Queue Up Your Regrets: Achieving the Dynamic Capacity Region of Multiplayer Bandits

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Abstract

Consider N cooperative agents such that for T turns, each agent n takes an action a_n and receives a stochastic reward $r_n(a_1, \ldots, a_N)$. Agents cannot observe the actions of other agents and do not know even their own reward function. The agents can communicate with their neighbors on a connected graph G with diameter $d(G)$. We want each agent n to achieve an expected average reward of at least λ_n over time, for a given quality of service (QoS) vector $\boldsymbol{\lambda}$. A QoS vector $\boldsymbol{\lambda}$ is not necessarily achievable. By giving up on immediate reward, knowing that the other agents will compensate later, agents can improve their achievable capacity region. Our main observation is that the gap between $\lambda_n t$ and the accumulated reward of agent n, which we call the QoS regret, behaves like a queue. Inspired by this observation, we propose a distributed algorithm that aims to learn a max-weight matching of agents to actions. In each epoch, the algorithm employs a consensus phase where the agents agree on a certain weighted sum of rewards by communicating only $O(d(G))$ numbers every turn. Then, the algorithm uses distributed successive elimination on a random subset of action profiles to approximately maximize this weighted sum of rewards. We prove a bound on the sum of expected QoS regrets of all agents, that holds if $\boldsymbol{\lambda}$ is a safety margin ε_T away from the boundary of the capacity region, where $\varepsilon_T \to 0$ as $T \to \infty$. This bound implies strong stability of the QoS regrets. The tunable parameter ε_T appears in the expected QoS regret bound and controls the tradeoff between the delay in obtaining the QoS and the size of the capacity region. Based on that, we prove that our algorithm can achieve any $\boldsymbol{\lambda}$ in the interior of the dynamic capacity region, with a sublinear expected QoS regret for all agents (i.e., mean rate stability).

1 Introduction

Consider a group of artificial cooperative agents interacting in a shared environment. Each agent has a reward $r_n(\boldsymbol{a})$ that is a function of the actions of all agents $\boldsymbol{a}$ and represents a local performance metric. Examples include wireless devices, robots in a facility, or autonomous vehicles. The designer wants to program these agents with a distributed protocol that guarantees at least a certain average reward for each agent, which we term its QoS. Wireless devices interact by transmitting and interfering with each other over multiple channels, and the wireless protocol guarantees the advertised throughput for all of them. Multitasking robots interact by cooperating to make progress on their tasks at a certain rate, following a distributed learning algorithm with this objective in mind. The main challenge in designing such a distributed protocol or algorithm is that the agents are local and do not know enough about each other to guarantee the QoS of their peers.

In data-driven decision making, agents do not know their reward function, and can only observe its value at the current action profile $\boldsymbol{a}(t)$, known as bandit feedback. The agents also do not know the

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reward functions of their peers, or even their bandit feedback. As a result, agents have to learn both how to optimize their reward function, and when to prioritize other agents instead.

The QoS is often measured over time. Throughput in communication networks is only averaged over seconds, and progress in an automated factory is only measured over minutes. When resources are involved, time-sharing is sometimes better than simultaneous access. The optimal policy is often more complicated. For example, consider four agents and one resource, such that each agent can access the resource or not. If Agents 1 and 2 access the resource together, they each get zero reward, and the same is true for Agents 3 and 4. Otherwise, an agent gets a reward of 1 by accessing the resource. To achieve any feasible QoS, the agents need to learn when to access, but also, who to access with. *The main objective of this paper is to design a distributed multi-agent algorithm that can achieve any (dynamically) feasible QoS based only on bandit feedback and minimal communication.*

1.1 Our Contributions

We define the QoS regret of agent n by $R_n(t) = \max\left\{0, \lambda_n t - \sum_{\tau=1}^{t} r_n(\boldsymbol{a}(\tau))\right\}$, given a QoS vector $\boldsymbol{\lambda}$. **Our main observation is that $R_n(t)$ behaves like a queue**. This observation inspires an algorithm that learns to approximate a max-weight matching between agents to actions to guarantee the stability of these "queues". A perfect centralized max-weight policy [1] plays the $\boldsymbol{a}$ that maximizes $\sum_{n=1}^{N} b_n R_n(t) r_n(\boldsymbol{a})$ for some positive $\{b_n\}$. To approximate and allow for a distributed architecture, our algorithm divides the time horizon into epoch of length E_T. In each epoch, agents use simultaneous distributed averaging (i.e., consensus) steps to agree on $\sum_{n=1}^{N} b_n R_n(t) r_n(\boldsymbol{a}(t+1))$ from recent turns, on a row stochastic communication matrix W. Then, agents use a distributed and scalable successive elimination to approximately maximize $\sum_{n=1}^{N} b_n R_n(t) r_n(\boldsymbol{a})$. Our consensus step does not require a doubly stochastic W and converges in a finite number of steps.

We show that for large T, our algorithm guarantees stability for any achievable QoS vector $\boldsymbol{\lambda}$ with sublinear QoS regrets, while only requiring each agent to communicate $O(d(G))$ numbers each turn, where $d(G)$ is the diameter of the communication graph G. Furthermore, for finite T, we bound the sum of expected QoS regrets as a function the epoch length E, and characterize the safety margin ε_E away from the boundary of the capacity region needed to guarantee strong stability.

1.2 Related Work

Our work brings together game theory and queuing theory. Aspects of learning in queuing were studied in [2–5]. Queuing games were studied in [6], where each agent chooses which queue to join. In contrast, our analysis uses the observation that the QoS regret behaves like a queue, but no real backlogs exist. We consider general discrete games that may have nothing to do with queues.

