How to Leverage Predictive Uncertainty Estimates for Reducing Catastrophic Forgetting in Online Continual Learning

Anonymous authors
Paper under double-blind review

Abstract

Many real-world applications require machine-learning models to be able to deal with non-stationary data distributions and thus learn autonomously over an extended period of time, often in an online setting. One of the main challenges in this scenario is the so-called catastrophic forgetting (CF) for which the learning model tends to focus on the most recent tasks while experiencing predictive degradation on older ones. In the online setting, the most effective solutions employ a fixed-size memory buffer to store old samples used for replay when training on new tasks. Many approaches have been presented to tackle this problem and conflicting strategies are proposed to populate the memory. Are the easiest-to-forget or the easiest-to-remember samples more effective in combating CF? Furthermore, it is not clear how predictive uncertainty information for memory management can be leveraged in the most effective manner. Starting from the intuition that predictive uncertainty provides an idea of the samples' location in the decision space, this work presents an in-depth analysis of different uncertainty estimates and strategies for populating the memory. The investigation provides a better understanding of the characteristics data points should have for alleviating CF. Then, we propose an alternative method for estimating predictive uncertainty via the generalised variance induced by the negative log-likelihood. Finally, we demonstrate that the use of predictive uncertainty measures helps in reducing CF in different settings.

1 Introduction

Typical machine learning models assume to work in a single-task static scenario where multiple epochs are performed over the same data until convergence. In many real-world situations, however, this setting is too limiting. As an example, let us consider the problem of product recommendation. Since trends may change and new types of products may arrive, new product categories need to be classified. In this context, a typical learning model would fail because the standard setup does not account for the continuous addition of new classes. For this reason, *online* Continual Learning (online-CL) has been constantly more explored. In online-CL, a single model is required to learn continuously from a sequence of tasks that comes as a stream of tiny batches which can be processed only once (Aljundi et al., 2019b; 2017; Mai et al., 2022). Given the overlap between old and new information, the model tends to forget about the past knowledge to focus more on the newest tasks, leading to a performance degradation on previous tasks. This challenge is usually referred to as catastrophic forgetting (CF) (McCloskey & Cohen, 1989; Ratcliff, 1990). Many approaches have been developed to prevent catastrophic forgetting and the most successful ones employ a memory buffer for rehearsal (Chaudhry et al., 2019; Soutif-Cormerais et al., 2023). Memory-based strategies tackle CF by training the model on both current samples and some old samples stored in a limited size memory. In this context, what differentiates each memory-based approach are the retrieval strategy and the update strategy (Mai et al., 2022) — i.e., how to populate and update the memory with meaningful and representative samples, and how to efficiently sample from the memory respectively. Although many approaches have been developed in this direction – ranging from using random approaches (Chaudhry et al., 2019) to exploiting the gradient (Lopez-Paz & Ranzato, 2017; Chaudhry et al., 2018b; Aljundi et al., 2019b)

or the loss (Belilovsky et al., 2019) information – the effect of predictive uncertainty estimates for memory management is largely unexplored. One of the few methods in this direction is presented in Bang et al. (2021), where the authors propose a combination of predictive uncertainty and data augmentation at the memory-level to generate a memory of diverse samples – from the most uncertain to the most representative ones – for each class. Despite remarkable results, the contribution of the uncertainty score on the overall performance is not easily quantifiable. Furthermore, considering the quickly expanding landscape of recently rehearsal-based methods proposed, it is difficult to identify a clear strategy to exploit the memory at its best. In fact, contrasting strategies can be found in the literature. For instance, Kumari et al. (2022) suggest working on instances close to the decision boundary, thus considering the marginal samples as the most important for alleviating CF. Contrarily, in Hurtado et al. (2023) and Yoon et al. (2021), the proposed strategies focus on populating the memory with the most representative samples for each class. Hence, it is unclear which type of data points would reduce CF in a consistent manner: Are the easiest-to-forget or the easiest-to-remember samples more effective in combating CF? To answer these and other questions, this paper investigates the contribution of predictive uncertainty scores on reducing CF. Intuitively, using predictive uncertainty to populate the memory represents a solution for identifying the location of the instances in the decision space and considering either the most representative – i.e., the k samples with the *lowest* uncertainty (bottom-k) – or the marginal ones (i.e., the top-k with the *highest* uncertainty). In the first part of this work, we evaluate and compare different combinations of uncertainty scores and sorting on CIFAR-10 and CIFAR-100 (Krizhevsky et al., 2009), two datasets commonly used in CL. The objective is to understand how different uncertainty estimates behave when used in different ways under same conditions. In order to achieve the intended goal, following a recent trend on research transparency and comparability (Mundt et al., 2021), the evaluation framework will be freed from any other 'trick' that could create ambiguity in the assessment. The investigation will focus on describing the characteristics of the considered uncertainty scores, while providing an in-depth analysis of the effect of the metrics under a memory-based regime. In addition to the comparison of established uncertainty scores, we propose a new selection strategy based on recent advances in uncertainty quantification via the logit space. More specifically, we introduce a memory management strategy based on the Bregman Information (BI) as a generalised variance measure, which stems directly from a bias-variance decomposition of the model loss (Gruber & Buettner, 2023).
In the second part of our work, we evaluate our findings on more challenging and realistic scenarios. In realistic setups, recent tasks have less data points available than older tasks as the time to collect instances is considerably shorter than for previous tasks. Considering this behaviour, we will focus on long-tailed (LT) datasets with this characteristic. We test our findings on two datasets with controllable degrees of data imbalance, as well as a real-world imbalanced dataset for classification of biomedical images (Yang et al., 2023). The goal of this work is not to propose a new method that outperforms the state-of-the-art, but rather to present practical insights from an uncertainty-aware perspective on the desirable characteristics samples should have to alleviate catastrophic forgetting in both standard and realistic scenarios.

Thus, the main contributions of this paper can be summarised as follows:

- We conduct a systematic evaluation of established predictive uncertainty scores to assess their effectiveness in mitigating catastrophic forgetting (CF) in online continual learning for image classification tasks.
- We introduce an alternative method for estimating predictive uncertainty by using the generalized variance derived from the classification loss function.
- We analyze the key characteristics that samples need to possess to effectively reduce CF from an uncertainty-aware perspective.
- We demonstrate that selecting the most representative samples (bottom-k) to populate the memory consistently reduces CF, as compared to choosing marginal samples (top-k), as illustrated in Figure 1.
- Different from conventional evaluation pipelines, we validate our findings on both standard benchmarks and realistic scenarios, including challenging settings like medical image classification with imbalanced data.

