Error Compensated Distributed SGD can be Accelerated

Anonymous Author(s)
Affiliation
Address
email

Abstract

Gradient compression is a recent and increasingly popular technique for reducing the communication cost in distributed training of large-scale machine learning models. In this work we focus on developing efficient distributed methods that can work for any compressor satisfying a certain contraction property, which includes both unbiased (after appropriate scaling) and biased compressors such as RandK and TopK. Applied naively, gradient compression introduces errors that either slow down convergence or lead to divergence. A popular technique designed to tackle this issue is error compensation/error feedback. Due to the difficulties associated with analyzing biased compressors, it is not known whether gradient compression with error compensation can be combined with Nesterov's acceleration. In this work, we show for the first time that error compensated gradient compression methods can be accelerated. In particular, we propose and study the error compensated loopless Katyusha method, and establish an accelerated linear convergence rate under standard assumptions. We show through numerical experiments that the proposed method converges with substantially fewer communication rounds than previous error compensated algorithms.

1 Introduction

When training very large scale supervised machine learning problems, such as those arising in the context of federated learning [Konečný et al., 2016b, McMahan et al., 2017, Konečný et al., 2016a] (see also recent surveys [Li et al., 2019, Kairouz, 2019]), distributed algorithms are indispensable. In such settings, communication is generally much slower than (local) computation, which makes it the key bottleneck in the design of efficient distributed systems. There are several ways to tackle this issue, including reliance on large mini-batches [Goyal et al., 2017, You et al., 2017], asynchronous learning [Agarwal and Duchi, 2011, Lian et al., 2015, Recht et al., 2011], local updates [Ma et al., 2017, Stich, 2020, Khaled et al., 2020, Hanzely and Richtárik, 2020, Woodworth et al., 2020] and communication compression (e.g., quantization and sparsification) [Alistarh et al., 2017, Bernstein et al., 2018, Mishchenko et al., 2019, Seide et al., 2014, Wen et al., 2017].

Communication compression. In this work, we focus on the last of these techniques: *communication compression*. The key idea here is to apply a lossy compression transformation/operator to the messages before they are communicated so as to save on communication time. While compression reduces the communicated bits in each communication round, it introduces errors, and this generally leads to an increase in the number of communication rounds needed to find a solution of any predefined accuracy. Still, compression has been found useful in practice, as the trade-off often seems to prefer compression to no compression.

Contractive and unbiased compressors. There are two large families of such compression operators: *contraction* and *unbiased* compressors [Beznosikov et al., 2020]. A (possibly) randomized map

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$Q : \mathbb{R}^d \to \mathbb{R}^d$ is called a *contraction compressor* if there exists a constant $0 < \delta \leq 1$ such that

$$\mathbb{E}\left[\|x - Q(x)\|^2\right] \leq (1-\delta)\|x\|^2, \qquad \forall x \in \mathbb{R}^d. \tag{1}$$

Further, we say that a randomized map $\tilde{Q} : \mathbb{R}^d \to \mathbb{R}^d$ is an *unbiased compressor* if there exists a constant $\omega \geq 0$ such that

$$\mathbb{E}\left[\tilde{Q}(x)\right] = x \quad \text{and} \quad \mathbb{E}\left[\|\tilde{Q}(x)\|^2\right] \leq (\omega+1)\|x\|^2, \qquad \forall x \in \mathbb{R}^d. \tag{2}$$

It is well known that (see, e.g., [Beznosikov et al., 2020]) after appropriate scaling, any unbiased compressor satisfying (2) becomes a contraction compressor. Indeed, it is easy to verify that for any $\tilde{Q}$ satisfying (2), $\frac{1}{\omega+1}\tilde{Q}$ is a contraction compressor satisfying (1) with $\delta = 1/(\omega+1)$. Thus we can construct corresponding contraction compressor from any unbiased compressor. However, there are empirically very powerful contractive compressors, such as TopK (described below), which do not arise this way, and because of this, the class of contractive compressors is important in practice. For illustration purposes, we now define two canonical examples of contraction compressors. Let $1 \leq K \leq d$. The TopK compressor is defined as

$$(\mathrm{TopK}(x))_{\pi(i)} = \begin{cases} (x)_{\pi(i)} & \text{if } i \leq K, \\ 0 & \text{otherwise,} \end{cases}$$

where π is a permutation of $\{1, 2, ..., d\}$ such that $(|x|)_{\pi(i)} \geq (|x|)_{\pi(i+1)}$ for $i = 1, ..., d-1$. The RandK compressor is defined as

$$(\mathrm{RandK}(x))_i = \begin{cases} (x)_i & \text{if } i \in S, \\ 0 & \text{otherwise,} \end{cases}$$

where S is chosen uniformly from the set of all K element subsets of $\{1, 2, ..., d\}$. For TopK and RandK compressors, we have $\delta \geq K/d$ [Stich et al., 2018]. Some frequently used unbiased compressors include random dithering [Alistarh et al., 2017], random sparsification [Stich et al., 2018], and natural compression [Horváth et al., 2019b]. For more examples of contraction and unbiased compressors, we refer the reader to [Beznosikov et al., 2020].

Error compensation. While contractive compressors can be more powerful in practice, they are not unbiased, and this leads to severe difficulties in the algorithm design space, and poses challenges for theoreticians trying to understand the convergence behavior of distributed algorithms using contractive compressors. Indeed, while there is no issue with the use of contractive compressors in the non-distributed setting (which is of course not interesting from a practical point of view) in combination with standard gradient-type methods, serious convergence issues arise in the distributed setting. Indeed, classical methods such gradient descent can diverge (even exponentially fast) when the TopK compressor is used to compress uplink gradient information [Beznosikov et al., 2020]. Fortunately, this issue can be resolved using the *error compensation (EC)*[1] mechanism, which can be traced back to at least Seide et al. [2014]. However, this mechanism remained a heuristic until recently. To highlight the difficulty of analyzing EC, we remark that the first theoretical breakthroughs applied to the single node regime only [Stich et al., 2018, Stich and Karimireddy, 2019]. More recently, theoretical progress was made in the crucially important distributed setting as well [Beznosikov et al., 2020, Gorbunov et al., 2020]. However, we are still very far in our understanding of optimization algorithms using contractive compressors.

