth Karimireddy, Satyen Kale, Mehryar Mohri, Sashank Reddi, Sebastian Stich, and Ananda Theertha Suresh. Scaffold: Stochastic controlled averaging for federated learning. In *International Conference on Machine Learning (ICML)*, 2020.

Sai Praneeth Karimireddy, Lie He, and Martin Jaggi. Learning from history for Byzantine robust optimization. In *International Conference on Machine Learning (ICML)*, 2021.

Sai Praneeth Karimireddy, Lie He, and Martin Jaggi. Byzantine-robust learning on heterogeneous datasets via bucketing. In *International Conference on Learning Representations (ICLR)*, 2022.

Pang Wei Koh and Percy Liang. Understanding black-box predictions via influence functions. In *International Conference on Machine Learning (ICML)*, 2017.

Jakub Konečnỳ, H Brendan McMahan, Felix X Yu, Peter Richtárik, Ananda Theertha Suresh, and Dave Bacon. Federated learning: Strategies for improving communication efficiency. *arXiv preprint arXiv:1610.05492*, 2016.

Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. 2009.

Leslie Lamport, Robert Shostak, and Marshall Pease. The Byzantine generals problem. *ACM Trans. Program. Lang. Syst.*, 4(3):382–401, 1982.

Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86:2278–2324, 1998.

Sam Leroux, Steven Bohez, Elias De Coninck, Tim Verbelen, Bert Vankeirsbilck, Pieter Simoens, and Bart Dhoedt. Multi-fidelity matryoshka neural networks for constrained IoT devices. In *International Joint Conference on Neural Networks (IJCNN)*, 2016.

Bo Li, Yining Wang, Aarti Singh, and Yevgeniy Vorobeychik. Data poisoning attacks on factorization-based collaborative filtering. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2016.

Mu Li, David G Andersen, Jun Woo Park, Alexander J Smola, Amr Ahmed, Vanja Josifovski, James Long, Eugene J Shekita, and Bor-Yiing Su. Scaling distributed machine learning with the parameter server. In *11th USENIX Symposium on Operating Systems Design and Implementation (OSDI 14)*, 2014.

Saeed Mahloujifar, Mohammad Mahmoody, and Ameer Mohammed. Data poisoning attacks in multi-party learning. In *International Conference on Machine Learning (ICML)*, 2019.

Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Aguera y Arcas. Communication-efficient learning of deep networks from decentralized data. In *International Conference on Artificial Intelligence and Statistics (AISTATS)*, 2017.

Shike Mei and Xiaojin Zhu. Using machine teaching to identify optimal training-set attacks on machine learners. In *AAAI Conference on Artificial Intelligence*, 2015.

Michel Métivier. *Semimartingales*. Walter de Gruyter, 1982.

John F. Nash. Equilibrium points in n-person games. *Proceedings of the National Academy of Sciences*, 36(1):48–49, 1950.

Krishna Pillutla, Sham M Kakade, and Zaid Harchaoui. Robust aggregation for federated learning. *IEEE Transactions on Signal Processing*, 70:1142–1154, 2022.

Shashank Rajput, Hongyi Wang, Zachary Charles, and Dimitris Papailiopoulos. DETOX: A redundancy-based framework for faster and more robust gradient aggregation. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2019.

Ali Ramezani-Kebrya, Iman Tabrizian, Fartash Faghri, and Petar Popovski. Mixtailor: Mixed gradient aggregation for robust learning against tailored attacks. *arXiv preprint arXiv:2207.07941*, 2022.

Yevgeny Seldin, Csaba Szepesvári, Peter Auer, and Yasin Abbasi-Yadkori. Evaluation and analysis of the performance of the EXP3 algorithm in stochastic environments. In *European Workshop on Reinforcement Learning*, 2013.

Shai Shalev-Shwartz et al. Online learning and online convex optimization. *Foundations and Trends® in Machine Learning*, 4(2):107–194, 2012.

Chengshuai Shi and Cong Shen. Federated multi-armed bandits. In *AAAI Conference on Artificial Intelligence*, 2021.

Jinhyun So, Başak Güler, and A Salman Avestimehr. Byzantine-resilient secure federated learning. *IEEE Journal on Selected Areas in Communications*, 39(7):2168–2181, 2020.

Lili Su and Nitin H Vaidya. Fault-tolerant multi-agent optimization: optimal iterative distributed algorithms. In *Proceedings of the 2016 ACM symposium on principles of distributed computing*, 2016.

Jingwei Sun, Ang Li, Louis DiValentin, Amin Hassanzadeh, Yiran Chen, and Hai Li. FL-WBC: Enhancing robustness against model poisoning attacks in federated learning from a client perspective. *Advances in Neural Information Processing Systems (NeurIPS)*, 34:12613–12624, 2021.

Ziteng Sun, Peter Kairouz, Ananda Theertha Suresh, and H. Brendan McMahan. Can you really backdoor federated learning? *CoRR*, abs/1911.07963, 2019.

Hao Wang, Zakhary Kaplan, Di Niu, and Baochun Li. Optimizing federated learning on non-iid data with reinforcement learning. In *IEEE INFOCOM 2020-IEEE Conference on Computer Communications*, 2020a.

Hongyi Wang, Kartik Sreenivasan, Shashank Rajput, Harit Vishwakarma, Saurabh Agarwal, Jy-yong Sohn, Kangwook Lee, and Dimitris Papailiopoulos. Attack of the tails: Yes, you really can backdoor federated learning. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2020b.

Han Xiao, Kashif Rasul, and Roland Vollgraf. Fashion-mnist: a novel image dataset for benchmarking machine learning algorithms. *arXiv preprint arXiv:1708.07747*, 2017.

Chulin Xie, Keli Huang, Pin-Yu Chen, and Bo Li. Dba: Distributed backdoor attacks against federated learning. In *International Conference on Learning Representations (ICLR)*, 2019.

Cong Xie, Oluwasanmi Koyejo, and Indranil Gupta. Fall of empires: Breaking Byzantine-tolerant SGD by inner product manipulation. In *Uncertainty in Artificial Intelligence*, 2020.

Yi-Rui Yang and Wu-Jun Li. BASGD: Buffered asynchronous SGD for byzantine learning. In *International Conference on Machine Learning (ICML)*, 2021.

Dong Yin, Yudong Chen, Ramchandran Kannan, and Peter Bartlett. Byzantine-robust distributed learning: Towards optimal statistical rates. In *International Conference on Machine Learning (ICML)*, 2018.