Multiplayer bandits is an emerging field that studies how agents can learn to achieve a common goal [7–10] given bandit feedback of their unknown reward functions. Most works on multiplayer bandits assume the collision model, in which when multiple agents choose the same arm, they all receive zero reward [11–16]. Instead of the sum of rewards, our objective is to guarantee that the empirical average reward of each agent is at least a certain QoS. Multi-player QoS bandits were studied in [17], assuming the collision model. However, [17] only achieves QoS vectors $\boldsymbol{\lambda}$ for which there exists an $\boldsymbol{a}$ such that $\mathbb{E}\{r_n(\boldsymbol{a})\} \geq \lambda_n$ for all n. Compared to [17], our work considers a general discrete game. But even for the collision model, for the example on the top of this page, two agents get zero reward in **any** static solution like [17], while we can achieve $\lambda_n = 0.5$ for all n.

Dynamic multiplayer bandits, where agents can join at different times, were studied in [11, 14, 16] under the collision model. More related to our work, [18] combined queuing with multiplayer bandits. In [18], agents have a random arrival of packets to transmit to multiple servers, under the collision model and a constant service rate for each server. In contrast, our scenario has no actual queues and the "service rates" are the rewards of a general discrete game. Consequently, we need to learn a max-weight matching where in [18] a maximal matching of agents to servers is optimal.

In power control, [19] can achieve any target QoS vector that is achievable by a static transmission power profile. The interaction between the wireless links, through the interference, is a game with the transmission powers as the actions. From this point of view, our work generalizes [19] in two ways: from power control to a general game (but discrete), and from a static to a dynamic solution.

2 Problem Formulation

We consider a general multiplayer multi-armed bandit game. At every turn $t \geq 1$, for T turns, each agent n in the set $\mathcal{N} = \{1, ..., N\}$ picks an action $\boldsymbol{a}_n(t) \in \{0, \ldots, K-1\}$. The expected reward for agent n is an unknown function of the actions of all agents $\mu_n : \mathcal{A}_1 \times ... \times \mathcal{A}_N \to \mathbb{R}$. Each agent n only observes a stochastic reward, $r_n(\boldsymbol{a}(t)) \in [0,1]$, such that $\mu_n(\boldsymbol{a}) = \mathbb{E}\{r_n(\boldsymbol{a})\}$ and $z_n(t) \triangleq r_n(\boldsymbol{a}(t)) - \mu_n(\boldsymbol{a}(t))$ is independent over time. Agents only know the action they played and cannot observe the actions of others.

Our goal is to design a distributed algorithm for the cooperative agents, that dictates the actions $a_n(1), \ldots, a_n(T)$ of each agent n, such that ideally

$$\mathbb{E}\left\{\min_n \left(\frac{1}{T}\sum_{t=1}^{T} r_n(\boldsymbol{a}(t)) - \lambda_n\right)\right\} \geq 0 \tag{1}$$

i.e., the expected empirical average reward of agent n is at least λ_n.

The objective in (1) is only possible for certain QoS vectors $\lambda_1, \ldots, \lambda_N$. To stress the importance of a dynamic algorithm, we note that the achievability region of (1) can be much larger than the static one of $\max_{\boldsymbol{a}}\min_n (\mu_n(\boldsymbol{a}) - \lambda_n) \geq 0$, which is all $\lambda \leq \lambda^*$ for $\lambda^* = \max_{\boldsymbol{a}}\min_n \mu_n(\boldsymbol{a})$. As an example, consider a game with $N = 2$ and $K = 2$, such that $\mu_1(1,0) = 1$, $\mu_1(0,0) = \mu_1(0,1) = \mu_1(1,1) = 0$ and $\mu_2(0,1) = 1$, $\mu_2(0,0) = \mu_2(1,0) = \mu_2(1,1) = 0$. Then $\lambda^* = 0$ which is obtained by any constant a_1, a_2. However, if the agents play $(1,0)$ in even turns and and $(0,1)$ in odd ones then $\frac{1}{T}\sum_{t=1}^{T}\mu_1(\boldsymbol{a}(t)) = \frac{1}{T}\sum_{t=1}^{T}\mu_2(\boldsymbol{a}(t)) = \frac{1}{2}$. This simple example arises often when time-sharing of resources is better than simultaneous access. In general, the optimal strategies can involve softer time-sharing or time-sharing between groups of agents, based on their characteristics.

This motivates defining the dynamic capacity region [1]:

Definition 1. Let $\varepsilon \geq 0$. The approximate dynamic capacity region of the game $\mathcal{G} = \left(\mathcal{N}, \{\mathcal{A}_n\}_{n\in\mathcal{N}}, \{\mu_n\}_{n\in\mathcal{N}}\right)$ is defined as the set:

$$C_\varepsilon(\mathcal{G}) = \left\{(\lambda_1, \ldots, \lambda_N) \mid \exists \boldsymbol{p} \in \Delta^{K^N} \text{ s.t. } \sum_{\boldsymbol{a}} \boldsymbol{p}(\boldsymbol{a})\mu_n(\boldsymbol{a}) > \lambda_n + \varepsilon, \forall n\right\} \tag{2}$$

where $\boldsymbol{p}(\boldsymbol{a})$ is the probability of $\boldsymbol{a}$. The (exact) dynamic capacity region is then $C(\mathcal{G}) \triangleq C_0(\mathcal{G})$.

A learning algorithm that has no knowledge of the game will need time to achieve (1), so we can only hope to achieve (1) up to some T-dependent error, which we term the QoS regret.

