STRUCTURED ENERGY NETWORK AS A DYNAMIC LOSS FUNCTION
A CASE STUDY WITH MULTI-LABEL CLASSIFICATION

Anonymous authors
Paper under double-blind review

ABSTRACT

We propose SEAL which utilizes this energy network as a trainable loss function for a simple feedfoward network. Structured prediction energy networks (SPENs) (Belanger & McCallum, 2016; Gygli et al., 2017) have shown that a neural network (i.e. energy network) can learn a reasonable energy function over the candidate structured outputs. We find that rather than using SPEN as a prediction network, using it as a trainable loss function is not only computationally efficient but also results in higher performance. compared to SPENs in both training and inference time. As the energy loss function is trainable, we propose SEAL to be dynamic which can adapt energy function to focus on the region where feedforward model will be affected most. We find this to be effective in ablation study comparing SEAL to the static version (§5) where energy function is fixed after pretraining. We show the relation to previous work on the joint optimization model of energy network and feedforward model (INFNET) as we show that it is equivalent to SEAL using margin-based loss if INFNET relaxes their loss function. Based on the unique architecture of SEAL, we further propose a variant of SEAL that utilizes noise contrastive ranking (NCE) loss that by itself does not perform well as a structured energy network, but embodied in SEAL, it shows the greatest performance among the variants we study. We demonstrate the effectiveness of SEAL on 7 feature-based and 3 text-based multi-label classification datasets. The best version of SEAL that uses NCE ranking method achieves close to +2.85, +2.23 respective F1 point gain in average over cross-entropy and INFNET the feature-based datasets, excluding one outlier that has an excessive gain of +50.0 F1 points. Lastly, examining whether the proposed framework is effective on a large pre-trained model as well, we observe SEAL achieving +0.87 F1 point gain in average on top of BERT-based adapter model on text datasets.

1 INTRODUCTION

Structured prediction is a popular machine learning task wherein the model learns a mapping function from an input $\mathbf{x}$ to a multivariate-structured output $\mathbf{y}$. Popular examples of this include image segmentation (Müller, 2014), extracting parse trees or semantic role labels from a text (Palmer et al., 2010), and multi-label classification (Belanger & McCallum, 2016; Gygli et al., 2017). In structured prediction, the output space $\mathcal{Y}$ is often extremely large. For example, in multi-label classification, the size of $\mathcal{Y}$ is 2^L where model needs to predict output $\mathbf{y} \in \{0,1\}^L$. There are two key aspect that a model for structured prediction needs to balance: statistical efficiency and computational efficiency. Most models for structured prediction can be categorized into two categories: the feed-forward approach wherein one learns a neural network that models the direct mapping between the input and the structured output and the structured approach in which the model explicitly models the interactions (structure) in the output space.

The feed-forward approach learns all dimensions of $\mathbf{y}$ jointly in a conditionally independent manner given input $\mathbf{x}$, relying on the representational power of the network to capture the dependencies of output structure implicitly. While the feed-forward approach is computationally efficient, since it does not capture the relationships in the label space, it lacks statistical efficiency. The traditional structured approaches, on the other hand, model a joint probability distribution $P(\mathbf{x}, \mathbf{y})$ that can capture the label relationships. However, due to the intractability of modeling full joint distribution which captures

every possible interaction of output space, these approaches resort to limiting the interaction terms to local subsets of the output space (Lafferty et al., 2001; Ghamrawi & McCallum, 2005). In order to make the inference for structured models more efficient, a recent line of work replaces the joint probability with a structured energy function $E(\mathbf{x}, \mathbf{y})$ (LeCun et al., 2006), which can be thought of as unnormalized probability distribution, allowing the model to learn arbitrary global dependencies in the output space. There have been several works that propose efficient approximate inference procedures for structured energy network using gradient based inference (Belanger et al., 2017; Gygli et al., 2017; Rooshenas et al., 2019), hence also called 'prediction network'. These models showed noticeable gains in predictive performance over feed-forward models and graphical models that assume partial structure such as full pairwise potential (Chen et al., 2015; Schwing & Urtasun, 2015). Despite these efforts, the inference for energy-based models still remains relatively inefficient when compared to feed-forward approach. Moreover, the models using gradient based inference (GBI) are finicky to train. This raises a question: *Can the energy network be used in a way that is as expressive as a full joint probability, as efficient at inference as well as a feed-forward approach, and also stable and easy to train?* We believe that using structured energy networks as a parameterized dynamic loss functions for feed-forward networks, instead of a prediction network, can fulfil all these requirements.

In this paper, we propose a framework called **S**tructured **E**nergy **A**s **L**oss, or SEAL, that uses the trainable structured energy network (SEN) [1] as a loss function guiding the training of a feed-forward network. The key idea is to provide the feed-forward network access to rich relationships in the output space through a learned loss function, and do this in a dynamic manner by updating the loss based on the current outputs of the feed-forward network. We show that learning the loss function dynamically, is efficient, stable and produces the best performance. SEAL can be viewed as general-purpose framework where one can plugin various loss functions and architectures to train SEN as well as the feed-forward network. Through experiments (§4), we compare the impact of using different losses (eg. margin-based, regression-based, etc.) to update the SEN. We identify the benefits and shortcoming of each and propose the noise-contrastive ranking loss (NCEranking) which works best with the SEAL framework.

To summarize, we introduce a general framework SEAL that interprets structured energy networks (SEN) as a dynamic loss functions. Through empirical evaluation on the task of multi-label classification, we analyse the impact of various loss functions for updating SEN. Finally, we propose a NCEranking loss that is uniquely suited for the SEAL framework, and demonstrate its superior performance on 7 feature-based as well as 3 text-based multi-label classification datasets, when compared to simple feed-forward approach and various energy based models.

2 STRUCTURED ENERGY NETWORK AS LOSS (SEAL)

This section first describes the proposed framework **S**tructured **E**nergy network **A**s **L**oss (SEAL). Then, we introduce various energy-network-learning objectives that can be plugged into SEAL. Let $\mathcal{X}$ denote the input space, $\mathcal{Y} = \{0, 1\}^L$ the output space, and $\tilde{\mathcal{Y}} = [0, 1]^L$ the continuous relaxation of $\mathcal{Y}$. Then the structured energy $E_\Theta : \mathcal{X} \times \tilde{\mathcal{Y}} \rightarrow \mathbb{R}$ is defined to be a parameterized function of the input and the continuous relaxation of the output. The feedforward network $F_\Phi : \mathcal{X} \rightarrow \tilde{\mathcal{Y}}$ is defined to be a neural network that maps an input to the continuous relaxation of the output space. We denote j-th training instance pair as $(\mathbf{x}^{(j)}, \mathbf{y}^{(j)}) \in \mathcal{D}$ and $y_i \in \{0, 1\},\ \tilde{y}_i \in [0, 1]$ to denote i-th label dimension where $i \in \{1, \ldots, L\}$.

As shown in Figure 1, SEAL consists two loss functions: the energy loss L_E that trains structured energy network (Θ), and feedforward loss L_F that guides the training of the feedforward network (Φ). We first discuss how structured energy network implicitly affects training of the feedforward network by defining $\mathrm{L}_F(\Phi)$.

Given a training instance $(\mathbf{x}, \mathbf{y})$ the feedforward loss is defined as:

$$\mathrm{L}_F(\Phi) = \lambda_1 E_\Theta(\mathbf{x}, F_\Phi(\mathbf{x})) - \lambda_2 \sum_{j=1}^{L} \left[y_i \log F_\Phi(\mathbf{x})_i + (1 - y_i) \log \left(1 - F_\Phi(\mathbf{x})_i\right) \right] . \tag{1}$$

[1] The term comes from structured prediction energy network, minus 'prediction', as we do not use it for prediction anymore. For brevity, we use terms 'energy network' and 'strcutured energy newtork' interchangeably.

