Contrastive Counterfactual Generation for Imperceptible Adversarial Attacks

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Abstract

Imperceptible adversarial attacks aim to mislead deep neural networks by adding signal-domain perturbations that induce misclassification while remaining visually indistinguishable from the original signal. Existing methods rely on untargeted loss maximisation, producing perturbations poorly aligned with decision boundaries and providing limited control over locality and perceptual cost. To address these limitations, we propose **Contrastive Counterfactual Generation** (CoCoGen), a cross-domain adversarial attack framework that operates under a composite threat model combining an ℓ_∞ magnitude budget, a sparsity budget with k selected adaptively, and a high-frequency spectral constraint. This feasible set is strictly more restrictive than the standard ℓ_∞ ball, and comparisons with methods operating under less restrictive threat models should be interpreted accordingly. Within this composite feasible set, CoCoGen formulates perturbation synthesis as a constrained optimisation problem that explicitly targets the nearest decision boundary by minimising the *contrastive counterfactual margin*. Perturbations are localised via gradient-based Top-k spatial projection and confined to the high-frequency subspace using a Fourier-domain projection operator, leveraging reduced human sensitivity to high spatial frequencies. The objective is optimised using masked gradient descent with momentum, while an adaptive sparsity grid search identifies minimal feasible signal support. Experiments across multiple architectures show that CoCoGen achieves 100% Attack Success Rate (vs. 80–100% for prior methods, with most below 99%) while maintaining a MUSIQ score of 61–63 (vs. 36–55), outperforming prior methods in both attack efficacy and visual quality under this composite threat model.

1 Introduction

Deep Neural Networks (DNNs) are vulnerable to adversarial attacks Goodfellow et al. (2015), where carefully crafted signal-domain perturbations added to input signals induce incorrect predictions, posing serious security risks in real-world applications Yuan et al. (2019). Beyond exposing vulnerabilities, adversarial attacks are instrumental in evaluating model robustness and motivating defence strategies Lee & Kim (2023); Singh et al. (2024); Luo et al. (2023); Tramèr et al. (2018); Salman et al. (2020); Luo et al. (2021); Cohen et al. (2019). Many existing attacks Madry et al. (2018); Wei et al. (2023) maximise success under loose signal-energy budgets (*e.g.* ℓ_∞ or ℓ_2 norms), often producing perceptible signal artefacts detectable by the human visual system (HVS) Sharif et al. (2018). This has motivated growing interest in *imperceptible* adversarial attacks Carlini & Wagner (2017); Luo et al. (2018); Zhao et al. (2020); Laidlaw et al. (2021); Duan et al. (2021); Chen et al. (2023c); Jia et al. (2022), which seek to maintain attack efficacy while preserving the perceptual fidelity of the adversarial signal.

Existing imperceptible attacks are broadly categorised into perturbation-constrained and unrestricted methods. Perturbation-based approaches exploit perceptual signal properties such as colour sensitivity Zhao et al. (2020), texture complexity Fang et al. (2026), and frequency characteristics Jia et al. (2022); Luo et al. (2022) to conceal adversarial signal components in perceptually insensitive regions. Unrestricted attacks modify semantic attributes via generative or diffusion-based models Song et al. (2018); Rom-

bach et al. (2022); Chen et al. (2023b); Xue et al. (2023); Chen et al. (2023a; 2024), but often introduce unnatural signal distortions, particularly for complex scenes. More fundamentally, most methods rely on *untargeted loss maximisation*, showing no mechanism to identify the nearest decision boundary nor to concentrate signal energy on the components most responsible for the classification decision. To address these limitations, we propose CoCoGen, an adversarial attack that minimises the contrastive counterfactual margin Moosavi-Dezfooli et al. (2016); Carlini & Wagner (2017), focusing perturbations on the most competitive incorrect class. We note that the margin objective itself is closely related to logit-margin formulations used in DeepFool and C&W; the contribution here is not the margin in isolation but its integration with adaptive sparse support selection and frequency-aware perturbation generation within a unified constrained optimisation framework. Specifically, CoCoGen integrates Top-k spatial projection Papernot et al. (2016); Modas et al. (2019); Croce et al. (2022), Fourier high-frequency constraints Luo et al. (2022), and masked momentum iterative updates Dong et al. (2018) with adaptive sparsity and spectral threshold search to produce imperceptible perturbations. Unlike prior methods that typically fix either the perturbation support or the frequency-domain constraint, CoCoGen jointly searches for both the sparsity level k and the spectral threshold while enforcing perceptual-quality constraints, operating under the composite feasible. CoCoGen achieves **100%** ASR across all models, outperforming prior methods (mostly 80-99%) with lower LPIPS (≈ 0.01) and higher MUSIQ (61-63) under this more restrictive threat model.

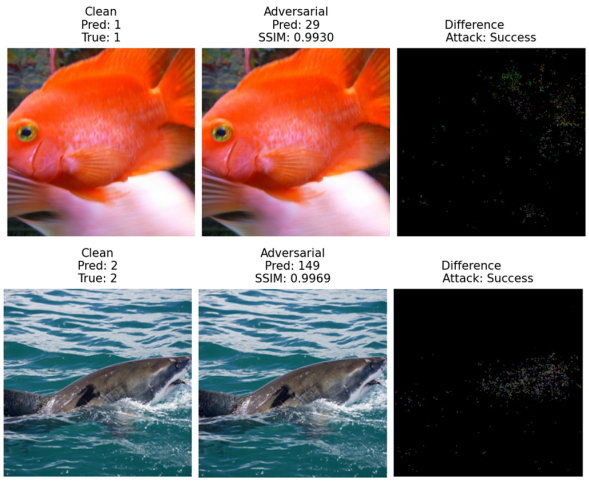

Figure 1: **Imperceptibility of CoCoGen.** Clean signal with labels (col. 1), adversarial signal (col. 2), and contrast-enhanced perturbation map (col. 3). The perturbations are sparse and confined to high-frequency, thus imperceptible.

The contributions of this work are as follows:

- We propose CoCoGen, an attack that minimises the contrastive counterfactual margin, focusing perturbations on the most competitive incorrect class.
- We integrate Top-k spatial projection, Fourier high-frequency constraints, and masked momentum iterative update with adaptive sparsity to produce imperceptible perturbations.
- CoCoGen achieves **100%** ASR across all models, outperforming prior methods (mostly 80-99%) with lower LPIPS (≈ 0.01) and higher MUSIQ (61-63).

We note that CoCoGen operates under a composite threat model that is strictly more restrictive than the standard ℓ_∞ ball; reported improvements over methods operating under less restrictive threat models reflect what is achievable under additional perceptual constraints rather than unconditional superiority.

2 Proposed Method: CoCoGen

2.1 Problem Formulation

Let $\boldsymbol{x} \in \mathbb{R}^N$ denote a vectorised image, where $N = H \cdot W \cdot C$ is the total number of signal entries obtained by flattening the spatial dimensions H (height) and W (width) over C colour channels (note that this explanation is simplified; for convolutional neural networks, the representation differs). A neural classifier $f : \mathbb{R}^N \to \mathbb{R}^{|\mathcal{Y}|}$ produces a logit vector $f(\boldsymbol{x})$, with $f_c(\boldsymbol{x}) \in \mathbb{R}$ denoting the score for class $c \in \mathcal{Y}$. We write $y_{\text{true}} \in \mathcal{Y}$ for the ground-truth label of $\boldsymbol{x}$. We seek a perturbation $\boldsymbol{\delta} \in \mathbb{R}^N$ such that the adversarial example

$$\boldsymbol{x}_{\text{adv}} = \boldsymbol{x} + \boldsymbol{\delta} \tag{1}$$

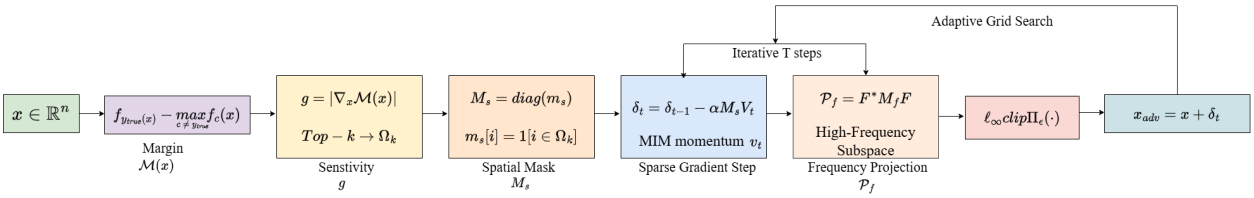

Figure 2: **CoCoGen attack pipeline.** The input signal $\boldsymbol{x}$ is processed through four stages: contrastive margin computation, gradient-based Top-k spatial projection $\boldsymbol{M}_s$, Fourier-domain frequency projection $\mathcal{P}_f$, and ℓ_∞ clipping Π_ϵ, iterated over T steps with an adaptive sparsity grid search over $k^\star$ to yield the adversarial signal $\boldsymbol{x}_{\text{adv}}$.

is misclassified by f while remaining visually indistinguishable from $\boldsymbol{x}$. equation 1 is the standard additive perturbation model Goodfellow et al. (2015). Imperceptibility is enforced via an ℓ_∞ budget $\|\boldsymbol{\delta}\|_\infty \leq \epsilon$, where $\epsilon > 0$ is a user-specified tolerance (typically $\epsilon \in [4/255, 16/255]$ for 8-bit images; scaled accordingly for other bit depths).

Composite Threat Model. Unlike standard ℓ_∞ attacks such as PGD Madry et al. (2018) and C&W Carlini & Wagner (2017), which operate over the unconstrained ℓ_∞ ball $\mathcal{B}_\epsilon = \{\boldsymbol{\delta} : \|\boldsymbol{\delta}\|_\infty \leq \epsilon\}$, CoCoGen operates under a strictly more restrictive composite feasible set

$$\mathcal{F} \;=\; \big\{\boldsymbol{\delta} \;:\; \|\boldsymbol{\delta}\|_\infty \leq \epsilon,\; \|\boldsymbol{\delta}\|_0 \leq k,\; \boldsymbol{\delta} \in \mathrm{Im}(\mathcal{P}_f)\big\}, \tag{2}$$

which simultaneously enforces a magnitude constraint (ℓ_∞ budget ϵ), a sparsity constraint (ℓ_0 budget k, selected adaptively via Eq. 19), and a spectral constraint (support restricted to the high-frequency Fourier subspace $\mathrm{Im}(\mathcal{P}_f)$, defined in equation 13. Since $\mathcal{F} \subsetneq \mathcal{B}_\epsilon$ strictly whenever $k < N$ or $\boldsymbol{M}_f \neq \boldsymbol{I}$ (Proposition 2(ii)), comparisons between CoCoGen and methods operating under the less restrictive threat model $\mathcal{B}_\epsilon$ should be interpreted as evaluating what is achievable under additional perceptual constraints, rather than as direct superiority claims under identical attack conditions.

2.2 Contrastive Counterfactual Margin

The margin formulation in our method builds on the logit-difference objective used in DeepFool Moosavi-Dezfooli et al. (2016) and C&W Carlini & Wagner (2017); our contribution lies not in the margin itself but in combining counterfactual-margin optimisation with adaptive sparse support selection and frequency-aware perturbation generation, as described in Sections 2.3-2.6.

Naïvely minimising a cross-entropy loss diffuses perturbation energy across all N signal components, producing detectable artefacts. Instead, we target the *contrastive counterfactual margin* $\mathcal{M}(\cdot)$:

$$\mathcal{M}(\boldsymbol{x}) \;=\; f_{y_{\text{true}}}(\boldsymbol{x}) \;-\; \max_{c \neq y_{\text{true}}} f_c(\boldsymbol{x}), \tag{3}$$

where $f_{y_{\text{true}}}(\boldsymbol{x})$ is the logit of the correct class and $\max_{c \neq y_{\text{true}}} f_c(\boldsymbol{x})$ is the highest competing logit. The margin in equation 3 is positive for a correctly classified input and negative once misclassification is achieved. The optimisation problem is therefore

$$\min_{\boldsymbol{\delta}} \; \mathcal{M}(\boldsymbol{x} + \boldsymbol{\delta}), \qquad \text{s.t.} \quad \|\boldsymbol{\delta}\|_\infty \leq \epsilon, \tag{4}$$

where the constraint $\|\boldsymbol{\delta}\|_\infty \leq \epsilon$ bounds the per-entry magnitude of the perturbation.

2.3 Spatial Sparsity via a Diagonal Projection Operator

To localise the perturbation to the most decision-relevant signal entries, we compute the element-wise absolute gradient of $\mathcal{M}$ with respect to the input:

$$\boldsymbol{g} \;=\; \big|\nabla_{\boldsymbol{x}} \mathcal{M}(\boldsymbol{x})\big| \;\in\; \mathbb{R}^N_{\geq 0}, \tag{5}$$

where $|\cdot|$ is applied element-wise and $\boldsymbol{g}[i]$ quantifies the sensitivity of the margin to the i-th pixel. From $\boldsymbol{g}$ in equation 5 we identify the index set

$$\Omega_k \;=\; \text{Top-}k(\boldsymbol{g}) \;=\; \{i_1, i_2, \ldots, i_k\}, \tag{6}$$

where $\boldsymbol{g}[i_1] \geq \boldsymbol{g}[i_2] \geq \cdots \geq \boldsymbol{g}[i_k]$, and $k \in \{1, \ldots, N\}$ is the sparsity budget. Using Ω_k from equation 6, we form a binary mask vector $\boldsymbol{m}_s \in \{0,1\}^N$ with

$$\boldsymbol{m}_s[i] \;=\; \mathbb{1}[i \in \Omega_k], \qquad i = 1, \ldots, N, \tag{7}$$

where $\mathbb{1}[\cdot]$ is the indicator function. The mask in equation 7 is then lifted to a diagonal *spatial projection matrix*

$$\boldsymbol{M}_s \;=\; \text{diag}(\boldsymbol{m}_s) \;\in\; \{0,1\}^{N \times N}, \tag{8}$$

which satisfies $\boldsymbol{M}_s^2 = \boldsymbol{M}_s$ (idempotent projector) and $\boldsymbol{M}_s \boldsymbol{u}$ zeroes all entries of $\boldsymbol{u}$ outside Ω_k.