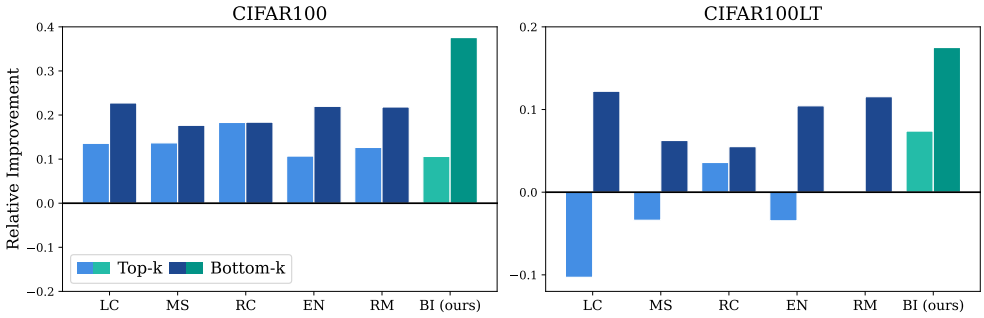

Figure 1: Relative CF improvement of different uncertainty scores over ER when using the easiest-to-forget (top-k) and the easiest-to-remember (bottom-k) samples. The bottom-k strategy (in darker colour) reduces CF in all cases. Furthermore, the proposed BI-based uncertainty estimate (in green) further diminishes CF compared to common uncertainty scores.

2 Related Work

2.1 Continual Learning

According to van de Ven et al. (2022), three different scenarios of incremental learning (IL) can be identified; 1) *Task-IL* where the task ID information is available at both training and testing time; 2) *Domain-IL* in which the learning task remains unchanged (e.g., binary classification) but there is a shift in the input distribution; and 3) *Class-IL* where the number of classes to discriminate can grow over time. Additionally, CL problems can be further categorised depending on whether the data can be accessed and processed multiple times (*offline*) or a single-pass through the data is expected (*online*) (Mai et al., 2022). In this work, following a popular trend in the literature, we focus on the most challenging scenario, i.e., online class-IL.

2.2 Class-Incremental Learning

Following the classification proposed by Mai et al. (2022), class-IL approaches can be grouped into the following categories: a) Regularization techniques that adjust the model parameter updates by incorporating penalty terms in the loss function (Aljundi et al., 2018; Lee et al., 2017), modifying parameter gradients during optimization (Chaudhry et al., 2018b; He & Jaeger, 2018), or employing knowledge distillation (Wu et al., 2019; Rannen et al., 2017); b) Memory-based techniques in which a fixed-size subset of past samples is stored for replay (Aljundi et al., 2019a; Chaudhry et al., 2019) or for regularization purposes (Nguyen et al., 2018; Tao et al., 2020) ; c) Generative-based techniques which involve training generative models to produce pseudo-samples that replicate the information from past tasks (Lesort et al., 2019; Shin et al., 2017), and d) Parameter-isolation-based techniques that allocate distinct model parameters to each task, either by activating only the relevant parameters for each task (Fixed Architecture) (Mallya & Lazebnik, 2018; Serra et al., 2018) or by adding new parameters while keeping the existing ones unchanged (Dynamic Architecture) (Yoon et al., 2018; Aljundi et al., 2017).

In online-CL, where data are received in mini-batches and the model is updated with high frequency, rehearsal-based methods are favoured over more complex solutions like generative methods due to their flexibility and reduced training time (Mai et al., 2022).

2.2.1 Population Strategies in Rehearsal-based CL

As already anticipated, there exist methods that use different metrics for deciding the samples to store in the memory. In Aljundi et al. (2019b), the gradient information is used as a feature to maximize the diversity of the samples in the memory. Similarly, in Yoon et al. (2021), gradient vectors are used to select the most representative and informative coreset at each iteration. Other methods exploit the loss information

to select samples based on their interference on the loss function (Aljundi et al., 2019a) or to analyze the loss region for improving generalization and minimizing CF (Verwimp et al., 2021). Sun et al. (2021) and Wiewel & Yang (2021) address the problem of memory diversity from an information-based perspective via entropy-based functions. As previously mentioned, the characteristics samples should have to be stored in the memory and alleviate CF are not clear. For instance, Hurtado et al. (2023) propose a method to eliminate outliers from the memory such that only the most representative samples for each class are kept via a label-homogeneity score. Intuitively, their approach inspect the neighbourhood of a given sample to check whether it is surrounded by samples from different classes or not. If the label homogeneity score is low (i.e., most of the nearest samples do not share the same class label), it means we are in the presence of an outlier or noisy sample and the considered sample should be discarded from memory sampling. Contrarily, in Kumari et al. (2022), the authors claim that the replay-phase should focus on *marginal samples* and propose a method to synthesize samples near the forgetting boundary that are confused with the current task. The approach consists on identifying the easiest-to-forget samples in the memory and then on moving those samples closer to the data points of the current task. Finally, the new perturbed samples are used for replay to cover marginal samples more consistently.

In both cases, the idea is similar to using uncertainty estimates to detect data points with specific characteristics – i.e., outliers or representative samples. Differently from their approaches, which require the samples' vector representation, our method exploits predictive uncertainty for the same purpose. It can be seen as a more principled way of identifying whether a sample is representative of its class or not. Nevertheless, despite the use of predictive uncertainty looks appealing, its usage for populating the memory with samples with desired properties is largely unexplored.

2.2.2 Uncertainty-aware Memory Management in CL

The idea of employing predictive uncertainty for memory management is borrowed from Active Learning (AL). In this context, uncertainty sampling (Lewis & Gale, 1994) is used to decide which samples from a pool of unlabeled data will be considered for labeling. The main assumption is that the most uncertain samples represent the most informative data points. Thus, by including them in the training set, we can improve the overall performance. Popular examples of uncertainty scores include, to name a few, entropy, smallest margin, and least confidence (Shannon, 1948; Campbell et al., 2000; Lewis & Gale, 1994; Culotta & McCallum, 2005).

Analogously, we can use estimates of predictive uncertainty for populating memory buffers in replay-based methods for online-CL. In Bang et al. (2021), the authors argue that samples stored in the memory should be representative of their own class, but also discriminative towards the remaining ones. Starting from this assumption, they evaluate the relative position of the samples in the decision space by estimating the predictive uncertainty of perturbed samples; exemplars with high uncertainty should be closer to the decision boundary, while the most certain ones should be located closer to the center of the corresponding class distribution. To ensure diversity, they use a step-sized sampling which guarantees to keep in the memory samples that span from the most uncertain to the most certain ones. Given a list P of perturbations, the authors propose to use the following uncertainty score $u_{RM}(x)$:

$$u_{RM}(x) = 1 - \frac{1}{P} \max_c S_c \tag{1}$$

where $S_c = \sum_{i=1}^{P} \mathbb{1}_c \operatorname{argmax}_{\hat{c}} p(y = \hat{c}|\tilde{x}_i)$. Eq. (1) represents an agreement score with respect to the perturbations. If all the perturbed versions of x are predicted with the same label, then $u_{RM}(x)$ will be 0. Otherwise, the higher is $u_{RM}(x)$ the most uncertain is the model about the considered sample.

The authors claim to work in the online setting. Nevertheless, the definition of online setting in the paper differs from the popular one. In their case, *online* means that the training stream is only processed once and the memory is updated at the end of each task. In the standard scenario, instead, *online* implies that only a mini-batch of data points is available to be processed (Mai et al., 2022), and the model and the memory are updated with high frequency (Soutif-Cormerais et al., 2023). In addition to this, to further enhance diversity, they employ data augmentation on the memory. Thus, the contribution of Eq. (1) on reducing CF is unclear.

