Brick-by-Brick: Combinatorial Construction with Deep Reinforcement Learning

Hyunsoo Chung[*1] **Jungtaek Kim**[*1] **Boris Knyazev**[23] **Jinhwi Lee**[14]
Graham W. Taylor[23] **Jaesik Park**[1] **Minsu Cho**[1]

[1]POSTECH [2]University of Guelph [3]Vector Institute [4]POSCO
{hschung2,jtkim}@postech.ac.kr

Abstract

Discovering a solution in a combinatorial space is prevalent in many real-world problems but challenging due to diverse complex constraints and the vast number of possible combinations. In a similar vein, we introduce a novel formulation, *combinatorial construction*, which requires a building agent to assemble unit primitives (i.e., LEGO bricks) sequentially – every connection between two bricks must follow a fixed rule, while no bricks mutually overlap. To construct a target object, we provide incomplete knowledge about the desired target (i.e., 2D images) instead of exact and explicit volumetric information to the agent. Such a problem requires a comprehensive understanding of partial information and long-term planning to append a brick sequentially, which leads us to employ reinforcement learning. However, this approach has to consider variable-sized action space and the existence of a large number of invalid actions that result in an overlap between bricks. To resolve these issues, our model, dubbed *Brick-by-Brick*, adopts an action validation network that filters invalid actions to an actor-critic network. We then introduce a reinforcement-learning environment and demonstrate that our approach successfully learns to construct an unseen object conditioned on a single image or multiple views of a target object.

1 Introduction

A combinatorial space, typically characterized by discrete variables and their combinations, often induces interesting yet challenging problems such as traveling salesperson and minimum spanning tree [20, 6]. The main challenges lie in the vast number of possible combinations as well as complex constraints imposed on them. In a similar spirit, we suggest a novel problem formulation, *combinatorial construction*, that focuses on the real-world construction procedure. Given only incomplete target information (i.e., 2D images or multiple views of a target object) [25, 12], an agent sequentially assembles unit primitives (i.e., LEGO bricks). The proposed formulation is combinatorial since it engages repetitive placement of primitives, which leads to a large number of available solutions. Distinct property of our proposed formulation, however, is that the agent must build the solution incrementally by adding on to the partial solution. Specifically, a brick, which is a unit primitive of the object of interest, is placed on a discrete space by connecting to one of the previously assembled bricks. In addition, every connection between two bricks must follow a fixed rule while no bricks mutually overlap. Each assembly (i.e., action) executed by the agent is, thus, modeled as selecting one of the feasible connections to place a new brick.

The problem we introduce closely depicts how humans understand an object and adapt the acquired knowledge to a downstream task. Humans naturally analyze a 3D object by picturing its part-by-part

[*]Equal contribution.

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Table 1: Analysis of recent studies in terms of state representation, supervision, conditioning, target objects, and action validation. CE and IoU stand for cross-entropy and intersection over union with respect to volumetric comparisons. Implicit, sampling, and pretrained indicate respective strategies that filter invalid actions; see the corresponding section for their details.

Method	**State**	**Supervision**	**Conditioning**	**Target**	**Action Validation**
Hamrick et al. [11]	Image	Task-dependent	N/A	2D	Implicit
Bapst et al. [3]	Object/Image	Task-dependent	Object and/or image	2D	Implicit
Kim et al. [19]	Set	Overlap	Exact target volume	3D	Sampling
Thompson et al. [38]	Graph	Step-wise CE	One-hot class info.	3D	Implicit
Brick-by-Brick (B^3, ours)	Graph/Image	IoU	Image or set of images	3D	Pretrained

decomposition and consequently grasp a rich semantic understanding [15, 22]. In various fields, they utilize an inherent ability to decompose objects to effectively solve challenging tasks such as object classification [17], robot grasp planning [2], and part segmentation [31, 28]. Likewise, humans exploit this ability to solve the inverse problem – combinatorial construction. Given a desired object to be constructed and no strong supervision (i.e., ordered step-by-step instructions), humans can often still manage to build a valid target object by carefully planning or, sometimes improvising, the sequence of actions. Our environment, which corresponds to the proposed problem, is designed to learn and test such behavior with only partial information of the desired target available to the agent.

Successfully constructing an object in our setup requires a comprehensive understanding of incomplete target information with the current structured state of assembled bricks and long-term planning to append each brick efficiently. These requirements, along with the absence of sequence-level supervision, incentivize us to devise a reinforcement learning (RL) approach [3, 37]. In this domain, however, we must carefully handle both an indefinite action space and the existence of many invalid actions when applying RL [43]. In particular, both defining an action space that varies by the number of assembled bricks and distinguishing an invalid action that results in an overlap with other existing bricks quickly become intractable as more bricks are placed. To resolve the aforementioned issues, our model, dubbed Brick-by-Brick (B^3), adopts an action validation network that filters invalid actions to an actor-critic network. In addition to the novel RL formulation, we use graph-structured representation of the brick combination to interpret the assembling process as a sequential graph generation process.