Definition 2. The QoS regret of agent n at turn t is defined as

$$R_n(t) \triangleq \max\left\{0, \lambda_n t - \sum_{\tau=1}^{t} r_n(\boldsymbol{a}(\tau))\right\}. \tag{3}$$

The expected QoS regret encodes that an agent "regrets" the total accumulated reward it misses to have an expected empirical reward of at least λ_n. On the other hand, an agent with an average empirical reward of more than λ_n does not require anything more so it has zero QoS regret.

Our contributions are a novel distributed algorithm that can learn to achieve a target QoS regret vector, along with the following result guaranteeing our algorithm's performance:

Theorem 1. *Let $\mathcal{G}$ be a multiplayer multi-armed bandit game, played for T turns. Let the agents run Algorithm 1 on a communication graph G, with epoch length E_T such that $E_T \to \infty$ and $\frac{E_T}{T} \to 0$ as $T \to \infty$. If $\boldsymbol{\lambda} \in C(\mathcal{G})$, then for large enough T, we have $\mathbb{E}\{R_n(T)\} = O\left(\sqrt{E_T T}\right)$ for all n, where the expectation is with respect to the randomness of the algorithm. To achieve that, each agent only needs to communicate $O(d(G))$ numbers[1] per turn, where $d(G)$ is the diameter of G.*

[1] These numbers are convex combinations of rewards, and their timestamps. They can be quantized with finite resolution without affecting our analysis since we only approximately optimize the weighted sum of rewards anyway. The communication overhead is reported for comparison purposes with other schemes that communicate rewards.

The stability guarantee of Theorem 1 is asymptotic in T. For finite T, a tradeoff arises between $\mathbb{E}\{R_n(T)\}$ and the safety margin ε_T we have to take from the boundary of $C(\mathcal{G})$ to guarantee this $\mathbb{E}\{R_n(T)\}$. Theorem 2 in the next section analyzes this tradeoff explicitly and shows that indeed $\varepsilon_T \to 0$ as $T \to \infty$, which implies Theorem 1 above as a special case. By tuning the epoch length E and the size of the random subset S_E, we can make ε_T smaller for finite T, at the price of increasing the expected QoS regret, or vice versa.

3 Dynamic Multi-player Bandit Algorithm

To tackle the problem in (1), we exploit the similarity between the QoS regret and a queue. We can think of a deterministic arrival flow with rate λ_n and a service rate of $r_n(\boldsymbol{x}(t))$ for agent n at turn t. Then we can employ ideas from queuing theory to design our algorithm. In particular, max-weight matching of queues to service rates is a centralized algorithm that achieves optimal throughput [20]. Note that in our case the service rate (the reward) is a function of the actions of all agents. Hence, our algorithm has to learn to approximate a max-weight matching distributedly for our game model.

Our algorithm, detailed as Algorithm 1, divides the horizon into epochs. In each epoch, the agents treat their QoS regrets as fixed on their initial value, which is an approximation (Step 1). Based on these values the agents distributedly learn to maximize $\sum_{n=1}^{N} b_n R_n(t)\,\mu_n(\boldsymbol{a})$ for some positive $\{b_n\}$, using successive elimination. Much of the algorithm's efficiency stems from the fact that we do not care what $\{b_n\}$ are, as long as they are positive and constant over epochs.

Algorithm 1 Dynamic Multi-player Bandit QoS Learning

Initialization: Set the epoch index to $e = 0$ and the successive elimination index to $l = 0$. Let W be the communication matrix (Definition 3). Let $S_E = \min\left\{\lceil S_0 \log E \rceil, K^N, E\right\}$, for some $S_0 > 0$, be the random subset size and $D_E > 0$ be the delay parameter. Let $b_{i,j} = \left\{W^{D_E}\right\}_{i,j}$ for all i, j.

Input: $\lambda_n \geq 0$ and agent index n

For each epoch $e \geq 0$**, each agent** n **runs:**

1. Compute the QoS regret for epoch e, $R_n^e = \max\left\{0, \lambda_n t - \sum_{\tau=1}^{t} r_n(\boldsymbol{a}(\tau))\right\}$.
2. Randomize actions $a_n^1, \ldots, a_n^{S_E}$, uniformly and independently. Store their indices in the ordered set of candidate action profiles $\mathcal{B} = \{1, \ldots, S_E\}$, where $\mathcal{B}(l)$ is the l-th element.
3. Set $v_i = \hat{\mu}_i = 0$ for all $i = 1, \ldots, S_E$, set $\tilde{r}_n = 1$ and $T_e = eE + 1$.
4. **For** $T_e \leq t \leq T_e + E - 1$ (E turns):
   (a) Set $\theta_n^0 = \tilde{r}_n R_n^e$ and $\theta_m^0 = 0$ for all $m \neq n$.
   (b) Broadcast $\{(\theta_n^\tau, \tau)\}_{\tau=0}^{D_E-1}$ to neighbors and receive $\{(\theta_m^\tau, \tau)\}_{\tau=0}^{D_E-1}$ from neighbors.
   (c) **Parallel** D_E **Consensus Steps**: Set $\theta_n^{\tau+1} = \sum_{m=1}^{N} w_{n,m}\theta_m^\tau$, $\forall \tau = 0, \ldots, D_E - 1$.
   (d) **Agreement**: If you know θ_1^τ for $D_E \leq \tau \leq 2D_E - 1$ then broadcast (θ_1^τ, τ).
   (e) If you receive (θ_1^τ, τ) for $D_E \leq \tau \leq 2D_E - 1$ then set the weighted sum of rewards from τ turns ago as $S(t - \tau) = \theta_1^\tau$.
   (f) Play $a_n(t) = a_n^{\mathcal{B}(l)}$ and set $\tilde{r}_n = r_n(\boldsymbol{a}(t))$.
   (g) Update the number of visits $v_k \leftarrow v_k + 1$ to $\boldsymbol{a}^k = \left(a_1^k, \ldots, a_N^k\right)$ for $k = \mathcal{B}(l - 2D_E)$.
   (h) Compute the confidence radius $\rho_k = \sqrt{\frac{2\left(\sum_{n=1}^{N} b_{1,n} R_n^e\right)^2 \log E}{\max\{v_k, 1\}}}$ and
   $$\hat{\mu}_k \leftarrow \left(1 - \frac{1}{v_k}\right)\hat{\mu}_k + \frac{S(t - 2D_E)}{v_k} \quad , \quad L_k = \hat{\mu}_k - \rho_k \quad , \quad U_k = \hat{\mu}_k + \rho_k.$$
   (i) **Elimination**: If $l = |\mathcal{B}|$ then delete from $\mathcal{B}$ all i such that $U_i < \max_{j \in \mathcal{B}} L_j$, and set $l = 0$.
   (j) Update the successive elimination index $l \leftarrow l + 1$.
5. Update the epoch index $e \leftarrow e + 1$.