Naoya Yoshida, Takayuki Nishio, Masahiro Morikura, Koji Yamamoto, and Ryo Yonetani. Hybrid-FL for wireless networks: Cooperative learning mechanism using non-IID data. In *ICC 2020-2020 IEEE International Conference on Communications (ICC)*, 2020.

Syed Zawad, Ahsan Ali, Pin-Yu Chen, Ali Anwar, Yi Zhou, Nathalie Baracaldo, Yuan Tian, and Feng Yan. Curse or redemption? How data heterogeneity affects the robustness of federated learning. In *AAAI Conference on Artificial Intelligence*, 2021.

Yue Zhao, Meng Li, Liangzhen Lai, Naveen Suda, Damon Civin, and Vikas Chandra. Federated learning with non-iid data. *arXiv preprint arXiv:1806.00582*, 2018.

Zhuangdi Zhu, Junyuan Hong, and Jiayu Zhou. Data-free knowledge distillation for heterogeneous federated learning. In *International Conference on Machine Learning (ICML)*, 2021.

Martin Zinkevich. Online convex programming and generalized infinitesimal gradient ascent. In *International Conference on Machine Learning (ICML)*, 2003.

Notation. We use $\mathbb{E}[\cdot]$, $\|\cdot\|$, $\|\cdot\|_0$, and $\|\cdot\|_*$ to denote the expectation operator, Euclidean norm, number of nonzero elements of a vector, and dual norm, respectively. We use $|\cdot|$ to denote the length of a binary string, the length of a vector, and cardinality of a set. We use lower-case bold letters to denote vectors. Sets are typeset in a calligraphic font. The base-2 logarithm is denoted by $\log$, and the set of binary strings is denoted by $\{0,1\}^*$. We use $[n]$ to denote $\{1,\cdots,n\}$ for an integer n. We use Δ_M to denote the probability simplex in $\mathbb{R}^M$.

A Complete related work

Federated learning (FL). FL (McMahan et al., 2017; Konečnỳ et al., 2016) keeps training data decentralized in multiple clients which collaboratively train a model under the orchestration of a server (Kairouz et al., 2021). For the server, such clients are often more unpredictable and especially more vulnerable to the attacks. Secure aggregation protocols (Bonawitz et al., 2017; So et al., 2020) ensure that the server computes aggregated updates without revealing the original data. In this paper, we focus on training-time attacks and corresponding aggregation rules.

Training-time attacks. Standard attacks can be broadly classified into training-time attacks (poisoning attacks) (Biggio et al., 2012; Li et al., 2016; Jagielski et al., 2018; Bhagoji et al., 2019; Huang et al., 2011; Mei & Zhu, 2015; Alfeld et al., 2016; Koh & Liang, 2017; Mahloujifar et al., 2019; Gu et al., 2019; Xie et al., 2019; Wang et al., 2020b; Yang & Li, 2021; Karimireddy et al., 2021; Data & Diggavi, 2021; Carlini & Terzis, 2021; Allen-Zhu et al., 2021) and inference-time attacks (evasion attacks) (Goodfellow et al., 2014; Carlini & Wagner, 2017). Because the server in FL trains the model across various unreliable clients with private datasets, FL usually suffers from training-time attacks (Biggio et al., 2012; Bhagoji et al., 2019; Sun et al., 2019; Bagdasaryan et al., 2020). A strong adversary can potentially participate in every training round, and meanwhile it can adapt its attacks to an updated model. One class of training-time attacks concerned in this work is *model update poisoning*. In model poisoning attack, an adversary can control some clients and can directly manipulate their outputs trying to bias the global model towards the opposite direction (Kairouz et al., 2021). If Byzantine clients have access to the updates of honest clients, they can tailor their attacks and make them difficult to detect (Fang et al., 2020; Xie et al., 2020; Lamport et al., 1982; Blanchard et al., 2017; Goodfellow et al., 2014; Bagdasaryan et al., 2020).

Robust aggregation and Byzantine resilience. To improve robustness under general Byzantine clients, a number of robust aggregation schemes have been proposed, which are mainly inspired by robust statistics such as median-based aggregators (Yin et al., 2018; Chen et al., 2017), Krum (Blanchard et al., 2017), trimmed mean (Yin et al., 2018). Krum (Blanchard et al., 2017) and coordinate-wise median (Comed) (Yin et al., 2018; Chen et al., 2017) are two main aggregation rules used in this paper. Krum is a squared-distance-based aggregation rule and it aggregates the gradients that minimize the sum of squared distances to its $n-f-2$ closest vectors where n denotes the total number of clients and f is the number of adversarial ones. Comed is a median-based aggregator and it selects the gradient closest to the median of each dimension.

Except of statistical aggregation rules, there are still many related works like server-side verification, client-side self-clipping etc. From the perspective of the server, Fang et al. (2020); Xie et al. (2020); Cao & Lai (2019); Cao et al. (2020) propose some server-side verification methods using auxiliary data. Specifically, Fang et al. (2020) assume the server has a small validation dataset and uses error rates to reject harmful gradients. In (Xie et al., 2020; Cao & Lai, 2019), the server asks a small clean dataset from clients and filters out unreliable gradients. Cao et al. (2020) utilize the ReLU-clipped cosine-similarity between local gradients and the standard one calculated by a small clean dataset as the weight for aggregation. Moreover, Karimireddy et al. (2021) and Alistarh et al. (2018) propose history-aided aggregators, and an expandable framework proposed by Ramezani-Kebrya et al. (2022) utilizes randomization to improve robustness. None of them selects a proper aggregation rule proactively during training as our framework $\mathrm{RobustTailor}$. We note that all aggregation rules shown here can be added to the pool of $\mathrm{RobustTailor}$ because a public dataset is available in our assumption and any aggregation rule can use it. In addition, client-side clipping methods are proposed by Sun et al. (2021) and Sun et al. (2019), and client-side momentum SGD is considered by Karimireddy et al. (2021) and El Mhamdi et al. (2021). However, the ability of clients is not the focus of our paper and we will consider it in future work.

Past work has shown that these aggregators can defend successfully under specific conditions (Blanchard et al., 2017; Chen et al., 2017; Su & Vaidya, 2016). However, Fang et al. (2020) and Xie et al. (2020) argue that Byzantine-resilient aggregators can fail when an informed adversary tailors a careful attack. Therefore, developing a robust and efficient algorithm under such strong tailored attacks is essential to improve security of FL, which is the goal of this paper.

