Learning Absorption Rates in Glucose-Insulin Dynamics from Meal Covariates

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Abstract

Traditional models of glucose-insulin dynamics rely on heuristic parameterizations chosen to fit observations within a laboratory setting. However, these models cannot describe glucose dynamics in daily life. One source of failure is in their descriptions of glucose absorption rates after meal events. A meal's macronutritional content has nuanced effects on the absorption profile, which is difficult to model mechanistically. In this paper, we propose to learn the effects of macronutrition content from glucose-insulin data and meal covariates. Given macronutrition information and meal times, we use a neural network to predict an individual's glucose absorption rate. We use this neural rate function as the control function in a differential equation of glucose dynamics, enabling end-to-end training. On simulated data, our approach is able to closely approximate true absorption rates, resulting in better forecast than heuristic parameterizations, despite only observing glucose, insulin, and macronutritional information. Our work readily generalizes to meal events with higher-dimensional covariates, such as images, setting the stage for glucose dynamics models that are personalized to each individual's daily life.

1 Introduction

Type-1 diabetes is a chronic condition of glucose dysregulation that affects 9 million people around the world. Decades of research have produced dozens of glucose-insulin dynamics models in order to understand the condition and help diabetics manage their daily lives. These models are typically developed using physiological knowledge and validated in laboratory settings. However, these mechanistic models are incomplete; they are not flexible enough to fit observations outside of controlled settings, due to unmodelled variables, unmodelled dynamics, and external influences. As a result, these mechanistic models fail to fully describe an individual's glycemic response to external inputs like nutrition.

Standard models, such as Dalla Man et al. [10], focus on the glycemic impact of carbohydrates in a meal—carbohydrates are broken down into glucose molecules, then absorbed into blood. However, these models typically ignore other macronutrients, such as fat, fiber, and protein, which are known to contribute substantially to the amount and timing of glucose absorption into the blood. Indeed, this phenomenon is the basis for the glycemic index of various foods. In reality, individual glycemic responses to nutrition go beyond such a simple characterization. For example, Zeevi et al. [28] identified multiple patient sub-groups with different glycemic responses to complex foods.

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In our paper, we propose a method that can leverage real-world nutrition and glucose-insulin measurements to improve the fidelity of existing mechanistic models. While we tailor this approach to the specific application of type-1 diabetes, we note that our methodology fits within a broad paradigm of hybrid modeling of dynamical systems [19, 22, 24, 27]. These approaches can improve mechanistic ODEs using flexible components that learn from observations of the system and its external controls.

2 Background on modelling glucose-insulin dynamics

Our paper builds on the tradition of modelling physiological dynamics via ordinary differential equations (ODEs), [5, 10, 16, 21, 26]. Traditional models consider ODEs of the form $\dot{x}(t) = f(t, x(t)) + u(t)$, where $x \in \mathbb{R}^n$ denotes physiologic states, $f : \mathbb{R}^n \to \mathbb{R}^n$ encodes mechanistic knowledge of their interactions, and $u : \mathbb{R} \to \mathbb{R}^n$ represents external time-varying inputs into the system. Significant effort has gone towards identifying u from insulin, exercise, and meal data, but u is typically represented via a gastrointestinal ODE model [9, 11] or via hand-chosen functional forms [14, 15, 20]. Both approaches for representing meals depend only on carbohydrate consumption and do not consider other macronutrient quantities.

Our paper considers the minimal model of glucose-insulin dynamics by Bergman et al. [5]:

$$\dot{G}(t) = -c_1[G(t) - G_b] - G(t)X(t) + u_G(t) \tag{1a}$$

$$\dot{X}(t) = -c_2 X(t) + c_3[I(t) - I_b] \tag{1b}$$

$$\dot{I}(t) = -c_4[I(t) - I_b] + u_I(t) \tag{1c}$$

where $x = (G, X, I)$ and $u = (u_G, u_I)$. Here, $G : \mathbb{R} \to \mathbb{R}$ represents plasma glucose concentration, $I : \mathbb{R} \to \mathbb{R}$ represents plasma insulin concentration, $X : \mathbb{R} \to \mathbb{R}$ represents the effect of insulin on glucose, $G_b, I_b \in \mathbb{R}$ represent basal glucose and insulin levels, respectively, and $c_1, c_2, c_3, c_4 \in \mathbb{R}$ represent rate constants for the interactions. Importantly, $u_G : \mathbb{R} \to \mathbb{R}$ represents the appearance of glucose in the blood (e.g. absorbed from nutrition in the gut) and $u_I : \mathbb{R} \to \mathbb{R}$ represents the appearance of insulin in the blood (e.g. absorbed from subcutaneous injection or drip). See Gallardo-Hernández et al. [13] for a modern exposition and the units of each quantity.