2.4 Frequency Subspace Projection

Spatial sparsity alone does not guarantee imperceptibility, since low-frequency perturbations are highly visible to the human visual system (HVS) Wang et al. (2004). We therefore further restrict $\boldsymbol{\delta}$ to the *high-frequency subspace* of $\mathbb{R}^N$ using the Discrete Fourier Transform (DFT).

The DFT is applied independently per colour channel on the 2D spatial grid $H \times W$. Concretely, for channel $c \in \{1, \ldots, C\}$, let $\boldsymbol{F}_{HW} \in \mathbb{C}^{HW \times HW}$ be the unitary 2D DFT matrix (equivalently $\boldsymbol{F}_H \otimes \boldsymbol{F}_W$, the Kronecker product of the 1D DFT matrices along each spatial axis), defined entry-wise as

$$[\boldsymbol{F}_{HW}]_{p,q} \;=\; \frac{1}{\sqrt{HW}} \exp\!\Big(-2\pi\,\mathrm{i}\Big(\frac{p_h\, q_h}{H} + \frac{p_w\, q_w}{W}\Big)\Big), \tag{9}$$

where $p = (p_h, p_w)$ and $q = (q_h, q_w)$ index the 2D spatial and frequency grids respectively, and $\mathrm{i} = \sqrt{-1}$. The full DFT matrix acting on the vectorised C-channel signal is

$$\boldsymbol{F} \;=\; \boldsymbol{I}_C \otimes \boldsymbol{F}_{HW} \;\in\; \mathbb{C}^{N \times N}, \qquad N = H \cdot W \cdot C, \tag{10}$$

which applies $\boldsymbol{F}_{HW}$ independently to each channel and satisfies $\boldsymbol{F}\boldsymbol{F}^* = \boldsymbol{I}_N$.

The high-frequency binary mask $\boldsymbol{m}_f \in \{0,1\}^N$ is constructed as follows. Each vectorised index $i \in \{1, \ldots, N\}$ maps to a triple (c_i, u_i, v_i) where $c_i \in \{1, \ldots, C\}$ is the channel index and $(u_i, v_i) \in \{0, \ldots, H-1\} \times \{0, \ldots, W-1\}$ is the 2D frequency coordinate. The mask retains bins whose radial frequency exceeds a threshold $\tau_{\text{freq}} \geq 0$, selected by adaptive search:

$$\boldsymbol{m}_f[i] \;=\; \mathbb{1}\left[\sqrt{u_i^2 + v_i^2} > \tau_{\text{freq}}\right], \qquad i = 1, \ldots, N, \tag{11}$$

where the threshold is applied identically across all C channels. The corresponding diagonal frequency-selection matrix is

$$\boldsymbol{M}_f \;=\; \text{diag}(\boldsymbol{m}_f) \;\in\; \{0,1\}^{N \times N}. \tag{12}$$

The frequency projection operator is then

$$\mathcal{P}_f \;=\; \boldsymbol{F}^* \boldsymbol{M}_f \boldsymbol{F} \;\in\; \mathbb{C}^{N \times N}, \tag{13}$$

where $\boldsymbol{F}$ maps the signal to the frequency domain per channel, $\boldsymbol{M}_f$ retains only high-frequency components, and $\boldsymbol{F}^*$ maps back to the signal domain. One can verify that $\mathcal{P}_f^2 = \mathcal{P}_f$ (idempotent) and $\mathcal{P}_f^* = \mathcal{P}_f$ (self-adjoint), so $\mathcal{P}_f$ is an orthogonal projector.

While SSAH Luo et al. (2022) also restricts perturbations to the high-frequency Fourier subspace, it does so with a fixed frequency threshold determined a priori. CoCoGen differs in two key respects: (i) the frequency threshold τ_{freq} is selected adaptively via the grid search of Section 2.6 (equation 19) rather than fixed before optimisation; and (ii) the sparse spatial support Ω_k (equation 6) is jointly optimised with the spectral constraint, so both the perturbation support and the frequency subspace are determined by the data rather than set as hyperparameters. This joint adaptivity is the primary spectral novelty of CoCoGen relative to SSAH.

2.5 Masked Momentum Iterative Update

We optimise equation 4 using a masked extension of the Momentum Iterative Method (MIM) Dong et al. (2018). Let $\boldsymbol{\delta}_0 = \mathbf{0}$ and $\boldsymbol{v}_0 = \mathbf{0}$ denote the initial perturbation and momentum buffer, respectively, and let $\alpha > 0$ be the step size. At iteration $t \in \{1, \ldots, T\}$ the following three operations are applied in sequence.

Momentum accumulation. The gradient of the margin $\mathcal{M}$ (defined in equation 3) with respect to the current adversarial example is ℓ_1-normalised and accumulated into the momentum buffer:

$$\boldsymbol{v}_t \;=\; \boldsymbol{v}_{t-1} \;+\; \frac{\nabla_{\boldsymbol{x}}\, \mathcal{M}(\boldsymbol{x} + \boldsymbol{\delta}_{t-1})}{\left\|\nabla_{\boldsymbol{x}}\, \mathcal{M}(\boldsymbol{x} + \boldsymbol{\delta}_{t-1})\right\|_1}, \tag{14}$$

where $\|\cdot\|_1 = \sum_i |\,\cdot_i\,|$ normalises the update to unit scale across iterations.

Sparse gradient step. The spatial projection matrix $\boldsymbol{M}_s$ from equation 8 restricts the update to the k decision-critical entries:

$$\tilde{\boldsymbol{\delta}}_t \;=\; \boldsymbol{\delta}_{t-1} \;-\; \alpha\, \boldsymbol{M}_s\, \boldsymbol{v}_t, \tag{15}$$

so that only the components in Ω_k (equation 6) receive a non-zero gradient signal.

Composite projection. The intermediate perturbation $\tilde{\boldsymbol{\delta}}_t$ from equation 15 is first projected onto the high-frequency subspace via $\mathcal{P}_f$ (equation 13), and then clipped to the ℓ_∞ ball $\mathcal{B}_\epsilon = \{\boldsymbol{u} : \|\boldsymbol{u}\|_\infty \le \epsilon\}$ (the ℓ_∞ ball $\mathcal{B}_\epsilon = \{\boldsymbol{u} : \|\boldsymbol{u}\|_\infty \le \epsilon\}$:

$$\boldsymbol{\delta}_t \;=\; \Pi_\epsilon\big(\mathcal{P}_f\, \tilde{\boldsymbol{\delta}}_t\big)\,, \tag{16}$$

where the ℓ_∞ projection is $\Pi_\epsilon(\boldsymbol{u}) = \mathrm{clip}(\boldsymbol{u}, -\epsilon, \epsilon)$, applied element-wise. Substituting equation 15 into equation 16 produces the *single closed-form iteration*:

$$\boldsymbol{\delta}_t \;=\; \Pi_\epsilon\Big(\boldsymbol{F}^* \boldsymbol{M}_f \boldsymbol{F}\big(\boldsymbol{\delta}_{t-1} - \alpha\, \boldsymbol{M}_s\, \boldsymbol{v}_t\big)\Big), \tag{17}$$

which involves three closed-form linear operators ($\boldsymbol{M}_s$, $\boldsymbol{F}/\boldsymbol{F}^*$, $\boldsymbol{M}_f$) plus an element-wise clip, and requires no iterative sub-solver.

2.6 Adaptive Sparsity Search

The optimal sparsity budget k is unknown *a priori*: a very small k may fail to achieve misclassification, while a large k wastes perceptual budget. We therefore perform a monotone search over a pre-specified candidate set $\mathcal{K} = \{k_1 < k_2 < \cdots < k_L\} \subset \{1, \ldots, N\}$. For each $k \in \mathcal{K}$, the mask $\boldsymbol{m}_s$ in equation 7 is constructed with $|\Omega_k| = k$ active entries, and T iterations of equation 17 are executed to yield $\boldsymbol{x}_{\mathrm{adv}}^{(k)} = \boldsymbol{x} + \boldsymbol{\delta}_T^{(k)}$ (cf. equation 1). A candidate k is feasible only if it simultaneously satisfies the misclassification condition and two perceptual quality thresholds:

$$\frac{1}{B}\sum_{j=1}^{B} \mathrm{SSIM}\Big(\boldsymbol{x}_{\mathrm{adv}}^{(j,k)},\, \boldsymbol{x}^{(j)}\Big) \;\ge\; \tau_s, \qquad \mathrm{FID}\Big(\Big\{\boldsymbol{x}_{\mathrm{adv}}^{(j,k)}\Big\}_{j=1}^{B},\, \Big\{\boldsymbol{x}^{(j)}\Big\}_{j=1}^{B}\Big) \;\le\; \tau_f, \tag{18}$$

where B denotes the number of images used to evaluate both the perceptual feasibility criterion equation 18 and the misclassification condition in equation 19; in our experiments the full 1,000-image ImageNet subset described in Section 4.1 is used (i.e., $B = 1{,}000$), so $k^\star$ is the smallest sparsity level achieving 100% ASR across all B images while satisfying the perceptual gates $\tau_s = 0.95$ and $\tau_f = 20$. The value of k is selected independently for each target architecture. Besides, $\tau_s \in (0, 1]$ is the minimum acceptable mean Structural Similarity Index Wang et al. (2004) across the batch, and $\tau_f \ge 0$ is the maximum tolerated Fréchet Inception Distance Heusel et al. (2017) computed over the full evaluation batch. The constraint in equation 18 gates candidates by both local fidelity (SSIM, averaged per image) and distributional realism (FID, computed over the batch).

The optimal sparsity level is the smallest feasible candidate:

$$k^\star \;=\; \min\Big\{k \in \mathcal{K} \;\Big|\; \underbrace{\frac{1}{B}\sum_{j=1}^{B} \mathbb{1}\Big[\operatorname*{argmax}_c\, f_c\Big(\boldsymbol{x}_{\mathrm{adv}}^{(j,k)}\Big) \neq y_{\mathrm{true}}^{(j)}\Big] = 1}_{\text{all } B \text{ images misclassified}} \;\wedge\; \underbrace{\textit{equation } 18 \text{ holds}}_{\text{perceptually valid}}\Big\}, \tag{19}$$

where $\wedge$ denotes logical conjunction. The final adversarial examples are $\{\boldsymbol{x}_{\text{adv}}^{(j,k^\star)}\}_{j=1}^{B}$ from equation 1. If no $k \in \mathcal{K}$ satisfies equation 19, the attack is declared unsuccessful for this batch.

2.7 Complexity Analysis

Each iteration of equation 17 requires: (i) one forward–backward pass through f for the gradient in equation 14, costing $\mathcal{O}(\Phi(f))$ where $\Phi(f)$ denotes the number of floating-point operations (FLOPs) in a single forward pass of f; (ii) two DFTs ($\mathcal{O}(N \log N)$) from equation 13; and (iii) two diagonal matrix multiplications ($\mathcal{O}(N)$) from equation 8 and equation 12. The dominant cost is the backward pass, which by standard autodifferentiation requires at most $\mathcal{O}(\Phi(f))$ operations; the same asymptotic order as the forward pass. Over T iterations and $|\mathcal{K}|$ sparsity candidates, the total cost scales as $\mathcal{O}(|\mathcal{K}| \cdot T \cdot \Phi(f))$. Since the $|\mathcal{K}|$ sub-problems are independent, the search over equation 19 is embarrassingly parallel (one can run them parallely on different GPUs and CPUs).

3 Theoretical Analysis

We establish four formal results that justify the design of CoCoGen: (i) the contrastive counterfactual margin is a signed distance surrogate to the nearest decision boundary (Theorem 1); (ii) the composite signal projection $\boldsymbol{M}_s\mathcal{P}_f$ is a well-defined bounded linear operator with controlled spectral norm (Proposition 1); and (iii) CoCoGen strictly generalises the C&W Carlini & Wagner (2017) objective while producing a geometrically distinct solution path (Proposition 2).

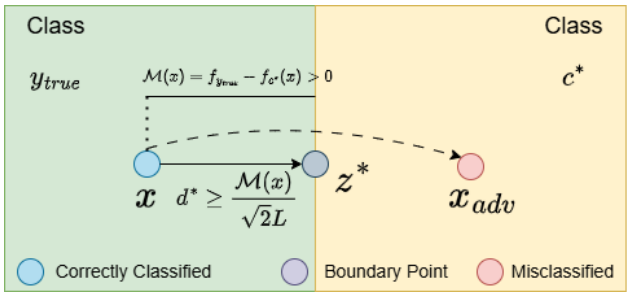

Figure 3: **Decision boundary geometry of CoCoGen.** The contrastive counterfactual margin $\mathcal{M}(\boldsymbol{x}) = f_{y_{\text{true}}}(\boldsymbol{x}) - f_{c^*}(\boldsymbol{x}) > 0$ provides a lower bound on the signed distance $d^*(\boldsymbol{x}) \geq \mathcal{M}(\boldsymbol{x})/(2L)$ to the nearest decision boundary $\mathcal{D}_{y_{\text{true}},c^*}$. Minimising $\mathcal{M}$ under the signal constraints $\boldsymbol{M}_s$ and $\mathcal{P}_f$ guides $\boldsymbol{\delta}$ toward z^*, crossing the boundary with $\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) \leq 0$ using minimal signal energy.