Here, the first term involving the energy captures interaction across label space whereas binary cross-entropy simply considers the predictions for each label independently.

Since the training of feedforward network depends on the quality of the parameterized energy network, it is critical to find the parameters Θ that produce the best loss surface for training the feedforward network. This can be done in two ways: by first training the energy separately, or by training the energy and the feedforward network simultaneously. SEAL uses the latter, which is shown to perform better (see §5) than the former, which we denote as as SEAL-static for distinction .

In SEAL-static, we first estimate the energy network parameter $\widehat{\Theta}$ over training data and optimize Φ by plugging $\widehat{\Theta}$ into L_F. Given $(\mathbf{x}, \mathbf{y}) \in \mathcal{D}$, Φ is trained as equation 2,equation 3.

$$\min_{\Phi} \frac{1}{|\mathcal{D}|} \sum_{\mathcal{D}} \mathrm{L}_F(\Phi) \tag{2}$$

$$\text{s.t.} \qquad \widehat{\Theta} = \arg\min_{\Theta} \frac{1}{|\mathcal{D}|} \sum_{\mathcal{D}} \mathrm{L}_E(\mathbf{x}, \mathbf{y}, \tilde{\mathbf{y}}; \Theta). \tag{3}$$

We finally present SEAL which learns $\widehat{\Theta}_t$ dynamically at step t. To do so, we alternate the optimization steps of L_E eq.(3) and L_F eq.(2). Given a training instance $(\mathbf{x}, \mathbf{y}) \in B_t$, a complete training step is given as:

$$\Theta_t \leftarrow \Theta_{t-1} - \nabla_{\Theta} \frac{1}{|B_t|} \sum_{B_t} \mathrm{L}_E \left(\mathbf{x}, \mathbf{y}, F_{\Phi_{t-1}}(\mathbf{x}); \Theta\right) \tag{4}$$

$$\Phi_t \leftarrow \Phi_{t-1} - \nabla_{\Phi} \frac{1}{|B_t|} \sum_{B_t} \mathrm{L}_F(\Phi) \tag{5}$$

Note that, from (3) to (4), arbitrary $\tilde{\mathbf{y}}$ got replaced with $F_{\Phi_{t-1}}(\mathbf{x})$ in estimating Θ_t so that estimation of energy surface depends on model output $F_{\Phi_{t-1}}(\mathbf{x})$. Further, the update step for feedforward network Φ relies on Θ_t rather than static $\widehat{\Theta}$. In both SEAL and SEAL-static, test-time predictions are performed solely using the feedfoward network as $\hat{y}_i = \mathbf{1}(F_{\Phi}(x)_i \geq 0.5)$.

The motivation for proposing dynamic loss function in SEAL is as following. It would be useful to learn perfect energy surface, if possible, by estimating $\widehat{\Theta}$ that is static. However, doing so is challenging as we are estimating energy surface on the joint space of $\mathbf{x}, \tilde{\mathbf{y}}$, a high-dimensional countinuous space with limited data and resource. Instead, we hypothesize that it is more important to concentrate resource in depicting accurate energy surface around the input, output pair $(\mathbf{x}, F_{\Phi}(x))$. In SEAL, utilizing the fact that we have access to trainable loss function, we dynamically adapt energy surface so that the region of interest, which involves most significantly in training feedforward, network are well represented.

We now discuss various energy loss (margin-based, regression-based, and constrastive-sampling-based) that can be plugged into L_E in SEAL. For brevity, we use $\mathrm{L}_E(\Theta)$ and $\mathrm{L}_E(\mathbf{x}, \mathbf{y}, \tilde{\mathbf{y}}; \Theta)$ interchangeably where again $(\mathbf{x}, \mathbf{y})$ denotes labeled data and $\tilde{\mathbf{y}}$ denotes probability vector. We defer the description of specific energy network structure to the experiment section (§4) as SEAL framework can work with arbitrary network structures.

Margin-based ($\mathbf{L}_{E-\text{margin}}$) SPEN (Belanger & McCallum, 2016) learned structured energy network with SSVM loss (Taskar et al., 2004; Tsochantaridis et al., 2004) so that energy functions learn to have sufficient energy difference, larger than the margin $\Delta(\tilde{\mathbf{y}}, \mathbf{y})$, between arbitrary output $\tilde{\mathbf{y}}$ and true output $\mathbf{y}$. To examine the effect of the margin-based loss in SEAL, we follow SPEN and utilize the SSVM loss as $\mathrm{L}_{E-\text{margin}} = \sum_{\mathbf{x},\mathbf{y}} \max_{\tilde{\mathbf{y}}} \left[\Delta(\tilde{\mathbf{y}}, \mathbf{y}) - E_{\Theta}(\mathbf{x}, \tilde{\mathbf{y}}) + E_{\Theta}(\mathbf{x}, \mathbf{y})\right]_+$.

Regression-based ($\mathbf{L}_{E-\text{regression}}$) Deep Value Network (DVN) (Gygli et al., 2017) attempts to learn an energy network which directly outputs a score that is similar to the metric $s(\tilde{\mathbf{y}}, \mathbf{y})$ of interest, such as F1 score, that compares arbitrary $\tilde{\mathbf{y}}$ with true output $\mathbf{y}$. Following DVN, by making score $s(\cdot)$[2]

[2]In this paper, we adopt soft F1 score $s(\tilde{\mathbf{y}}, \mathbf{y})$ from Gygli et al. (2017) that is defined on the continuos $\tilde{\mathbf{y}} \in [0, 1]^L$.

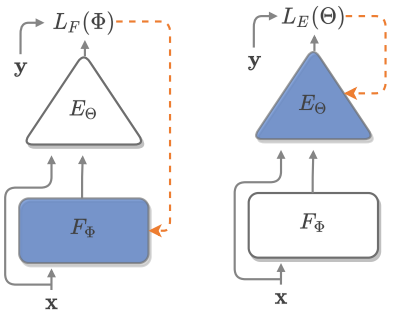


Figure 1: Overview of the SEAL framework. The figure on the right shows the update to the energy network (equation 5) and the figure on the left shows the update to the feed-forward network (equation 4)

Algorithm 1: SEAL Algorithm

Require: $(\mathbf{x}, \mathbf{y})$: Training Instance
Require: F_Φ: Feedforward Network
Require: `sampling`: True/False flag
Require: optimizer$_\Theta$, optimizer$_\Phi$
Require: T: No. of steps
  $t \leftarrow 0$
  $\Theta_0 \leftarrow$ Kaiming Initialization (Uniform)
  $\Phi_0 \leftarrow$ Kaiming Initialization (Uniform)
  while $t < T$ **do**
    if `sampling` **then**
      $S \leftarrow \{\mathbf{y}^{(i)}, i = 1, \ldots, K | \mathbf{y}^{(i)} \sim F_{\Phi_t}(\mathbf{x})\}$
    else
      $S \leftarrow \{F_{\Phi_t}(\mathbf{x})\}$ ▷ singleton set
    end if
    $t \leftarrow t + 1$
    Update Θ as Eqn. 4 using $\tilde{\mathbf{y}} \in S$ and optimizer$_\Theta$
    Update Φ as Eqn. 5 using optimizer$_\Phi$
  end while

and $-E(\cdot)$ to be between $[0, 1]$, we express regression loss as cross entropy

$$\mathrm{L}_{E-\text{regression}} = -s(\tilde{\mathbf{y}}, \mathbf{y}) \log -E_\Theta(\mathbf{x}, \tilde{\mathbf{y}}) - (1 - s(\tilde{\mathbf{y}}, \mathbf{y})) \log (1 + E_\Theta(\mathbf{x}, \tilde{\mathbf{y}})) \,. \tag{6}$$