Heterogeneous data. In the real world applications, many issues such as heterogeneous data become significant (Kairouz et al., 2021; Karimireddy et al., 2020). Karimireddy et al. (2022) find that robust learning algorithms of FL may fail under iid setting. Several algorithms are proposed to tackle non-iid data (Yoshida et al., 2020; Zhao et al., 2018; Karimireddy et al., 2022; Wang et al., 2020a; Data & Diggavi, 2021; Zhu et al., 2021). Besides, data heterogeneity easily leads to backdoor attacks, which can be viewed as a kind of training-time attacks (Xie et al., 2019; Bagdasaryan et al., 2019; Zawad et al., 2021). Therefore, establishing robustness under non-iid setting is also an important indicator for an aggregator.

Game theory in FL. *Online Convex Optimization* (OCO) framework (Zinkevich, 2003) is widely influential in the learning community (Hazan et al., 2016; Shalev-Shwartz et al., 2012), and *Bandit Convex Optimization* (BCO) as an extension of OCO was proposed by Awerbuch & Kleinberg (2008) for decision making with limited feedback. Bandit paradigms paralleling FL framework are proposed by Shi & Shen (2021) and its extension under Byzantine attacks is proposed by Demirel et al. (2022). However, they account for uncertainties from both arm and client sampling rather than robust aggregation in this paper. In this paper, we frame robust distributed learning problem as a game and consider the *bandit feedback model*.

B Robustness of aggregator

In this section, we define a general robustness definition of an aggregation rule against an attack. Note that our definition covers a broad range of settings with *general pure and mixed aggregation* along with *general pure and mixed attack* strategies. Our robustness notion leads to almost sure convergence guarantees to a local minimum of F in FL, which is equivalent to being immune to training-time attacks.

Definition 4 (Robustness of an aggregator to an attack program)**.** *Let $\mathbf{x} \in \mathbb{R}^d$ denote a machine learning model. Let $\mathbf{g}_i(\mathbf{x}) = \nabla f_i(\mathbf{x}) \in \mathbb{R}^d$ be independent honest updates for $i \in [n]$. Let $G(\mathbf{x})$ denote a function that draws an honest client i uniformly at random followed by outputting an unbiased stochastic gradient of $\nabla f_i(\mathbf{x})$ over that client such that $\mathbb{E}[G(\mathbf{x})] = \nabla F(\mathbf{x})$ where $\mathbb{E}$ is over both random client and samples. Let AG denote an arbitrary aggregation rule, which can be a mixed aggregation strategy selecting an aggregator from $\mathcal{A} = \{\mathrm{AG}_1, \ldots, \mathrm{AG}_M\}$ based on simulation. The output of AG is given by $\breve{g}(\mathbf{x}) = \mathrm{AG}(\{\mathbf{g}'\}_{i=1}^n)$. Note that $\{\mathbf{g}'\}_{i=1}^n$ includes both honest and compromised updates. The compromised updates are the output of an attack program $\mathrm{AT}(\{\mathbf{g}_i\}_{i=1}^{n-f}, \mathcal{A})$. Note that AT can be a pure or mixed attack strategy.*

The mixed aggregation rule AG is Byzantine-resilient to AT if $\breve{g}(\mathbf{x})$ satisfies $\mathbb{E}[\breve{g}(\mathbf{x})]^\top \nabla F(\mathbf{x}) > 0$ and $\mathbb{E}[||\breve{g}(\mathbf{x})||^r] \leq K_r \mathbb{E}[||G(\mathbf{x})||^r]$ for $r = 2, 3, 4$ and some constant K_r.

Suppose $\{\eta_t\}_{t=1}^\infty$ in Algorithm 3 satisfies $\sum_t \eta_t = \infty$ and $\sum_t \eta_t^2 < \infty$. For a *nonconvex loss function*, which is three times differentiable with continuous derivatives, bounded from below, and satisfies global confinement assumption in (Bottou, 1998, Section 5.1), *general pure and mixed aggregation and attack strategies* satisfying Definition 4, and *general non-iid data distribution across clients*, we can establish almost sure convergence ($\nabla F(\mathbf{x}_t) \to 0$ a.s.) of the output of AG in Algorithm 3 along the lines of (Bottou, 1998; Fisk, 1965; Métivier, 1982).

Note that to achieve $\mathbb{E}[\breve{g}(\mathbf{x})]^\top \nabla F(\mathbf{x}) > 0$ shown above, it requires both the distance between $\nabla F(\mathbf{x})$ and the estimate of the honest update $\widetilde{\mathbf{g}}$ and the distance between $\widetilde{\mathbf{g}}$ and the expected output of Algorithm 2, i.e., $\mathbb{E}[\breve{g}(\mathbf{x})]$, are small. Let θ_1 denote the angle between $\nabla F(\mathbf{x})$ and $\widetilde{\mathbf{g}}$, and let θ_2 denote the angle between $\widetilde{\mathbf{g}}$ and $\mathbb{E}[\breve{g}(\mathbf{x})]$, given by $\arg\cos\left(\frac{\widetilde{\mathbf{g}}^\top \nabla F(\mathbf{x})}{\|\widetilde{\mathbf{g}}\| \cdot \|\nabla F(\mathbf{x})\|}\right)$ and $\arg\cos\left(\frac{\widetilde{\mathbf{g}}^\top \mathbb{E}[\breve{g}(\mathbf{x})]}{\|\widetilde{\mathbf{g}}\| \cdot \|\mathbb{E}[\breve{g}(\mathbf{x})]\|}\right)$, respectively. If $\theta_1 + \theta_2 < \pi/2$, then we have $\mathbb{E}[\breve{g}(\mathbf{x})]^\top \nabla F(\mathbf{x}) > 0$. Following the arguments in Appendix B, almost sure convergence of Algorithm 3 is guaranteed as long as $\theta_1 + \theta_2 < \pi/2$. This condition can be satisfied assuming 1) the public data donated by clients is representative of the underlying data distribution of honest clients, which controls θ_1, and 2) the number of Byzantine

clients is sufficiently small, which controls θ_2. We defer derivation of the explicit necessary condition for almost sure convergence to future work.

C Details of Exp3

The bandit feedback model considers the following iterate game.