In the remainder of this work, we systematically investigate the effect of established uncertainty scores - including u_{RM} - as well as the newly proposed Bregman Information on combating catastrophic forgetting in a realistic online-CL setting.

3 Preliminaries and Notation

Following the notation proposed in Bang et al. (2021), we assume to have a set $\boldsymbol{\mathcal{C}} = \{c_1, \ldots, c_n\}$ of n different classes. Each class can be randomly assigned to a task t. T_t represents a subset of classes determined by an assign function $\psi(c)$ such that $T_t = \{c|\psi(c) = t\}$. For each task t, we have an associated dataset $D_t = \{(x_i, y_i)\}_{i=1}^{n_t}$ with x_i an input sample, y_i the corresponding class label, and n_t the number of training samples. In the proposed *online* setting, we assume that samples for each task t arrive sequentially in a stream of mini-batches $b_t = \{(x_i, y_i) \in D_t\}_{i=1}^{bs}$, with each mini-batch available for a single pass. It is important to notice that, for each task t, n_t can vary depending on whether we are working on a class-balanced setting or not. Finally, for replay, we introduce a fixed-size memory buffer $\mathcal{M}$ to store a portion of samples from past tasks. The memory is updated with high-frequency whenever a mini-batch from the stream of data is processed. Following the standard procedure in online-CL, for replay, we assume to extract from the memory a number of samples equal in size to the batch size.

4 Methodology

To facilitate a fair evaluation of the framework, we focus on the assessment of the uncertainty metrics under same conditions. In this way, we eliminate any ambiguous effect which could be inherited from other methodological choices. As anticipated in Section 1, there are two main objectives when dealing with memory-based approaches: 1) memory populating, and 2) memory sampling.

In the following subsections, we describe the available strategies for each of the two objectives, and the uncertainty scores considered in our evaluation. Finally, we propose an alternative uncertainty estimate that overcomes the limitations of the most popular uncertainty metrics.

4.1 Memory Management

Memory Population. Apart from the criteria-based population strategies presented in Section 2.2.1, random strategies have demonstrated to work surprisingly well in online-CL. *Reservoir* (Vitter, 1985) is a random sampling technique without replacement to select n samples from a pool of N samples where N is unknown and $N > n$. This strategy is used in Chaudhry et al. (2019) as a way to populate a memory of size M from a stream of data points with unknown length. In this way, each data point has a probability of being included in the memory equal to $\frac{M}{N}$. *Class-balanced Reservoir*, instead, is based on a class-balanced random sampling strategy (Chrysakis & Moens, 2020). This is important to consider the case where the stream of data is highly imbalanced. Indeed, class-imbalance may further deteriorate the predictive performance of the framework.

Different from the population strategy proposed in Chrysakis & Moens (2020), where each class-memory set is populated and updated randomly, we introduce a class-balanced memory management based on predictive uncertainty estimates. This is similar to Bang et al. (2021), where the intent is to keep in the memory samples with desired characteristics for each class.

Memory Sampling. Starting from the second task, we need a strategy to sample data points from the memory. The easiest way is *random sampling*, which selects at random a subset from the available data points in the memory buffer excluding the ones from the current task. The size of the replay set is equal to the batch size. Another option would be to extract past samples with specific characteristics. For this purpose, we can exploit uncertainty estimates. However, this solution represents an effective strategy only in the case the memory is populated at random. Furthermore, given the size of the memory buffer and the frequency of memory updates, computing uncertainty estimates for the whole buffer may be computationally expensive and not feasible during training.

In our case, since the memory is populated with samples with specific characteristics, the random sampling is sufficient.

4.2 Uncertainty Metrics

4.2.1 Predictive Uncertainty Estimation

Following previous works (Bang et al., 2021; Wang et al., 2022), to compute predictive uncertainty estimations for a given input image x, we employ the ensembling technique Test-Time Augmentation (TTA) (Wang et al., 2019). The set of transformations applied are the ones usually employed for image classification. A detailed list of the transformations used in our evaluation can be found in the supplementary. After selecting the set of transformations, we apply them on the selected input. In this way, we create a set of P perturbed inputs $\tilde{x}$ which are used at test-time to compute their corresponding logits $\hat{z}$. Finally, the generated logits are used by the uncertainty estimates in different ways to compute the uncertainty score $u(x)$.

4.2.2 Predictive Uncertainty Scores

In our investigation, we consider the following popular uncertainty metrics:

- *Least Confidence (LC)* (Culotta & McCallum, 2005) determines the level of predictive uncertainty by examining the samples with the smallest predicted class probability. A model is less certain about the sample if it associates the most probable class $y_{(1)}$ with a low probability.

$$u(x)_{LC} = 1 - \frac{1}{P}\sum_{i=1}^{P} p(y_{(1)} = c|\tilde{x}_i) \tag{2}$$

- *Margin Sampling (MS)* (Campbell et al., 2000) calculates predictive uncertainty by computing the difference between the most probable predicted class $y_{(1)}$ and the second largest one $y_{(2)}$. If the difference is low, the model is uncertain.

$$u(x)_{MS} = 1 - \frac{1}{P}\sum_{i=1}^{P} \big(p(y_{(1)} = c|\tilde{x}_i) - p(y_{(2)} = c|\tilde{x}_i)\big) \tag{3}$$

- *Ratio of Confidence (RC)* (Campbell et al., 2000) is similar to MS. In this case, the ratio between the probabilities of the two most probable classes is computed. The closer the ratio is to 1, the more uncertain the model is about the considered sample.

$$u(x)_{RC} = \frac{1}{P}\sum_{i=1}^{P} \left(\frac{p(y_{(2)} = c|\tilde{x}_i))}{p(y_{(1)} = c|\tilde{x}_i))}\right) \tag{4}$$

- *Entropy (EN)* (Shannon, 1948), differently from the above scores, estimates the uncertainty considering the whole probability distribution. If the predicted probabilities are similar to each other (i.e., tend to an uniform distribution), the model is not certain about the sample. The formulation reads as follows:

$$u(x)_{EN} = -\frac{1}{P}\sum_{i=1}^{P} \left(\sum_{j} p(y_j = c|\tilde{x}_i)\log(p(y_j = c|\tilde{x}_i))\right) \tag{5}$$

In addition to these metrics, we will also consider the agreement score $u(x)_{RM}$ reported in Eq. (1).

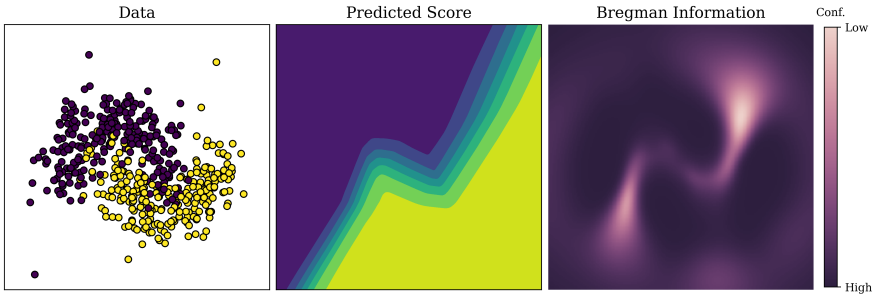

Figure 2: Illustration of the behaviour of the Bregman Information for uncertainty estimation in classification tasks. Given the focus on epistemic uncertainty, BI is low in the presence of high data density areas – i.e., where the uncertainty about the data generating process is low.