Overall, we summarize our contributions as follows:

(i) We propose a novel problem formulation, combinatorial construction, that closely resembles a real-world object construction process that engages repetitive placement of components;

(ii) We design an RL agent for combinatorial construction, dubbed Brick-by-Brick (B^3), to effectively address both growing action space and invalid actions;

(iii) We implement the corresponding environment based on OpenAI Gym and introduce new novel evaluation scenarios that vary by their incomplete partial target information.

2 Combinatorial Construction

To formulate the combinatorial construction problem, we start by defining a unit primitive that is used to construct a 3D object and an action space that determines where to assemble the next primitive.

As a unit primitive, we utilize a 2×4 brick, which has eight studs and their fit cavities. This design choice yields a consistently varying action space, implying that if we add one brick to the current state of brick combination, we can efficiently define the next action space. We want to emphasize that with only six 2×4 bricks, we can create *915,103,765 combinations* [9]. Accordingly, our choice of the primitive does not make our problem a trivial task; instead, every decision of where we place the next primitive can deteriorate the quality of the final result because there exists a plethora of wrong paths.

With our specific choice of unit primitive, we can define an action space for determining the next action and evaluating the future states. However, since every assembly step gradually expands an action space, naïve approaches to defining a growing action space are not appropriate for our problem;

an action space with redundant actions [43] is not applicable due to varying action space, and an action sampling approach [13] is also not suitable due to nominal or invalid actions. Thus, we define a *successive* action space composed of a two-step decision: (i) choosing a *pivot brick* and (ii) choosing an *offset* from the pivot brick.

Before explaining a pivot brick, we first assume a simplified assembly scenario that follows an Eulerian path[2] – new brick is always placed by connecting to the last assembled brick. This enables us to define a finite action space, though most objects are infeasible to assemble with the Eulerian path. To broaden search space by generalizing the Eulerian path, one of previously assembled bricks is chosen as a pivot brick. Then, B^3 decides an offset from the pivot brick, which describes how the next brick is placed relative to the pivot. Because of homogeneous brick type, the number of available offsets is finite and consistent where we do not consider the validity of such offsets in a certain state – for 2×4 bricks, there exist a maximum of 92 available offsets.

Due to the disallowance of overlap between bricks, our agent must consider an *invalid action* among available actions during assembly. Identifying the validity of actions from the current brick combination becomes intractable as more bricks are placed; the complexity of this process is $\mathcal{O}(|A_{\text{off}}|t^2)$, where A_{off} is an action space for offsets and t is the cardinality of assembled bricks at a given step; see the supplementary material for qualitative results on this complexity. Such expensive overhead for validation naturally leads us to adopt an action validation network that learns to identify invalid actions with a single forward pass.

As described in Section 1 and this section, our problem has interesting but challenging characteristics derived from the assumptions on discrete placement, a connectivity rule, disallowance of overlap, and ultimately invalid actions. We, therefore, present a comparison to other existing studies in terms of state, supervision (or a reward function), conditioning, target objects, and action validation, as shown in Table 1. Compared to the previous work [11, 3, 19, 38], our method B^3 constructs an object in 3D with incomplete target information and pretrained action validation component; see Section 4 and Section 5 for a detailed description.

3 Brick-by-Brick

In this section, we briefly introduce the definition of RL and the corresponding framework for combinatorial construction, where an agent places a brick sequentially. We then explain the details of our model that learns to select appropriate actions given only partial information of the desired target so that the assembled 3D object resembles the target. Moreover, to cope with an enormous number of invalid actions and decisions of such invalid actions, we propose an action validation network. See Figure 1 for the overall pipeline of B^3.

Definition. In a standard RL framework, there exists an agent that interacts with the environment by iteratively making decisions given an observation of the environment. This follows general decision making procedure of a Markov decision process (MDP), where a transition function satisfies the Markov property, i.e., $p(s_{t+1}|s_0, s_1, \ldots, s_t, a_t) = p(s_{t+1}|s_t, a_t)$, where s_t and a_t are a state and an action at timestep t, respectively.

In our problem setting, we only consider a finite horizon MDP formally defined as a tuple of (S, A, P, R, γ), where $S = \{s_t\}$ is a set of states, $A = \{a_t\}$ is a set of actions, $R : S \times A \to \mathbb{R}$ is a reward function, $P : S \times A \to S$ is a transition function, and $\gamma \in [0, 1)$ is a discount factor. The goal of the agent is to learn a policy $\pi(a_t|s_t)$ that maximizes the expected future cumulative reward. We introduce our detailed MDP formulation for combinatorial construction in the following sections.

3.1 Problem Formulation

Given target information $\mathcal{T}$, the agent aims to construct a 3D target object $\mathbf{T}$ by assembling bricks sequentially, one brick for each t-th step. Each t-th brick is represented by its pose $(\mathbf{x}_t, d_t)$, where $\mathbf{x}_t \in \mathbb{Z}^3$ is the center coordinate of the brick in 3D space and $d_t \in \{0, 1\}$ denotes one of two possible directions, meaning that its longer axis is aligned along either x axis or y axis in 3D space.

[2] It is a path that visits all the vertices without revisiting the edges visited before.