End

3.1 Communication

To know the reward values of others, agents need to communicate. This is done on the communication graph G and weighted by a row-stochastic communication matrix W, defined next:

Definition 3. Let G be the adjacency matrix of an undirected connected graph, such that $G_{n,m} = 1$ if agent n and agent m can communicate or that $n = m$. Let ν_n be the degree of agent n in G. Let W be a matrix such that $w_{n,m} > 0$ if and only if $G_{n,m} = 1$ and $\sum_{m=1}^{N} w_{n,m} = 1$ for all n.

A key design question is how much should agents know about each other to achieve (1). A max-weight matching at turn t is an $\boldsymbol{a}$ that maximizes $\sum_{n=1}^{N} b_n R_n(t) \mu_n(\boldsymbol{a})$ for some positive $\{b_n\}$. A naive choice is then to guarantee that all agents know all rewards before taking an action. This will require each agent to send $O(d(G) N)$ numbers every turn - one for every agent and every turn of a reward value that is still crossing G. This is infeasible in large-scale networks. Instead, our algorithm requires a communication overhead of $O(d(G))$, without sacrificing performance. This dramatic save in communication is possible since to guarantee stability, agents do not need to have the same coefficient b_n in the sum. Hence, in Algorithm 1, agents run the consensus protocol only until all local averages include all of them, even if not with equal b_n.

In contrast to the distributed optimization literature [21], we do not need the communication matrix W to be doubly stochastic, which is hard to guarantee distributedly [22]. Since W is not doubly stochastic, our consensus cannot converge to the average reward. In fact, since we stop the consensus early after exactly D_E turns, it does not converge at all. However, after our consensus step, each agent has a weighted sum of rewards that includes all agents that have a positive QoS, with weights that depend on W. In the next D_E turns, agents propagate (Steps 4(e) and 4(d)) the weighted sum of rewards that Agent 1 obtained, such that they all eventually know it with zero error.

Our algorithm introduces delays so that agents can agree on $\sum_{n=1}^{N} b_{1,n} R_n(t) r_n(\boldsymbol{a}(t+1))$, that Agent 1 obtained after the distributed averaging step (Steps 4(b) and 4(c)). If the diameter of G is $d(G)$, then a minimum of $d(G)$ turns is necessary to guarantee that all agents contributed to $\sum_{n=1}^{N} b_{1,n} R_n(t) r_n(\boldsymbol{a}(t+1))$. However, it is unnecessary to continue the consensus after this point, which is a major advantage of our consensus scheme over the classical one.

In Steps 4(b)-4(e), agents pipeline D_E simultaneous consensus algorithms that cannot be mixed since they are on the rewards generated at different turns from different action profiles.

In a connected graph $d(G) \leq N$ so $D_E = N$ is always sufficient. However, usually a tighter bound on $d(G)$ is available. For a random Erdős–Rényi graph $G(N, p)$, $d(G) = O\left(\frac{\log N}{\log(Np)}\right)$ with high probability [23], and $G(N, p)$ is connected with high probability for $p = \frac{c \log N}{N}$ with $c > 1$.

In our scenario, an apriori bound on $\sum_{n=1}^{N} b_{1,n} R_n^e r_n(\boldsymbol{a}(t))$, the effective reward for the successive elimination, is unknown to the agents but is needed to adjust the confidence radius ρ_k. Hence, at the beginning of each epoch (Step 4(a)), agents start a consensus averaging on $\{R_n^e\}_n$ to obtain $\sum_{n=1}^{N} b_{1,n} R_n^e$, which bounds $\sum_{n=1}^{N} b_{1,n} R_n^e r_n(\boldsymbol{a}(t))$, but varies between epochs.