Noise-contrastive ranking ($\mathrm{L}_{E-\text{NCEranking}}$ and $\mathrm{L}_{E-\text{ranking}}$) In SEAL, we are interested in capturing the output region that feedforward network has a high probability. We ask, rather than just taking a single point $\tilde{\mathbf{y}} = F_\Phi(\mathbf{x})$ from a feedforward network, whether sampling many discrete binary vectors from $\tilde{\mathbf{y}}$ and taking all those samples into consideration for learning E_Θ could lead in better estimation of energy surface. Motivated from noise contarstive estimation (NCE) (Ma & Collins), we ask whether energy network E_Θ trained to contrast the true output $\mathbf{y}$ from groups of K negative samples drawn from feedforward output $F_\Phi(\mathbf{x})$ can induce a good loss for teaching Φ. The intuition is as the $\tilde{\mathbf{y}} = F_\Phi(\mathbf{x})$ becomes better and better, the loss surface teaching feedforward network should become more fine grained as $\tilde{\mathbf{y}}$ and samples from $\tilde{\mathbf{y}}$ will be already close to the true $\mathbf{y}$.

Before we discuss $\mathrm{L}_{E-\text{ranking}}$ of our choice, we first review original form of NCE ranking loss from Ma & Collins. For K samples $\mathbf{y}^{(k)} \sim P_N,\ k = 1, \ldots, K$ drawn from noise distribution P_N and rewriting $\mathbf{y}$as $\mathbf{y}^{(0)}$ without loss of generality, NCE ranking loss is defined as

$$\log \frac{\exp\left(s(\mathbf{x}, \mathbf{y}^{(0)}; \Theta)\right)}{\sum_{k=1}^{K} \exp\left(s(\mathbf{x}, \mathbf{y}^{(k)}; \Theta)\right)}, \quad \text{where} \quad s(\mathbf{x}, \mathbf{y}; \Theta) = -E_\Theta(\mathbf{x}, \mathbf{y}) - \log P_{\mathrm{N}}(\mathbf{y}). \tag{7}$$

Minimization of this loss makes $P(y|x; \Theta) = \frac{\exp(-E_\Theta(x,y))}{\sum_{y \in \mathcal{Y}} \exp(-E_\Theta(x,y))}$ as an unbiased estimator of true distribution $P(Y|X)$. Thus, minimizing $-E_\Theta(\mathbf{x}, \mathbf{y})$ with respect to $\mathbf{y}$ in $\mathrm{L}_F$would be equivalent to maximizing the estimate of $-\log P(\mathbf{y}|\mathbf{x})$.

From the presented NCE ranking method, we propose to use $P(\mathbf{y}|\mathbf{x}; \Phi_t) = \prod_i P(\mathbf{y}_i|\mathbf{x}; \Phi_t)$ in place of $P_N(\mathbf{y})$. The novelty of this proposal is that we view the output of feedforward network that we train as a noise distribution, and that the energy model that constrasts noise distribution teaches the noise distribution as well through our SEAL framework (equation 4). In short, we get $\mathrm{L}_{E-\text{NCEranking}}$ when we plug in $s(\mathbf{x}, \mathbf{y}; \Theta) = -E_\Theta(\mathbf{x}, \mathbf{y}) - \log P_\Phi(\mathbf{y}|\mathbf{x}; \Phi_t)$ into equation 7 , i.e .

$$\log \frac{\exp\left(s(\mathbf{x}, \mathbf{y}^{(0)}; \Theta)\right)}{\sum_{k=1}^{K} \exp\left(s(\mathbf{x}, \mathbf{y}^{(k)}; \Theta)\right)}, \quad \text{where} \quad s(\mathbf{x}, \mathbf{y}; \Theta) = -E_\Theta(\mathbf{x}, \mathbf{y}) - \log P_\Phi(\mathbf{y}|\mathbf{x}; \Phi_t). \tag{8}$$

This achieves two benefits that we have in mind in using NCE. First benefit is that it captures the region of interest by sampling from the output distribution of Φ_t. Second benefit is it brings more efficient NCE method. NCE method is known to work best when the noise distribution is close to

the data distribution (Gutmann & Hirayama, 2012) but not exactly the same. In our SEAL learning procedure, we hyptothesize that $P(\mathbf{y}|\mathbf{x};\Phi_t)$ becomes closer to the data distribution and stabilizes to a certain distribution as it starts to converge.

Lastly, motivated that NCE can estimate the true distribution, we propose yet another loss function that can estimate the difference between the true probability $P(\mathbf{y}|\mathbf{x})$ and feedforward network probability. We show in appendix A, if we use plain ranking loss that sets $s(\mathbf{x},\mathbf{y};\Theta) = -E_\Theta(\mathbf{x},\mathbf{y})$ in equation 7, unbiased estimator of $P(\mathbf{y}|\mathbf{x})$ now becomes $\frac{P_N(\mathbf{y})\exp(-E_\Theta(\mathbf{x},\mathbf{y}))}{\sum_{\mathbf{y}\in\mathcal{Y}} P_N(\mathbf{y})\exp(-E_\Theta(\mathbf{x},\mathbf{y}))}$. This means if we take a derivative of $E_\Theta$with respect to $\mathbf{y}$, then we are taking derivative with respect to $-\log P(\mathbf{y}|\mathbf{x}) + \log P_N(\mathbf{y})$. With the trick of replacing the $P_N(\mathbf{y})$ with $P(\mathbf{y}|\mathbf{x})$ in SEAL, we hypothesize that loss captured by energy network can focus only on the difference between true and feedforward probability which maybe end up being more informative surface. In short, we get plain ranking loss $\mathrm{L}_{E-\text{ranking}}$ by plugging $s(\mathbf{x},\mathbf{y};\Theta) = -E_\Theta(\mathbf{x},\mathbf{y})$ into equation 7 , which results in equation 9 ,

$$\log \frac{\exp\left(-E(\mathbf{x},\mathbf{y}^{(0)};\Theta)\right)}{\sum_{k=1}^{K}\exp\left(-E(\mathbf{x},\mathbf{y}^{(k)};\Theta)\right)}. \tag{9}$$

3 Related Work

Learning with dynamic loss: Wu et al. (2018) and Huang et al. (2019) attempts to learn a dynamic loss defined by neural network that is tailored to the metric used for the task. Both these models try to to learn a loss function that directly increases task-specific metric , such as BLEU or 0-1 accuracy, by evaluating it on validation set. Since metric m is usually non-differentiable Huang et al. (2019) views this problem as a reinforcement learning problem and attempts to learn a loss function that maximizes the expected reward. On the other hand, Wu et al. (2018) relaxes the metric function to be differentiable and views the feed-forward model update and loss-model update as a connected chain of computation graph, so that loss function parameter (Θ) gets updated in relation to feed-forward model (Φ) improving the relaxed metric score.

There are three major differences between these models and SEAL. First, energy surface of SEAL is not limited to a specific parametric form, like $\sigma(\mathbf{y}^T\Theta\log F_\Phi(x))$ for Wu et al. (2018) [3]. Instead, SEAL simply uses a scalar valued neural network to express loss function $-E_\Theta(\cdot)$ which allows it to use a much larger function class. Second, these models require manual design of state vector, for example the state vector used by Wu et al. (2018) consists of iteration number, validation accuracy, and current loss-function parameter Θ, whereas SEAL alleviates this requirement by using the complete output of the feed-forward network. Lastly, these models do were designed for multi-class classification, and it is non-trivial to extend them for tasks with extremly large label space like structured prediction.