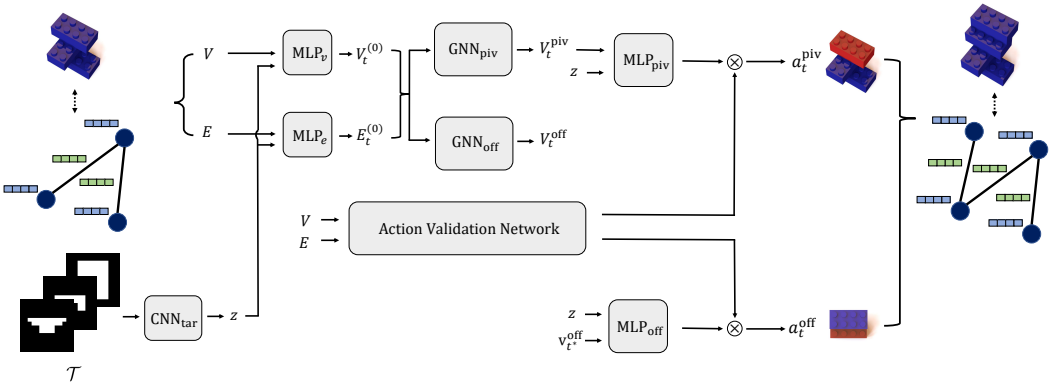


Figure 1: An overview of our proposed method B^3. State input $s_t = (G_t, \mathcal{T})$ is embedded and passed through CNNs, GNNs, and MLPs to predict action a_t that consists of pivot brick indicator a_t^{piv} and offset from the pivot brick a_t^{off}. Moreover, an action validation network helps to filter invalid actions from the action space where we determine the next brick to assemble. The red brick in both action for the pivot brick a_t^{piv} and action for the offset a_t^{off} indicates the chosen brick.

Target Information. As described earlier, target information $\mathcal{T}$, which is a single binary image or a set of three binary images from different views of a target object, is given as partial target information. In practice, obtaining (incomplete) partial information is easier than accessing a 3D target. The goal of our task, thus, is to create a sequence of unit bricks by inferring a target object from abstract information in a combinatorial manner.

State. Each t-th state s_t of the MDP is represented by a tuple of a directed graph G_t composed of t bricks and target information $\mathcal{T}$, i.e., $s_t = (G_t, \mathcal{T})$. The graph is defined as $G_t = (V_t, E_t)$, where $V_t = \{\mathbf{v}_i\}_{i=1}^t$ is a set of t bricks, i.e., $\mathbf{v}_i = (\mathbf{x}_i, d_i) \in \mathbb{Z}^4$, and $E_t = \{\mathbf{e}_{ij}\}_{i,j=1}^t$ is a set of the offset vectors between two connected nodes, i.e., $\mathbf{e}_{ij} = (\mathbf{x}_i - \mathbf{x}_j, d_i \oplus d_j) \in \mathbb{Z}^4$. Note that nodes are connected by edges according to sequential actions and relative offsets in pose are used for edge features in order to induce translational and orientational equivariance. Since all the edges are bi-directional, we omit the arrows when displaying graphs in Figure 1.

Action. As noted earlier, we define a successive action space of choosing the pivot brick first and the corresponding offset next. Formally, with t bricks assembled, we define an action $a_t = (a_t^{\text{piv}}, a_t^{\text{off}})$ where a_t^{piv} is to select a pivot brick and a_t^{off} is to select an offset with respect to the pivot brick. The pose of the next brick is then $(\mathbf{x}^{\text{piv}} + \Delta\mathbf{x}, (d^{\text{piv}} + \Delta d) \mod 2)$, where $(\mathbf{x}^{\text{piv}}, d^{\text{piv}})$ and $(\Delta\mathbf{x}, \Delta d)$ are determined by a_t^{piv} and a_t^{off}, respectively. In choosing actions a_t^{piv} and a_t^{off}, we exclude invalid actions: (i) choosing a pivot brick near which no additional brick can be placed and (ii) choosing an offset for the next brick that overlaps with existing bricks. By the decision on invalid actions using the action validation network, we mask out all the probabilities of invalid actions, re-normalize the distributions over actions a_t, and then sample one of valid actions.

Transition Function. Given state s_t and action a_t, our transition function $p(s_{t+1}|s_t, a_t)$ is designed to determine the next state s_{t+1} by deterministically updating s_t based on a_t. The node of the new brick $\mathbf{v}_{t+1}$ is created so that $V_{t+1} = V_t \cup \{\mathbf{v}_{t+1}\}$. The edges between the new brick and existing bricks in physical contact via studs are created so that $E_{t+1} = E_t \cup \{\mathbf{e}_{(i)(t+1)}\}_{i \in \mathcal{N}_{t+1}} \cup \{\mathbf{e}_{(t+1)(i)}\}_{i \in \mathcal{N}_{t+1}}$, where $\mathcal{N}_{t+1}$ denotes the set of bricks in direct contact with the new brick $\mathbf{v}_{t+1}$. As a result, the graph in the state s_{t+1} is updated to $G_{t+1} = (V_{t+1}, E_{t+1})$.

