FINDING GENERAL EQUILIBRIA IN MANY-AGENT ECONOMIC SIMULATIONS USING DEEP REINFORCEMENT LEARNING

Anonymous authors
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ABSTRACT

Real economies can be seen as a sequential imperfect-information game with many heterogeneous, interacting strategic agents of various agent types, such as consumers, firms, and governments. Dynamic general equilibrium models are common economic tools to model the economic activity, interactions, and outcomes in such systems. However, existing analytical and computational methods struggle to find explicit equilibria when all agents are strategic and interact, while joint learning is unstable and challenging. Amongst others, a key reason is that the actions of one economic agent may change the reward function of another agent, e.g., a consumer's expendable income changes when firms change prices or governments change taxes. We show that multi-agent deep reinforcement learning (RL) can discover stable solutions that are ϵ-Nash equilibria for a meta-game over agent types, in economic simulations with many agents, through the use of structured learning curricula and efficient GPU-only simulation and training. Conceptually, our approach is more flexible and does not need unrealistic assumptions, e.g., market clearing, that are commonly used for analytical tractability. Our GPU implementation enables training and analyzing economies with a large number of agents within reasonable time frames, e.g., training completes within a day. We demonstrate our approach in real-business-cycle models, a representative family of DGE models, with 100 worker-consumers, 10 firms, and a government who taxes and redistributes. We validate the learned meta-game ϵ-Nash equilibria through approximate best-response analyses, show that RL policies align with economic intuitions, and that our approach is constructive, e.g., by explicitly learning a spectrum of meta-game ϵ-Nash equilibria in open RBC models.

1 INTRODUCTION

Real-world economies can be seen as a general-sum sequential imperfect-information game with many heterogeneous strategic agents, including consumer-workers, firms, and governments. Dynamic general equilibrium models (DGE) describe the economic incentives, activity, interactions, and constraints of (a large collection of individual) agents to model and predict *macro-economic* outcomes, such as productivity, equality, and growth (Heer & Maussner, 2009). Following *micro-economic theory* (Mas-Colell et al., 1995), the behavior of each agent is driven by incentives ("rewards"), e.g., consumers maximize utility ("happiness"). For economic analysis, it is key to find the strategic equilibria. e.g., at an ϵ-Nash equilibrium, no agent would unilaterally change its behavior, assuming, for example, that all agents act rationally and optimally (Nisan et al., 2007).[1]

Existing analytical and computational methods often struggle to find explicit DGE equilibria. First, the nonlinear structure of DGE models makes it challenging to find optimal agent policies explicitly. For example, an often-used analytical approach is to solve a linearized version of the DGE dynamics, akin to a Taylor expansion. However, a linearized DGE model may have fewer or different equilibria compared to the full DGE model. Furthermore, they may only be good approximations around steady-state equilibria and not be valid in the presence of shocks, e.g., when agents change their behavior

[1]This could be expanded to more closely mimic human behavior through "boundedly rational" agents that may act suboptimally or whose affordances or objectives encode cognitive biases or limits (Kahneman, 2003).

dramatically in time (Boneva et al., 2016; Atolia et al., 2010). Second, formal solutions methods, e.g., backwards induction and related dynamic programming methods, often cannot be solved explicitly and only characterize optimal policies implicitly (Stokey et al., 1989). Third, computational methods can suffer from the curse of dimensionality, e.g., when there is a large number of agents. In particular, agent incentives are often misaligned and pose a complex general-sum game. However, enumerating and selecting equilibria in general-sum games is an unsolved challenge (Bai et al., 2021).

Multi-agent deep reinforcement learning (RL) (Sutton & Barto, 2018) is a powerful framework to model agent behavior and has several key advantages for economic analysis. Deep neural network policies can learn nonlinear predictive patterns over high-dimensional state-action spaces, while deep value functions can model complex reward function landscapes over long time horizons. Notably, deep RL has achieved superhuman performance in high-dimensional, sequential games (Silver et al., 2017; Vinyals et al., 2019; OpenAI, 2018), suggesting it can learn (approximate) strategic equilibria in complex settings. Moreover, one does not have to approximate nonlinear world dynamics (e.g., as in the DGE model); RL has been successful in highly nonlinear environments (Tassa et al., 2018).

However, *deep RL is still not commonly used in economics*, although recent work has started to explore its use in economics (Zheng et al., 2020; Trott et al., 2021; Hill et al., 2021; Chen et al., 2021). In particular, deep MARL poses several challenges in the context of economic simulations.

First, a general MARL challenge is that each agent faces a non-stationary environment *when multiple agents learn simultaneously*; each agent's experienced reward may depend on what other agents do. For example, a consumer may not be able to buy a desired good if there is not enough supply generated by the corresponding firm. Second, economic agents typically have private information that is not available to other agents, e.g., a worker's skill level and behavioral policy (parameters) are not observable by firms. This means that learning has to be (partially) decentralized, as in Dec-POMDPs (Oliehoek & Amato, 2016). Moreover, a key feature of economic models is that *the actions of one economic agent may change the shape of the reward function* of another agent, e.g., a consumer's expendable income changes when firms change prices or governments change taxes. Hence, agents change the optimization problem for other agents during training. This makes joint learning unstable: individually training policies easily get stuck in trivial equilibria where no labor or production occurs.