> *One of the key open problems in this area—the problem we address in this paper in the affirmative— is whether it is possible to design provably accelerated gradient-type methods, in the sense of Nesterov [Nesterov, 1983, 2004], that work with contractive compressors.*

2 Problem Description and Summary of Contributions

We formally describe the distributed optimization problem we solve in Section 2.1, and then summarize our key contributions in Section 2.2.

[1] Error compensation is also known under the name error feedback.

2.1 Problem Description

We consider the composite finite-sum optimization problem

$$\min_{x\in\mathbb{R}^d} P(x) := \left\{ \frac{1}{n}\sum_{\tau=1}^{n} f^{(\tau)}(x) + \psi(x) \right\}, \tag{3}$$

where $f(x) := \frac{1}{n}\sum_\tau f^{(\tau)}(x)$ is an average of n smooth convex functions, distributed over n compute nodes, and $\psi : \mathbb{R}^d \to \mathbb{R}\cup\{+\infty\}$ is a proper closed convex function representing a possibly nonsmooth regularizer. On each node τ, $f^{(\tau)}(x)$ is an average of m smooth convex functions, i.e., $f^{(\tau)}(x) = \frac{1}{m}\sum_{i=1}^{m} f_i^{(\tau)}(x)$, representing the average loss over the training data stored on this node.

We denote the smoothness constants of functions f, $f^{(\tau)}$ and $f_i^{(\tau)}$ using symbols L_f, $\bar{L}$ and L, respectively.[2] These constants are in general related as follows:

$$L_f \le \bar{L} \le nL_f, \qquad \bar{L} \le L \le m\bar{L}. \tag{4}$$

While we specifically focus on the case when $m = 1$, our results are also new in the $m = 1$ case, and hence this regime is relevant as well. We assume throughout that problem (3) has at least one optimal solution x^*.

We will optionally use the following additional assumption for the contraction compressor. The assumption is not necessary, but when used, it will lead to better complexity.

Assumption 2.1. *$\mathbb{E}[Q(x)] = \delta x$ and $\mathbb{E}[Q_1(x)] = \delta_1 x$ for all $x \in \mathbb{R}^d$, where δ and δ_1 are compression parameters of Q and Q_1, respectively.*

It is easy to verify that RandK compressor satisfies Assumption 2.1 with $\delta = {}^{K}\!/_{d}$, and $\frac{1}{\omega+1}\tilde{Q}$, where $\tilde{Q}$ is any unbiased compressor, also satisfies Assumption 2.1 with $\delta = {}^{1}\!/_{(\omega+1)}$.

2.2 Contributions

We now summarize the main contributions of our work.

Acceleration for error compensation. We develop a new communication efficient algorithm (Algorithm 1) for solving the distributed optimization problem (3) which we call *Error Compensated Loopless Katyusha* (ECLK). ECLK is the *first accelerated* error compensated method (this is true even in the non-distributed setting), which we believe is a significant contribution to the field. ECLK can alternatively be seen as an error compensated variant of the loopless Katyusha [Kovalev et al., 2020].

Iteration complexity. We obtain the *first accelerated linear convergence rate* for error compensated methods using contraction operators. Let $p \le \mathcal{O}(\delta_1)$, where $p \in (0, 1]$ is a parameter of the method described later and δ_1 is the compression parameter of Q_1 in Algorithm 1 (see Assumption 2.1). Then the iteration complexity of ECLK is

$$\mathcal{O}\left(\left(\frac{1}{\delta} + \frac{1}{p} + \sqrt{\frac{L_f}{\mu}} + \sqrt{\frac{L}{\mu p n}} + \frac{1}{\delta}\sqrt{\frac{(1-\delta)\bar{L}}{\mu p}} + \sqrt{\frac{(1-\delta)L}{\mu p \delta}}\right)\log\frac{1}{\epsilon}\right). \tag{5}$$

This is an improvement over the previous best known results for error compensated methods in [Beznosikov et al., 2020] and [Gorbunov et al., 2020], where *nonaccelerated* linear rates were obtained for the smooth case ($\psi \equiv 0$). Moreover, in oder to obtained a linear rate, Beznosikov et al. [2020] required the assumption that $\nabla f^{(\tau)}(x^*) = 0$ for all τ, and also need full gradients to be computed by all nodes. The complexity of EC-LSVRG-DIANA in [Gorbunov et al., 2020] is

$$\mathcal{O}\left(\left(\omega + m + \frac{L}{\delta\mu}\right)\log\frac{1}{\epsilon}\right). \tag{6}$$

When the last term in (6) is dominant and the last two terms in (5) are dominant, the complexity of ECLK is better than that of EC-LSVRG-DIANA if $p \ge \mathcal{O}\left(\frac{(1-\delta)\bar{L}\mu}{L^2} + \frac{(1-\delta)\delta\mu}{L}\right)$.

[2] Function $f : \mathbb{R}^d \to \mathbb{R}$ is smooth if it is differentiable, and has L-Lipschitz gradient: $\|\nabla f(x) - \nabla f(y)\| \le L\|x - y\|$ for any $x, y \in \mathbb{R}^d$. L is called the *smoothness constant* of f.

If we invoke additional assumptions (Assumption 2.1) on the contraction compressor, the iteration complexity of ECLK is improved to

$$\mathcal{O}\left(\left(\frac{1}{\delta}+\frac{1}{p}+\sqrt{\frac{L_f}{\mu}}+\sqrt{\frac{L}{\mu p n}}+\frac{1}{\delta}\sqrt{\frac{(1-\delta)L_f}{\mu p}}+\sqrt{\frac{(1-\delta)L}{\mu p \delta n}}\right)\log\frac{1}{\epsilon}\right).$$

This is indeed an improvement since $L_f \leq \bar{L}$ (see (4)), and because of the extra scaling factor of n in the last term. If $\delta = \delta_1 = 1$, i.e., if no compression is used, we recover the iteration complexity of the accelerated method L-Katyusha [Qian et al., 2019].