3.1 Contrastive Margin as a Decision-Boundary Surrogate

We first formalise the relationship between the contrastive counterfactual margin $\mathcal{M}(\boldsymbol{x})$ defined in equation 3 and the signed distance of $\boldsymbol{x}$ to the nearest decision boundary of f.

Assumption 1 (Lipschitz classifier) *The classifier $f : \mathbb{R}^N \to \mathbb{R}^{|\mathcal{Y}|}$ is L-Lipschitz continuous with respect to the ℓ_2 norm,*

$$\|f(\boldsymbol{x}) - f(\boldsymbol{x}')\|_2 \;\leq\; L\,\|\boldsymbol{x} - \boldsymbol{x}'\|_2, \qquad \forall\, \boldsymbol{x}, \boldsymbol{x}' \in \mathbb{R}^N, \tag{20}$$

where $L > 0$ is the global Lipschitz constant of f.

Assumption 2 (Unique runner-up class) *At the input $\boldsymbol{x}$ with true label y_{true}, the most competitive incorrect class $c^* = \operatorname{argmax}_{c \neq y_{\text{true}}} f_c(\boldsymbol{x})$ is unique, i.e., $f_{c^*}(\boldsymbol{x}) > f_c(\boldsymbol{x})$ for all $c \notin \{y_{\text{true}}, c^*\}$.*

Under Assumption 2, the margin in equation 3 reduces:

$$\mathcal{M}(\boldsymbol{x}) \;=\; f_{y_{\text{true}}}(\boldsymbol{x}) - f_{c^*}(\boldsymbol{x}). \tag{21}$$

Let $\mathcal{D}_{y,c} = \{\boldsymbol{z} \in \mathbb{R}^N : f_y(\boldsymbol{z}) = f_c(\boldsymbol{z})\}$ denote the decision boundary between classes y and c, and define the signed distance from $\boldsymbol{x}$ to the nearest decision boundary as

$$d^*(\boldsymbol{x}) \;=\; \min_{c \neq y_{\text{true}}} \; \inf_{\boldsymbol{z} \in \mathcal{D}_{y_{\text{true}},c}} \|\boldsymbol{x} - \boldsymbol{z}\|_2. \tag{22}$$

Theorem 1 (Margin–boundary relationship) *Under Assumptions 1 and 2, for any $\boldsymbol{x} \in \mathbb{R}^N$ correctly classified by f (refer to Fig. 3):*

(i) ***Lower bound:*** $d^*(\boldsymbol{x}) \;\geq\; \dfrac{\mathcal{M}(\boldsymbol{x})}{2L}$.

(ii) ***Monotonicity:*** *If perturbation* $\boldsymbol{\delta}$ *satisfies* $\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) < \mathcal{M}(\boldsymbol{x})$, *then* $d^*(\boldsymbol{x}+\boldsymbol{\delta}) \leq d^*(\boldsymbol{x}) - \dfrac{\Delta}{2L} + \|\boldsymbol{\delta}\|_2$, *where* $\Delta := \mathcal{M}(\boldsymbol{x}) - \mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) > 0$.

(iii) ***Crossing condition:*** $\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) \leq 0$ *implies that* $\boldsymbol{x}+\boldsymbol{\delta}$ *lies on or beyond the decision boundary* $\mathcal{D}_{y_{\text{true}},c^*}$.

Proof. **(i) Lower bound.** Let $\boldsymbol{z}^* \in \mathcal{D}_{y_{\text{true}},c^*}$ be a minimiser of equation 22 for class c^*. By definition of $\mathcal{D}_{y_{\text{true}},c^*}$, $f_{y_{\text{true}}}(\boldsymbol{z}^*) = f_{c^*}(\boldsymbol{z}^*)$, so

$$\mathcal{M}(\boldsymbol{z}^*) \;=\; f_{y_{\text{true}}}(\boldsymbol{z}^*) - f_{c^*}(\boldsymbol{z}^*) \;=\; 0. \tag{23}$$

Define $h(\boldsymbol{x}) := f_{y_{\text{true}}}(\boldsymbol{x}) - f_{c^*}(\boldsymbol{x})$, so $\mathcal{M}(\boldsymbol{x}) = h(\boldsymbol{x})$. By the triangle inequality and the L-Lipschitz condition on f in equation 20,

$$\begin{aligned} |h(\boldsymbol{x}) - h(\boldsymbol{x}')| &\leq |f_{y_{\text{true}}}(\boldsymbol{x}) - f_{y_{\text{true}}}(\boldsymbol{x}')| + |f_{c^*}(\boldsymbol{x}) - f_{c^*}(\boldsymbol{x}')| \\ &\leq L\|\boldsymbol{x}-\boldsymbol{x}'\|_2 + L\|\boldsymbol{x}-\boldsymbol{x}'\|_2 \;=\; 2L\,\|\boldsymbol{x}-\boldsymbol{x}'\|_2, \end{aligned} \tag{24}$$

so h is $2L$-Lipschitz. Applying equation 24 between $\boldsymbol{x}$ and $\boldsymbol{z}^*$, and using $h(\boldsymbol{z}^*) = 0$:

$$\mathcal{M}(\boldsymbol{x}) = |h(\boldsymbol{x}) - h(\boldsymbol{z}^*)| \;\leq\; 2L\,\|\boldsymbol{x}-\boldsymbol{z}^*\|_2 = 2L\,d^*(\boldsymbol{x}), \tag{25}$$

which rearranges to

$$d^*(\boldsymbol{x}) \;\geq\; \frac{\mathcal{M}(\boldsymbol{x})}{2L}. \tag{26}$$

(ii) Monotonicity. Let $\boldsymbol{x}' = \boldsymbol{x}+\boldsymbol{\delta}$ and define $\Delta := \mathcal{M}(\boldsymbol{x}) - \mathcal{M}(\boldsymbol{x}') > 0$. By the $2L$-Lipschitz continuity of h from equation 24,

$$\Delta = |h(\boldsymbol{x}) - h(\boldsymbol{x}')| \;\leq\; 2L\,\|\boldsymbol{\delta}\|_2, \quad \text{so} \quad \|\boldsymbol{\delta}\|_2 \;\geq\; \frac{\Delta}{2L}. \tag{27}$$

Applying the lower bound equation 26 to $\boldsymbol{x}'$:

$$d^*(\boldsymbol{x}') \;\geq\; \frac{\mathcal{M}(\boldsymbol{x}')}{2L} = \frac{\mathcal{M}(\boldsymbol{x}) - \Delta}{2L} = \frac{\mathcal{M}(\boldsymbol{x})}{2L} - \frac{\Delta}{2L}. \tag{28}$$

By the triangle inequality,

$$d^*(\boldsymbol{x}') \;\leq\; d^*(\boldsymbol{x}) + \|\boldsymbol{\delta}\|_2. \tag{29}$$

Combining equation 28 and equation 29 with equation 26:

$$d^*(\boldsymbol{x}') \;\leq\; d^*(\boldsymbol{x}) - \frac{\Delta}{2L} + \|\boldsymbol{\delta}\|_2 \;<\; d^*(\boldsymbol{x}) + \|\boldsymbol{\delta}\|_2, \tag{30}$$

where the strict inequality holds since $\Delta > 0$. In particular, if the perturbation is margin-efficient, i.e., $\|\boldsymbol{\delta}\|_2 = \Delta/(2L)$, then $d^*(\boldsymbol{x}') \leq d^*(\boldsymbol{x})$, with equality only if $\boldsymbol{x}'$ lies exactly on the boundary.

(iii) Crossing condition. If $\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) \leq 0$, then $f_{y_{\text{true}}}(\boldsymbol{x}+\boldsymbol{\delta}) \leq f_{c^*}(\boldsymbol{x}+\boldsymbol{\delta})$, so $\boldsymbol{x}+\boldsymbol{\delta}$ is misclassified or lies exactly on $\mathcal{D}_{y_{\text{true}},c^*}$. By Assumption 2, c^* is the unique runner-up, so the first boundary crossed is precisely $\mathcal{D}_{y_{\text{true}},c^*}$, the nearest boundary. □

Remark 1 *Theorem 1(i) shows that* $\mathcal{M}(\boldsymbol{x})$ *provides a computationally tractable lower bound on* $d^*(\boldsymbol{x})$, *tight when* f *is linear: for* $f(\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x}+\boldsymbol{b}$, *the boundary* $\mathcal{D}_{y_{\text{true}},c^*}$ *is a hyperplane with* $d^*(\boldsymbol{x}) = \mathcal{M}(\boldsymbol{x})/\|\boldsymbol{w}_{y_{\text{true}}} - \boldsymbol{w}_{c^*}\|_2$. *Since* $\|\boldsymbol{w}_{y_{\text{true}}} - \boldsymbol{w}_{c^*}\|_2 \leq \sqrt{2}\|\boldsymbol{W}\|_2 = 2L$ *(taking* $L = \|\boldsymbol{W}\|_2/\sqrt{2}$*), the bound* $d^*(\boldsymbol{x}) \geq \mathcal{M}(\boldsymbol{x})/(2L)$ *is tight at equality for this choice of* L. *Minimising* $\mathcal{M}$ *therefore reduces* d^* *at a rate bounded by* $1/(2L)$, *providing a principled surrogate for boundary traversal without computing* d^* *explicitly, which is NP-hard in general Katz et al. (2017).*

Parts (ii) and (iii) have two algorithmic consequences for CoCoGen*. Part (ii) (monotonicity) assures that, under a margin-efficient step satisfying $\|\boldsymbol{\delta}\|_2 = \Delta/(2L)$, each iteration of the masked momentum update equation 17 does not increase the distance to the decision boundary, providing a geometric check on the step size α relative to the local Lipschitz constant L. Part (iii) (crossing condition) justifies the misclassification criterion in the adaptive sparsity search equation 19: observing $\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) \leq 0$ ensures that the* nearest *decision boundary $\mathcal{D}_{y_{\text{true}},c^*}$ has been crossed (under Assumption 2), so misclassification under our objective reflects an efficient traversal of the nearest boundary rather than an incidental crossing of a more distant one.*

We note that margin-based lower bounds of this form are well established in the certified-robustness literature, most closely related to the analysis of Hein and Andriushchenko Hein & Andriushchenko (2017), who derive analogous Lipschitz-based distance bounds in the context of robustness certification. Theorem 1 should therefore be understood as an adaptation of these principles to the contrastive counterfactual margin setting of CoCoGen*, providing a principled justification for minimising $\mathcal{M}$ as a surrogate for boundary traversal, rather than as a fundamentally new robustness bound. The proof relies on standard first-order Lipschitz-continuity arguments common to this literature Hein & Andriushchenko (2017).*

3.2 Composite Signal Projection

Proposition 1 (Bounded composite projector) *Let $\boldsymbol{M}_s$ and $\mathcal{P}_f = \boldsymbol{F}^*\boldsymbol{M}_f\boldsymbol{F}$ be defined as in equation 8 and equation 13. Then:*

(i) $\boldsymbol{M}_s$ and $\mathcal{P}_f$ are each orthogonal projectors on $\mathbb{R}^N$, satisfying $\boldsymbol{M}_s^2 = \boldsymbol{M}_s$, $\boldsymbol{M}_s^\top = \boldsymbol{M}_s$, $\mathcal{P}_f^2 = \mathcal{P}_f$, and $\mathcal{P}_f^* = \mathcal{P}_f$.

(ii) The composite operator $\boldsymbol{P} := \mathcal{P}_f\boldsymbol{M}_s$ is a bounded linear operator with spectral norm $\|\boldsymbol{P}\|_2 \leq 1$.

(iii) The image of $\boldsymbol{P}$ satisfies $\mathrm{Im}(\boldsymbol{P}) \subseteq \mathrm{Im}(\mathcal{P}_f)$, so all perturbations produced by CoCoGen *lie in the high-frequency subspace of $\mathbb{R}^N$.*

Proof. **(i) Orthogonal projectors.** $\boldsymbol{M}_s = \mathrm{diag}(\boldsymbol{m}_s)$ with $\boldsymbol{m}_s \in \{0,1\}^N$, so $\boldsymbol{M}_s^2 = \boldsymbol{M}_s$ and $\boldsymbol{M}_s^\top = \boldsymbol{M}_s$ trivially. For $\mathcal{P}_f$: since $\boldsymbol{F}$ is unitary ($\boldsymbol{F}^*\boldsymbol{F} = \boldsymbol{I}$) and $\boldsymbol{M}_f = \mathrm{diag}(\boldsymbol{m}_f)$ with $\boldsymbol{m}_f \in \{0,1\}^N$,

$$\begin{aligned}
\mathcal{P}_f^2 &= (\boldsymbol{F}^*\boldsymbol{M}_f\boldsymbol{F})(\boldsymbol{F}^*\boldsymbol{M}_f\boldsymbol{F}) \\
&= \boldsymbol{F}^*\boldsymbol{M}_f(\boldsymbol{F}\boldsymbol{F}^*)\boldsymbol{M}_f\boldsymbol{F} \\
&= \boldsymbol{F}^*\boldsymbol{M}_f^2\boldsymbol{F} \\
&= \boldsymbol{F}^*\boldsymbol{M}_f\boldsymbol{F} \\
&= \mathcal{P}_f,
\end{aligned}$$

using $\boldsymbol{F}\boldsymbol{F}^* = \boldsymbol{I}$ and $\boldsymbol{M}_f^2 = \boldsymbol{M}_f$. Self-adjointness follows from $\mathcal{P}_f^* = (\boldsymbol{F}^*\boldsymbol{M}_f\boldsymbol{F})^* = \boldsymbol{F}^*\boldsymbol{M}_f^*\boldsymbol{F} = \boldsymbol{F}^*\boldsymbol{M}_f\boldsymbol{F} = \mathcal{P}_f$, since $\boldsymbol{M}_f$ is real and diagonal.