Contributions. To address these challenges, we develop MARL techniques that enable finding equilibria in DGE models with many strategic agents.

We **run both simulation and RL on a GPU**, following the WarpDrive framework (Lan et al., 2021), so training converges in hours (rather than days). This allows us to analyze DGE models with hundreds of heterogeneous RL agents, a scale that has not been studied before with strategic agents.

From an **algorithmic** perspective, we generalize the curriculum approach from Zheng et al. (2020) to structured learning curricula for simulations with multiple agent types. This approach yields non-trivial solutions *more stably* compared to using agent-independent RL. We also show our approach is *sound*, e.g., by showing that an RL government can improve social welfare vs fixed baselines.

From a **modeling** perspective, our approach is more flexible and can find stable solutions in a variation of real-business-cycle (RBC) models (Pierre Danthine & Donaldson, 1993), a family of DGE models. We find equilibria in closed and open RBC economies, the latter has a price-taking export market.

From an **empirical game theory** perspective, we show that our MARL approach can explictly find ϵ-Nash equilibria for the meta-game over agent types, without approximating the DGE model. Here, the meta-game equilibrium is a set of agent policies such that no *agent type* can unilaterally improve its reward by more than ϵ (its *best-response*). Previous solutions only could do so implicitly or for approximations of the DGE dynamics. Furthermore, we learn a *spectrum of equilibria* in open RBC economies, i.e., explicitly learn the equilibrium behavioral policies.

More general types of equilibria may exist in DGE models, but can be hard to validate. For instance, there may be asymmetric Stackelberg-like equilibria, where a leader (e.g., the government) acts first (e.g., sets taxes), and the followers (e.g., consumers and firms) respond. This differs from symmetric Nash equilibria where all agents act simultaneously. However, analyzing the equilibria in Stackelberg-like, general-sum games is an open challenge (Bai et al., 2021). For instance, with 2 or more (heterogeneous) followers, finding the Stackelberg best-response to a fixed leader requires finding multi-agent equilibria for the followers. This is computationally expensive and there is no known

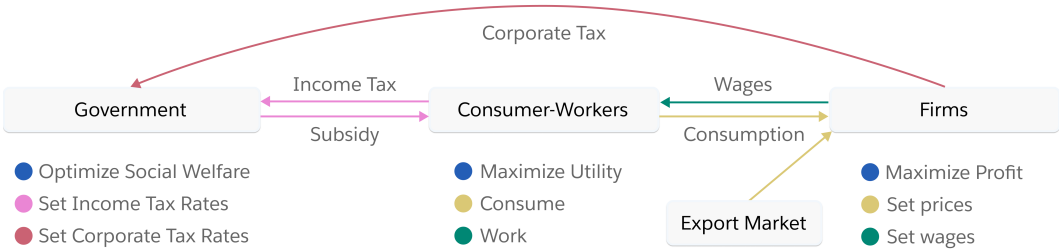

Figure 1: **RBC model with consumers, firms, and governments.** Arrows represent money flow. Consumer-workers earn wages through work and consume goods from firms. They also strategically choose *which* firm to work for and *which basket of goods* to buy, but this is not explicitly visualized. Firms produce goods, pay wages, and set a price for their goods. They also invest a fixed fraction of profits to increase capital. The government taxes labor income and firm profits, and redistribute the tax revenue through subsidies to the consumer-workers. Firms can also sell goods to an external export market, which acts as a price-taker that is willing to consume goods at any price.

provably convergent algorithm for multi-level Stackelberg games. Instead, we use MARL to converge to a stable solution and analyze best-responses to evaluate to what extent it is an "equilibrium".

Moreover, *which* equilibrium agents converge to ("equilibrium selection") may depend on, e.g., the 1) world dynamics, 2) initial conditions and policies, 3) learning algorithm, and 4) policy model class. Our framework allows us to study how these factors relate to which equilibria can be found.

2 Case Study: Real-Business-Cycle Models

We show how RL can find equilibria in real-business-cycle models (RBC), a representative DGE model in which consumers earn income from labor and buy goods, firms produce goods using capital and labor, and the government taxes income and profits (Pierre Danthine & Donaldson, 1993), see Figure 1. RBC models are stylized and may not fully describe reality (Summers et al., 1986). However, RBC models are a suitable environment to validate the use of RL, as they feature heterogenous agents with nonlinear economic interactions, making it challenging to find equilibria. Below, we describe the RBC dynamics; for low-level details, see Appendix C.

Agent Types. Formally, our RBC model can be seen as a Markov Game (Littman, 1994) with multiple agent types and partial observability. The simulations proceeds in finite-length episodes with T timesteps. Each timestep represents a quarter of 3 months. At each timestep t, we simulate

- firms and their goods who use the labor of the worker-consumers to each produce a different good, indexed by i,
- consumers, indexed by j, who work and consume goods, and
- a government who sets a tax rate on income from labor and on revenue from selling goods.