3 Error compensated L-Katyusha

3.1 Description of the method

In this section we describe our method: error compensated L-Katyusha (see Algorithm 1). The search direction in L-Katyusha in the distributed setting ($n \geq 1$) at iteration k is

$$\frac{1}{n}\sum_{\tau=1}^{n}\left(\nabla f_{i_k^\tau}^{(\tau)}(x^k) - \nabla f_{i_k^\tau}^{(\tau)}(w^k) + \nabla f^{(\tau)}(w^k)\right),$$

where i_k^τ is sampled uniformly and independently from $[m] := \{1, 2, ..., m\}$ on the τ-th node for $1 \leq \tau \leq n$, x^k is the current iteration, and w^k is the current reference point. Whenever ψ is nonzero in problem (3), $\nabla f(x^*)$ is nonzero in general, and so is $\nabla f^{(\tau)}(x^*)$ (it could be nonzero even if $\nabla f(x^*) = 0$ in the non-regularized case). Thus, compressing the direction

$$\nabla f_{i_k^\tau}^{(\tau)}(x^k) - \nabla f_{i_k^\tau}^{(\tau)}(w^k) + \nabla f^{(\tau)}(w^k) \tag{7}$$

directly on each node would cause nonzero noise even if x^k and w^k converged to the optimal solution x^*. On the other hand, since $f_i^{(\tau)}$ is L-smooth, $\nabla f_{i_k^\tau}^{(\tau)}(x^k) - \nabla f_{i_k^\tau}^{(\tau)}(w^k)$ could be small if x^k and w^k are close enough. For the last term $\nabla f^{(\tau)}(w^k)$, we introduce a vector h_τ^k to learn it iteratively in a similar way in VR-DIANA [Horváth et al., 2019a]. However, we use a contraction compressor rather than an unbiased one. More precisely, we take the update $h_\tau^{k+1} = h_\tau^k + Q_1(\nabla f^{(\tau)}(w^k) - h_\tau^k)$, where Q_1 is a contraction compressor. Now we would expect $\nabla f^{(\tau)}(w^k) - h_\tau^k$ could converge to zero as $k \to +\infty$. We substract h_τ^k from the search direction in (7) to get

$$g_\tau^k = \nabla f_{i_k^\tau}^{(\tau)}(x^k) - \nabla f_{i_k^\tau}^{(\tau)}(w^k) + \nabla f^{(\tau)}(w^k) - h_\tau^k,$$

and will add h_τ^k back after aggregation.

Next we apply the compression and error feedback techniques, i.e., we compress the vector $\frac{\eta}{\mathcal{L}_1}g_\tau^k + e_\tau^k$ on each node to get $\tilde{g}_\tau^k = Q(\frac{\eta}{\mathcal{L}_1}g_\tau^k + e_\tau^k)$, where Q is a contraction compressor. The accumulated error e_τ^{k+1} is updated by the compression error at iteration k. On each node, a scalar u_τ^k is also maintained, and only u_1^k will be updated. The summation of u_τ^k for $1 \leq \tau \leq n$ is u^k, and we use u^k to control the update frequency of the reference point w^k. Each node sends the two compressed vectors $\tilde{g}_\tau^k$, $Q_1(\nabla f^{(\tau)}(w^k) - h_\tau^k)$, and a scalar u_τ^{k+1} to the other nodes. After aggregating $\tilde{g}^k = \frac{1}{n}\sum_{\tau=1}^n \tilde{g}_\tau^k$, we add the average vector $h^k = \frac{1}{n}\sum_{\tau=1}^n h_\tau^k$ after multiplying the stepsize $\frac{\eta}{\mathcal{L}_1}$ to it to get the search direction $\tilde{g}^k + \frac{\eta}{\mathcal{L}_1}h^k$. We also use the following standard proximal operator for the update of z^k:

$$\mathrm{prox}_{\eta\psi}(x) := \arg\min_y \left\{\frac{1}{2}\|x-y\|^2 + \eta\psi(y)\right\}.$$

The reference point w^k will be updated if $u^{k+1} = 1$. It is easy to see that w^k will be updated with probability p at each iteration. All nodes maintain the same copies of x^k, w^k, y^k, z^k, $\tilde{g}^k$, h^k, and u^k.

We need the following assumptions in this section.

Assumption 3.1. *The two compressors Q and Q_1 in Algorithm 1 are contraction compressors with parameters δ and δ_1, respectively.*

Assumption 3.2. *$f_i^{(\tau)}$ is L-smooth, $f^{(\tau)}$ is $\bar{L}$-smooth, f is L_f-smooth and μ_f-strongly convex, and ψ is μ_ψ-strongly convex, where $\mu_f \geq 0$, $\mu_\psi \geq 0$, and $\mu := \mu_f + \mu_\psi > 0$.*

Algorithm 1 Error Compensated Loopless Katyusha (ECLK)

Parameters: stepsize parameters $\eta = \frac{1}{3\theta_1} > 0$, $\mathcal{L}_1 > 0$, $\sigma_1 = \frac{\mu_f}{2\mathcal{L}_1} \geq 0$, $\theta_1, \theta_2 \in (0,1)$; probability $p \in (0,1]$

Initialization: $x^0 = y^0 = z^0 = w^0 \in \mathbb{R}^d$; $u_\tau^0 = 0 \in \mathbb{R}$; $e_\tau^0 = 0 \in \mathbb{R}^d$; $h_\tau^0 \in \mathbb{R}^d$; $u^0 = \sum_{\tau=1}^n u_\tau^0$; $h^0 = \frac{1}{n}\sum_{\tau=1}^n h_\tau^0$

for $k = 0, 1, 2, ...$ **do**

  for $\tau = 1, ..., n$ **do**

    Sample i_k^τ uniformly and independently in $[m]$ on each node

    $g_\tau^k = \nabla f_{i_k^\tau}^{(\tau)}(x^k) - \nabla f_{i_k^\tau}^{(\tau)}(w^k) + \nabla f^{(\tau)}(w^k) - h_\tau^k$