4.2.3 Uncertainty Quantification in the Logit Space

Inspecting the metrics described above, we can make two main observations: 1) all the metrics need a normalization step to squash the logits in the $[0, 1]$ range of the probability space; 2) the majority of them compute the uncertainty relying on the largest predicted probabilities only. In both cases, with different magnitudes, there is a loss of information compared to using the logit space. Importantly, such confidence scores are a reliable measure of predictive uncertainty only when the model is well calibrated, which is not the case in many real-world scenarios (Gruber & Buettner, 2023; Ovadia et al., 2019; Tomani & Buettner, 2021).

Furthermore, the confidence scores detailed in Section 4.2.2 mostly capture the aleatoric uncertainty which is inherent in the data (Wimmer et al., 2023). For this reason, we believe it would be favorable to focus on samples for which the uncertainty about the data generating process (i.e., the epistemic uncertainty) is low. To provide an intuitive explanation behind this decision, let us consider Figure 2 where we have a graphical representation of the behaviour of the two different sources of uncertainty. With commonly used confidence scores, regions with low confidence correspond to areas close to the decision boundary irrespective of the presence or not of outliers. Contrarily, when exploiting measures mostly based on epistemic uncertainty, low confidence regions correspond to areas in which the data density is low, i.e., where the uncertainty about the data generating process is high. Consequently, if we are interested in the high confidence samples, this would result in selecting different samples depending on the employed score. For confidence scores, given their distance from the decision boundary, we may obtain outliers although not being representative of the class of interest.

In light of these considerations, following recent advances for computing uncertainty estimates in the logit space for classification tasks, we suggest to employ an uncertainty estimator based on the Bregman Information (BI) (Gruber & Buettner, 2023). The authors demonstrate that BI can be used to quantify the variance of the model at the sample level through deep ensembles or TTA. This approach is particularly appealing from a theoretical standpoint, since it stems from a general bias-variance decomposition of proper scoring rules which gives rise to the Bregman Information as the variance term of the loss.
For the commonly used cross-entropy loss, this variance term – that can be directly associated to the epistemic uncertainty – can be estimated via P TTA perturbations as follows:

$$u(x)_{BI} = \frac{1}{P} \sum_{i=1}^{P} \text{LSE}(\hat{z}_i) - \text{LSE}\left(\frac{1}{P} \sum_{i=1}^{p} \hat{z}_i \right), \tag{6}$$

where $\hat{z}_i \in \mathbb{R}^c$ and $\text{LSE}(x_1, \ldots, x_n) = \ln \sum_{i=1}^{n} e^{x_i}$ represent the logits generated by the model and the *LogSumExp* (LSE) function respectively. Intuitively, a large value of $u(x)_{BI}$ means that the logits predicted across the perturbations are very different and, thus, the model is not confident about the considered sample.

4.2.4 Uncertainty Scores Ranking

Depending on the characteristics we want to extract from the images, we can use the uncertainty estimates in different ways. In particular, in our experiments we consider three different strategies to select the set of images for populating the memory depending on the uncertainty scores. By sampling images with the highest uncertainty (*top-k*), we aim at extracting the easiest-to-forget samples which are closer to the decision boundaries and/or represent outliers. With the *step-sized* strategy, we aim at sampling a diverse set of images, spanning from the most representative to the most uncertain ones. Finally, with the *bottom-k* approach we aim at finding the most representative set of images for each of the classes seen in the past tasks. Note that, as explained above, different uncertainty scores select different images based on their greater focus on epistemic or aleatoric uncertainty.

5 Experiments

5.1 Datasets and Settings

5.1.1 Datasets

To understand the change in the performance when using different uncertainty strategies, we employ two datasets commonly used in online-CL for image classification, CIFAR-10 and CIFAR-100 (Krizhevsky et al., 2009). To configure the online-CIL setup, we randomly assign with different random seeds a set of classes to 5 tasks. Thus, each task T_t has 2 (CIFAR-10) or 20 (CIFAR-100) classes designated. Once the best strategy is identified, we evaluate our findings on class-imbalanced scenarios to assess the performance under more realistic conditions. First, we employ two artificially controlled imbalanced datasets, CIFAR10-LT and CIFAR100-LT (Cao et al., 2019). As explained in Section 1, with this experiment we want to replicate a realistic scenario in which recent tasks contain less data than the older ones. For this, the most appropriate type of imbalance is the long-tailed (LT) one (Cui et al., 2019) which follows an exponential decay to choose the sample size of each class. Specifically, for both datasets, we decided to use an imbalance factor ρ equal to 0.1. This means that, if the largest class contains, e.g., 500 data points, then the smallest one consists of 50 instances. Finally, we decide to focus on biomedical image analysis using the microscopic peripheral blood cell images dataset (BloodCell) which consists of 8 classes annotated by expert clinical pathologists (Acevedo et al., 2020; Yang et al., 2023). To reflect the intended realistic conditions, we assign 2 classes for each tasks by increasing size. In this way, older tasks have more data points than the most recent ones.

5.1.2 Experimental Settings

The baseline for assessing the goodness of the proposed strategies will be Experience Replay (ER) (Chaudhry et al., 2019). ER consists of a reservoir approach for the memory management part, and a random sampling for replay. We decide to use ER because, despite its simplicity, it is surprisingly competitive compared to more sophisticated and newer approaches. This is corroborated by recent empirical surveys finding that newly introduced methods perform very similarly to the common Experience Replay (ER) method (Soutif-Cormerais et al., 2023). In addition to the standard strategy, we will also consider the class-balanced version (CBR) proposed in Chrysakis & Moens (2020) and previously described in Section 4.1. Furthermore, we assess the results in comparison with Monte Carlo Dropout (MC) (Gal & Ghahramani, 2016) to consider a different uncertainty estimation method.

In all the experiments, we employ a slim version of Resnet18 (He et al., 2016) – as done in previous works (Hurtado et al., 2023; Lopez-Paz & Ranzato, 2017; Soutif-Cormerais et al., 2023; Kumari et al., 2022) –, and use the SGD optimizer with a learning rate of 0.1. Following standard practice, we set the batch size equal to 10. We set the memory size to different values for each dataset to evaluate a variety of memory configurations considering both large or small buffers.

For evaluation, following previous works, we employ the *Last Accuracy* (A) and *Last Forgetting* (F) – defined in Chaudhry et al. (2018a). *Last* refers to the calculation of the metrics at the end of the training on all tasks. Let denote with $a^{t,i}$ and T, the accuracy of task i after learning task t and the total number of tasks respectively.