Each agent iteratively receives an observation $o_{i,t}$ of the world state s_t, executes an action $a_{i,t}$ sampled from its policy π_i, and receives a reward $r_{i,t}$. The environment *state* $\boldsymbol{s}$ consists formally of all agent states and a general world state (see Appendix C for details). At each timestep t, all agent simultaneously execute actions. However, some actions only apply to the next timestep $t+1$. For example, the government sets taxes *that will apply at the next timestep* $t+1$. These tax rates are observable by the firms and consumer-workers at timestep t; hence, they can condition their behavioral policy in response to the government policy. Similarly, at timestep t, firms set prices and wages that will apply at the next timestep $t+1$. Consumer-workers can observe these prices and wages at timestep t and can condition their behavioral policy in response. This setup is akin to a Stackelberg leader-follower structure, where, for example, the government (leader) moves first, and the firms and consumer (followers) and move second (von Stackelberg et al., 2010). This typically gives a strategic advantage to the followers: they have more information to condition their policy on.

Consumer-Workers. Individual people both consume and work; we will refer to them as *consumer-workers*. At each timestep t, person j works $l_{j,t}$ hours and consumes $c_{i,j,t}$ units of good i. Each person chooses to work for a specific firm i at each timestep. Consumer-workers also (attempt to) consume $\hat{c}_{i,j,t}$. However, consumers cannot borrow or outspend their budget: if the cost of attempted consumption exceeds the budget, then we scale consumption so that $\sum_i p_i \hat{c}_{t,i,j} = B_j$.

Moreover, the realized consumption depends on the available supply of goods. The total demand for good i is $\hat{c}_{i,t} = \sum_j \hat{c}_{i,j,t}$. If there is not enough supply, we ration goods proportionally:

$$c_{i,j,t} = \min\left(1, \frac{y_{i,t}}{\hat{c}_{i,t}}\right) \hat{c}_{i,j,t}. \tag{1}$$

Consuming and working change a consumer's budget $B_{j,t}$. Consumer j has labor income $z_{j,t} = \sum_i l_{i,j,t} w_{i,t}$; each firm pays a wage $w_{i,t}$. The cost of consumption is $\sum_i p_{i,t} \cdot c_{i,j,t}$ Moreover, workers pay income tax $\tau_t \cdot z_{j,t}$; the total tax revenue R_t is redistributed evenly back to workers. In all, consumer budgets change as:

$$B_{t+1,j} = B_{j,t} + (1-\tau_t) z_{j,t} + \frac{R_t}{J} - \sum_i p_{i,t} \cdot c_{i,j,t}. \tag{2}$$

Each consumer optimizes its behavioral policy to maximize utility:

$$\max_{\pi_j} \mathbb{E}_{c,l\sim\pi_j}\left[\sum_t \gamma_c^t \sum_i u(c_{i,j,t}, l_{i,j,t}, \theta_j)\right], \quad u(c,l,\theta) = \frac{(c+1)^{1-\eta} - 1}{1-\eta} - \frac{\theta}{2} l_t, \tag{3}$$

where γ_c is the consumer's discount factor. The utility function is a sum of isoelastic utility over consumption and a linear disutility of work with coefficient θ_j that can vary between workers.

Firms. At each timestep t, a firm receives labor from workers, produces goods, sells goods, and may invest in capital. At each timestep t, it sets a price $p_{t+1,i}$ for its good and chooses a wage $w_{t+1,i}$ to pay, both *effective at the next timestep* $t+1$. If a firm invests $\Delta k_{i,t}$ in capital, its capital increases as $k_{i,t+1} = k_{i,t} + \Delta k_{i,t}$. Using its capital $k_{i,t}$ and total labor $L_{i,t} = \sum_j l_{i,j,t}$ (hours worked), a firm i produces Y units of good i, modeled using the *production function*:

$$Y_{i,t} = A_i K_{i,t}^{1-\alpha} L_{i,t}^{\alpha}, \tag{4}$$

where $0 \leq \alpha \leq 1$ sets the importance of capital relative to labor. Also, consumers buy $C_{i,t} = \sum_j c_{i,j,t}$ units of good i. Accordingly, inventories change as $y_{t+1,i} = y_{i,t} + Y_{i,t} - C_{i,t}$. Inventories are always positive, as only actually produced goods can be consumed. The firm's profit (or loss) is:

$$P_{i,t} = p_{i,t} C_{i,t} - w_{i,t} L_{i,t} - \Delta k_{i,t}. \tag{5}$$

Firms pay tax on their profit, so their budget changes as:

$$B_{t+1,i} = B_{i,t} + (1-\sigma_t) P_{i,t}, \tag{6}$$

where σ is the corporate tax rate. The government receives $\sigma_t P_{i,t}$. Firms may borrow and temporarily be in debt (negative budget), but should have non-negative budget at the end of an episode (*no-Ponzi condition*). This may allow firms to invest more, which may lead to higher future economic growth. Each firm optimizes its behavioral policy to maximize profits as in Equation 5:

$$\max_{\pi_i} \mathbb{E}_{p,w,\Delta k\sim\pi_i}\left[\sum_t \gamma_f^t P\left(p_{j,t}, w_{j,t}, \Delta k_{j,t}\right)\right], \tag{7}$$

where γ_f is the firm's discount factor. The firm gets a negative penalty if it violates *no-Ponzi*.

