EFFECTIVE OFFLINE REINFORCEMENT LEARNING VIA CONSERVATIVE STATE VALUE ESTIMATION

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ABSTRACT

Offline RL promises to learn effective policies from static experience datasets without further interaction, which expect to perform well in the online environment. However, it faces up to a major challenge of value over-estimation introduced by the distributional drift between the dataset and the current learned policy, which leads to learning failure in practice. The common approach is to add a penalty term to reward or value estimation in the Bellman iterations, which has given rise to a number of successful algorithms such as CQL. Meanwhile, to avoid extrapolation on unseen states, existing methods focus on conservative Q-function estimation. In this paper, we propose CSVE, a new approach that directly imposes penalty on out-of-distribution states. We prove that for the evaluated policy, our conservative state value estimation satisfies: (1) over the state distribution that samples penalizing states, it lower bounds the true values in expectation, and (2) over the marginal state distribution of data, it is no more than the true values in expectation plus a constant decided by sampling error. Further, we develop a practical actor-critic algorithm in which the critic does the conservative value estimation by additionally sampling and penalizing the states *around* the dataset, while the actor applies advantage weighted updates to improve the policy. We evaluate in classic continual control tasks of D4RL, showing that our method performs better than the conservative Q-function learning methods (e.g., CQL) and is strongly competitive among recent SOTA methods.

1 INTRODUCTION

Reinforcement Learning (RL), which learns to act by interacting with the environment, has achieved remarkable success in various tasks. However, in most real applications, it is impossible to learn online from a weak policy and exploration may be risky. Instead, offline RL((Fujimoto et al., 2019; Lange et al., 2012)) avoids this problem by learning the policy solely from historical data.

The naive offline RL approach, which directly uses online RL algorithms to learn from interaction data, suffers from the problems of value over-estimation and policy extrapolation on unseen states or actions. To conquer these issues, two types of approaches, conservative value function estimation and constrained policy optimization, are proposed and widely investigated. The value-based methods, e.g., Conservative Q-Learning (CQLKumar et al. (2020)), avoid the value over-estimation problem by systematically underestimating the values of points (i.e., state-action pairs) that are not seen in the dataset. However, for the large joint space of states and actions, such a conservative value estimation is challenging and in practice, they are often not complete. On the other hand, constrained policy optimization, which constrains the learned policy is not distant from behaviour policy that collected the data, and often suffers the problem of less exploration and thus bad performance.

In this paper, we propose a new offline approach, CSVE(Conservative State Value Estimation). The main idea is to derive a conservative value estimation on states rather than joint state-actions, which can leads to tighter bounds on true value function than methods with pessimism on Q values, and develop a model-based method to derive th policy. Our main contributions are as follows.

- We propose an approach of conservative state value estimation, and prove that it lower bounds the real state values in expectation over any state distribution that is used to sample penalizing states, and is up-bounded by the real values in expectation over the marginal

state distribution of the dataset plus a constant term depending on sampling errors. It gives a tighter estimation bound than that of CQL and thus in potential can derive a better policy.

- We propose a practical method to approximately estimate the state values in the offline context, and learn the policy in an advantage weighting updates approach. In particular, we use the fitted dynamics model to generalize over in-distribution space and sample out-of-distribution points directly reachable from the dataset.
- We evaluate our method in continuous control tasks of Gym (Brockman et al., 2016) and Adroit (Rajeswaran et al., 2017) in D4RL (Fu et al., 2020) benchmark, showing that CSVEperforms better compared to previous methods based on conservative Q-value estimation, and is strongly competitive among main SOTA offline RL algorithms.

2 PRELIMINARIES

Offline Reinforcement Learning. Consider the Markov Decision Process $M := (\mathcal{S}, \mathcal{A}, P, r, \rho, \gamma)$, which consists of the state space $\mathcal{S}$, the action space $\mathcal{A}$, the transition model $P : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$, the reward function $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$, the initial state distribution ρ and the discount factor $\gamma \in (0, 1]$. A stochastic policy $\pi : \mathcal{S} \to \Delta(\mathcal{A})$ takes an action in probability given the current state. A transition is the tuple (s_t, a_t, r_t, s_{t+1}) where $a_t \sim \pi(\cdot|s_t)$, $s_{t+1} \sim P(\cdot|s_t, a_t)$ and $r_t = r(s_t, a_t)$. We assume that the reward values satisfy $|r(s, a)| \leq R_{max}, \forall s, a$. A trajectory under π is the random sequence $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_T)$ which consists of continuous transitions starting from $s_0 \sim \rho$.

The standard RL is to learn a policy $\pi \in \Pi$ that maximize the future cumulative rewards $J_\pi(M) = \mathbb{E}_{M,\pi}[\sum_{t=0}^{\infty} \gamma^t r_t]$ via active interaction with the environment M. At any time t, for the policy π, the value function of state is defined as $V^\pi(s) := \mathbb{E}_{M,\pi}[\sum_{k=0}^{\infty} \gamma^{t+k} r_{t+k} | s_t = s]$, and the Q value function is $Q^\pi(s, a) := \mathbb{E}_{M,\pi}[\sum_{k=0}^{\infty} \gamma^{t+k} r_{t+k} | s_t = s, a_t = a]$. The Bellman operator is a function projection: $\mathcal{B}^\pi Q(s, a) := r(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a), a' \sim \pi(\cdot|s')}[Q(s', a')]$, or $\mathcal{B}^\pi V(s) := \mathbb{E}_{a \sim \pi(\cdot|s)}[r(s, a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)}[V(s')]]$, which leads to iterative value updates. Bellman consistency implies that $V^\pi(s) = \mathcal{B}^\pi V^\pi(s), \forall s$ and $Q^\pi(s) = \mathcal{B}^\pi Q^\pi(s, a), \forall s, a$. In practice with function approximation, we use the empirical Bellman operator $\hat{\mathcal{B}}^\pi$ where the former expectations are estimated with data.

The offline RL is to learn the policy π from a static dataset $D = \{(s, a, r, s')\}$ consisting of transitions collected by any behaviour policy, aiming to behave well in the online environment. Note that, unlike the standard online RL, offline RL cannot interact with the environment during learning.

Conservative Value Estimation. One main challenge in offline RL is the over-estimation of values introduced by extrapolation on unseen states and actions, which may make the learned policy collapse. To address this issue, conservatism or pessimism are used in value estimation, e.g. CQL learns a conservative Q-value function by penalizing the value of unseen actions on states:

$$\begin{aligned} \hat{Q}^{k+1} \leftarrow \arg\min_{Q} \; & \alpha \left(\mathbb{E}_{s \sim D, a \sim \mu(a|s)}[Q(s, a)] - \mathbb{E}_{s \sim D, a \sim \hat{\pi}_\beta(a|s)}[Q(s, a)] \right) \\ & + \frac{1}{2} \mathbb{E}_{s,a,s' \sim D}[(Q(s, a) - \hat{\beta}_\pi \hat{Q}^k(s, a))^2] \end{aligned} \tag{1}$$

where $\hat{\pi}_\beta$ and π are the behaviour policy and learnt policy separately, μ is any arbitrary policy different from $\hat{\pi}_\beta$, and α the factor for trade-off of conservatism.

Constrained Policy Optimization. To address the issues of distribution drift between learning policy and behaviour policy, one approach is to constrain the learning policy close to the behaviour policy (Bai et al., 2021; Wu et al., 2019; Nair et al., 2020; Levine et al., 2020; Fujimoto et al., 2019). Here we take AWAC (Nair et al. (2020)) which adopts an implicit KL divergence to constrain the distance of policies as example:

$$\pi^{k+1} \leftarrow \arg\max_{\pi} \mathbb{E}_{s,a \sim D} \left[\log \pi(a|s) \frac{1}{Z(s)} \exp\left(\frac{1}{\lambda} A^{\pi^k}(s, a) \right) \right] \tag{2}$$

where A^{π^k} is the advantage of policy π^k, and Z the normalization constant for s.

Model-based Offline RL. In RL, the model is an approximation of the MDP M. We denote a model as $\hat{M} := (\mathcal{S}, \mathcal{A}, \hat{P}, \hat{r}, \rho, \gamma)$, where $\hat{P}$ and $\hat{r}$ are approximations of P and R respectively. In

the setting of offline RL, the model is used to roll out and augment data (Yu et al., 2020; 2021) or act as a surrogate of real environment to interact with agent (Kidambi et al., 2020). In this paper, we use model to sample the next states that are approximately reachable from the dataset.