Structured Prediciton Energy Networks: Structured prediction energy networks (Belanger & McCallum, 2016) and its variants (Gygli et al., 2017; Rooshenas et al., 2019) (referred to as SPENs from here on) learn an energy network $E_\Theta : \mathcal{X}\times\tilde{\mathcal{Y}} \to \mathbb{R}$ and finds $\mathbf{y} = \arg\min_y E(\mathbf{x},\mathbf{y})$ by applying gradient-based inference (GBI). To train energy network, SPENs apply GBI adversarially in order to find $\mathbf{x}$, $\mathbf{y}$pair that violated the minimality of the energy the most. We found this training procedure to be a bit unstable as its success depends on the initial point of GBI. We discuss detailed form of loss function for SPEN (Belanger & McCallum, 2016) and DVN (Gygli et al., 2017) in §2 and provide these as baselines in this work.

Jointly learning Θ and Φ: The Inference network (INFNET) proposed in Tu & Gimpel (2018) is the first work that tried to learn structured energy network and feed-forward network jointly. As the name suggests, INFNET is focused in bringing efficient inference-time feedforward network that can mimic behaviour of SPEN while bypassing expensive gradient-based inference at test time. However, just like SPENs, INFNET also resorts to finding adversarial point that has the maximum energy violation. This results in adversarial loss similar to GAN (Goodfellow et al., 2014) where INFNET maximizes and energy network minimizes the SSVM loss. While the end goal of INFNET is in

[3]The papers (Wu et al., 2018; Huang et al., 2019) use Φ instead of Θ. We change it to Θ to put notations in consistent manner with this paper.

Dataset	Domain	#Instances			#Labels	Input Type	Label Taxonomy
		Train	Val	Test			
Expr FUN	Gene Ontology	1636	849	1288	500	Continuous	Forest
Spo FUN	Gene Ontology	1600	837	1266	500	Continuous	Forest
Bibtex	Text	4407	1491	1497	159	Binary	-
Cal500	Music	283	105	114	174	Continuous	-
Delicious	Text	9690	3207	3194	983	Binary	-
Genbase	Biology	398	132	132	27	Binary	-
Eurlex-ev	Text	11557	3876	3881	3993	Binary	-
BGC	Text	58715	14785	18394	142	Raw Text	Forest
RCV	Text	13890	9260	781265	104	Raw Text	DAG
NYT	Text	175299	177067	180659	2109	Raw Text	-

Table 1: Statistics of the datasets used in the experiments.

efficient inference for finding output with lowest energy, it also has to serve as the adversarial sampler for training energy network. Having to serve two roles, INFNET has train-, test-time objective mismatch as (Tu et al., 2020), which is resolved in Tu et al. (2020) by using augmented INFNET structure, wherein two separate feed-forward networks are used, one for prediction and the other as the adversarial sampler. However, INFNET's reliance on margin-based loss still leaves train-, test-time mismatch. In contrast, SEAL has completely independent loss functions for updating the energy and feed-forward network. On feed-forward (Φ) update, SEAL simply update the model in the direction that reduces energy defined by structured energy network. INFNET is a special case of SEAL (i.e. SEAL with $\mathrm{L}_{E-\text{margin}}$) if the margin is removed from the SSVM loss for the feedforward model udpate. In fact, Tu et al. (2020) reports that this special case leads to more efficient and stable learning of feedforward network in their ablation study. We use this special case of INFNET to represent SEAL with $\mathrm{L}_{E-\text{margin}}$in the main experiment.

4 EXPERIMENTS (RESULTS)

In our experiments, we use 10 multi-label classification datasets (shown in Table 1) covering varied label space, size, as well as input characteristics. The first 7 are small and medium sized feature based dataset, while the last three Blurb Genre Collection (BGC), RCV (Reuters 1), and New York Times (NYT) are large datasets with raw text as the input.[4]

Energy Network: We use the same structure for the energy network $E_\Theta(x, y)$ as described in Belanger & Mccallum (2016), wherein the energy is the sum of local $E_\Theta^{\text{local}}(x, y)$ and global $E_\Theta^{\text{global}}(x, y)$ energies defined as:

$$E_\Theta^{\text{local}}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{L} y_i \mathbf{b}_i^\top T_E(\mathbf{x}) \ , \qquad E_\Theta^{\text{global}}(\mathbf{x}, \mathbf{y}) = \mathbf{v}^\top \sigma(\mathbf{M}\mathbf{y}), \tag{10}$$

where $\mathbf{b}_i, \mathbf{v}, \mathbf{M}, T_E$ contain learnable parameters, $\sigma(z) = \log(1 + e^z)$ is the softplus activation function, and T_E is a feature network for structured energy network.

Feature Network: We use the same feature network structure for both the energy and feedforward network denoted as T_E, T_F respecitvely. For the the feature-based datasets, the feature network $T : \mathbb{R}^d \to \mathbb{R}^h$ consists of multi-layer perceptron with with softmax non-linearities. For the raw text based datasets, we use pre-trained BERT (Devlin et al., 2019) with adapter (Houlsby et al., 2019; Pfeiffer et al., 2020) as the feature network.

Feedforward Network: Given a feature network T_F that generates features in $\mathbb{R}^h$, the feedforward network $F_\Phi(x) = \mathbf{G}T_F(x)$, where $\mathbf{G} \in \mathbb{R}^{L \times h}$ is a matrix consisting of learnable embeddings, one row of embedding for each label.

[4]Due to their enormous size, we use a subset of NYT training set, which is still the largest dataset in our experiments.

method	samples	discrete input datasets			continuous input datasets			
		bibtex	**delicious**	**genbase**	**cal500**	**eurlexev**	**expr_fun**	**spo_fun**
cross-entropy	x	42.40	30.16	47.37	33.58	42.19	37.48	27.97
energy only								
SPEN	x	**42.99**	26.76	32.50	39.36	**41.75**	**36.43**	27.20
DVN	x	42.73	**29.59**	**78.43**	**48.21**	28.90	32.19	**29.97**
NCE	o	17.93	16.78	20.64	38.87	0.19	27.12	19.33
SEAL								
margin	x	42.86	29.75	96.53	36.69	41.83	37.91	28.43
regression	x	43.74	29.79	96.95	37.97	41.65	38.12	28.89
regression-s	o	44.53	29.87	96.81	38.95	42.32	37.84	28.02
NCEranking	o	**44.76**	34.79	**97.32**	**41.62**	**42.78**	**38.21**	28.70
ranking	o	44.20	**36.04**	96.60	40.71	**42.78**	38.03	**29.68**

Table 2: Performance of SEAL framework (row 5-9) compared to feedforward network trained with cross-entropy (row 1) and sturctured energy networks (row 2-4) that are learned with $\mathrm{L}_E$described in earlier section. We observe feedforward network learned with SEAL is almost always better than cross-entropy learned model. We also observe while $\mathrm{L}_{E-\text{margin}}$and $\mathrm{L}_{E-\text{regression}}$does not show significant difference, $\mathrm{L}_{E-\text{NCEranking}}$and $\mathrm{L}_{E-\text{ranking}}$has stronger performance in general.

Training: The models for feature-based dataset are trained on a single TitanX GPU with 14GB CPU memory and the models for BCG, RCV and NYT are trained on m40 GPU. We use separte ADAM optimizer (Kingma & Ba, 2014) for energy network (Θ) and for feedforward network (Φ) which optimizes parameters in alternating fashion[5]. For each minimization loop, we take n_E, n_F gradient steps respectively for estimating Θ_t and Φ_t. We leave these n_E, n_F as hyperparameter to be tuned. The best hyper-parameters for each model are found using Bayesian search[6] and report it in Appendix C.