Government. The government, or social planner, indexed by p, sets corporate and income tax rates, and receives total tax revenue $R_t = \sigma_t \sum_j z_{j,t} + \tau_t \sum_i P_{i,t}$. As a modeling choice, this revenue is redistributed evenly across consumer-workers; as such, the government's budget is always 0. The government optimizes its policy π_p to maximize social welfare:

$$\max_{\pi_p} \mathbb{E}_{\tau,\sigma\sim\pi_p}\left[\sum_t \gamma_p^t \text{swf}\left(\tau_t, \sigma_t, \boldsymbol{s}_t\right)\right], \tag{8}$$

where $\text{swf}\left(\boldsymbol{s}_t\right)$ is the social welfare at timestep t, and γ_p is the government's discount factor. In this work, we consider two definitions of social welfare (although many other definitions are possible): 1) *total consumer utility* (after redistribution), or 2) *total consumer utility and total firm profit*.

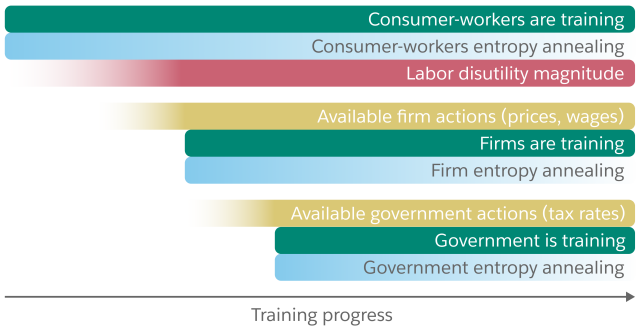

Figure 2: **Structured learning curricula.** Colored bars show how annealing activates over time. Consumers are always training, use a decaying entropy regularization, and increasingly experience their labor disutility. Firms start with fixed prices and wages, and are slowly allowed to change these. Once the full price and wage range is available, firms start to train. Similarly, tax rates start at zero and are slowly allowed to change. Once the full tax range is available, the government starts to train.

Open and Closed Economies via Export Markets. We consider both open and closed economies. In the open economy, firms can also sell goods to an external market which acts as a *price-taker*: their demand does not depend on the price of a good. Operationally, export happens after worker-consumers consume. The export market has a minimum price p_{export} and a cap q_{export}. If the price of good i is greater than the minimum price ($p_{i,t} > p_{\text{export}}$) then the additional export consumption is $c_{t,\text{export}} = \min(q_{\text{export}}, y_{i,t} - C_{i,t})$, at price $p_{i,t}$, i.e., the exported quantity is insensitive to the price.

From a learning perspective, the export market prevents firms from seeing extremely low total demand for their good, e.g., when prices are exorbitantly high and consumers do not want or cannot consume the good. In such cases, an on-policy learner that represents a firm may get stuck in a suboptimal solution with extremely high prices and no production as consumers cease to consume in response.

3 Multi-Agent RL and Structured Curricula

A key idea of our approach is the use of structured multi-agent curricula to stabilize learning, where each individual agent uses standard RL. These curricula consist of staged training, annealing of allowed actions, and annealing of penalty coefficients, see Figure 2.

Base RL. Firms and governments use 3-layer fully-connected neural network policies $\pi(\boldsymbol{a}|\boldsymbol{s})$, each layer using 128-dim features, that map states to action distributions. Consumer policies are similar, using separate heads for each action type, i.e., the joint action distribution is factorized and depends on a shared neural network feature $\varphi_t(s_t)$: $\pi(a_1, a_2, \ldots|s) = \pi(a_1|\varphi, s)\pi(a_2|\varphi, s)\ldots$ (omitting t and s_t for clarity). Any correlation between actions is modeled implicitly through φ_t. We also learn a value function $V(\varphi_t)$ for variance reduction purposes. We compare policies trained using policy gradients (Williams, 1992) or PPO Schulman et al. (2017). See Appendix D for further training details. Both simulation and RL ran on a GPU with WarpDrive (Lan et al., 2021), see Appendix F.

Intuition for Structured Curricula. We define curricula based on these observations about our multi-agent learning problem: 1) during exploration, many actions may reduce utility, while few actions increase utility, 2) high prices or tax rates can eliminate all gains in utility, even though the consumer-worker did not change its policy, 3) for stable learning, agents should not adapt their policy too quickly when experiencing large negative (changes in) utility, and 4) in a Stackelberg game, the followers (e.g., consumers, firms) should get enough time to learn their best response to the leader's policy (e.g., the government). We now operationalize these intuitions below.

Staged Learning and Action Space Annealing. All policies are randomly initialized. We first allow consumers to train, without updating the policies of other agents. Initially, firm and government actions are completely fixed; prices and wages start at non-zero levels. We then anneal the range of firm actions without training the randomly initialized policy. This allows consumers to learn to