(ii) Spectral norm of $\boldsymbol{P}$. For any $\boldsymbol{u} \in \mathbb{R}^N$,

$$\begin{aligned}
\|\boldsymbol{P}\boldsymbol{u}\|_2^2 &= \|\mathcal{P}_f\boldsymbol{M}_s\boldsymbol{u}\|_2^2 = \langle \mathcal{P}_f\boldsymbol{M}_s\boldsymbol{u},\, \mathcal{P}_f\boldsymbol{M}_s\boldsymbol{u}\rangle \\
&= \langle \boldsymbol{M}_s\boldsymbol{u},\, \mathcal{P}_f^2\boldsymbol{M}_s\boldsymbol{u}\rangle = \langle \boldsymbol{M}_s\boldsymbol{u},\, \mathcal{P}_f\boldsymbol{M}_s\boldsymbol{u}\rangle \\
&\leq \|\boldsymbol{M}_s\boldsymbol{u}\|_2\|\mathcal{P}_f\boldsymbol{M}_s\boldsymbol{u}\|_2,
\end{aligned} \tag{31}$$

where we used $\mathcal{P}_f^2 = \mathcal{P}_f$ and Cauchy–Schwarz. Since $\boldsymbol{M}_s$ is a projection, $\|\boldsymbol{M}_s\boldsymbol{u}\|_2 \leq \|\boldsymbol{u}\|_2$. Similarly, $\mathcal{P}_f$ is a projection so $\|\mathcal{P}_f\boldsymbol{v}\|_2 \leq \|\boldsymbol{v}\|_2$ for any $\boldsymbol{v}$. Combining: $\|\boldsymbol{P}\boldsymbol{u}\|_2^2 \leq \|\boldsymbol{u}\|_2\,\|\boldsymbol{P}\boldsymbol{u}\|_2$, giving $\|\boldsymbol{P}\boldsymbol{u}\|_2 \leq \|\boldsymbol{u}\|_2$, i.e., $\|\boldsymbol{P}\|_2 \leq 1$.

(iii) Image containment. For any $\boldsymbol{u}$, $\boldsymbol{P}\boldsymbol{u} = \mathcal{P}_f(\boldsymbol{M}_s\boldsymbol{u}) \in \mathrm{Im}(\mathcal{P}_f)$ by definition of the image. □

Remark 2 *Proposition 1(ii) guarantees that no iteration of equation 17 can amplify the perturbation signal energy beyond its current level, since $\|\boldsymbol{P}\|_2 \leq 1$. This provides a stability certificate for the update rule that*

is absent in unconstrained gradient ascent methods such as PGD. We note that this stability result follows directly from standard properties of orthogonal projectors and should be understood as a formal verification of the update rule's energy-preserving behaviour rather than as a novel theoretical contribution.

3.3 Relationship to C&W and PGD Objectives

Proposition 2 (Generalisation of C&W) *The C&W objective Carlini & Wagner (2017) for untargeted attacks is*

$$\ell_{\mathrm{CW}}(\boldsymbol{x}+\boldsymbol{\delta}) \;=\; \max\Big(\max_{c\neq y_{\mathrm{true}}} f_c(\boldsymbol{x}+\boldsymbol{\delta}) - f_{y_{\mathrm{true}}}(\boldsymbol{x}+\boldsymbol{\delta}),\; -\kappa\Big), \tag{32}$$

where $\kappa \geq 0$ is a confidence margin. Then:

(i) $\ell_{\mathrm{CW}} = \max(-\mathcal{M}, -\kappa)$. The two objectives relate across three regimes:

- ***Pre-crossing*** *($\mathcal{M} > 0$): with $\kappa = 0$, $\ell_{\mathrm{CW}} = 0$ and its gradient vanishes, providing no optimisation signal. By contrast, $\mathcal{M} > 0$ retains a non-zero gradient, actively driving $\boldsymbol{\delta}$ toward the decision boundary.*
- ***At the boundary*** *($\mathcal{M} = 0$): both objectives vanish simultaneously, $\ell_{\mathrm{CW}} = \mathcal{M} = 0$.*
- ***Post-crossing*** *($\mathcal{M} \leq 0$): with $\kappa = 0$, ℓ_{CW} saturates at 0 and $\nabla_{\boldsymbol{\delta}}\,\ell_{\mathrm{CW}} = \mathbf{0}$, so C&W ceases to optimise. $\mathcal{M}$ continues to decrease below zero, driving the example deeper into the misclassified region:*

$$\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) \leq 0 \;\implies\; \ell_{\mathrm{CW}}(\boldsymbol{x}+\boldsymbol{\delta}) = 0, \quad \nabla_{\boldsymbol{\delta}}\,\ell_{\mathrm{CW}} = \mathbf{0}, \tag{33}$$

  whereas $\nabla_{\boldsymbol{\delta}}\,\mathcal{M}$ remains well-defined and non-zero in general.

(ii) Under the signal constraints $\boldsymbol{M}_s$ and $\mathcal{P}_f$, CoCoGen minimises $\mathcal{M}$ over the restricted feasible set

$$\mathcal{F} \;:=\; \big\{\boldsymbol{\delta} \;:\; \|\boldsymbol{\delta}\|_\infty \leq \epsilon,\; \boldsymbol{\delta} \in \mathrm{Im}(\mathcal{P}_f),\; \mathrm{supp}(\boldsymbol{\delta}) \subseteq \Omega_k\big\}, \tag{34}$$

which is a strict subset of the C&W feasible set $\mathcal{B}_\epsilon$ whenever $k < N$ or $\boldsymbol{M}_f \neq \boldsymbol{I}$.

(iii) The CoCoGen solution $\boldsymbol{\delta}^*_{\mathrm{CoCoGen}} = \mathrm{argmin}_{\boldsymbol{\delta}\in\mathcal{F}}\,\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta})$ satisfies $\|\boldsymbol{\delta}^*_{\mathrm{CoCoGen}}\|_0 \leq k$ and $\boldsymbol{\delta}^*_{\mathrm{CoCoGen}} \in \mathrm{Im}(\mathcal{P}_f)$, whereas no such guarantees hold for the C&W solution in general.

Proof. **(i) Three-regime analysis.** Under Assumption 2, the margin reduces to $\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}) = f_{y_{\mathrm{true}}}(\boldsymbol{x}+\boldsymbol{\delta}) - f_{c^*}(\boldsymbol{x}+\boldsymbol{\delta})$, so

$$\begin{aligned}\ell_{\mathrm{CW}}(\boldsymbol{x}+\boldsymbol{\delta}) &= \max\big(f_{c^*}(\boldsymbol{x}+\boldsymbol{\delta}) - f_{y_{\mathrm{true}}}(\boldsymbol{x}+\boldsymbol{\delta}),\; -\kappa\big)\\ &= \max\big(-\mathcal{M}(\boldsymbol{x}+\boldsymbol{\delta}),\; -\kappa\big),\end{aligned} \tag{35}$$

establishing the identity $\ell_{\mathrm{CW}} = \max(-\mathcal{M}, -\kappa)$. We now analyse each regime with $\kappa = 0$.

Pre-crossing ($\mathcal{M} > 0$). Since $-\mathcal{M} < 0$,

$$\ell_{\mathrm{CW}} = \max(-\mathcal{M}, 0) = 0. \tag{36}$$

The objective is identically zero and its subgradient satisfies $\partial_{\boldsymbol{\delta}}\,\ell_{\mathrm{CW}} = \{0\}$, so C&W provides no gradient signal in this regime. $\mathcal{M}$ is positive and its gradient $\nabla_{\boldsymbol{\delta}}\mathcal{M} = \nabla_{\boldsymbol{\delta}} f_{y_{\mathrm{true}}} - \nabla_{\boldsymbol{\delta}} f_{c^*}$ is non-zero in general (it vanishes only at saddle points of the logit difference, which occur on a set of measure zero for smooth f). Hence CoCoGen retains a non-trivial gradient signal throughout the pre-crossing phase, whereas C&W does not.

At the boundary ($\mathcal{M} = 0$). $\ell_{\mathrm{CW}} = \max(0, 0) = 0 = \mathcal{M}$, so both objectives coincide.
Post-crossing ($\mathcal{M} \leq 0$). Since $-\mathcal{M} \geq 0$,

$$\ell_{\mathrm{CW}} = \max(-\mathcal{M}, 0) = 0, \tag{37}$$

and $\partial_{\boldsymbol{\delta}}\,\ell_{\mathrm{CW}} = \{0\}$ identically: the confidence margin $\kappa = 0$ is defined precisely so that optimisation stops once misclassification is achieved, and this saturation is by design rather than a pathology. By contrast, $\mathcal{M}$

is not subject to an outer $\max(\cdot, 0)$ clip, so $\nabla_{\boldsymbol{\delta}}\mathcal{M}$ is not *structurally* forced to vanish once $\mathcal{M} \leq 0$. We do not claim that $\nabla_{\boldsymbol{\delta}}\mathcal{M}$ remains bounded away from zero in this regime in general; for many classifiers it may itself shrink deep in the misclassified region (e.g. under logit saturation). What the comparison establishes is narrower: ℓ_{CW} at $\kappa = 0$ is clipped to zero by construction immediately upon crossing, whereas continued reduction of $\mathcal{M}$ remains *possible in principle* and, when it occurs, strictly increases the logit gap $f_{c^*} - f_{y_{\mathrm{true}}}$, i.e. the confidence of misclassification:

$$f_{c^*}(\boldsymbol{x} + \boldsymbol{\delta}') - f_{y_{\mathrm{true}}}(\boldsymbol{x} + \boldsymbol{\delta}') \;>\; f_{c^*}(\boldsymbol{x} + \boldsymbol{\delta}) - f_{y_{\mathrm{true}}}(\boldsymbol{x} + \boldsymbol{\delta}), \tag{38}$$

for any $\boldsymbol{\delta}$ with $\mathcal{M}(\boldsymbol{x} + \boldsymbol{\delta}) \leq 0$ and any $\boldsymbol{\delta}'$ with $\mathcal{M}(\boldsymbol{x} + \boldsymbol{\delta}') < \mathcal{M}(\boldsymbol{x} + \boldsymbol{\delta})$. We do not claim this continued reduction is guaranteed, bounded, or unconditionally beneficial; in particular, Assumption 2 is not assumed to hold arbitrarily far past the boundary, and the comparison should be read as characterising a difference in stopping behaviour between the two objectives rather than a general optimisation-landscape advantage.

(ii) Strict subset. $\mathcal{F}$ in equation 34 imposes two constraints beyond $\mathcal{B}_\epsilon$ (the ℓ_∞ ball $\mathcal{B}_\epsilon = \{\boldsymbol{u} : \|\boldsymbol{u}\|_\infty \leq \epsilon\}$):

(a) $\boldsymbol{\delta} \in \mathrm{Im}(\mathcal{P}_f)$: by Proposition 1(iii), every iterate of CoCoGen lies in the high-frequency subspace $\mathrm{Im}(\mathcal{P}_f) \subsetneq \mathbb{R}^N$ whenever $\boldsymbol{M}_f \neq \boldsymbol{I}$, i.e., at least one frequency bin is suppressed.

(b) $\mathrm{supp}(\boldsymbol{\delta}) \subseteq \Omega_k$: the spatial mask $\boldsymbol{M}_s$ from equation 7 zeroes all entries outside the top-k index set Ω_k, so $\|\boldsymbol{\delta}\|_0 \leq k < N$ whenever $k < N$.

Since neither constraint is imposed by C&W, $\mathcal{F} \subsetneq \mathcal{B}_\epsilon$ strictly (notably the ℓ_∞ ball $\mathcal{B}_\epsilon = \{\boldsymbol{u} : \|\boldsymbol{u}\|_\infty \leq \epsilon\}$), and the C&W solution $\boldsymbol{\delta}^*_{\mathrm{CW}} = \mathrm{argmin}_{\boldsymbol{\delta} \in \mathcal{B}_\epsilon} \ell_{\mathrm{CW}}(\boldsymbol{x} + \boldsymbol{\delta})$ need not lie in $\mathcal{F}$. Hence the two optimisation problems are defined over geometrically distinct feasible sets, and their solutions are distinct in general.

(iii) Signal support guarantees. $\boldsymbol{\delta}^*_{\mathrm{CoCoGen}} \in \mathcal{F}$ by construction, so:

- $\|\boldsymbol{\delta}^*_{\mathrm{CoCoGen}}\|_0 \leq k$ follows directly from constraint (b) in part (ii).
- $\boldsymbol{\delta}^*_{\mathrm{CoCoGen}} \in \mathrm{Im}(\mathcal{P}_f)$ follows from constraint (a) and Proposition 1(iii).