The last accuracy A is then defined as follows:

$$A = \frac{1}{T} \sum_{i=1}^{T} a^{T,i}. \tag{7}$$

Last Forgetting (F) is defined as the difference between the peak knowledge (i.e., the maximum accuracy) captured about a particular task during the learning process and its accuracy at the end of the learning process. This provides an idea of the information retained about a certain task after learning new tasks. The last forgetting (F) is computed as:

$$F = \frac{1}{T-1} \sum_{t=1}^{T-1} \max_{l \in \{1,\dots,T-1\}} a^{l,j} - a^{T,j}, \qquad \forall j < T. \tag{8}$$

All the experiments are run on three different random seeds.

5.2 Empirical Results

Tables 1 and 2 show the results for all the uncertainty metrics and sorting strategies employed for populating the memory. From the values reported in the tables, we can observe that selecting the most representative samples for each class ('Bottom' column) consistently improves the results in terms of both accuracy (A) and forgetting (F) across all the considered uncertainty scores and memory sizes. Thus, our results indicate that the *easiest-to-remember* samples are beneficial in improving the predictive performance of the learning model and in reducing considerably CF.

The comparison among the different uncertainty strategies suggests that BI outperforms the other metrics in most cases. While the difference among the accuracy values is less pronounced, BI stands out in its effectiveness at mitigating CF. Given the reduced information loss compared to the most popular uncertainty metrics, we think that BI is more sensitive to changes in the predictive space and thus able to detect changes when subtle differences are present. We can notice that MC delivers the smallest forgetting value (F) in most of the cases. However, this is because MC performs poorly in the predictive task (see Figure 3) and considering how forgetting is computed, the resulting value is inevitably smaller. The poor performance of MC compared to other strategies is probably given by the online nature of the setting since, as reported in the original paper, dropout takes longer to converge (Gal & Ghahramani, 2016). Thus, a single-epoch setup may be insufficient for training convergence. All these findings are evident in Figure 3 where we can observe that BI is able to maintain competitive predictive performance on all tasks at the end of the training compared to other strategies.

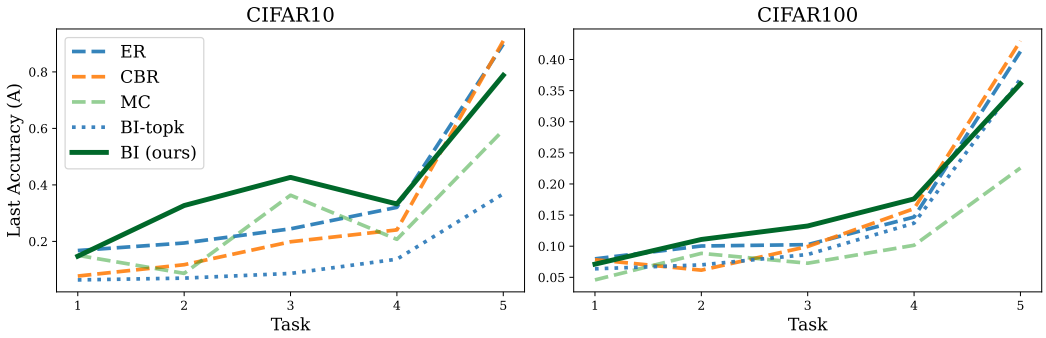

Figure 3: Average task accuracy at the end of the learning procedure for different strategies and datasets. One can observe how the proposed strategy based on the BI perform consistently across tasks and helps maintaining competitive performance on old tasks and thus reducing catastrophic forgetting.

Table 1: Comparison of last accuracy (A) and last forgetting (F) on CIFAR10.

Score		M=500			M=1000		
ER		26.38 ± 2.27 74.92 ± 4.93			36.50 ± 2.16 57.42 ± 5.93		
CBR		24.65 ± 2.10 76.70 ± 5.29			30.88 ± 1.89 66.40 ± 5.58		
		Top	Step	Bottom	Top	Step	Bottom
LC	A(↑)	25.27 ± 0.93	30.51 ± 3.32	34.49 ± 2.91	30.70 ± 2.22	38.03 ± 2.98	38.30 ± 2.98
	F(↓)	72.88 ± 2.76	58.07 ± 6.23	53.11 ± 5.85	64.48 ± 4.09	48.04 ± 6.54	41.71 ± 5.68
MS	A(↑)	26.33 ± 2.07	30.16 ± 0.99	32.50 ± 2.31	33.29 ± 1.41	38.85 ± 4.31	39.36 ± 1.39
	F(↓)	71.08 ± 5.00	61.19 ± 5.74	56.03 ± 4.50	59.75 ± 1.87	48.89 ± 8.57	43.84 ± 3.42
RC	A(↑)	27.84 ± 2.33	30.57 ± 2.43	32.49 ± 3.86	36.35 ± 1.35	37.68 ± 1.17	39.63 ± 0.80
	F(↓)	66.13 ± 7.78	59.70 ± 6.71	54.73 ± 7.66	54.53 ± 4.54	48.08 ± 3.69	41.9 ± 3.53
EN	A(↑)	23.72 ± 1.50	28.76 ± 1.76	35.20 ± 1.52	26.57 ± 0.98	36.13 ± 1.56	35.78 ± 4.99
	F(↓)	77.04 ± 3.78	61.20 ± 3.33	46.12 ± 3.02	71.66 ± 3.78	50.37 ± 3.44	42.65 ± 8.07
RM	A(↑)	26.92 ± 1.75	28.57 ± 0.80	**35.71 ± 4.12**	32.87 ± 2.56	38.23 ± 0.95	38.57 ± 2.09
	F(↓)	71.45 ± 2.92	61.16 ± 4.40	51.01 ± 6.60	62.01 ± 4.36	47.21 ± 3.68	42.59 ± 2.79
MC	A(↑)	25.28 ± 2.05	28.72 ± 2.57	27.52 ± 4.84	28.96 ± 2.69	32.87 ± 3.31	28.04 ± 1.58
	F(↓)	44.97 ± 5.56	47.05 ± 5.57	**38.01 ± 3.38**	45.10 ± 3.13	39.18 ± 6.23	40.85 ± 5.87
BI	A(↑)	26.41 ± 1.17	28.88 ± 1.96	34.10 ± 6.24	28.62 ± 2.14	37.44 ± 2.87	**40.44 ± 3.09**
	F(↓)	72.68 ± 3.81	61.21 ± 6.12	43.03 ± 5.65	69.02 ± 4.78	47.68 ± 5.96	**37.79 ± 6.07**

Table 2: Comparison of last accuracy (A) and last forgetting (F) on CIFAR100.