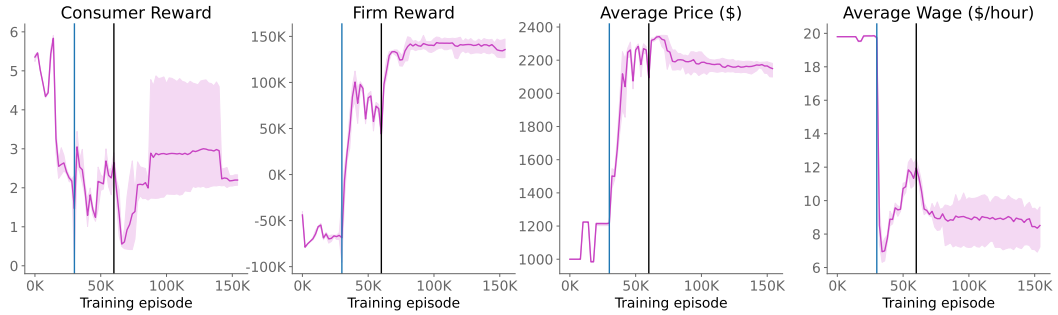

Figure 3: **Structured Curricula and Training, Open Economy, 3 runs.** In each plot, the blue (black) vertical line shows when the firms (government) start training, see Figure 2. **Left two plots:** Consumer and firm rewards during training. All runs converged to an approximate equilibrium, as confirmed by a best-response analysis. The outcomes were chosen to be qualitatively similar across runs. Once firms start training, their reward significantly increases. When the government starts training, firms get even higher reward, as the social welfare definition includes the firm's objective (profits). **Right two plots:** Average wages and prices (over firms) during training. Firms increase prices rapidly and lower wages once they start training.

best respond to the full scope of prices and wages, without firms strategically responding. Once firm action annealing is complete, we allow the firm to train jointly with the consumers. We then perform the same process, gradually allowing the government to increase its corporate and income tax rates, so that firms and consumers can react to a wide range of tax rates. Once the annealing process is complete, we allow the government to train to maximize welfare.

Penalty Coefficient Annealing. In addition to the action annealing, we anneal two penalty coefficients. First, we slowly increase the consumers' disutility of work over time, which avoids disincentivizing work early in the training process. Second, as each agent starts training, we assign a high value (0.5) for the entropy coefficient in their policy gradient loss, and gradually anneal it down over time to a minimum of 0.01. This ensures that when the firm or government policies start training, their "opponent" policies are able to learn against them without being too quickly exploited.

4 Learning RBC Equilibria and Method Validation

We show that our approach is sound and can explicitly find spectra of approximate RBC equilibria. We study variations of our RBC model with 100 consumers and 10 firms. For all simulation parameter settings, see Table 1 and Appendix C. We repeat all experiments with 3 random seeds.

Structured Curricula during Training. Figure 3 shows a representative result of training using our structured curricula. Although all RL agents aim to improve their objective, some agents may see it decrease as the system converges towards an equilibrium, e.g., the consumers in Figure 3. Empirically, we found that training collapses to trivial solutions much more often without curricula.

Best Responses and Local Equilibrium Analysis We abstract the RBC as a normal-form meta-game between three players representing the agent types, i.e., the consumers, firms, and government. We test whether a set of learned policies is an ϵ-Nash equilibrium for the meta-game by evaluating whether or not they are approximate best responses. Recall that agents of the same type share policy weights (e.g., consumers), but use agent-specific inputs to their policy, To find an approximate best-response, we train each agent type separately for a significant number of RL iterations, holding other agent types fixed, and see if this improves their reward by more than ϵ. If so, the policies are not an equilibrium; if not, this suggests we found at least a *local* equilibrium. In general, we find empirically that best-responses improve rewards much more at the start compared to the end of training. Throughout, we report results where the policies have found an approximate equilibrium at the end of training.

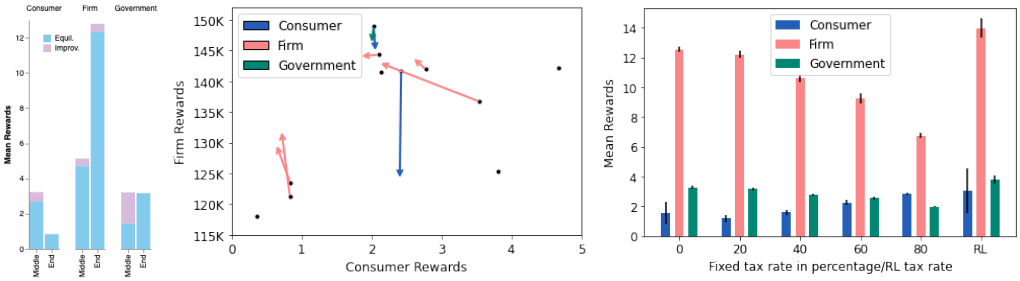

Figure 4: **Left: Best-Responses during Training.** The stacked bar chart shows the improvement in the mean rewards of an agent type (consumer, firm, and government, resp.) when only that agent type is training and the other agents policies are fixed. For firms and governments, the mean rewards are measured in units of 10^4 and 10^3, respectively. We compare the best response improvement in the middle and at the end of training. The improvement from best response is significant in the middle and much less at the end, indicating that training is closer to an equilibrium at the end. **Middle: Outcomes under Best-Response.** After training, typically neither consumer nor firm rewards change significantly under a best-response analysis. This holds generally for the approximate equilibria reported in this work. In the figure, we display only those which change by more than 1%. **Right: Comparing Fixed and RL Government Policies.** Using fixed tax rates for baselines, we measure the mean rewards for all agent types. Again, for firms and governments, the mean rewards are measured in units of 10^4 and 10^3, respectively. The mean rewards for the consumers increase with tax rates, whereas for the firms they decrease. An RL government can improve the mean reward for both consumers and firms simultaneously by appropriately adapting tax rates across rounds. RL policies increase the social welfare by almost 15% over the best baseline policy with a fixed tax rate.