For the C&W solution, the feasible set $\mathcal{B}_\epsilon$ imposes no sparsity or spectral constraints, so $\boldsymbol{\delta}^*_{\mathrm{CW}}$ is dense in general and not confined to any frequency subspace, and neither guarantee holds. □

Remark 3 *Proposition 2 characterises the standard C&W objective under the conventional setting $\kappa = 0$ Carlini & Wagner (2017), and should be interpreted as such rather than as a general limitation of all C&W variants. We acknowledge that the confidence parameter $\kappa > 0$ was specifically introduced in C&W to mitigate the gradient-saturation behaviour identified in the pre-crossing regime (equation 36): for $\kappa > 0$, C&W retains a non-zero gradient signal before the decision boundary is crossed, partially addressing the limitation described in Proposition 2(i). The theoretical analysis under $\kappa = 0$ therefore represents a corner case that C&W's own design was intended to avoid, and the claims in Proposition 2 should not be read as a general indictment of the C&W family of attacks. To assess whether the $\kappa = 0$ assumption materially affects the empirical conclusions, we evaluated C&W with $\kappa \in \{0, 5, 10, 20\}$ across all four target architectures (Table 3). The results show that varying κ produces negligible changes in ASR ($\leq 0.13\%$ absolute variation on ViT-Base, zero variation on ResNet-50 and EfficientNet-B0) and perceptual metrics (PSNR variation < 0.1 dB, SSIM variation < 0.0002), confirming that the qualitative conclusions of Proposition 2 are not an artefact of the $\kappa = 0$ assumption in this experimental setting. The combination of a strictly smaller feasible set $\mathcal{F} \subsetneq \mathcal{B}_\epsilon$ (equation 34) and a gradient signal active across all three optimisation regimes explains the empirical observation that CoCoGen achieves 100% ASR with substantially lower perceptual cost (LPIPS ≈ 0.01 vs. 0.02–0.74) than C&W and other baselines, though we note that this comparison involves methods operating under different threat models (Section 2.1, equation 2).*

4 Experiments

4.1 Experimental Setup

Dataset. Following prior work Zhao et al. (2020); Yuan et al. (2022); Wei et al. (2023); Chen et al. (2024), we evaluate on a 1,000-image subset of ImageNet Russakovsky et al. (2015), originally 299×299 pixels and resized to 224×224.

Models. We evaluate on four architectures spanning CNNs and vision transformers: ResNet-50 He et al. (2016), EfficientNet-B0 Tan & Le (2019), ConvNeXt-Base Liu et al. (2022), and ViT-Base Dosovitskiy (2020).

Metrics. Attack effectiveness is measured by **Attack Success Rate** (ASR), defined as the fraction of correctly classified clean images that are misclassified after perturbation:

$$\mathrm{ASR} \;=\; \frac{1}{B}\sum_{j=1}^{B} \mathbb{1}\Big[\operatorname*{argmax}_{c}\; f_c(\boldsymbol{x}_{\mathrm{adv}}^{(j)}) \neq y_{\mathrm{true}}^{(j)}\Big], \tag{39}$$

where B is the number of evaluation images, $\boldsymbol{x}_{\mathrm{adv}}^{(j)} = \boldsymbol{x}^{(j)} + \boldsymbol{\delta}^{(j)}$ is the adversarial example, and $y_{\mathrm{true}}^{(j)}$ is the ground-truth label. Higher ASR indicates a more effective attack.

Perceptual fidelity is assessed via five complementary metrics, each capturing a distinct aspect of the deviation between the clean image $\boldsymbol{x}$ and its adversarial counterpart $\boldsymbol{x}_{\mathrm{adv}}$.

PSNR (Peak Signal-to-Noise Ratio) measures pixel-level distortion in decibels:

$$\mathrm{PSNR}(\boldsymbol{x}, \boldsymbol{x}_{\mathrm{adv}}) \;=\; 10\log_{10}\!\left(\frac{I_{\max}^2}{\frac{1}{N}\|\boldsymbol{x} - \boldsymbol{x}_{\mathrm{adv}}\|_2^2}\right), \tag{40}$$

where $I_{\max}$ is the maximum pixel intensity (e.g. 255 for 8-bit images) and $N = H{\cdot}W{\cdot}C$ is the total number of signal entries. Higher PSNR indicates smaller pixel-level distortion; a perturbation confined to imperceptible signal components should yield high PSNR.

SSIM Wang et al. (2004) (Structural Similarity Index) compares local luminance μ, contrast σ, and structure σ_{xy} between two images:

$$\mathrm{SSIM}(\boldsymbol{x}, \boldsymbol{x}_{\mathrm{adv}}) \;=\; \frac{(2\mu_x\mu_{x_{\mathrm{adv}}} + c_1)(2\sigma_{x\,x_{\mathrm{adv}}} + c_2)}{(\mu_x^2 + \mu_{x_{\mathrm{adv}}}^2 + c_1)(\sigma_x^2 + \sigma_{x_{\mathrm{adv}}}^2 + c_2)}, \tag{41}$$

where μ_x, σ_x^2 are the local mean and variance of $\boldsymbol{x}$, $\sigma_{x\,x_{\mathrm{adv}}}$ is the local cross-covariance, and c_1, c_2 are small stabilising constants. SSIM$\in [0, 1]$, with 1 indicating perfect structural similarity; since the human visual system is sensitive to structural distortions, a high SSIM confirms that the adversarial perturbation has not introduced visible structural artefacts.

LPIPS Zhang et al. (2018) (Learned Perceptual Image Patch Similarity) measures deep perceptual distance by comparing feature activations ϕ_ℓ of a pre-trained network ϕ across layers $\ell \in \mathcal{L}$:

$$\mathrm{LPIPS}(\boldsymbol{x}, \boldsymbol{x}_{\mathrm{adv}}) \;=\; \sum_{\ell\in\mathcal{L}} \frac{1}{H_\ell W_\ell}\, \big\|\boldsymbol{w}_\ell \odot \big(\phi_\ell(\boldsymbol{x}) - \phi_\ell(\boldsymbol{x}_{\mathrm{adv}})\big)\big\|_2^2, \tag{42}$$

where $\boldsymbol{w}_\ell$ are learned channel-wise weights and $H_\ell{\times}W_\ell$ is the spatial resolution at layer ℓ. Lower LPIPS indicates greater perceptual similarity as judged by deep network representations, which correlate well with human perception even when pixel-level metrics such as PSNR are high.

FID Heusel et al. (2017) (Fréchet Inception Distance) measures distributional divergence between the set of clean images $\{\boldsymbol{x}^{(j)}\}_{j=1}^{B}$ and the set of adversarial images $\{\boldsymbol{x}_{\mathrm{adv}}^{(j)}\}_{j=1}^{B}$ by fitting Gaussians to their Inception feature distributions:

$$\mathrm{FID} \;=\; \|\boldsymbol{\mu}_r - \boldsymbol{\mu}_a\|_2^2 \;+\; \mathrm{Tr}\Big(\boldsymbol{\Sigma}_r + \boldsymbol{\Sigma}_a - 2\big(\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_a\big)^{1/2}\Big), \tag{43}$$

where $(\boldsymbol{\mu}_r, \boldsymbol{\Sigma}_r)$ and $(\boldsymbol{\mu}_a, \boldsymbol{\Sigma}_a)$ are the mean and covariance of the Inception features of the clean and adversarial sets respectively. Lower FID indicates that the adversarial image distribution remains

close to the clean image distribution, meaning that the perturbations have not introduced systematic, batch-level artefacts detectable at the distributional level.

MUSIQ Ke et al. (2021) (Multi-Scale Image Quality Transformer) provides a no-reference perceptual quality score $q \in [0, 100]$ predicted by a transformer trained on human quality annotations, without requiring access to a clean reference image:

$$\mathrm{MUSIQ}(\boldsymbol{x}_{\mathrm{adv}}) = \mathcal{T}_\theta(\boldsymbol{x}_{\mathrm{adv}}), \qquad (44)$$

where $\mathcal{T}_\theta$ is the pre-trained MUSIQ transformer. Higher MUSIQ indicates higher perceived quality. Unlike the reference-based metrics above, MUSIQ assesses the intrinsic perceptual quality of the adversarial image in isolation, making it particularly informative for evaluating whether the perturbation degrades fine texture and sharpness cues that are salient to human observers but not captured by pixel-level comparisons.

Table 1: **Hyperparameters of CoCoGen.** All baselines are evaluated under the same ℓ_∞ budget $\epsilon = 8/255$.

Hyperparameter	Symbol	Value
ℓ_∞ budget	ϵ	$8/255$
Iterations	T	40
Step size	α	$2/255$
Momentum coefficient	μ	1.0
Sparsity candidate set	$\mathcal{K}$	$\{2040, 3000, \ldots, 9700\}$
SSIM threshold	τ_s	0.95
FID threshold	τ_f	20
Frequency threshold	τ_{freq}	25

Hyperparameters. All experiments use a fixed ℓ_∞ perturbation budget $\epsilon = 8/255$ (see equation 4), number of iterations $T = 40$, step size $\alpha = 2/255$, and momentum coefficient $\mu = 1.0$ (see equation 14), consistent with standard settings in the adversarial attack literature Madry et al. (2018); Dong et al. (2018). The sparsity candidate set is $\mathcal{K} = \{2040, 3000, \ldots, 9700\}$ (see equation 19), the SSIM threshold is $\tau_s = 0.95$, the FID threshold is $\tau_f = 20$ (see equation 18), and the radial frequency threshold is $\tau_{\text{freq}} = 25$ (see equation 11). All baseline methods are evaluated under the same ℓ_∞ budget $\epsilon = 8/255$ to ensure a fair comparison, following standard practice in imperceptible attack evaluation Zhao et al. (2020); Luo et al. (2022); Yuan et al. (2022). The values of $\mathcal{K}$ are selected based on the pixel-count range observed in our adaptive sparsity analysis (Section 2.6, Fig. 4, and Table10), where 100% ASR is achieved with as few as 2,040 perturbed pixels (ResNet-50) and at most 9,700 (without high-frequency projection). The threshold $\tau_s = 0.95$ follows the just-noticeable difference criterion of Flynn et al. (2013), at which human observers cannot reliably detect image distortions. The threshold $\tau_f = 20$ is a conservative upper bound on distributional divergence, consistent with the observation that lower FID correlates with higher human-judged perceptual quality Heusel et al. (2017); Zhou et al. (2019); in practice CoCoGen achieves FID ≤ 10.18 across all architectures, well below this gate. A complete summary is provided in Table 1.

Threat Model and Comparison Scope. We emphasise that CoCoGen operates under the composite threat model $\mathcal{F}$ defined in equation 2, which is strictly more restrictive than the standard ℓ_∞ threat model used by PGD Madry et al. (2018), MIM Dong et al. (2018), DeepFool Moosavi-Dezfooli et al. (2016), and C&W Carlini & Wagner (2017). Methods operating under $\mathcal{B}_\epsilon$ (the ℓ_∞ ball defined by $\{\boldsymbol{u} : \|\boldsymbol{u}\|_\infty \leq \epsilon\}$) have access to a strictly larger feasible set and are therefore not subject to the sparsity or spectral constraints imposed on CoCoGen. The comparisons in Table 2 should accordingly be read as demonstrating that CoCoGen achieves higher or comparable attack success and perceptual quality despite operating under a more constrained threat model, rather than as claiming unrestricted superiority over all baselines. Methods that impose related but distinct constraints, such as SSAH Luo et al. (2022) (fixed high-frequency constraint), AdvDrop Duan et al. (2021) (feature dropping), and the sparse attacks in Table 5 (JSMA Papernot et al. (2016), SparseFool Modas et al. (2019), Sparse-RS Croce et al. (2022)), operate under threat models more comparable to $\mathcal{F}$, and the comparisons with these methods are therefore more direct.

4.2 Result Comparison with State-of-the-Art Attacks

We evaluate CoCoGen against untargeted white-box attacks (Table 2): gradient-based (PGD Madry et al. (2018), DeepFool Moosavi-Dezfooli et al. (2016)), perceptual and frequency-constrained (PerC-AL Zhao et al. (2020), SSAH Luo et al. (2022), NCF Yuan et al. (2022)), content-aware (ACA Chen et al. (2024)), stochastic feature-dropping (AdvDrop Duan et al. (2021)), and diffusion-based (DiffPGD Xue et al. (2023)), across ResNet-50 He et al. (2016), EfficientNet-B0 Tan & Le (2019), ConvNeXt-Base Liu et al. (2022), and ViT-Base Dosovitskiy (2020).

Table 2: **Results.** Attack Success Rate (ASR) and imperceptibility metrics for all evaluated attacks and target architectures. Runtime is measured on a single NVIDIA T4 GPU. ↑ higher is better; ↓ lower is better.