3.2 Distributed and Scalable Successive Elimination

In the second part of every epoch (Steps 4(f)-4(j)), agents use the values they got in the communication step to run distributed successive elimination on a random subset of the action profiles. In our case, the effective reward of "arm" $\boldsymbol{a}$ at epoch e is $\sum_{n=1}^{N} b_{1,n} R_n^e r_n(\boldsymbol{a})$. In Step 2, each agent prepares a sequence of S_E actions independently and uniformly at random. This induces a sequence of S_E action profiles that all agents agree on their indices $\mathcal{B} = \{1, \ldots, S_E\}$. Therefore, all agents count the same number of visits $v_{\boldsymbol{a}}(t)$ for all $\boldsymbol{a}$ in Step 4(g). Moreover, all agents agree on the same weighted sum of rewards for the action profile they played $2D_E$ turns ago, denoted by $\hat{\mu}_k$ for $k = \mathcal{B}(l - 2D_E)$. Hence, all agents compute the same confidence radius ρ_k in Step 4(h), for all the action profiles indexed in $\mathcal{B}$. As a result, all agents agree on when to eliminate an action profile from $\mathcal{B}$ in Step 4(i) and will skip it in their next cycle over the action profiles, which is indexed by l.

Restricting the search to a random subset of action profiles is done to improve the scalability of the algorithm, as we discuss in detail in Section 7 of the appendix. For $S_E = K^N$, no randomization is needed as agents can go over all the action profiles sequentially.

3.3 Stability Guarantees

Theorem 2 below proves stability results on the sum of QoS regrets of the agents. The proof is given in the appendix. If the epoch length E_T increases with T, then the theorem shows that $\mathbb{E}\left\{R_n\left(T\right)\right\} = O\left(\sqrt{E_T T}\right)$ for any n and any $\boldsymbol{\lambda} \in C\left(\mathcal{G}\right)$. However, this does not provide a guarantee on the fraction of time during which the regret of a specific agent is "large". A highly fluctuating QoS regret means that an agent has to wait a long time to be "compensated" for when it was patient in favor of others. Hence, we also provide a general bound on $\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n=1}^{N} b_{1,n}\mathbb{E}\left\{R_n\left(t\right)\right\}$, for positive $\{b_{1,n}\}$ that are specified below. Using this bound, the first part of Theorem 2 assumes a fixed epoch length E and that $\boldsymbol{\lambda} \in C_{\varepsilon_E}\left(\mathcal{G}\right)$ for ε_E as specified below, so we are far enough from the boundary of $C\left(\mathcal{G}\right)$. In this case, we show that $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n=1}^{N}\mathbb{E}\left\{R_n\left(t\right)\right\} < \infty$, so the QoS regret of any agent is never too large. In the queuing literature, these stability notions are known as mean rate stability (i.e., sublinear regret) and strong stability.

Theorem 2. *Let $\mathcal{G}$ be a multiplayer multi-armed bandit game, played for T turns. Let $\boldsymbol{\lambda} \in C\left(\mathcal{G}\right)$. Let the agents run Algorithm 1 with a row-stochastic W, a connected G with diameter $d\left(G\right)$, epochs of length E, a random subset of action profiles of size $S_E \leq \lceil S_0 \log E \rceil$ for some $S_0 > 0$ and $D_E \geq d\left(G\right)$. Let $b_{i,j} = \left\{W^{D_E}\right\}_{i,j}$. Define the set of approximately optimal action profiles*

$$\mathcal{A}^*\left(\boldsymbol{\alpha}\right) = \left\{\boldsymbol{a} \mid \sum_{n=1}^{N}\alpha_n\mu_n\left(\boldsymbol{a}\right) \geq \max_{\boldsymbol{a}'}\sum_{n=1}^{N}\alpha_n\mu_n\left(\boldsymbol{a}'\right) - \sqrt{\frac{\log E}{E}}\right\} \tag{4}$$

and

$$\varepsilon_E = 34S_0\sqrt{\frac{\log^3 E}{E}} + 2\frac{D_E}{E} + 2\frac{S_0\log E + 1}{E^2} + E^{-\frac{S_0 \min_{\boldsymbol{\alpha}\in\Delta^N}\left|\mathcal{A}^*\left(\boldsymbol{\alpha}\right)\right|}{K^N}}. \tag{5}$$

Define

$$\delta\left(\boldsymbol{\lambda}, E\right) = \max_{\{\boldsymbol{a}(t)\}_{t=1}^{T}}\min_{n}\left(\frac{1}{T}\sum_{t=1}^{T}\mu_n\left(\boldsymbol{a}\left(t\right)\right) - \lambda_n - \varepsilon_E\right). \tag{6}$$

If $\boldsymbol{\lambda} \in C_{\varepsilon_E}\left(\mathcal{G}\right)$ then the QoS regrets (Definition 2) satisfy

$$\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n=1}^{N} b_{1,n}\mathbb{E}\left\{R_n\left(t\right)\right\} \leq \frac{1 + \left(\max_n \lambda_n\right)\left(2 + \varepsilon_E\right)E}{\delta\left(\boldsymbol{\lambda}, E\right)} \tag{7}$$

where $\mathbb{E}\left\{\cdot\right\}$ is over the rewards and the random subsets, $b_{1,n} > 0$ for all n, which implies strong stability, i.e., $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n=1}^{N}\mathbb{E}\left\{R_n\left(t\right)\right\} < \infty$. Furthermore, if we pick E_T such that $E_T \to \infty$ and $\frac{E_T}{T} \to 0$ as $T \to \infty$, then for any $\boldsymbol{\lambda} \in C\left(\mathcal{G}\right)$, we have for all n that $\mathbb{E}\left\{R_n\left(T\right)\right\} = O\left(\sqrt{E_T T}\right)$, and hence $\lim_{T\to\infty}\frac{\mathbb{E}\{R_n(T)\}}{T} = 0$ (mean rate stability).