3 CONSERVATIVE STATE VALUE ESTIMATION

In the offline setting, the value overestimation is a major problem that results in failing to learn a reasonable policy (Levine et al., 2020; Fujimoto et al., 2019). In contrast to previous work that lower-bounds state-action values (Kumar et al., 2020; Yu et al., 2021), we directly lower bound the state values. All proofs of our theorems can be found in the Appendix A.

3.1 CONSERVATIVE OFF-POLICY EVALUATION

We aim to conservatively estimate the value $V^\pi(s)$ of a target policy π given a dataset D to avoid overestimation of out-of-distribution states s_{ood}. To achieve this, we penalize the V-values evaluated on states that is more likely to be out-of-distribution and pushing up the V-values on states that is in the distribution of the dataset, which is achieved through the following iteration:

$$\hat{V}^{k+1} \leftarrow \arg\min_V \frac{1}{2}\mathbb{E}_{s\sim d_u(s)}[(\hat{\mathcal{B}^\pi}\hat{V}^k(s) - V(s))^2] + \alpha(\mathbb{E}_{s'\sim d(s)}V(s') - \mathbb{E}_{s\sim d_u(s)}V(s)). \quad (3)$$

Here, $d_u(s)$ is the discounted state distribution of D, $d(s)$ is any state distribution, and $\hat{\mathcal{B}^\pi}$ is the empirical Bellman operator(see appendix for more details). Considering the setting without function approximation, by setting the derivative of Eq. 3 as zero, the V function found by approximate dynamic programming in iteration k can be obtained:

$$\hat{V}^{k+1}(s) = \hat{\mathcal{B}^\pi}V^k(s) - \alpha[\frac{d_{(s)}}{d_u(s)} - 1]\ \forall s, k. \quad (4)$$

Theorem 1. *For any $d(s)$ with $\operatorname{supp} d \subseteq \operatorname{supp} d_u$ ($d \neq d_u$), with a sufficiently large α (i.e., $\alpha \geq \mathbb{E}_{s\sim d(s)}\mathbb{E}_{a\sim\pi(a|s)}\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}}/\mathbb{E}_{s\sim d(s)}[\frac{d(s)}{d_u(s)} - 1]$), the expected value of the estimation $\hat{V}^\pi(s)$ under $d(s)$ is the lower bound of the true value, that is: $\mathbb{E}_{s\sim d(s)}[\hat{V}^\pi(s)] \leq \mathbb{E}_{s\sim d(s)}[V^\pi(s)]$.*

$\hat{V}^\pi(s) = \lim_{k\to\infty}\hat{V}^k$ is the converged value estimation with the static dataset D, and $\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}}$ is related to sampling error introduced by the use empirical rather than Bellman operator. If the counts of each state-action pair is greater than zero, $|D(s,a)|$ denotes a vector of size $|\mathcal{S}||\mathcal{A}|$ containing counts for each state-action pair. If the counts of this state action pair is zero, the corresponding $\frac{1}{\sqrt{|D(s,a)|}}$ is large but finite value. We assume that with probability $\geq 1-\delta$, the sampling error is less than $\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}}$, while $C_{r,t,\delta}$ is a constant (See appendix for more details.) Note that if the sampling error is ignorable, $\alpha > 0$ can guarantee the lower bound results.

Theorem 2. *The expected value of the estimation $\hat{V}^\pi(s)$ under the state distribution of the original dataset is the lower bound of the true value plus the term of irreducible sampling error, that is: $\mathbb{E}_{s\sim d_u(s)}[\hat{V}^\pi(s)] \leq \mathbb{E}_{s\sim d_u(s)}[V^\pi(s)] + \mathbb{E}_{s\sim d_u(s)}(I-\gamma P^\pi)^{-1}\mathbb{E}_{a\sim\pi(a|s)}\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}}$.*

, where P^π refers to the transition matrix coupled with policy π (see Appendix for details).

Now we show that, during iterations, the gap between the value of in-distribution state and out-of-distribution state in the estimated V-function is higher than in the true V-functions.

Theorem 3. *At any iteration k, with a large enough α, our method expands the difference in expected V-values under the chosen state distribution and the dataset state distribution, that is: $\mathbb{E}_{s\sim d_u(s)}[\hat{V^k}(s)] - \mathbb{E}_{s\sim d(s)}[\hat{V^k}(s)] > \mathbb{E}_{s\sim d_u(s)}[V^k(s)] - \mathbb{E}_{s\sim d(s)}[V^k(s)]$.*

In the policy extraction part, this property enables our policy to take actions a in state $s(s\sim D)$ that remains in distribution instead of out of distribution, given that our estimated V-function does not overestimate the erroneous out-of-distribution states compared to the in-distribution states.

Now we present four remarks to explain how the above theorems guide applications of Eq. 3 in offline RL algorithms.

Remark 1. In Eq. 3, if $d = d_u$, the penalty on out-of-distribution states degenerates, which means that the policy should not reach states with low support in data, and consequently never explore the unseen actions at the state. Indeed, AWAC Nair et al. (2020) adopts this setting. We show that with proper choice of d different from d_u, our method performs better than AWAC in practice.

Remark 2. Theorem 2 implies that under d_u, the marginal state distribution of data, the expectation estimated value of π should either be lower than the true value, or higher than the true value but within a threshold. This fact motivates our advantage weighted policy update method in Eq. 11.

Remark 3. Theorem 1implies that under d, say the discounted state distribution of any policy, the expectation estimated value of π should lower bounds the true value. This fact motivates our policy improvement method of unifying advantage weighted update with a bonus exploration in Eq. 12.

Remark 4. Theorem 3 states $\mathbb{E}_{s\sim d(s)}[V^k(s)] - \mathbb{E}_{s\sim d(s)}[\hat{V}^k(s)] > \mathbb{E}_{s\sim d_u(s)}[V^k(s)] - \mathbb{E}_{s\sim d_u(s)}[\hat{V}^k(s)]$. That is to say, under the distribution d, the amount of value under-estimation in expectation is larger than that of the behaviour policy d_u. With proper choice of d, it is safe and effective to derive a new and potentially better policy with $\hat{V}^k$. Our algorithm choose the distribution of model predictive next-states as d, i.e., $s' \sim d$ is implemented by $s \sim D, a \sim \pi(\cdot|s), s' \sim \hat{P}(\cdot|s,a)$, which effectively builds a soft 'river' with low values around the dataset.

Comparison to previous results: Different from previous work Kumar et al. (2020) which penalize the Q function of out-of-distribution actions, we directly penalize out-of-distribution state values, which results in a better lower bound. In Kumar et al. (2020) , the resulting estimate V function is the point-wise lower bound. While in our work, only the expectations of the V-function is the lower bound. In Yu et al. (2021), by using roll-outs generated by models, the resulting V function estimate is the lower bound under the initial state distribution, i.e., $E_{s\sim u_0(s)}[\hat{V}^\pi(s)] \leq E_{s\sim u_0(s)}[V^\pi(s)]$, where u_0 is the initial state distribution. While in our work, we relax this distribution to be any state distribution that is different from the discounted state distribution in the dataset.