Score		M=1000			M=2000		
ER		**14.23 ± 1.00** 29.32 ± 0.15			16.84 ± 0.96 22.67 ± 1.35		
CBR		13.38 ± 0.58 31.66 ± 0.30			16.59 ± 0.41 23.05 ± 1.29		
		Top	Step	Bottom	Top	Step	Bottom
LC	A(↑)	13.33 ± 0.16	12.52 ± 0.65	14.07 ± 0.36	15.63 ± 1.28	15.06 ± 0.69	15.44 ± 1.31
	F(↓)	25.18 ± 1.51	27.50 ± 1.02	22.67 ± 3.02	19.71 ± 2.82	20.56 ± 1.93	17.50 ± 3.48
MS	A(↑)	13.69 ± 1.25	11.69 ± 1.07	13.34 ± 0.22	15.76 ± 0.93	14.73 ± 0.92	16.32 ± 1.11
	F(↓)	24.45 ± 1.82	27.05 ± 1.20	25.64 ± 3.62	20.23 ± 2.76	18.14 ± 1.26	17.50 ± 3.81
RC	A(↑)	13.65 ± 0.97	12.65 ± 0.07	13.94 ± 0.78	16.69 ± 1.77	14.30 ± 0.80	16.03 ± 0.88
	F(↓)	25.71 ± 2.99	27.55 ± 0.48	24.99 ± 3.48	17.15 ± 3.25	20.72 ± 1.50	17.68 ± 2.86
EN	A(↑)	13.00 ± 0.64	12.79 ± 0.30	13.42 ± 0.87	14.18 ± 1.77	14.49 ± 0.73	14.42 ± 0.78
	F(↓)	25.30 ± 2.25	27.82 ± 1.80	23.20 ± 4.00	20.93 ± 3.83	19.59 ± 2.58	17.43 ± 2.61
RM	A(↑)	12.99 ± 1.35	12.24 ± 0.22	13.65 ± 0.75	15.62 ± 0.99	14.33 ± 0.64	16.39 ± 0.52
	F(↓)	26.01 ± 3.18	28.08 ± 0.98	24.27 ± 3.52	19.49 ± 4.25	19.25 ± 1.55	16.68 ± 3.67
MC	A(↑)	9.48 ± 1.92	10.49 ± 1.05	10.77 ± 2.71	10.99 ± 1.16	12.36 ± 1.64	10.68 ± 2.75
	F(↓)	16.77 ± 2.42	15.09 ± 1.41	**7.50 ± 2.53**	15.98 ± 1.45	11.91 ± 0.90	**9.08 ± 4.36**
BI	A(↑)	13.38 ± 0.59	12.27 ± 0.69	13.81 ± 1.00	14.52 ± 0.54	14.33 ± 1.41	**17.03 ± 0.30**
	F(↓)	25.43 ± 3.23	25.50 ± 1.15	19.41 ± 1.74	20.87 ± 3.51	17.68 ± 1.63	13.30 ± 3.65

Table 3: Comparison of last accuracy (A) and last forgetting (F) on long-tailed imbalanced datasets.

Score		CIFAR10-LT		CIFAR100-LT		BloodCell	
		M=200	M=500	M=500	M=1000	M=40	M=160
ER	A(↑)	22.06 ± 1.13	31.62 ± 1.65	8.22 ± 0.36	9.30 ± 0.22	**58.82 ± 6.62**	64.08 ± 11.98
	F(↓)	76.19 ± 3.17	52.09 ± 3.21	20.16 ± 0.9	15.78 ± 0.62	49.60 ± 8.23	28.50 ± 5.75
CBR	A(↑)	22.38 ± 2.84	27.34 ± 5.84	8.10 ± 0.70	8.75 ± 0.33	55.74 ± 2.24	70.11 ± 2.66
	F(↓)	77.41 ± 5.76	62.59 ± 7.66	21.82 ± 1.98	18.32 ± 1.84	57.53 ± 2.76	29.56 ± 5.80
LC	A(↑)	31.16 ± 1.24	32.5 ± 1.05	7.15 ± 0.73	8.93 ± 1.05	51.19 ± 5.49	66.08 ± 1.78
	F(↓)	42.31 ± 2.81	37.59 ± 2.01	17.53 ± 1.71	13.98 ± 2.66	27.33 ± 0.64	17.03 ± 3.05
MS	A(↑)	29.62 ± 1.70	33.48 ± 1.23	7.20 ± 0.57	9.11 ± 0.66	55.28 ± 5.04	68.34 ± 3.38
	F(↓)	47.31 ± 2.64	39.46 ± 2.45	19.36 ± 1.79	14.43 ± 1.85	34.41 ± 5.90	14.21 ± 2.80
RC	A(↑)	31.09 ± 0.79	32.48 ± 1.21	7.86 ± 0.45	9.47 ± 0.59	51.88 ± 3.59	63.92 ± 4.31
	F(↓)	44.38 ± 1.21	40.65 ± 1.75	18.51 ± 1.38	15.33 ± 2.04	28.43 ± 2.91	21.20 ± 7.00
EN	A(↑)	**31.34** ± 0.49	32.53 ± 1.89	7.02 ± 0.49	8.48 ± 0.71	49.32 ± 3.86	69.38 ± 1.13
	F(↓)	41.98 ± 2.06	32.70 ± 3.08	17.79 ± 2.18	14.34 ± 2.99	32.37 ± 2.22	11.41 ± 1.36
RM	A(↑)	28.50 ± 2.04	33.98 ± 2.75	7.89 ± 0.90	9.90 ± 1.12	57.13 ± 3.18	67.97 ± 4.19
	F(↓)	51.20 ± 5.82	39.17 ± 3.95	18.36 ± 1.27	13.54 ± 1.64	31.93 ± 4.18	14.29 ± 7.72
MC	A(↑)	25.84 ± 4.55	23.16 ± 1.03	7.42 ± 2.12	7.76 ± 2.23	55.90 ± 0.18	51.90 ± 5.25
	F(↓)	43.03 ± 10.72	**28.75 ± 1.55**	**7.84 ± 2.62**	**8.74 ± 2.69**	**16.96 ± 0.61**	9.9 ± 6.06
BI	A(↑)	30.35 ± 1.01	**34.37 ± 1.26**	**8.93 ± 1.02**	**10.35 ± 0.58**	58.08 ± 7.08	**71.88 ± 2.33**
	F(↓)	**40.66 ± 1.06**	34.93 ± 0.50	16.31 ± 1.48	13.27 ± 1.10	18.27 ± 7.25	**8.71 ± 5.00**

To provide a graphical intuition of the differences between the sorting strategies and their plausible effect on the learning capabilities of the model, Figure 4 depicts a subset of class-specific samples stored in the memory at the end of the training for the two opposite population strategies, i.e. bottom-k and top-k. By inspecting the images, we note that the ones selected via the bottom-k strategy show some consistency and are easily identifiable as birds. On the opposite side, the top-k samples present images with a different perspective, zooming, or background making it more difficult, even for the human eye, to identify a bird. For this reason, we believe that the easiest-to-remember images are more useful to the model for recalling the past information while the easiest-to-forget data points may hamper the generalization of the model – since we could be in the presence of noisy images or outliers.

Finally, in Table 3 we evaluate our findings in the more realistic long-tailed data imbalance scenario. In both cases (artificial and real data imbalance), the results support our findings from the standard balanced setting: BI substantially reduces CF while delivering competitive or improved predictive performance.