Definition 5 (Bandit setting)**.** *The player is given a decision set* $[N]$*. At each iteration* $k = 1, \ldots, K$*:*

1. *the player picks* $i_k \in [N]$*.*
2. *the adversary picks a loss vector* ℓ^k*.*
3. *the player observes and suffers the loss at index* i_k*, i.e.* $\ell^k(i_k)$*.*

Exp3, as shown abstractly in Algorithm 4, enjoys a so called no-regret property in this setting. We employ Exp3 from both the perspective of a simulated server and simulated attacker to find a robust aggregation rule in Algorithm 2. In Appendix E we show how to convert the no-regret properties into a convergence guarantee.

Algorithm 4 Exp3

Input: Updating rate λ and $\tilde{\lambda}$, iteration rounds K, N

Initialize weight vector $w^0(i) = 1$ for $i = 1, \ldots, N$.

for $k = 1$ ***to*** K **do**

  Set $W^k = \sum_{i=1}^{N} w^k(i)$, and set for $i = 1, \ldots, N$

$$\mathbf{p}(i) = (1 - \lambda)\frac{w^k(i)}{W^k} + \lambda\frac{1}{N}$$

  Draw i_k randomly accordingly to the probabilities $\mathbf{p}$.

  Receive loss ℓ^k.

  Set for $i = 1, \ldots, N$

$$\hat{\ell}^k(i) = \begin{cases} \ell^k/\mathbf{p}(i), & \text{if } i = i_k; \\ 0, & \text{otherwise.} \end{cases}$$

$$w^{k+1}(i) = w^k(i)\exp(-\tilde{\lambda}\hat{\ell}^k(i)/N).$$

D Simulation of adversary

In this section, we show the simulation of the adversary. We term *adversarial simulation* as $\mathrm{AttackTailor}$, which outputs an appropriate AT at each iteration. The specific steps from the perspective of the adversary is shown in Algorithm 5. After observing $n - f$ honest gradients, the server performs K-round simulation and obtains a final probability distribution $\mathbf{q}$. By sampling from $\mathbf{q}$, the server selects an attack. Then, f Byzantine clients create and send the compromised gradients to the server. The steps for simulating the attack procedure are summarized in Algorithm 6.

Importantly, the main difference between the adversary's simulation compared with that of server is that the adversary does simulation based on realistic honest gradients while the server has access only to *noisy estimates* of true gradients. Hence, unlike typical games and simulation setups, the adversary has an additional advantage over the server, which is due to *information asymmetry*.

Algorithm 5 AttackTailor

Input: Updating rates λ_1, λ_2, $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, simulation rounds K, gradients of honest clients $\{\mathbf{g}_i\}_{i=1}^{n-f}$, $\mathcal{A}$ and $\mathcal{F}$

Initialize weight vector $w_1^0(i) = 1$ for $i \in [M]$ and $w_2^0(j) = 1$ for $j \in [S]$.

for $k = 1$ ***to*** K **do**

  Set $\mathbf{p}^k(\widetilde{\mathrm{AG}}_i) = (1-\lambda_1)\frac{w^k(i)}{\sum_{i=1}^M w^k(i)} + \lambda_1 \frac{1}{M}$ for $i = 1, \ldots, M$.

  Set $\mathbf{q}^k(\widetilde{\mathrm{AT}}_j) = (1-\lambda_2)\frac{v^k(j)}{\sum_{j=1}^S v^k(j)} + \lambda_2 \frac{1}{S}$ for $j = 1, \ldots, S$.

  Sample $\mathrm{AG}^k \sim \mathbf{p}^k$ and $\mathrm{AT}^k \sim \mathbf{q}^k$ respectively.

  Estimate the loss $\ell^k = \widetilde{\ell}(\mathrm{AG}^k, \mathrm{AT}^k, \{\mathbf{g}_i\}_{i=1}^{n-f})$.

  Set for $i = 1, \ldots, M$

$$\hat{\ell}_1^k(i) = \frac{\mathbb{I}\{\widetilde{\mathrm{AG}}_i = \mathrm{AG}^k\}}{\mathbf{p}^k(\widetilde{\mathrm{AG}}_i)}\ell^k, \quad w_1^{k+1}(i) = w_1^k(i)\exp(-\tilde{\lambda}_1 \hat{\ell}_1^k(i)/M).$$

  Set for $j = 1, \ldots, S$

$$\hat{\ell}_2^k(j) = \frac{\mathbb{I}\{\widetilde{\mathrm{AT}}_j = \mathrm{AT}^k\}}{\mathbf{q}^k(\widetilde{\mathrm{AT}}_j)}\ell^k, \quad w_2^{k+1}(j) = w_2^k(j)\exp(\tilde{\lambda}_2 \hat{\ell}_2^k(j)/S).$$

Set for $i = 1, \ldots, M$

$$\mathbf{q}_j = \frac{\sum_{k=1}^K \mathbf{q}^k(\mathrm{AT}_j)}{K}.$$

Sample $\mathrm{AT} \sim \mathbf{q}$.

Output: AT.

Algorithm 6 Adversary's attack

Input: Learning rate η_t, n workers, f compromised workers, iteration rounds T, $\mathcal{A}$ and $\mathcal{F}$

for $t = 1$ ***to*** T **do**

  Observe all gradients of honest workers $\{\mathbf{g}_i\}_{i=1}^{n-f}$.

  Call Algorithm to attack $\mathrm{AT}^t = \mathrm{AttackTailor}(\{\mathbf{g}_i\}_{i=1}^{n-f}, \mathcal{A}, \mathcal{F})$.

  Produce f gradients for compromised clients. Set for $j \in [f]$

$$\mathbf{b}_j = \mathrm{AT}^t(\{\mathbf{g}_i\}_{i=1}^{n-f}, \mathcal{A}).$$

  Send compromised gradients $\{\mathbf{b}_j\}_{j=1}^f$ to the server.