4 QoS Regret Stability Analysis

In this section, we break our analysis required to prove Theorem 2 into lemmas and postpone the proofs to the appendix. The proof technique is inspired by the optimality proof of max-weight matching [20]. However, the QoS regret $R_n\left(t\right)$ does not behave like a standard queue. The main difference is that the service rate of agent n, which is its reward function, is not selected by a centralized decision-maker but is a function of the actions of the distributed agents. Since $\lambda_n t - \sum_{\tau=1}^{t} r_n\left(\tau\right)$ can get very negative, "empty queues" ($R_n\left(t\right) = 0$) are much more likely in our problem. Moreover, our "arrival process" is deterministic, of $\lambda_n < 1$ reward points every turn for agent n, and the backlog is not an integer. As detailed next, some of these differences require careful analysis or affect the algorithm design. For example, while a sublinearly increasing queue (in T) is unstable, in our case, as long as $\frac{R_n(T)}{T} \to 0$, agent n will asymptotically achieve its desired QoS.

Definition 4. Define the matrix $B = W^{D_E}$, and denote its (i,j) element by $b_{i,j}$. Define the following Lyapunov function such that $L\left(0\right) = 0$ and for all $t \geq 1$

$$L\left(t\right) \triangleq \sum_{n=1}^{N} b_{1,n}R_n^2\left(t\right). \tag{8}$$

$L(t)$ only uses the coefficients of Agent 1 since all agents, that know their IDs, agree on the weighted sum of rewards that Agent 1 sees. This forced agreement is done so that agents will use the exact same weighted sum of rewards. However, Agent 1 does not know more than others and can be replaced by any of them. If $D_E \geq d(G)$ is large then the consensus error is small and all agents see approximately the same weighted sum of rewards, so forcing agreement is not needed in practice.

The first lemma bounds the Lyapunov drift in each turn.

Lemma 1. *The Lyapunov drift at turn $t \geq 0$ satisfies:*

$$L(t+1) - L(t) \leq 1 + 2\sum_{n=1}^{N} b_{1,n} R_n(t) \left(\lambda_n - r_n\left(\boldsymbol{a}(t+1)\right)\right). \tag{9}$$

The proof of Theorem 2 follows by upper bounding $\mathbb{E}\{L(T)\} - \mathbb{E}\{L(0)\}$. Lemma 1 shows that this can be done by lower bounding $\sum_{n=1}^{N} b_{1,n} R_n(t) \mu_n(\boldsymbol{a}(t+1))$. The policy that does that is the maximum weight matching, given by $\boldsymbol{a}^*(t+1) = \arg\max_{\boldsymbol{a}} \sum_{n=1}^{N} b_{1,n} R_n(t) \mu_n(\boldsymbol{a})$. The two main obstacles our algorithm has to overcome to approximate max-weight matching are the distributed nature of the agents, and the unknown reward functions (bandit feedback). Distributed agents cannot deduce $\boldsymbol{a}^*(t+1)$ instantly every turn since they need time to agree on the weighted sum of rewards. For this reason, our algorithm works in epochs. Working in epochs means that our algorithm treats the QoS regret $R_n(t)$ as constant on its initial value $R_n^e \triangleq R_n(T_e)$ throughout epoch e. The agents can then aim to solve the following problem instead in every epoch

$$\boldsymbol{a}_e^* = \arg\max_{\boldsymbol{a}} \sum_{n=1}^{N} b_{1,n} R_n^e \mu_n(\boldsymbol{a}). \tag{10}$$

The next lemma bounds the resulting error from solving (10) instead of the max-weight matching.

Lemma 2. *Define $e(t)$ as the epoch such that $T_{e(t)} \leq t \leq T_{e(t)} + E - 1$. Then*

$$\sum_{t=0}^{T-1} \sum_{n=1}^{N} b_{1,n} \mu_n(\boldsymbol{a}_e^*) R_n^{e(t)} \geq \sum_{t=0}^{T-1} \sum_{n=1}^{N} b_{1,n} \mu_n(\boldsymbol{a}^*(t+1)) R_n(t) - ET \max_n \lambda_n. \tag{11}$$

In addition to solving (10) instead of the max-weight matching, working in epochs incurs an additive error of $O(\lambda_{\max} ET)$ again while replacing $R_n(t)$ with R_n^e while bounding the drift in (9).

The other loss of Algorithm 1 stems from learning $\boldsymbol{a}_e^*$ with bandit feedback. While dividing into epochs creates an additive error of $O(\lambda_{\max} ET)$, learning the optimal action profile $\boldsymbol{a}_e^*$ creates a multiplicative error, so the larger the QoS regrets R_n^e are, the larger this error is.

For this reason, we need to employ an algorithm that achieves sublinear regret in learning $\boldsymbol{a}_e^*$. Algorithm 1 induces successive elimination on a random subset of S_E action profiles, with $\sum_{n=1}^{N} b_{1,n} R_n^e r_n(\boldsymbol{a})$ as the reward function. The next lemma bounds the error (regret) of this successive elimination. Since agents only agree on $\sum_{n=1}^{N} b_{1,n} R_n^e r_n(\boldsymbol{a}(t))$ $2D_E$ turns later, our successive elimination suffers from delayed feedback. While not our focus, we chose successive elimination over UCB since it is more robust to delays, as was shown in [24]. The restriction to a random subset of S_E action profiles is done to improve scalability, but it leads to a worse regret than in the single-agent standard multi-armed bandits. Nevertheless, the regret is still sublinear in E and therefore good enough to prove our stability results.