Reward Function. In contrast to the tasks where rigorous evaluations are available, it is difficult to seek out an appropriate countermeasure to quantify the object assembled by combinatorial construction, especially, in the context of graph generative model [26]. To mitigate such an issue, we exploit the property of a voxel representation. More concretely, given a desired object, we first create voxels in a closed space and determine the occupancy of voxels with a target object, after normalizing it to

the bottom center of voxels. Then, we transform the combination of currently assembled bricks into the occupancy of the voxels defined with the target object and measure the overlap between them:

$$\Delta\text{IoU}(\mathbf{C}_t, \mathbf{T}) = \frac{\text{vol}(\mathbf{C}_t \cap \mathbf{T})}{\text{vol}(\mathbf{C}_t \cup \mathbf{T})} - \frac{\text{vol}(\mathbf{C}_{t-1} \cap \mathbf{T})}{\text{vol}(\mathbf{C}_{t-1} \cup \mathbf{T})}, \tag{1}$$

where $\mathbf{C}_t$, $\mathbf{C}_{t-1}$, and $\mathbf{T}$ are the occupied voxels at timestep t, timestep $t-1$, and a desired target, respectively. In addition, $\text{vol}(\cdot)$ is a function that measures a volume. The step-wise reward function is then ΔIoU if the new brick overlaps at least 50% with the occupied voxels of target object else 0. Consequently, our agent will learn the ordering and placement of the bricks to construct the object, without explicit supervision by maximizing Eq. (1), as will be described in the subsequent section.

3.2 Sequential Construction

In this section, we describe how we process G_t and $\mathcal{T}$, which comprise a state, with various neural networks including convolutional neural networks and graph neural networks. The overview of this construction procedure is illustrated in Figure 1.

Node and Target Embeddings. Given a state $s_t = (G_t, \mathcal{T})$ where $G_t = (V_t, E_t)$, we first use a convolutional neural network (CNN) to extract features $\mathbf{z}$ from the target:

$$\mathbf{z} = \text{CNN}_{\text{tar}}(\mathcal{T}). \tag{2}$$

If the partial information is given as a set of images, the feature $\mathbf{z}$ is obtained by first applying CNN to each image separately and then concatenating to a single vector.

For node and edge features, an MLP embeds them to higher-dimensional features with $\mathbf{z}$ extracted from the target:

$$\mathbf{v}_i^{(0)} = \text{MLP}_v([\mathbf{v}_i, \mathbf{z}]), \quad \text{and} \quad \mathbf{e}_{ij}^{(0)} = \text{MLP}_e([\mathbf{e}_{ij}, \mathbf{z}]), \tag{3}$$

for all $i, j \in \{1, \dots, t\}$, where $[,]$ and $^{(0)}$ denote concatenation and the first layer, respectively.

Eq. (2) and Eq. (3) can be viewed as pre-processing inputs to feed in a graph neural network (GNN). Inspired by [4], we apply a variant of graph networks (GNs) in which a global graph feature is omitted because our network produces outputs from the representation of the nodes of graph. At the ℓ-th layer of GNNs, we first update edge features, and then aggregate message vectors for each node, and finally update node features:

$$\mathbf{e}_{ij}^{(\ell+1)} = \text{MLP}_e^{(\ell)}\left([\mathbf{v}_i^{(\ell)}, \mathbf{v}_j^{(\ell)}, \mathbf{e}_{ij}^{(\ell)}]\right), \tag{4}$$

$$\mathbf{m}_i^{(\ell)} = \sum_{j \in \mathcal{N}_i} \text{aggregate}\left(\mathbf{e}_{ij}^{(\ell+1)}\right), \tag{5}$$

$$\mathbf{v}_i^{(\ell+1)} = \text{MLP}_v^{(\ell)}\left([\mathbf{v}_i^{(\ell)}, \mathbf{m}_i^{(\ell)}]\right), \tag{6}$$

where $\mathcal{N}_i$ is the neighborhood nodes of $\mathbf{v}_i$, and $\text{aggregate}(\cdot)$ is the aggregation function that computes a message for each node by aggregating the features of its neighboring nodes. Note that MLP_v, MLP_e, $\text{MLP}_v^{(\ell)}$, and $\text{MLP}_e^{(\ell)}$ have their own learnable parameters.

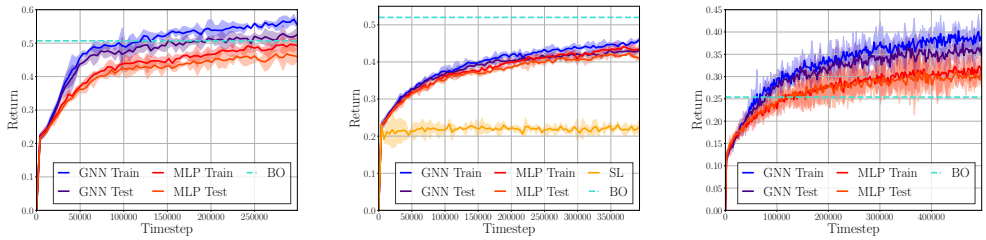
Action Selection. In order to obtain richer representations that are useful for predicting a_t, we employ two separate GNNs: GNN_{piv} and GNN_{off}, with L layers in total, to produce sets of node embeddings, V_t^{piv} and V_t^{off} for selecting a pivot and relative displacement:

$$V_t^{\text{piv}} = \text{GNN}_{\text{piv}}(V_t^{(0)}, E_t^{(0)}), \quad \text{and} \quad V_t^{\text{off}} = \text{GNN}_{\text{off}}(V_t^{(0)}, E_t^{(0)}), \tag{7}$$

where $V_t^{(0)} = \{\mathbf{v}_i^{(0)}\}_{i=1}^t$ and $E_t^{(0)} = \{\mathbf{e}_{ij}^{(0)}\}_{i,j=1}^t$ are the sets of node and edge features obtained by Eq. (3). Each layer of both GNNs updates node and edge features using Eq. (4), Eq. (5), and Eq. (6). Finally, the set of node and edge features, V_t^{piv} and V_t^{off} along with a target feature $\mathbf{z}$ are used to compute a prediction for the next action a_t^{piv} and a_t^{off}:

$$p\big(a_t^{\text{piv}}\big) = \sigma\Big(\text{MLP}_{\text{piv}}\big([V_t^{\text{piv}}, \mathbf{z}]\big)\Big), \quad \text{and} \quad p\big(a_t^{\text{off}}\big) = \sigma\Big(\text{MLP}_{\text{off}}\big([\mathbf{v}_{i^*}^{\text{off}}, \mathbf{z}]\big)\Big), \tag{8}$$

where σ is a softmax function and $\mathbf{v}_{i^*}^{\text{off}}$ is the node feature selected by the index i^* of a_t^{piv}.



(a) MNIST Class 0 (b) Randomly-Assembled (c) ModelNet Airplane

Figure 2: Episode return curves vs. timesteps in different setups. The curves measured by training and test episodes are reported by repeating 3 times with different seeds.

Action Validation. To prune massive invalid actions efficiently, one of the following approaches can be adopted: (i) implicit approach, which an agent is trained with early terminated episodes and attempts to produce a valid action without a separate component; (ii) action sampling approach, which samples valid actions from action space and then choose one of the sampled actions as the next action; (iii) approach with additional component for action validation, which utilizes a surrogate for confirming the validity of actions and masks out invalid actions. As described in [3, 38], the implicit approach suffers from an issue that most episodes are terminated early. Moreover, the action sampling approach, which is applied in [19], requires identifying valid actions at every iteration and defining adaptive action prediction heads for the sampled actions.

In this paper, we thus train a GNN, of which the head corresponds to the level of validity, by utilizing the graph-structured representation. The structures of both pivot and offset validation networks are identical to the networks described in Eq. (8), but the last activation is a sigmoid function and no target feature $\mathbf{z}$ is used. Importantly, these networks can be pretrained by the ground-truth action validity, which is obtained by randomly-assembled objects, and moreover such a pretrained network can be applied in training an actor-critic network, *without re-training*. Note that, if our action validation network fails to filter invalid actions and one of such actions is selected by the agent, the corresponding episode is terminated. Unlike the implicit approach, training the agent with the action validation network is efficient because the validation network still masks out the majority of invalid actions.

Training. We adopt the proximal policy optimization (PPO) algorithm [35], which is one of the state-of-the-art on-policy algorithms. In particular, we optimize the clipped surrogate objective over parameters $\boldsymbol{\theta}$:

$$\mathcal{L}(\boldsymbol{\theta}) = \mathbb{E}\left[\min(r_t(\boldsymbol{\theta})\hat{A}_t, \text{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon)\hat{A}_t)\right], \tag{9}$$

where $r_t(\boldsymbol{\theta})$ is a probability ratio between the previous and updated policy, clip is a clipping function between the second and the third arguments, and $\hat{A}_t$ is an advantage function [34]. To calculate the advantage of a state s_t, our model employs a value network, $\text{MLP}_{\text{val}}([\mu(V_t^{\text{piv}}), \mu(V_t^{\text{off}}), \mathbf{z}])$, where $\mu(\cdot)$ is a global average function over instances in a given set.

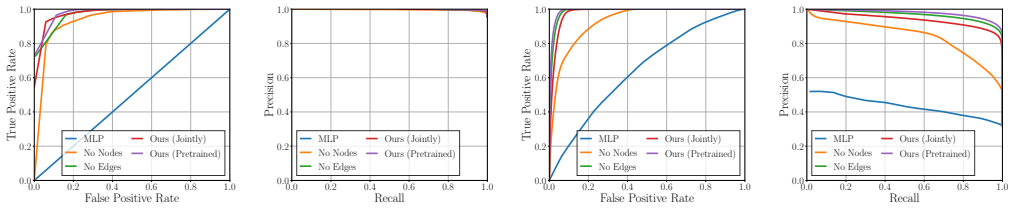
4 Experimental Results

As shown in Table 1, the prior studies do not evaluate their models in a scenario conditioned on the abstract expression of target objects. In a spirit of generalization in machine learning, we show the possibility of an image-conditioned 3D object construction scheme and the effectiveness of B^3 in three scenarios: (i) MNIST construction, (ii) randomly-assembled object construction, and (iii) ModelNet construction.