Modelling nutrition absorption from discrete meal events. When simulating the daily management of diabetes, the *continuous* functions u_G, u_I are typically derived from observed *discrete* events (e.g. meals and insulin injections). Each discrete-time event $e_i = (t_i, m_i)$ consists of a timestamp t_i and a covariate m_i. If e_i is a meal event, m_i may consist of macronutritional information, an image of the food, or both. Pharmacodynamics models are often used to map the insulin dose to a continuous absorption profile u_I that is compatible with the above model. However, the dependence of glucose absorption u_G on full macronutritional content of a meal event is less well-understood; thus *we focus on modelling u_G in this paper*.

Mechanistic u_G models often derive u_G as the solution to another set of heuristic ODEs[10]. However, this approach introduces additional handcrafted parameterizations to explain quantities that are unobservable outside of the lab setting, such as the glucose concentration in the stomach over time after a meal. A simpler yet effective approach is to directly model u_G phenomenologically, and estimate it from data [14, 20]. Instead of deriving u_G from an intricate model of the human body, this approach represents u_G directly using a parametric function adapted from data.

3 Phenomenologically modelling the absorption rate

Let each meal event i be $e_i = (t_i, m_i)$ where $t_i \in \mathbb{R}$ is the meal time and $m_i \in \mathbb{R}^M$ is a vector of meal covariates, such as its macronutrition content or even a photo of the food. We assume we have data on a set E of these meal events. For each meal i, we associate a parametric function $a_i : \mathbb{R}_+ \to \mathbb{R}_+$, such that $a_i(t)$ is the absorption rate of the meal at time t. The overall control function u_G is then a sum over the events:

$$u_G(t) = \sum_{i=1}^{|E|} a_i(t). \tag{2}$$

a_i is usually compactly supported, since meals only affects glucose locally in time. Decomposing u_G into a sum allows us to model the effect of each meal individually, instead of all at once.

A simple heuristic choice is a square function $a_i(t) = g_i \mathbb{1}_{[0,w)}(t - t_i)/w$ where w is the width of the square as a free parameter and $g_i \in \mathbb{R}$ is the amount of glucose produced from the meal. Another choice is the bump function $a_i(t) = g_i \mathbb{1}_{[0,\infty)}(t - t_i)(e^{-b_1(t-t_i)} - e^{-b_2(t-t_i)})/b_3$ where b_1 and b_2 are free parameters and b_3 is a normalization constant [1, 2]. For both choices, g_i must be estimated by the patient or by a nutritionist (e.g. when m_i is a food image), which can be highly inaccurate. More importantly, the *shape* of these parameterizations does not depend on m_i, even though foods vary in absorption profiles.

A neural phenomenological model. The form of Equation (2) suggests a natural extension that takes advantage of the flexiblity of neural networks. Given a meal event $e_i = (t_i, m_i)$, we model its absorption rate using a neural network a_θ such that

$$a_i(t) = g_i \cdot a_\theta(t - t_i, m_i) \mathbb{1}_{[0,\infty)}(t - t_i). \tag{3}$$

We make use of the estimated glucose content g_i following prior approaches since it is often already available in the meals dataset, and gives an expert-informed glucose absorption scale factor. Alternatively, g_i can be included as another input to a_θ instead of being a multiplicative constant. Even if the estimated g_i is inaccurate, a_θ has the flexiblity to rescale g_i based on the observed m_i. Most importantly, our parameterization differs in that its *shape* can adapt to the meal covariates m_i. We share one neural network a_θ across all meal events, allowing it to generalize to macronutritional information similar to, but not exactly the same as, meals from the training set. Altogether, Equations (1),(2),(3) define our neural differential equation model.