Lemma 3. *Let $R_n^e = R_n(T_e)$ and define the set of approximately optimal action profiles as*

$$\mathcal{A}_e^* = \left\{ \boldsymbol{a} \mid \frac{\sum_{n=1}^{N} b_{1,n} R_n^e \mu_n(\boldsymbol{a})}{\sum_{n=1}^{N} b_{1,n} R_n^e} \geq \max_{\boldsymbol{a}'} \frac{\sum_{n=1}^{N} b_{1,n} R_n^e \mu_n(\boldsymbol{a}')}{\sum_{n=1}^{N} b_{1,n} R_n^e} - \sqrt{\frac{\log E}{E}} \right\}. \tag{12}$$

Let $\boldsymbol{a}_e^* \in \mathcal{A}_e^*$. Define the filtration $\mathcal{F}_e = \sigma(\{\boldsymbol{a}(t) \mid t \leq T_e\})$. Then Algorithm 1 maintains

$$\mathbb{E}\left\{ \sum_{t=0}^{T-1} \sum_{n=1}^{N} b_{1,n} R_n^e \left(r_n\left(\boldsymbol{a}_{e(t)}^*\right) - r_n\left(\boldsymbol{a}(t+1)\right) \right) \mid \mathcal{F}_e \right\} \leq$$

$$\left[34 S_0 \log E \sqrt{E \log E} + 2D_E + 2\frac{S_E}{E} + E^{1 - \frac{S_0 |\mathcal{A}_e^*|}{K^N}} \right] \sum_{n=1}^{N} b_{1,n} R_n^e \tag{13}$$

where $\mathbb{E}\{\cdot \mid \mathcal{F}_e\}$ is over the rewards and the e-th subset of action profiles.

5 Numerical Simulations

Our algorithm is useful for any application where cooperative agents want to learn to achieve a target QoS over time. The main advantage of our algorithm over static multi-agent learning is enabling dynamic strategies such as time-sharing between groups of agents. In the following, we tested our algorithm for wireless channel access and multitasking robots, two applications where smart time-sharing can significantly improve performance. For each experiment, we plot the sum of QoS regrets averaged over 100 realizations with one standard deviation shaded region.

5.1 Wireless Channel Access

Consider N devices and $K-1$ channels, such that each device can choose to transmit a packet on channel $k \geq 1$ by playing $a_n = k$ or not to transmit at all by playing $a_n = 0$. We consider a "soft collisions" model where the success probability of transmitting a packet for device n on channel k is $p_n^k = \frac{g_{n,n}^k}{1+\sum_{m=1}^N g_{m,n} 1\{a_m=k\}}$ for some channel gains $\{g_{n,m}\}$ and $\left\{g_{n,n}^k\right\}$. This model generalizes the collision model [11, 13, 14], where multiple devices that pick the same channel all receive zero reward. As in practice, the geometry of the devices dictate how much they interfere with each other.

Fig. 1(a) shows the sum of QoS regrets over time for $N = 4$ and $K = 3$. We used $S_E = 128$, $d(G) = 4$ and $E = 50S_E$. The channel gains were $g_{1,4} = g_{1,3} = g_{2,4} = g_{3,2} = 0$ and $g_{1,2} = g_{3,4} = 100$ and $g_{1,1}^2 = g_{2,2}^2 = g_{3,3}^3 = g_{4,4}^3 = 1$, such that $g_{n,m} = g_{m,n}$ for all n, m, and $g_{1,1}^3 = g_{2,2}^3 = g_{3,3}^2 = g_{4,4}^2 = 0.1$. Hence devices 1 and 2 are in proximity, and devices 3 and 4 are also in proximity, but far away from 1 and 2. Then, devices 1 and 2 prefer channel $k = 2$ while devices 3 and 4 prefer channel $k = 3$. The QoS vector was $\lambda_n = 0.499$ for all n, which cannot be achieved with a static solution, but only if devices 1 and 3 transmit simultaneously each on their preferred channel, and in the next turn devices 2 and 4 do the same, and so on in a round-robin manner. After a while, the QoS regrets stabilized on low values.

Fig. 1(b) shows the sum of QoS regrets over time for $N = 100$ and $K = 2$ (one channel). The channel gains $\{g_{n,m}\}$ and $\left\{g_{n,n}^k\right\}$ were chosen independently and uniformly at random on $[0, 1]$. We used $S_E = 10^4$, $E = 5S_E$, and $d(G) = 4$. The QoS of agent n was $\lambda_n = 1.9\frac{p_n^1 - 0.02}{N}$, which is 90% more than what an almost perfect time-sharing can achieve. The stability of the QoS regrets is again evident, in agreement with the theory. Note that while there are 2^{100} action profiles, considering only a random subset of 10^4 action profiles was enough to achieve this QoS.

5.2 Multitasking Robots

Consider N robots and $K = N$ tasks, such that the n-th robot is in charge of the n-th task. Each robot n has a skill of s_n^k for the k-th task. Each turn, each robot can choose to work on the k-th task by playing $a_n = k$. The reward of robot n is $r_n(\boldsymbol{a}) = \sum_{m=1}^N 1_{\{a_m=n\}} s_m^n + z_n$, which models the rate at which this task progresses, where z_n is a uniform noise on $[-0.2, 0.2]$ that is i.i.d. over time. Hence, only one robot can measure the performance of a certain task, but the others can still assist.

Fig. 2(a) shows the sum of QoS regrets over time for $N = 4$ and $K = 4$. We used $E = 5K^N = 5120$, $d(G) = 3$. The QoS vector was $\boldsymbol{\lambda} = (0.8, 0.2, 0.4, 0.4)$ and the skill matrix was $\begin{pmatrix} 0.5 & 0.4 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.2 & 0.3 \\ 0.25 & 0.5 & 0.6 & 0.3 \\ 0.25 & 0.4 & 0.2 & 0.6 \end{pmatrix}$ (e.g., $s_1^4 = 0.2$). As a result, λ_1 is larger than the maximal skill of a single robot so the robots had to cooperate on their tasks to achieve this QoS vector. Additionally, no static solution can support this $\boldsymbol{\lambda}$.