4.1 Results for feature based datasets

The experiment results across 7 feature-based dataset are presented in Table 2. The table consists of feedforward network only trained with cross-entropy (row 1), energy networks (row 2-4) described in previous section that is evaluated with GBI, and lastly feedforward network trained within SEAL framework with the dispaly of $\mathrm{L}_E$type used (row 5-9).

SEAL outperforms cross-entropy loss (CE) on almost all datasets, avergae gains over CE ranging from +0.6 (SEAL-$\mathrm{L}_{E-\text{margin}}$) - +3 (SEAL-$\mathrm{L}_{E-\text{NCEranking}}$) F1 points in average when the CE results are around 30-50 F1. We excluded highest gain in computing the average. The highest gain came form genbase, where the gains are around +50 F1 points (near 100% improvement).

Among various L_Es, $\mathrm{L}_{E-\text{NCEranking}}$ and $\mathrm{L}_{E-\text{ranking}}$ performs the best when $\mathrm{L}_{E-\text{margin}}$ and $\mathrm{L}_{E-\text{regression}}$ performs similary in avergae. $\mathrm{L}_{E-\text{NCEranking}}$ and $\mathrm{L}_{E-\text{ranking}}$ are the only ones that always outperforms CE in all 7 datasets whereas $\mathrm{L}_{E-\text{margin}}$ and $\mathrm{L}_{E-\text{regression}}$sometimes performs slightly worse, but on par, than CE in eurlex and delicious.

With the high gains of $\mathrm{L}_{E-\text{NCEranking}}$ and $\mathrm{L}_{E-\text{ranking}}$, we are likely to conclude using samples rather than a single point can capture the output surface of the feedforward network better. To examine this hypothesis in more detail, we test sampling approaches on regression-based loss as well. We define $\mathrm{L}_{E-\text{regression-s}} = \sum_{\tilde{\mathbf{y}} \in S} \mathrm{L}_{E-\text{regression}}(\mathbf{x}, \mathbf{y}^*, \tilde{\mathbf{y}}; \Theta)$ given sample set S. We take two approaches in collecting these samples: 1. discrete binary vectors drawn from probability vector $\tilde{\mathbf{y}}$ as done in $\mathrm{L}_{E-\text{NCEranking}}$ and 2. continuous perturbation of $\tilde{\mathbf{y}}$ with gaussian noise. We found that discrete samples did not make any notable changes between $\mathrm{L}_{E-\text{regression}}$ and $\mathrm{L}_{E-\text{regression-s}}$, however, found that using continuous samples made +0.4 F1 improvements in average and report $\mathrm{L}_{E-\text{regression-s}}$ as the version which uses continuous sample. We conlcude that using samples are helpful in capturing

[5]The code we used to train and evaluate our models is available a https://anonymous.4open.science/r/SEAL/README.md.

[6]We use Weights & Biases (Biewald, 2020) for hyperparameter search

method	text datasets		
	BGC	**RCV**	**NYT**
cross-entropy	81.15	87.18	77.4
SEAL (NCEranking)	**81.64**	**87.82**	**78.87**

Table 3: Test F1 for text datasets.

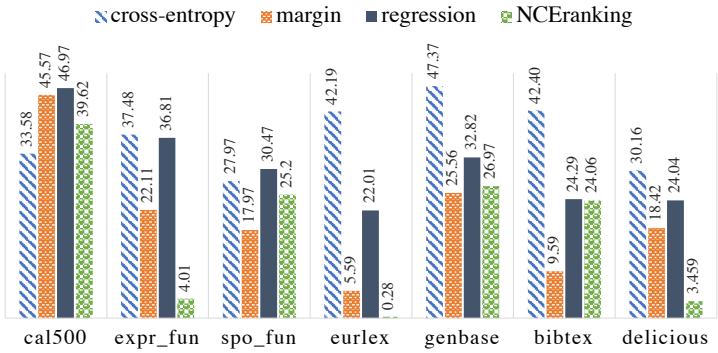


Figure 2: The performance of the margin, regression and NCEranking for SEAL without cross-entropy loss term on L_F, i.e. when $\lambda_1 = 1, \lambda_2 = 0$ in equation 1, compared to the model that only learns with cross-entropy.

better energy surface for traininig feedforward network, however, effect and characteristics of sample might differ per types of L_E. We believe the sampling is more effective on $L_{E-\text{NCEranking}}$, $L_{E-\text{ranking}}$ as groups of samples contribute in relative manner in learning surfaace in contrast to $L_{E-\text{regression}}$.

To further study effect of different L_E, we examine whether there is a correlation in performance between the isolate structured energy network and SEAL which utilizes idential L_E. For training isolated structured energy network with margin-based and regression-based energy loss, we follow training approaches of SPEN (Belanger & McCallum, 2016) and DVN (Gygli et al., 2017). For NCE, we need access to noise distribution P_N. we set pretrained feedforward network probability $P(Y|X;\Phi)$ as $P_N(Y)$. We do not see much of correlation between the two, in other words, characteristics of L_E *that trains best structured energy network* and L_E *that trains best feedforward network* are different. While NCE seems to perform worst when utilizing $L_{E-\text{NCEranking}}$ in isolated manner, the constrastive characteristic of $L_{E-\text{NCEranking}}$ and $L_{E-\text{ranking}}$that is tailored to feedforward network seems to be helpful. Regardless of L_E types, we also obesreve that it is much more stable and usually higher-performing to train with SEAL than to train energy network separately. In Appendix B, we also show compare individual energy network as a prediction model compared to SEAL in detail.

4.2 Results for large text datasets

In order to test the effectiveness of SEAL in the scope of large pre-trained models, we examine text datasets pre-trained BERT (Devlin et al., 2019) with adapter (Houlsby et al., 2019; Pfeiffer et al., 2020) as the feature network. The NYT subset we used has almost 17x larger training data than the largest feature-based data. SEAL with $L_{E-\text{NCEranking}}$gains +1.47 F1 points when compared to cross-entropy fine tuning. This shows that SEAL is effective in large models that utilizes large datasets as well. In average, SEAL gains +0.87 F1 points over cross entropy in text datset as shown in Table 3.

5 Further analysis

SEAL without cross entropy: Figure 2 shows the performance of SEAL when the cross-entropy loss (CE) is completely removed from equation 1. We first observe that surprisingly that SEAL itself without resorting to CE learns a model that sometimes, cal500 and spo_fun, outperforms model

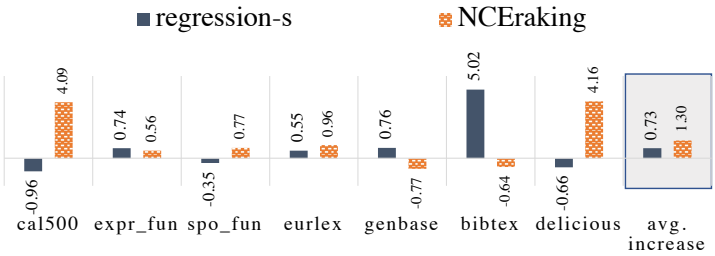


Figure 3: Change in test f1 when moving from SEAL-static to SEAL-dynamic.

trained with CE. However, we also observe that SEAL without CE is unstable as performance varies a lot across different datasets. This tells us that SEAL requires a signal that can bring the model $F_\Phi(\cdot)$ into reasonable region first in order to demonstrate stable performance shown in Table 2. Another interesting observation is that SEAL-NCErannking model when it is often weakest without CE can give the largest improvement when combined with CE together in SEAL as shown in Table 2. In short, we find CE to bring a stable synergy with energy network loss within SEAL: surpassing each individual's performance significantly when combined together. While there could be multiple techniques to stabilize learning of SEAL besides adding CE loss, including pretraining techniques, we leave those intersting studies as future work.