    $\tilde{g}_\tau^k = Q(\frac{\eta}{\mathcal{L}_1} g_\tau^k + e_\tau^k), \quad h_\tau^{k+1} = h_\tau^k + Q_1(\nabla f^{(\tau)}(w^k) - h_\tau^k)$

    $e_\tau^{k+1} = e_\tau^k + \frac{\eta}{\mathcal{L}_1} g_\tau^k - \tilde{g}_\tau^k, \quad u_\tau^{k+1} = 0$ for $\tau = 2, ..., n$

    $u_1^{k+1} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$

    Send $\tilde{g}_\tau^k$, $Q_1(\nabla f^{(\tau)}(w^k) - h_\tau^k)$, and u_τ^{k+1} to the other nodes

    Receive $\tilde{g}_\tau^k$, $Q_1(\nabla f^{(\tau)}(w^k) - h_\tau^k)$, and u_τ^{k+1} from the other nodes

    $\tilde{g}^k = \frac{1}{n}\sum_{\tau=1}^n \tilde{g}_\tau^k, \quad u^{k+1} = \sum_{\tau=1}^n u_\tau^{k+1}$

    $z^{k+1} = \text{prox}_{\frac{\eta}{(1+\eta\sigma_1)\mathcal{L}_1}\psi}\left(\frac{1}{1+\eta\sigma_1}\left(\eta\sigma_1 x^k + z^k - \tilde{g}^k - \frac{\eta}{\mathcal{L}_1} h^k\right)\right)$

    $y^{k+1} = x^k + \theta_1(z^{k+1} - z^k)$

    $w^{k+1} = \begin{cases} y^k & \text{if } u^{k+1} = 1 \\ w^k & \text{otherwise} \end{cases}$

    $x^{k+1} = \theta_1 z^{k+1} + \theta_2 w^{k+1} + (1 - \theta_1 - \theta_2) y^{k+1}$

    $h^{k+1} = h^k + \frac{1}{n}\sum_{\tau=1}^n Q_1(\nabla f^{(\tau)}(w^k) - h_\tau^k)$

  end for

end for

3.2 Convergence analysis: preliminaries

First, we introduce some perturbed vectors which will be used in the convergence analysis. The perturbed vector is usually used in the analysis of error compensated methods [Karimireddy et al., 2019]. In Algorithm 1, let $e^k = \frac{1}{n}\sum_{\tau=1}^n e_\tau^k$, $g^k = \frac{1}{n}\sum_{\tau=1}^n g_\tau^k$, and $\tilde{x}^k = x^k - \frac{1}{1+\eta\sigma_1} e^k$, $\tilde{z}^k = z^k - \frac{1}{1+\eta\sigma_1} e^k$ for $k \geq 0$. Then $e^{k+1} = \frac{1}{n}\sum_{\tau=1}^n \left(e_\tau^k + \frac{\eta}{\mathcal{L}_1} g_\tau^k - \tilde{g}_\tau^k\right) = e^k + \frac{\eta}{\mathcal{L}_1} g^k - \tilde{g}^k$, and from the optimality condition for the proximal operator, we have

$$\begin{aligned}
\tilde{z}^{k+1} &= z^{k+1} - \tfrac{1}{1+\eta\sigma_1} e^{k+1} \\
&= \tfrac{1}{1+\eta\sigma_1}\left(\eta\sigma_1 x^k + z^k - \tilde{g}^k - \tfrac{\eta}{\mathcal{L}_1} h^k\right) - \tfrac{\eta \partial\psi(z^{k+1})}{(1+\eta\sigma_1)\mathcal{L}_1}) - \tfrac{e^{k+1}}{1+\eta\sigma_1} \\
&= \tfrac{1}{1+\eta\sigma_1}\left(\eta\sigma_1 x^k + z^k - e^k - \tfrac{\eta}{\mathcal{L}_1} g^k - \tfrac{\eta}{\mathcal{L}_1} h^k\right) - \tfrac{\eta \partial\psi(z^{k+1})}{(1+\eta\sigma_1)\mathcal{L}_1} \\
&= \tfrac{1}{1+\eta\sigma_1}\left(\eta\sigma_1 \tilde{x}^k + \tilde{z}^k - \tfrac{\eta}{\mathcal{L}_1} g^k - \tfrac{\eta}{\mathcal{L}_1} h^k\right) - \tfrac{\eta \partial\psi(z^{k+1})}{(1+\eta\sigma_1)\mathcal{L}_1}. \qquad (8)
\end{aligned}$$

The above relation plays a vital role in the convergence analysis, and allows us to follow the analysis of original L-Katyusha. In particular, we will use $\tilde{z}^k$ to construct Lyapunov functions. Next we define some notations which will be used to construct Lyapunov functions. Define $\tilde{\mathcal{Z}}^k := \frac{\mathcal{L}_1 + \eta\mu/2}{2\eta}\|\tilde{z}^k - x^*\|^2$, $\mathcal{Y}^k := \frac{1}{\theta_1}(P(y^k) - P^*)$, and $\mathcal{W}^k := \frac{\theta_2}{pq\theta_1}(P(w^k) - P^*)$ for $k \geq 0$, where $P^* := P(x^*)$. From the update rule of w^k in Algorithm 1, it is easy to see that

$$\mathbb{E}_k[\mathcal{W}^{k+1}] = (1-p)\mathcal{W}^k + \tfrac{\theta_2}{q}\mathcal{Y}^k, \qquad (9)$$

for $k \geq 0$. Let $B_f(x, y) := f(x) - f(y) - \langle \nabla f(y), x - y\rangle$ be the Bregman divergence and $\mathbb{E}_k[\cdot]$ denote the expectation conditional on x^k, y^k, z^k, w^k, h_τ^k, u_τ^k, and e_τ^k. The following lemma reveals the evolution of the other two terms $\tilde{\mathcal{Z}}^k$ and $\mathcal{Y}^k$.