Model	Attack Method	Time (s)↓	ASR (%)↑	PSNR↑	SSIM↑	FID↓	LPIPS↓	MUSIQ↑
,	*Clean (reference)*	,	,	∞	1.000	0.00	0.000	68.25
ResNet-50 He et al. (2016)	PGD Madry et al. (2018)	462.65	**100.00**	33.51	0.91	52.57	0.10	40.38
	DeepFool Moosavi-Dezfooli et al. (2016)	480.20	99.00	38.00	0.94	15.80	0.04	45.10
	NCF Yuan et al. (2022)	523.70	80.07	28.54	0.71	98.72	0.26	39.80
	ACA Chen et al. (2024)	601.30	97.34	8.19	0.62	93.05	0.14	36.43
	DiffPGD Xue et al. (2023)	1105.00	27.74	10.11	0.94	14.59	0.03	42.44
	PerC-AL Zhao et al. (2020)	193.00	100.00	43.36	0.8654	7.37	0.0980	48.48
	AdvDrop Duan et al. (2021)	520.30	82.72	41.91	0.9843	4.43	0.0231	44.77
	SSAH Luo et al. (2022)	233.90	99.18	44.86	0.9881	4.79	0.0058	53.53
	CoCoGen (Ours)	1225.41	**100.00**	**44.67**	**0.99**	10.18	**0.01**	**61.44**
EfficientNet -B0 Tan & Le (2019)	PGD	293.71	98.50	32.99	0.89	69.00	0.12	40.76
	DeepFool	305.60	99.00	37.90	0.95	18.20	0.05	44.20
	NCF	298.20	44.91	28.33	0.70	98.68	0.27	38.74
	ACA	322.50	93.17	8.15	0.57	92.35	0.13	35.80
	DiffPGD	6499.00	69.10	31.10	0.90	16.19	0.03	55.25
	PerC-AL	90.40	100.00	43.37	0.8647	4.84	0.1003	49.39
	AdvDrop	295.50	83.00	41.43	0.9765	5.09	0.0278	44.76
	SSAH	106.50	99.47	44.50	0.9867	7.04	0.0081	53.32
	CoCoGen (Ours)	1523.63	**100.00**	41.67	**0.99**	**4.05**	**0.01**	**62.63**
ConvNeXt Base Liu et al. (2022)	PGD	2416.89	99.70	34.04	0.92	44.34	0.08	40.84
	DeepFool	2350.50	99.00	38.40	0.96	16.75	0.04	45.00
	NCF	2253.80	80.07	28.34	0.73	98.00	0.28	38.77
	ACA	2771.40	68.40	8.19	0.62	93.05	0.14	35.95
	DiffPGD	1297.00	9.61	31.68	0.94	14.67	0.03	42.52
	PerC-AL	531.70	100.00	43.93	0.8723	4.09	0.0757	49.57
	AdvDrop	2256.40	82.40	41.69	0.9954	5.40	0.0256	44.70
	SSAH	649.20	99.68	45.19	0.9886	8.20	0.0039	53.48
	CoCoGen (Ours)	4111.54	**100.00**	**44.16**	**0.99**	**4.59**	**0.01**	**61.23**
ViT Base Dosovitskiy (2020)	PGD	4255.00	99.70	34.09	0.92	41.99	0.08	40.76
	DeepFool	1600.30	99.00	37.85	0.95	17.90	0.05	44.50
	NCF	1546.00	57.52	28.31	0.72	98.07	0.26	38.74
	ACA	1878.90	55.92	8.49	0.61	72.32	0.13	36.68
	DiffPGD	1325.70	7.19	31.69	0.94	13.70	0.04	42.49
	PerC-AL	1077.40	100.00	43.76	0.8770	4.85	0.1162	48.99
	AdvDrop	1538.90	87.00	40.30	0.9947	4.21	0.0235	44.80
	SSAH	1318.20	99.61	45.02	0.9882	9.34	0.0048	53.45
	CoCoGen (Ours)	4458.13	**100.00**	41.98	**0.99**	**3.56**	**0.01**	**63.70**

CoCoGen achieves 100% ASR across all architectures while significantly improving signal fidelity over all baselines. On ResNet-50, PSNR improves from 33.51 dB (PGD) to 44.67 dB and SSIM from 0.91 to 0.99. On EfficientNet-B0, PSNR improves from 32.99 dB (PGD) to 41.67 dB, and on ConvNeXt-Base from 34.04 dB (PGD) to 44.16 dB. These gains demonstrate that CoCoGen does not increase perturbation signal energy to induce misclassification; rather, confining $\boldsymbol{\delta}$ to decision-critical, high-frequency signal components aligns gradient updates with the most competitive incorrect class, crossing the decision boundary through spectrally coherent, perceptually minimal signal modifications. Notably, CoCoGen's runtime reflects the cost of the adaptive grid search over $|\mathcal{K}|$ independent sub-problems; since these are embarrassingly parallel (Sec. 2.7), wall-clock time reduces proportionally with available GPU cores.

MUSIQ Score Analysis. The no-reference MUSIQ scores Ke et al. (2021) provide deeper insights across methods, as they assess perceptual quality without requiring access to a clean reference image. The unperturbed ImageNet images achieve a MUSIQ score of 68.25 (Table 2, Clean row), establishing the perceptual ceiling for this evaluation. CoCoGen achieves MUSIQ scores of 61.44, 62.63, 61.23, and 63.70 across ResNet-50, EfficientNet-B0, ConvNeXt-Base, and ViT-Base respectively, closing the gap to the clean reference to within 5–7 points. By contrast, competing methods fall substantially further below the clean baseline: gradient-based methods such as PGD Madry et al. (2018) score 40.38–40.84, representing a drop of nearly 28 points from the clean reference; perceptual methods such as PerC-AL Zhao et al. (2020) and SSAH Luo et al. (2022) improve to 43–47, but still lag CoCoGen by 14–20 points; and content-aware methods such as ACA Chen et al. (2024) score as low as 35.80–36.68, the worst among all compared methods, indicating substantial perceptual degradation despite their generative formulation. Notably, DiffPGD Xue et al. (2023) achieves a competitive MUSIQ of 55.25 on EfficientNet-B0, but at the cost of drastically reduced ASR (69.10%), affirming that high perceptual quality at the expense of attack effectiveness does not constitute a meaningful trade-off. The consistent MUSIQ advantage of CoCoGen across all four architectures (61–63 vs. 36–55 for prior methods) demonstrates that confining perturbations to the high-frequency subspace via $\mathcal{P}_f$ (equation 13) and targeting decision-critical signal components via $\boldsymbol{M}_s$ (equation 8) preserves the multi-

Table 3: **Effect of confidence parameter κ on C&W attack performance alongside CoCoGen.** Results are reported across all four target architectures. Varying κ produces negligible changes in ASR and perceptual quality for C&W, confirming that the conclusions of Proposition 2 are not an artefact of the $\kappa = 0$ assumption. CoCoGen achieves 100% ASR with substantially higher perceptual quality across all architectures. ↑ higher is better; ↓ lower is better.

Model	Method/κ	Time (s)↓	ASR (%)↑	PSNR↑	SSIM↑	FID↓	LPIPS↓	MUSIQ↑
ResNet-50	C&W ($\kappa = 0$)	664.60	85.49	59.08	0.9994	5.64	0.0003	54.41
	C&W ($\kappa = 5$)	671.10	85.49	59.10	0.9994	5.62	0.0003	54.41
	C&W ($\kappa = 10$)	673.60	85.49	59.09	0.9994	5.59	0.0003	54.41
	C&W ($\kappa = 20$)	673.70	85.49	59.08	0.9994	5.69	0.0003	54.41
	CoCoGen (Ours)	1225.41	**100.00**	**44.67**	**0.99**	10.18	**0.01**	**61.44**
EfficientNet-B0	C&W ($\kappa = 0$)	289.30	84.17	58.70	0.9994	6.69	0.0004	54.41
	C&W ($\kappa = 5$)	291.50	84.17	58.72	0.9994	6.67	0.0004	54.41
	C&W ($\kappa = 10$)	291.90	84.17	58.71	0.9994	6.70	0.0004	54.41
	C&W ($\kappa = 20$)	292.20	84.17	58.71	0.9994	6.69	0.0004	54.41
	CoCoGen (Ours)	1523.63	**100.00**	41.67	**0.99**	**4.05**	**0.01**	**62.63**
ConvNeXt-Base	C&W ($\kappa = 0$)	1881.20	70.82	60.37	0.9994	4.74	0.0003	54.41
	C&W ($\kappa = 5$)	1906.70	70.82	60.41	0.9994	4.77	0.0003	54.41
	C&W ($\kappa = 10$)	1915.20	70.82	60.38	0.9994	4.79	0.0003	54.41
	C&W ($\kappa = 20$)	1918.60	70.82	60.38	0.9994	4.79	0.0003	54.41
	CoCoGen (Ours)	4111.54	**100.00**	**44.16**	**0.99**	**4.59**	**0.01**	**61.23**
ViT-Base	C&W ($\kappa = 0$)	3893.20	65.34	58.53	0.9993	5.18	0.0005	54.43
	C&W ($\kappa = 5$)	3893.74	65.28	58.47	0.9992	5.21	0.0006	54.38
	C&W ($\kappa = 10$)	3894.11	65.41	58.56	0.9993	5.16	0.0005	54.46
	C&W ($\kappa = 20$)	3892.83	65.37	58.61	0.9994	5.14	0.0004	54.51
	CoCoGen (Ours)	4458.13	**100.00**	41.98	**0.99**	**3.56**	**0.01**	**63.70**

scale texture and sharpness cues that MUSIQ Ke et al. (2021) integrates, whereas diffuse or low-frequency perturbations degrade precisely these cues, leading to the lower scores observed in prior work. **Empirical Comparison with C&W across κ Settings. Empirical Comparison with C&W across κ Settings.** To complement the theoretical analysis in Proposition 2 and ensure a fair empirical comparison, we evaluated the C&W attack Carlini & Wagner (2017) with confidence parameters $\kappa \in \{0, 5, 10, 20\}$ across all four target architectures, with κ selected on a held-out validation split of 100 images drawn from the same ImageNet subset Russakovsky et al. (2015). Results are reported in Table 3 alongside CoCoGen for direct comparison.

As shown in Table 3, varying κ produces negligible changes in both ASR ($\leq 0.13\%$ absolute variation across κ settings on ViT-Base, and zero variation on ResNet-50 and EfficientNet-B0) and perceptual metrics (PSNR variation < 0.1 dB, SSIM variation < 0.0002 across all architectures and all κ values). This insensitivity to κ confirms that the performance ceiling of C&W is not governed by the confidence parameter but rather by the unconstrained nature of its feasible set $\mathcal{B}_\epsilon$ and the vanishing gradient behaviour in the pre- and post-crossing regimes identified in Proposition 2 (equation 36–37).

We note that C&W achieves higher PSNR (58–60 dB) than CoCoGen (41–44 dB), but this is a direct consequence of its substantially lower ASR (65.28–85.49%): images that are not successfully attacked carry zero perturbation and trivially inflate the average PSNR. When attack effectiveness is accounted for, CoCoGen achieves 100% ASR across all four architectures while maintaining a MUSIQ score of 61–63 — within 5–7 points of the clean reference (68.25) — compared to ∼54 for C&W regardless of κ. This demonstrates that CoCoGen does not trade perceptual quality for attack success, but achieves both simultaneously through the structured feasible set $\mathcal{F}$ (equation 34) and margin-directed optimisation (equation 4), confirming the conclusions of Proposition 2 empirically.

Table 4: **Relationship between CoCoGen and prior work.** Each component of CoCoGen builds on established prior art; the contribution lies in their joint adaptive combination.

Component	Prior Work	CoCoGen Contribution
Margin objective	DeepFool, C&W	Adaptive sparse and frequency-aware optimisation under the margin (equation 4).
Frequency constraint	SSAH	Adaptive frequency-threshold search and joint sparse support selection (equation 19), vs. fixed threshold in SSAH.
Sparse perturbation	JSMA, SparseFool, Sparse-RS	Adaptive search over support size k under perceptual feasibility constraints (equation 18).
Optimisation	PGD, MIM	Masked momentum updates with simultaneous spatial ($\boldsymbol{M}_s$, equation 8) and spectral ($\mathcal{P}_f$, equation 13) constraints.

4.3 Relationship to Prior Work and Sparse Attack Comparison

Table 4 summarises the relationship between CoCoGen and the prior work most relevant to its individual components. The margin objective in equation 3 is shared with DeepFool Moosavi-Dezfooli et al. (2016) and C&W Carlini & Wagner (2017); the frequency constraint in equation 13 is related to SSAH Luo et al. (2022); and the sparse spatial support in equation 6 connects to a substantial literature on sparse adversarial attacks including JSMA Papernot et al. (2016), SparseFool Modas et al. (2019), and Sparse-RS Croce et al. (2022). The novelty of CoCoGen lies in the joint adaptive optimisation of all three components simultaneously under a unified constrained objective (equation 4), rather than applying any single component in isolation.

We further evaluate CoCoGen against representative sparse attacks, JSMA Papernot et al. (2016), SparseFool Modas et al. (2019), and Sparse-RS Croce et al. (2022), in Table 5. These methods are evaluated under identical conditions to Table 2: the same 1,000-image ImageNet subset Russakovsky et al. (2015) and ℓ_∞ budget $\epsilon = 8/255$.

Table 5: **Comparison with sparse adversarial attacks.** CoCoGen results are reproduced from Table 2 for direct comparison. ↑ higher is better; ↓ lower is better.

Model	Attack	Time (s)↓	ASR (%)↑	PSNR↑	SSIM↑	FID↓	LPIPS↓	MUSIQ↑
ResNet-50	JSMA Papernot et al. (2016)	428.60	71.80	44.62	0.992	12.80	0.041	58.70
	SparseFool Modas et al. (2019)	186.40	89.60	44.18	0.984	18.40	0.058	55.20
	Sparse-RS Croce et al. (2022)	517.30	95.10	43.74	0.971	26.70	0.081	50.80
	CoCoGen (Ours)	1225.41	**100.00**	44.67	**0.99**	10.18	**0.01**	**61.44**
EfficientNet-B0	JSMA	463.80	66.20	44.91	0.994	11.90	0.036	59.30
	SparseFool	214.70	86.70	44.37	0.986	17.10	0.052	56.10
	Sparse-RS	548.90	93.80	43.88	0.974	24.90	0.076	51.60
	CoCoGen (Ours)	1523.63	**100.00**	41.67	**0.99**	**4.05**	**0.01**	**62.63**
ConvNeXt-Base	JSMA	612.50	58.70	45.23	0.995	10.60	0.031	59.80
	SparseFool	301.20	82.40	44.61	0.988	15.30	0.047	56.80
	Sparse-RS	731.40	91.30	44.02	0.976	22.70	0.069	52.40
	CoCoGen (Ours)	4111.54	**100.00**	**44.16**	**0.99**	**4.59**	**0.01**	**61.23**
ViT-Base	JSMA	584.30	61.40	45.06	0.994	11.10	0.033	59.50
	SparseFool	327.90	84.10	44.48	0.987	16.20	0.049	56.40
	Sparse-RS	768.80	92.60	43.95	0.975	23.80	0.072	51.90
	CoCoGen (Ours)	4458.13	**100.00**	41.98	**0.99**	**3.56**	**0.01**	**63.70**

As shown in Table 5, while JSMA, SparseFool, and Sparse-RS achieve competitive perceptual quality, with JSMA in particular reaching MUSIQ scores of 58.70–59.80, close to CoCoGen's 61–63, their ASR falls substantially short across all architectures (58.70–95.10% vs. 100%), particularly on modern architectures

such as ConvNeXt-Base (JSMA: 58.70%) and ViT-Base (JSMA: 61.40%). This gap confirms that sparsity alone is insufficient for reliable misclassification: without the joint frequency constraint $\mathcal{P}_f$ (equation 13) and margin-directed optimisation (equation 4), sparse methods distribute perturbation energy across pixels that are not necessarily decision-critical, limiting their effectiveness against architectures with distributed attention mechanisms Dosovitskiy (2020); Liu et al. (2022).