For evaluation metric, we measure the episode return or IoU between the constructed object and the desired target at the end of each episode:

$$\text{IoU}(\mathbf{C}_N, \mathbf{T}) = \frac{\text{vol}(\mathbf{C}_N \cap \mathbf{T})}{\text{vol}(\mathbf{C}_N \cup \mathbf{T})}, \tag{10}$$

where N is the total number of bricks and $\mathbf{T}$ is the voxel representation of the target object. The maximum number of bricks to be placed depends on $\mathcal{T}$ and is pre-defined. After exhausting the brick



(a) Pivot ROC (b) Pivot PR (c) Offset ROC (d) Offset PR

Figure 3: ROC and PR curves for the action validation network. All the results are measured using the test dataset of randomly-assembled objects.

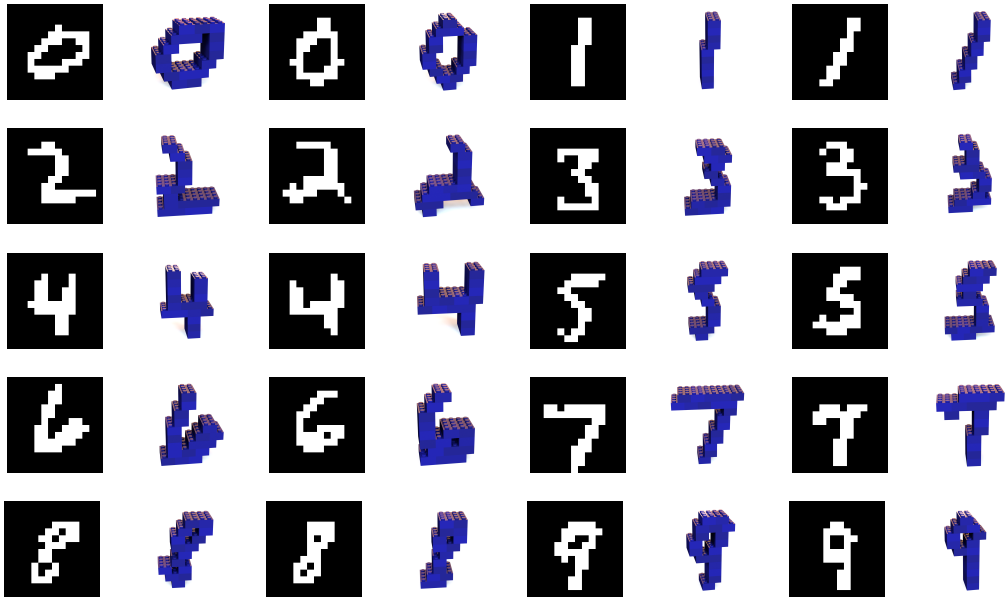
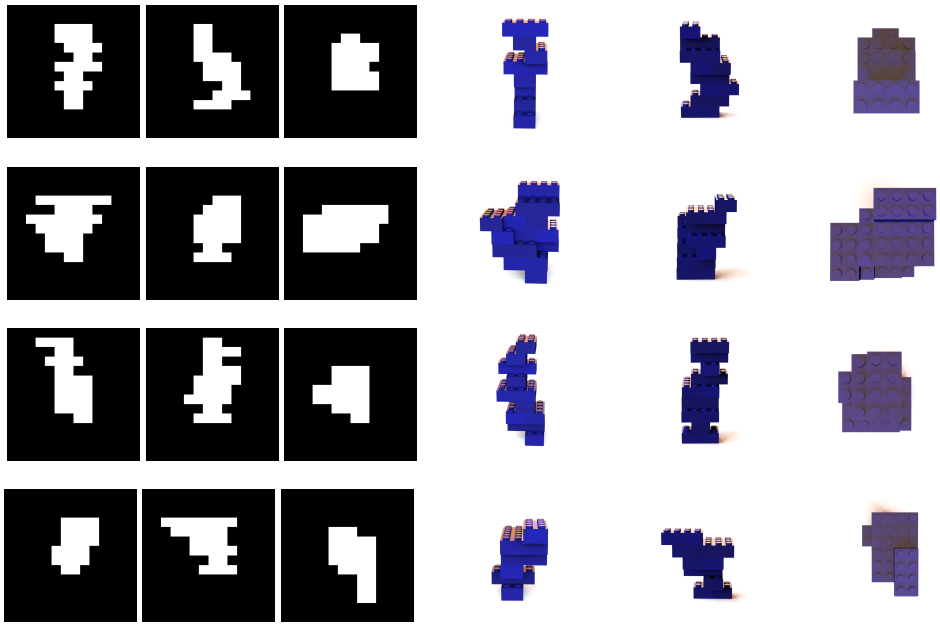


Figure 4: Qualitative results on MNIST construction. Our model is separately trained on each class, and target images are unseen during training.

budget, we terminate the episode and compute the final IoU. Unless otherwise specified, we report the average performance over 3 random seeds, each of which is trained for a fixed timestep budget.