3.2 Safe Policy Improvement Guarantees

Following prior works (Laroche et al. (2019); Kumar et al. (2020); Yu et al. (2021)), we show that our method has the safe policy improvement guarantees against the data-implied behaviour policy. We first show that our method optimizes a penalized RL empirical objective:

Theorem 4. *Let $\hat{V}^\pi$ be the fixed point of Equation 3, then $\pi^*(a|s) = \arg\max_\pi \hat{V}^\pi(s)$ is equivalently obtained by solving:*

$$\pi^*(a|s) \leftarrow \arg\max_\pi J(\pi, \hat{M}) - \alpha \frac{1}{1-\gamma} \mathbb{E}_{s\sim d^\pi_{\hat{M}}(s)}[\frac{d(s)}{d_u(s)} - 1] \tag{5}$$

Building upon Theorem 4, we show that our method provides a ζ-safe policy improvement over π_β

Theorem 5. *Let $\pi^*(a|s)$ be the policy obtained in Equation 5. Then, it is a ζ-safe policy improvement over $\hat{\pi}^\beta$ in the actual MDP M, i.e., $J(\pi^*, M) \geq J(\hat{\pi}^\beta, M) - \zeta$ with high probability 1- δ, where ζ is given by:*

$$\zeta = 2(\frac{C_{r,\delta}}{1-\gamma} + \frac{\gamma R_{max} C_{T,\delta}}{(1-\gamma)^2}) \mathbb{E}_{s\sim d^\pi_{\hat{M}}(s)}[\frac{\sqrt{|\mathcal{A}|}}{\sqrt{|\mathcal{D}(s)|}} \sqrt{\mathbb{E}_{a\sim\pi(a|s)}(\frac{\pi(a|s)}{\pi_\beta(a|s)})}] - \underbrace{(J(\pi^*, \hat{M}) - J(\hat{\pi}_\beta, \hat{M}))}_{\geq \alpha \frac{1}{1-\gamma} \mathbb{E}_{s\sim d^\pi_{\hat{M}}(s)}[\frac{d(s)}{d_u(s)} - 1]}. \tag{6}$$

4 Methodology

In this section, we propose a practical Actor-Critic method for computing conservative value estimation function by approximately solving Equation 3 and taking advantage weighted policy updates. It is mainly motivated by the theoretic results, as explained by the four remarks in section 3.1. Besides, the full algorithm of deep learning implementation is presented in section B.

4.1 CONSERVATIVE VALUE ESTIMATION

Given the access to a dataset D collected by some behaviour policy π_β, our aim is to estimate the value function V^π for a target policy π. As stated in section 3, to prevent the value overestimation, we instead learn a conservative value function $\hat{V}^\pi$ that lower bounds the real values of π by adding a penalty on out-of-distribution states into the flow of Bellman projections. Our method consists of the following iterative updates of Equations 7- 9, where $\overline{\hat{Q}^k}$ is the target network of $\hat{Q}^k$.

$$\begin{aligned}\hat{V}^{k+1} \leftarrow \arg\min_V L_V^\pi(V; \overline{\hat{Q}^k}) = \ & \alpha \left(\mathbb{E}_{s\sim D, a\sim\pi(\cdot|s), s'\sim\hat{P}(s,a)}[V(s')] - \mathbb{E}_{s\sim D}[V(s)]\right) \\ & + \mathbb{E}_{s\sim D}\left[(\mathbb{E}_{a\sim\pi(\cdot|s)}[\overline{\hat{Q}^k}(s,a)] - V(s))^2\right]\end{aligned} \tag{7}$$

$$\hat{Q}^{k+1} \leftarrow \arg\min_Q L_Q^\pi(Q; \hat{V}^{k+1}) = \mathbb{E}_{s,a,s'\sim D}\left[\left(r(s,a) + \gamma\hat{V}^{k+1}(s') - Q(s,a)\right)^2\right] \tag{8}$$

$$\overline{\hat{Q}^{k+1}} \leftarrow \omega\overline{\hat{Q}^k} + (1-\omega)\hat{Q}^{k+1} \tag{9}$$

The RHS of Eq. 7 is an approximation of Eq. 3, where the first term gives out-of-distribution states a penalty, and the second term follows the definition of V values and Q values. In Eq. 8, the RHS is TD errors estimated on transitions in the dataset D. Note that the target term here uses the sum of the immediate reward $r(s,a)$ and the next step state's value $\hat{V}^{k+1}(s')$. In Eq. 9, the target Q values are updated with a soft interpolation factor $\omega \in (0,1)$. $\overline{\hat{Q}^k}$ changes slower than $\hat{Q}^k$, which makes the TD error estimation in Eq. 7 more stable.

Constrained policy. Note that in RHS of Eq. 7, we use $a \sim \pi(\cdot|s)$ in expectation. To safely estimate the target value of $V(s)$ by $\mathbb{E}_{a\sim\pi(\cdot|s)}[\overline{\hat{Q}}(s,a)]$, almost always requires $\text{supp}(\pi(\cdot|s)) \subset \text{supp}(\pi_\beta(\cdot|s))$. We achieves this by the *advantage weighted policy update*, which forces $\pi(\cdot|s)$ have significant probability mass on actions taken by π_β in data, as detailed in section 3.2.

Model-based out-of-distribution state sampling. In Eq. 7, we set d in Eq. 3 as the distribution of the predictive next-states from D by following π. It is beneficial in practive. On one hand, this method is efficient to sample only the states that are approximately reachable from D by one step, rather than to sample the whole state space. On the other hand, we only need the model to do one-step prediction such that no bootstrapped errors due to long horizon are introduced. Following previous work (Janner et al., 2019; Yu et al., 2020; 2021), we implement the probabilistic dynamics model using an ensemble of neural networks $\{p\theta^1, \ldots, p\theta^B\}$. Each neural network produces a Gaussian distribution over the next state and reward: $P_\theta^i(s_{t+1}, r|s_t, a_t) = \mathcal{N}(u_\theta^i(s_t,a_t), \sigma_\theta^i(s_t,a_t))$.

Adaptive penalty factor α. The pessimism level is controlled by the parameter $\alpha \geq 0$. In practice, we set α adaptive during training as follows, which is similar as that in CQL(Kumar et al. (2020))

$$\max_{\alpha\geq 0}[\alpha(\mathbb{E}_{s'\sim d}[V_\psi(s')] - \mathbb{E}_{s\sim D}[V_\psi(s)] - \tau)] \tag{10}$$

, where τ is a budget parameter. If the expected difference in V-values is less than τ, α will decrease. Otherwise, α will increase and penalize the out of distribution state values more aggressively.

4.2 ADVANTAGE WEIGHTED POLICY UPDATES

After learning the conservative $\hat{V}^{k+1}$ and $\hat{Q}^{k+1}$ (or $\hat{V}^\pi$ and $\hat{Q}^\pi$ when converged), we improve the policy by the following advantage weighted policy update (Nair et al., 2020).

$$\begin{aligned}\pi \leftarrow \arg\min_{\pi'} L_\pi(\pi') = -\mathbb{E}_{s,a\sim D}\left[\log \pi'(a|s) \exp\left(\beta\hat{A}^{k+1}(s,a)\right)\right] \\ \text{where } \hat{A}^{k+1}(s,a) = \hat{Q}^{k+1}(s,a) - \hat{V}^{k+1}(s).\end{aligned} \tag{11}$$

Eq. 11 updates the policy π to amounts of weighted maximum likelihood which are computed by re-weighting state-action samples in D with estimated advantage $\hat{A}^{k+1}$. As discussed in the

AWAC (Nair et al., 2020), this method avoids explicitly estimating the behaviour policy and its resulted sampling errors which is an import issue in the offline RL setting (Kumar et al., 2020).

Implicit policy constraints. We adopt the advantage weighted policy updates which imposes an implicit KL divergence constraints between π and π_β. This policy constraint is necessary to guarantee that the next state s' in Equation 7 can be safely generated through policy π. As derived in Nair et al. (2020) (Appendix A), the Eq. 11 is an parametric solution of the following problem:

$$\max_{\pi'} \mathbb{E}_{a\sim\pi'(\cdot|s)}[\hat{A}^{k+1}(s,a)] \quad s.t.\ D_{\mathrm{KL}}(\pi'(\cdot|s) \parallel \pi_\beta(\cdot|s)) \leq \epsilon, \quad \int_a \pi'(a|s)da = 1.$$

Note that $D_{\mathrm{KL}}(\pi' \parallel \pi_\beta)$ is an reserve KL divergence with respect to π', which is mode-seeking ((Shlens, 2014)). When treated as Lagrangian it forces π' allocate its probability mass to the maximum likelihood supports of π_β, re-weighted by the estimated advantage. In other words, for the space of A where $\pi_\beta(\cdot|s)$ has no samples, $\pi'(\cdot|s)$ has almost zero probability mass too.

Bonus of Exploration on Near States. As suggested by remarks in Section 3.1, in practice allowing the policy explore the predicated next states transition $(s \sim D)$ following $a \sim \pi'(\cdot|s))$ leads to better test performance. With this kind of exploration, the policy is updated as follows.

$$\pi \leftarrow \arg\min_{\pi'} L^+_\pi(\pi') = L_\pi(\pi') - \lambda \mathbb{E}_{s\sim D, a\sim\pi'(s), s'\sim\hat{P}(s,a)} \left[r(s,a) + \hat{V}^{k+1}(s') \right] \tag{12}$$

The second term is an approximation to $E_{s\sim d_\pi(s)}[V^\pi(s)]$, while the first term is the approximation of $E_{s\sim d_u(s)}[V^\pi(s)]$. While the choice of λ is ultimately just a hyper-parameter, we balance between optimistic policy optimization (in maximizing V) and constrained policy update (the first term) by adjusting λ.