Effect of dynamic loss computation: We now compare the effect of dyanmic loss learning by comparing SEAL to SEAL-static in Figure 3. We observe that pretrained energy networks with SEAL-static also results in competitve results, often better performing than the simple cross-entropy loss. However, we oberve that in average, the dynamic loss learnig of SEALoutperforms SEAL-static for $\text{L}_{E-\text{regression}}$, and $\text{L}_{E-\text{NCEranking}}$.

Effect of applying ranking loss directly on F_Φ With the large improvement that $\text{L}_{E-\text{NCEranking}}$ and $\text{L}_{E-\text{ranking}}$ has, we examine whether the gain is actually coming from trained energy-netwrok loss or simply from the ranking loss itself which can be directly applied to feedforward network F_Φ We conduct this ablation study over relatively small datasets: genbase, cal500 and delicious. While we observed a near +10 F1 point in genbase, we saw decrease in performance in cal500 (-1.0 F1) and delicious (-2.3 F1) compared to plain cross-entropy model. With this we again see that genbase is an outlier where capturing structure really helps, however, we confirm that the ranking loss without energy network is not effective in training feedforward model F_Φ. This again confirms that the energy network E_Θ plays a major role in training F_Φ.

6 Conclusion

We proposed SEAL: a framework that can adopt structured energy network as a trainable and adaptable loss function for training feedforward network. Through examining different energy losses that trains energy network, we show that SEAL is a general framework which effective under different loss functions and different architectures, such as MLP and BERT-based adapter. Through extensive result on 7 feature-based and 3 text-based datasets, we show that SEAL brings synergy between isolated energy newtork and cross entropy function: performing better and in more stable manner when combined together. Lastly, through ablation study, we examine the benefit of dyanmic loss learing which in average brings a gain over static version of SEAL, i.e. SEAL-static. This reserach opens up a door for various future work, including but not lmited to, usage of energy loss in unsupervised manner and different architectures of energy network. In the long horizon, the authors are interested in whether general pretrained scoring neural networks, such as BLEURTSellam et al. (2020) and BERT-score(Zhang et al., 2019), could serve as a loss function as it provides a backpropagatable gradients to the output space.

REFERENCES

David Belanger and Andrew Mccallum. Structured Prediction Energy Networks. Technical report, 2016.

David Belanger and Andrew McCallum. Structured prediction energy networks. In *International Conference on Machine Learning*, pp. 983–992. PMLR, 2016.

David Belanger, Bishan Yang, and Andrew Mccallum. End-to-End Learning for Structured Prediction Energy Networks. Technical report, 2017.

Lukas Biewald. Experiment tracking with weights and biases, 2020. URL `https://www.wandb.com/`. Software available from wandb.com.

Liang-Chieh Chen, Alexander Schwing, Alan Yuille, and Raquel Urtasun. Learning deep structured models. In *International Conference on Machine Learning*, pp. 1785–1794. PMLR, 2015.

Jacob Devlin, Ming Wei Chang, Kenton Lee, and Kristina Toutanova. BERT: Pre-training of deep bidirectional transformers for language understanding. *NAACL HLT 2019 - 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies - Proceedings of the Conference*, 1:4171–4186, 2019. URL `http://arxiv.org/abs/1810.04805`.

Nadia Ghamrawi and Andrew McCallum. Collective multi-label classification. In *Proceedings of the 14th ACM international conference on Information and knowledge management*, pp. 195–200, 2005.

Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. *Advances in neural information processing systems*, 27, 2014.

Michael Gutmann and Jun-ichiro Hirayama. Bregman divergence as general framework to estimate unnormalized statistical models. *arXiv preprint arXiv:1202.3727*, 2012.

Michael Gygli, Mohammad Norouzi, and Anelia Angelova. Deep value networks learn to evaluate and iteratively refine structured outputs. In *34th International Conference on Machine Learning, ICML 2017*, volume 3, pp. 2160–2170, 2017. ISBN 9781510855144. URL `https://goo.gl/8OLufh`.

Neil Houlsby, Andrei Giurgiu, Stanislaw Jastrzebski, Bruna Morrone, Quentin De Laroussilhe, Andrea Gesmundo, Mona Attariyan, and Sylvain Gelly. Parameter-efficient transfer learning for NLP. In Kamalika Chaudhuri and Ruslan Salakhutdinov (eds.), *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pp. 2790–2799. PMLR, 09–15 Jun 2019. URL `https://proceedings.mlr.press/v97/houlsby19a.html`.

Chen Huang, Shuangfei Zhai, Walter Talbott, Miguel Bautista Martin, Shih-Yu Sun, Carlos Guestrin, and Josh Susskind. Addressing the loss-metric mismatch with adaptive loss alignment. In *International Conference on Machine Learning*, pp. 2891–2900. PMLR, 2019.

Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.

John Lafferty, Andrew McCallum, and Fernando CN Pereira. Conditional random fields: Probabilistic models for segmenting and labeling sequence data. 2001.

Yann LeCun, Sumit Chopra, Raia Hadsell, M Ranzato, and F Huang. A tutorial on energy-based learning. *Predicting structured data*, 1(0), 2006.

Zhuang Ma and Michael Collins. Noise Contrastive Estimation and Negative Sampling for Conditional Models: Consistency and Statistical Efficiency. Technical report.

Andreas Christian Müller. Methods for learning structured prediction in semantic segmentation of natural images. 2014.

Martha Palmer, Daniel Gildea, and Nianwen Xue. Semantic role labeling. *Synthesis Lectures on Human Language Technologies*, 3(1):1–103, 2010.

Jonas Pfeiffer, Andreas Rücklé, Clifton Poth, Aishwarya Kamath, Ivan Vulić, Sebastian Ruder, Kyunghyun Cho, and Iryna Gurevych. Adapterhub: A framework for adapting transformers. In *Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing: System Demonstrations*, pp. 46–54, 2020.

Amirmohammad Rooshenas, Dongxu Zhang, Gopal Sharma, and Andrew McCallum. Search-guided, lightly-supervised training of structured prediction energy networks. *Advances in Neural Information Processing Systems*, 32:13522–13532, 2019.

Alexander G Schwing and Raquel Urtasun. Fully connected deep structured networks. *arXiv preprint arXiv:1503.02351*, 2015.

Thibault Sellam, Dipanjan Das, and Ankur P Parikh. Bleurt: Learning robust metrics for text generation. In *Proceedings of ACL*, 2020. URL `https://arxiv.org/abs/2004.04696`.

Ben Taskar, Carlos Guestrin, and Daphne Koller. Max-Margin Markov Networks. *Advances in neural information processing systems 16 (2004): 25*, 2004.

Ioannis Tsochantaridis, Thomas Hofmann, Thorsten Joachims, and Yasemin Altun. Support vector machine learning for interdependent and structured output spaces. In *Proceedings of the twenty-first international conference on Machine learning*, pp. 104, 2004.

Lifu Tu and Kevin Gimpel. Learning approximate inference networks for structured prediction, 2018. ISSN 23318422.

Lifu Tu, Richard Yuanzhe Pang, and Kevin Gimpel. Improving joint training of inference networks and structured prediction energy networks. In *SPNLP*, 2020.