Note that the meta-game over agent types is different from the game over individual agents. In particular, a meta-game best-response (where *all* agents of the same type can best-respond) may not be an individual's best-response. A meta-game best-response may feature competitive-coordination dynamics between agents of the same type, e.g., consumers, such that their collective behavior decreases the reward for their agent type. However, this may feature adverse behaviors for some individual agents, e.g., there may be free-riders that benefit from the collective performance while not putting in effort themselves. As we consider settings with a large number of agents (e.g., 100 consumers), we limit our analysis to meta-game approximate best-responses.

Comparing with Baseline Government Policies. To show that our approach is sound, we show that RL tax policies lead to improved social welfare compared with several different manually defined fixed tax rate policies. As observed in Figure 5, RL policies generate a social welfare ranging from 3000 to 4000, depending on the equilibrium (as noted above, multiple equilibria could be reached). Compared to that, the best social welfare achieved using fixed tax rates was 3160. Figure 4 shows the social welfare achieved under different various fixed tax rates. We note that social welfare improves by almost 15% for the best equilibrium under RL tax policy over the best equilibrium with fixed tax rates. This indicates that the RL policy is able to appropriately adjust the taxes across different rounds to improve average social welfare.

Exploring RBC Equilbria. Figure 5 visualizes and describes the learned RBC equilibria in open and closed RBC economies (with and without an export market). In both cases, we find multiple equilibria, although learning mostly converges to low-welfare equilibria in the closed economy. The open RBC model admits a wide spectrum of approximate equilibria with distinct outcomes (e.g., consumer utility, hours worked, prices, and taxes), and trends that align with economic intuitions. To study the differences between equilibria, Figure 6 shows a detailed comparison between two rollouts at two different equilibria. This reveals distinct equilibrium strategies, e.g., firms can profit by either focusing on consumers or the export market.

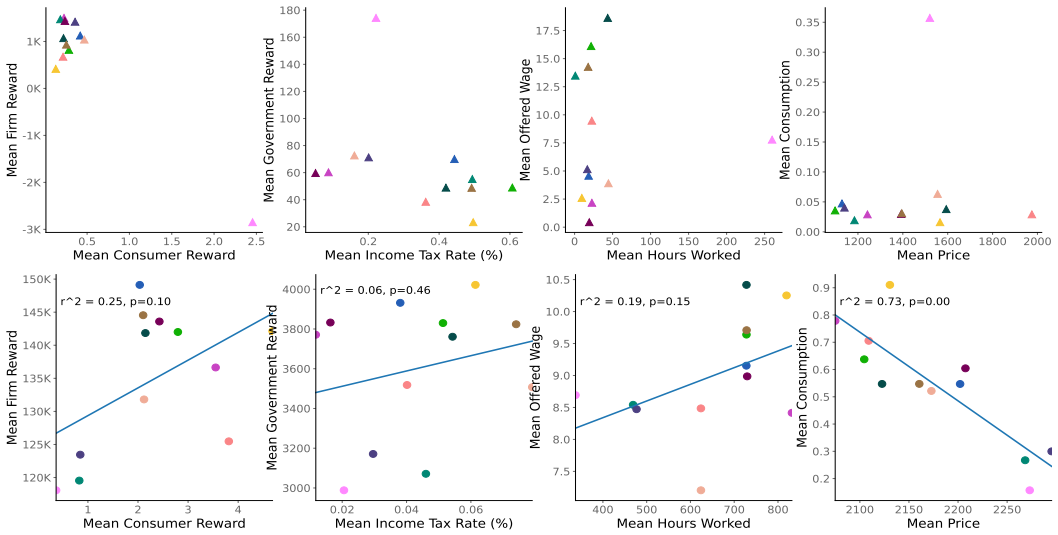

Figure 5: **Learned Equilbria in Open (Bottom) and Closed (Top) RBC Models.** Outcomes at convergence for the same experiments in Figure 4. Each point represents an approximate equilibrium, verified by an approximate best-response analysis. Points of the same color and shape correspond to the same run. **Top:** In the closed economy, training often converges to equilibria with low consumer reward and little economic production. In particular, social welfare (government reward) does not increases with higher tax rates, average labor does not change even if wages are increased, and consumption is across all prices. An exception is an equilibrium with significantly higher social welfare, labor, and consumption. This suggests multiple equilibria do exist, but non-trivial equilibria are harder to learn in the closed economy. **Bottom:** We learn multiple distinct, non-trivial equilibria in an open economy. We observe multiple interesting and explainable economic relationships. Blue lines show linear regressions to the data. Consumer and firm rewards are positively correlated ($r^2 = 0.25$), e.g., if consumers earn more, they can consume more, yielding higher profits. Furthermore, higher prices decrease mean consumption ($r^2 = 0.73$), while lower wages decrease mean hours worked ($r^2 = 0.19$). Finally, there is no strong signal that higher taxes improve social welfare ($r^2 = 0.06$).