5 Experiments

The primary goal of this section is to investigate whether the proposed tighter conservative value estimation leads to performance improvement. Besides, we would like to ascertain when further exploration has benefits and how well CSVE performs compared with SOTA algorithms.

We evaluate our method on classical continuous control tasks of Gym(Brockman et al., 2016) and Adroit(Rajeswaran et al., 2017) in the standard D4RL (Fu et al. (2020)) benchmark. The Gym control tasks include HalfCHeetah, Hopper and Walker2D, each with 5 datasets collected by following different types of policies (random, medium, medium-replay, medium-expert, and expert). The Adroid tasks include Pen, Hammer, Door and Relocate, each with 3 dataset collected by different policies (human, cloned, and expert).

Our method, namely CSVE, and the compared baselines are CQL(Kumar et al., 2020), AWACNair et al. (2020), PBRL(Bai et al., 2021) and other SOTA algorithms (MOPO(Yu et al., 2020), TD3-BC(Fujimoto & Gu, 2021), UWAC(Wu et al., 2021), BEAR(Kumar et al., 2019)) whose performance results are public or have high-quality open implementations. CQL which estimates the conservative Q values on state-action pairs rather than states, is the direct comparing method to ours. AWAC is one special case of our Eq. 3 when $d = d_u$. PBRL is a very strong performant in offline RL, but is quite costly on computation since it uses the ensemble of hundreds of sub-models.

5.1 Overall Performance

We first test on the Gym control tasks. We train our methods for 1 million steps and report the final evaluation performance. The overall results are shown in Table 1. Compared to CQL, our method has better performance on 11 of 15 tasks and similar performance on others. In particular, our method shows consistent advantage on the datasets that generated by following random or sub-optimal policies (random and medium). Compared to AWAC, our method has better performance on 9 out 15 tasks and comparable performance on others, which demonstrates the effect of our further exploration beyond cloning the behaviour policy. In particular, our method shows an obvious advantage in extrating the best policy on data of mixed policy (Medium Expert) while AWAC can not. Finally, our method performs close to PBRL, even PBRL has almost orders of more model capacity and computation cost.

		AWAC	CQL	MOPO	TD3-BC	PBRL	CSVE (ours)
Random	HalfCheetah	12.1	17.5 ± 1.5	35.9 ± 2.9	11.0 ± 1.1	13.1 ± 1.2	26.7 ± 2.0
	Hopper	8.7	7.9 ± 0.4	16.7 ± 12.2	8.5 ± 0.6	31.6 ± 0.3	27.0 ± 8.5
	Walker2D	2.2	5.1 ± 1.3	4.2 ± 5.7	1.6 ± 1.7	8.8 ± 6.3	6.1 ± 0.8
Medium	HalfCheetah	50.0	47.0 ± 0.5	73.1 ± 2.4	48.3 ± 0.3	58.2 ± 1.5	48.6 ± 0.0
	Hopper	96.0	53.0 ± 28.5	38.3 ± 34.9	59.3 ± 4.2	81.6 ± 14.5	99.4 ± 5.3
	Walker2D	89.1	73.3 ± 17.7	41.2 ± 30.8	83.7 ± 2.1	90.3 ± 1.2	82.5 ± 1.5
Medium Replay	HalfCheetah	43.4	45.5 ± 0.7	69.2 ± 1.1	44.6 ± 0.5	49.5 ± 0.8	54.8 ± 0.8
	Hopper	97.8	88.7 ± 12.9	32.7 ± 9.4	60.9 ± 18.8	100.7 ± 0.4	91.7 ± 0.3
	Walker2D	80.0	81.8 ± 2.7	73.7 ± 9.4	81.8 ± 5.5	86.2 ± 3.4	78.5 ± 1.8
Medium Expert	HalfCheetah	62.8	75.6 ± 25.7	70.3 ± 21.9	90.7 ± 4.3	93.6 ± 2.3	93.1 ± 0.3
	Hopper	87.2	105.6 ± 12.9	60.6 ± 32.5	98.0 ± 9.4	111.2 ± 0.7	95.2 ± 3.8
	Walker2D	73.7	107.9 ± 1.6	77.4 ± 27.9	110.1 ± 0.5	109.8 ± 0.2	109.0 ± 0.1
Expert	HalfCheetah	10.3	96.3 ± 1.3	81.3 ± 21.8	96.7 ± 1.1	96.2 ± 2.3	93.8 ± 0.1
	Hopper	111.6	96.5 ± 28.0	62.5 ± 29.0	107.8 ± 7	110.4 ± 0.3	111.2 ± 0.6
	Walker2D	110.6	108.5 ± 0.5	62.4 ± 3.2	110.2 ± 0.3	108.8 ± 0.2	108.5 ± 0.0

Table 1: Performance comparison on Gym control tasks. The results of CSVE is over three seeds) and we reinplement AWAC using d3rlpy. Results of other algorithms are from Table 2 in Bai et al. (2021).

		BC	BEAR	UWAC	CQL	TD3-BC	PBRL	CSVE (ours)
Human	Pen	34.4	-1.0	10.1 ± 3.2	37.5	0.0	35.4 ± 3.3	105.0 ±6.1
	Hammer	1.5	0.3	1.2 ± 0.7	4.4	0.0	0.4 ± 0.3	3.2 ± 2.9
	Door	0.5	−0.3	0.4 ± 0.2	9.9	0.0	0.1 ± 0.0	3.1 ± 2.7
	Relocate	0.0	-0.3	0.0 ± 0.0	0.2	0.0	0.0 ± 0.0	0.1 ± 0.0
Cloned	Pen	56.9	26.5	23.0 ± 6.9	39.2	0.0	74.9 ± 9.8	55.2 ± 6.1
	Hammer	0.8	0.3	0.4 ± 0.0	2.1	0.0	0.8 ± 0.5	0.5 ± 0.2
	Door	-0.1	−0.1	0.0 ± 0.0	0.4	0.0	4.6 ± 4.8	1.3 ± 1.1
	Relocate	-0.1	-0.3	−0.3 ± 0.0	-0.1	0.0	-0.1 ± 0.0	−0.3 ± 0.0
Expert	Pen	85.1	105.9	98.2 ± 9.1	107.0	0.3	135.7 ± 3.4	142.9 ± 10.6
	Hammer	125.6	127.3	107.7 ± 21.7	86.7	0.0	127.5 ± 0.2	126.5 ± 0.4
	Door	34.9	103.4	104.7 ± 0.4	101.5	0.0	95.7 ± 12.2	104.3 ± 0.9
	Relocate	101.3	98.6	105.5 ± 3.2	95.0	0.0	84.5 ± 12.2	103.0 ± 1.1

Table 2: Performance comparison on Adroit tasks. The results of CSVE are over three seeds. Results of other algorithms are from Table 2 in Bai et al. (2021).