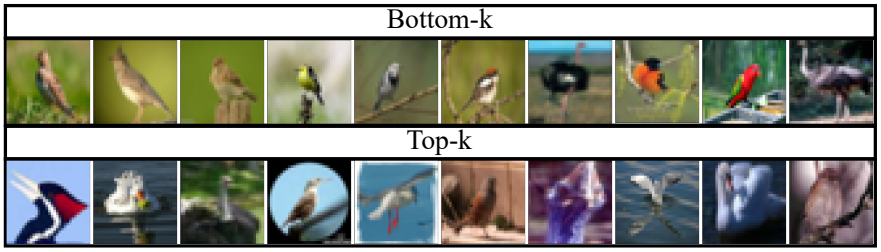

Figure 4: Class-specific samples stored in the memory according to different sorting strategies: *bottom-k* selects prototypical examples of the considered class facilitating the recollection of the class characteristics and reducing CF; *top-k* selects the most uncertain and hard samples making difficult to learn clear discriminative patterns and to reduce CF.

6 Conclusion

In this paper, we propose to leverage estimates of predictive uncertainty for memory management. Starting from the examination of the properties and behaviours of popular uncertainty estimates, we identify the limitations of common strategies and propose to use an alternative estimate based on the Bregman Information. Given the conflicting nature of the strategies available in the literature for populating the memory, our investigation seeks to a) clarify which characteristics of the samples alleviate CF in a consistent manner, and b) to assist practitioners in selecting the appropriate memory management strategy. Our findings indicate that the most representative samples – i.e., the easiest-to-remember or the least uncertain ones – are the most effective at mitigating CF (Figure 3) while maintaining competitive performance in classification tasks (Tables 1 and 2). Furthermore, the newly proposed BI-based uncertainty score for memory management shows its superiority in reducing CF compared to well-established uncertainty scores and other memory-based strategies (Figure 1). Due to its emphasis on epistemic uncertainty, the memory contains samples that are representative of their respective class distributions (Figure 4) while being close enough to the decision boundary, helping the learning model to distinctly shape the boundaries between the classes during training. The results on various realistic setups (Table 3) – not addressed in related work – further support our findings and demonstrate that the use of BI-based predictive uncertainty estimates for reducing CF in online-CL is appealing and well-motivated even in more challenging scenarios.

References

Andrea Acevedo, Anna Merino, Santiago Alférez, Ángel Molina, Laura Boldú, and José Rodellar. A dataset of microscopic peripheral blood cell images for development of automatic recognition systems. *Data in brief*, 30, 2020.

Rahaf Aljundi, Punarjay Chakravarty, and Tinne Tuytelaars. Expert gate: Lifelong learning with a network of experts. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 3366–3375, 2017.

Rahaf Aljundi, Francesca Babiloni, Mohamed Elhoseiny, Marcus Rohrbach, and Tinne Tuytelaars. Memory aware synapses: Learning what (not) to forget. In *Proceedings of the European conference on computer vision (ECCV)*, pp. 139–154, 2018.

Rahaf Aljundi, Eugene Belilovsky, Tinne Tuytelaars, Laurent Charlin, Massimo Caccia, Min Lin, and Lucas Page-Caccia. Online continual learning with maximal interfered retrieval. *Advances in neural information processing systems*, 32, 2019a.

Rahaf Aljundi, Min Lin, Baptiste Goujaud, and Yoshua Bengio. Gradient based sample selection for online continual learning. *Advances in neural information processing systems*, 32, 2019b.

Jihwan Bang, Heesu Kim, YoungJoon Yoo, Jung-Woo Ha, and Jonghyun Choi. Rainbow memory: Continual learning with a memory of diverse samples. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 8218–8227, 2021.

Eugene Belilovsky, Massimo Caccia, Min Lin, Laurent Charlin, Tinne Tuytelaars, Rahaf Aljundi, et al. Online continual learning with maximally interfered retrieval. *Advances in Neural Information Processing Systems,(NeurIPS)*, 2019.

Colin Campbell, Nello Cristianini, Alex Smola, et al. Query learning with large margin classifiers. In *ICML*, volume 20, pp. 0, 2000.

Kaidi Cao, Colin Wei, Adrien Gaidon, Nikos Arechiga, and Tengyu Ma. Learning imbalanced datasets with label-distribution-aware margin loss. *Advances in neural information processing systems*, 32, 2019.

Arslan Chaudhry, Puneet K Dokania, Thalaiyasingam Ajanthan, and Philip HS Torr. Riemannian walk for incremental learning: Understanding forgetting and intransigence. In *Proceedings of the European conference on computer vision (ECCV)*, pp. 532–547, 2018a.

Arslan Chaudhry, Marc'Aurelio Ranzato, Marcus Rohrbach, and Mohamed Elhoseiny. Efficient lifelong learning with a-gem. In *International Conference on Learning Representations*, 2018b.

Arslan Chaudhry, Marcus Rohrbach, Mohamed Elhoseiny, Thalaiyasingam Ajanthan, P Dokania, P Torr, and M Ranzato. Continual learning with tiny episodic memories. In *Workshop on Multi-Task and Lifelong Reinforcement Learning*, 2019.

Aristotelis Chrysakis and Marie-Francine Moens. Online continual learning from imbalanced data. In *International Conference on Machine Learning*, pp. 1952–1961. PMLR, 2020.

Yin Cui, Menglin Jia, Tsung-Yi Lin, Yang Song, and Serge Belongie. Class-balanced loss based on effective number of samples. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 9268–9277, 2019.

Aron Culotta and Andrew McCallum. Reducing labeling effort for structured prediction tasks. In *AAAI*, volume 5, pp. 746–751, 2005.

Yarin Gal and Zoubin Ghahramani. Dropout as a bayesian approximation: Representing model uncertainty in deep learning. In *international conference on machine learning*, pp. 1050–1059. PMLR, 2016.

Sebastian Gruber and Florian Buettner. Uncertainty estimates of predictions via a general bias-variance decomposition. In *International Conference on Artificial Intelligence and Statistics*, pp. 11331–11354. PMLR, 2023.

Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770–778, 2016.

Xu He and Herbert Jaeger. Overcoming catastrophic interference using conceptor-aided backpropagation. In *International Conference on Learning Representations*, 2018.

Julio Hurtado, Alain Raymond-Sáez, Vladimir Araujo, Vincenzo Lomonaco, Alvaro Soto, and Davide Bacciu. Memory population in continual learning via outlier elimination. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pp. 3481–3490, 2023.

Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. *online*, 2009.

Lilly Kumari, Shengjie Wang, Tianyi Zhou, and Jeff A Bilmes. Retrospective adversarial replay for continual learning. *Advances in Neural Information Processing Systems*, 35:28530–28544, 2022.

Sang-Woo Lee, Jin-Hwa Kim, Jaehyun Jun, Jung-Woo Ha, and Byoung-Tak Zhang. Overcoming catastrophic forgetting by incremental moment matching. *Advances in neural information processing systems*, 30, 2017.

Timothée Lesort, Hugo Caselles-Dupré, Michael Garcia-Ortiz, Andrei Stoian, and David Filliat. Generative models from the perspective of continual learning. In *2019 International Joint Conference on Neural Networks (IJCNN)*, pp. 1–8. IEEE, 2019.