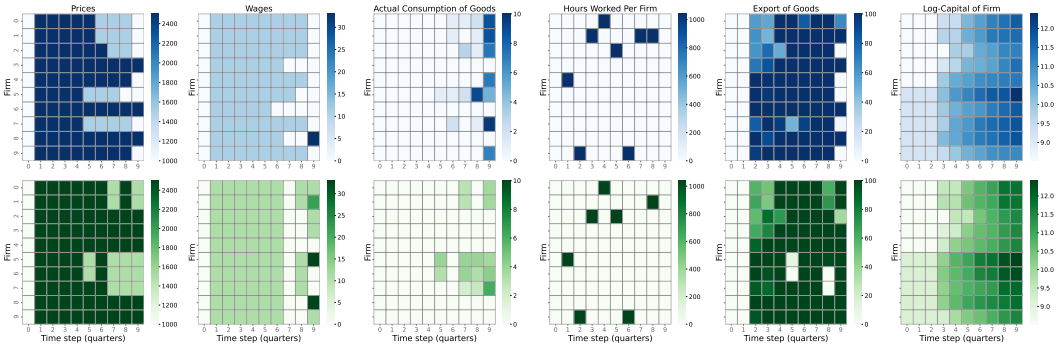

Figure 6: **Open Economy Rollouts, Two Different Equilibria.** We show actions and states for two representative runs at two qualitatively different equilibria. Heatmaps for the same metric use the same range of intensities, e.g., for prices in the top and bottom row. We observe that firms have different strategies: some set prices high and rely on exporting goods (for example, firm 3); others set prices lower and also sell to consumers (for example, firm 0). Consumers respond sensibly, only consuming when prices are low and only working when wages are not 0. The differences in initial firm endowments as well as evolution during the episode lead to different final capital levels.

5 Related Work

Computational Approaches to Find Equilibria. Leibo et al. (2017) studied meta-game equilbria in sequential social dilemmas, showing that RL policies can be categorized into high-level strategy types akin to cooperation or defection in the classic social dilemma. Counterfactual regret minimization has yielded superhuman poker bots (Brown & Sandholm, 2018), an imperfect-information game; however, it can be computationally inefficient. Empirical game-theoretic analysis studies equilibria through agent simulations (Wellman, 2006), but is limited to games with interchangable agents with identical affordances (Tuyls et al., 2018) and do not easily scale to settings with heterogeneous agents.

Higher-order gradients (Foerster et al., 2017) and first-order gradient adjustments (Balduzzi et al., 2018) have sought to promote convergence to non-trivial equilibria. However, these methods make strong assumptions on what agents know, e.g., that agents can see the policy weights of other agents and/or know the full reward function, and may deform the original equilibria.

RL for Economic Simulation. Agent-based modeling (ABM) studies economic simulations with individual agents. Early work mostly used rule-based agents or RL in finite state ABMs to study emergent phenomena (Bonabeau, 2002). A recent economics survey of ABMs (Haldane & Turrell, 2019) outlined the potential for RL agents, following its success in ML.

Several recent works have explored this idea. The AI Economist used RL to design optimal taxes to improve social welfare in high-dimensional simulations, where both agents and governments use RL policies (Zheng et al., 2020). RL also yields interpretable economic and public health policies in pandemic simulations (Trott et al., 2021). Danassis et al. (2021) takes a similar approach: they explore a multi-agent environment with harvesters participating in a common fishery, and a centralized price setter, and demonstrate that the price setter can outperform prices set by a market equilibrium.

Other contemporary work includes Chen et al. (2021), which studies monetary policy with a single representative household RL agent, and Hill et al. (2021), which learns the value function of consumers in RBC models. These works are more limited, compared to ours. First, they use RL for one agent type, while the other agents (e.g., firms) use simple and fixed policies. Second, they assume markets always clear at each time step, i.e., prices are set to ensure supply and demand are balanced. However, this is an unrealistic assumption and causes slow simulations, requiring solving a nonlinear optimization problem at each timestep. In our work, we do not enforce market clearing constraints.

RL and Stackelberg Games. In a Nash equilibrium, every player is playing a best response against every other player, so that no player can improve their outcome by unilaterally deviating. By contrast, in a Stackelberg game, one player is a leader who has the opportunity to move first; followers observe that move and choose their best response. As discussed in Zheng et al. (2020), there are aspects of Stackelberg games in economic simulations: a government who sets taxes or a firm setting wages and prices must make a choice anticipating a response by other agents. One popular setting for Stackelberg equilibria is in security games (An et al., 2017). In such games, the leader represents a defender who chooses where to deploy patrols; the attacker wishes to evade the patrols and knows the strategy chosen by the leader. Several works have studied the use of RL techniques in such settings (Wang et al., 2019; Trejo et al., 2016; Kamra et al., 2018; Bai et al., 2021).

6 Discussion

Daskalakis et al. (2009) showed that computing equilibria for general-sum games is hard in terms of computational complexity, even for simple matrix games. There is no theoretical guarantee that our framework can find *all* equilibria in sequential economic games. However, our best-response analysis suggests our framework does discover local equilibria. Hence, our current framework is at least an exploratory tool for finding qualitatively different equilibria, e.g., by varying initial conditions, environment parameters, or conditioning sampling, and provides exciting avenues for future research.

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A Reproducibility Statement

We discuss specific training details for all experiments such as hyperparameters in the Appendix. Upon acceptance, all code for this project will be made open source and publicly available for reproducibility purposes and further research. Upon request, we will share our code with reviewers and area chairs for review.