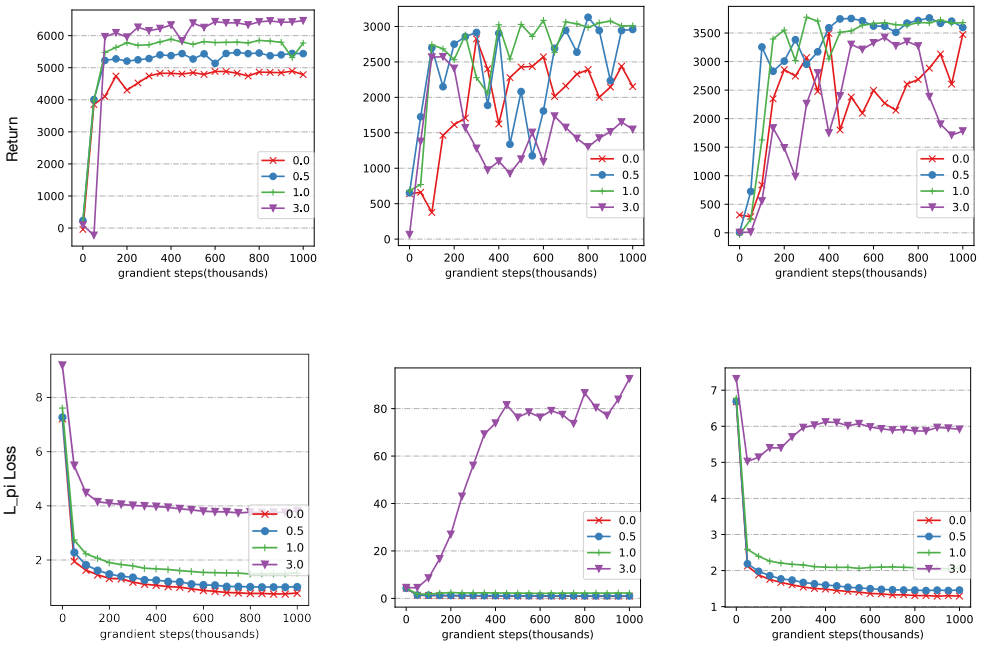
We now evaluate our method on the Adroit tasks. For CSVE, we report the final evaluation results after training in 0.1 million steps. The full results are reported in Table2. For the expert datasets, all methods (except TD3-BC) including simple BC (behaviour cloning) can perform well, among which ours is the most competitive on all four tasks. For human and cloned datasets, almost all methods fail to learn effective policies on three tasks except the Pen task. For the Pen task, CSVEis the only one that succeeds to learn a good policy on the human dataset, while it can learn a medium policy on the cloned dataset as BC and PBRL.

5.2 Effect of Exploration near Dataset Distributions

As discussed in Section 3.1 and 4.2, proper choice of exploration on the distribution (d) beyond data (d_u) should help policy improvement. The factor λ in Eq. 12 controls the trade-off on such 'bonus' exploration and complying the data-implied behaviour policy.

Let us take the medium-replay type of datasets to analyze its effect. In the experiments, with fixed $\beta = 0.1$, we investigate λ values of $\{0.0, 0.5, 1.0, 3.0\}$. As shown in the upper figures in Fig. 1,

λ shows obvious effect to policy performance and variances during training. In general, there is a value under which increasing λ leads to performance improvement, while above which further increasing λ hurts performance. For example, with $\lambda = 3.0$ in hopper-medium-replay task and walker2d-medium-replay task, the performance get worse than with smaller λ values. The value of λ is task-specific, and we find that its effect is highly related to the loss in Eq. 11 which can be observed by comparing bottom and upper figures in Fig. 1. Thus, in practice, we can choose proper λ according to the above loss without online interaction.



(d) halfcheetah-medium-replay (e) hopper-medium-replay (f) walker2d-medium-replay

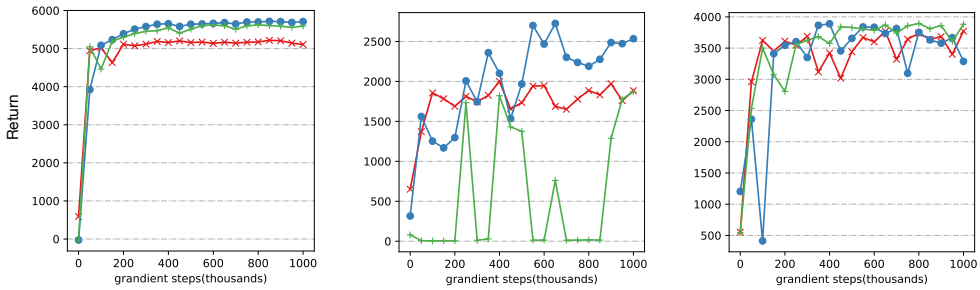
Figure 1: Effect of λ to Return (upper figures) and L_π loss in Eq. 11 (bottom figures)

5.3 Sensitiveness of hyper-parameters

Another main hyper-parameter of our method is β which trades off between behaviour cloning and policy optimization. For smaller values, the objective behaves similarly to behavior cloning (weights are close for all actions), while for larger values, it attempts to recover the maximum of the Q-function. To quantitatively analyze its effect, we test different β from $\{0.1, 3, 10\}$ in mujoco tasks with the medium-type datasets, whose results are shown in Fig. 2. We can see that λ has effect on the policy performance during training. Empirically, we found out that in general, $\beta = 3.0$ is suitable for such medium type datasets. Besides, in practice, by default we use $\beta = 3.0$ for random and medium task while 0.1 for medium-replay, medium-expert and expert datasets.

6 Related work

Offline RL (Fujimoto et al., 2019; Levine et al., 2020) aims to learn a reasonable policy from a static dataset collected by arbitrary policies, without further interactions with the environment. Compared to interactive RL, offline RL suffers two critical inherent issues, i.e., the distribution drift introduced by off-policy learning and the out-of-distribution extrapolation in value estimation (Ostrovski et al., 2021; Levine et al., 2020). The common mind of offline RL algorithms is to incorporate conservatism or regularization into the online RL algorithms. Here we briefly review the prior work with a comparison to ours.



(a) halfcheetah-medium-v2 (b) hopper-medium-v2 (c) walker2d-medium-v2

Figure 2: The effect of β in medium tasks.

Conservative value estimation: Prior offline RL algorithms regularize the learning policy close to the data or explicitly estimated behaviour policy) and penalize the exploration to the out-of-distribution region, via distribution correction estimation (Dai et al., 2020; Yang et al., 2020), policy constraints with support matching (Wu et al., 2019) and distributional matching Fujimoto et al. (2019); Kumar et al. (2019), applying policy divergence based penalty on Q-functions (Kostrikov et al., 2021; Wang et al., 2020) or uncertainty-based penalty (Agarwal et al., 2020) on Q-functions and conservative Q-function estimation (Kumar et al., 2020). Besides, model-based algorithms (Yu et al., 2020) directly estimate dynamics uncertainty and translated it into reward penalty. Different from these prior work that imposes conservatism on state-action pairs or actions, ours directly does such conservative estimation on states and requires no explicit uncertainty quantification.

With learned conservative value estimation, an offline policy can be learned via implicit derivation from a state-action joint distribution or in Q-Learning and actor-critic framework. In this paper, our implementation adopts the method proposed in AWAC (Nair et al., 2020; Peng et al., 2019).

Model-based algorithms: Model-based offline RL learns the dynamics model from the static dataset and uses it to quantify uncertainty (Yu et al., 2020), data augmentention (Yu et al., 2021) with roll-outs, or planning (Kidambi et al., 2020; Chen et al., 2021). Such methods typically rely on wide data coverage when planning and data augmentation with roll-outs, and low model estimation error when estimating uncertainty, which is often difficult to satisfy in reality and leads to policy instability. Instead, we use the model to sample the next-step states only reachable from data, which has no such strict requirements on data coverage or model bias.

Theoretical results: Our theoretical results are derived from conservative Q-value estimation (CQL) and safe policy improvement (Laroche et al., 2019). Besides, COMBO (Yu et al., 2021) gives a result of conservative but tighter value estimation than CQL, when dataset is augmented with model-based roll-outs. Compared to our result, COMBO's lower bounds additionally assume same initial state distribution which may not always satisfy in continuous control.

7 Discussion

In this paper, we propose a new approach for offline RL based on conservative value estimation on states and discussed how the theoretical results could lead to the new RL algorithms. In particular, we developed a practical actor-critic algorithm, in which the critic does conservative state value estimation by incorporating the penalty of the model predictive next-states into Bellman iterations, and the actor does the advantage weighted policy updates with a bonus of exploring states with conservative values. Experimental evaluation shows that our method performs better than alternative methods based on conservative Q-function estimation and is competitive among the SOTA methods, confirming our theoretical analysis well. Moving forward, we hope to explore the design of more powerful algorithms based on conservative state value estimation.

REFERENCES

Rishabh Agarwal, Dale Schuurmans, and Mohammad Norouzi. An optimistic perspective on offline reinforcement learning. In *International Conference on Machine Learning*, pp. 104–114. PMLR, 2020.

Chenjia Bai, Lingxiao Wang, Zhuoran Yang, Zhi-Hong Deng, Animesh Garg, Peng Liu, and Zhaoran Wang. Pessimistic bootstrapping for uncertainty-driven offline reinforcement learning. In *International Conference on Learning Representations*, 2021.

Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. Openai gym. *arXiv preprint arXiv:1606.01540*, 2016.