E Proof of Lemma 1

Proof. Defined $\bar{i} \sim \bar{\mathbf{p}}$ to be uniformly sampled from $\{i_k\}_{k=1}^K$, and $\bar{j} \sim \bar{\mathbf{q}}$ to be uniformly sampled from $\{j_k\}_{k=1}^K$. Using the no-regret property from (8),

$$\begin{aligned}
\mathbb{E}[L(\bar{i}, j)] &= \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^K L(i_k, j)\right] \leq \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^K L(i_k, j_k)\right] + \frac{1}{K}\mathcal{R}_K^{\mathrm{j}} \\
\mathbb{E}[L(i, \bar{j})] &= \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^K L(i, j_k)\right] \geq \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^K L(i_k, j_k)\right] - \frac{1}{K}\mathcal{R}_K^{\mathrm{i}},
\end{aligned} \tag{12}$$

for any $i \in [M]$ and $j \in [S]$, where the expectation is taken over $\bar{i}$ and $\bar{j}$ and the randomness of the algorithms. Subtracting the two equations,

$$\mathbb{E}[L(\bar{i}, j)] - \mathbb{E}[L(i, \bar{j})] \leq \frac{1}{K}(\mathcal{R}_K^{\mathrm{j}} + \mathcal{R}_K^{\mathrm{i}}) =: \varepsilon. \tag{13}$$

Observe that by first evoking the inequality with $i \sim \bar{\mathbf{p}}$ and secondly with $j \sim \bar{\mathbf{q}}$, we see that $(\bar{\mathbf{p}}, \bar{\mathbf{q}})$ is an ε-approximate Nash equilibrium, i.e.,

$$\mathbb{E}[L(\bar{i}, j)] - \varepsilon \leq \mathbb{E}[L(\bar{i}, \bar{j})] \leq \mathbb{E}[L(i, \bar{j})] + \varepsilon. \tag{14}$$

We are interested in the i-players performance $\mathbb{E}[L(\bar{i}, j)]$ which we can relate to the mixed strategy Nash equilibrium defined as $\mathbb{E}[L(i^\star, j)] \leq \mathbb{E}[L(i^\star, j^\star)] \leq \mathbb{E}[L(i, j^\star)]$ where $i^\star \sim \mathbf{p}^\star$ and $j^\star \sim \mathbf{q}^\star$. By picking $i \sim \mathbf{p}^\star$ in (13) we get,

$$\begin{aligned} \mathbb{E}[L(\bar{i}, j)] &\leq \mathbb{E}[L(i, \bar{j})] + \frac{1}{K}(\mathcal{R}_K^{\mathrm{j}} + \mathcal{R}_K^{\mathrm{i}}) \\ &= \mathbb{E}[L(i^\star, \bar{j})] + \frac{1}{K}(\mathcal{R}_K^{\mathrm{j}} + \mathcal{R}_K^{\mathrm{i}}) \\ &\leq \mathbb{E}[L(i^\star, j^\star)] + \frac{1}{K}(\mathcal{R}_K^{\mathrm{j}} + \mathcal{R}_K^{\mathrm{i}}), \end{aligned} \tag{15}$$

where the last inequalities follows by the definition of a Nash equilibrium above. The claim follows by writing the expectation on the RHS in terms of $\mathbf{p}^\star$ and $\mathbf{q}^\star$. □

F Proof of Lemma 3

Proof. Let both player i and player j in Lemma 1 employ the no-regret algorithm Exp3 such that Lemma 2 applies and consequently $\mathcal{R}_K^{\mathrm{i}}$ and $\mathcal{R}_K^{\mathrm{j}}$ in (8) reduce to

$$\begin{aligned} \mathcal{R}_K^{\mathrm{i}} &= 2\sqrt{\mathrm{KM}\log M} \\ \mathcal{R}_K^{\mathrm{j}} &= 2\sqrt{\mathrm{KS}\log S}. \end{aligned} \tag{16}$$

Substituting (16) into Lemma 1, we have

$$\mathbb{E}_{\bar{i}}\left[L(\bar{i}, j)\right] \leq \mathbb{E}_{i^\star \sim \mathbf{p}^\star, j^\star \sim \mathbf{q}^\star}\left[L(i^\star, j^\star)\right] + 2\frac{\sqrt{\mathrm{M}\log M} + \sqrt{\mathrm{S}\log S}}{\sqrt{K}}. \tag{17}$$

Notice that Algorithm 2 is an instance of two simultaneously played Exp3 algorithms where $i = \mathrm{AG}$, $j = \mathrm{AT}$ and $L(i, j) = \widetilde{\ell}\left(\mathrm{AG}, \mathrm{AT}, \{\mathbf{g}_i'\}_{i=1}^n\right)$. It follows from (17) that

$$\begin{aligned} \mathbb{E}_{\mathrm{AG},\mathcal{G}}\left[\widetilde{\ell}\left(\mathrm{AG}, \mathrm{AT}, \{\mathbf{g}_i'\}_{i=1}^n\right)\right] \leq \mathbb{E}_{\mathrm{AG}^\star \sim \mathbf{p}^\star, \mathrm{AT}^\star \sim \mathbf{q}^\star, \mathcal{G}}\left[\widetilde{\ell}(\mathrm{AG}^\star, \mathrm{AT}^\star, \{\mathbf{g}_i'\}_{i=1}^n)\right] \\ + 2\frac{\sqrt{M\log M} + \sqrt{S\log S}}{\sqrt{K}} \end{aligned} \tag{18}$$

where AG is the average iterate as defined in Algorithm 2. We can concisely write the Nash equilibrium on the R.H.S. of (18) in terms of the payoff matrix $\mathcal{L}$ from Sim-MinMax defined componentwise as $\mathcal{L}(\mathrm{AG}, \mathrm{AT}) = \widetilde{\ell}(\mathrm{AG}, \mathrm{AT}, \{\mathbf{g}_i'\}_{i=1}^n)$. This completes the proof. □

G Experimental details and additional experiments

In this section, we provide the training hype-parameters and show a series of additional experiments.

G.1 Details of implementation

Both MNIST (Lecun et al., 1998) and FMNIST (Xiao et al., 2017) datasets contain 60000 training samples and 10000 test samples. Each sample is a 28 by 28 pixel grayscale image. The details of training hype-parameters are shown in Table 2. The network architecture is a fully connected neural network with two fully connected layers (Leroux et al., 2016). The number of neurons is 100 and 10 for the first and second layer, respectively. All experiments have been run on a cluster with Xeon-Gold processors and V100 GPUs.