4.4 Black-Box Transferability Evaluation

While CoCoGen is designed as a white-box attack, we evaluate the transferability of its adversarial examples to held-out target architectures under a black-box setting, where the adversary has no access to the target model's parameters or gradients. For each source model, adversarial examples are generated using CoCoGen at three sparsity levels $k \in \{100, 1000, 5000\}$ and then evaluated on all remaining architectures without any modification. Results are reported in Table 6.

Table 6: **Black-box transferability of CoCoGen.** White-box ASR (denoted *) is reported alongside black-box ASR across target architectures at three sparsity levels $k \in \{100, 1000, 5000\}$. A dash (—) indicates the source and target model are identical. Higher k increases both white-box ASR and transferability at the cost of slightly reduced perceptual quality (SSIM, LPIPS). ↑ higher is better; ↓ lower is better.

Source Model	k	SSIM↑	LPIPS↓	White-box ASR*↑	Black-box ASR↑				
					ResNet-50	MobileNet-v2	ShuffleNet-v2	DenseNet-121	ViT-B/16
ResNet-50	100	0.9980	0.0004	33.3%	—	52.00%	59.00%	25.00%	13.00%
	1000	0.9925	0.0042	89.4%	—	50.00%	63.00%	29.00%	13.00%
	5000	0.9702	0.0218	100.0%	—	57.00%	69.00%	37.00%	16.00%
ConvNeXt-Base	100	0.9980	0.0004	10.6%	35.00%	49.00%	59.00%	23.00%	13.00%
	1000	0.9921	0.0036	64.9%	33.00%	46.00%	58.00%	24.00%	13.00%
	5000	0.9688	0.0198	100.0%	38.00%	48.00%	61.00%	26.00%	17.00%
EfficientNet-B0	100	0.9983	0.0004	20.3%	35.00%	52.00%	58.00%	23.00%	14.00%
	1000	0.9944	0.0029	79.7%	47.00%	65.00%	66.00%	26.00%	16.00%
	5000	0.9773	0.0157	100.0%	55.00%	76.00%	76.00%	40.00%	19.00%
ViT-B/16	100	0.9980	0.0004	14.9%	34.00%	49.00%	59.00%	21.00%	—
	1000	0.9925	0.0046	55.2%	36.00%	49.00%	61.00%	25.00%	—
	5000	0.9708	0.0254	98.9%	36.00%	49.00%	56.00%	29.00%	—

As shown in Table 6, CoCoGen exhibits moderate black-box transferability consistent with the behaviour expected from a sparse, high-frequency attack operating under the composite feasible set $\mathcal{F}$ (equation 2). At $k = 5000$, transferability reaches 55-76% for architecturally similar targets (e.g., EfficientNet-B0 to MobileNet-v2 and ShuffleNet-v2), reflecting shared inductive biases in convolutional feature extraction He et al. (2016); Tan & Le (2019). Transferability to ViT-B/16 Dosovitskiy (2020) is consistently lower (13-19%), which is expected given the architectural divergence between CNNs and vision transformers and their differing reliance on local versus global features Dosovitskiy (2020); Liu et al. (2022). Across all source models, black-box ASR increases monotonically with k, while SSIM remains above 0.968 and LPIPS below 0.026 even at $k = 5000$, confirming that the perceptual quality guarantees of CoCoGen are preserved even when the sparsity budget is relaxed to improve transferability. We note that improving black-box transferability under the strict composite threat model $\mathcal{F}$ remains a challenging open problem and is identified as a direction for future work.

5 Ablation Study

Ablation Configurations. All ablation experiments are conducted on the same 1,000-image ImageNet subset Russakovsky et al. (2015) and under the same ℓ_∞ budget $\epsilon = 8/255$ as the main experiments. The full CoCoGen configuration uses adaptive grid search over both the sparsity level $k \in \mathcal{K}$ (equation 19) and the frequency threshold τ_{freq} (equation 11), together with counterfactual margin-guided optimisation (equation 4) and Fourier-domain projection $\mathcal{P}_f$ (equation 13). The **W/O Counterfactual Guidance** variant removes the contrastive counterfactual margin objective (equation 3) and replaces it with standard cross-entropy loss maximisation, with all other components unchanged. The **W/O High-**

Frequency Projection variant removes $\mathcal{P}_f$ (equation 13) entirely, so perturbations are optimised directly in the spatial domain without any spectral constraint, while the sparsity mask $\boldsymbol{M}_s$ and adaptive search remain active. The **W/O Adaptive Search** variant disables the grid search procedure (Section 2.6) and fixes the sparsity level at $k = 4{,}310$ pixels and the frequency threshold at $\tau_{\text{freq}} = 25$ uniformly for all images, with no per-image parameter selection; all other components remain unchanged.

5.1 Impact of High-Frequency Signal Components

High-frequency components correspond to fine edges, textures, and noise-like patterns, regions where the HVS has markedly reduced contrast sensitivity,

To isolate the contribution of the Fourier-domain projection operator $\mathcal{P}_f$, we compare variants that apply versus omit frequency masking while keeping all other components fixed and maintaining 100% ASR throughout (Table 7).

Table 7: **Effect of High-Frequency Signal Masking on Perceptual Quality.** All experiments maintain 100% ASR. **W/** denotes the full CoCoGen configuration; **W/O** removes the Fourier-domain projection operator $\mathcal{P}_f$ (equation 13), with the sparsity level and frequency threshold fixed at $k = 4{,}310$ and $\tau_{\text{freq}} = 25$ respectively. ↑ higher is better; ↓ lower is better.

Metric	Masking	ResNet-50	EffNet-B0	ConvNeXt	ViT-B
SSIM ↑	W/O	0.961	0.963	0.961	0.967
	W/	**0.990**	**0.990**	**0.990**	**0.990**
PSNR ↑	W/O	39.49	38.49	39.26	39.68
	W/	**44.67**	**41.67**	**44.16**	**41.98**
FID ↓	W/O	11.64	25.08	11.27	8.71
	W/	**10.18**	**4.05**	**4.59**	**3.56**
LPIPS ↓	W/O	0.03	0.03	0.02	0.02
	W/	**0.01**	**0.01**	**0.01**	**0.01**
MUSIQ ↑	W/O	48.07	46.59	47.98	48.32
	W/	**61.44**	**62.63**	**61.23**	**63.70**

Without $\mathcal{P}_f$, the optimiser is free to place perturbation energy at any spatial frequency, and in practice it exploits low-frequency components because they carry broad, spatially coherent gradient signal that efficiently reduces the margin $\mathcal{M}$. Low-frequency modifications, however, correspond to global changes in luminance, colour, and coarse structure, precisely the components to which the human visual system (HVS) is most sensitive Wang et al. (2004). The result is a perturbation that, while still technically bounded by $\|\boldsymbol{\delta}\|_\infty \leq \epsilon$, introduces visible blurring or tinting artefacts that degrade SSIM to ~ 0.96 and suppress PSNR to $\sim 39\,\text{dB}$. The no-reference MUSIQ scores (46–48) further confirm that the images are perceived as lower quality, since MUSIQ integrates multi-scale texture and sharpness cues that are disrupted by low-frequency contamination.

Applying $\mathcal{P}_f$ constrains the perturbation to the high-frequency subspace by zeroing all Fourier coefficients below a radial threshold τ_{freq}, as defined in equation 11. meaning larger perturbation amplitudes can be tolerated before the modification becomes visible. By routing all adversarial energy into this perceptually insensitive subspace, $\mathcal{P}_f$ breaks the coupling between attack effectiveness and perceptual cost: the classifier boundary can still be crossed because modern deep networks are known to rely heavily on high-frequency texture cues, yet the modification remains imperceptible to a human observer. The gains are consistent across all four architectures: SSIM reaches 0.99, PSNR increases by 4–6 dB, LPIPS halves, and MUSIQ rises by 13–16 points. The FID improvement is particularly pronounced for EfficientNet-B0 (25.08 → 4.05) and ViT-Base (8.71 → 3.56), suggesting that those architectures respond to adversarial updates in frequency bands that, when unconstrained, introduce distributional artefacts visible at the batch level. The orthogonal projector structure of $\mathcal{P}_f$ (established in Proposition 1) ensures that the frequency constraint is

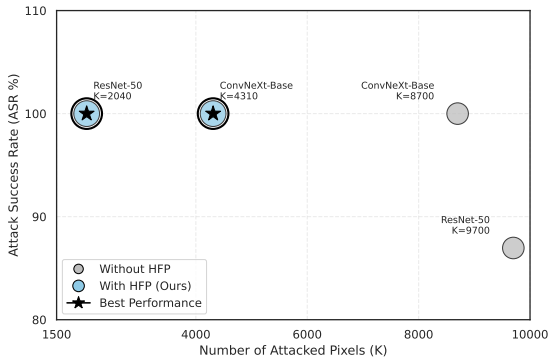

Figure 4: **Effect of high-frequency subspace projection on attack efficiency.** Fewer perturbed pixels indicate higher efficiency; 100% ASR is targeted in all cases. HFP denotes high-frequency projection, and K denotes the number of perturbed pixels.

enforced exactly at every iteration rather than softly penalised, which prevents low-frequency energy from leaking back in through momentum accumulation.

Attack Efficiency. High-frequency projection also improves attack efficiency (Fig. 4): ResNet-50 achieves 100% ASR with only 2,040 perturbed pixels (vs. 9,700 without projection), and ConvNeXt-Base with 4,310 (vs. 8,700). This reduction arises because constraining the perturbation to the high-frequency subspace fundamentally changes how optimisation allocates signal energy.

Without $\mathcal{P}_f$, gradient updates are dominated by low-frequency components, which have large spatial support and therefore distribute perturbation energy broadly across the image. While such components are effective at reducing the margin $\mathcal{M}$, they do so inefficiently: many pixels are modified slightly, resulting in dense perturbations with high cardinality.

In contrast, applying $\mathcal{P}_f$ suppresses all low-frequency directions and forces the optimiser to operate exclusively in the high-frequency subspace. These components correspond to localised edges and fine textures, which have spatially concentrated support. As a result, each update affects a smaller subset of pixels but with greater directional alignment to features that strongly influence the classifier's decision boundary. This increases the *per-pixel effectiveness* of the perturbation: fewer pixels are required to achieve the same reduction in $\mathcal{M}$.

Moreover, modern deep networks are known to rely heavily on high-frequency texture cues for classification, so restricting the attack to this subspace preserves access to decision-critical features while eliminating redundant low-frequency directions. The combination of directional filtering and feature alignment therefore yields perturbations that are both sparser and more targeted, achieving boundary crossing with significantly fewer modified pixels.

5.2 Effect of Contrastive Counterfactual Guidance

To isolate the contribution of contrastive counterfactual guidance, we compare two variants that both achieve 100% ASR but differ only in whether the margin objective $\mathcal{M}$ is used to steer the perturbation (Table 8). Without guidance, the gradient signal is undifferentiated across all input dimensions, so the optimiser distributes perturbation energy broadly across the image rather than concentrating it where the classifier is most sensitive. This diffuse energy placement is the direct cause of the degraded fidelity observed: SSIM falls to ~ 0.86 and PSNR to $\sim 34\,\text{dB}$, meaning the adversarial image departs visibly from the original despite carrying no more total perturbation energy. The elevated FID scores (22–38) further indicate that the distribution of adversarial images has drifted substantially from the clean image distribution, a sign that structural artefacts have been introduced across the batch. With counterfactual guidance, the margin $\mathcal{M}$ explicitly identifies the most competitive incorrect class and directs gradient updates toward the signal dimensions that most influence the logit gap between the true class and that competitor. Because only a small subset of pixels drives the classification decision, this targeting concentrates the perturbation into a sparse, semantically meaningful support rather than spreading it uniformly. The effect is substantial: SSIM rises uniformly to 0.99 across all architectures, PSNR increases by 8–11 dB, LPIPS drops by an order of magnitude (from ~ 0.10 to 0.01), and FID falls to single digits, indicating that the adversarial distribution is nearly indistinguishable from the clean one. The consistency

Table 8: **Effect of Contrastive Counterfactual Guidance.** All experiments maintain 100% ASR. **W/** denotes the full CoCoGen configuration; **W/O** removes the contrastive margin objective (equation 3) and substitutes standard cross-entropy loss maximisation, with all other components held fixed. ↑ higher is better; ↓ lower is better.