Xiong-Hui Chen, Yang Yu, Qingyang Li, Fan-Ming Luo, Zhiwei Qin, Wenjie Shang, and Jieping Ye. Offline model-based adaptable policy learning. *Advances in Neural Information Processing Systems*, 34:8432–8443, 2021.

Bo Dai, Ofir Nachum, Yinlam Chow, Lihong Li, Csaba Szepesvári, and Dale Schuurmans. Coindice: Off-policy confidence interval estimation. *Advances in neural information processing systems*, 33: 9398–9411, 2020.

Justin Fu, Aviral Kumar, Ofir Nachum, George Tucker, and Sergey Levine. D4rl: Datasets for deep data-driven reinforcement learning. *arXiv preprint arXiv:2004.07219*, 2020.

Scott Fujimoto and Shixiang Shane Gu. A minimalist approach to offline reinforcement learning. *Advances in neural information processing systems*, 34:20132–20145, 2021.

Scott Fujimoto, David Meger, and Doina Precup. Off-policy deep reinforcement learning without exploration. In Kamalika Chaudhuri and Ruslan Salakhutdinov (eds.), *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pp. 2052–2062. PMLR, 09–15 Jun 2019. URL https://proceedings.mlr.press/v97/fujimoto19a.html.

Michael Janner, Justin Fu, Marvin Zhang, and Sergey Levine. When to trust your model: Model-based policy optimization. *Advances in Neural Information Processing Systems*, 32, 2019.

Rahul Kidambi, Aravind Rajeswaran, Praneeth Netrapalli, and Thorsten Joachims. Morel: Model-based offline reinforcement learning. *Advances in neural information processing systems*, 33: 21810–21823, 2020.

Ilya Kostrikov, Rob Fergus, Jonathan Tompson, and Ofir Nachum. Offline reinforcement learning with fisher divergence critic regularization. In *International Conference on Machine Learning*, pp. 5774–5783. PMLR, 2021.

Aviral Kumar, Justin Fu, Matthew Soh, George Tucker, and Sergey Levine. Stabilizing off-policy q-learning via bootstrapping error reduction. *Advances in Neural Information Processing Systems*, 32, 2019.

Aviral Kumar, Aurick Zhou, George Tucker, and Sergey Levine. Conservative q-learning for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 33:1179–1191, 2020.

Sascha Lange, Thomas Gabel, and Martin Riedmiller. Batch reinforcement learning. In *Reinforcement learning*, pp. 45–73. Springer, 2012.

Romain Laroche, Paul Trichelair, and Remi Tachet Des Combes. Safe policy improvement with baseline bootstrapping. In *International Conference on Machine Learning*, pp. 3652–3661. PMLR, 2019.

Sergey Levine, Aviral Kumar, George Tucker, and Justin Fu. Offline reinforcement learning: Tutorial, review, and perspectives on open problems, 2020. URL https://arxiv.org/abs/2005.01643.

Ashvin Nair, Abhishek Gupta, Murtaza Dalal, and Sergey Levine. Awac: Accelerating online reinforcement learning with offline datasets, 2020. URL https://arxiv.org/abs/2006.09359.

Georg Ostrovski, Pablo Samuel Castro, and Will Dabney. The difficulty of passive learning in deep reinforcement learning. *Advances in Neural Information Processing Systems*, 34:23283–23295, 2021.

Xue Bin Peng, Aviral Kumar, Grace Zhang, and Sergey Levine. Advantage-weighted regression: Simple and scalable off-policy reinforcement learning. *CoRR*, abs/1910.00177, 2019. URL https://arxiv.org/abs/1910.00177.

Aravind Rajeswaran, Vikash Kumar, Abhishek Gupta, Giulia Vezzani, John Schulman, Emanuel Todorov, and Sergey Levine. Learning complex dexterous manipulation with deep reinforcement learning and demonstrations. *arXiv preprint arXiv:1709.10087*, 2017.

Takuma Seno and Michita Imai. d3rlpy: An offline deep reinforcement learning library. *arXiv preprint arXiv:2111.03788*, 2021.

Jonathon Shlens. Notes on kullback-leibler divergence and likelihood. *CoRR*, abs/1404.2000, 2014. URL http://arxiv.org/abs/1404.2000.

Ziyu Wang, Alexander Novikov, Konrad Zolna, Josh S Merel, Jost Tobias Springenberg, Scott E Reed, Bobak Shahriari, Noah Siegel, Caglar Gulcehre, Nicolas Heess, et al. Critic regularized regression. *Advances in Neural Information Processing Systems*, 33:7768–7778, 2020.

Yifan Wu, George Tucker, and Ofir Nachum. Behavior regularized offline reinforcement learning. *arXiv preprint arXiv:1911.11361*, 2019.

Yue Wu, Shuangfei Zhai, Nitish Srivastava, Joshua Susskind, Jian Zhang, Ruslan Salakhutdinov, and Hanlin Goh. Uncertainty weighted actor-critic for offline reinforcement learning. *arXiv preprint arXiv:2105.08140*, 2021.

Mengjiao Yang, Ofir Nachum, Bo Dai, Lihong Li, and Dale Schuurmans. Off-policy evaluation via the regularized lagrangian. *Advances in Neural Information Processing Systems*, 33:6551–6561, 2020.

Tianhe Yu, Garrett Thomas, Lantao Yu, Stefano Ermon, James Y Zou, Sergey Levine, Chelsea Finn, and Tengyu Ma. Mopo: Model-based offline policy optimization. *Advances in Neural Information Processing Systems*, 33:14129–14142, 2020.

Tianhe Yu, Aviral Kumar, Rafael Rafailov, Aravind Rajeswaran, Sergey Levine, and Chelsea Finn. Combo: Conservative offline model-based policy optimization. *Advances in neural information processing systems*, 34:28954–28967, 2021.

A PROOFS

We first redefine notation for clarity and then provide the proofs of the results in the main paper.

Notation. Let $k \in N$ denote an iteration of policy evaluation(in Section 3.2). V^k denotes the true, tabular (or functional) V-function iterate in the MDP, without any correction. $\hat{V}^k$ denotes the approximate tabular (or functional) V-function iterate.

We first present the bound on using empirical Bellman operator compared to the true Bellman operator.

Accounting for the sampling error

The empirical Bellman operator can be expressed as follows:

$$(\hat{\mathcal{B}}^{\pi}\hat{V}^k)(s) = E_{a\sim\pi(a|s)}\hat{r}(s,a) + \gamma\sum_{s'} E_{a\sim\pi(a|s)}\hat{P}(s'|s,a)[\hat{V}^k(s')] \tag{13}$$

where $\hat{r}(s,a)$ is the empirical average reward obtained in the dataset when performing action a at state s . The true Bellman operator can be expressed as follows:

$$(\mathcal{B}^\pi V^k)(s) = E_{a\sim\pi(a|s)} r(s,a) + \gamma \sum_{s'} E_{a\sim\pi(a|s)} P(s'|s,a)[V^k(s')] \tag{14}$$

Following previous work Kumar et al. (2020), we make the following assumptions that: P^π is the transition matrix coupled with policy, specifically, $P^\pi V(s) = E_{a'\sim\pi(a'|s'),s'\sim P(s'|s,a')}[V(s')]$

Assumption 1. *$\forall s,a \in \mathcal{M}$, the following relationships hold with at least $(1-\delta)$ ($\delta \in (0,1)$) probability,*

$$|r - r(s,a)| \leq \frac{C_{r,\delta}}{\sqrt{|D(s,a)|}}, ||\hat{P}(s'|s,a) - P(s'|s,a)||_1 \leq \frac{C_{P,\delta}}{\sqrt{|D(s,a)|}} \tag{15}$$

Under this assumption, the absolute difference between the empirical Bellman operator and the actual one can be calculated as follows:

$$|(\hat{\mathcal{B}}^\pi)\hat{V}^k - (\mathcal{B}^\pi)\hat{V}^k)| = E_{a\sim\pi(a|s)}|r - r(s,a) + \gamma \sum_{s'} E_{a'\sim\pi(a'|s')}(\hat{P}(s'|s,a) - P(s'|s,a))[\hat{V}^k(s')]| \tag{16}$$

$$\leq E_{a\sim\pi(a|s)}|r - r(s,a)| + \gamma|\sum_{s'} E_{a'\sim\pi(a'|s')}(\hat{P}(s'|s,a') - P(s'|s,a'))[\hat{V}^k(s')]| \tag{17}$$

$$\leq E_{a\sim\pi(a|s)} \frac{C_{r,\delta} + \gamma C_{P,\delta} 2R_{max}/(1-\gamma)}{\sqrt{|D(s,a)|}} \tag{18}$$

Thus, the estimation error due to sampling error can be bounded by a constant as a function of $C_{r,\delta}$ and $C_{t,\delta}$. We define this constant as $C_{r,T,\delta}$.