Lijun Wu, Fei Tian, Yingce Xia, Yang Fan, Tao Qin, Lai Jian-Huang, and Tie-Yan Liu. Learning to Teach with Dynamic Loss Functions. In *Advances in Neural Information Processing Systems*, volume 31. Curran Associates, Inc., 2018. URL `https://papers.nips.cc/paper/2018/hash/8051a3c40561002834e59d566b7430cf-Abstract.html`.

Tianyi Zhang, Varsha Kishore, Felix Wu, Kilian Q Weinberger, and Yoav Artzi. Bertscore: Evaluating text generation with bert. *arXiv preprint arXiv:1904.09675*, 2019.

A NCE RANKING METHOD WITH $s(\mathbf{x}, \mathbf{y}; \Theta) = -E_\Theta(\mathbf{x}, \mathbf{y})$

As pointed out by Ma & Collins as well, If we set $s(\mathbf{x}, \mathbf{y}; \Theta) = -E_\Theta(\mathbf{x}, \mathbf{y})$, we can regard this this learning with $E'_\Theta(\mathbf{x}, \mathbf{y}) = E_\Theta(\mathbf{x}, \mathbf{y}) + \log P_{\mathrm{N}}(\mathbf{y})$. As pointed out earlier, this leads the model to estimate $P(\mathbf{y}|\mathbf{x}$ with

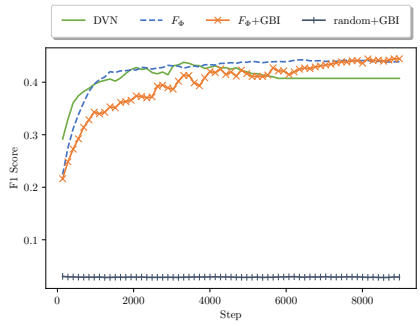
$$\frac{\exp\left(-E'_\Theta(x, y)\right)}{\sum_{y \in \mathcal{Y}} \exp\left(-E'_\Theta(x, y)\right)}.$$

Plugging in $E_\Theta{}'$ as $E_\Theta(\mathbf{x}, \mathbf{y}) + \log P_{\mathrm{N}}(\mathbf{y})$ once again, we can see that $P(Y|X)$ is estimated by

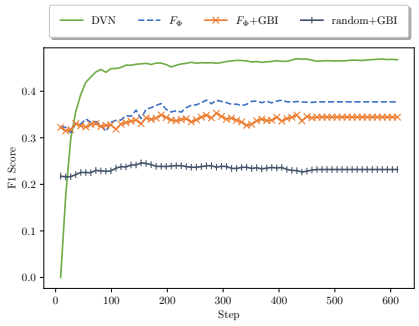
$$\frac{P_N(\mathbf{y}) \exp\left(-E_\Theta(\mathbf{x}, \mathbf{y})\right)}{\sum_{\mathbf{y} \in \mathcal{Y}} P_N(\mathbf{y}) \exp\left(-E_\Theta(\mathbf{x}, \mathbf{y})\right)}$$

B COMPARISON OF SEAL AND SEPARATE ENERGY NETWORK

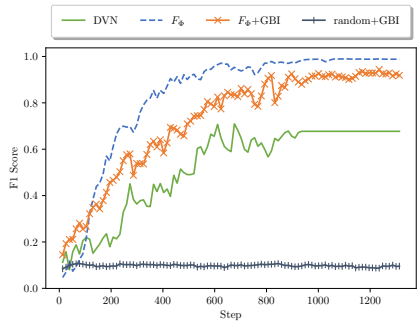
In the following figures, we compare the F1 scores of SEAL framework with $\mathrm{L}_{E-\text{regression}}$ (Figure 4), $\mathrm{L}_{E-\text{NCEranking}}$ (Figure 5) and $\mathrm{L}_{E-\text{margin}}$ (Figure 6) on validation set with its energy-only counterpart. We further report validation F1 scores for the output of GBI starting from the samples drawn from $\hat{y}$ and randomly. We show this comparison to evaluate the performance of the energy network on points around $\hat{y}$ and random points.



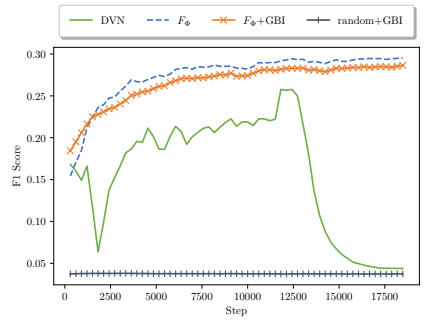
(a) bibtex



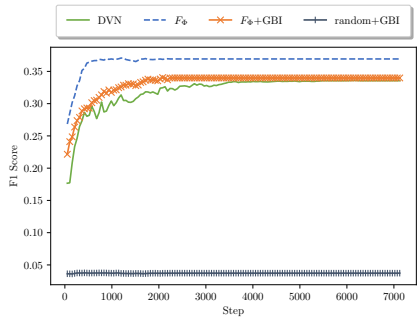
(b) cal500



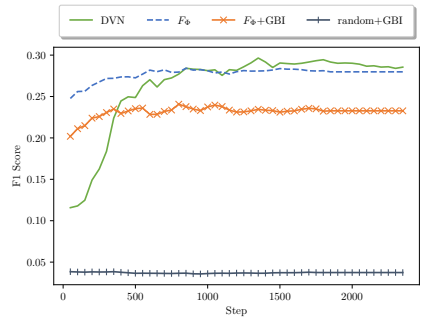
(c) genbase



(d) delicious

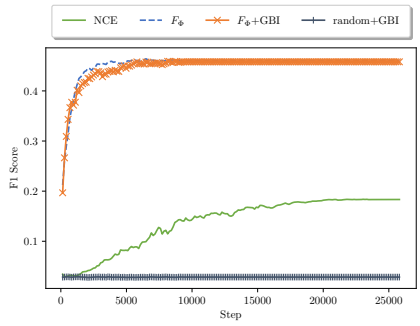


(e) expr_fun

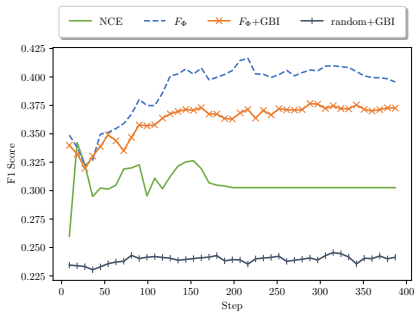


(f) spo_fun

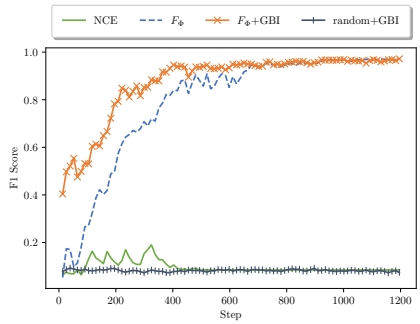
Figure 4: Validation F1 performance of SEAL framework with $\mathrm{L}_{E-\text{regression}}(F_\Phi)$ compared to energy-only (DVN). Further, we compare validation F1 performance of GBI starting from samples drawn from feedforward network's output (F_Φ+GBI) and samples drawn randomly (random + GBI).
.