End-to-end training on partial observations. Having defined our parametric function, we now discuss how to learn the parameters θ in a setting that is realistic to settings outside of the laboratory. Recent technologies like continuous glucose monitors and artificial pancreases enable real-time measurements of glucose levels and insulin dosage. However, most of a patient's physiological state is unobserved. Within Equation (1), we do not observe insulin I and its effect X.

Let x be the state of our differential equation from Equation (1). We assume our temporal data consists of noisy partial observations over time $\{(t_k, y_k)\}_{k=1}^T$, where $y_k = Hx(t_k) + \varepsilon$. We assume the projection operator $H : (G, X, I) \mapsto (G, 0, 0)$ and ε is a zero-mean i.i.d. noise process.. Given initial condition $x(t_0) = x_0$, we can numerically integrate Equation (1) with a given u_I and our parameterized $u_G(\cdot\,; \theta)$ to obtain an estimate $\hat{y}(t_k) = H\hat{x}(t_k)$ where $\hat{x}(t_k) = \text{Integrate}(f, u, x_0, t_0, t_k)$. We then minimize the mean squared error objective $L(\theta) = \sum_{k=1}^T \|\hat{y}(t_k) - y_k\|_2^2/T$ with respect to θ to fit our parametric model [12]. However, this procedure requires us to know x_0, which is not fully observed in practice.

Many methods exist for performing such under-determined state and parameter estimation; often, the state-estimation component is performed using filtering or smoothing [6–8, 19, 25], but can also be learnt through other data-driven [4, 17] or gradient-descent [23] methods. In our experiments, we estimate an initial state x_0 by using a sequence of F observations $(G(t_{-F+1}), G(t_{-F+2}), \ldots, G(t_0))$ as a forcing function when forward integrating Equation (1), described in Section 4.3 of Levine and Stuart [19]. This simple procedure was sufficient for our model to learn a good θ, likely due to the rapidly decaying autocorrelation of (1).

4 Experiments

We evaluate our proposed method on simulated data. We simulate 28 days worth of glucose, insulin, and meal data for one virtual patient using Equation (1). We evaluate our method against baseline methods with and without glucose observation noise. We also evaluate each method in the realistic setting where the *time* of each meal is noisily reported, since in daily life, the recorded meal time is often only approximately correct.

Data generation. For each day, we generate four meals: breakfast, lunch, dinner, and a late snack. Meals occur uniformly at random within 6-9AM, 11AM-2:30PM, 5-8PM, and 10-11PM, respectively. Each meal contains a glucose amount uniformly random within 5-65g, 20-70g, 40-100g, and 5-15g respectively. For each meal event i, we convert grams of glucose to plasma glucose concentration, assuming the individual has 50dl of blood, and use the result as g_i. To simulate different absorption profiles, each meal is a convex mixture of three "absorption templates". Each template j is given

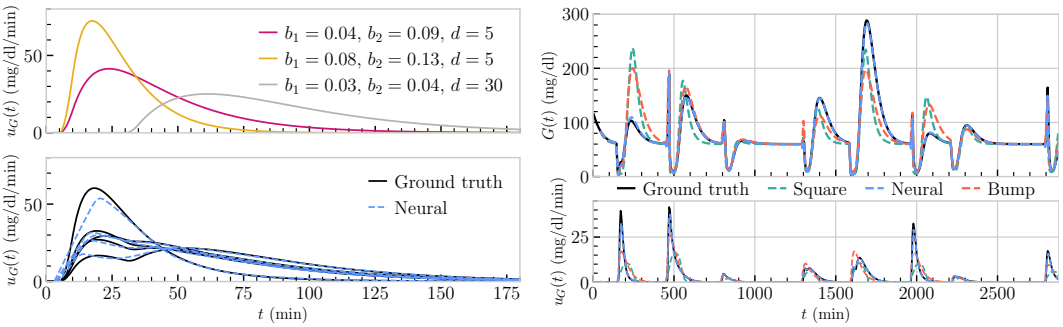

Figure 1: *Left*: (Top) "Absorption templates" used to generate u_G. (Bottom) 5 Samples of ground truth and learned u_G for meals from the test set. *Right*: Glucose forecast and predicted absorption rates of each model over a 2 day window from the test trajectory.