Thus we obtain:

$$\forall V, s \in D, |\hat{\mathcal{B}}^\pi V(s) - \mathcal{B}^\pi V(s)| \leq E_{a\sim\pi(a|s)} \frac{C_{r,t,\delta}}{(1-\gamma)\sqrt{|D(s,a)|}} \tag{19}$$

Next we provide an important lemma.

Lemma 1. *(Interpolation Lemma) For any $f \in [0,1]$, and any given distribution $\rho(s)$, let d_f be an f-interpolation of ρ and D, i.e.,$d_f(s) := fd(s) + (1-f)\rho(s)$, let $v(\rho, f) := E_{s\sim\rho(s)}[\frac{\rho(s)-d(s)}{d_f(s)}]$, then $v(\rho,f)$ satisfies $v(\rho,f) \geq 0$.*

The proof can be found in Yu et al. (2021). By setting f as 1, we have $E_{s\sim\rho(s)}[\frac{\rho(s)-d(s)}{d(s)}] > 0$.

Proof of Theorem 1: The V function of approximate dynamic programming in iteration k can be obtained as:

$$\hat{V}^{k+1}(s) = \hat{\mathcal{B}}^\pi \hat{V}^k(s) - \alpha[\frac{d(s)}{d_u(s)} - 1] \; \forall s, k \tag{20}$$

The fixed point:

$$\hat{V}^\pi(s) = \hat{\mathcal{B}}^\pi \hat{V}^\pi(s) - \alpha[\frac{d(s)}{d_u(s)} - 1] \leq \mathcal{B}^\pi \hat{V}^\pi(s) + E_{a\sim\pi(a|s)} \frac{C_{r,t,\delta} R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}} - \alpha[\frac{d(s)}{d_u(s)} - 1] \tag{21}$$

Thus we obtain:

$$\hat{V}^\pi(s) \leq V^\pi(s) + (I - \gamma P^\pi)^{-1} E_{a\sim\pi(a|s)} \frac{C_{r,t,\delta} R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}} - \alpha(I - \gamma P^\pi)^{-1}[\frac{d(s)}{d_u(s)} - 1] \tag{22}$$

, where P^π is the transition matrix coupled with the policy π and $P^\pi V(s) = E_{a'\sim\pi(a'|s')s'\sim P(s'|s,a')}[V(s')]$.

Then the expectation of $V^\pi(s)$ under distribution $d(s)$ satisfies:

$$\begin{aligned} E_{s\sim d(s)}\hat{V}^\pi(s) \leq & E_{s\sim d(s)}(V^\pi(s)) + E_{s\sim d(s)}(I-\gamma P^\pi)^{-1}E_{a\sim\pi(a|s)}\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}} \\ & -\alpha \underbrace{E_{s\sim d(s)}(I-\gamma P^\pi)^{-1}[\frac{d(s)}{d_u(s)}-1])}_{>0} \end{aligned} \tag{23}$$

When $\alpha \geq \frac{E_{s\sim d(s)}E_{a\sim\pi(a|s)}\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}}}{E_{s\sim d(s)}[\frac{d(s)}{d_u(s)}-1])}$, $E_{s\sim d(s)}\hat{V}^\pi(s) \leq E_{s\sim d(s)}(V^\pi(s))$

Proof of Theorem 2: The expectation of $V^\pi(s)$ under distribution $d(s)$ satisfies:

$$\begin{aligned} E_{s\sim d_u(s)}\hat{V}^\pi(s) \leq & E_{s\sim d_u(s)}(V^\pi(s)) + E_{s\sim d_u(s)}(I-\gamma P^\pi)^{-1}E_{a\sim\pi(a|s)}\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}} \\ & -\alpha E_{s\sim d_u(s)}(I-\gamma P^\pi)^{-1}[\frac{d(s)}{d_u(s)}-1]) \end{aligned} \tag{24}$$

Noticed that the last term:

$$\sum_{s\sim d_u(s)}(\frac{d_f(s)}{d_u(s)}-1) = \sum_s d_u(s)(\frac{d_f(s)}{d_u(s)}-1) = \sum_s d_f(s) - \sum_s d_u(s) = 0 \tag{25}$$

We obtain that:

$$E_{s\sim d_u(s)}\hat{V}^\pi(s) \leq E_{s\sim d_u(s)}(V^\pi(s)) + E_{s\sim d_u(s)}(I-\gamma P^\pi)^{-1}E_{a\sim\pi(a|s)}\frac{C_{r,t,\delta}R_{max}}{(1-\gamma)\sqrt{|D(s,a)|}} \tag{26}$$

Proof of Theorem 3: Recall that the expression of the V-function iterate is given by:

$$\hat{V}^{k+1}(s) = \mathcal{B}^{\pi^k}\hat{V}^k(s) - \alpha[\frac{d(s)}{d_u(s)}-1]\forall s,k \tag{27}$$

Now the expectation of $V^\pi(s)$ under distribution $d_u(s)$ is given by:

$$E_{s\sim d_u(s)}\hat{V}^{k+1}(s) = E_{s\sim d_u(s)}\left[\mathcal{B}^{\pi^k}\hat{V}^k(s) - \alpha[\frac{d(s)}{d_u(s)}-1]\right] = E_{s\sim d_u(s)}\mathcal{B}^{\pi^k}\hat{V}^k(s) \tag{28}$$

The expectation of $V^\pi(s)$ under distribution $d(s)$ is given by:

$$E_{s\sim d(s)}\hat{V}^{k+1}(s) = E_{s\sim d(s)}\mathcal{B}^{\pi^k}\hat{V}^k(s) - \alpha[\frac{d(s)}{d_u(s)}-1] = E_{s\sim d(s)}\mathcal{B}^{\pi^k}\hat{V}^k(s) - \alpha E_{s\sim d(s)}[\frac{d(s)}{d_u(s)}-1] \tag{29}$$

Thus we can show that:

$$\begin{aligned} E_{s\sim d_u(s)}\hat{V}^{k+1}(s) - E_{s\sim d(s)}\hat{V}^{k+1}(s) &= E_{s\sim d_u(s)}\mathcal{B}^{\pi^k}\hat{V}^k(s) - E_{s\sim d(s)}\mathcal{B}^{\pi^k}\hat{V}^k(s) + \alpha E_{s\sim d(s)}[\frac{d(s)}{d_u(s)}-1] \\ &= E_{s\sim d_u(s)}V^{k+1}(s) - E_{s\sim d(s)}V^{k+1}(s) - E_{s\sim d(s)}[\mathcal{B}^{\pi^k}(\hat{V}^k - V^k)] \\ &\quad + E_{s\sim d_u(s)}[\mathcal{B}^{\pi^k}(\hat{V}^k - V^k)] + \alpha E_{s\sim d(s)}[\frac{d(s)}{d_u(s)}-1] \end{aligned} \tag{30}$$

By choosing α:

$$\alpha > \frac{E_{s\sim d(s)}[\mathcal{B}^{\pi^k}(\hat{V}^k - V^k)] - E_{s\sim d_u(s)}[\mathcal{B}^{\pi^k}(\hat{V}^k - V^k)]}{E_{s\sim d(s)}[\frac{d(s)}{d_u(s)}-1]} \tag{31}$$

We have $E_{s\sim d_u(s)}\hat{V}^{k+1}(s) - E_{s\sim d(s)}\hat{V}^{k+1}(s) > E_{s\sim d_u(s)}V^{k+1}(s) - E_{s\sim d(s)}V^{k+1}(s)$ hold.

Proof of Theorem 4: $\hat{V}$ is obtained by solving the recursive Bellman fixed point equation in the empirical MDP, with an altered reward, $r(s,a) - \alpha[\frac{d(s)}{d_u(s)} - 1]$, hence the optimal policy $\pi^*(a|s)$ obtained by optimizing the value under Eq. 4.