D Gale Lewis and William A Gale. A sequential algorithm for training text classifiers. In *Proceedings of SIGIR-94*, pp. 3–12, 1994.

David Lopez-Paz and Marc'Aurelio Ranzato. Gradient episodic memory for continual learning. *Advances in neural information processing systems*, 30, 2017.

Zheda Mai, Ruiwen Li, Jihwan Jeong, David Quispe, Hyunwoo Kim, and Scott Sanner. Online continual learning in image classification: An empirical survey. *Neurocomputing*, 469:28–51, 2022.

Arun Mallya and Svetlana Lazebnik. Packnet: Adding multiple tasks to a single network by iterative pruning. In *Proceedings of the IEEE conference on Computer Vision and Pattern Recognition*, pp. 7765–7773, 2018.

Michael McCloskey and Neal J Cohen. Catastrophic interference in connectionist networks: The sequential learning problem. In *Psychology of learning and motivation*, volume 24, pp. 109–165. Elsevier, 1989.

Martin Mundt, Steven Lang, Quentin Delfosse, and Kristian Kersting. Cleva-compass: A continual learning evaluation assessment compass to promote research transparency and comparability. In *International Conference on Learning Representations*, 2021.

Cuong V Nguyen, Yingzhen Li, Thang D Bui, and Richard E Turner. Variational continual learning. In *International Conference on Learning Representations*, 2018.

Yaniv Ovadia, Emily Fertig, Jie Ren, Zachary Nado, David Sculley, Sebastian Nowozin, Joshua Dillon, Balaji Lakshminarayanan, and Jasper Snoek. Can you trust your model's uncertainty? evaluating predictive uncertainty under dataset shift. *Advances in neural information processing systems*, 32, 2019.

Amal Rannen, Rahaf Aljundi, Matthew B Blaschko, and Tinne Tuytelaars. Encoder based lifelong learning. In *Proceedings of the IEEE international conference on computer vision*, pp. 1320–1328, 2017.

Roger Ratcliff. Connectionist models of recognition memory: constraints imposed by learning and forgetting functions. *Psychological review*, 97(2):285, 1990.

Joan Serra, Didac Suris, Marius Miron, and Alexandros Karatzoglou. Overcoming catastrophic forgetting with hard attention to the task. In *International conference on machine learning*, pp. 4548–4557. PMLR, 2018.

Claude Elwood Shannon. A mathematical theory of communication. *The Bell system technical journal*, 27 (3):379–423, 1948.

Hanul Shin, Jung Kwon Lee, Jaehong Kim, and Jiwon Kim. Continual learning with deep generative replay. *Advances in neural information processing systems*, 30, 2017.

Albin Soutif-Cormerais, Antonio Carta, Andrea Cossu, Julio Hurtado, Vincenzo Lomonaco, Joost Van de Weijer, and Hamed Hemati. A comprehensive empirical evaluation on online continual learning. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pp. 3518–3528, 2023.

Shengyang Sun, Daniele Calandriello, Huiyi Hu, Ang Li, and Michalis Titsias. Information-theoretic online memory selection for continual learning. In *International Conference on Learning Representations*, 2021.

Xiaoyu Tao, Xinyuan Chang, Xiaopeng Hong, Xing Wei, and Yihong Gong. Topology-preserving class-incremental learning. In *Computer Vision–ECCV 2020: 16th European Conference, Glasgow, UK, August 23–28, 2020, Proceedings, Part XIX 16*, pp. 254–270. Springer, 2020.

Christian Tomani and Florian Buettner. Towards trustworthy predictions from deep neural networks with fast adversarial calibration. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 35, pp. 9886–9896, 2021.

Gido M van de Ven, Tinne Tuytelaars, and Andreas S Tolias. Three types of incremental learning. *Nature Machine Intelligence*, 4(12):1185–1197, 2022.

Eli Verwimp, Matthias De Lange, and Tinne Tuytelaars. Rehearsal revealed: The limits and merits of revisiting samples in continual learning. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pp. 9385–9394, 2021.

Jeffrey S Vitter. Random sampling with a reservoir. *ACM Transactions on Mathematical Software (TOMS)*, 11(1):37–57, 1985.

Guotai Wang, Wenqi Li, Michael Aertsen, Jan Deprest, Sébastien Ourselin, and Tom Vercauteren. Aleatoric uncertainty estimation with test-time augmentation for medical image segmentation with convolutional neural networks. *Neurocomputing*, 338:34–45, 2019.

Guotai Wang, Wenqi Li, Michael Aertsen, Jan Deprest, Sebastien Ourselin, and Tom Vercauteren. Test-time augmentation with uncertainty estimation for deep learning-based medical image segmentation. 2022.

Felix Wiewel and Bin Yang. Entropy-based sample selection for online continual learning. In *2020 28th European Signal Processing Conference (EUSIPCO)*, pp. 1477–1481. IEEE, 2021.

Lisa Wimmer, Yusuf Sale, Paul Hofman, Bernd Bischl, and Eyke Hüllermeier. Quantifying aleatoric and epistemic uncertainty in machine learning: Are conditional entropy and mutual information appropriate measures? In *Uncertainty in Artificial Intelligence*, pp. 2282–2292. PMLR, 2023.

Yue Wu, Yinpeng Chen, Lijuan Wang, Yuancheng Ye, Zicheng Liu, Yandong Guo, and Yun Fu. Large scale incremental learning. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, pp. 374–382, 2019.

Jiancheng Yang, Rui Shi, Donglai Wei, Zequan Liu, Lin Zhao, Bilian Ke, Hanspeter Pfister, and Bingbing Ni. Medmnist v2-a large-scale lightweight benchmark for 2d and 3d biomedical image classification. *Scientific Data*, 10(1):41, 2023.

Jaehong Yoon, Eunho Yang, Jeongtae Lee, and Sung Ju Hwang. Lifelong learning with dynamically expandable networks. In *International Conference on Learning Representations*, 2018.

Jaehong Yoon, Divyam Madaan, Eunho Yang, and Sung Ju Hwang. Online coreset selection for rehearsal-based continual learning. In *International Conference on Learning Representations*, 2021.

A Appendix

A.1 List of Augmentations

The list of augmentations used for estimating the variance of the classification loss via TTA is reported in Figure 5.

```
transform_cands = [
    CutoutAfterToTensor(args, 1, 10),
    CutoutAfterToTensor(args, 1, 20),
    v2.RandomHorizontalFlip(),
    v2.RandomVerticalFlip(),
    v2.RandomRotation(degrees=10),
    v2.RandomRotation(45),
    v2.RandomRotation(90),
    v2.ColorJitter(brightness=0.1),
    v2.RandomPerspective(),
    v2.RandomAffine(degrees=20, translate=(0.1, 0.3), scale=(0.5, 0.75)),
    v2.RandomResizedCrop(args.input_size[1:], scale=(0.8, 1.0), ratio=(0.9, 1.1), antialias=True),
    v2.RandomInvert()
        ]
```

Figure 5: List of the augmentations applied to the images for computing the uncertainty via TTA.