Lemma 3.3. *If $\mathcal{L}_1 \geq L_f$ and $\theta_1 + \theta_2 \leq 1$, then $\mathbb{E}_k\left[\tilde{\mathcal{Z}}^{k+1} + \mathcal{Y}^{k+1}\right]$ can be upper bounded by*

$$\frac{\mathcal{L}_1}{\mathcal{L}_1+\eta\mu/2}\tilde{\mathcal{Z}}^k + (1-\theta_1-\theta_2)\mathcal{Y}^k + pq\mathcal{W}^k + \left(\frac{\mathcal{L}_1}{2\eta} + \frac{\mu_f}{2}\right)\|e^k\|^2 + \left(\frac{\mathcal{L}_1}{2\eta} + \frac{\mu}{2}\right)\mathbb{E}_k\left[\|e^{k+1}\|^2\right]$$
$$-\frac{1}{\theta_1}\left(\theta_2 - \frac{2L}{n\mathcal{L}_1}\right)B_f(w^k, x^k) - \frac{1-\theta_1-\theta_2}{\theta_1}B_f(y^k, x^k).$$

Because of the compression, we have the additional error terms $\|e^k\|^2$ and $\|e^{k+1}\|^2$ in the evolution of $\tilde{\mathcal{Z}}^k$ and $\mathcal{Y}^k$ in Lemma 3.3. To upper bound these error terms, we need to analyze the evolution of $\frac{1}{n}\sum_{\tau=1}^n e_\tau^k$ and $\frac{1}{n}\sum_{\tau=1}^n \|h_\tau^k - \nabla f^{(\tau)}(w^k)\|^2$ in the next two lemmas.

Lemma 3.4. *The quantity $\mathbb{E}_k\left[\frac{1}{n}\sum\limits_{\tau=1}^n \|e_\tau^{k+1}\|^2\right]$ is upper bounded by the expression*

$$\left(1-\frac{\delta}{2}\right)\frac{1}{n}\sum_{\tau=1}^n \|e_\tau^k\|^2 + \frac{4(1-\delta)\eta^2}{\delta n\mathcal{L}_1^2}\sum_{\tau=1}^n \|\nabla f^{(\tau)}(w^k) - h_\tau^k\|^2 + \frac{2(1-\delta)\eta^2}{\mathcal{L}_1^2}\left(\frac{4\bar{L}}{\delta} + L\right)B_f(w^k, x^k).$$

Lemma 3.5. *The quantity $\frac{1}{n}\sum_{\tau=1}^n \mathbb{E}_k[\|h_\tau^{k+1} - \nabla f^{(\tau)}(w^{k+1})\|^2]$ is upper bounded by the expression*

$$\left(1-\frac{\delta_1}{2}\right)\frac{1}{n}\sum_{\tau=1}^n \|h_\tau^k - \nabla f^{(\tau)}(w^k)\|^2 + 4\bar{L}p\left(1+\frac{2p}{\delta_1}\right)B_f(y^k, x^k) + 4\bar{L}p\left(1+\frac{2p}{\delta_1}\right)B_f(w^k, x^k).$$

Under the additional Assumption 2.1, we need to analyze the evolution of $\|e^k\|^2$ and $\|h^k - \nabla f(w^k)\|^2$ as well, which can be found in the appendix.

3.3 Convergence analysis: main results

With the above lemmas at hand, we are ready to construct suitable Lyapunov functions which enable us to prove linear convergence. First, we construct the Lyapunov function Φ^k for the general case. Let $\mathcal{L}_2 := \frac{6L}{n} + \frac{112(1-\delta)\bar{L}}{3\delta^2} + \frac{28(1-\delta)L}{3\delta} + \frac{224(1-\delta)\bar{L}p}{\delta^2\delta_1}\left(1+\frac{2p}{\delta_1}\right)$, and for $k \geq 0$ define

$$\Phi^k := \tilde{\mathcal{Z}}^k + \mathcal{Y}^k + \mathcal{W}^k + \frac{4\mathcal{L}_1}{\delta\eta}\cdot\frac{1}{n}\sum_{\tau=1}^n \|e_\tau^k\|^2 + \frac{56(1-\delta)}{3\theta_1\delta^2\delta_1\mathcal{L}_1}\cdot\frac{1}{n}\sum_{\tau=1}^n \|h_\tau^k - \nabla f^{(\tau)}(w^k)\|^2.$$

Then we can get the linear convergence of Φ^k in the following theorem.

Theorem 3.6. *Let Assumption 3.1 and Assumption 3.2 hold. If $\mathcal{L}_1 \geq \max\{L_f, 3\mu\eta\}$, $\theta_1 + 2\theta_2 \leq 1$, and $\theta_2 \geq \frac{\mathcal{L}_2}{3\mathcal{L}_1}$, then we have*

$$\mathbb{E}\left[\Phi^k\right] \leq \left(1 - \min\left(\frac{\mu}{\mu+6\theta_1\mathcal{L}_1}, \theta_1+\theta_2-\frac{\theta_2}{q}, p(1-q), \frac{\delta}{6}, \frac{\delta_1}{6}\right)\right)^k \Phi^0, \ \ \forall k \geq 0.$$

If Assumption 2.1 holds, we define the Lyapunov function Ψ^k as follows. Let $\mathcal{L}_3 := \frac{6L}{n} + \frac{112(1-\delta)L_f}{3\delta^2} + \frac{224(1-\delta)\bar{L}}{\delta^2 n} + \frac{308(1-\delta)L}{3\delta n} + \frac{224(1-\delta)p}{\delta^2\delta_1}\left(L_f + \frac{9\bar{L}}{n}\right)\left(1+\frac{2p}{\delta_1}\right)$, and for $k \geq 0$ define

$$\begin{aligned}\Psi^k \quad := \quad & \tilde{\mathcal{Z}}^k + \mathcal{Y}^k + \mathcal{W}^k + \frac{4\mathcal{L}_1}{\delta\eta}\|e^k\|^2 + \frac{28\mathcal{L}_1(1-\delta)}{\delta\eta n}\cdot\frac{1}{n}\sum_{\tau=1}^n \|e_\tau^k\|^2 \\ & + \frac{56(1-\delta)\eta}{\delta^2\delta_1\mathcal{L}_1}\|h^k - \nabla f(w^k)\|^2 + \frac{504(1-\delta)\eta}{\delta^2\delta_1 n\mathcal{L}_1}\cdot\frac{1}{n}\sum_{\tau=1}^n \|h_\tau^k - \nabla f^{(\tau)}(w^k)\|^2.\end{aligned}$$

Then we can get the linear convergence of Ψ^k in Theorem 3.7.