Fig. 2(b) shows the sum of QoS regrets for $N = 8$ and $K = 8$. We used $S_E = 1000$, $d(G) = 7$ and $E = 10S_E$, $\boldsymbol{\lambda} = (0.83, 0.79, 0.78, 0.71, 0.85, 0.76, 0.74, 0.86)$ and picked s_n^k uniformly and independently at random on $[0.2, 1]$. Note that while there are 8^8 action profiles, considering only a random subset of 10^3 action profiles was enough to achieve this QoS.

In both figures, the sum of QoS regrets increases slowly with T, which for a traditional queue would mean instability but in our case means that the empirical average rewards converge to $\boldsymbol{\lambda}$.

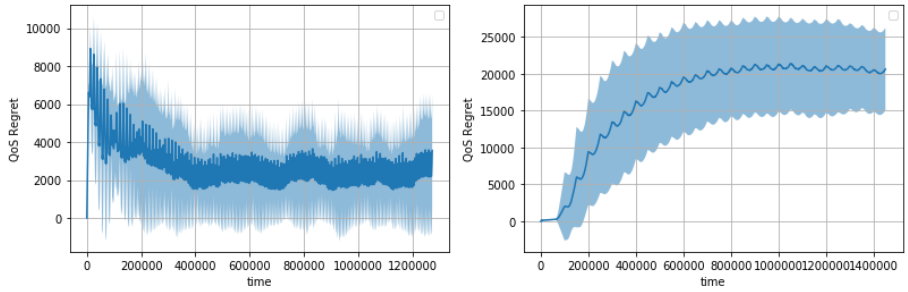

Figure 1: Sum of QoS regrets: channel access, $N = 4, K = 3$ (left) and $N = 100, K = 2$ (right)

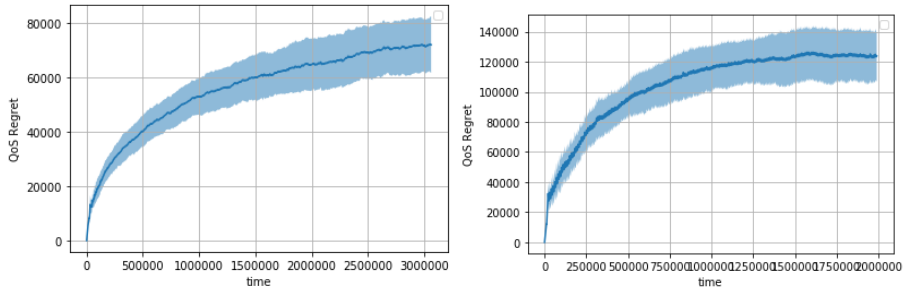

Figure 2: Sum of QoS regrets: multitasking robots, $N = 4$ (left) and $N = 8$ (right)

6 Conclusion

We studied how a team of agents can improve the Quality of Service (QoS) of each by inducing a dynamic behavior between them. The main observation that inspired our algorithm is that the QoS regret of each agent behaves like a queue. Then, our distributed algorithm learns to approximate a max-weight matching of agents to actions to maximize the "throughout" going out of these virtual queues. To that end, our algorithm works in epochs and employs simultaneous consensus averaging and distributed successive elimination on a random subset of action profiles and with delayed feedback. Our main result shows that our algorithm can achieve all QoS vectors in the capacity region for large enough T, and characterizes the tradeoff between the capacity region and the delay in obtaining the QoS for finite T. Simulations demonstrate the effectiveness of our algorithm for channel access in wireless networks and multitasking robots.

We considered a general discrete game where we cannot avoid going over a list of candidate action profiles to find an approximately optimal one. However, applications often focus on much more structured games. For example, in congestion games, best-response dynamics are guaranteed to converge to a Nash equilibrium. Hence, if the game is designed such that all Nash equilibria are efficient, an exhaustive search is not needed. Another alternative is to let the agents directly solve their combinatorial optimization problem as a team, using its special structure. Examples are the assignment problem that can be solved in $O\left(N^4\right)$ time, unconstrained submodular minimization, and shortest path [25]. Taking this approach, the challenge is to design a distributed algorithm that can solve this problem based on bandit feedback alone.

Our work introduces an analogy between multi-agent QoS and queuing. By further studying this analogy we can leverage more tools from queuing theory to design dynamic distributed learning algorithms with strong performance guarantees. For example, scheduling algorithms with delay guarantees [26] can help to design QoS multi-agent algorithms with better delay performance. Another example is to employ the "drift plus penalty" technique [20] to optimize an objective such as the sum of rewards while still guaranteeing some QoS for all agents.

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Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] Limitations of the model are described in Section 2 and Section 3, and some mitigation is suggested in Section 7.
   (c) Did you discuss any potential negative societal impacts of your work? [N/A] As a theory paper, the negative societal impacts are rather ambiguous.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes]
   (b) Did you include complete proofs of all theoretical results? [Yes]
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] All the details of our algorithm are provided in Section 3. The algorithm is extremely simple to implement so no code is provided (given the instructions we do provide, it is not needed to reproduce the results). Section 5 includes all the details needed to reproduce the numerical experiments, which are of secondary importance since they are only meant to validate the theoretical findings.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [N/A]
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes]
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [N/A] The simulations can be easily run on a personal computer.
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [N/A]
   (b) Did you mention the license of the assets? [N/A]
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   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]