by delayed bump function $a^j(t) \propto g_i \mathbb{1}_{[0,\infty)}(t - t_i - d)(e^{-b_1(t-t_i-d)} - e^{-b_2(t-t_i-d)})$, each with its own set of parameters $(b_1, b_2, d) \in \{(0.04, 0.09, 5\text{min}), (0.08, 0.13, 5\text{min}), (0.03, 0.04, 30\text{min})\}$, visualized in Figure 1. The templates represent regular absorption, fast absorption, and slow absorption, respectively. The macronutrition of meal i is then the vector of mixture coefficients $m_i \in \mathbb{R}^3$ such that meal i has absorption profile $a_i(t) = \sum_{j=1}^{3} m_{ij} a^j(t)$. To ensure a_i is smooth, we average each value $a_i(t)$ with a grid of 50 points from the past 5 minutes.

For each meal time t_i, we simulate an insulin bolus dose at a time sampled from $\mathcal{N}(t_i, (10\text{min})^2)$. We sample a glucose to insulin conversion for each meal from $\mathcal{N}(7\text{g/U}, (1\text{g/U})^2)$. To simulate imperfect measurements, we add a relative $N(0, 0.05^2)$ observation noise. To simulate imperfect meal time recordings, we add $N(5\text{min}, (2.5\text{min})^2)$ noise to meal times. We use a square function u_I, corresponding to a constant insulin absorption rate, over 30 minutes, which we assume to be known to every model. We use parameters from Andersen and Højbjerre [3] for Equation 1, and we use Euler integration with a step size of 0.1 minutes to produce an observation every 5 minutes.

Experimental setup. We split our generated data temporally into 3 disjoint training, validation, and testing trajectories. We optimize using Adam [18] for 1000 iterations, with a half-period cosine learning rate schedule following a linear ramp up to 0.2 over the first 30 iterations. We use minibatches of 512 sequences of 4 hour windows (48 observations) and use 10 observations for estimating the initial condition. We minimize the mean squared error on the observed glucose values with respect to the parameters θ of a_θ, keeping the other parameters of Equation (1) fixed. We parameterize our neural a_θ using a feedforward network with 2 hidden layers of 64 units and GELU activations. We found that appropriately scaling the input and outputs of a_i is crucial for stable optimization.

	Exact timestamps		Noisy timestamps	
a_i	Exact observations	Noisy observations	Exact observations	Noisy observations
Neural	0.95mg/dl	3.66mg/dl	1.48mg/dl	3.63mg/dl
Bump	9.52mg/dl	10.11mg/dl	9.53mg/dl	10.24mg/dl
Square	11.60mg/dl	11.53mg/dl	11.65mg/dl	11.56mg/dl

Table 1: Forecast RMSE computed over all possible 4 hour windows of the test set trajectory, reflecting the window size used for training.

Evaluations. We compare our neural absorption function against the two common parameterizations of u_G from Section 3, fit via gradient-based optimization. We approximate the piece-wise constant square function using a difference of sigmoids; otherwise the width cannot be learned. Our neural model is able to closely approximate the ground truth u_G, especially in the tails, as shown in Figure 1 (left). This results in significantly better forecasts, and our neural model closely tracks the ground truth glucose values and absorption rates, *even extrapolating to durations much longer than what was seen in training*. We visualize such long term forecasts in Figure 1 (right). We also report the forecast RMSE on the test set in Table 1. Our neural model attains lower forecast errors across all settings. In the noiseless case, our neural model is 10x more accurate than heuristic parameterizations. The RMSEs generally increase as we add noise, though the bump and square functions are already such poor forecasters that noise does not worsen their errors significantly.

5 Discussion

Our experiments show that our proposed method is a promising way to learn absorption profiles that depend on macronutritional information. Our approach readily generalizes to handle arbitrary meal covariates beyond macronutritional information, such as food images or descriptions. Although this paper only uses synthetic data, our method can complement any glucose dynamics model of real-world data. Learning accurate dynamics from data, however, remains a challenging problem. We see our method as a vital component in future data-driven hybrid models of glucose-insulin dynamics.

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