Theorem 3.7. *Let Assumption 2.1 , Assumption 3.1 and Assumption 3.2 hold. If $\mathcal{L}_1 \geq \max\{L_f, 3\mu\eta\}$, $\theta_1 + 2\theta_2 \leq 1$, and $\theta_2 \geq \frac{\mathcal{L}_3}{3\mathcal{L}_1}$, then we have*

$$\mathbb{E}\left[\Psi^k\right] \leq \left(1 - \min\left(\frac{\mu}{\mu+6\theta_1\mathcal{L}_1}, \theta_1+\theta_2-\frac{\theta_2}{q}, p(1-q), \frac{\delta}{6}, \frac{\delta_1}{6}\right)\right)^k \Psi^0, \ \ \forall k \geq 0.$$

From Theorems 3.6 and 3.7, by choosing the parameters appropriately, we can obtain the iteration complexity for ECLK.

Theorem 3.8. *Let Assumption 3.1 and Assumption 3.2 hold. Let* $\mathcal{L}_1 = \max(\mathcal{L}_4, L_f, 3\mu\eta)$, $\theta_2 = \frac{\mathcal{L}_4}{3\max\{L_f, \mathcal{L}_4\}}$, *and*

$$\theta_1 = \begin{cases} \min\left(\sqrt{\frac{\mu}{\mathcal{L}_4 p}}\theta_2, \theta_2\right) & \text{if } L_f \le \frac{\mathcal{L}_4}{p} \\ \min\left(\sqrt{\frac{\mu}{L_f}}, \frac{p}{3}\right) & \text{otherwise} \end{cases}.$$

(i) Let $\mathcal{L}_4 = \mathcal{L}_2$. *Then with some* $q \in [\frac{2}{3}, 1)$, *we have* $\mathbb{E}[\Phi^k] \le \epsilon\Phi^0$ *for*

$$k \ge \mathcal{O}\left(\left(\frac{1}{\delta} + \frac{1}{\delta_1} + \frac{1}{p} + \sqrt{\frac{L_f}{\mu}} + \sqrt{\frac{\mathcal{L}_2}{\mu p}}\right)\log\frac{1}{\epsilon}\right).$$

In particular, if $p \le \mathcal{O}(\delta_1)$, *then the iteration complexity becomes*

$$k \ge \mathcal{O}\left(\left(\frac{1}{\delta} + \frac{1}{p} + \sqrt{\frac{L_f}{\mu}} + \sqrt{\frac{L}{\mu p n}} + \frac{1}{\delta}\sqrt{\frac{(1-\delta)\bar{L}}{\mu p}} + \sqrt{\frac{(1-\delta)L}{\mu p \delta}}\right)\log\frac{1}{\epsilon}\right). \tag{10}$$

(ii) Let $\mathcal{L}_4 = \mathcal{L}_3$. *If Assumption 2.1 holds, then for some* $q \in [\frac{2}{3}, 1)$, *we have* $\mathbb{E}[\Psi^k] \le \epsilon\Psi^0$ *for* $k \ge \mathcal{O}\left(\left(\frac{1}{\delta} + \frac{1}{\delta_1} + \frac{1}{p} + \sqrt{\frac{L_f}{\mu}} + \sqrt{\frac{\mathcal{L}_3}{\mu p}}\right)\log\frac{1}{\epsilon}\right)$. *If* $p \le \mathcal{O}(\delta_1)$, *the iteration complexity becomes*

$$k \ge \mathcal{O}\left(\left(\frac{1}{\delta} + \frac{1}{p} + \sqrt{\frac{L_f}{\mu}} + \sqrt{\frac{L}{\mu p n}} + \frac{1}{\delta}\sqrt{\frac{(1-\delta)L_f}{\mu p}} + \sqrt{\frac{(1-\delta)L}{\mu p \delta n}}\right)\log\frac{1}{\epsilon}\right). \tag{11}$$

From Theorem 3.8, it is easy to see that the optimal p for ECLK is $\Theta(\delta_1)$.

Comparison to ADIANA. ADIANA [Li et al., 2020] is an accelerated and compressed distributed method with any unbiased compressor. The iteration complexity of ADIANA is $\mathcal{O}\left(\omega\left(1 + \sqrt{\frac{\bar{L}}{n\mu}}\right)\log\frac{1}{\epsilon}\right)$ when $n \le \omega$, and $\mathcal{O}\left(\left(\omega + \sqrt{\frac{\bar{L}}{\mu}} + \sqrt{\sqrt{\frac{\omega}{n}}\frac{\omega\bar{L}}{\mu}}\right)\log\frac{1}{\epsilon}\right)$ when $n > \omega$. If we choose Q and Q_1 in ECLK to be the contraction compressor $\frac{1}{\omega+1}\tilde{Q}$ where $\tilde{Q}$ is any unbiased compressor, then $\delta = \delta_1 = \frac{1}{\omega+1}$, and thus the complexity in (11) becomes

$$\mathcal{O}\left(\left(\omega + \sqrt{\frac{L_f}{\mu}} + \sqrt{\frac{(\omega+1)L}{\mu n}} + (\omega+1)\sqrt{\frac{\omega L_f}{\mu}} + \sqrt{\frac{\omega(\omega+1)L}{\mu n}}\right)\log\frac{1}{\epsilon}\right),$$

for $p = \delta_1$. In this case, the dependency of ω of the iteration complexity of ADIANA is better than that of ECLK. However, the full gradients are computed by all nodes at each iteration in ADIANA, which could be slower than the communication when m is very large. Furthermore, the contraction compressor could be more efficient than the unbiased compressor in practice, i.e., for the same level of compression, δ could be larger than $1/(\omega+1)$. This efficiency of the contraction compressor is the motivation to study the error compensated methods, and in numerical experiments, ECLK is comparable to ADIANA.