B Ethics Statement

Our work proposes a framework to model economies using Multi-Agent Reinforcement Learning and thus may be used to draw implications about the real world. Our findings and used simulations are purely for research purposes and should not be used to make decisions in real-world systems. Furthermore, our framework should not be used to explore methods to increase discrimination or unfairness in real-world systems.

C More Details on the RBC

Settings. Table 1 lists all the simulation parameters. Table 2 lists all the training hyperparameters.

Parameter	**Symbol**	**Values**
Labor disutility	θ	0.01
Pareto quantile function scale parameter	-	4.0
Initial firm endowment	B	2200000
Export market minimum price	-	500
Export market maximum quantity	-	100
Production function values	α	0.2 to 0.8, increments of 0.2
Initial capital	K	5000 or 10000
First round wages	w	0
First round prices	p	1000
Initial inventory	y	0

Table 1: **Simulation Parameters.**

More implementation details follow:

- For consumers, consumption choices range from 0 to 10 units for each good and work choices from 0 to 1040 hours in increments of 260.
- Consumers have a CRRA utility function with parameter 0.1, and a disutility of work of 0.01.

Parameter	Values
Learning Rate	0.001
Learning Rate (Government)	0.0005
Optimizer	Adam
Initial entropy	0.5
Minimum entropy annealing coefficient	0.1
Entropy annealing decay rate	10000
Batch Size	128
Max gradient norm	2.0
PPO clipping parameter	0.1 or 0.2
PPO updates	2 or 4
Consumer reward scaling factor	5
Firm reward scaling factor	30000
Government reward scaling factor	1000

Table 2: **Training Hyperparameters.**

- For firms, price choices range from 0 to 2500 in units of 500; wage choices from 0 to 44 in units of 11.
- The 10 firms are split into two groups, receiving either 5000 or 10000 units of capital. Within these groups, firms receive a production exponent ranging from 0.2 to 0.8 in increments of 0.2. Thus each firm has a different production "technology".
- Firms invest 10% of their available budget (if positive) in each round to increase their capital stock.
- Government taxation choices range from 0 to 100% in units of 20%, for both income tax and corporate tax rates.
- The government can either value only consumers when calculating its welfare ("consumer-only") or value welfare of both consumers and firms ("total"), with firm welfare down-weighted by a factor of 0.0025 (to be commensurate with consumers).
- We set the minimum price at which firms are willing to export to be either 500 or 1000, and the quota for each firm's good to a variety of values: 10, 50, 100, or 1000.
- For consumers, consumption choices range from 0 to 10 units for each good and work choices from 0 to 1040 hours in increments of 260.

Observation. The environment *state* $\boldsymbol{s}$ consists formally of all agent states and a general world state. Each agent observes can observe their own information and the global state:

$$\boldsymbol{s}_{\text{global}} = \left(t, \{y_{i,t}\}_i, \{p_{i,t}\}_i, \{w_{i,t}\}_i, \{o_{i,t}\}_i\right). \tag{9}$$

Here $y_{i,t}$ is the available supply of good i, $p_{i,t}$ is the price, $w_{i,t}$ is the wage. The extra information $o_{i,t}$ includes whether good i was overdemanded at the previous timestep and tax information.

In addition, consumer-workers observe private information about their own state: $(B_{i,t}, \theta)$ A firm i also observes its private information: $(B_{i,t}, k_{i,t}, (0, \ldots, 1, \ldots, 0), \alpha)$, including a one-hot vector encoding its identity and its production function shape parameter α. The government only sees the global state.

D Training Details

For all experiments, we clip gradient ℓ_2 norms to a maximum of 2.0, use a Huber loss for the value function head, and center and standardize estimated advantages. For PPO, we use the clipping based surrogate loss approach rather than a KL penalty, closely following the baseline implementation of Kostrikov (2018).

E IMPLEMENTATION DETAILS

Budget Constraints. We implement budget constraints on the consumers by proportionally scaling down the resulting consumption of all goods to fit within a consumer's budget. Thus, the consumer actions really represent attempted consumption – if the budget is small or stock is limited, the actual consumption enjoyed by the consumer may be lower. Firm budgets are allowed to go negative (borrowing money). However, because the firm's goal is to maximize profit, they are incentivized to take actions will will be profitable, increasing their budget.

Scaling of Observables. The scales of rewards and state variables can vary widely in our simulation, even within time steps in a single episode. If the scales of loss functions or input features are very large or small, learning becomes difficult. We directly scale rewards and some state features by constant factors. For certain state features which have very large ranges (item stocks and budgets) we encode each digit of the input as a separate dimension of the state vector.

F GPU IMPLEMENTATION

We followed the WarpDrive framework (Lan et al., 2021) to simulate our DGE and do RL on a single GPU. We implemented the environment dynamics of our model as a CUDA kernel in order to increase the speed at which we can collect samples. We assigned one thread per agent (consumer, firm, or government); the threads communicate and share data using block-level shared memory. Multiple environment replicas run in multiple blocks, allowing us to reduce variance by training on large mini-batches of rollout data. We use PyCUDA (Klöckner et al., 2012) to manage CUDA memory and compile kernels. The policy network weights and rollout data (states, actions, and rewards) are stored in PyTorch tensors; the CUDA kernel reads and modifies these tensors using pointers to the GPU memory, thereby working with a single source of data and avoiding slow data copying.