To show the effectiveness of our method, we first analyze our action validation network and test other baseline methods and B^3 in different scenarios. In all construction tasks, we compare B^3 to the MLP-based model where all GNNs are replaced with MLPs, and to the Bayesian optimization-based approach (BO) that sequentially optimizes the step-wise reward in terms of IoU to search for an optimal construction sequence. As presented in Table 1, BO uses exact volumetric information for both training and test target objects because it cannot assemble an object with only partial information. For each scenario, the episode returns of the BO model are averaged over both training and test datasets. In addition, we compare B^3 to the supervised learning method trained with the cross-entropy loss between predicted and ground-truth sequences, specifically, in the randomly-assembled object construction. Since sequence-level supervision is used, the performance of the supervised learning method is only measured on the test dataset. Details can be found in the supplementary material.

Action Validation Network. We test our action validation network by creating training and test datasets. The training dataset is composed of 200,000 brick combinations and their ground-truth action masks, and the test dataset is composed of 30,000 brick combinations and their ground-truth action masks. Importantly, the range of the size of a brick combination in the training dataset is $[1, 20]$, and the range in the test dataset is $[1, 30]$. Even though the test dataset contains larger brick combinations than the training dataset, the performance of the action validation network in terms of



(a) Target images (b) Constructed objects from three viewpoints

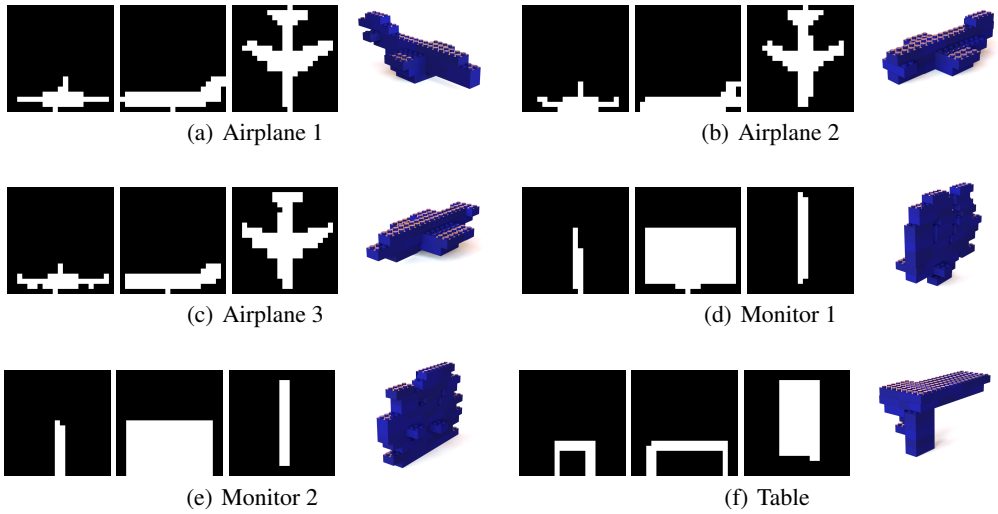
Figure 5: Qualitative results on randomly-assembled object construction. Targets are obtained from the test dataset, and each row represents a pair of target information and constructed object.

precision and recall is satisfactory by predicting a combination within unseen range, as presented in Figure 3. Our graph neural network outperforms other baselines including MLP and GNNs either without node feature or without edge feature. In addition, a pretrained network, which can be reusable in different scenarios, is slightly better than the action validation network that is jointly trained with training episodes. See the supplementary material for more details of action validation network.

MNIST Construction. In each episode, an agent is provided with an image from the MNIST dataset and is provided to create a 3D object resembling the digit target. Similar to [32], we binarize the MNIST dataset to convert a real-valued number to either 0 or 1, for brevity of the calculation of IoU. To create a 3D target object with a 2D MNIST image, we first rescale an image to half of the original size and then expand an image along the channel dimension, in order to assemble with 2×4 bricks, i.e., an image of size 28×28 is transformed to a 3D object of size $14 \times 14 \times 4$. Furthermore, we limit possible offset candidates to 6 different types of which the values according to the channel dimension are fixed to the same value. Training and test datasets are established by choosing one of the ten classes in the binarized MNIST and splitting images from that class. In particular, 500 images from one of available classes are chosen, further divided into 400 samples for a training dataset and 100 samples for a test dataset.

Due to a space limit, we only report the average reward performance on class 0 in Figure 2(a). Results for other classes are available in the supplementary material. The gap between the training and test sets on episode returns is marginal, which implies that our model B^3 generalizes to unseen targets well. In addition, both our training and test results are better compared to both the BO and MLP-based models. We provide visualizations of constructed objects for the test dataset of classes 0 through 9 in Figure 4. More qualitative results are also provided in the supplementary material. In general, our agent successfully constructs objects of unseen instances. This can be understood as our agent catches distinctive details in the target information and reflects in a construction process.

Randomly-Assembled Object Construction. Contrary to the experiments of MNIST construction, this task focuses on building objects that require more than one image to fully understand the structure. Accordingly, the agent must construct an object with three 14×14 images from different viewpoints, which are initially given as the target information. Objects in this experiment are artificially generated by connecting bricks at random. The total number of bricks is also chosen randomly between 10 to



(a) Airplane 1 (b) Airplane 2

(c) Airplane 3 (d) Monitor 1

(e) Monitor 2 (f) Table

Figure 6: Results on ModelNet construction. Targets are obtained from the test dataset. The first three panels and the last panel of each figure show the target images and the constructed example.