G.2 Additional experiments

To further validate the performance of RobustTailor, we set up additional experiments:

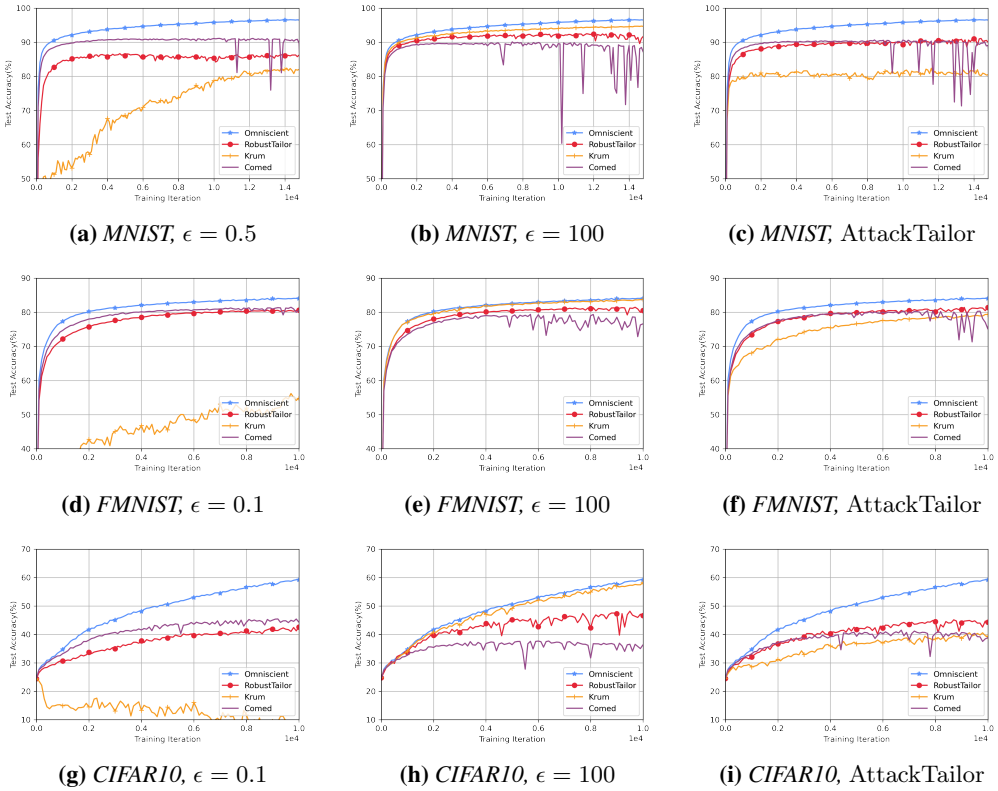
- Training on 3 datasets.
- Non-iid settings with 3 different heterogeneous degree.

Table 2: Training Hyper-parameters for Fashion-MNIST and MNIST

Hyper-parameter	MNIST	FMNIST	CIFAR10
Learning Rate	0.01	0.003	0.002
Batch Size	50	50	80
Total Iterations	15K	10K	10K
Simulation Rounds	10	10	10
λ_1, λ_2	0.3	0.3	0.3
$\tilde{\lambda}_1$, $\tilde{\lambda}_2$	0.3	0.3	0.3

- More Byzantines.
- More aggregators (Geomed, Trimmedmean, Bulyan) added against single attack.
- Unknown attacks for the server including same kind of attacks and different kind of attacks.
- The impact of the proportion of public data.

Different datasets. We train a CNN model on MNIST (Lecun et al., 1998), Fashion-MNIST (FMNIST) (Xiao et al., 2017) and CIFAR10 (Krizhevsky et al., 2009) under iid setting. We summarize the training results against 3 attacks here and they are consensus with the results shown in Section 5.



(a) *MNIST,* $\epsilon = 0.5$ **(b)** *MNIST,* $\epsilon = 100$ **(c)** *MNIST,* AttackTailor

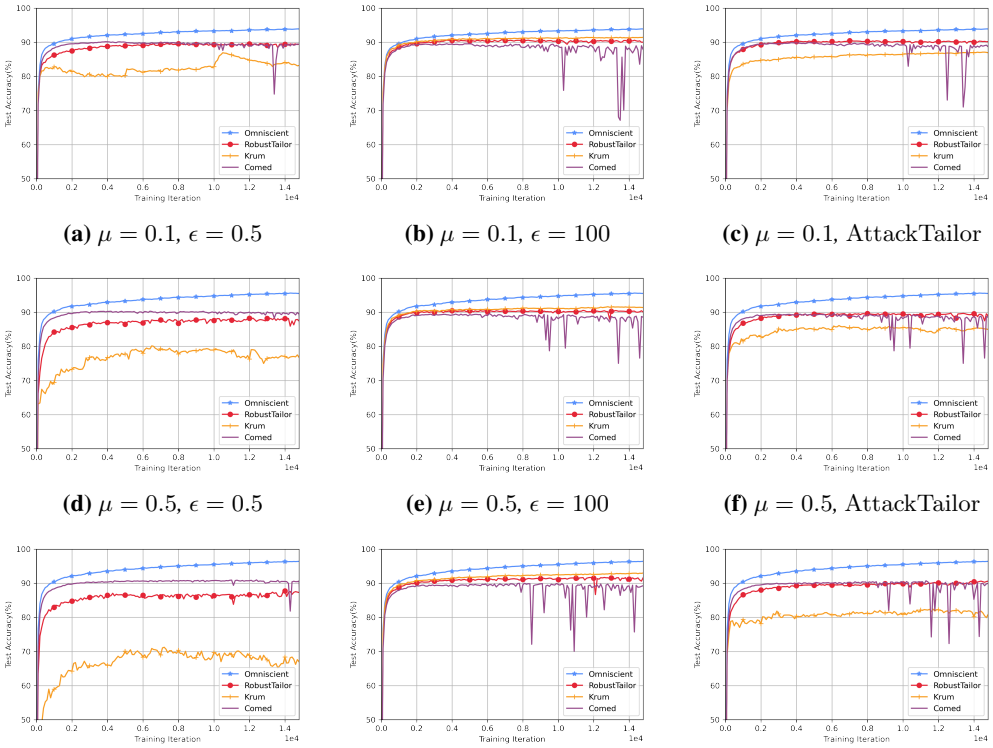
(d) *FMNIST,* $\epsilon = 0.1$ **(e)** *FMNIST,* $\epsilon = 100$ **(f)** *FMNIST,* AttackTailor

(g) *CIFAR10,* $\epsilon = 0.1$ **(h)** *CIFAR10,* $\epsilon = 100$ **(i)** *CIFAR10,* AttackTailor

Figure 5: ***iid setting on three datasets.*** RobustTailor *includes Krum and Comed.* AttackTailor *includes* $\epsilon = 0.1/0.5$ *and* $\epsilon = 100$.