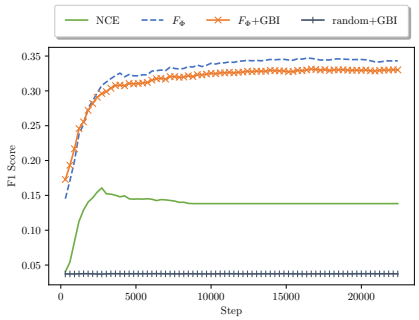
(a) bibtex



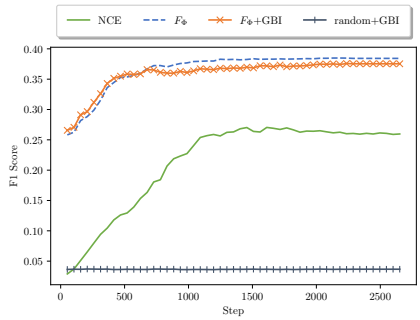
(b) cal500



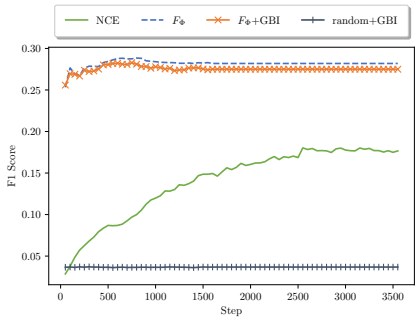
(c) genbase



(d) delicious

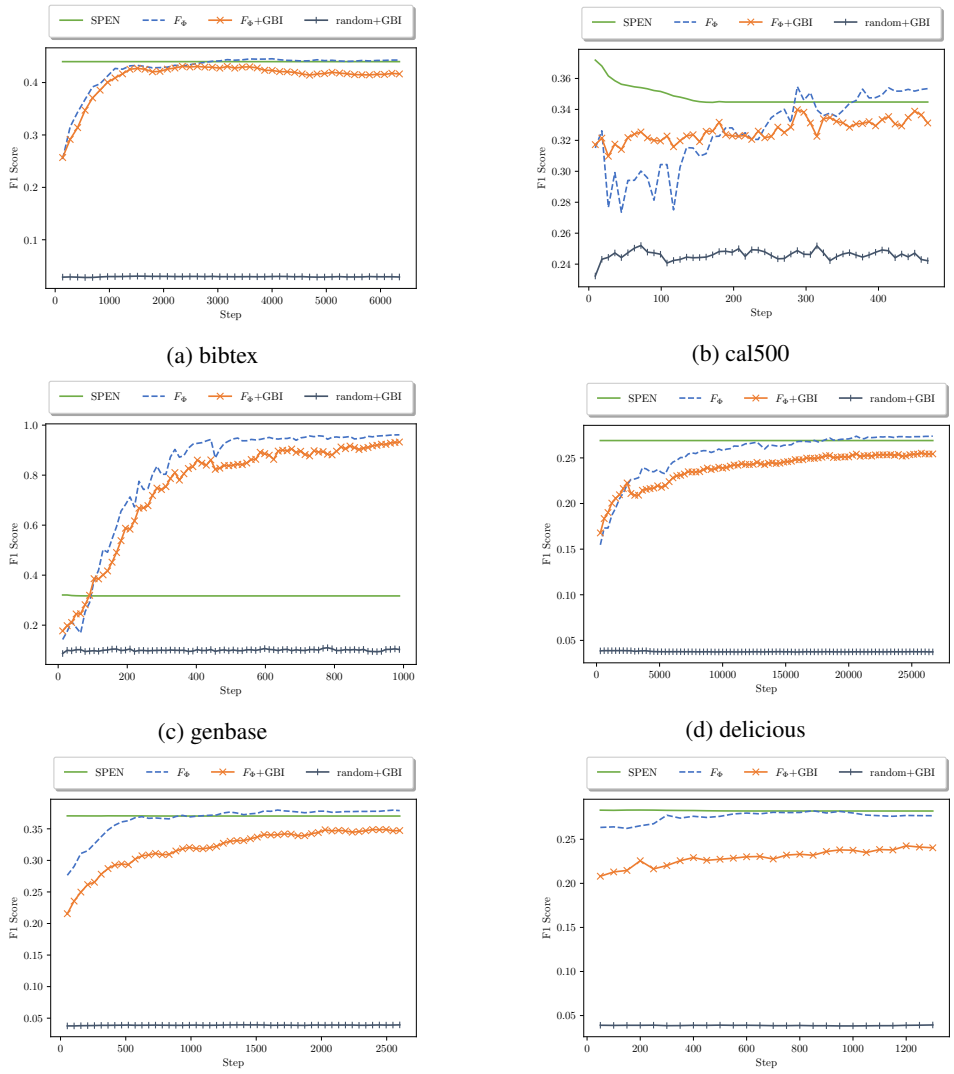


(e) expr_fun



(f) spo_fun

Figure 5: Validation F1 performance of SEAL framework with $\mathrm{L}_{E-\mathrm{NCEranking}}(F_\Phi)$ compared to energy-only (NCE). Further, we compare validation F1 performance of GBI starting from samples drawn from feedforward network's output (F_Φ+GBI) and samples drawn randomly (random + GBI).



(a) bibtex

(b) cal500

(c) genbase

(d) delicious

(e) expr_fun

(f) spo_fun

Figure 6: Validation F1 performance of SEAL framework with $\mathrm{L}_{E-\text{margin}}(F_\Phi)$ compared to energy-only (SPEN). Further, we compare validation F1 performance of GBI starting from samples drawn from feedforward network's output (F_Φ+GBI) and samples drawn randomly (random + GBI).

C HYPERPARAMETERS

The following tables tabulate the hyper-parameters for SEAL framework for each dataset. We further fix the number of layers, layer-dimensions and dropout in feed-forward network based on the configuration of the best model trained using cross-entropy.

method	λ_1	num samples	Θ_{lr}	Φ_{lr}
margin	0.001	n/a	0.0002	0.001
regression	0.002	n/a	0.0003	0.001
regression-s	0.03	20	0.0004	0.001
NCEranking	0.002	100	0.0005	0.001
ranking	0.002	100	0.000015	0.001

Table 4: genbase

method	λ_1	num samples	Θ_{lr}	Φ_{lr}
margin	0.001	n/a	0.0005	0.008
regression	1.4	n/a	0.006	0.007
regression-s	9	5	0.008	0.006
NCEranking	0.4	40	0.007	0.005
ranking	8	80	0.003	0.0001

Table 5: cal500

method	λ_1	num samples	Θ_{lr}	Φ_{lr}
margin	0.0002	n/a	0.002	0.004
regression	8	n/a	0.0001	0.00015
regression-s	9	5	0.0001	0.003
NCEranking	0.9	20	0.002	0.001
ranking	1	60	0.0008	0.0001

Table 6: delicious

method	λ_1	num samples	Θ_{lr}	Φ_{lr}
margin	0.001	n/a	0.0003	0.002
regression	0.002	n/a	0.0006	0.004
regression-s	0.01	20	0.0004	0.007
NCEranking	0.5	40	0.001	0.002
ranking	0.3	40	0.0003	0.002

Table 7: eurlex

method	λ_1	num samples	Θ_{lr}	Φ_{lr}
margin	0.01	n/a	0.00003	0.001
regression	0.02	n/a	0.0003	0.0006
regression-s	2	30	0.007	0.002
NCEranking	4	80	0.00005	0.0002
ranking	1	40	0.008	0.0005

Table 8: expr_fun

method	λ_1	num samples	Θ_{lr}	Φ_{lr}
margin	0.08	n/a	0.0001	0.002
regression	9	n/a	0.001	0.00015
regression-s	0.02	10	0.0002	0.0009
NCEranking	0.7	60	0.008	0.0005
ranking	1.5	40	0.004	0.002

Table 9: spo_fun

method	λ_1	num samples	Θ_{lr}	Φ_{lr}
margin	0.0002	n/a	0.00005	0.001
regression	3	n/a	0.005	0.003
regression-s	9	10	0.005	0.001
NCEranking	5	20	0.0004	0.001
ranking	9	40	0.00005	0.001

Table 10: bibtex