15. For available offset types, we only utilize connection types that occupy four or more studs and only allow a new brick to be placed on top of the pivot brick so that the resulting target becomes more distinguishable; see the supplementary material for the details. In this scheme, the total number of offsets is 16. Finally, we sample images from 800 target objects for a training dataset while 200 target objects are used for a test dataset.

As shown in Figure 2(b), our model achieves a return comparable to the MLP-based model while a slightly lower return compared to BO. This is because the number of bricks used is relatively small compared to other test suites. However, we observe that our agent is still capable of associating the target object in 3D space from multiple images as illustrated in Figure 5. Our model learns to assemble bricks in a way that the images of a resulting object match the initially given images fairly well. Furthermore, the model trained with the supervised learning method does not generalize to unseen images of the test dataset. This clearly demonstrates the effectiveness of applying reinforcement learning compared to sequence-level supervision.

ModelNet Construction. Similar to randomly-assembled object construction, the agent is given 3 images of a realistic target from the ModelNet dataset [40]. To adjust the difficulty of this task, we find an object that is able to limit the maximum budget of bricks to less than 60. As a result, we choose airplane, monitor, and table categories from the ModelNet dataset. Moreover, we use offset types that connect with four or more studs and allow a new brick to be placed above and below the pivot brick. This task is the most challenging due to the excessive search space compared to MNIST construction and randomly-assembled object construction, and assesses the agent's ability to generate a real-world target object.

Due to a space limit, we only provide training and testing curves for airplane class as shown in Figure 2(c); see the supplementary material for the curves for monitor and table categories. Despite the difficulty raised from the large search space and long sequence, the result demonstrates that B^3 is capable of learning the construction process of real-world objects. BO with a limited budget achieves lower return compared to B^3 since the search space is too big to explore with limited computation. By comparing the constructed object to images of the desired target in Figure 6, it can be observed that B^3 generally captures overall shape of the target. Though, our model tends to struggle to catch fine-grained details such as wings of the airplane or legs of the table. However, for example, a table with only three legs (i.e., one leg missing) or two legs in a diagonal direction would perfectly match with the same three input views. It implies that if we provide more complete target information than three different views of target object, our agent can construct the target object more precisely.

5 Related Work

In this section, we briefly cover related work on the task solved in this work.

3D Object Generation. Following the studies on 2D object generation, e.g., the work by Dosovitskiy et al. [8], 3D object generation is often achieved in holistic manner [39, 1, 14, 41, 30]. They generate a 3D object in a single feed-forward operation which limits exploitation of intermediate structures. Compared to these holistic methods, Kim et al. [19] propose an approach to tackle a combinatorial assembly problem by using Bayesian optimization [5], not a learning-based method. Unlike [19], Thompson et al. [38] apply a graph-structured generative model in the combinatorial 3D object generation task, by training to match a ground-truth sequence of LEGO bricks.

Graph-based Reinforcement Learning. A common learning-based technique for creating a graph is to use an autoregressive model such as recurrent neural networks [42, 24], adversarial networks [7], and variational autoencoders [18, 33, 36]. Unlike these directions, Simm et al. [37] solve this idea of generating molecules with reinforcement learning such that generated molecules are placed in the Cartesian coordinate. The key difference to our work is that we are sequentially generating 3D shapes which have a much larger search space. Furthermore, Bapst et al. [3] show that an RL agent can learn physical construction in 2D space, and utilize rich visual information as well as a graph-structured representation, in order to define a state and a search space.

Image-Conditioned Reinforcement Learning. Ganin et al. [10] propose an approach to synthesizing a program for 2D images when either an unconditional or conditional scenario is assumed. Their method generates an image by sequentially conducting an action in a MuJoCo environment. Nair et al. [29] propose a goal-conditioned RL approach the goal of which is provided by visual information. Huang et al. [16] suggest a method to paint a palette with stokes where a target image is conditionally given, by utilizing an RL algorithm.

Brick Assembly Optimization. The brick assembly problem satisfying pre-defined constraints is a longstanding topic in computer graphics. Lee et al. [23] tackle LEGO brick layout optimization by a genetic algorithm. Similarly, Luo et al. [27] solve building sculptures safely with LEGO brick by stability aware refinement. Zhang et al. [44] propose the method for generating component-based building instructions that is safe based on segmentation models. Kozaki et al. [21] tackle a similar problem of brick assembly from images with the octree voxel-based model.

6 Conclusion

We have proposed a novel problem formulation, combinatorial construction, which asks an agent to construct an object sequentially. We adopt reinforcement learning by defining a state as graph-structured representation to express assembled bricks and their connections, where incomplete target information is given. In addition, we develop our algorithm with a successive action space that does not depend on the number of bricks already constructed and a reward function that measures overlap between a target object and the current state. Through extensive experiments, we demonstrate that our method can construct objects in various construction scenarios, and provide the detailed analysis of our action validation network.

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