Non-iid settings. We also extend our consideration to more realistic settings with non-iid data distribution across clients. We use the *heterogeneous degree* $\mu \in [0, 1]$ to represent the level of disparity among clients' local data. To be specific, we construct a setting, in which 100μ % of local data for each client is drawn in a non-identical but independent manner from a particular class corresponding to the client index and $100(1-\mu)$ % of the local data is drawn iid from all classes. A small μ represents low disparity while a large μ means significant disparity among clients. Fig. 6

shows three non-iid settings including $\mu = 0.1, 0.5, 0.9$. We surprisingly observe that RobustTailor shows a satisfactory level of robustness even under heterogeneous data settings.



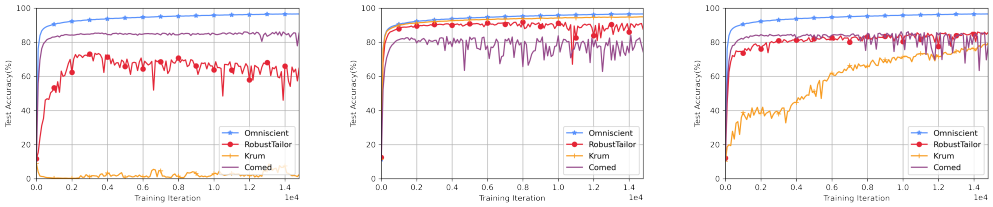
(a) $\mu = 0.1$, $\epsilon = 0.5$ **(b)** $\mu = 0.1$, $\epsilon = 100$ **(c)** $\mu = 0.1$, AttackTailor

(d) $\mu = 0.5$, $\epsilon = 0.5$ **(e)** $\mu = 0.5$, $\epsilon = 100$ **(f)** $\mu = 0.5$, AttackTailor

(g) $\mu = 0.9$, $\epsilon = 0.5$ **(h)** $\mu = 0.9$, $\epsilon = 100$ **(i)** $\mu = 0.9$, AttackTailor

Figure 6: ***Non-iid setting.*** *Larger μ means higher heterogeneous degree.*

More Byzantines. Fig. 7 shows the results when there are 4 Byzantines in 12 total clients under three different attacks. Except the number of compromised clients, which is four instead of two, the setting is same as that in Fig. 1. We observe that both Krum and Comed are sensitive to the number of Byzantine clients while RobustTailor is much more stable. Specifically, Krum has lower accuracy closing to zero, and Comed shows more obvious fluctuations.

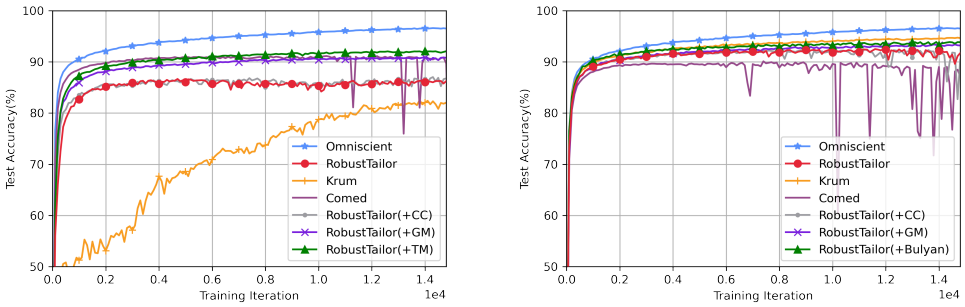


(a) $\epsilon = 0.5$ **(b)** $\epsilon = 100$ **(c)** AttackTailor

Figure 7: ***4 Byzantines.*** *There are $n = 12$ total workers including $f = 4$ Byzantine workers.*

More aggregators against single attacks. To show the intuitive results of RobustTailor, we just construct the server's pool with two aggregators including Krum and Comed in the main text. However, RobustTailor could outperform both Krum and Comed simultaneously when additional aggregators are put into the server's pool. Trimmedmean (TM) (Yin et al., 2018), Geomed (GM) (Pillutla et al., 2022), and Bulyan (Guerraoui et al., 2018) are also statistic-based Byzantine-resilient aggregators and all of them can be added in RobustTailor. When Bulyan is included, each Bulyan aggregator uses a different aggregator from Krum, Comed, TM, and GM for either the selection phase or in the

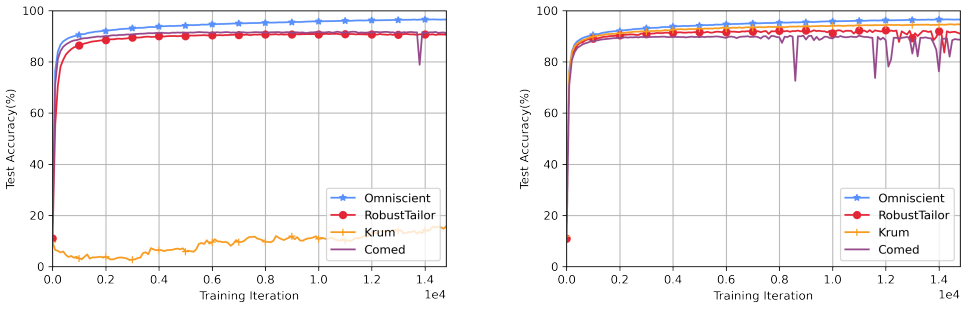
aggregation phase. For each class, we generate 16 aggregators, each with a randomly generated ℓ_p norm from one to 16. RobustTailor selects one aggregator from the entire pool of 64 aggregators uniformly at random at each iteration. Moreover, centered clipping (CC) (Karimireddy et al., 2021) as a history-aided aggregator was proposed recently and it can also be added in RobustTailor framework. Fig. 8 shows results when more aggregators added into RobustTailor framework and they perform better.