Proof of Theorem 5: The proof of this statement is divided into two parts. We first relates the return of π^* in the empirical MDP $\hat{M}$ with the return of π_β , we can get:

$$J(\pi^*, \hat{M}) - \alpha\frac{1}{1-\gamma}\mathbb{E}_{s\sim d^{\pi^*}_{\hat{M}}(s)}[\frac{d(s)}{d_u(s)} - 1] \geq J(\pi_\beta, \hat{M}) - 0 = J(\pi_\beta, \hat{M}) \tag{32}$$

The next step is to bound the difference between $J(\pi_\beta, \hat{M})$ and $J(\pi_\beta, M)$ and the difference between $J(\pi^*, \hat{M})$ and $J(\pi^*, M)$. We quote a useful lemma from Kumar et al. (2020) (Lemma D.4.1):

Lemma 2. *For any MDP M, an empirical MDP $\hat{M}$ generated by sampling actions according to the behavior policy π_β and a given policy π,*

$$|J(\pi, \hat{M}) - J(\pi, M)| \leq (\frac{C_{r,\delta}}{1-\gamma} + \frac{\gamma R_{max} C_{T,\delta}}{(1-\gamma)^2})\mathbb{E}_{s\sim d^{\pi^*}_{\hat{M}}(s)}[\frac{\sqrt{|\mathcal{A}|}}{\sqrt{|\mathcal{D}(s)|}}\sqrt{E_{a\sim\pi(a|s)}(\frac{\pi(a|s)}{\pi_\beta(a|s)})}] \tag{33}$$

Setting π in the above lemma as π_β, we get:

$$|J(\pi_\beta, \hat{M}) - J(\pi_\beta, M)| \leq (\frac{C_{r,\delta}}{1-\gamma} + \frac{\gamma R_{max} C_{T,\delta}}{(1-\gamma)^2})\mathbb{E}_{s\sim d^{\pi^*}_{\hat{M}}(s)}[\frac{\sqrt{|\mathcal{A}|}}{\sqrt{|\mathcal{D}(s)|}}\sqrt{E_{a\sim\pi^*(a|s)}(\frac{\pi^*(a|s)}{\pi_\beta(a|s)})}] \tag{34}$$

, given that $\sqrt{E_{a\sim\pi^*(a|s)}[\frac{\pi^*(a|s)}{\pi_\beta(a|s)}]}$ is a pointwise upper bound of $\sqrt{E_{a\sim\pi_\beta(a|s)}[\frac{\pi_\beta(a|s)}{\pi_\beta(a|s)}]}$(Kumar et al. (2020)). Thus we get,

$$\begin{aligned} J(\pi^*, \hat{M}) \geq J(\pi_\beta, \hat{M}) - 2(\frac{C_{r,\delta}}{1-\gamma} + \frac{\gamma R_{max} C_{T,\delta}}{(1-\gamma)^2})\mathbb{E}_{s\sim d^{\pi^*}_{\hat{M}}(s)}[\frac{\sqrt{|\mathcal{A}|}}{\sqrt{|\mathcal{D}(s)|}}\sqrt{E_{a\sim\pi^*(a|s)}(\frac{\pi^*(a|s)}{\pi_\beta(a|s)})}] \\ + \alpha\frac{1}{1-\gamma}\mathbb{E}_{s\sim d^{\pi}_{\hat{M}}(s)}[\frac{d(s)}{d_u(s)} - 1] \end{aligned} \tag{35}$$

, which completes the proof.

Here, the second term is sampling error which occurs due to mismatch of $\hat{M}$ and M; the third term denotes the increase in policy performance due to CSVE in $\hat{M}$. Note that when the first term is small, the smaller value of α are able to provide an improvement compared to the behavior policy.

B CSVE ALGORITHM

Now we put all in section 4 together and describe the practical deep offline reinforcement learning algorithm. In particular, the dynamic model model, value functions and policy are all parameterized with deep neural networks and trained via stochastic gradient decent methods. The pseudo code is given in Alg. 1.

C IMPLEMENTAION DETAIL

We implement our method based on an offline deep reinforcement learning library d3rlpy (Seno & Imai, 2021). The code is available at https://github.com/iclr20234089/code4098.

The detailed hyper-parameters are provided in Table 3

Algorithm 1: CSVE Algorithm

Input : Data $D = \{(s, a, r, s')\}$
Parameters: Q_θ, V_ψ, π_ϕ, $Q_{\overline{\theta}}$, M_ν
Hyperparameters: α, λ, learning rates $\eta_\theta, \eta_\psi, \eta_\phi, \omega$
begin

```
// Train transition model with the static dataset D
```

$M_\nu \leftarrow$ *train*(D);

```
// Train the conservative value and policy functions
```

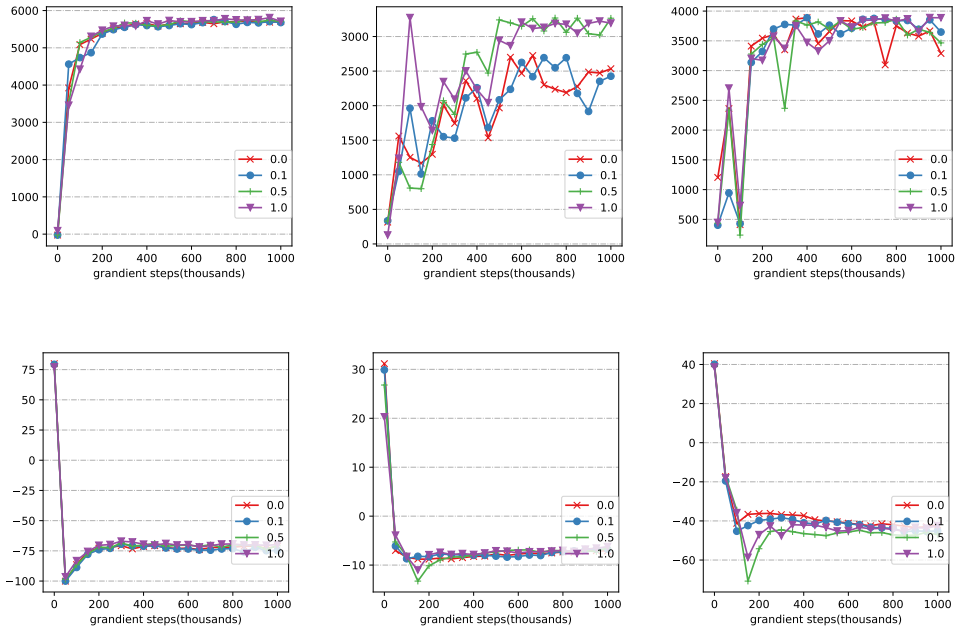
Initialize function parameters $\theta_0, \psi_0, \phi_0, \overline{\theta}_0 = \theta_0$;
foreach *step* $k = 1 \rightarrow N$ **do**
  $\psi_k \leftarrow \psi_{k-1} - \eta_\psi \nabla_\psi L_V^\pi(V_\psi; \overline{\hat{Q}_{\theta_k}})$;
  $\theta_k \leftarrow \theta_{k-1} - \eta_\theta \nabla_\theta L_Q^\pi(Q_\theta; \hat{V}_{\psi_k})$;
  $\phi_k \leftarrow \phi_{k-1} - \eta_\phi \nabla_\phi L_\pi^+(\pi_\phi)$;
  $\overline{\theta}_k \leftarrow \omega \overline{\theta}_{k-1} + (1 - \omega)\theta_k$;

Hyper-parameters	Value and description
B	5, number of ensembles in dynamics model
α	10, to control the penalty of out-of-distribution states
τ	10, budget parameter in Eq. 10
β	In Gym domain, 3 for random and medium tasks, 0.1 for the other tasks; In Adroit domain, 30 for human and cloned tasks, 0.01 for expert tasks
γ	0.99, discount factor.
H	1 million for Mujoco while 0.1 million for Adroit tasks.
w	0.005, target network smoothing coefficient.
lr of actor	3e-4, policy learning rate
lr of critic	1e-4, critic learning rate

Table 3: Hyer-parameters

D EXTENDED EXPERIMENTAL RESULTS

We also investigated λ values of $\{0.0, 0.1, 0.5, 1.0\}$ in the medium tasks. The results are shown in Fig. 3.



(d) halfcheetah-medium (e) hopper-medium (f) walker2d-medium

Figure 3: Return (upper sub-figures) and loss in Eq. 11 (bottom sub-figures) during training with different λ values