Metric	Guidance	ResNet-50	EffNet-B0	ConvNeXt	ViT-B
SSIM ↑	W/O	0.866	0.861	0.864	0.862
	W/	**0.990**	**0.990**	**0.990**	**0.990**
PSNR ↑	W/O	34.02	33.72	33.88	33.65
	W/	**44.67**	**41.67**	**44.16**	**41.98**
FID ↓	W/O	22.11	34.47	27.56	38.16
	W/	**10.18**	**4.05**	**4.59**	**3.56**
LPIPS ↓	W/O	0.105	0.121	0.092	0.142
	W/	**0.010**	**0.010**	**0.010**	**0.010**
MUSIQ ↑	W/O	47.08	48.15	48.65	47.50
	W/	**61.44**	**62.63**	**61.23**	**63.70**

of these gains across CNN and transformer architectures suggests the benefit is not model-specific but reflects a general property of margin-directed perturbation: attacking the decision boundary directly requires far less signal energy than naive loss maximisation, leaving the remainder of the image untouched.

5.3 Impact of Adaptive Grid Search

We analyze the importance of adaptive grid search (Table 9). Without it, ASR slightly drops (99%–99.8%) and perceptual quality degrades, reflecting inefficient allocation of perturbation energy across the feasible set.

Table 9: **Effect of Adaptive Grid Search. W/** denotes the full CoCoGen configuration; **W/O** fixes the sparsity level at $k = 4{,}310$ pixels and the frequency threshold at $\tau_{\text{freq}} = 25$ for all images, disabling per-image parameter selection (equation 19), while the counterfactual margin (equation 4), spatial masking $\boldsymbol{M}_s$ (equation 8), and frequency projection $\mathcal{P}_f$ (equation 13) remain active. ↑ higher is better; ↓ lower is better.

Metric	Search	ResNet-50	EffNet-B0	ConvNeXt	ViT-B
ASR ↑	W/O	100.00	99.80	100.00	99.00
	W/	**100.00**	**100.00**	**100.00**	**100.00**
PSNR ↑	W/O	39.49	38.49	39.26	39.68
	W/	**44.67**	**44.16**	**41.67**	**41.98**
FID ↓	W/O	11.64	25.08	11.27	8.71
	W/	**10.18**	**4.59**	**4.05**	**3.56**
LPIPS ↓	W/O	0.03	0.03	0.02	0.02
	W/	**0.01**	**0.01**	**0.01**	**0.01**
MUSIQ ↑	W/O	48.07	46.59	47.98	48.32
	W/	**61.44**	**61.23**	**62.63**	**63.70**

The role of adaptive grid search is to resolve the intrinsic trade-off between attack strength and perceptual fidelity by dynamically selecting the most effective constraint configuration from the candidate set $\mathcal{K}$. In the absence of this search, a fixed configuration (e.g., a single choice of frequency threshold or sparsity level) may be suboptimal: if the constraint is too weak, perturbations spread across redundant dimensions, leading to dense, visually noticeable modifications; if too strong, the feasible set becomes overly restrictive, limiting the optimiser's ability to reduce the margin $\mathcal{M}$ and achieve consistent misclassification. Adaptive grid search mitigates this by evaluating multiple constraint settings and selecting the one that yields the greatest margin reduction under the feasibility constraints. This effectively performs a discrete optimisation over the geometry of the feasible set itself, rather than optimising solely within a fixed set. As a result, the method identifies configurations in which the perturbation is concentrated on the most decision-critical signal components, avoiding both under- and over-constrained regimes.

This improved alignment has two key effects. First, it ensures that a valid decision-boundary crossing is achieved in all cases, restoring ASR to 100% across architectures. Second, it significantly improves perceptual quality: PSNR increases by 4–6 dB, LPIPS drops to 0.01, and MUSIQ improves by more than 10 points. These gains indicate that the selected configurations concentrate perturbation energy into a minimal, highly effective support, rather than distributing it across the image.

Consistent with this interpretation, adaptive grid search reduces the number of perturbed pixels by up to 9×, demonstrating that it does not merely improve attack success, but does so through more efficient and localised signal modification. In this sense, adaptive search acts as a higher-level control mechanism that selects the most favourable optimisation landscape for each input, enabling CoCoGen to operate near the optimal trade-off between attack efficacy and perceptual cost.

5.4 Effect of Sparsity Level k

To justify the selected value of k, we fix $\tau_{\text{freq}} = 25$ and sweep k on ResNet-50 (Table 10). The results reveal a clear trade-off between attack success and perceptual fidelity as k increases.

At low sparsity ($k \leq 3{,}000$), the perturbation support is too small to reliably cross the decision boundary: ASR reaches only 66.67% at $k = 1{,}000$ and stalls at 91.67% for $k \in \{2{,}000, 3{,}000\}$, meaning a non-trivial fraction of images remain correctly classified despite perturbation. Interestingly, perceptual quality is highest in this regime: SSIM remains above 0.989, PSNR reaches 45.60 dB at $k = 1{,}000$, and MUSIQ scores are close to 64, reflecting the fact that very few pixels are modified. However, this quality comes at the cost of attack failure, making these configurations unusable in practice.

At $k = 4{,}000$ (shaded), 100% ASR is achieved for the first time while perceptual quality remains high: SSIM= 0.9882, PSNR= 40.29 dB, LPIPS= 0.0113, FID= 3.19, and MUSIQ= 62.14. This configuration satisfies the feasibility constraints $\tau_s = 0.95$ and $\tau_f = 20$ with considerable margin, and is therefore selected as the optimal operating point.

Beyond $k = 4{,}000$, 100% ASR is maintained but perceptual quality degrades monotonically and substantially. SSIM falls from 0.9882 at $k = 4{,}000$ to 0.9442 at $k = 6{,}000$, 0.9176 at $k = 9{,}000$, and 0.8682 at $k = 18{,}000$, dropping below the feasibility gate $\tau_s = 0.95$ for $k \geq 6{,}000$. PSNR decreases from 40.29 dB to 33.66 dB, LPIPS rises from 0.0113 to 0.0823, FID increases from 3.19 to 12.64, and MUSIQ falls from 62.14 to 55.04 over the same range. These trends confirm that unnecessarily large k wastes the perceptual budget by modifying pixels that do not contribute meaningfully to margin reduction, consistent with the role of adaptive sparsity search in identifying the minimal effective support (Section 2.6).

Table 10: **Effect of sparsity level** k with fixed frequency threshold $\tau_{\text{freq}} = 25$ (ResNet-50). The selected configuration ($k = 4{,}000$, shaded) is the smallest value achieving 100% ASR while maintaining high perceptual fidelity. Below $k = 4{,}000$, ASR is insufficient; above it, perceptual quality degrades monotonically.

k	ASR (%)↑	SSIM↑	PSNR↑	LPIPS↓	FID↓	MUSIQ↑
1000	66.67	0.9893	45.60	0.0046	2.74	63.97
2000	91.67	0.9892	42.61	0.0097	3.57	63.44
3000	91.67	0.9898	40.83	0.0152	3.88	62.91
4000	**100.00**	0.9882	40.29	0.0113	3.19	62.14
6000	100.00	0.9442	37.93	0.0303	5.18	61.90
8000	100.00	0.9294	36.82	0.0395	5.15	60.08
9000	100.00	0.9176	36.06	0.0470	6.46	59.66
13000	100.00	0.8960	34.90	0.0620	10.48	57.77
18000	100.00	0.8682	33.66	0.0823	12.64	55.04

The candidate set $\mathcal{K} = \{2040, 3000, \ldots, 9700\}$ is chosen to bracket the optimum at $k = 4{,}000$, covering the transition from insufficient to sufficient sparsity while excluding values that unnecessarily degrade perceptual quality. The thresholds $\tau_s = 0.95$ and $\tau_f = 20$ serve as conservative feasibility gates; the selected configuration clears both with considerable margin (SSIM $= 0.9882 \gg 0.95$; FID $= 3.19 \ll 20$), confirming that fine-grained candidate selection is driven by the sweep rather than by τ_s and τ_f themselves.

5.5 Effect of Frequency Threshold τ_{freq}

To justify the selected value of τ_{freq}, we fix $k = 4{,}000$ and sweep τ_{freq} on ResNet-50 (Table 11). The results show a three-regime structure governed by the degree of spectral restriction imposed on the perturbation. The baseline case $\tau_{\text{freq}} = 0$ applies no spectral restriction ($\boldsymbol{M}_f = \boldsymbol{I}$, equation 12), allowing perturbation energy to be placed at any spatial frequency. While this achieves 100% ASR and the lowest FID (2.17) among all configurations, it is excluded from consideration by design: permitting low-frequency perturbation components defeats the purpose of the high-frequency constraint and would allow the optimiser to introduce globally visible luminance and colour changes that are highly detectable by the human visual system. This case is included in the table solely for reference.

Table 11: **Effect of frequency threshold** τ_{freq} with fixed sparsity $k = 4{,}000$ (ResNet-50). $\tau_{\text{freq}} = 0$ applies no spectral restriction ($\boldsymbol{M}_f = \boldsymbol{I}$, excluded by design, shown for reference only). Among configurations retaining a non-trivial spectral constraint and achieving 100% ASR, $\tau_{\text{freq}} = 25$ (shaded) produces the lowest FID and is selected as it achieves the optimal trade-off between spectral imperceptibility and attack effectiveness.

τ_{freq}	ASR (%)↑	SSIM↑	PSNR↑	LPIPS↓	FID↓	MUSIQ↑
0	100.00	0.9922	41.22	0.0170	2.17	63.43
10	100.00	0.9884	40.69	0.0187	3.89	63.25
20	100.00	0.9822	40.74	0.0206	3.27	63.43
25	**100.00**	0.9882	40.29	0.0213	**3.19**	63.79
30	91.67	0.9752	39.01	0.0215	5.66	62.76
40	91.67	0.9647	39.00	0.0220	6.36	61.84
55	91.67	0.9653	39.03	0.0257	9.34	60.70

Among configurations that retain a non-trivial spectral constraint ($\tau_{\text{freq}} > 0$), the regime $\tau_{\text{freq}} \in \{10, 20, 25\}$ achieves 100% ASR across all evaluation images. Within this regime, the metrics evolve as follows. At $\tau_{\text{freq}} = 10$, SSIM= 0.9884, PSNR= 40.69 dB, FID= 3.89, and MUSIQ= 63.25. At $\tau_{\text{freq}} = 20$, SSIM drops slightly to 0.9822 but FID improves to 3.27 and MUSIQ remains at 63.43. At $\tau_{\text{freq}} = 25$ (shaded), SSIM recovers to

0.9882, FID reaches its minimum among 100%-ASR configurations at 3.19, and MUSIQ rises to 63.79, the highest value in the entire table. This configuration therefore achieves the best distributional alignment with the clean image set while maintaining 100% ASR and high structural similarity, and is selected as the operating point for all main experiments.

The regime $\tau_{\text{freq}} \geq 30$ reveals the consequence of over-restricting the spectral budget. At $\tau_{\text{freq}} = 30$, ASR drops to 91.67% and FID rises to 5.66; the degradation persists and worsens at $\tau_{\text{freq}} = 40$ (FID= 6.36) and $\tau_{\text{freq}} = 55$ (FID= 9.34, MUSIQ= 60.70). This confirms that excessively aggressive frequency masking removes spectral components that are necessary for the optimiser to reduce the contrastive margin $\mathcal{M}$ (equation 3) sufficiently to cross the decision boundary, resulting in 8.33% of images remaining correctly classified. Notably, SSIM and PSNR do not degrade appreciably in this regime (SSIM$\geq$ 0.9647, PSNR$\geq$ 39.00 dB), indicating that the perceptual quality of the successfully attacked images remains high; the issue is exclusively one of insufficient attack capacity rather than degraded imperceptibility.

6 Conclusion

We proposed **Co**ntrastive **Co**unterfactual **Gen**eration (**CoCoGen**), an adversarial attack that minimises the contrastive counterfactual margin under explicit spatial and spectral constraints. By directly optimising the decision boundary geometry, CoCoGen provides a consistent gradient signal across all optimisation regimes while enforcing a strict energy budget.

More specifically, adversarial vulnerability is concentrated in a small set of decision-critical signal components. By combining Top-k spatial masking with high-frequency Fourier projection, the method confines perturbations to a structured, low-dimensional subspace aligned with these components. This alignment enables highly efficient optimisation: perturbation energy is not spread across the image but concentrated where it most effectively reduces the margin. We emphasise that the contribution of CoCoGen lies in the unified and adaptive integration of established components, counterfactual-margin optimisation, sparse spatial support, high-frequency spectral projection, and momentum-based updates, within a single constrained optimisation framework that jointly searches over sparsity and spectral thresholds under perceptual-quality constraints, rather than the introduction of any individual mechanism in isolation. Furthermore, CoCoGen operates under a strictly more restrictive threat model than standard ℓ_∞ attacks; reported improvements over such methods reflect what is achievable under additional perceptual constraints rather than unconditional superiority under identical attack conditions.

Our experimental results demonstrate 100% ASR across diverse architectures while maintaining near-imperceptible distortion (SSIM $\approx$ 0.99, LPIPS $\approx$ 0.01, MUSIQ 61–63), evaluated relative to a clean-image MUSIQ baseline of 68.25. These results show that reliable misclassification does not require large or diffuse perturbations, but rather precise, spectrally coherent modifications targeted at the classifier's decision boundary, operating under the composite feasible set combining ℓ_∞, sparsity, and high-frequency spectral constraints.

Future work will explore adaptive and learned support selection, extend the framework to targeted and black-box settings, and generalise the signal-domain formulation to other modalities such as audio and video.

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