(a) $\epsilon = 0.5$ **(b)** $\epsilon = 100$

Figure 8: ***More aggregators added into to*** RobustTailor ***structure against single attacks.***

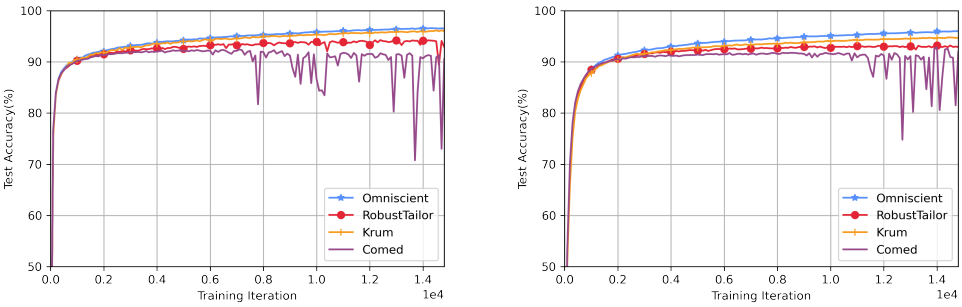
Unknown attacks for the server. In our assumption, the server knows all attacks in the adversary's pool. What will happen if there is an attack out of the server's expectation? Fig. 9 and Fig. 10 give the results. When an attack is not expected by the server but is of *the same type* as the desired attack, Fig. 9 proves that RobustTailor still performs well since $\epsilon = 0.1/150$ attack is similar with $\epsilon = 0.5/100$ shown in Fig. 1. When an attack is *different* with the server's expectation, Fig. 10 shows that RobustTailor can keep a great performance surprisingly. This demonstrates that RobustTailor as a mixed framework is hard to be totally ruined, since it is difficult for the adversary to design a tailored attack which can ruin several aggregation rules simultaneously compared to break a single aggregation rule.



(a) $\epsilon = 0.1$ **(b)** $\epsilon = 150$

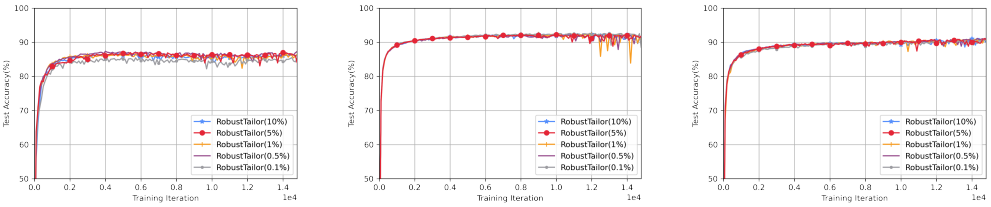
Figure 9: ***Same kind of attacks out of the server's expectation.***

Proportion of public data. Because RobustTailor ask for clients to donate a small amount of data as the public dataset, we want to minimise data leakage as much as possible while maintaining effectiveness. Therefore, detecting the impact of the proportion of public data is necessary. In our experiments, we assume every client donates 5% of data to the server. Fig. 11 shows the performance of RobustTailor with different proportion of public data under 3 attacks. It is obvious that the amount of public data has little impact on RobustTailor and even very small proportion of data donated by clients (*e.g.,* 0.1%) can help RobustTailor achieve a great performance. This also strongly proves Remark 1 that the public dataset just need to represent clients' data distribution.



(a) *Mimic* **(b)** *Alittle*

Figure 10: ***Different kind of attacks out of the server's expectation.***



(a) $\epsilon = 0.5$ **(b)** $\epsilon = 100$ **(c)** AttackTailor

Figure 11: ***The impact of the proportion of public data.***

H COMPUTATIONAL COMPLEXITY

The computational complexity bound depends on the simulation of the inner loop (including simulation rounds K, aggregator set $\mathcal{A}$, and attack set $\mathcal{F}$) and problem dimensions of the outer loop (including number of clients n and the dimension of gradients). We show the theoretical analysis below and utilize empirical results to prove that it is worth to trade a little more computational complexity for a great robust model.

Theoretical analysis For $\mathrm{RobustTailor}$ with K simulation rounds, our algorithm approaches a min-max equilibrium the best aggregation rule at the rate $O(K^{-1/2})$. In addition, the computational cost of RobustTailor is influenced by the server's aggregation rules and the adversary's attacks. If n clients submit d-dimensional vectors, Krum's expected time complexity is $O(n^2 d)$ (Blanchard et al., 2017) and Comed's is $O(nd)$ (Pillutla et al., 2022).

For more fine-grained complexity analysis, suppose $\{\tilde{T}_1, \ldots, \tilde{T}_M\}$ denote the number of elementary operations to run each aggregation rule within the set of M aggregation rules. The *worst-case* runtime complexity of $\mathrm{RobustTailor}$ per simulation round is determined by $\max_{i\in[M]} \tilde{T}_i$. However, the average complexity per round is the expected value of the number of elementary operations where the expectation is over the distribution of how likely each aggregator is chosen during simulation, which can be estimated empirically. Let us use $\tilde{p}_i$ to denote the probability of choosing $\mathcal{A}_i$. The average complexity per round is given by $\overline{T} = \sum_{i=1}^{M} \tilde{T}_i \tilde{p}_i$. Finally, the overall time complexity of $\mathrm{RobustTailor}$ is given by $O(\overline{T} K^{-1/2})$.

Moreover, the number of elementary operations for simulation can be much smaller than applying the actual aggregator on the model during training assuming the size of the public data is very small, which is typically the case in practice(Yoshida et al., 2020; Zhao et al., 2018). Note that our algorithm just adds computation complexity to the server while all clients remain the same cost based on their models and datasets. Therefore, it is worthwhile to trade slightly longer training time for a significantly improved training procedure w.r.t. robustness.

Empirical results We show computation costs and accuracy for different aggregation rules after running 15k iterations in reality, whose results are also shown in Fig. 1. We can see that $\mathrm{RobustTailor}$

still maintains a stable and high accuracy when facing a powerful adversary although it needs more computation time. However, Krum cannot reach a high accuracy and Comed shows a very unstable performance with lots of fluctuations when facing a strong adversary with $\mathrm{AttackTailor}$. We note that compared with undesirable models of Krum and Comed, $\mathrm{RobustTailor}$ improves accuracy and stability drastically at the cost of slightly increased training time.

Table 3: Computational Complexity based on MNIST after running 15k iterations

Aggregator	**Time**	**Accuracy**
Omniscient	34 min	96.63%
$\mathrm{RobustTailor}$	96 min	85.87%
Krum	37 min	82.13%
Comed	52 min	90.74%

(a) $\epsilon = 0.5$

Aggregator	**Time**	**Accuracy**
Omniscient	34 min	96.63%
$\mathrm{RobustTailor}$	96 min	92.03%
Krum	37 min	94.74%
Comed	52 min	60.39%-88.80%

(b) $\epsilon = 100$

Aggregator	**Time**	**Accuracy**
Omniscient	34 min	96.63%
$\mathrm{RobustTailor}$	190 min	89.72%
Krum	90 min	80.98%
Comed	121 min	71.34%-89.75%

(c) $\mathrm{AttackTailor}$