4 Experiments

In this section, we experimentally study the performance of error compensated L-Katyusha (ECLK) used with several contraction compressors on the logistic regression problem for binary classification,

$$x \mapsto \log\left(1 + \exp(-y_i A_i^T x)\right) + \frac{\lambda}{2}\|x\|^2,$$

where $\{A_i, y_i\}$ are the training data points. We use the datasets `a5a`, `a9a`, `w6a`, `w8a`, `phishing`, and `mushrooms` from the LIBSVM library [Chang and Lin, 2011]. The regularization parameter was set to $\lambda = 10^{-3}$. The number of nodes in our experiments is $n = 20$, and the optimal solution is obtained by running the uncompressed L-Katyusha for 10^5 iterations. More experiments can be found in the appendix.

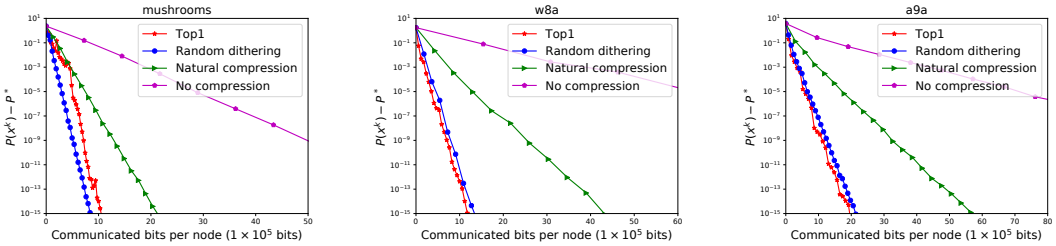

Figure 1: The communication complexity performance of ECLK used with compressors: Top1 vs random dithering vs natural compression vs no compression. Datasets: `mushrooms`, `w8a`, and `a9a`.

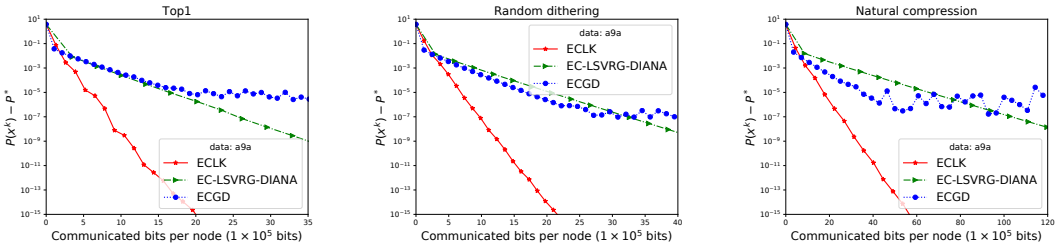

Figure 2: The communication complexity performance of ECGD vs EC-LSVRG-DIANA vs ECLK for Top1, Random dithering, and Natural compression on `a9a` data set.

Compressors. We use three contraction compressors: the TopK compressor with $K = 1$, or the compressor $\frac{1}{\omega+1}\tilde{Q}$, where $\tilde{Q}$ is either the unbiased random dithering compressor with $s = \sqrt{d}$ levels [Alistarh et al., 2017] or the natural compressor [Horváth et al., 2019b]. For the Top1 compressor, the number of communicated bits for the compressed vector is $64 + \lceil \log d \rceil$, and $\delta = 1/d$. For the random dithering compressor with $s = \sqrt{d}$, the number of communicated bits for the compressed vector is $2.8d + 64$, and $\omega = 1$ [Alistarh et al., 2017] . For the natural compressor, the number of communicated bits for the compressed vector is $12d$, and $\omega = 1/8$ [Horváth et al., 2019b].

Parameter setting. In all the experiments, we search for the optimal stepsize for all tested algorithms. In particular, we use the parameter setting in Theorem 3.8 (i) for ECLK. We calculate the theoretical L_f, $\bar{L}$, and L as L_f^{th}, $\bar{L}^{th}$, and L^{th}, respectively. Then we choose $L_f = t \cdot L_f^{th}$, $\bar{L} = t \cdot \bar{L}^{th}$, and $L = t \cdot L^{th}$, and search for the best t in the set $t \in \{10^{-k} \mid k = 0, 1, 2, ...\}$. Q and Q_1 are chosen to be the same contraction compressor for ECLK. We set $p = \delta$ for ECLK and EC-LSVRG-DIANA [Gorbunov et al., 2020], except in Section 4.4. EC-LSVRG-DIANA requires the use of an additional unbiased compressor; for this we make use of random dithering.

4.1 Top1 vs random dithering vs natural compression vs no compression

First, we compare the uncompressed L-Katyusha method with ECLK with three contraction compressors: Top1, random dithering, and natural compression; see Figure 1. ECLK is orders of magnitude better in terms of communication complexity.

4.2 Comparison with EC-LSVRG-DIANA and ECGD

Next, we compare ECLK with EC-LSVRG-DIANA [Gorbunov et al., 2020] and error compensated GD (ECGD) using the Top1, random dithering, and natural compressor. Note that ECGD is a special case of error compensated SGD [Stich et al., 2018] with $m = 1$, where the full gradient $\nabla f^{(\tau)}(x^k)$ is calculated on each node. Figures 2 and 3 show that ECGD can only converge to a neighborhood of the optimal solution, and that ECLK is considerably faster than EC-LSVRG-DIANA.

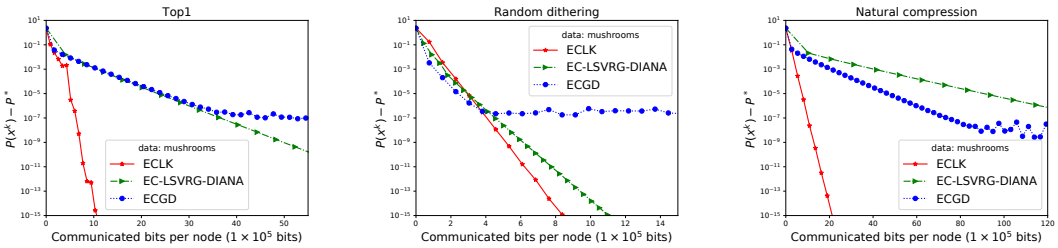

Figure 3: The communication complexity performance of ECGD vs EC-LSVRG-DIANA vs ECLK for Top1, Random dithering, and Natural compression on `mushrooms` data set.

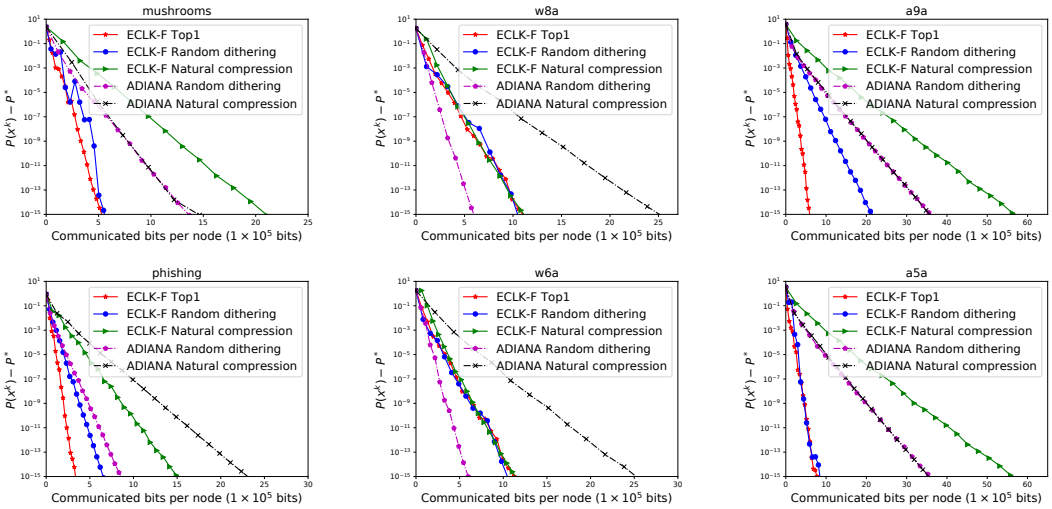

Figure 4: The communication complexity performance of ECLK vs ADIANA for Top1, Random dithering, and Natural compression on `mushrooms`, `w8a`, `a9a`, `phishing`, `w6a`, and `a5a` data sets.

4.3 Comparison with ADIANA

Let us denote by ECLK-F the special case of ECLK with $m = 1$, i.e., the full gradient $\nabla f^{(\tau)}(x^k)$ is calculated on each node. We compare ECLK-F with the accelerated variant of DIANA, called ADIANA [Li et al., 2020], for six data sets. For ADIANA we use two unbiased compressors: random dithering and natural compression. Figure 4 shows that in terms of communication complexity, ECLK-F is comparable to ADIANA, and can be better than ADIANA in some cases.

4.4 Impact of the update frequency of the reference point

Finally, we test the impact of the update frequency of the reference point p for ECLK with Top1 compressor; see Figure 5. We choose five values for p: $\delta/3$, δ, 3δ, 9δ, and 1. Figure 5 shows that the performance of $p = \delta$ is usually better than $p = \delta/3$. As we increase p from $p = \delta$, the performance of ECLK could be improved. However, if p is too large, the performance of ECLK is no better than $p = \delta$, generally.

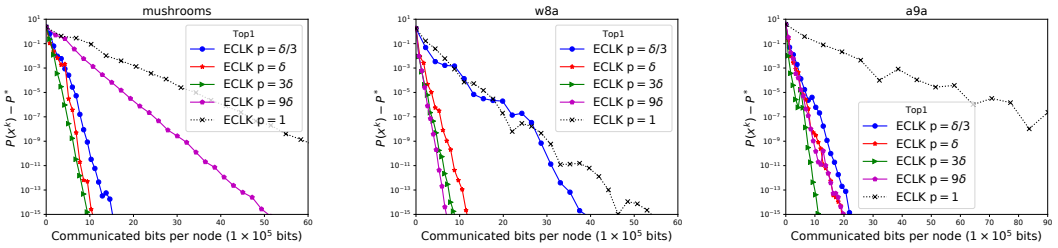

Figure 5: The communication complexity performance of ECLK with the Top1 compressor and $p \in \{\delta/3, \delta, 3\delta, 9\delta, 1\}$ on the `mushrooms`, `w8a`, and `a9a` data sets.

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Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] We need Assumptions 3.1 and 3.2 for our results.
   (c) Did you discuss any potential negative societal impacts of your work? [N/A]
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] See Assumptions 2.1, 3.1, and 3.2.
   (b) Did you include complete proofs of all theoretical results? [Yes] We include the complete proofs in the Appendix.
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [Yes] Data sets are from the LIBSVM library. Code and instructions are in the supplemental material.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See Section 4
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [No] We used a fixed random seed to make sure the numerical results can be reproduced.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No] We run the experiments on a laptop, and we did not count the time. Hence the results are independent of the amount of compute and the type of resources.
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes] We used the LIBSVM library and cited.
   (b) Did you mention the license of the assets? [Yes]
   (c) Did you include any new assets either in the supplemental material or as a URL? [No]
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [No]
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]